Outsourcing, 
Contracts 
and 
Growth*

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Abstract
We study the decision of firms between vertical integration and outsourcing in a dynamic setting with product innovation. As in static models, outsourcing brings specialization efficiencies by reducing marginal costs of production, while bearing extra costs arising from search and incomplete contracts. We show that static effects map into growth effects and investigate the impact of outsourcing on long-run economic performance.

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1 Introduction

A substantial growth in the outsourcing of activities in industrial countries is the most recent form of a greater division of labor (Feenstra, 1998). When corporations began selling their factories and relocating manufacturing in the 80s and 90s to boost efficiency and focus on specialization, most insisted that important R&D would remain in-house. Today, leading multinationals are turning toward a new approach to innovation, one that employs global networks of partners. The likes of Dell, Hewlett-Packard Co., Motorola, and Philips have started buying complete designs of some digital devices from Asian developers, modifying them to their own specifications, and using their own brand names. Dell, for example, does little of its own design for notebook PCs, digital TVs, or other products. Hewlett-Packard Co. contributes key technology and some design input to all its products but relies on outside partners to co-develop everything from servers to printers. Motorola buys complete designs for its cheapest phones but controls all of the development of high-end handsets. Asian contract manufacturers and independent design houses have become key players in nearly every technological device, from laptops and high-definition TVs to MP3 music players and digital cameras. While the electronics sector is the most significant example of search for offshore help with innovation, the concept is spreading to nearly every sector of the economy. Boeing is, for instance, working with India’s HCL Technologies to co-develop software for everything from the navigation systems and landing gear to the cockpit controls. Pharmaceuticals as GlaxoSmithKline and Eli Lilly are teaming up with Asian biotech research companies in a bid to cut the average $500 million cost of bringing a new drug to market. Procter & Gamble also wants half of its new product ideas to be generated from outside by 2010, compared with 20% now (Engardio and Einhorn, 2005).

The growing importance of outsourcing has generated an intense debate on the costs and benefits of industrial fragmentation. Within the international trade literature, a recent strand of research tries to investigate the phenomenon of outsourcing as the result of a trade-off between the governance costs of complex vertical organizations and the contractual costs of networks of independent specialized upstream and downstream producers. Such networks are tainted by problems of contractual incompleteness stemming from the lack of ex-post verifiability by third parties as the quality of deliverables is too costly to observe by courts. In Grossman and Helpman (2002) unobservable intermediate input quality is an issue in so far as only high quality inputs can be processed whereas low quality inputs are useless even though supplied at zero cost. Related models can be classified in terms of their relative focus on three decisions: the ‘own-
ership decision’ on whether production should be in-house or outsourced; the ‘location decision’ on where to place production; the ‘organization decision’ on how to structure the production process in different stages. The first decision is studied, for example, by McLaren (2000) and Grossman and Helpman (2002) for a closed economy and by Antras (2003), Grossman and Helpman (2003), Feenstra and Hanson (2003 and 2004) for an open economy. The second decision is analyzed by Grossman and Helpman (2005). The third decision by Antras and Helpman (2004).2

While all these studies focus on the static effects of outsourcing, we investigate instead its dynamic effects. The aim is to understand what drives the emergence and the performance of the new approach to innovation based on global networks of partners.

In so doing, we introduce innovation and growth in the static outsourcing model by Grossman and Helpman (2002). These authors study the industrial organization of a sector in which the varieties of a horizontally differentiated good are produced by monopolistically competitive firms. Production involves two stages, intermediate supply and final assembly. Firms choose whether to enter as intermediate suppliers, final assemblers or vertically integrated firms by paying the corresponding organization-specific fixed costs. Vertical integration bears additional costs due to more complex governance and limited specialization. Specialized suppliers face, instead, additional costs of searching and contracting with complementary partners. Contracts themselves are incomplete due to input characteristics that are unverifiable by third parties, which leads to bargaining between intermediate suppliers and final assemblers after the former have produced their inputs. The fear of being held up during the bargaining process causes the intermediate suppliers to underproduce and this reduces the joint surplus of specialized firms with respect to vertically integrated ones. Accordingly, the choices of firms in terms of organizational modes depends on the balance between the costs generated by, on the one hand, the complex governance and the limited specialization of vertical integration and, on the other hand, the search uncertainty and the incomplete contracts of specialized producers.

Grossman and Helpman (2002) show that the bargaining power of partners plays a key role in the fragmentation of production. Outsourcing is preferred when specialized final assemblers have a good chance of finding specialized intermediate supplier; when product differentiation is weak so that the profit share of revenues of vertically integrated firms is small relative to the share appropriated by final assemblers through bargaining; when vertical revenues are relatively small due to large gains from specialization and mild intermediate underproduction thanks to strong supplier bargaining power; when the entry costs for specialized assembly are relatively cheap compared with those for vertically integrated production. The matching probability of firms entering as specialized assemblers depends itself negatively on their relative R&D costs and positively on their relative profits margins with respect to intermediate suppliers. The

2See, e.g., Gattai, 2005, for a recent survey.
authors investigate two cases. When high relative costs and low relative profits make final entrants relatively scarce, they are sure to be matched. In this case the incentive to outsource is maximized for intermediate bargaining power. In the opposite case, when low relative costs and high relative profits make final entrants relatively abundant, their matching is uncertain and the impact of bargaining power on the propensity to outsource is unambiguously positive. The reason is that, by fostering intermediate entry and hampering final entry, stronger supplier bargaining power raises the matching probability of final assemblers.

To the static set-up of Grossman and Helpman (2002) we add dynamic product innovation. Whatever their organizational choice, to enter the market firms need blueprints for production. These are invented by perfectly competitive labs. They are protected by infinitely lived patents and come in three organization-specific types depending on whether they are designed for vertical integration, intermediate supply and final assembly. We show that the steady state of the dynamic model is isomorphic to the static equilibrium of Grossman and Helpman (2002) once their fixed entry costs are interpreted as the marginal costs of innovation. Our analysis, therefore, complements their work by providing microfounded transitionary dynamics.

Most parameters have the same impact independently from industrial organization. Weaker product differentiation reduces product development. The reason is thinner profit margins, which discourage innovation and force firms to employ more workers in order to cover the fixed R&D costs through larger scale of production. Also faster depreciation has a negative impact on product development as it reduces the incentive to innovate and diverts labor from alternative uses. Differently, stronger time preference has a negative impact on product variety since it biases intertemporal decisions towards consumption and away from saving. Finally, higher costs of innovation have a negative impact on product variety whereas a larger economy supports proportionately larger product variety. As to supplier bargaining power, if assembler entrants are relatively scarce and thus readily matched, stronger power is associated with poorer product variety. The reason is the following. On the one hand, the return to assembly falls and this discourages the creation of new assembler blueprints. On the other hand, the return to intermediates rises and this encourages both intermediate production and the creation of new intermediate blueprints. The matching probability of assemblers is nonetheless unaffected as they are always sure of being matched. Thus, larger intermediate suppliers and lower returns to assembly reduce product variety. If assembler entrants are relatively abundant and thus not necessarily matched, stronger bargaining power is associated with smaller expenditures and richer product variety. This stems from the fact that, in addition to the effects already discussed, as more suppliers are encouraged to enter, assembler matching probability rises, which fosters innovation.

By further introducing learning externalities in innovation, we are able to study how the organizational choices of firms affect the long run growth rate of the economy. In particular, we model learning as organization specific: cumulate experience in vertically integrated and specialized production reduces the
invention costs of vertically integrated and specialized blueprints respectively. The conditions that support outsourcing are the same as before and, again, under outsourcing there are two cases depending on the endogenous relative scarcity of final assemblers and intermediate suppliers. In both cases growth is fostered by weak time preference, slow depreciation, large size of the economy, small R&D cost, and pronounced product differentiation. The role of bargaining power differs, instead, in the two cases. When assembler entrants are surely matched, stronger supplier power is associated with slower growth. This is due to the fact that the return to assembly falls, thus discouraging the creation of new assembler blueprints. When there is uncertainty about assembler entrants finding partners, stronger supplier power increases assembler matching probability, which promotes growth. Crucially, innovation is higher and growth is faster when assemblers are sure of being matched, which highlights the importance of lively R&D in intermediate inputs.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 determines the equilibrium of the model when growth fades away in long run due to decreasing returns to innovation. Section 4 investigates the behavior of the model when endogenous growth is sustained in the long run by learning spillovers. Section 5 concludes.

2 A Model of Product Innovation

To study the organizational choice between vertical integration and outsourcing in a dynamic environment, we merge the organization model with incomplete contracts by Grossman and Helpman (2002) with the innovation model with horizontal product differentiation by Grossman and Helpman (1991).

2.1 Demand

There are $L$ infinitely-lived households who share the same preferences defined over the consumption of a horizontally differentiated good $C$. The utility function is assumed to be instantaneously Cobb-Douglas and intertemporally CES with unit elasticity of intertemporal substitution:

$$U = \int_{0}^{\infty} e^{-\rho t} \ln C(t) dt,$$

(1)

where $\rho > 0$ is the rate of time preference and

$$C(t) = \left[ \int_{0}^{n(t)} c(i, t)^{\alpha} di \right]^{1/\alpha}$$

is a CES quantity index in which $c(i, t)$ is the consumption of variety $i$, $n(t)$ is the mass (‘number’) of varieties produced and $\alpha$ is an inverse measure of the degree of product differentiation between varieties. In particular, if $\sigma$ is defined as the constant own- and cross-price elasticity of demand, then $\alpha = 1 - 1/\sigma$. 5
Households have perfect foresight and they can borrow and lend freely in a perfect capital market at instantaneous interest rate $R(t)$.

Given the chosen functional forms, multi-stage budgeting can be used to solve the utility maximization problem. This allows the households’ decisions to be modeled as a two-stage sequence. In the first stage, in each period they allocate their income flow between savings and expenditures. This yields a time path of total expenditures $E(t)$ that obeys the Euler equation of a standard Ramsey problem:

$$\frac{\dot{E}(t)}{E(t)} = R(t) - \rho, \quad (2)$$

where we have used the fact that the intertemporal elasticity of substitution equals unity. By definition, $E(t) = P(t)C(t)$ where $P(t)$ is the exact price index associated with the quantity good $C(t)$:

$$P(t) \equiv \left[ \int_0^{\mu(t)} p(i, t)^{\alpha/(1-\alpha)} di \right]^{(1-\alpha)/\alpha} \quad (3)$$

In the second stage, households allocate their expenditures across all varieties, which yields instantaneous demand functions for each variety:

$$c(i, t) = A(t)p(i, t)^{-1/(1-\alpha)}, \quad i \in [0, n(t)] \quad (4)$$

where $p(i, t)$ is the price of variety $i$ and

$$A(t) = \frac{E(t)}{P(t)^{-\alpha/(1-\alpha)}} \quad (5)$$

is aggregate demand. To simplify notation, from now on we leave the time dependance of variables implicit when this does not generate confusion.

2.2 Supply

The economy is endowed with two factors. Labor is inelastically supplied by households. Each household supplies one unit of labour and we call $L$ the number of households as well as the total endowment of labor. Labor is chosen as numeraire. The other factor is knowledge capital in the form blueprints for the production of differentiated varieties. These blueprints are protected by infinitely lived patents that depreciate at the constant rate $\delta$.

There are two sectors, production and innovation (R&D). Innovation is performed by perfectly competitive labs that invent different types of blueprints for vertically integrated processes and fragmented ones. Each of the former processes requires a blueprint with marginal cost of invention equal to $k_v$. Each of the latter require two blueprints: one for an intermediate component with marginal R&D cost equal to $k_m$ and one for final assembly with marginal R&D
cost equal to $k_s$ with $k_s + k_m \leq k_v$. We call $v$, $m$, and $s$ the numbers of the three types of blueprints available at time $t$.

As to production, varieties are supplied by monopolistically competitive firms that buy the corresponding patents from R&D labs and hire an amount of labor proportionate to output. Therefore, each firm produces under increasing returns to scale as the price of its patent generates a fixed cost and the wage bill a variable cost. Depending on the patent it has chosen to buy, a firm can enter in three alternative modes: as a vertically integrated firm, as an intermediate supplier or as a final assembler. The first needs $\lambda$ units of labour per unit of final output; the second needs $1 \leq \lambda$ units of labor per unit of intermediate component; the third needs one unit of intermediate component per unit of final output. Accordingly, fragmented production is cheaper in terms of both fixed and marginal costs. The reason is lower governance costs and productivity gains from specialization.

Fragmented production (henceforth, ‘outsourcing’) faces, however, additional costs that result from search frictions and incomplete contracting. After buying their patents, specialized entrants of each type must find a suitable partner in a matching process that may not always end in success. Moreover, intermediate suppliers also suffer hold-up problems due to contractual incompleteness. In particular, after matching each intermediate supplier produces a relation-specific input. This input has no value outside the relation and its quality is unverifiable by third parties (e.g. input quality is too costly to observe by courts). As in Grossman and Helpman (2002, 2003, 2005), unobservable input quality is an issue in so far as only high quality inputs can be processed whereas low quality inputs are useless even though supplied at zero cost. This implies that the final assembler can refuse payment after the intermediate has been produced and the parties have to bargain on the division of the joint surplus that will materialize after final assembly. This gives rise to a hold-up problem in so far as, the variety-specific input having no alternative use at the bargaining stage, its production cost is sunk. The transaction costs involved in ex-post bargaining may then cause both parties to underinvest in their contractual relation, thus reducing their joint surplus.

Specifically, define $\dot{s} = ds/dt$ and $\dot{m} = dm/dt$ the flows of new entrants as final assemblers and suppliers respectively, that is, the numbers of new assembler and supplier blueprints invented at time $t$. Let $f(\dot{s}, \dot{m}) = \min(\dot{s}, \dot{m})$ be a constant-return matching function that at time $t$ determines the number of new supplier-assembler matches given the number of entrants as final assemblers ($\dot{s}$) and the number entrants as intermediate suppliers ($\dot{m}$). Then, if we define $r \equiv m/s$, $\eta(r) \equiv f(\dot{s}, \dot{m}) / s$ is the matching probability of an assembler entrant while $\eta(r) / r$ is the matching probability of a supplier entrant. Accordingly, the relative abundance of the two types of entrants determines their probabilities of being matched. Two scenarios will arise. In the first, intermediate entrants are

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3This assumption prevents vertically integrated firms from buying inputs from specialized suppliers.
relatively scarce, so they are sure of being matched while assembler entrants are not. In the second, the roles are reversed. We assume that blueprints that go unmatched are instantaneously destroyed.

After a match is formed, each pair of firms bargain on the division of their joint surplus, given by the prospective revenues of the corresponding variety. Since both parties cannot find a replacement, there is bilateral monopoly so that they will eventually agree on a share that makes both better off than if they had not met. We denote the bargaining weight of the intermediate input producer by $\omega$.

To summarize, in each period $t$ the sequence of actions is the following. First, R&D takes place and firms choose their mode of entry by buying the corresponding patents. Second, prospective parties of an outsourcing agreement search for partners, which could end in a successful or an unsuccessful match. Unmatched entrants exit and their blueprints are destroyed. Third, each matched intermediate producer manufactures the input needed by its partner. Fourth, parties bargain over the division of total revenues from final sales and inputs are handed over to assemblers. Fifth, final assembly takes place and the final goods are sold to households together with those supplied by vertically integrated firms.

3 Organization and Product Variety

At time $t$ the instantaneous equilibrium is found by solving the model backwards from final production to R&D for given numbers of blueprints for vertically integrated firms $v$, intermediate suppliers $m$ and final assemblers $s$.

3.1 Production

Varieties can be sold to final customers by two types of firms: vertically integrated firms and final assemblers. A typical vertically integrated firm faces a demand curve derived from (4) and a marginal cost equal to $\lambda$. It chooses its scale by maximizing its operating profit

$$\pi_v = p_v y_v - \lambda x_v,$$

where $x_v$ is the amount of the intermediate input produced and $y_v = x_v$ is final output. Optimal final output and price are then given by:

$$x_v = y_v = A \left( \frac{\alpha}{\lambda} \right)^{\frac{1}{1-\alpha}},$$

and

$$p_v = \frac{\lambda}{\alpha}.$$  

Replacing these values in (6) results in operating profit equal to

$$\pi_v = (1 - \alpha) A \left( \frac{\alpha}{\lambda} \right)^{\frac{\alpha}{1-\alpha}}.$$
which is a decreasing function of the elasticity of substitution $1/(1 - \alpha)$ and of the marginal cost $\lambda$.

Turning to outsourcing modes, there is a one-to-one equilibrium relationship between the number of matched assemblers, the number of matched intermediate suppliers, and the number of outsourced varieties, which are all equal to $f$. The joint surplus of a matched pair of entrants is given by the revenues from the final sales of the corresponding variety $p_s y_s$. This is divided according to the bargaining weights of the parties. Accordingly, a share $(1 - \omega)$ goes to the final assembler:

$$\pi_s = (1 - \omega)p_s y_s.$$  \hspace{1cm} (10)

The remaining $\omega p_s y_s$ goes to the intermediate supplier.

Moving one step backwards, the intermediate producer must decide how much input $x_m$ to produce anticipating a share of revenues $\omega p_s y_s$ while bearing a cost of $x_m$ units of labor. Therefore, the intermediate supplier maximizes

$$\pi_m = \omega p_s y_s - x_m.$$  \hspace{1cm} (11)

which implies intermediate and final outputs equal to

$$x_m = y_s = A (\alpha \omega)^\frac{1}{1-\alpha}.$$  \hspace{1cm} (12)

with associated prices

$$p_m = \frac{1}{\omega}, p_s = \frac{1}{\alpha \omega}.$$  \hspace{1cm} (13)

Note that $p_v/p_s = \lambda \omega$, which is the ratio of the efficiency loss of vertical integration to that of outsourcing. The former stems from complex governance and lack of specialization, the latter from intermediate underproduction due to hold-up fears. Using these results in (10) and (11) gives the operating profits of matched final assemblers and intermediate suppliers:

$$\pi_s = (1 - \omega) A (\alpha \omega)^\frac{\omega}{1-\alpha}$$  \hspace{1cm} (14)

and

$$\pi_m = (1 - \alpha) \omega A (\alpha \omega)^\frac{\omega}{1-\alpha}$$  \hspace{1cm} (15)

Finally, when both vertically integrated firms and final assemblers are active, substituting (8) and (13) into (3) and (5) allow us to write aggregate demand as

$$A = \frac{E}{v (\frac{\omega}{\lambda})^\frac{\omega}{\alpha} + f (\alpha \omega)^\frac{\omega}{1-\alpha}}.$$  \hspace{1cm} (16)

where $v$ is the number of vertically integrated firms and $f$ is the number of matched pairs of specialized producers that are active at time $t$. 

9
3.2 Innovation

Going backwards, we reach now the entry stage. Here labs invent new blueprints at marginal costs $k_v$, $k_m$ and $k_s$ depending on the organizational modes. Their output determines the laws of motion of $v$ and $f$. For vertically integrated firms, we have

$$\dot{v} = \frac{L^I_v}{k_v} - \delta v$$

(17)

where $\dot{v} \equiv dv/dt$, $L^I_v$ is labor employed in inventing new blueprints for vertically integrated production, $1/k_v$ is its productivity, and $\delta$ is the depreciation rate.

For specialized pairs we have

$$\dot{f} = \eta(r) s - \delta f$$

with

$$r \equiv \frac{m}{s} = \frac{L^I_f}{k_f}, \quad m = \frac{L^I_m}{k_m}$$

(18)

where $\dot{f} \equiv df/dt$, $L^I_f$ and $L^I_m$ are labor employed in inventing new final assembler and intermediate supplier blueprints, while $1/k_f$ and $1/k_m$ are their respective productivities.

Labs pay their researchers by borrowing at the interest rate $R$ while knowing that the resulting patents will generate instantaneous dividends equal to the expected profits of the corresponding firms. Since specialized entrants are not sure of being matched, (14) and (15) imply that the expected dividends of intermediate and final assembly patents are respectively:

$$\pi^e_m = (1 - \alpha) \frac{r}{\eta(r)} \omega A (\alpha \omega)^{\frac{m}{\omega}}$$

(19)

and

$$\pi^e_s = \eta(r) (1 - \omega) A (\alpha \omega)^{\frac{m}{\omega}}.$$  

(20)

Then, if we call $J_j$ the asset value of a patent with for $j = v, s, m$, arbitrage in the capital market implies

$$R = \frac{\pi^e_j}{J_j} + \frac{\dot{J}_j}{J_j} - \delta.$$

(21)

where $\dot{J}_j \equiv dJ_j/dt$ is the capital gain.

Due to perfect competition in R&D, patents are priced at marginal cost, which requires:

$$J_j = k_j$$

(22)

The value of a patent is, therefore, constant through time so that $\dot{J}_j = 0$. When substituted into (21), these results give

$$R + \delta = \frac{\pi^e_v}{k_v} = \frac{\pi^e_s}{k_s} = \frac{\pi^e_m}{k_m}.$$  

(23)
which pins down the interest rate in the Euler equation (2).

Finally, the aggregate resource constraint (or full employment condition) closes the characterization of the instantaneous equilibrium. Since labour is used in innovation and in intermediate production by both vertically integrated and specialized producers, we have \( L = L^I_v + L^I_s + L^I_m + v\lambda x_v + f x_m \). By (7), (12), (17) and (18), the condition can be rewritten as

\[
L = k_v \left( \dot{v} + \delta v \right) + k_s \dot{s} + k_m \dot{m} + v \lambda A \left( \frac{\alpha}{\lambda} \right) + f A (\alpha \omega) \]

(24)

3.3 Equilibrium

The first thing to notice is that in any instant \( t \) there is never simultaneous invention of both vertically integrated and specialized blueprints. This would be the case if all equalities in (23) held at the same time. This is generally impossible. To see this, consider first that new outsourcing agreements are signed only if there is new creation of both intermediate supplier and final assembler blueprints, which requires

\[
\pi_v e_m k_m = \pi_s e_s k_s \Leftrightarrow r = \frac{k_s (1 - \alpha) \omega}{k_m (1 - \omega)}
\]

(25)

where we have used (19) and (20). Thus, the two types of specialized blueprints have to be invented in fixed proportion. Second, if also vertically integrated patents are simultaneously invented, it must be

\[
\frac{\pi_v}{k_v} = \frac{\pi_s}{k_s} \Leftrightarrow \frac{(1 - \alpha) \lambda^{-\frac{\alpha}{\lambda - \omega}}}{k_v} = \frac{\eta(\tau) (1 - \omega) \omega^{\frac{\alpha}{\lambda - \omega}}}{k_s}
\]

(26)

Since both sides are constant, this equality is satisfied only for a zero-measure set of parameter values. Hence, in general, vertically integrated and specialized blueprints are not invented together in equilibrium. In particular, only the former are created when

\[
\frac{(1 - \alpha) \lambda^{-\frac{\alpha}{\lambda - \omega}}}{k_v} > \frac{\eta(\tau) (1 - \omega) \omega^{\frac{\alpha}{\lambda - \omega}}}{k_s}
\]

(27)

and only the latter when the reverse is true.

3.3.1 Vertical Integration

When condition (27) holds, \( L^I_s = L^I_m = 0 \), so no specialized blueprints are ever created: \( \dot{s} = \dot{m} = 0 \) and \( f = 0 \). Then, using (6), (16) and (23), the full employment condition (24) and the Euler condition (2) can be respectively rewritten as:

\[
L = k_v \left( \dot{v} + \delta v \right) + \alpha E
\]

(28)
and

\[
\frac{\dot{E}}{E} = (1 - \alpha) \frac{E}{k_v v} - \rho - \delta \tag{29}
\]

Equations (28) and (29) form a two-dimensional dynamic system that has a unique steady-state in \(E\) and \(v\) and it is saddle-path stable (see Appendix). The steady state values can be obtained by setting \(\dot{v} = \dot{E} = 0\) in (28) as well as (29) and by solving the resulting system:

\[
E^* = \frac{\rho + \delta}{\delta + \alpha \rho} L, \quad \nu^* = \frac{1 - \omega}{\delta + \alpha \rho} k_v L.
\tag{30}
\]

Due to depreciation, the steady state mass of firms is maintained through ongoing innovation. When all firms are destroyed instantaneously (\(\delta = 1\)), these results are the same as in the dynamic model by Grossman and Helpman (1991). In addition, when there is no time discounting (\(\rho = 0\)), they are the same as in the static model by Grossman and Helpman (2002).

### 3.3.2 Outsourcing

When condition (27) does not hold, \(L^I = 0\), so no vertically integrated blueprints are ever created: \(\dot{v} = 0\) and \(v = 0\). Then, using (6), (16), (18), (23) and (25), the full employment condition (24) and the Euler condition (2) can be respectively rewritten as:

\[
L = \frac{k_s + k_m}{\eta(r)} \left( f + \delta f \right) + \omega E \tag{31}
\]

and

\[
\frac{\dot{E}}{E} = \frac{\eta(r) (1 - \omega) E}{k_s f} - \rho - \delta \tag{32}
\]

This dynamic system has a unique steady-state and it is saddle-path stable (see Appendix). The associated level of expenditures and number of firms can be obtained by setting \(\dot{f} = \dot{E} = 0\) in (31) as well as (32) and by solving the resulting system:

\[
E^* = \frac{\rho + \delta}{\delta + \omega \alpha \rho} L, \quad f^* = \frac{1 - \omega}{\delta + \omega \alpha \rho} \frac{L}{k_s}.
\tag{33}
\]

which depend on the matching probability of assembler entrants \(\eta(r)\). Hence, there are two cases. If there are fewer assembler than intermediate entrants
(\(\tau > 1\)), then the former are surely matched, so \(\eta(\tau) = 1\). Accordingly, (33) becomes:

\[ E^*_s = \frac{\rho + \delta}{\delta + \omega \alpha \rho} L, \quad f^*_s = \frac{1 - \omega}{\delta + \omega \alpha \rho} \frac{L}{k_s} \]  

(34)

If there are more assembler than intermediate entrants (\(\tau < 1\)) then the latter are surely matched, so \(\eta(\tau) / \tau = 1\). This allows us to write (33) as:

\[ E^*_m = \frac{\rho + \delta}{\delta + \omega \alpha \rho} L, \quad f^*_m = \frac{(1 - \alpha) \omega}{\delta + \omega \alpha \rho} \frac{L}{k_m} \]  

(35)

Since \(E^*_m = E^*_s\) and \(f^*_m = \tau f^*_s\), \(\tau < 1\) implies that expenditures are the same in the two case but the number of firms are higher when assemblers are sure of being matched. Again, due to depreciation, the steady state masses of firms are maintained through ongoing innovation. Note that the static equilibrium in Grossman Helpman (2002) corresponds to the steady state of our model when \(\rho = 0\) (no time discounting) and \(\delta = 1\) (all firms die every period).

3.4 Comparative Statics

Economic intuition on the driving forces behind the choice of the organizational mode is boosted by remembering that \(\alpha = (1 - 1/\sigma)\) and rewriting (27) as

\[
\frac{1}{\eta(\tau)} < \frac{1/\sigma}{1 - \omega} \left( \frac{\lambda}{1/\omega} \right)^{1-\sigma} > \frac{k_v}{k_s} \tag{36}
\]

with

\[ \eta(\tau) = \min(1, \frac{k_s}{k_m}, \frac{1/\sigma}{1/\omega - 1}) \]  

(37)

If condition (36) holds, all entrants choose vertical integration. Of course, this is the case when the relative benefits of vertical integration dominate the relative costs. The left hand side of (36) shows that the former come from three sources: the fact that specialized final assemblers face matching uncertainty whereas vertically integrated firms do not (1/\(\eta(\tau)\)); the relative profit margin ((1/\(\sigma\)) / (1 - \(\omega\))) and the relative total revenues ((\(\lambda \omega\))\(^{1-\sigma}\)). The right hand side shows, instead, that the relative costs of vertical integration derive from the costs of innovation (\(k_v/k_s\)). Then, vertical integration is chosen when specialized final assemblers have low chances of finding specialized intermediate suppliers (small \(\eta(\tau)\)); when product differentiation is strong so that the profit share of revenues of vertically integrated firms (large 1/\(\sigma\)) is large relative to the share appropriated by final assemblers through bargaining (small 1 - \(\omega\)); when vertically integrated revenues are relatively large due to small gains from specialization (small \(\lambda\)) and severe
intermediate underproduction caused by weak supplier bargaining power (small \(\omega\)); when the blueprints for vertically integrated production are relatively cheap compared with those for specialized assembly (small \(k_v/k_s\)).

Equation (37) reveals that the matching probability of specialized assemblers depends itself on the relative R&D costs \(k_s/k_m\) and profits margins \((1/\sigma) / (1/\omega - 1)\) of final assemblers and intermediate suppliers. When their R&D costs are relatively large and margins relatively small, the minority of entrants are final assemblers, so they are surely matched \((\eta(\tau) = 1)\). In this case, their matching probability is unaffected by marginal parameter changes. All of them have unambiguous impacts on the propensity to vertical integration except \(\omega\). The reason is that stronger supplier bargaining power has two opposite effects: it promotes intermediate production but at the same time hampers final production. While the first effect fosters outsourcing, the second hampers it. Higher demand elasticity (large \(\sigma\)) fosters the second effect because demand reacts a lot to small price differences, so high intermediate prices due to large \(\omega\) map into small final quantities sold. The best scenario for outsourcing strikes the optimal balance between those two effects. This happens at \(\omega = \alpha = (1 - 1/\sigma)\) where the left hand side of (36) is minimized.

When their R&D costs are relatively small and margins relatively large, final assemblers are the majority of entrants, which reduces their chances of being matched \((\eta(\tau) < 1)\). This makes condition (36) more easily fulfilled, its exact expression becoming \(\omega^\sigma \lambda^{\sigma - 1} < k_m/k_v\). Now also the impact of \(\omega\) on the propensity to vertical integration is unambiguously negative. The reason is that, by fostering intermediate entry and hampering final entry, stronger supplier bargaining power (larger \(\omega\)) raises the matching probability of final assemblers.

Turning to steady state outcomes, most parameters have the same impact independently from industrial organization. Larger elasticity of substitution (larger \(\sigma\)) reduces both expenditures and variety. The reason is thinner profit margins, which discourage innovation and force firms to employ more workers in order to cover the fixed R&D costs through larger scale of production. Also faster depreciation (larger \(\delta\)) has a negative impact on both expenditures and variety as it reduces the incentive to innovate and diverts labor from alternative uses. Differently, stronger time preference (larger \(\rho\)) has a negative impact on product variety but a positive one on expenditures since it biases intertemporal decisions towards consumption and away from saving. Finally, higher costs of innovation (larger \(k_v, k_s,\) or \(k_m\)) have no impact on expenditures and a negative impact on product variety whereas a larger economy (larger \(L\)) supports proportionately larger expenditures and product variety.

A parameter is peculiar to outsourcing: the bargaining weight \(\omega\) that determines the revenue share of intermediate suppliers. If \(\tau > 1\) so that all assembler entrants are matched \((\eta(\tau) = 1)\), larger \(\omega\) is associated with lower expenditures and poorer product variety. The reason is the following. On the one hand, the return to assembly falls and this discourages the creation of new assembler blueprints. On the other hand, the return to intermediates rises and this encourages both intermediate production and the creation of new intermediate
blueprints. The matching probability of assemblers is nonetheless unaffected as they are always sure of being matched. Thus, larger intermediate suppliers and lower returns to assembly reduce both expenditures and product variety. If $\pi < 1$ so that assembler entrants are not sure of being matched ($\eta(\pi) = \pi < 1$), larger $\omega$ is associated with smaller expenditures and richer product variety. This stems from the fact that, in addition to the effects already discussed, as more suppliers are encouraged to enter, now assembler matching probability rises, which fosters innovation.

4 Organization and Growth

We assume now that R&D faces a learning curve: the larger the number of a certain type of blueprints that have been successfully introduced in the past, the more productive researchers are in inventing that type of blueprints. What matters is not only the number of invented patents but, for specialized blueprints, also the number of those that have been actually matched and used in production. In particular, as in Grossman and Helpman (1991), we consider a linear learning curve such that the marginal costs of innovation become $k_v/v$, $k_m/f$, and $k_s/f$ depending on the types of blueprints. Given this functional form, some initial stocks of implemented blueprints is needed to have finite costs of innovation at all times. We call them $v_0 > 0$ and $f_0 > 0$ for vertically integrated and specialized blueprints respectively.

Learning implies that the values of blueprints are not constant anymore. As innovation cumulates, it becomes increasingly cheaper to create new patents. Being priced at marginal cost, their values fall through time. Specifically, we have $J_v = k_v/v$, $J_m = k_m/f$ and $J_s = k_s/f$, which imply

$$\frac{\dot{J}_v}{J_v} = -\frac{\dot{v}}{v}, \quad \frac{\dot{J}_m}{J_m} = -\frac{\dot{f}}{f}$$

Accordingly, the arbitrage condition in the capital market (21) becomes

$$R = \pi_v \frac{\dot{v}}{J_v} - \frac{\dot{v}}{v} - \delta$$

$$R = \pi_j \frac{\dot{f}}{J_j} - \frac{\dot{f}}{f} - \delta, \quad j = m, s$$

As shown by Grossman and Helpman (1991), with only vertically integrated firms the model as no transitionary dynamics and jumps instantaneously to its balanced growth path. Simple inspection reveals that, by analogy, the same property applies when only specialized firms or all types of firms are simultaneously active. Along the balanced growth path all variables either grow at the same rate or do not grow at all. Therefore, for both vertical and specialized
blueprints to be generated at the same time, \( \frac{f_f}{v} = \frac{v_f}{v} = g \) must hold. Under this constraint, at all times \( v = v_0 e^{gt} \) and \( f_0 e^{gt} \). Then, substituting \( J_v = k_v/v \) and \( J_j = k_j/f \) for \( j = m, s \) into (39) and (40) leads to

\[
\frac{\pi_v v_0 e^{gt}}{k_v} = \frac{\pi_j f_0 e^{gt}}{k_j}, \quad j = m, s
\]

As before, new outsourcing agreements are signed only if there is new creation of both intermediate supplier and final assembler blueprints, which requires (25) to hold: the two types of specialized blueprints have to be invented in fixed proportion. Moreover, also vertically integrated blueprints are simultaneously invented if

\[
\frac{\pi_v v_0 e^{gt}}{k_v} = \frac{\pi_s f_0 e^{gt}}{k_s} \iff \frac{(1 - \alpha) \lambda^\frac{\pi v}{\pi s} v_0}{k_v} = \frac{\eta(\tau)(1 - \omega) \omega^\frac{\pi v}{\pi s} f_0}{k_s}
\]

Both sides being constant, this equality is satisfied only for a zero-measure set of parameter values. Therefore, in general, vertically integrated and specialized blueprints are not invented together in equilibrium. In particular, only the former are created when

\[
\frac{(1 - \alpha) \lambda^\frac{\pi v}{\pi s} v_0}{k_v} > \frac{\eta(\tau)(1 - \omega) \omega^\frac{\pi v}{\pi s} f_0}{k_s}
\]

and only the latter when the reverse is true. With respect to (27) the novelty lies in the role of initial relative experience in innovation. Higher initial experience in vertically integrated \((v_0)\) or in specialized processes \((f_0)\) make new blueprints of the same types cheaper to invent. Clearly, (42) boils down to (27) when the initial stocks of successfully implemented blueprints are the same for both types, which we assume from now on. In this case, as discussed in Section (3.4), vertical integration is selected when specialized final assemblers have low chances of finding specialized intermediate suppliers \((\text{small } \eta(\tau))\); when product differentiation is strong so that the profit share of revenues of vertically integrated firms \((\text{large } 1/\sigma)\) is large relative to the share appropriated by final assemblers through bargaining \((\text{small } 1 - \omega)\); when vertical revenues are relatively large due to small gains from specialization \((\text{small } \lambda)\) and severe intermediate underproduction caused by weak supplier bargaining power \((\text{small } \omega)\); when the blueprints for vertically integrated production are relatively cheap compared with those for specialized assembly \((\text{small } k_v/k_s)\).

### 4.1 Vertical Integration

When condition (27) holds, no labour is allocated to specialized innovation \((L^I_s = L^I_m = 0)\), so no new specialized patent is ever created: \( \dot{s} = \dot{m} = 0 \) and asymptotically \( f = 0 \). Along a balance growth path \( v/v = g \) and \( \dot{E} = 0 \). This allows us to write the full employment condition (24) and the Euler condition
(2) as:

\[ L = k_v (g_v + \delta) + \alpha E \]

and

\[ \frac{\dot{E}}{E} = \frac{(1 - \alpha)}{k_v} - g_v - \rho - \delta \]

These can be solved to yield the equilibrium values of expenditures and growth:

\[ E_v^G = L + \rho k_v, \quad g_v^G = \frac{(1 - \alpha)}{k_v} - \alpha \rho - \delta. \]  

(43)

Setting \( \delta = 0 \) gives the same result that Grossman and Helpman (1991) derive for their vertically integrated firms. Under vertical integration growth is boosted by weak time preference (small \( \rho \)), slow depreciation (small \( \delta \)), large size of the economy (large \( L \)), small R&D cost (small \( k_v \)), and pronounced product differentiation (small \( \alpha \)). While a large size of the economy also gives large expenditures, weak time preference and small R&D cost depress them.

4.2 Outsourcing

When condition (27) does not hold, no labour is allocated to vertical innovation \( (L_v^I = 0) \), so no vertically integrated blueprints are ever created: \( \dot{v} = 0 \) and asymptotically \( v = 0 \). Along a balance growth path \( \dot{f} = g_f \) and \( \dot{E} = 0 \). This allows us to write the long run full employment condition (24) and the Euler condition (2) as:

\[ L = \frac{k_s + k_m \ov{\eta}}{\eta \ov{\tau}} (g_f + \delta) + \alpha \omega E \]

and

\[ 0 = \frac{\eta \ov{\tau} (1 - \omega)}{k_s} E - g_f - \rho - \delta \]

Given the definition of \( \ov{\tau} \) in (25), these can be solved together to yield

\[ E = L + \rho \frac{k_s}{\eta \ov{\tau}} \frac{1 - \omega \alpha}{1 - \omega} \frac{L}{k_s} - \rho \omega \alpha - \delta \]

which depend on the matching probability of assembler entrants \( \eta(\ov{\tau}) \). Hence, there are two cases. If there are fewer assembler than intermediate entrants \( (\ov{\tau} > 1) \), then the former are surely matched, so \( \eta(\ov{\tau}) = 1 \). Accordingly, (44) becomes:

\[ E_v^G = L + \rho k_s \frac{1 - \omega \alpha}{1 - \omega}, \quad g_v^G = \frac{(1 - \omega)}{k_s} - \rho \omega \alpha - \delta \]

(45)
If there are more assembler than intermediate entrants ($\tau < 1$), then the latter are surely matched, so $\eta(\tau) / \tau = 1$. This allows us to write (44) as:

$$E^G_m = L + \rho k_m \frac{1 - \omega}{(1 - \alpha) \omega}, \quad g^G_m = (1 - \alpha) \omega \frac{L}{K_m} - \rho \omega \alpha - \delta$$  \hspace{1cm} (46)

Since $\tau < 1$ yields $\eta(\tau) = \tau < 1$, expressions (44) imply that $E^G_s < E^G_m$ and $g^G_s > g^G_m$: expenditures are lower and growth is higher when assemblers are sure of being matched. As under vertical integration, in both cases growth is fostered by weak time preference (small $\rho$), slow depreciation (small $\delta$), large size of the economy (large $L$), small R&D cost (small $k_s$ or $k_m$), and pronounced product differentiation (small $\alpha$). A large size of the economy also supports large expenditures whereas weak time preference as well as small R&D cost depress them.

However, with respect to vertical integration, there are two main differences. The first is the role of the bargaining weight $\omega$. When assembler entrants are surely matched ($\tau > 1$), larger $\omega$ is associated with larger expenditures and slower growth. This is due to the fact that the return to assembly falls, thus discouraging the creation of new assembler blueprints. When there is uncertainty about assembler entrants finding partners ($\tau < 1$), larger $\omega$ increases their matching probability, which reduces expenditures and promotes growth ($dg_m/d\omega > 0$ provided that $g_m > 0$). The second difference is the impact of product differentiation on expenditures. The reason is that the annuity value of the initial stock of blueprints depends positively on the dividends to assembler patents and negatively on the matching probability of new assembler entrants. When matching is certain ($\tau > 1$), little differentiation (large $\alpha$) depresses dividends and thus expenditures. When matching is uncertain ($\tau < 1$), little differentiation depresses the matching probability more than the dividends, which sustains expenditures.

5 Conclusion

We have proposed a theoretical framework to investigate the origin and the performance of the new approach to innovation that relies on increasingly global networks of partners.

The underlying idea is that outsourcing brings about gains in terms of both lower costs of governance and higher benefits of specialization in upstream and downstream production. It is, however, associated with larger transaction costs due to incomplete contracts and hold-up problems.

We have shown that the bargaining power of upstream and downstream parties at the production stage feeds back to R&D incentives, thus affecting innovation and growth. The reason is that specialized production also require specialized innovation whose returns depend on the bargaining outcomes. The framework shows that, in the presence of search friction and incomplete contracts, growth through innovation networks is maximized when upstream R&D
is more dynamic than downstream R&D in terms of both blueprint creation and destruction.

References


Appendix - Stability of Steady States

We show that in Section 3.3 the steady states with vertical integration and outsourcing are both saddle-path stable. Consider the former case. The corresponding dynamic system is formed by (28) and (29). The Jacobian matrix evaluated at (30) is:

\[ D_v = \begin{bmatrix} \delta + \rho & -\frac{(\delta + \rho)^2 k_v}{(1-\alpha)} \\ \frac{\alpha}{v} & -\delta \end{bmatrix} \]

with eigenvalues

\[
\lambda_v^1 = \rho - \frac{\sqrt{(1-\alpha)[(\rho + 2\delta)^2 + \alpha \rho (3\rho + 4\delta)]}}{(1-\alpha)} < 0
\]

\[
\lambda_v^2 = \rho + \frac{\sqrt{(1-\alpha)[(\rho + 2\delta)^2 + \alpha \rho (3\rho + 4\delta)]}}{(1-\alpha)} > 0
\]

As the first eigenvalue is negative while the second is positive, the dynamic system is saddle-path stable.

Consider now the outsourcing case. The associated dynamic system is formed by (31) and (32). There are two Jacobian matrices depending on whether intermediate suppliers or final assemblers are surely matched. Evaluated at the corresponding steady states (34) and (35), the matrices are given by:

\[ D_s \equiv \begin{bmatrix} \frac{\delta + \rho}{\alpha (1-\omega)} & -\frac{(\delta + \rho)^2 k_s}{(1-\omega)} \\ -\frac{\alpha}{\omega (1-\omega)} & -\delta \end{bmatrix} \text{ for } \tau > 1 \]

\[ D_m \equiv \begin{bmatrix} \frac{\delta + \rho}{\alpha \omega (1-\omega)} & -\frac{(\delta + \rho)^2 k_m}{\omega (1-\omega)} \\ -\frac{\alpha \omega}{k_m (1-\omega)} & -\delta \end{bmatrix} \text{ for } \tau < 1 \]

which both have eigenvalues

\[
\lambda_s^1 = \rho - \frac{\sqrt{(1-\alpha \omega)[(1-\alpha \omega)(\rho + 2\delta)^2 - 4(\rho + \delta)^2]}}{(1-\alpha \omega)} < 0
\]

\[
\lambda_s^2 = \rho + \frac{\sqrt{(1-\alpha \omega)[(1-\alpha \omega)(\rho + 2\delta)^2 - 4(\rho + \delta)^2]}}{(1-\alpha \omega)} > 0
\]

As the first eigenvalue is negative while the second is positive, the dynamic system is saddle-path stable.