Does production fall in a small country when trade costs decrease? - Home market effect in the face of multinationals.

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Abstract
We show that for large trade costs, all firms are multinationals and countries’s production levels are proportional to their sizes. A fall in trade costs reduces the number of multinationals and increases the number of single-plant firms in a large country. Depending on trade costs, the number of single-plant firms in the small country may increase or not. Despite the existence of a home market effect according to which the large country hosts a disproportionate share of firms and of production, the fall in trade costs may also be beneficial to the small country that may produce more.

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1 Introduction

According to the home market effect, a large country hosts a more than proportionate share of firms from an industry that produces differentiated goods (see Krugman, 1980; Helpman and Krugman, 1985; Head, Mayer and Ries, 2002; Ottaviano and Thisse, 2004; Yu, 2005). This effect is magnified when trade costs are lower. To understand this result, consider first prohibitive trade costs in a world with two countries. Under reasonable assumptions, profits are proportional to earnings, which, in autarchy, are themselves proportional to the number of individuals living in each country. When firms can freely move across countries, they will locate in proportion to the populations of each country so that profits are equalized: there is no home market effect. Now consider a fall in trade costs. In each country, firms now suffer from their competitors located in the other country. In the small country, firms now face the competition (attenuated by trade costs) of the large number of firms located in the large country. Their profit is strongly affected. In the large country, firms now face the competition of the small number of firms located in the small country. Their profit is less affected. Therefore, some firms from the small country will re-locate to the large country where profits are higher: firms disproportionately locate in the large country and this disproportion increases with trade openness.\footnote{Note that the literature explains why the large country will host more firms than the small country but it does not provide an intuition for why the share of firms is smaller than the share of populations in the small country.}

Now consider ‘horizontal’ multinationals. When trade costs are prohibitive, firms duplicate their production in each country and serve each market from their local plant. For lower trade costs, the costs of exporting production from a single plant are lower than the cost of duplication. Therefore, firms concentrate their production into a single country (see e.g. Brainard, 1993, 1997; Markusen and Venables, 2000).

Is a small country likely to promote a fall in trade costs? According to the above analysis, this country would run the risk that multinationals close some of their plants in the small country and concentrate instead their production into the large country. Still, the fall in trade costs also allows firms from the small country to produce more because they can export more easily to the large country. Total production (and employment) in the small country may increase because firms produce more, but it may also decrease because fewer firms locate in that country. This is what we examine in this paper.

We consider a model with two countries of unequal sizes and with a sector producing a good under increasing returns to scale. We examine the structure of production of the firms that operate within this sector and we analyze where they locate their production. We adopt a footloose capital model according to which one unit of capital is required to run a firm and the returns of that capital are distributed among capital owners who are spread across both countries.
modeling creates a rich set of equilibria, which we can derive analytically:2 (i) for large trade costs, all firms are multinationals; (ii) for smaller trade costs, some firms are multinationals and the others are single-plant firms in the large country; (iii) for smaller trade costs, some firms are multinationals and the others are single-plant firms in the large or in the small country; (iv) for smaller trade costs, some firms are single-plant firms in the large or in the small country; (v) for smaller trade costs, all firms are single-plant firms in the large country.

As in Raybaudi-Massilia (2000), Ekholm and Forslid (2001), and Behrens and Picard (2005), our analysis confirms that the home market effect is attenuated by multinationals. For large trade costs all varieties are produced in both countries (all firms are multinationals) and the small country does not suffer in terms of varieties produced locally. Moreover, we show that the share of total production in one country is equal to its share in population. The home market effect vanishes. For lower trade costs, some firms concentrate their production in a single-plant and the home market effect re-appears: the share of single-plant firms and the share of production in the small country are always smaller than its share in population. A reduction of trade costs increases this effect because it affects competition in the small country more strongly than in the large country.

Concerning the levels of production (and employment) in each country, we show that a reduction in trade costs makes the large country more accessible to single-plant firms located in the small country, which induces them to produce more. It also increases competition from firms located in the large country, which reduces the market share (and the production) of each single-plant firm on its (small) domestic market. A reduction in trade costs also induces firms to change their organization and their location. In many equilibria, this change is detrimental to the small country because fewer varieties are produced in the small country. Therefore, total production and employment in the increasing returns to scale sector are likely to decrease in the small country.

Still, we show that when multinationals and single-plant firms in both countries coexist (equilibrium (iii)), total production (and total employment) can increase in the small country. Indeed, some multinationals concentrate their production in the small country, which raises production in that country. Other multinationals concentrate their production in the large country, which reduces production in the small country. However, the reduction in production is attenuated because with trade costs, the new single-plant firms in the large country sell

2In our framework, the number of varieties is exogenously determined by the amount of capital, which is fixed. The alternative would be to let the number of firms change with the level of trade costs. Unfortunately, this alternative assumption prevents equilibria with the coexistence of single-plant firms and multinationals; moreover a change in trade costs would provoke catastrophic changes in firms structure (see Brainard, 1987; Elberfeld, Götz and Stähler, 2005). Our assumption limits the analysis because the number of varieties is fixed, but it allows a deeper analysis because it allows smooth changes in firm’s structure following a change in trade costs.
less in the small country than they were used to sell when they were multinationals. As a consequence, we show that total production may increase in the small country. Improving freeness of trade may increase production and employment in a small country, even though the home market effect favors the large market.

We present the model in the next section. Then we assume that firms cannot split their production in two plants (no multinational), which allows us to present in a simple way the intuition behind the home market effect. Finally we solve the complete model with multinationals.

2 The model

The model builds on Dixit-Stiglitz (1977) with two countries. Country \( r \) is large and populated by \( L_r \) individuals whereas country \( s \) is smaller with only \( L_s < L_r \) individuals.

**Technology** There are two sectors: the traditional sector that produces a good under constant returns to scale and the modern sector with increasing returns to scale and product differentiation. Each firm active in the increasing returns to scale sector can concentrate its production in a single plant, or it can duplicate its production in two plants, one in each country. The firm faces a fixed cost \( f \) per plant. Whatever its number of plants, the firm must also use one unit of capital to develop its own variety. Each individual owns \( N / (L_r + L_s) \) units of capital, so that the number of varieties is equal to \( N \). The capital can be used indifferently in both countries, as in the footloose capital model introduced by Martin and Rogers (1995). Firms compete for the available units of capital, so that capital costs exhaust profits. We denote \( \Pi \) the worldwide capital revenues that are shared among the individuals.

There are no costs to trade the traditional good between both countries. We take this good as the numeraire. The modern good is costly to trade with a proportion \( \phi \in (0, 1) \) of the output shipped from one country arriving in the other country (the standard iceberg trade cost hypothesis). We normalize the marginal product of labor to one in both sectors. Moreover, we assume that the individuals are able to work in both sectors. As a consequence individuals earn the same (unit) wage in both sectors in both countries.

**Preferences** Preferences are represented by a Cobb-Douglas function with a share \( \mu \in (0, 1) \) of income spent on the modern goods and a share \( 1 - \mu \) spent on the traditional good. The modern good is a composite made of a continuum of differentiated varieties \( i \in [0, N] \) with a constant elasticity of substitution

\(^3\)See Picard, Thiss and Toulemonde (2004) for a general discussion on the role played by the location of capital owners on firms’ location.
between varieties, $\sigma$. In each country $k \in (r, s)$, the demand for each variety is iso-elastic:

$$C_k(i) = D_k [p'_k(i)]^{-\sigma} \quad \text{where} \quad D_k \equiv \mu Y_k (P_k)^{\sigma-1} \quad \text{and} \quad P_k \equiv \left[ \int_0^N p'_k(i)^{-\sigma} di \right]^{-1/(\sigma-1)}$$

(1)

The income available in country $k$ is denoted $Y_k$; it is equal to the labor income, $L_k$, and the country’s share of capital revenues, $\Pi * L_k / (L_r + L_s)$. The price index of the varieties sold in that country is $P_k$ and $p'_k(i)$ is the price paid by a household located in country $k$ for one unit of variety $i$. This price is equal to the mill price, $p_k(i)$, if variety $i$ is produced in country $k$ but it is larger than the mill price if variety $i$ is produced in the other country $l$: $p'_k(i) = p_l(i) / \phi$. For notational convenience we define $\Phi \equiv \phi^{\sigma-1}$, which Baldwin et al. (2003) call freeness of trade.

**Price setting** There are $N_r$ firms producing only in country $r$, $N_s$ firms producing only in country $s$ and $N_t$ multinational firms that have a plant in both countries, with $N_r + N_s + N_t = N$. On each market, the firm sets its price as a constant markup over its marginal costs because the demand is iso-elastic and because each firm considers the price indexes as given under the assumption of a continuum of firms. The marginal cost of production is equal to one for any type of firms. Therefore, the mill price is equal to $\sigma / (\sigma - 1)$ for all varieties and the price index in country $r$ can be written as

$$P_r = \frac{\sigma}{\sigma - 1} (N_r + N_t + \Phi N_s)^{1/(\sigma-1)}$$

(2)

In the following, we first examine the firms’ behavior when multinationals are forbidden. This allows us to isolate the home market effect precisely. Then we analyze the behavior of firms when they can split their production between the two countries.

**3 Firms location with single-plant firms only**

Consider a single-plant firm that is located in country $r$. Given the price setting, its profits gross of capital costs are $\pi_r(i) = q_r(i) / (\sigma - 1) - f$ where $q_r(i)$ is its total production, which is equal to the demand from $r$ and $s$. Consumers from $r$ buy $C_r(i)$ whereas consumers from $s$ must buy $C_s(i) / \phi$ units to consume $C_s(i)$ units. At an interior equilibrium (with some firms in each region), firms must make the same profits in both regions. It is readily checked that at the interior location equilibrium,

$$\pi_r(i) = \pi_s(i) \iff q_r(i) = q_s(i) \iff \frac{Y_r}{Y_s} \left( \frac{P_r}{P_s} \right)^{\sigma-1} = 1$$
We use (2) with $N_t = 0$, $N_s = N - N_r$, and the definition of nominal earnings, $Y_k = L_k + \Pi \ast L_k / (L_r + L_s)$, to find the interior location equilibrium:

$$\frac{N_r}{N} = \frac{L_r}{L_r + L_s} + \frac{\Phi}{1 - \Phi} \frac{L_r - L_s}{L_r + L_s}$$

(3)

This interior equilibrium holds only if $\Phi \leq L_s / L_r$. Otherwise, the proportion of firms in country $r$ would become larger than 1. For $\Phi > L_s / L_r$, the output of a firm located in $r$ is larger than its potential output in the small country $s$; in such a case, the profit is larger in country $r$ and all firms locate in the large country.

From the above expression, it is clear that the large country hosts a more than proportionate share of firms. This is the home market effect. Moreover, the easier it is to trade, the larger is this effect: the proportion of firms in the large market increases with $\Phi$.

To understand this result, note that the difference between the two countries is twofold: a difference in sizes ($Y_r / Y_s = L_r / L_s$ for any location of firms and for any trade costs) and a difference in competition if the mass of firms differs across countries. Consider prohibitive trade costs ($\Phi = 0$). In country $k$, each firm sells $\mu Y_k / N_k$; the numerator $\mu Y_k$ denotes the market size whereas the denominator $N_k$ denotes the competition. Obviously, the output per firm is equalized across countries if and only if the partition of firms between countries is equal to the partition of populations. In that case, firms located in the small country face a weak competition in a small market whereas firms located in the large country face a fierce competition in a large market.

Now, consider a decrease in trade costs (increase in $\Phi$). This does not change the market sizes but this affects the competition on each market. In the small market (say Luxembourg), competition increases significantly because firms located on this market now face the competition of the large number of firms located on the large market (say Germany). By contrast, competition increases less in the large market because there are only few firms in the small market that can now sell in the large market (there are few firms in Luxembourg compared to the number of firms in Germany). As a result, everything else equal, output per firm falls more in the small market than in the large market and the large market becomes more profitable. The opening of trade between a small and a large country (Luxembourg and Germany) increases substantially competition in the small market and weakly affects the large market. Therefore, some firms from the small market move to the large market, which decreases competition in the small market and increases it in the large market. Hence, the equalization of profits across countries is restored through a move of firms from the small to the large market which now hosts a more than proportionate mass of firms.
4 Firms location with any types of firms

We now extend the analysis and allow firms to become multinational (or to stay single-plant firms). The analysis requires the comparison of the profits made by the different firms:

\[ \pi_k = \frac{1}{\sigma} \left( \frac{\sigma - 1}{\sigma} \right) \sigma^{-1} \left( D_k + \Phi D_l \right) - f, \quad k, l \in \{r, s\} \text{ and } k \neq l, \]

\[ \pi_l = \frac{1}{\sigma} \left( \frac{\sigma - 1}{\sigma} \right) \sigma^{-1} \left( D_r + D_s \right) - 2f. \]

Sales of single-plant firms are lower than those of multinationals, but fixed costs of single-plant firms are also smaller. It is the trade-off between these two differences that determines firms’ organization and location. To compare the profits, we need to write them as functions of the partition of firms \((N_r, N_s, N_t)\). To this aim, we need to derive the values of \(D_k = \mu Y_k (P_k)^{\sigma - 1}\) We can use (2) to get rid of \(P_k\) and we use the definition \(Y_k = L_k + \Pi L_k / (L_r + L_s)\) and the value of gross profits \(\Pi\) to get rid of \(Y_k\). The gross profits \(\Pi\) are equal to \(\mu (Y_r + Y_s) / \sigma - (N + N_t) f\), i.e., it is a markup \((1/\sigma)\) times the sales (i.e., times the earnings spent on the modern good, \(\mu(Y_r + Y_s)\)) minus the fixed costs (\(N_t f\) denotes the extra fixed cost incurred by multinationals).\(^4\) Accordingly,

\[ D_k = \frac{\sigma}{\sigma - \mu} \left( \frac{\sigma - 1}{\sigma} \right) \sigma^{-1} \left( 1 - \frac{N + N_t f}{L_r + L_s} \right) \frac{\mu L_k}{N_k + \Phi N_l + N_t} \] where \(k, l \in \{r, s\}, k \neq l\) (4)

We will distinguish two cases in the analysis. One in which the two countries do not differ too much in size, and the other in which they substantially differ in size. The critical condition for separating the two cases is the following:

\[ \frac{L_r - L_s}{L_r + L_s} < \frac{\sigma - \mu}{\mu} \frac{N f}{L_r + L_s - N f} \] (5)

The comparison of profits is done in the Appendix. The results show that under (5), five types of equilibria may exist: (i) for large trade costs, all firms are multinationals; (ii) for smaller trade costs, some firms are multinationals and the others are single-plant firms in the large country; (iii) for smaller trade costs, some firms are multinationals and the others are single-plant firms in the large or in the small country; (iv) for smaller trade costs, some firms are single-plant firms in the large or in the small country; (v) for smaller trade costs, all firms are single-plant firms in the large country. If (5) does not hold, then equilibria (iii) and (iv) vanishes. Figure 1a summarizes the results when (5) holds whereas Figure 1b represents firms’ location otherwise. The critical values of freeness to trade \((\Phi_1, \ldots, \Phi_4)\) are given in the Appendix.

\(^4\)A sufficient condition for \(\Pi\) to be positive is \(\mu (L_r + L_s) > 2f N \sigma\) which is found by setting \(N_t = N\) and \(Y_r = L_r, Y_s = L_s\) in \(\Pi\).
We first examine firms’ organization and location. As expected (and formally shown in the Appendix), the number of multinationals decreases with trade openness, whereas the number of single-plant firms in the large country increases with openness. Interestingly, we distinguish two cases for the evolution of the number of single-plant firms in the small country. For small differences between the two countries, the number of single-plant firms in the small country increases with trade openness when openness is low whereas it decreases with trade openness when openness is large. When the difference between both countries is strong enough, the small country never hosts single-plant firms. The location equilibrium results from a trade-off between two forces. On the one hand, firms tend to switch from multinationals to single-plant firms when trade costs are reduced because it becomes less costly to export the production. On the other hand, a reduction in trade costs exacerbates the home market effect: as we have seen in previous section, single plant firms tend to agglomerate in the large market because the small market suffers more from the extra competition induced by the fall in trade costs. This second effect is stronger, the larger is the difference
between the sizes of the two countries\textsuperscript{5} and the smaller are trade costs. If the second effect is not too strong (i.e. the two countries do not differ too much in sizes), then a reduction in trade costs first induces some multinationals to become single-plant firms, some of them settle in the small countries, whereas a further reduction in trade costs make competition so strong in the small country that all single-plant firms move to the large country. If the two countries differ substantially in sizes, then multinational firms always settle in the large country when they switch to single-plant firms.

Let us now focus on the number of varieties produced in each country. For large trade costs, all firms are multinationals. Hence, both countries produce the same number of varieties. For lower trade costs, the number of multinationals decreases; the number of single-plant firms in the small country may increase, but one can check that it increases by a smaller amount.\textsuperscript{6} Thus, trade openness reduces the number of varieties produced in the small country. Moreover, when single-plant firms exist, they are over-represented in the large country: $N_r/N_s > L_r/L_s$, which is reminiscent of the home market effect (see the Appendix for a formal proof).

We now examine the effects of trade costs on the production per inhabitant in each country. In the Appendix, we show that when all firms are multinationals, the home market effect vanishes because the production per inhabitant is equalized across countries. By contrast, when single-plant firms exist, the production per person is larger in the large country (see also the Appendix). Thus, the large country hosts a more than proportionate share of the production when some firms have a single-plant. This is the home market effect.

So far, the results confirm that the larger market benefits from trade liberalization by hosting a share of economic activity that is at least equal to its share of population. However, the existence of multinational firms tend to attenuate this effect. Now, we examine to what extent a reduction in trade costs is detrimental to the economic activity in the small country. What happens with the total production of the good produced under increasing returns? A reduction in trade costs has several effects: (1) it makes the foreign country more accessible to single-plant firms, which induces them to produce more; (2) it increases competition from foreign firms which reduces the market share (and the production) of each single-plant firm on its domestic market; (3) it induces some firms to change their organization, and/or their location.

Under regime (i), it is straightforward to check that the output of a firm is independent of trade costs because none of the firms export. A change in trade costs does not affect firms’ organization nor production, so that total production is unchanged in each country.

\textsuperscript{5}When countries have approximately the same size, they face more or less the same competition.

\textsuperscript{6}The only regime under which $N_s + N_t$ could in principle increase is regime (iii) and it is readily checked with the values of $N_s$ and $N_t$ in the Appendix that $N_s + N_t$ decreases with $\Phi$. 

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Under regime (v), all firms are located in the large country and export to the small country. A decrease in trade costs is equivalent to a reduction in the price paid by consumers of the small country, which induces them to buy more of these goods: production increase in the large country whereas it remains nil in the small country.

Under regime (iv), the production of a firm from the large country is the same as the production of a firm from the small country. A reduction in trade costs make their production more attractive for foreign consumers (effect (1)) but it also increases the competition from foreign firms (effect (2)). Under the specific functional forms that we use, both effects cancel and the production of each firm is independent of trade costs (and equal to \( \mu (\sigma - 1) (L_r + L_s - fN) / [N (\sigma - \mu)] \)).

According to the third effect, some firms move from the small to the large country. Therefore, total production increases in the large country and decreases in the small country when trade costs are reduced. This is the standard result when multinationals are not considered.

Regimes (ii) and (iii) give expressions of production levels that are tedious to work with (but these expressions are still analytically tractable). Let us first examine regime (ii). The small country hosts only plants of multinationals. The reduction in trade costs does not make the large country more accessible for them because they do not export anyway (effect (1) is absent). By contrast, single plant-firms from the large country can now export more easily to the small country: the market share of the plants of multinationals in the small country is reduced (second effect). Summing up, these two effects contribute to the reduction of the production of each plant located in the small country. According to the third effect, some multinationals change their organization and become single-plant firms in the large country. On the one hand, in the small country, multinationals face less competition because firms that become single-plant firms in the large country now face trade costs to export to the small country. This effect is opposite to the two previous effects as it tends to increase the production of each plant in the small country. One can show that the net effect is positive: production of each plant in the small country increases with trade openness (it is equal to \( f (\sigma - 1) / (1 - \Phi) \)). On the other hand, there are fewer plants in the small country, which tends to reduce total production in that country. Accordingly, the net effect on total production is ambiguous. Using the value of \( N_t \) in regime (ii), it is possible to show that total production in the small country may increase or decrease. Simulations suggests however that it usually decreases: the small country suffers from liberalization in regime (ii).

Finally, let us examine regime (iii), which is the richer regime as the three types of firms coexist. In contrast to regime (ii), the first effect is present: trade liberalization makes the large country more accessible to single-plant firms in the small country. The second effect is similar to regime (ii): the small country is more accessible to single-plant firms from the large country. Accordingly, the net effect is ambiguous but it is more favorable to the small country than under regime (ii).
According to the third effect, some multinationals change their organization and become single-plant firms. Some become single-plant firms in the large country, which entails the same consequences as under regime (ii). However, in contrast to regime (ii), other firms concentrate their production in the small country, which raises the production in that country. Summing up, this regime is more favorable to the small country than regime (ii). Indeed, it can be shown that trade liberalization is likely to increase total production in the small country. Thus, even though the number of varieties produced in the small country decreases and its output relative to the output of the large country decreases, the absolute value of output may increase in the small country after an increase in trade openness. This is illustrated in the following Figure.

Figure 2: total production in the small country as a function of trade openness. ($\sigma = 5, \mu = .7, N = 1, f = .2, L_s = 2, L_r = 2.1$)

5 Conclusion

The paper develops a model in which firms must choose how many plants they operate and where they locate their plants. For large transport costs, all firms are multinationals and there is no home market effect. For lower transport costs, a home market effect appears as the large country hosts a disproportionate share of firms and share of production. We examine the effects of a fall in transport costs. We show that more firms locate in the large country and multinational firms concentrate their production in a single country. The home market effect is strengthened. Nevertheless, we show that total production in the small country may increase when each country hosts single-plant firms and multinationals. Thus, in that case, both countries increase their production, even though the larger country benefits more from trade liberalization. When trade costs are so low that all multinational firms concentrate their production, trade liberalization is beneficial to the large country only.
6 References


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7 Appendix

7.1 Location equilibria with any type of firms

Equilibria of type (i). We set $N_t = N, N_r = 0$ and $N_s = 0$ in (4), which we plug in $\pi_r, \pi_s$ and $\pi_t$. Then we check that $\pi_t > \pi_r$ and $\pi_t > \pi_s$ if and only if $\Phi < \Phi_1$.

Equilibria of type (ii). We set $N_s = 0$ in (4), which we plug in $\pi_r, \pi_s$ and $\pi_t$. To have coexistence of multinationals and single-plant firms in the large country, it must be that $\pi_t = \pi_r$, which requires

$$\frac{N_t}{N} = 1 - \frac{1}{\sigma(L_r + L_s) - \mu L_r} \left[ \frac{(\sigma - \mu)(L_r + L_s)}{(1 - \Phi) - \mu L_r(L_r + L_s - 2fN)} \right], \quad \frac{N_r}{N} = 1 - \frac{N_t}{N}$$

It is readily checked that $N_r > 0$ if and only if $\Phi > \Phi_1$ and $N_t > 0$ if and only if $\Phi < \Phi_2$. Also, to have $N_s = 0$ it must be that, at the equilibrium, $\pi_r > \pi_s$ which requires $\Phi < \Phi_2$. Finally, $\Phi_1 > \Phi_2$. Hence, under (5), equilibria of type (ii) exists if and only if $\Phi \in (\Phi_1, \Phi_2)$.

Equilibria of type (iii). It must be that $\pi_t = \pi_r = \pi_s$. These two conditions requires that

$$\frac{N_r}{N} = \frac{L_r}{L_r + L_s} + \frac{\Phi (L_r - L_s)}{1 - \Phi L_r + L_s} - \frac{L_s}{L_r + L_s} \frac{N_t}{N}, \quad \frac{N_s}{N} = \frac{L_s}{L_r + L_s} + \frac{\Phi L_s - L_r}{1 - \Phi L_r + L_s} - \frac{L_r}{L_r + L_s} \frac{N_t}{N}, \quad \frac{N_t}{N} = 1 - \frac{2(\sigma - \mu)}{\sigma(1 - \Phi)} + \frac{\mu(L_r + L_s - 2fN)}{\sigma N f}$$

It is readily checked that $N_s > 0$ if and only if $\Phi > \Phi_2$ and that $N_t > N_s$. Also, $N_t > 0$ if and only if $\Phi < \Phi_3$. Hence, all types of firms coexist if $\Phi_2 < \Phi < \Phi_3$. Note that the interval exists ($\Phi_3 > \Phi_2$) if and only if (5) is fulfilled.

Finally, it is readily checked that $N_r$ and $N_s$ increase with $\Phi$ whereas $N_t$ decreases with $\Phi$.

Equilibria of type (iv). We set $N_t = 0$. To have single-plant firms in both countries, it must be that $\pi_r = \pi_s$, which requires

$$\frac{N_r}{N} = \frac{L_r}{L_r + L_s} + \frac{\Phi (L_r - L_s)}{1 - \Phi L_r + L_s} - \frac{L_s}{L_r + L_s} \frac{N_t}{N}, \quad \frac{N_s}{N} = 1 - \frac{N_r}{N}$$

Note that the second inequality is always fulfilled under the first inequality.
$N_r$ is always positive whereas $N_s$ is positive if and only if $\Phi < \Phi_4$. Finally, the profit of a multinational must be smaller than that of a single-plant firm: $\Phi > \Phi_3$. Hence, single-plant firms coexist if $\Phi_3 < \Phi < \Phi_4$ where it can be checked that $\Phi_4 > \Phi_3$ if and only if (5) is fulfilled.

Note that is readily checked that $N_r$ increases with $\Phi$ whereas $N_s$ decreases with $\Phi$.

**Equilibria of type (v).** We set $N_r = N$, $N_s = 0$, and $N_t = 0$. We check that $\pi_r > \pi_s$ if and only if $\Phi > \Phi_4$ whereas $\pi_r > \pi_t$ if and only if $\Phi > \Phi'_2$. Moreover, $\Phi_4 > \Phi'_2$ if and only if (5) is fulfilled.

### 7.2 Critical values of $\Phi$

\[
\Phi_1 \equiv 1 - \frac{Nf (\sigma - \mu)}{\mu L_s} \frac{L_r + L_s}{L_r + L_s - 2Nf}
\]
\[
\Phi'_2 \equiv \left[1 + \frac{Nf (\sigma - \mu)}{\mu L_s} \frac{L_r + L_s}{L_r + L_s - Nf}\right]^{-1}
\]
\[
\Phi_2 \equiv 1 - \frac{Nf (\sigma - \mu)}{2 (\sigma - \mu) Nf} \frac{L_r + L_s}{L_r + L_s - 2Nf}
\]
\[
\Phi_3 \equiv 1 - \frac{2 (\sigma - \mu) Nf}{L_r \sigma (L_r + L_s - 2Nf) + Nf \mu}
\]
\[
\Phi_4 \equiv \frac{L_s}{L_r}
\]

### 7.3 Proof that $N_r/N_s > L_r/L_s$

Under regimes (ii) and (v), the result is trivial ($N_s = 0$ and $N_r > 0$). Under regime (iii), we can use the equilibrium values of $N_r$, $N_s$ and $N_t$ to show that $N_r L_s - N_s L_r = (L_r - L_s) [N_t (1 - \Phi) + N \Phi]/(1 - \Phi) > 0$. Under regime (iv), $N_s = N - N_r$, and we can use the value of $N_r$ to establish that $N_r L_s - N_s L_r = (L_r - L_s) N \Phi/(1 - \Phi) > 0$.

### 7.4 Proof that the production in the large country is disproportionately larger than in the small country

The production in the large country is equal to the production of single-plant firms in that country plus the production of the plants of multinational firms that are established in that country: \((\sigma - 1)/\sigma)^{t} (N_r (D_r + \Phi D_s) + N_t D_r)\). A similar result holds in the small country and the ratio of production in both...
countries can be written as
\[
N_r \Phi \frac{D_s}{D_r} + (N_r + N_t) \\
N_s \Phi + (N_s + N_t) \frac{D_s}{D_r}
\]

Moreover, using (4), we have that
\[
\frac{D_s}{D_r} = \frac{L_s N_r + \Phi N_s + N_t}{L_r N_s + \Phi N_r + N_t}
\]

Hence, the ratio of production in both countries is
\[
\frac{(N_r + N_t) (N_s + N_t + \Phi N_r) (L_r/L_s) + \Phi N_r (N_r + N_t + \Phi N_s)}{(N_s + N_t) (N_r + N_t + \Phi N_s) + \Phi N_s (N_s + N_t + \Phi N_r) (L_r/L_s)}
\]

This is larger than \(L_r/L_s\) if and only if
\[-(L_r/L_s)^2 \Phi N_s (N_s + N_t + \Phi N_r) + \Phi (N_r - N_s) N (L_r/L_s) + \Phi N_r (N_r + N_t + \Phi N_s) \geq 0\]

Using \(N_t = N - N_r - N_s\), this condition is equivalent to
\[(N_r - N_s (L_r/L_s)) (N - N_r + \Phi N_r) + N_r (1 - \Phi) (N_r - N_s) \geq 0\]

Since \(N_r - N_s (L_r/L_s) > 0\) under regimes (ii)-(v), this condition always holds. Under regime (i), this condition holds with equality: the ratio of production is equal to the ratio of population.