Endogenous FDI Spillovers: Do You Want to Keep Your Recipe to Yourself?

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Abstract

This paper aims to explore the role of spillovers in the strategic choice for a MNE in a duopoly model, especially focusing on endogenous FDI spillovers with spillover-prevention costs. After discussing the exogenous spillover case, this paper shows that with a quadratic spillover-prevention cost function, the FDI-performing firm may choose a positive level of spillovers, and also shows the determinants of such optimal spillovers. As a government policy of FDI host country, this paper shows that it may induce more FDI spillovers by taxing on the profits of the firm in the country and by using the tax revenue to subsidize the FDI-performing firm. As an extension of the model, this paper explores a n FDI-performing firm case, and shows that as the number of the home firms goes infinity, the FDI condition converges to that with exogenous spillovers while the cost disadvantage to the foreign firm disappears in case of FDI.

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1 Introduction

From the 1990s, many firms in developed countries have performed FDI in China and other east and southeast Asian countries. However, some companies have chosen to keep their production facilities in their home countries, although they are very competitive and sell their products in many countries, including the FDI host countries cited above. For instance, some Japanese manufacturers producing digital consumer electronics such as large-screen liquid crystal display (LCD) TV keep their production facilities in Japan, reported that they intent not to disclose their technology to rival companies abroad. At the same time, FDI host countries want multinational enterprises (MNEs) to transfer their superior technology to domestic counterparts. Some MNEs reply to host countries’ request positively while others do negatively. The letter case sometimes let MNEs to hesitate FDI. What causes such a difference? One concept which help us consider these two problems together is FDI spillovers, i.e. a positive externality from MNEs to firms in the FDI host countries.

In the international trade literature, various factors are discussed as causes of outward FDI, such as heterogeneity in productivity among domestic firms (Helpman et al. 2004), networks to sell products to buyers from the same country (Greaney 2003), expectation of demand growth in the FDI host country (Rob and Vettas 2003), and marginal-cost differences between the home and FDI host countries associated with a cost of FDI (Horstmann and Markusen 1992, Xing and Zhao 2003, Yomogida 2006). Among these factors, the cost factor seems to be relevant with FDI spillovers, because without cost or product-quality disadvantage, FDI host countries do not necessarily need technology transfer associated with FDI. However, to my knowledge, no trade literature discusses FDI spillovers as a determinant of FDI theoretically, although many empirical studies on this issue exist (Dimelis 2005, for instance). Another line of the literature about spillovers is R&D spillovers in industrial organization, such as research joint venture (Greenlee 2005) and spatial model a la Hotelling in terms of product quality (Piga and Poyago-Theotoky 2005).

This paper aims to explore the role of FDI spillovers in the strategic choice for MNEs. Especially, this paper focuses on endogenous FDI spillovers. That is, this paper examines the case when a MNE may determine the level of spillovers with FDI by itself. If preventing all spillovers incurs no costs, it is obvious that the MNE does so. However, if no spillovers is not free, the MNE might seek an optimal level of spillovers, which could be positive.

The most important contribution of this paper will be to discuss endogenous FDI spillovers with the MNE’s decision on the plant location, i.e. exports or FDI. Suppose that no spillovers occur with exports. Then a MNE faces two problems to be solved, plant location and the degree of spillovers in case of FDI. Note that these two choices are not independent. When the MNE determines the degree of FDI spillovers, at the same time it must check whether the profits with FDI at the optimal level of spillovers are really higher than those with exports. This paper explores such issues thoroughly.
This paper aims to explore the role of spillovers in the strategic choice for a MNE in a duopoly model, especially focusing on endogenous FDI spillovers with spillover-prevention costs. After discussing the exogenous spillover case, this paper shows that with a quadratic spillover-prevention cost function, the FDI-performing firm may choose a positive level of spillovers, and also shows the determinants of such optimal spillovers. As a government policy of FDI host country, this paper shows that it may induce more FDI spillovers by taxing on the profits of the firm in the country and by using the tax revenue to subsidize the FDI-performing firm. As an extension of the model, this paper explores a \( n \) FDI-performing firm case, and shows that as the number of the home firms goes infinity, the FDI condition converges to that with exogenous spillovers while the cost disadvantage to the foreign firm disappears in case of FDI.

The rest of this article is arranged as follows. Section two describes the model. First as a benchmark, exogenous spillovers are discussed. Then endogenous case with a quadratic spillover-prevention function is analyzed. As a welfare analysis, section three discusses a government policy, spillover subsidy. Section four extends the model to a \( n \) FDI-performing firm case. Finally, section five concludes this paper.

2 Model

This paper develops a duopoly model based on Horstmann and Markusen (1992), who analyze endogenous plant location in a two-firm, two-market model. Consider two countries, home and foreign, and a home firm plans to enter the foreign market either by exports or FDI. Assume that to export products to the foreign country, the home firm must pay a unit trade cost of \( t \), which is an incentive for the home firm to perform FDI. After the decision on plant location, the home firm compete with a foreign firm by quantity of the same product. A inverse demand in the foreign country is assumed; \( P = a - Q \) where \( P \) is the price of the product, \( Q \) is the total quantity, and \( a \) is a positive constant.

Suppose that marginal costs of the both firms are constant. The marginal cost of the home firm is \( c \), and that of the foreign firm is \( (1 + d)c \), where \( d \) is a positive constant. Thus, the home (foreign) firm has a cost (dis)advantage and the degree of its (dis)advantage is captured by the parameter \( d \). Suppose that FDI reduces the cost disadvantage to the foreign firm by \( s \) while exports does not at all.\(^1\) After FDI, the marginal cost of the foreign firm is \( (1 + d - s)c \). Such a decrease in the marginal cost is referred as “FDI spillovers” in this paper. Such spillovers make FDI less attractive for the home firm.

First, as a benchmark, a case of exogenous spillovers is discussed. In this case, a degree of spillovers \( s \) is exogenous for both the home and foreign firms, as well as

\(^1\)One might assert that exports also make some spillovers although the degree is lower than that with FDI. However, for the tractability of the model, zero spillovers are assumed in this paper.
the trade cost \( t \) and other exogenous variables. Then endogenous spillovers, which the home firm may determine, are examined.

### 2.1 Benchmark: Exogenous Spillovers

The profits of the home and foreign firms in each of the plant location of the home firm are as follows.

\[
\pi_h = \begin{cases} 
(a - x - y)x - (c + t)x & \text{No FDI, i.e. exports,} \\
(a - x - y)x - cx & \text{FDI.}
\end{cases} \tag{1}
\]

\[
\pi_f = \begin{cases} 
(a - x - y)y - (1 + d)cy & \text{No FDI,} \\
(a - x - y)y - (1 + d - s)cy & \text{FDI.}
\end{cases} \tag{2}
\]

\( \pi_h \) and \( \pi_f \) are profits of the home and foreign firms respectively. \( x \) and \( y \) are quantity produced by the home and foreign firms respectively.

From the first order conditions, the quantity produced by each firm in each case is the following. When the home firm chooses exports (Case \( N \)),

\[
x^N = \frac{a + (d - 1)c - 2t}{3}, \tag{3}
\]

\[
y^N = \frac{a - (2d + 1)c + t}{3}. \tag{4}
\]

On the other hand, when the home firm chooses FDI (Case \( F \)),

\[
x^F = \frac{a + (d - s - 1)c}{3}, \tag{5}
\]

\[
y^F = \frac{a - \{2(d - s) + 1\}c}{3}. \tag{6}
\]

By inserting the equilibrium outputs in each case to the profits (1) and (2), it is shown that \( \pi_h = (x^i)^2 \), where \( i = N, F \), and that \( \pi_f = (y^j)^2 \), where \( j = N, F \). This implies that comparing the outputs of a firm in the two cases is enough to compare the profits in the two cases. From equations (3) and (5), equilibrium outputs of the home firm in the two cases, in order for the home firm to prefer FDI, the following inequality must hold;

\[
s c < 2t. \tag{7}
\]

The left hand side of the inequality is the reduction of the foreign firm’s marginal cost due to FDI spillovers, i.e. the cost of FDI for the home firm, while the right hand side is two times the cost of exports. Thus the home firm compares the cost of FDI with that of exports and chooses FDI when the former is less than the latter. Although equation (7) is a condition for FDI with exogenous spillovers, it is interesting to compare this condition with the counterpart in case of endogenous spillovers, as we see in the next section.
2.2 Endogenous Spillovers

Suppose that before deciding the plant location, the home firm may determine the degree of FDI spillovers $s$ by itself. If perfect prevention of spillovers, i.e. $s = 0$, is possible without any costs, it is obvious from equation (5) that the home firm definitely does so. However, if lower spillovers need additional costs, an optimal value of $s$, which is positive, might exist.

Assume that the spillover-prevention cost function is quadratic. Then the profits of the home firm in case of FDI is

$$(a - x - y)x - cx - e(d - s)^2$$

where $e$ is a positive constant. The formulae of the profits of the home firm in case of exports and those of the foreign firms in either cases are not changed. Now the model has two periods; decision on the degree of spillovers in period one, and plant location and quantity competition in period two. Thus the model is solved by backward induction.

2.2.1 Period Two: Plant Location and Quantity Competition

In period two, the home firm chooses its optimal quantity associated with its optimal plant location, and the foreign firm chooses its optimal quantity. These decisions are made for a given level of spillovers with FDI, determined by the home firm in period one. This implies that the first order conditions in period two are exactly the same as those in the benchmark case. Thus, equations (3) to (6), the equilibrium outputs of the two firms with exogenous spillovers, hold with endogenous spillovers too.

2.2.2 Period One: Optimal Degree of Spillovers with FDI

Substituting the equilibrium outputs of the two firms in case of FDI (equations 5 and 6) into the profits of the two firms (the second lines of equations 1 and 2) yields the profits of the two firms in period one if the home firm chooses FDI in period two;

$$\pi_h^{Period \ One} = \left[\frac{a + (d - s - 1)c}{3}\right]^2 - c(d - s)^2$$

$$\pi_f^{Period \ One} = \left[\frac{a - \{2(d - s) + 1\} c}{3}\right]^2$$

The home firm chooses $s$ to maximize its profits. From the first order condition, the optimal $s$ is

$$s^* = d - \frac{c(a - c)}{9e - c^2}.$$
$s^*$ may be positive.\footnote{1} The lower bound for $s^*$ is zero due to the definition of the spillovers. The cost advantage parameter $d$ and prevention-cost parameter $e$ have positive effects on $s^*$. When the home firm has a large cost advantage, it may afford more spillovers. When the spillover prevention costs more, the home firm must allow more spillovers to save costs. The demand parameter $a$ has a negative effect. One possibility for this negative effect is that a larger $a$ implies a larger profit opportunity. The home firm attempts to keep its cost advantage to utilize this opportunity.

Inserting $s^*$ into equations (8) and (9) yields equilibrium profits of the two firms, denoted by $\pi_h^*$ and $\pi_f^*$ respectively;

$$
\pi_h^* = \frac{(a-c)^2e}{9e-c^2},
$$

(11)

$$
\pi_f^* = \left[\frac{(a-c)(3e-c^2)}{9e-c^2}\right]^2
$$

(12)

Note that $d$ does not appear in either equations (11) or (12). The reason is that the home firm actually chooses an optimal level of after-spillover cost disadvantage to the foreign firm for a given $d$, $d - s^*$, which equation (10) implies.

One important question is whether the equilibrium profits of the home firm in case of FDI are larger than the profits in case of exports. If the home firm prefers FDI, the following inequality must hold;

$$
\frac{(a-c)^2e}{9e-c^2} > \frac{(a+(d-1)c-2t)^2}{9e-c^2}.
$$

The left hand side is the equilibrium profits in case of FDI (equation 11), and the right hand side is those in case of exports, equal to the squared equilibrium output (equation 3). From the above inequality, the following expression similar with inequality (7), the condition for FDI with exogenous spillovers, is derived;

$$
dc + (a-c)\left(1 - \sqrt{\frac{9e}{9e-c^2}}\right) < 2t.
$$

(13)

One might ask how endogeneizing spillovers change the condition for FDI. Suppose that spillovers are exogenous and the level is equal to $s^*$ (equation 10). Then equation (7), the FDI condition with exogenous spillovers, is changed to;

$$
s^*c = dc - \frac{c^2(a-c)}{9e-c^2} < 2t.
$$

(14)

From the assumptions in footnote two, it is shown that the left hand side of inequality (14) is less than that in inequality (13), the FDI condition with endogenous spillovers. The following proposition summarizes the result; see Appendix A for the proof.

\footnote{1}{(1) $a$ is much greater than $c$, and (2) $e > \frac{c^2}{9}$ are assumed. The first assumption is to secure the positive output and profits of the each firm, and the second assumption is necessary for the second order condition to be held.}
Proposition 1

With a quadratic cost function of FDI-spillover prevention, endogeneizing spillovers increases the threshold of the trade cost for FDI.

Proposition 1 implies that endogeneizing spillovers makes FDI less likely. Note that this result is due to a kind of “overprevention of spillovers” for the quantity competition in period two. That is, the home firm wants to keep its cost advantage for the quantity competition in period two. However, the Cournot equilibrium is a Nash equilibrium. Thus the home firm prevents FDI spillovers too much, which is a kind of prisoner’s Dilemma. Moreover, such overprevention of spillovers makes the costs of FDI higher, which results in a rise of hurdle for FDI in terms of the trade costs. In the next section, policies by the government of FDI host country that may improve this situation by mitigating this overprevention problem, which could be good even for the home firm, are discussed.

3 Policies of FDI Host Country

Suppose that the exogenous variables \((c, d, e, t)\) satisfies inequality (13). That is, the home firm chooses FDI. Under this assumption, the following situation is considered; the government of the FDI host country, i.e. the foreign country, attempts to increase the profits of the foreign firm. From equation (9), an increase in \(s\) always rises the profits of the foreign firm.

One possible policy is a subsidy for the spillovers. Suppose that before the home firm determines the degree of spillovers, the foreign government gives a subsidy \(T\) to the home firm if the home firm promises to give extra spillovers \(\tau\) to the foreign firm, in addition to \(s^*\), the optimal degree of spillovers for the home firm. The subsidy is financed by taxing on the profits of the foreign firm. Then the question is whether a combination of \((T, \tau)\) that increases the after tax profits of the foreign firm while never decreasing the profits of the home firm really exists. First, a numerical example shows that such \((T, \tau)\) exist. Then, this issue is explored analytically.

3.1 Numerical Example

Consider the following example; \((a, c, d, e, t) = (9, 1, 10, 1, 5)\). With these parameters, \(s^* = 9\) and inequality (13) holds. \(\pi_h^*\) is equal to 8, and \(\pi_f^*\) is equal to 4. Denote after-subsidy profits of the home firm by \(\pi_{h}^{**}\), and after-tax/extra spillovers profits of the foreign firm by \(\pi_{f}^{**}\). Formally,

\[
\pi_{h}^{**} = \left[ \frac{a + (d - s^* - \tau - 1)c}{3} \right]^2 - e(d - s^* - \tau)^2 + T
\]

(15)

\[
\pi_{f}^{**} = \left[ \frac{a - \{2(d - s^* - \tau) + 1\}c}{3} \right]^2 - T
\]

(16)

\(^3\)As a measure of the welfare of the foreign country, the sum of consumer surplus and the profits would be desirable. However, as the first step to discuss the policies of FDI host country, the profit measure is used in this section.
Inserting the above parameters and $\tau = 1$ into equations (15) and (16) yields

$$\pi^{**}_h = \frac{64}{9} + T,$$

$$\pi^{**}_f = \frac{64}{9} - T.$$  

These values imply that any $T$ satisfying $\frac{28}{9} > T \geq \frac{8}{9}$ strictly increases $\pi^{**}_f$ while never decreasing $\pi^{**}_h$. Thus with this policy the FDI host-country government may increases the profits of the foreign firm by inducing more FDI spillovers.

### 3.2 General Case

Substituting $s^*$ into equations (15) and (16) and using the definitions of profits in period one without tax or subsidy (equations 8 and 9), the differences in profits of the two firms before and after the policies of the host-country government are as follows;

$$\pi^{**}_h - \pi^*_h = \left(\frac{c^2}{9} - e\right) \tau^2 + T.$$

$$\pi^{**}_f - \pi^*_f = \frac{4c^2}{9} \tau^2 + \frac{4c(a - c)(9e - 3c^2)}{9(9e - c^2)} \tau - T.$$

In order for the government policy to be effective, the profits of the home firm must not decrease while the profits of the foreign firm must increase. In other words, the following two inequalities must hold.

$$\left(\frac{c^2}{9} - e\right) \tau^2 + T \geq 0. \quad (17)$$

$$\frac{4c^2}{9} \tau^2 + \frac{4c(a - c)(9e - 3c^2)}{9(9e - c^2)} \tau - T > 0. \quad (18)$$

The following proposition claims that pairs of $(T, \tau)$ which satisfy these two inequalities exist.

**Proposition 2** Suppose that the government of FDI host country subsidizes the FDI-performing home firm if the home firm gives extra FDI spillovers to the foreign firm. By this policy, the government may increase the profits of the foreign firm while never decreasing the profits of the home firm.

Proof: see Appendix B

### 4 Extension: $n$ Home Firms

So far the case of duopoly, i.e. one home firm, is discussed. As an extension of the model, consider a case where $n (\geq 1)$ identical firms exist in the home country
and they play the two-period location-production game discussed in the previous sections.

If more than one firm exist in the home country, a natural question is how to formulate FDI spillovers. It seems plausible that the degree of spillovers increases as more home firms perform FDI. However, the upper bound for the degree of spillovers is \( d \), the degree of cost disadvantage to the foreign firm. Therefore, assume the following structure of FDI spillovers, i.e. the average degree of spillovers among the home firms:

\[
s = \frac{1}{n} \sum_{j=1}^{n} s_j
\]

where \( s_j \) is the degree of spillovers from home firm \( j \) \((j = 1, \ldots, n)\). With this formula of FDI spillovers, the profits of home firm \( i \) and foreign firm when all of the home firms choose FDI are as follows.

\[
\pi_{h,i} = (a - \sum_{j=1}^{n} x_i - y)x_i - cx_i - e(d - s_i)^2.\quad i = 1, \ldots, n. \tag{19}
\]

\[
\pi_f = (a - \sum_{j=1}^{n} x_i - y)y - (1 + d - \frac{1}{n} \sum_{j=1}^{n} s_j)cy. \tag{20}
\]

From the first order conditions in period two and the symmetry of the model among the home firms, the equilibrium outputs of the home firm \( i \) and the foreign firm are as follows.

\[
x^F_i = x^F = \frac{a + (d - s - 1)c}{n + 2}.\quad i = 1, \ldots, n. \tag{21}
\]

\[
y^F = \frac{a - \{2(d - s) + 1\}c}{n + 2}. \tag{22}
\]

The equilibrium outputs in case of exports are derived by replacing “3” in the denominators of equations (3) and (4) with \( n + 2 \). Substituting the equilibrium outputs (equations 21 and 22) into the profits of the home firm \( i \) (equation 19) yields its objective function in period one.

\[
\pi_{h,i}^{\text{Period One}} = \left[ a + \left( d - \frac{1}{n} \sum_{j=1}^{n} s_j - 1 \right) c \right]^2 - e(d - s_i)^2.\quad i = 1, \ldots, n. \tag{23}
\]

From the first order conditions, a reaction function for the home firm \( i \) with respect to the sum of spillovers from all the other home firms is derived;

\[
s_i = \frac{(n + 2)^2 de - c\{a + (d - 1)c\}}{(n + 2)^2 e - c^2} + \frac{c^2}{(n + 2)^2 ne - c^2} \sum_{j \neq i} s_j.\quad i = 1, \ldots, n. \tag{24}
\]

Assume \( (n + 2)^2 e > c^2 \). This assumption corresponds to the second assumption in footnote two, i.e. the assumption for the second order condition in the duopoly.
case. Note that the degree of spillovers by home firm $i$ increases as the sum of counterparts of the all other firms. Therefore, FDI spillovers are strategic complements among the home firms.

From the first order conditions and the symmetry of the model among the home firms, the equilibrium degree of FDI spillovers by the home firm $i$ is;

$$s^*_i = s^* = d - \frac{c(a - c)}{(n + 2)^2ne - c^2}. \quad i = 1, \ldots, n. \quad (25)$$

Note that $s^*$ increases as $n$ increases, which implies that if more home firm enter the foreign market by FDI, the equilibrium degree of spillovers by the each firm, which is equal to the total spillovers, increases.

Does an increase in the number of the home firms affect the condition for the home firms to choose FDI? The each home firm chooses FDI if the profits with FDI are higher than those with exports;

$$\frac{(n + 2)^2n^2e - c^2}{(n + 2)^2ne - c^2} > \left[ \frac{a + (d - 1)c - 2t}{n + 2} \right]^2. \quad (26)$$

The left hand side, the profits with FDI, are derived by substituting $s^*$ (equation 25) into the objective function of the home firm $i$ in period one (equation 23). The right hand side is the profits with exports, equal to the squared equilibrium output with exports. The above inequality is changed to;

$$dc + (a - c) \left( 1 - \frac{(n + 2)^2ce(2n - c) - c^2}{(n + 2)^2ne - c^2} \right) < 2t. \quad (26)$$

This is the FDI condition for the $n$-home firm case. When $n = 1$, it is reduced to equation (13). One important property with the generalized FDI condition is that as the number of the home firms goes to infinity, this condition is reduced to the FDI condition with ”exogenous” $s$ (equation 7).\textsuperscript{4}

**Proposition 3** As the number of the home firm $n$ goes to infinity, the $n$-firm FDI condition with endogenous spillovers converges to the condition with exogenous spillovers, and the cost disadvantage to the foreign firm disappears in case of FDI.

### 5 Conclusions

First by a duopoly and then by an oligopoly model, this paper explores roles of FDI spillovers as a strategic variable for firms entering the foreign market. This paper shows that (1) Endogeneizing spillovers make FDI less likely, compared to the exogenous case, (2) Subsidizing spillovers may increase the profits of the firm in the FDI host country, and (3) Increasing the number of firms in FDI-source

\textsuperscript{4}The right hand sides of both equations (25) and (26) converge to $dc$ as $n$ goes infinity. Thus, $s^*c = dc < 2t$ holds. The first equation implies that maximum spillovers occur.
country affects both the level of spillovers and the threshold of the trade cost for FDI.

This paper assumes identical firms in the FDI source country in its oligopoly model. However, if the firms are heterogeneous in production costs, the each firm’s behavior might be different. Moreover, one popular policy by FDI host countries, mandating a joint venture with the foreign firm to the home firm, considered as a way of inducing more spillovers, is not discussed in this paper, Examining effects of cost heterogeneity either in source or in host countries and relating them with a joint venture may be possible further research agendas.

Appendix A. Proof for Proposition 1

Suppose that the left hand side of inequality (14), the FDI condition with exogenous spillovers $s^*$, is less than that in inequality (13), the condition with endogenous spillovers. Then the following inequality holds;

$$(a - c) \left(1 - \sqrt{\frac{9e}{9e - c^2}}\right) > -\frac{c^2(a - c)}{9e - c^2}.$$  

This inequality is changed to;

$$\frac{9e}{9e - c^2} > \sqrt{\frac{9e}{9e - c^2}}$$

From the assumption in footnote two, $\frac{9e}{9e - c^2} > 1$. Thus the above inequality holds.

QED.

Appendix B. Proof for Proposition 2

Summing the both sides of inequalities (17), the condition for the profits of the home firm not to decrease, and (18), the condition for the profits of the foreign firm to increase, respectively to erase $T$, the amount of subsidy, and multiplying 9 for both sides yield the following inequality;

$$(5c^2 - 9e)\tau^2 + \frac{4c(a - c)(9e - 3c^2)}{9e - c^2} \tau > 0. \quad (27)$$

It is shown that $\tau$, tax on the foreign firm, satisfying inequality (27) exist. Depending on the signs of $5c^2 - 9e$ and $9e - 3c^2$, three cases may occur.\footnote{The assumption in footnote two does not determine the signs of these values. Although the signs can be assumed with extra assumptions, these assumptions do not have concrete theoretical background.}
1. $5c^2 < 9e$

Inequality (27) is changed to

$$\tau \left( \tau + \frac{4c(a - c)(9e - 3c^2)}{(9e - c^2)(5c^2 - 9e)} \right) < 0,$$

since $5c^2 - 9e < 0$. Because the second term inside the brackets is negative, any $\tau$ less than $\frac{4c(a - c)(9e - 3c^2)}{(9e - c^2)(9e - 5c^2)}$, which is positive, satisfies this inequality.

2. $3c^2 < 9e < 5c^2$

Inequality (27) is changed to

$$\tau \left( \tau + \frac{4c(a - c)(9e - 3c^2)}{(9e - c^2)(5c^2 - 9e)} \right) > 0.$$

Because the second term inside the brackets is positive, this inequality holds for any positive $\tau$.

3. $9e < 3c^2$

Inequality (27) is changed to

$$\tau \left( \tau + \frac{4c(a - c)(9e - 3c^2)}{(9e - c^2)(5c^2 - 9e)} \right) > 0.$$

Because the second term inside the brackets is negative, any $\tau$ greater than $\frac{4c(a - c)(3c^2 - 9e)}{(9e - c^2)(9e - 5c^2)}$, which is positive, satisfies this inequality.

To find a $T$ satisfying inequalities (17) and (18), the following procedure can be used. First, for a $\tau$ satisfying inequality (27), say $\tau^*$, find $T$ equal to $\left( e - \frac{c^2}{9} \right) \tau^{*2}$, denoted by $T^*$. Then, this $T$ satisfies inequality (18), because substituting $T^*$ into $T$ yields inequality (27), which holds at $\tau^*$. Note that as the numerical example suggests, depending on the exogenous variables, $T^*$ could be greater than $\left( e - \frac{c^2}{9} \right) \tau^{*2}$, which implies $(T^*, \tau^*)$ increases the profits of the home firm too.

QED.

References


