Key International Trade Theorems and Large Shocks

Ronald W. Jones
University of Rochester

Much of the theory of international trade makes use of relatively small dimensional models and engages in comparative statics exercises - comparing equilibria disturbed by small (infinitesimal) changes. However, in some circumstances the direction of change in key variables is altered once the size of the disturbing shock becomes sufficiently large. Such non-monotonic behavior is shown to exist in a pair of cases in which the shock is a change in a relative price ratio as well as in a pair of cases in which the initiating change is technical progress or a transfer of technology. Both sets reveal the importance of required alterations in production and trade patterns.

1. Commodity Price Changes: Stolper-Samuelson for Finite Shocks

One of the most celebrated theorems in international trade theory is that of Stolper and Samuelson (1941). Although much of their analysis refers to finite changes along a contract curve of a production box, with accompanying changes in marginal physical productivities, the primary result, viz. that an increase in a commodity’s price will unambiguously improve the real return to the factor intensively used there (and worsen the return to the other factor), is the same as if the calculus had been used for infinitesimal changes. But what can be said of real wage alterations when relative commodity prices change so much that complete specialization in production takes place in a context of international trade (so that both commodities can continue to be consumed)?
Consider Figure 1, with the upward sloping curves, 1 and 2, illustrating the given technology for the two commodities, viz, how capital/labor ratios utilized in production would increase should there be increases in wage/rental ratios (w/r). An initial given set of commodity prices shows, with the heavily drawn sections, that production patterns depend upon the economy’s capital/labor endowment ratio (K/L). Suppose the economy’s endowment ratio is initially at point A, with positive production of both labor-intensive commodity 1 and capital-intensive commodity, 2. Let the first commodity serve as numeraire and consider a small increase in the price of capital-intensive commodity 2. This will lower the wage/rental ratio as shown by point F on new (dashed) line BC. However, a larger shock that lowers the range of possible factor endowment ratios that would support production of both commodities to DE would, for the original economy, lower the wage/rental ratio to point G, and keep it there despite further increases in commodity 2’s price. The economy remains completely specialized in producing capital-intensive commodity 2.

The change in the real wage rate in the move from A to F is unambiguous – the increase in the price of the capital-intensive commodity lowers the nominal wage and thus lowers the real wage rate. The real wage continues to fall until further price increases for commodity 2 lead to complete specialization in production at point G. Even larger price increases do not alter the marginal productivity of either factor, but do serve now to increase the real wage rate. The rationale is that although marginal productivities are not affected, the values of the marginal products are raised so that both the nominal wage rate and the return to capital increase in precisely the same proportion as does the
price of the second commodity. As long as laborers consume some of both commodities (which trade allows), the real wage rate must increase.

Figure 2 illustrates explicitly the relationship between the (relative and absolute) price of the second commodity and the real wage rate, which has value shown by point $A$ at the initial price, $p_2^0$. As $p_2$ increases to $p_2'$, the real wage falls, in Stolper-Samuelson fashion, to point $G$. Further increases in commodity 2’s price cause the nominal wage rate to increase in the same proportion (shown by the dashed ray), and the real wage to increase by a smaller proportion (given by the share of consumption in the first commodity). A price increase exceeding $p_2''$ would serve to raise labor’s real wage above its initial value at point $A$.

The Stolper-Samuelson result is illustrated for small changes anywhere along the $AG$ portion, when in this two-factor case both commodities are produced. But this effect changes sign once the pattern of production is altered, in this case to one of complete specialization in producing the commodity that has risen in price. And a large enough increase in the price of the capital-intensive commodity serves to raise the real wage rate.

2. Large Price Shocks and Endogenous Production Structures

In post-Ricardian competitive trade theory two production structures have been emphasized. The first is the Heckscher-Ohlin theory, primarily used in the $2 \times 2$ setting familiar from the Stolper-Samuelson paper (1941), the factor-price equalization literature, and the flood of work following the publication of the famous Leontief Paradox (1953). The second is the Specific-Factors Model, earlier described by Haberler (1936) and developed more formally in Samuelson (1971) and Jones (1971, 1975).
The Heckscher-Ohlin structure is typified not only in the 2x2 setting, but as well in settings in which the number of productive factors matches the number of produced commodities once an economy is engaged in trade. Without further structure, the general \((nxn)\) setting yields scant comparative statics results, but one structural setting that is amenable to detailed analysis makes use of the Gruen and Corden (1971) construct for a three-commodity, three-factor setting, wherein one industry (say textiles) uses labor and capital and in another sector a pair of other industries uses labor as well, but also a third factor, land, not used in textile production. This idea was extended in Jones and Marjit (1992) to an “even” \((n+1)\times(n+1)\) setting in which \((n-1)\) of these sectors each produces a single industry and employs a unique form of capital as well as economy-wide homogeneous labor, and the \(n^{th}\) sector utilizes yet a different type of capital, as well as labor, to support a pair of industries in a 2x2 Heckscher-Ohlin structure. This sector is described as the 2x2 nugget. By contrast, the specific-factors structure generalizes easily to higher dimensions, e.g. to one in which there are \(n\) sectors in the economy, each supporting a single industry, utilizing a specific form of capital as well as homogeneous labor, making this an \((n+1)\times n\) setting.

The initial scenario in the Jones and Marjit (1992) paper envisions an autarky equilibrium in which in each of \(n\) sectors a unique type of capital exists, and is used in production by a variety (any number) of industries all using labor as well to produce commodities. As such a country joins the world market to engage in free trade, competition ruthlessly cuts down on the number of industries that can survive. One possibility is that given world commodity prices, in each sector the only surviving industry is the one exhibiting the largest return to the type of capital used by industries in
that sector. (This mirrors the reduction by trade in a Ricardian setting for a small country to a single “best” industry). However, another possibility is that in one of the sectors there emerges a “nugget” in which a pair of industries uses the same type of capital used by all industries in that sector in autarky. Which of these production structures emerges depends on commodity prices faced by the economy in the world market and the economy’s own factor endowment base. This is what is illustrated for the 2x2 case in Figure 1. The country might, with trade, produce two commodities, such as at $A$ or at $F$, depending on the commodity price ratio, or a single commodity, such as at $G$. That is, large shocks in commodity prices (or in factor endowments) can change, endogenously, the production structure from a specific-factors one to a Gruen-Corden type of Heckscher-Ohlin setting with one of the sectors supporting a 2x2 “nugget”. Could there be more than one such sector? Not if commodity prices are independently given in world markets – a country need not produce with trade more commodities than it has factors of production. As illustrated by Jones and Marjit, a sufficiently large price shock could move the structure from a Heckscher-Ohlin one in which a nugget appears in the $i^{th}$ sector to a Heckscher-Ohlin one in which a different nugget appears in the $j^{th}$ sector. Alternatively, the price shock could alter the production structure to one of the Specific-Factors variety.

3. Technical Progress and Wage Rates in the Heckscher-Ohlin Model

For infinitesimal improvements in productivity, the effect on factor returns depends only on the relative extent of cost reductions at initial factor prices (call these $\delta_1$ and $\delta_2$) and not at all on the bias (labor-saving or capital-saving) of the change in productivity.
Thus the competitive profit equations of change in the case of technical progress with given commodity prices are shown in equations (1):

\[ \sim_{L1} \hat{w} + \sim_{K1} \hat{r} = \doteq_{1} \]

\[ \sim_{L2} \hat{w} + \sim_{K2} \hat{r} = \doteq_{2} \]

Distributive factor shares are indicated by the \( \sim \)'s. The \( \doteq \)'s play the same role in the solution for the change in the factor price ratio, \( w/r \), as commodity price changes in section 1’s discussion. Of course if commodity prices are given, and the extent of technical progress in the second (capital-intensive) sector exceeds that in the first industry, the real wage rate need not fall. Solving separately for \( \hat{w} \), the change in the real wage if commodity prices are given, reveals that even though capital’s share in commodity 2 exceeds that in the first commodity, the real wage will increase if (assuming both \( \doteq \)'s are positive) the discrepancy between rates of technical progress in the two sectors is smaller than that between capital distributive shares. That is, the real wage improves with technical progress favoring the capital-intensive commodity if:

\[ \doteq_{2}/\doteq_{1} < \sim_{K2}/\sim_{K1} \]

The extent of the bias in technical progress does matter if technical progress at constant commodity prices is finite in size. The possibilities are illustrated in Figure 3, showing technical progress in the second industry that is of the Hicksian capital-saving variety, \( \text{viz.} \), for any given wage/rental ratio, commodity 2 would be produced with more labor-intensive techniques than initially. To widen the set of possible outcomes a third commodity, even more capital-intensive than the second commodity, has been added.
Given world commodity prices, the heavy broken line indicates the initial wage/rental ratios that would correspond to various endowment capital/labor ratios, and the dashed broken line the post-technical progress locus. Three initial situations, $A$, $A'$, and $A''$ are considered, and in all of them the country’s endowment bundle has it producing commodities 1 and 2. If initially at $A$, technical progress in capital-intensive commodity 2 serves to lower the wage/rental ratio to $B$, the standard result. By contrast, for initial endowment ratios shown by $A'$ or $A''$, Hicksian labor-using technical progress of finite size has the country switching completely out of producing labor-intensive commodity 1 into being specialized in commodity 2 at $B'$ or, in the move from $A''$ to $B''$, moving from producing commodities 1 and 2 to producing commodities 2 and 3. In either of these cases the country’s relative wage rate has improved by the technical progress taking place in the capital-intensive second commodity. And, since commodity prices are held constant, these moves represent improvements in the real wage rate as well as the relative wage rate.1

The exercise concerning technological change when there is no effect on commodity prices would be one extreme possibility. The other would be represented by a case in which commodity prices fall by the full extent of the degree of cost-cutting involved in technological progress. Here the effect on the real wage rate is very simple to analyze. In the pair of equations shown in (1) the right hand side in each would vanish, since each $\hat{p}_i$ would be matched by a value of $\hat{p}_i$ of equivalent size but opposite sign. Although the

---

1 Another surprising result for finite changes in technology is reported in Findlay and Jones (2000). For the two-commodity case, compare two alternative types of technical progress that take place in the labor-intensive commodity. Both are of the same Hicksian extent, but in one case it is purely labor saving (at initial factor prices) and in the other it is purely capital saving. In either case the real wage increases, but such an increase is even greater in the case in which technical progress is of the pure labor-saving type. The rationale is that in this latter case factor intensities in the two industries are brought closer together, serving to enhance the effect of technical progress on factor prices.
wage/rental ratio would not change, the lower price level would benefit laborers as consumers. The real wage would unambiguously increase.

4. Price Changes and Internationally Mobile Capital

Return to the 2x2 Heckscher-Ohlin setting, but now consider a world in which (i) commodity prices are exogenously given to a pair of countries (Home and Foreign) and (ii) one of the two factors of production, capital, is mobile between these countries although fixed in total supply. Although these countries do not share the same technology, assume that commodity 2 is the capital-intensive commodity in both countries. With international capital mobility, the rate of return to capital \((r)\) is equalized between Home and Foreign. Now suppose that if both countries were to be incompletely specialized in production, Foreign would be able to produce the second commodity at a relatively lower cost than the first compared with Home. Alternatively phrased, when both countries face the same international price ratio for commodities, the rate of return to capital in Foreign would exceed that in Home if both countries were to be incompletely specialized in production. However, with free trade in commodities and international mobility of capital, the kind of technological advantage that Foreign possesses in producing capital-intensive commodity 2 precludes both countries being incompletely specialized at the same time.

This setting corresponds to that studied in Jones and Ruffin (1975) and Jones (2000). Figure 4 illustrates the world production patterns that emerge with different possible terms of trade for commodities. If the relative price of the second commodity in world markets is very low, neither country will engage in its production. With both Home and
Foreign producing just the first commodity, capital mobility between countries ensures that its rate of return (in terms of commodity 1, the numeraire) is equal to the physical marginal product of capital in each country. As \( p_2/p_1 \) attains higher values, eventually one of the countries can start to produce the second commodity. This will be Foreign, because it has a comparative advantage in commodity 2’s production if both countries face the same rate of return to capital.

In Figure 4 on the stretch for which Foreign produces both commodities, but Home is still specialized to the first, Stolper-Samuelson type of results in Foreign hold: As commodity 2’s relative price increases, there is a magnified increase in the rate of return to capital (faced by both countries). In Foreign, not only are resources taken from use in the first industry, capital is shipped from Home (thus causing the increase at home in its rate of return). This situation changes when Foreign becomes completely specialized in producing the second commodity, and in a further stretch of the terms of trade each country is completely specialized – Home to commodity one and Foreign to the second commodity.\(^2\) Increases in \( p_2/p_1 \) still indicate that more of commodity 2 should be produced, and the response is a further flow of capital to Foreign. However, \( p_2/p_1 \) eventually reaches a height that will allow Home to put some resources into the production of commodity 2. At this point one can ask the following question: A further increase in \( p_2/p_1 \) is a signal that the world (of these two countries) wants to produce more of commodity 2. Which country has become the better producer? Admittedly Foreign would be the lower cost producer if both countries produced both goods. Despite this

---

\(^2\) In this stretch note that the percentage change in the return to capital is smaller than that in the commodity terms of trade. The reason is that the structure of the two-country model in this range is like that of the specific-factors model, with the specific factors being the labor force in each country, each devoted to producing a different commodity, and the mobile factor being capital.
technological advantage in Foreign’s favor, Home has something Foreign does not: a hinterland, where resources are being used to produce commodity 1. Therefore if more commodity 2 is indicated by the rise in its relative price (from the value indicated by point A in Figure 5), capital starts flowing back towards Home, where its use in producing commodity 2 is joined by both labor and capital released by the first sector. The capital flow towards Home as \( p_2/p_1 \) increases continues until 2’s relative price has risen to such an extent that both countries put all resources into 2’s production.

This necessary non-monotonic behavior of capital flows with respect to the commodity terms of trade is illustrated in Figure 5. Point A indicates the terms of trade in which Foreign has a maximum amount of capital located within its country.\(^3\) Consider the terms of trade indicated by point B. Here a small increase in capital-intensive commodity 2’s relative price encourages an outflow of capital to the country (Foreign) that has a technologically-given comparative advantage in 2’s production (for common return to capital). This would be the predicted result. However, a sufficient finite increase in commodity 2’s relative price could result in a net flow of capital back to Home. The critical turning point (A) is the point at which the pattern of production at Home changes in that it becomes a viable producer of the second commodity, with resources available in production of the other commodity (1) to join a capital inflow from Foreign. Once again, non-monotonic behavior (in this case of the direction of international capital flows instead of real wages) is associated with changes in the pattern of production.

\(^3\) A double dose of non-monotonicity would result if Foreign’s superiority in production of the second commodity held only for some range of returns to capital, with Home possessing a superiority in a different range (Jones and Ruffin, 1975). The possibility that there could be some value of the return to capital for which the two countries might have equal relative costs of production was stressed earlier by Kemp and Inada (1969) and Chipman (1971).
5. Technology Transfer in a Ricardian Setting

A recent contribution to the debate about winners and losers in the process of globalization was that of Paul Samuelson (2004). Basically he pointed out the possibility that when some countries (such as China) gained by increasing production of some commodities, other countries (those producing these same commodities) would lose because of a deterioration in their terms of trade. In Samuelson’s view, some pro-globalizers were pushing their case too far by neglecting this terms-of-trade effect. The Ricardian model, with each country requiring only labor and its own technology to produce any commodity, was selected by Samuelson to focus on this issue. In a pair of articles (Ruffin and Jones, 2006; Jones and Ruffin, 2006) the case was made that if a foreign less developed country actually stole a more developed country’s superior technology for the commodity in which that country had its greatest comparative advantage, a Ricardian model would show that the developed country might gain. This contrast with the Samuelson result points out the difference between a small foreign improvement in something the advanced country produces and an improvement sufficiently large that the developed country ceases its production. Whereas the advanced country would lose in the case of a small improvement, it might gain if its own industry is completely wiped out! This is not an argument that would be embraced by

---

Bhagwati, Panagariya, and Srinivasan (2004) comment on Samuelson’s article, downplaying the extent of such foreign takeovers. The effects of small terms-of-trade changes on real incomes in the two countries version of the Ricardian model was earlier spelled out in Jones (1979). As for large changes, both Kemp and Shimomura (1988) and Beladi, Jones and Marjit (1997) pointed out the gains a country could achieve even by giving away without compensation a superior technology it possesses for a commodity for which it relies entirely upon imports.
most in the media, but it does follow from a further application of the basic Ricardian model.

Here I skip over the details of the argument in order to set the stage for the fundamental intuition. Let Home denote the advanced country, whose technology in all commodities is assumed to be superior to that in Foreign, especially in the first commodity. To simplify, suppose the units in which commodities are measured are selected (arbitrarily) so that each commodity requires exactly one unit of Home labor to produce. Furthermore always select a commodity that Home produces as numeraire so that the nominal home wage rate is always unity. Home’s real wage is another matter, for that depends upon the commodity price level. In general there are two effects on this price level of a transfer (without compensation) of Home’s superior technology in the first commodity to Foreign: Foreign’s wage rate might increase, and with it the price paid by Home for all commodities produced only in Foreign and imported by Home. On the other hand, when Home’s first industry is completely wiped out, the price of this commodity faced by Home consumers falls to $w^*$, Foreign’s new wage rate, which is less than unity (Home’s nominal wage rate). This turns into a classic “on the one hand, and on the other hand” situation so typically found in the economics profession. It turns out that much depends upon the size of Foreign’s labor force compared with that at Home (assumed fixed). Not surprisingly, in general the greater is $L^*$ (Foreign’s labor force), the lower must be its wage rate, $w^*$, in order for Foreign to be competitive in a larger number of commodities. In Ricardian models with a finite number of commodities, there are stretches of constant $w^*$ corresponding to both countries producing a commodity in common.
More can be said if special demand conditions are specified. Let both countries have the same Cobb-Douglas preference orderings with the same (constant) shares of income spent on each commodity. This assumption was made both by Samuelson and in the Ruffin and Jones papers. In this case there are specific values for $L^*/L$ for which the technology transfer does not alter the Foreign wage rate at all. Obviously for such values the price index faced by Home consumers must fall, and Home’s real wage must increase as a consequence of unrequited technology transfer. In general, however, Foreign’s wage rate rises, and with it the prices of all commodities imported by Home. If Foreign and Home produce a commodity in common and Foreign’s technology gets a bit better, such that the production pattern stays the same (with Home’s output slightly reduced), Home’s real income must fall, as Samuelson (2004) suggested. However, with a bigger improvement in Foreign technology, such that the Home industry is completely wiped out, Home’s real income could improve.\footnote{In the Appendix to Jones and Ruffin (2006) a particular 4-commodity case is examined, leading to cyclical patterns in the price index as $L^*/L$ increases, always showing regions in which the price index falls.} Once again the difference between small and large shocks rests upon the possibility that the pattern of production is altered by the large shock.\footnote{An example in which lack of monotonicity results not from a change in the pattern of production but, instead, in an alternation in factor flows is given in Jones and Marjit (1995). There the wage rate of national skilled workers is tied to that of foreign skilled workers also employed in an enclave. However, once a sufficient number of national workers are trained in skills, all foreign workers depart and this link to foreign wage rates is cut. As a consequence, national skilled wages stop rising and start to fall.}

6. Concluding Remarks

Half a century ago Tjalling Koopmans (1957) made a plea for the use of more fundamental mathematics than calculus in economic theory:
“….calculus used as a scanning device for optimal positions is myopic: it permits comparison only with neighboring positions.” (p. 175)

The use of calculus to detect optimal positions is not the focus of the present paper. Instead, when examining how a market equilibrium is disturbed it is common practice in the theory of international trade to use the calculus in order to sign the derivative of some variable in which interest is focused (such as wage rates or real incomes) when there has been a small (infinitesimal) change to equilibrium (such as caused by a change in relative commodity prices). This procedure is extremely valuable, but it does not account for the possibility that the direction of change in key variables may be altered for sufficiently large shocks to the original equilibrium. That is, the response may not be monotonic. In this paper the signal that such a sign change may occur is an alteration in the pattern of production.

A key observation made in international trade is that such trade frees up a country’s production pattern from its consumption pattern, so that a country typically produces only a small range of commodities for international markets, and the compositional pattern of this range can easily be altered by finite shocks to the equilibrium. As a consequence it proves instructive to investigate when key theorems in the theory of international trade, established by focusing on small disturbances, may need to be qualified if the disturbance to equilibrium is sufficiently large as to change the pattern of production. Here I have considered four such cases and in each a result may be obtained that runs counter to the standard result appropriate for small changes. If a country is initially producing both a

---

7 Emphasis is on the difference between small and large changes. Some properties of infinitesimal changes, such as the second-order nature of substitution effects for factors on costs, are not at issue. International trade theory has always had primary focus on a situation in which the calculus would not be appropriate, viz., the movement from autarky to trade, in which no gains are registered unless there is a finite movement away from autarky.
labor-intensive commodity and a capital-intensive commodity a small increase in the relative price of the capital-intensive commodity must worsen the real wage rate, whereas a large enough price change could increase the real wage. In another scenario in which two countries, Home and Foreign, have different technologies, and capital is internationally mobile, a small increase in the relative price of the capital-intensive commodity might cause a capital flow to Foreign, but a larger price increase could cause capital to flow back to Home. The other pair of cases involves technical progress. In a Heckscher-Ohlin setting in which a country is initially producing two commodities, a small improvement in the technology for producing the capital-intensive commodity would lower the real wage rate, but a larger improvement that is labor-saving could improve the real wage. In a Ricardian setting with two countries, Home and Foreign, if Foreign gets a bit better at producing a commodity both countries produce in common, Home is made worse off. But if Foreign gets so much better that it wipes out the Home industry, Home might be made better off. Once again a change in the pattern of production is important in bringing about this different outcome, but many disturbances in a trading world are finite and large enough to force a change in the range of commodities produced by each country.
References:

Beladi, Hamid, Ronald W. Jones and Sugata Marjit: “Technology for Sale,”


Chipman, John S: “International Trade with Capital Mobility: A Substitution Theorem,”


Findlay, Ronald and Ronald W. Jones: “Factor Bias and Technological Progress,”

*Economics Letters*, v. 68, no. 3, September, 2000, pp. 303-308.


___________: “Technical Progress and Real Incomes in a Ricardian Trade Model,”


Figure 2
Figure 4
Figure 5