A Search and Matching Model of International Trade
– Preliminary and incomplete –

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1 Introduction

A search perspective on international trade

- Firm heterogeneity with respect to productivity goes some way in explaining export behavior (Melitz, ECM 2003), but

- ... calibration/simulation exercises predict to high a share of exporters (for given productivity-vintage-size clusters) (Bernard et al., AER 2003)

- Firms with identical productive characteristics may exhibit different ex post behavior due to search frictions

- Sunk costs in searching for a trading partner are large and a significant source of export persistence (Bernard and Jensen, 2004)

- Creation and destruction of trade relations is quite common empirically (Besedeš and Prusa, CanJoE 2006)

Production firms, trade partners

- Exporting entails additional costs, which are usually treated as fixed (see Melitz, ECM 2003)

- Exporting often requires intermediation: a panoply of possibilities ranging from trade brokerage, franchising agreements, exclusive dealership agreements, to joint venture FDI (see Feenstra and Hanson, 2004, or Schröder et al., 2005)

⇒ Producer/Partner matching problem

⇒ Off-market price determination (Nunn, 2005)

The idea

- Without some intermediation, trade is strictly impossible. Intermediation is meant to be the involvement of any partner in exporting

- However, the fundamental desirability of trade across countries creates profit opportunities for specialized intermediaries who act as market makers

- The number of active intermediaries reacts to market conditions: in a tight market, opportunities are relatively rare, and deal making is more expensive

- The number of intermediaries will play a crucial role in determining the effect of trade liberalization on trade volumes, utility, etc.
Our contribution

1. Merging the monopolistic competition model with a matching-function approach borrowed from equilibrium unemployment theory (Pissarides, 2000)

2. Analysis of a symmetric, two-country, general equilibrium at its steady state

3. Endogenizing market access costs through search costs

4. Capturing realistic features of trade empirics (see above)

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2 Model Setup

Consumer problem

The representative consumer solves

\[ U = \int_0^{N+M} c_z^{\frac{\sigma-1}{\sigma}} \mathrm{d}z, \sigma > 1 \] (1)

subject to the budget constraint

\[ Y \geq \int_0^N p_i^D c_i^D \mathrm{d}i + \int_N^{N+M} p_j^M c_j^M \mathrm{d}j, \] (2)

- Superscripts \( D, M \) refer to domestically produced vs. foreign produced varieties
- \( Y \) income, \( p \) price of a symmetric variety, \( c \) consumption quantity
Production firms

- Labor is only factor of production
- Fixed firm-specific setup costs, constant marginal input requirements:
  Conditional labor demand per unit of time
  \[ L_i = \psi_i + \alpha_i y_i \]  \hspace{1cm} (3)
- All producers are symmetric (drop \( i \))
- Producers sell quantity \( c^D \) on domestic market, or, if applicable, \( c^X \) on export market
- However, exporting only possible through an intermediary (a partner) that needs to be found through a search process

Matching

Number of matches in \([t, t + dt]\)
\[ Q = Q(X^S, M^S) \]  \hspace{1cm} (4)

- \( X^S = (1 - x)N \ldots \) number of unmatched (non-exporting) producers
- \( M^S \ldots \) number of unmatched partners
- \( Q(\cdot) \) is hom(1) and strictly concave in both arguments
- Market tightness: \( \theta \equiv M^S / X^S \)
- \( q(\theta) \ldots \) per unit of time Poisson rate of a successful match from unmatched partner perspective, \( q'(\theta) < 0 \)
- \( q(\theta)\theta \ldots \) per unit of time Poisson rate of a successful match from unmatched producer perspective

3 Equilibrium conditions

The value of a startup firm \( V^D \)
Bellman equation
\[ \rho V^D = -\psi^D + \pi^D + q(\theta) \theta (V^X - V^D) \]  \hspace{1cm} (5)

- \( \rho V^D \ldots \) per unit of time flow return to a newly created firm
- \( \psi^D \ldots \) flow fixed costs of producing for the domestic market
- \( \pi^D \ldots \) operating profit from selling on domestic market
- \( q(\theta)\theta \ldots \) Poisson rate of becoming an exporter in next \( dt \)
- \( V^X - V^D \ldots \) capital gain from switching status
- Focusing on steady states, we ignore \( \dot{V} \) terms
The value of an exporting firm $V^X$

$$\rho V^X = -\left(\psi^D + \psi^X\right) + \pi^D + \pi^X(p) + \lambda \left(V^D - V^X\right)$$  \hspace{1cm} (6)

- $\psi^D + \psi^X$ ... flow fixed costs of producing for the domestic and the export market
- $p$ ... price paid by the intermediary
- $\pi^X(p)$ ... operating profit from selling on export market
- $\lambda$ ... per unit of time Poisson ‘death’ rate of an existing relationship
- $\lambda \left(V^D - V^X\right)$ ... expected capital loss from losing export market in $dt$

The value of a newly created importer $V^S$

$$\rho V^S = -\phi + q(\theta) \left(V^T - V^S\right)$$  \hspace{1cm} (7)

- $\phi$ ... flow costs while searching
- $q(\theta) \left(V^T - V^S\right)$ expected capital gain from finding a producer

The value of an ongoing importer $V^T$

$$\rho V^T = -\psi^T + \pi^T(p) + \lambda \left(V^S - V^T\right)$$  \hspace{1cm} (8)

- $\psi^T$ ... flow fixed costs of importing
- $\pi^T(p)$ ... operating profits from selling imported variety
- $\lambda \left(V^S - V^T\right)$ expected capital loss from losing the supplier

Consumer behavior

- Normalizing $w = 1$ in both countries, household income is $Y = L$
- Using firms’ pricing rules, demand for imported varieties is

$$c^M = L p^{\sigma - 1} \sigma^{-\sigma} \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma}$$

- Similarly, demand for domestically produced varieties is

$$c^D = L p^{\sigma - 1} \alpha^{-\sigma} \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma}$$

- The aggregate price index $P$ is

$$P^{1-\sigma} = N \alpha^{1-\sigma} \left(\frac{\sigma}{\sigma - 1}\right)^{1-\sigma} + x N p^{1-\sigma} \left(\frac{\sigma}{\sigma - 1}\right)^{1-\sigma}$$
Nash bargaining

- When an importer searching a specific variety meets a producer that supplies such a variety, the prospective partners find themselves in a bilateralmonopoly.
- They negotiate a transaction price $p$

$$p = \arg \max \left( V^X(p) - V^D \right)^\beta \left( V^T(p) - V^S \right)^{1-\beta} \quad (9)$$
- $\beta \in (0,1)$...the producer's bargaining power
- First order condition

$$V^X(p) = \frac{\beta}{1-\beta} \left( \frac{-V^X_p(p)}{V^T_p(p)} \right) V^T(p) \quad (10)$$

Steady state flow condition

- Share of exporting firms evolves according to

$$\dot{x} = \theta q(\theta) \left( 1 - x \right) - \lambda x \quad (11)$$
- We focus on steady state equilibria where $\dot{x} = 0$, hence

$$x = \frac{\theta q(\theta)}{\theta q(\theta) + \lambda} \quad (12)$$

Full employment

$$L = \psi^D N + \psi^X x N + \alpha c^D N + \alpha c^X x N \quad (13)$$

Note on search costs: as in Pissarides (2000), perfect international capital mobility at given $\rho$

Focus on symmetric equilibrium

- Both countries are identical in all respects
- $c^X = c^M (1 + \tau)$...each country export production less iceberg trade costs ($\tau > 0$) is consumed in the other country
- $xN = M$...numbers of exported and imported varieties are the same
- Balanced trade $p^X \times c^X \times x \times N = p^M \times c^M \times M$ holds as an identity
4 A partial equilibrium perspective on search and matching

Assumptions

- Goods prices are given: $p^D$ for domestically produced and $p^M$ for imported varieties
- Quantities are given and normalized to one ($e^D = e^X = 1$): profit maximization only works via the transaction price $p$
- Number of producers are given: $N$

Operating profits

- No variable costs: costs of producing are captured by $\psi^D$ and $\psi^X$ respectively
- Operating profits from selling domestically $\pi^D = p^D$
- Operating profit of an exporting firm $\pi^X = p$
- Operating profit of an ongoing importer $\pi^T = \frac{p^M}{1+\tau} - p^D$

Three equilibrium conditions

1. Free entry of intermediaries (Zero profit condition line)

$$p = -(\rho + \lambda) \frac{\phi}{q(\theta)} + \psi^T - \frac{p^M}{1+\tau}$$

(14)

2. Nash bargaining (Price equation line)

$$p = (1 - \beta)\psi^X + \beta \left( \phi^T - \psi^T + \frac{p^M}{1+\tau} \right)$$

(15)

3. Steady-state condition

$$x = \frac{q(\theta)\theta}{q(\theta)\theta + \lambda}$$

(16)

three equations in three endogenous unknowns ($p, \theta, x$)
Solution

- To solve the system of three equations, we assume the matching function to be Cobb-Douglas

\[
q(\theta) = \frac{Q(X^S, M^{S*})}{M^{S*}} = \frac{(X^S)^\gamma (M^{S*})^{1-\gamma}}{M^{S*}} = \theta^{-\gamma}
\]

and set \( \gamma = \frac{1}{2} \)

- Equilibrium share of searching producers

\[
1 - x = \frac{\lambda}{\Gamma} \left( (\rho + \lambda)\phi + \sqrt{(\rho + \lambda)^2 \phi^2 + 2\beta \phi \Gamma} \right),
\]

where \( \Gamma = 2 \left[ (1 + \beta) \left( \psi^T - \frac{p^M}{1 + \tau} \right) - (1 - \beta) \psi^X \right] > 0 \)

- Equilibrium market tightness

\[
\theta = \frac{\lambda^2}{(1 - x)^2}
\]

- Equilibrium price

\[
p = -\frac{\phi (\rho + \lambda)}{\lambda} (1 - x) + \psi^T - \frac{p^M}{1 + \tau}
\]

Reduction of search costs

- A reduction of the flow search costs \( \phi \) rotates the price equation line clockwise and the ZPC line outward
- The market tightness increases
- \( q(\theta) \) does not shrink as fast as \( \phi \), so the expected search costs \( \frac{\phi}{q(\theta)} \) fall
- The effect on the ZPC line is stronger, so the price rises
- In the \( m-(1 - x) \)-space the ZPC line rotates anticlockwise and \( x \) rises
- A reduction of the flow search costs induces a rise in the trade volume \( V = px\bar{N} \)
Changes in the trade costs, the willingness to pay, and the fixed costs of a trader

- A reduction of trade costs $\tau$, an increase in the willingness to pay in the importing country $p^M$ and a reduction of the flow fixed costs of importing $\psi^T$ affect the endogenous variables via the same channels

- The price equation line is shifted to the north, the ZPC line to the southwest

- The market tightness decreases

- The price equation line shifts even more, so overall the price is reduced

- In the $m-(1-x)$-space the ZPC line rotates clockwise, causing $x$ to shrink

- In this scenario, the trade volume is reduced

Limitations of the partial equilibrium approach

- The partial equilibrium results are isomorph to Pissarides (2000), but

- ... no interaction with the foreign country

- ... no interaction with the consumer

$\Rightarrow$ solution not sustainable in general equilibrium model of international trade
5 General equilibrium analysis

Summary of equilibrium conditions

1. Free entry of producers: $G^1(p, c^X, c^D, \theta) = 0$
2. Free entry of intermediaries: $G^2(p, c^X, \theta) = 0$
3. Nash bargaining: $G^3(p, c^D, c^X) = 0$
4. Consumer behavior: $G^4(p, c^X, x, N) = 0$
5. Steady-state condition: $G^5(x, \theta) = 0$
6. Full employment: $G^6(c^X, c^D, x, N) = 0$

six equations in six endogenous unknowns $(p, c^X, c^D, \theta, x, N)$

Equilibrium free entry conditions

- Necessary conditions for existence of a steady state
  \[ \sigma > (1 - \beta)^{-1}, \psi^P > \psi^T \] \hfill (21)

- Free entry of producers: number of varieties (=firms) $N$ in each country adjusts endogenously so that $\rho V^D = 0$ and
  \[ \psi^P = \pi^D + \frac{q(\theta) \theta}{\rho + \lambda + q(\theta) \theta} \pi^X, \] \hfill (22)

  where $\psi^P$ covers the flow fixed costs for firms producing for the domestic market only or for both markets and
  \[ \pi^D = \frac{\alpha}{\sigma - 1} c^D, \pi^X = (p - \alpha) c^X \] \hfill (23)

- Free entry of intermediaries: number of trading firms (importers) $M$ adjusts endogenously so that $\rho V^S = 0$ and
  \[ \frac{\phi}{q(\theta)} = \frac{\pi^T - \psi^T}{\rho + \lambda}, \] \hfill (24)

  where
  \[ \pi^T = \frac{p}{\sigma - 1} c^X \] \hfill (25)
Firm profits and firm valuation

- Different to Melitz: no luck rentiers
- Net profits of exporters are positive, those of non-exporters are negative
- $V^X - V^D = 0$ if $\rho = 0$: asymptotically all firms spend the same fraction of time exporting. Only advantage of present exporters is that they are currently enjoy higher profits
- This is a testable implication specific to our model

A perspective on the transaction price $p$

- To simplify, assume the ratio of partial derivatives of the firm values w.r.t $p$ to be constant and $-1$. This assures the bargaining parties to have conflicting interests and implies the necessary condition
  \[ \sigma < \frac{p}{p - \alpha} \]  
  (26)

- Solving the Nash bargaining problem
  \[ p = K \left( \alpha - \frac{\beta}{1 - \beta} \frac{\psi^P}{c^X} + \frac{\pi^D}{c^X} \right) \geq 0 \]  
  (27)

  \[ K = \frac{(1 - \beta)(\sigma - 1)}{(1 - \beta)(\sigma - 1) - \beta} > 1, \text{ if } \sigma > \frac{1}{1 - \beta} \]  
  (28)

  \[ p > \alpha, \text{ if } K > 1 \text{ and } \psi^P - \pi^D \geq \frac{\beta}{1 - \beta} \psi^T \]  
  (29)

- In equilibrium, $p$ needs to cover the fixed costs of the intermediaries and help cover those of the producers
- Price is larger, the larger $\sigma$ and $\psi^P$
- Price increases in $\alpha$ if $c^X > c^D / (\sigma - 1)$ ($p$ can well fall in $\alpha$ due to larger domestic profits)
- Price increases in $\beta$ if $\psi^P - \pi^D + \alpha c^X \geq (\sigma - 1) \psi^T$ ($p$ is more likely to fall in $\beta$, the larger $\psi^T$ and/or $\sigma$ are)
The effect of ‘globalization’

- Globalization driven by technology: $\phi$ falls
- Globalization driven by institutional reform: $\lambda$ falls
- Different channels, but similar welfare implications
- Globalization scenarios different from decreasing trade costs as in Monoplastic Competition Model
- Trade volume is given by $x \times N \times c^X \times p$
- Intermediaries face endogenous expected search costs of $\phi/q(\theta)$
- Fall in $\phi$ and $\lambda$: intermediaries’ expected search costs $\phi/q(\theta)$ shrink, value of a ‘match’ falls, this is consistent only with reduced profits, with fixed markups this means that the value of trade per intermediary $pc^X$ falls

Calibration

- Again the matching function is assumed to be Cobb-Douglas $q(\theta) = \theta^{-\gamma}$
- Let $w = 1$ by choice of numeraire and $L = 1$
- We set an elasticity of substitution in line with Bernard et al. (AER, 2003)
  - According to the Hosius condition $\gamma$ equals $\beta = 0.5$
  - Let $\alpha = 0.5, \psi^P = 1.9, \psi^T = 1.2, \rho = 0.01, \tau = 0.01$
  - Let (a) $\phi = 0.4$ and (b) $\lambda = 1$
Drivers of trade volume as a function of $\lambda$

- If $\lambda$ is sufficiently large, all matches are destructed immediately. By the steady state condition $x$ almost zero (autarky)
- We analyse now a decreasing destruction rate $\lambda$
- By the steady state condition the share of exporters $x$ tends to 1, so the market tightness $\theta = \frac{M}{(1-x)N}$ increases, $q(\theta)$ decreases and $q(\theta)\theta$ increases. By free entry of producers $\pi^D$ and/or $\pi^X$ have to shrink as $\frac{q(\theta)\theta}{\rho+\lambda+q(\theta)\rho}$ rises (even with decreasing $\lambda$). $\pi^D$ is driven by $c^D$, while $\pi^X$ can adjust via $p$ and $c^X$: $p$ has a lower limit, namely $\alpha$. To drive down operating profits from exporting the price $p$ has to increase, while the export quantity $c^X$ decreases
- As the share of exporters rises faster than the export quantity shrinks, the number of varieties produced domestically has to shrink by the full employment condition
Openness \( \left( xNpc^x/Y \right) \) as a function of \( \lambda \)

- The drivers of openness are already described. Overall, the product of exporter share and price rises faster than the product of export quantity and number of domestically produced varieties shrinks.
- The number of imported varieties \( M \) is given by \( xN \), so the number of available varieties \( N + M \) is \( N(1+x) \). \( x \) increases faster than \( N \) shrinks, so \( N + M \) increases.
Effects on intermediaries, producers and welfare

- The effects on expected search costs, sales of intermediaries and profits on producers are rather clear.

- As $q(\theta)$ decreases, the expected search costs have to rise.

- Sales of intermediaries ($p_c X \times$ constant markup) have to shrink (as explained above), so intermediaries become smaller.

- The share of gross profit margin earned on foreign market is given by $\left(1 + \frac{p_c}{\alpha^*(p-\alpha)(1+\tau)}\right)^{-1}$. With $p < 1$ the term $\frac{p_c}{p-\alpha}$ shrinks in $p$ and the share of gross profit margin earned on foreign market rises.

- The rise in $x$ and $p$ offset the decrease in $N$, so $P^{1-\sigma} = N(\alpha^{1-\sigma} + xp^{1-\sigma})$ increases, $P$ decreases and the real wage $1/P$ increases. This is equivalent to a rise in indirect utility.
Drivers of trade volume as a function of $\phi$

- We analyse a decrease in flow fixed costs of searching
- A decrease in $\phi = \frac{q(\theta)(\pi^T - \psi^T)}{\rho + \lambda}$ causes $q(\theta)$ and/or $\pi^T$ to decrease
- If the matching probability decreases, the market tightness increases, and the share of exporters $x$ rises
- A decrease in $\pi^T$ is as above brought about by an increase in $p$ and a decrease in $c^X$
- It is noticeable, that for sufficient small $\phi$ the number of domestically produced varieties rises again. This might be driven by free entry of producers, especially by the properties of $\frac{q(\theta)\theta}{\rho + \lambda + q(\theta)p}$. For a certain $\theta$ this ratio rises faster than $\pi^X$ shrinks. It becomes easier for producers to cover their flow fix costs of producing, so more producers enter
Openness \((V = xNpc^{X/Y})\) as a function of \(\phi\)

- By full employment, \(c^D\) has to adjust as well
- The effect on the openness and the number of available varieties works as above
Effects on intermediaries, producers and welfare

- $\phi$ shrinks faster than $q(\theta)$, so the expected search costs fall in contrast to the scenario, where globalization is driven by institutional reform.

- The same implications as in the case of decreasing $\lambda$ apply for the size of intermediaries, the share of gross profit margins earned on foreign market and welfare.

6 Discussion

Key results

- For given productivities some producers export, some don’t (in line with evidence).

- Profit seeking intermediaries help overcome frictions, entry high if $\rho + \lambda$ high and/or market tightness high.

- Producer/intermediary matches are bilateral monopolies where the transaction price is set through bargaining and not through competitive pricing mechanism.

- Partial equilibrium perspective helps to understand the search and matching mechanism, while general equilibrium approach brings interaction with foreign country and consumers into the picture.
• Liberalization scenarios have potentially non-monotonic effects on number of varieties produced domestically

Extensions

• Within this project
  – Off-steady-state dynamics
  – Asymmetric equilibria
  – Analysis of variable trade costs
  – General Nash bargaining solution

• Additional work streams
  – Two-sided search
  – Endogenous match destruction
  – Different productivities across firms
  – Vertical integration of producers and intermediaries (foreign affiliates)
  – Incorporation of network effects (matching function with increasing returns to scale)

7 Conclusion

• The model shows a way to merge a Pissarides-type matching model with a standard Monopolistic Competition Model of international trade

• Contributes towards unpacking trade costs (Venables) beyond Rauch and co-authors

• Search costs are to be seen as relationship specific sunk costs and are qualitatively different to Melitz-type beachhead export costs
References


