Integrated Equilibrium in a Four-good Heckscher-Ohlin-Ricardo model*

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Abstract

This paper develops a four-good version of the Davis (1995) Heckscher-Ohlin-Ricardo model of international trade based on technological and factor endowment differences across countries. We develop several results from this model. First, the area defining the integrated equilibrium is smaller, the greater is the weight placed by consumers on the goods that have different technologies across countries, relative to the goods that have identical technologies across countries. Second, demand conditions can turn the Heckscher-Ohlin model and the Ricardian model, into special cases of the Heckscher-Ohlin-Ricardo model. Third, trade patterns in the four-good model may be different from those of Davis’s (1995) three-good model; in particular, increasing similarity in relative endowments across countries may under certain circumstances increase the volume of trade.

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1 Introduction

The integrated equilibrium has been widely used as an analytical tool in international trade theory since its popularisation by Dixit and Norman (1980), although the idea can be traced back to at least Samuelson (1949). It starts from an integrated world economy with free movement of goods and factors of production, then divides the world economy into countries, restricting the movement of factors of production, and asking whether free trade in goods alone can replicate the resource allocation and goods and factor prices that characterise the integrated equilibrium. Importantly, in a competitive model with perfect markets, the integrated equilibrium set of prices and quantities represents the most efficient allocation of resources in the world economy, in the sense that total world income is maximised. Whilst the original formulation of the integrated equilibrium was based on a perfectly competitive, comparative advantage framework, Helpman and Krugman (1985) show that it can also be used in models of international trade based on increasing returns and product differentiation, which has been used in explaining patterns of intra-industry trade (that is, trade in similar goods).

However, increasing returns to scale and product differentiation are not the only ways of modelling intra-industry trade. In an important paper, Davis (1995) showed that intra-industry trade can be the outcome of a model based on constant returns to scale and comparative advantage. The basic idea of the Davis Heckscher-Ohlin-Ricardo (HOR) model is that, when there are three goods, two of which are intra-industry in the sense of sharing identical production techniques, and one of the trading partners has an absolute (technological) advantage in producing one of these intra-industry goods, then if this country has sufficient factor endowments, it will produce the entire world’s output of that good. However, the remaining resources may result in the other country producing most of the world’s output of the other intra-industry good. There will then be two-way trade in this intra-industry good.

Davis (1995) showed that the integrated equilibrium can be replicated when the country that has a technological advantage in producing one of the intra-industry goods, is able to produce the entire integrated equilibrium supply of that good. However, what Davis (1995) did not show, is that the integrated equilibrium depends on consumer demand. We develop a simple, four-good version of the HOR model, and show that the demand for the technologically differentiated good rela-
tive to the demand for the technologically undifferentiated good within the same intra-industry pair, is what determines the dimensions of the integrated equilibrium. This has implications for whether or not the division of factors of production across countries maximises world income. As an immediate corollary, our model shows how the Heckscher-Ohlin and the Ricardian models can be obtained as special cases of the more general HOR model, depending on the relative demands for the technologically differentiated versus the technologically undifferentiated goods.

We then use our four-good setup to explore trade patterns within the integrated equilibrium. Trade patterns in our four-good model are determinate, but different from those in Davis's (1995) three-good model, and this difference is driven by the introduction of the fourth good, which is technologically differentiated across countries. In particular, under certain conditions, greater similarity in relative endowments across countries may lead to increasing trade volumes, as this may lead to greater differentiation in industrial structure across countries because of specialisation in production resulting from Ricardian technological differences.

Our results highlight the crucial role of demand in international trade. Often, demand is pushed into the background, with models focussing more on the endowments/technology/market structure side of the story. However, several models have given demand a key role, for example the early work of Linder (1961), Markusen (1986), Flam and Helpman (1987) on international trade between similar and dissimilar countries. More recently, the work of Chung (2003) and Mitra and Trindade (2005) have used demand conditions to provide an explanation for Trefler's (1995) celebrated "case of the missing trade", and to argue for the role of within-country inequality in determining the pattern of trade, respectively. More closely related to the present paper is Balboni (2005), who independently develops a demand side to the HOR model, but his paper is focussed on developing an explanation for vertical intra-industry trade in a three-good model, whereas our objective is to develop a four-good model to explain integrated equilibrium and the pattern of trade.

Section 2 first develops the model, then considers the implications of different weights in consumer demand for the different goods on the size of the integrated equilibrium, and how this allows us to obtain the Heckscher-Ohlin and Ricardian models as special cases of the HOR model. Section 3 analyses the trade patterns and trade volumes that arise in the model, and the final section provides some concluding comments.
2 The model

We develop a simple model with factor endowment and technological differences across countries. There are two countries, \( j = H, F \) (Home and Foreign), and four goods in two pairs, \( i \in \{X_1, X_2, Y_1, Y_2\} \). Each pair of products is produced using capital \( K \) and labour \( L \) using constant-returns-to-scale production functions, with goods in each pair having the same factor intensities, and the \( X \)-goods being capital-intensive relative to the \( Y \)-goods at any factor price ratio: \( \frac{K}{L}_X > \frac{K}{L}_Y \). Both countries are equally productive in goods \( X_2 \) and \( Y_2 \), whilst there are Hicks-neutral technological differences across countries in goods \( X_1 \) and \( Y_1 \), so that the production functions take the form:

\[
\begin{align*}
\text{Home :} & \quad Q^H_{X_1} = A^H_X (K^H_{X_1}, L^H_{X_1}) \quad Q^H_{X_2} = F (K^H_{X_2}, L^H_{X_2}) \quad Q^H_{Y_1} = A^H_Y (K^H_{Y_1}, L^H_{Y_1}) \quad Q^H_{Y_2} = G (K^H_{Y_2}, L^H_{Y_2}) \\
\text{Foreign :} & \quad Q^F_{X_1} = A^F_X (K^F_{X_1}, L^F_{X_1}) \quad Q^F_{X_2} = F (K^F_{X_2}, L^F_{X_2}) \quad Q^F_{Y_1} = A^F_Y (K^F_{Y_1}, L^F_{Y_1}) \quad Q^F_{Y_2} = G (K^F_{Y_1}, L^F_{Y_1})
\end{align*}
\]

(1)

(2)

where \( Q^j_i \) indicates production of good \( i \) in country \( j \). Factors of production are perfectly mobile across sectors within each country, but immobile across countries. All markets are perfectly competitive. We assume that the Hicks-neutral technology parameters \( A^j_i \geq 1 \), although this is not crucial for what follows. Davis’s (1995) setup is simply the one above with the additional constraint that \( A^H_Y = A^F_X = A^F_Y = 1 \).

One alternative interpretation of the model is that it is a modified, general equilibrium version of Falvey (1981). In that paper, Falvey showed that, within a single industry with different qualities, countries will specialise in different qualities based on the capital-intensity of different qualities, and the capital-abundance of different countries. This results in two-way trade in different qualities in the same industry. In our model, we may interpret each pair of goods as representing different qualities of the good, but in this case, countries will specialise in different varieties of each pair of goods based on technological advantage.

Returning to the model, consumer utility is assumed to be identical across coun-
tries, and takes a Cobb-Douglas form:

\[ U = \sum \alpha_i \log c_i \quad \sum \alpha_i = 1 \]  

(3)

so that the consumer spends a share \( \alpha_i \) of his income on each of the four goods.

### 2.1 Integrated equilibrium

Consider what happens when we allow for free trade in goods between the two countries. Following Davis (1995), we ask: what divisions of the world endowment of capital and labour are consistent with replicating the integrated equilibrium, that is, the resource allocation that would occur if both goods and factors of production are freely traded. In the integrated equilibrium factor price equalisation (FPE) holds across countries.

To replicate the integrated equilibrium, it must be the case that each country produces the world output of the good(s) in which it has an absolute technological advantage. Given the constant expenditure share on each good imposed by the Cobb-Douglas utility function, Proposition 1 is immediate:

**Proposition 1** In the integrated equilibrium, if \( A_X^H \neq A_X^F \) and \( A_Y^H \neq A_Y^F \), the ratio of the factors of production used in producing \( X_1 \) to that used in producing \( X_2 \) is equal to the relative expenditure shares on the \( X \)-goods, \( \frac{\alpha_{X_1}}{\alpha_{X_2}} \), and the ratio of the factors of production used in producing \( Y_1 \) to that used in producing \( Y_2 \) is equal to the relative expenditure shares on the \( Y \)-goods, \( \frac{\alpha_{Y_1}}{\alpha_{Y_2}} \).

**Proof.** We show the proof for the \( X \)-goods; the proof for the \( Y \)-goods is analogous. In the integrated equilibrium, the country that has the technological advantage in producing \( X_1 \), will produce the world output of \( X_1 \). In equilibrium, the ratio of the expenditure shares must be equal to the ratio of the total worldwide revenues of each industry, so that \( \frac{\alpha_{X_1}}{\alpha_{X_2}} = \frac{p_{X_1}A_X^sF(K_{X_1}^W,L_{X_1}^W)}{p_{X_2}F(K_{X_2}^W,L_{X_2}^W)} \), where \( A_X^s \) is the Hicks-neutral technology parameter for the country that has the technological advantage in \( X_1 \), and \( K_{X_1}^W \) is the total capital used in producing \( X_1 \) in the world. But in the integrated equilibrium, since the country with the technological advantage in \( X_1 \) at least potentially produces both goods \( X_1 \) and \( X_2 \), it must be the case that the relative price
of goods $X_1$ and $X_2$ is equal to the opportunity cost of production in this country: $rac{p_{X_1}}{p_{X_2}} = \frac{1}{A_X}$. Therefore, $\frac{\alpha_{X_1}}{\alpha_{X_2}} = \frac{F(K_{X_1}, L_{X_1})}{F(K_{X_2}, L_{X_2})} = \frac{K_{X_1}}{K_{X_2}} = \frac{L_{X_1}}{L_{X_2}}$, where the third equality comes from the fact that, in the integrated equilibrium, the capital-labour ratio of $X_1$ and $X_2$ are identical, and the second equality comes from the previous fact, and from our assumption of constant returns to scale. ■

Proposition 1 shows that it is the relative weights placed by consumers on the technologically-differentiated versus the technologically identical goods within each industry pair that determines the dimensions of the integrated equilibrium. A greater weight placed on the technologically differentiated good, reduces the size of the integrated equilibrium. This is because, to replicate the integrated equilibrium, countries with a technological advantage in producing a good, must produce the world output of that good. The greater the weight that consumers place on these goods, the greater the resources required to produce the world output of these goods, hence the more restricted is the possible allocation of resources that can replicate the integrated equilibrium. Notice also that the proof of Proposition 1 implies that the allocation of resources between the two $X$-industries is independent of the relative (and absolute) level of technology in each $X$-industry, and is also independent of the relative demand for the $Y$-industries.

Suppose for concreteness that $A_{HX}^H > A_{FX}^F$ and $A_{FY}^F > A_{HY}^H$, so that Home has an absolute (and comparative) technological advantage in producing good $X_1$, while Foreign has an absolute and comparative technological advantage in producing good $Y_1$. Therefore, to replicate the integrated equilibrium, Home must produce the world supply of $X_1$, and Foreign has to produce the world supply of $Y_1$, while the world supply of $X_2$ and $Y_2$ can be produced in either country in integrated equilibrium because of identical technologies across countries in these goods.

Figure 1 is a diagrammatic representation of Proposition 1 for this case. It shows the Dixit-Norman-Helpman-Krugman (DNHK) rectangle\(^1\). The dimensions of the box measure the world endowment of capital and labour, and the origins are for Home and Foreign. The line $O_HA$ represents the resources allocated to producing the $X$-goods, while the line $AO_F$ represents the resources allocated to producing the $Y$-goods, in the integrated equilibrium.

\(^1\)So-called because it was first popularised by Dixit and Norman (1980) and Helpman and Krugman (1985).
Given the case above, if the division of factor endowments between the two countries lies within the area \( ACDE \), integrated equilibrium can be replicated and hence factor price equalisation achieved through free international trade in goods alone, since we take into account the fact that Home must produce the world supply of good \( X_1 \) in which it has a technological advantage, and Foreign must produce the world supply of good \( Y_1 \), in which it has a technological advantage. The line \( O_HC \) represents the resources used in Home to produce the world supply of good \( X_1 \), while the remaining fraction \( CA \) is used by either country in producing \( X_2 \). Therefore, from Proposition 1, the ratio of the lengths of both lines is equal to the relative consumption weights on the two goods, \( \frac{O_HC}{CA} = \frac{\alpha_{X_1}}{\alpha_{X_2}} \). Similarly, the line \( O_FE \) represents the resources used in Foreign to produce the world supply of good \( Y_1 \), while the remaining fraction \( EA \) is used by either country in producing \( Y_2 \). As with the \( X \)-goods, the ratio of resources used in producing \( Y_1 \) to \( Y_2 \), is \( \frac{O_FE}{EA} = \frac{\alpha_{Y_1}}{\alpha_{Y_2}} \).

If technologies are identical across the two countries in all industries as in the standard Heckscher-Ohlin model, then the area within which integrated equilibrium can be replicated is \( O_HAO_FB \), since with identical technologies, there are no constraints on the location of production of any of the goods based on technological advantage. Therefore, we have Corollary 2:

**Corollary 2** The ratio of the area of the integrated equilibrium with technological differences across countries (Heckscher-Ohlin-Ricardo), to that with identical technologies (Heckscher-Ohlin), is equal to \( \frac{\alpha_{X_2}\alpha_{Y_2}}{(\alpha_{X_1}+\alpha_{X_2})(\alpha_{Y_1}+\alpha_{Y_2})} \).

Corollary 2 shows clearly that, with technological differences across countries in \( X_1 \) and \( Y_1 \), the integrated equilibrium depends on the relative demands for the four goods, and it vanishes when consumers do not demand at least one of the technologically identical goods; that is, if \( \alpha_{X_2} \) or \( \alpha_{Y_2} = 0 \). Also, if consumers demand only the goods which have identical technologies across countries \( (\alpha_{X_1} = \alpha_{Y_1} = 0) \), then the integrated equilibrium is exactly the same as that when only technologically identical goods exist.

Consider first the second case, where consumers only demand the goods which have identical technologies across countries. This is the case of the Heckscher-Ohlin model; only goods with identical technologies are produced. On the other hand, consider the first case, where if we impose the stronger condition that \( \alpha_{X_2} = \alpha_{Y_2} = \)
0, then consumers only demand the goods which are technologically differentiated across countries. This then becomes a version of the Ricardian model of trade based on relative technological differences across countries. In this case, factor price equalisation is unlikely to occur except as a coincidence. Therefore, we have the following corollary:

**Corollary 3** The Heckscher-Ohlin and the Ricardian models are special cases of the HOR model, when consumers place no weight on the technologically differentiated goods, $\alpha_{X_1} = \alpha_{Y_1} = 0$, and when they place no weight on the technologically identical goods, $\alpha_{X_2} = \alpha_{Y_2} = 0$, respectively.

### 2.2 Integrated equilibrium: Discussion

As we noted in our discussion of Proposition 1 above, consumer demand for the $Y$-goods relative to the $X$-goods does not affect the dimensions of the integrated equilibrium relative to the identical-technologies case. What the relative demand for $X$- and $Y$-goods does, is that it changes the overall shape of the parallelogram, but not the relative size of the integrated equilibrium. This is shown in Figure 2, which shows an increase in the weight placed by consumers on the $X$-goods relative to the $Y$-goods, with no change in the relative demands between $X_1$ and $X_2$, and between $Y_1$ and $Y_2$. The initial condition is shown by the solid lines, the new condition by the dashed lines. The increase in consumer demand for $X$-goods raises the price of the $X$-goods relative to that of the $Y$-goods, resulting in expanded supply of the $X$-goods and thus raising the relative demand for capital (since $X$ is capital-intensive relative to $Y$), and hence lowering the wage-rental ratio and lowering the capital-labour ratio used in the production of both types of goods.

The fact that there are no trade barriers across countries, also implies that it is world relative demands that matter for determining the size of the integrated equilibrium, not individual country demands. For instance, a home bias in consumption as a result of different per-capita incomes and non-homothetic preferences, while it affects the pattern of trade, does not change the size of the integrated equilibrium.

Changes in the Hicks-neutral technology parameters also have no impact on the relative size of the integrated equilibrium, except when relative technologies
are equal across countries. In this case, the integrated equilibrium is identical to that for the Heckscher-Ohlin model, as that is essentially what the model reduces to. Except for this case, however, technology has no impact on the size of the integrated equilibrium. For example, suppose that Home starts with a tiny technological advantage in good $X_1$ compared to Foreign. Then, to replicate the integrated equilibrium, Home must always produce the entire world supply of good $X_1$, so that the integrated equilibrium immediately and discontinuously reduces in size as a result of this additional constraint on the location of production. But suppose now that Home’s productivity in good $X_1$ doubles. This doubles the output of $X_1$ for any amount of input that is used. However, if both $X$-goods continue to be produced in Home, then free movement of factors of production across sectors implies that the relative price of the $X$-goods $\frac{p_{X_1}}{p_{X_2}}$ must halve. This preserves the relative expenditure shares of $X_1$ and $X_2$, without any change in the resource allocation, hence the area of the integrated equilibrium is also unchanged as a result of this technological change.

What changes in the Hicks-neutral technology parameters can do, however, is to change the division of resources across countries for which the integrated equilibrium can be replicated. Suppose for example that the technology parameters are such that Home has an absolute advantage in both $X_1$ and $Y_1$: $A^H_{X_1} > A^F_{X_1}$ and $A^H_{Y_1} > A^F_{Y_1}$. Then, the integrated equilibrium can be replicated only if Home produces the world supply of goods $X_1$ and $Y_1$. If this requires resources equal to $OHF$ for good $X_1$ and $FC$ for good $Y_1$, this means that the integrated equilibrium is now given by the area $CEO_FD$ in Figure 3.

### 3 Trade patterns

In this section we consider the pattern of trade in our four-good model. As with Davis (1995), we focus our attention on the integrated equilibrium, the dimensions of which have been characterised in the previous two sections. For concreteness, in this section assume that $\alpha_i = \frac{1}{4}$ for all $i$, so that the representative consumer spends an equal share of his income on each of the four goods, and therefore that the integrated equilibrium is a quarter the size of the integrated equilibrium in the HO model. Depending on the relative technologies between the two countries, there are four possible locations of the integrated equilibrium as shown in Figure 4, although
two of these locations are mirror images of the other two, so we focus only on the integrated equilibria indicated by regions I and II.

As the first step in the analysis, consider trade patterns in each of the four corners of regions I and II. Table 1 summarises the trade patterns in both regions. Take region I first. Here, Home has a technological advantage in $X_1$ and Foreign in $Y_1$, so that in the integrated equilibrium, Home always exports $X_1$ and Foreign always exports $Y_1$. At point A, Home is producing the world’s output of both $X_1$ and $X_2$, while Foreign is producing the world’s output of both $Y_1$ and $Y_2$. Hence Home will export $X_1$ and $X_2$ in return for imports of $Y_1$ and $Y_2$ from Foreign; this may be called pure inter-industry trade. At point B, Home is producing the world’s output of $X_1$, $X_2$ and $Y_2$ and so exports these three goods in return for imports of $Y_1$ from Foreign. At point C, Home exports $X_1$ and $Y_2$ in exchange for imports of $X_2$ and $Y_1$. Finally, at point D, Home exports $X_1$ in exchange for imports of $X_2$, $Y_1$ and $Y_2$. Trade patterns at points B and D may be referred to as partial intra-industry trade, while the trade pattern at point C may be called pure intra-industry trade.

If the integrated equilibrium is region II, then Home has a technological advantage in both $X_1$ and $Y_1$. Therefore, in the integrated equilibrium, Home always exports these two goods. At the common points B and C, the goods within each pair that each country exports depends on which country has the technological advantage, but the net factor content of trade is the same\(^2\). Thus, at point B, Home will export $X_1$, $Y_1$ and $X_2$ while Foreign will export $Y_2$. At point C, Home exports $X_1$ and $Y_1$ in exchange for $X_2$ and $Y_2$, and at point E, Home exports $X_1$, $Y_1$ and $Y_2$ while Foreign will export $X_2$. Therefore, trade at point C is once again pure intra-industry trade, while trade at points B and E is partial intra-industry trade.

We can delineate the integrated equilibrium into smaller areas. As we move from point A to point D, or from point B to point C, the trade pattern changes from one where Home exports $X_2$ to one where Foreign exports $X_2$. Therefore, somewhere between these points must be a resource allocation such that both countries are self-sufficient in $X_2$ and hence $X_2$ is not traded. Given our assumptions on technology and preferences, this can be represented by a line from Foreign’s origin $O_F$, denoted $X_2^{SS}$. Similarly, moving from point C to point E changes the trade pattern from

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\(^2\)Similarly to Helpman and Krugman (1985), the net factor content of trade can be shown by the difference between the endowment point and the consumption point, the latter of which always lies on the diagonal since we assume identical preferences across countries.
Foreign exporting $Y_2$ to Home exporting $Y_2$. Hence there is also a line defining the allocations of resources across countries such that both countries are self-sufficient in $Y_2$, denoted $Y_2^{SS}$ from Foreign’s origin. A similar argument can be made for self-sufficiency lines for $X_2$ and $Y_2$ emanating from Home’s origin $O_H$.

### 3.1 Trade volume

We can also consider the volume of trade. Differently from Davis (1995), to facilitate analysis, we divide the goods into technologically differentiated goods $X_1$ and $Y_1$, and technologically undifferentiated goods $X_2$ and $Y_2$. Consider first the integrated equilibrium in region I. In this region, Home produces the world output of $X_1$ and Foreign produces the world output of $Y_1$. Therefore, the volume of trade in $X_1$ is:

$$VT_{X_1} = p_{X_1} \left[ Q_{X_1} - s_H c_{X_1} \right] = p_{X_1} (1 - s_H) \frac{Q_{X_1}}{s_F} = p_{X_1} s_F \frac{Q_{X_1}}{s_H}$$

(4)

while the volume of trade in $Y_1$ is:

$$VT_{Y_1} = p_{Y_1} \left[ Q_{Y_1} - s_F c_{Y_1} \right] = p_{Y_1} (1 - s_F) \frac{Q_{Y_1}}{s_H} = p_{Y_1} s_H \frac{Q_{Y_1}}{s_F}$$

(5)

where $Q_i$ is the world output of good $i$, $c_i$ is the world consumption of good $i$, and $s_j$ is country $j$’s share of world income. As can be seen from both expressions, the volume of trade in $X_1$ within region I is increasing in Foreign’s share of world income, while the volume of trade in $Y_1$ is increasing in Home’s share of world income. Since in this section we are assuming identical expenditure shares in each good, the total volume of trade in $X_1$ and $Y_1$ is:

$$VT_{X_1} + VT_{Y_1} = s_F p_{X_1} \frac{Q_{X_1}}{s_H} + s_H p_{Y_1} \frac{Q_{Y_1}}{s_F} = (s_F + s_H) p_{X_1} \frac{Q_{X_1}}{s_F} = p_{X_1} \frac{Q_{X_1}}{s_H} = p_{Y_1} \frac{Q_{Y_1}}{s_F}$$

(6)

Given endowments, technologies and preferences, the total volume of trade in $X_1$ and $Y_1$ in region I is therefore a constant.

In region II, Home produces the world output of both $X_1$ and $Y_1$. Therefore,
the total volume of trade in these two goods is:

\[ VT_{X_1} + VT_{Y_1} = s_F p_{X_1} Q_{X_1} + s_F p_{Y_1} Q_{Y_1} = 2s_F p_{X_1} Q_{X_1} = 2s_F p_{Y_1} Q_{Y_1} \]  

(7)

Hence, within region II, the total volume of trade in \( X_1 \) and \( Y_1 \) is increasing in Foreign’s share of world income, reaching a maximum at point C when Foreign’s share of world income is maximised at \( s_F = \frac{1}{2} \). In Figure 5 which is an enlarged version of the top half of Figure 4, the solid arrow shows that the total volume of trade in these two goods is increasing in the direction shown. The level curves for this trade volume can be represented by lines that are parallel to the national income line that divides world income into Home and Foreign.

Turning now to goods \( X_2 \) and \( Y_2 \), the volume of trade in these two goods increases as one moves away from the self-sufficiency lines \( X_{2SS} \) and \( Y_{2SS} \). The level curves of the trade volume for these two goods are parallel to \( X_{2SS} \) and \( Y_{2SS} \). The dashed arrows in Figure 5 show the direction in which trade volume in these two goods is increasing. In region I, given identical expenditure shares on each good, the volume of trade in these two goods at each of points A to D are:

Point A: \( VT_{X_2} + VT_{Y_2} = s_F p_{X_2} Q_{X_2} + s_H p_{Y_2} Q_{Y_2} = p_{X_2} Q_{X_2} \)

Point B: \( VT_{X_2} + VT_{Y_2} = s_F p_{X_2} Q_{X_2} + s_F p_{Y_2} Q_{Y_2} = 2s_F p_{X_2} Q_{X_2} \)  

(8)

Point C: \( VT_{X_2} + VT_{Y_2} = s_H p_{X_2} Q_{X_2} + s_F p_{Y_2} Q_{Y_2} = p_{X_2} Q_{X_2} \)

Point D: \( VT_{X_2} + VT_{Y_2} = s_H p_{X_2} Q_{X_2} + s_H p_{Y_2} Q_{Y_2} = 2s_H p_{X_2} Q_{X_2} \)

Since at point B \( s_F < \frac{1}{2} \) and at point D \( s_H < \frac{1}{2} \), the total volume of trade in \( X_2 \) and \( Y_2 \) in region I is maximised at points A and C. We can perform a similar calculation for region II, giving the following results:

Point B: \( VT_{X_2} + VT_{Y_2} = s_F p_{X_2} Q_{X_2} + s_H p_{Y_2} Q_{Y_2} = p_{X_2} Q_{X_2} \)

Point C: \( VT_{X_2} + VT_{Y_2} = s_H p_{X_2} Q_{X_2} + s_H p_{Y_2} Q_{Y_2} = p_{X_2} Q_{X_2} \)  

(9)

Point E: \( VT_{X_2} + VT_{Y_2} = s_H p_{X_2} Q_{X_2} + s_F p_{Y_2} Q_{Y_2} = p_{X_2} Q_{X_2} \)

where the result for point C comes from the fact that here, \( s_H = \frac{1}{2} \). In region II, the total volume of trade in \( X_2 \) and \( Y_2 \) is maximised at each of points B, C and E.
Without getting into the details of deriving the level curves of trade volumes, we can draw some general conclusions. Considering only regions I and II, trade volume is maximised at two points in the DNHK rectangle: at points A and C. These are points of maximal differentiation between the two countries in terms of their production structure. This differs from Davis (1995), whose point of maximal differentiation between the two countries is at point A. The reason for this difference is that, because our model introduces a second Y-good, the two countries are no longer self-sufficient in good Y along the diagonal of the DNHK rectangle as they are in the Davis (1995) model. Therefore, one result of our model can be summarised in the following Proposition:

**Proposition 4** In the area bounded by the four self-sufficiency curves in $X_2$ and $Y_2$ from both countries’ origins, trade volume is maximised at the central point C, and is increasing in the direction of this point.

An immediate corollary of Proposition 4 is that, in region I, in the area closest to the central point C, increasing similarity in relative endowments across countries leads to greater trade volumes. This is a result which is very similar to Davis (1997), and suggests that even small technological differences across countries may be able to account for the large volumes of trade between countries in the developed world with similar relative endowments, compared with smaller volumes of trade between developed and less developed countries, between which endowment ratios are more different.

### 4 Conclusions

If the allocation of factors of production across countries lies within the integrated equilibrium, then free trade in goods alone can lead to factor price equalisation and the most efficient allocation of resources in the world economy. The possibility of replicating the integrated equilibrium depends crucially on the relative number of goods and factors of production (see e.g. Ethier (1984)), but also on the production technologies involved.

This paper shows how the size of the integrated equilibrium in the HOR model depends on consumer demand. A greater weight placed by consumers on goods
which are technologically differentiated relative to goods which have identical technologies across countries, places a tighter constraint on the production structure of the world economy, and hence reduces the size of the integrated equilibrium. We use this to show how varying a single set of parameters of the model, namely the coefficients on the Cobb-Douglas utility function, allows us to obtain the Heckscher-Ohlin and the Ricardian models as special cases of the more general HOR model. We also discuss the trade patterns of the model, and show that countries with identical endowment ratios may trade more with one another than countries with different endowment ratios.

Our result on the size of the integrated equilibrium suggests one possible reason why recent studies such as Bernard, Redding and Schott (2005) and Bernard, Redding, Schott and Simpson (2005) have found that relative factor prices are not equalised within countries. Even small differences in the productivity parameter can lead to differences in relative factor prices across locations as it will not be possible to replicate the integrated equilibrium. This then has implications for the efficiency of resource allocation across different regions of a country.
References


Figure 1: Dixit-Norman-Helpman-Krugman rectangle showing the integrated equilibrium in a 4-good Heckscher-Ohlin-Ricardo model.

Figure 2: Dixit-Norman-Helpman-Krugman rectangle showing an increase in consumer demand for the X-goods (movement from solid to dashed lines).
Figure 3: Dixit-Norman-Helpman-Krugman rectangle showing integrated equilibrium when Home has a technological advantage in both $X_1$ and $Y_1$.

Figure 4: Dixit-Norman-Helpman-Krugman rectangle showing trade patterns.
Figure 5: Trade volumes in the integrated equilibrium (arrows point in the direction of increasing trade volumes).

Table 1: Export patterns.

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