ILLEGAL IMMIGRATION, EMPLOYER SANCTIONS AND EQUILIBRIUM

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Abstract
We extend the Bond and Chen (1987) two-country model of illegal immigration using a constant elasticity of substitution (CES) production function. The objectives of this paper are twofold: (1) to analyze the characteristics of equilibrium (including existence) under CES production when capital is either mobile or immobile between countries, and (2) to re-evaluate the effects of restrictive internal inspections on domestic firms that intentionally hire illegal foreign labor. When capital is internationally immobile, it is found that a unique equilibrium exists. However, under capital mobility, the nature and characteristics of equilibrium depend on the form of the production function. Indeed, when capital is assumed to be internationally mobile, numerous equilibria exist, some of which display surprising characteristics. For example, given mobile capital, an increase in inspections may increase illegal immigration and decrease capital exports, provided that the equilibrium is saddle point stable. Contrary to existing work, it is also found that tighter employer sanctions impact wages in the labor-importing and labor-exporting countries differently.

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1. Introduction

Inequality in wages and employment opportunities between nations has encouraged the migration of residents from poor to rich countries. Wealthier countries have attempted to restrict this inflow of foreign migrants resulting in illegal immigration from poorer, less developed countries to wealthier, developed nations. Illegal immigration is usually considered the problem of the labor-importing or host country.

Two policy instruments, border patrol and internal enforcement, are used by the governments of developed countries to control the illegal entry and employment of foreign workers. Border patrol involves the prevention of attempted illegal migration at the border. Internal enforcement, or employer sanctions, targets foreign workers employed illegally by domestic firms. Sanctions against employers have been introduced in the United States and Japan, as well as several European countries, including Germany, France, Austria, the Netherlands, Sweden, Norway, Italy, and Hungary (see Brochmann and Hammar (1999) and Irlenkaueuser (2001)).

Ethier (1986a, b) broke new ground by analyzing the effects of border and internal enforcement policies in a one-country model using a crime-theoretic analysis (Becker (1968)). Ethier examines how a small country can use domestic border controls and internal enforcement to achieve domestic policy objectives with respect to illegal immigration and income distribution. He demonstrates that the wages of legal domestic workers might rise with stronger internal enforcement.

Bond and Chen (1987) extend the Ethier model incorporating two-countries, one-good and two-factors. They find that when capital is immobile, internal inspections reduce the level of illegal immigration increasing domestic wages and decreasing foreign wages. By using enforcement policy, the host country can increase its welfare provided that it is large enough to influence foreign wages and the marginal costs

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1 As well as from poor to rich regions within countries.
of enforcement are sufficiently low. When capital is internationally mobile, however, stricter immigration policies cause both domestic and foreign wages to rise.\(^2\)

The existing literature has little to say on the nature of equilibrium in models of illegal immigration. In particular, the existence and uniqueness of equilibrium tend not to be discussed. In this paper we seek to address this gap by fully characterizing all equilibria, both when capital is internationally mobile and when it is not. This is achieved by implementing the Bond and Chen model using a constant elasticity of substitution (CES) production function. In the process, we also re-examine the effects of restrictive internal inspections on domestic firms who intentionally hire illegal foreign workers.

When capital is internationally immobile, there is a unique equilibrium that is a stable node. This equilibrium corresponds to that discussed by Bond and Chen (1987). Higher levels of enforcement result in reduced illegal immigration; domestic labor benefits and foreign labor loses.

The introduction of capital mobility may yield multiple equilibria. In a static framework, a unique stable equilibrium arises when the labor-importing country’s elasticity of production substitution between labor and capital, is greater than that of the labor-exporting nation. However, when the host country’s elasticity of production substitution is less than that of the foreign country, either stable or unstable equilibria may result.

Within a dynamic framework, a variety of equilibria exist when capital is assumed to be internationally mobile. In an equilibrium that is saddle point stable, a rise in internal enforcement causes illegal immigration to rise and capital outflows to decline. This outcome is opposite to that obtained by Bond and Chen (1987) and, to the best of our knowledge, has not been observed in the literature before. Nevertheless, anecdotal evidence suggests that, while counter-intuitive, the positive relationship between tighter employer sanctions and illegal immigration is observed. In 1986, the United States Congress

\(^2\) Yoshida (2001) implements the Bond and Chen model using a Cobb-Douglas production function and a specification of the probability with which illegal foreign workers are detected by the host country’s immigration authority. Assuming that capital is internationally mobile, Yoshida confirms Bond and Chen’s results with respect to the effects of employer sanctions on domestic and foreign wages.
passed the “employer sanctions” provision of the Immigration Reform and Control Act. This law made it a crime for employers to “knowingly” employ an illegal immigrant. Nevertheless, Miller and Moore (1995) argue that between 1989 and 1993 the number of illegal immigrants apprehended at the Mexican border “rose steadily”. Moreover, a United States General Accounting Office analysis of twenty countries (USGAO, 1982) concluded that “employer sanctions were not an effective deterrent to illegal employment”. One reason for this failure is that employer sanctions may be circumvented through the use of fraudulent work authorization documentation. In such cases, employers can plausibly claim ignorance as to the illegal status of an employee. They can thus escape punishment under an employer sanctions regime by merely declining to investigate the veracity of a prospective employee’s paperwork.

In contrast, the existing literature overwhelmingly concludes that there is a negative relationship between the intensity of internal enforcement and the volume of illegal immigration. For example, Bucci and Tenorio (1996) find that internal inspections reduce the number of illegal migrants in the host country. Djajic (1997) argues that ‘tougher enforcement measures and/or fines may serve to deter future immigration flows, as well as induce (or compel by means of deportation) return migration, lowering the economy’s stock of illegal aliens’. Bandyopadhyay and Bandyopadhyay (1998), meanwhile, model the intensity of internal enforcement as a function of the proportion of illegal workers in the host country’s labor force. They conclude that border patrol curtails the supply of illegal immigration from the source country.

Under capital mobility, stricter internal enforcement can also cause domestic and foreign wages to decline when the equilibrium is saddle point stable. In contrast, Bond and Chen (1987) find that both domestic and foreign workers gain due to stricter enforcement.

Note that, in the case of mobile capital, our model reproduces Bond and Chen’s results for the special case in which the CES production function takes the Cobb-Douglas form. This suggests that the equilibrium relationship between internal enforcement and illegal immigration is not robust to different assumptions about the form of the production function. In what follows, we analyze the characteristics of equilibrium and the implications of stricter employer sanctions under CES production. The existing
literature does not explicitly model CES production. Nevertheless, this paper demonstrates that CES production can imply a positive (rather than negative) relationship between enforcement and illegal immigration.

The next section uses a CES production function to implement the Bond and Chen (1987) model of illegal immigration assuming that capital is internationally immobile. Section 3 introduces capital mobility within a static framework. Section 4 re-examines the mobile capital case in a dynamic setting. In each case, the impact of greater enforcement on the domestic and foreign economies is evaluated. Section 5 undertakes a comparative static analysis to examine the relationship between employer sanctions and factor prices. The final section offers some concluding remarks.

2. The Basic Model with Immobile Capital

In this section, we implement the Bond and Chen two-country, one good, two-factor model of illegal immigration assuming that both the labor-importing (capital abundant) country and the labor-exporting (labor abundant) country are characterized by CES production. Technology is permitted to differ between nations. Henceforth, the labor-importing country will be referred to as the “Host” nation, while the labor-exporting country will be called the “Foreign” nation.

We first consider the model under capital immobility in which there is illegal labor movement from the foreign country to the host nation. Labor is fully employed in both countries. Following Bond and Chen, the host country’s government enforces employer sanctions against domestic firms who hire illegal migrant workers. The probability, \( p \), that an illegal immigrant worker is arrested increases with the effort that the authority devotes to internal inspection:

\[
p = (E/R)^{1/\beta}, \quad p' > 0, \quad p'' < 0, \quad p(0) = 0, \quad p \leq 1,
\]

where \( E(\geq 0) \) is the effort expended on internal enforcement by an immigration authority, \( R(> 0) \) is
the initially endowed national budget of the host country, and $\beta$ is a parameter that is larger than unity.\(^3\)

The host country's government allocates a part of the national budget, $R$, to the immigration authority. Hence, $E \leq R$. If $E = 0$, there are no restrictions on the inflow of illegal migrants. If $E = R$, the probability of detection of illegal migrants is unity. Because technologies are assumed to differ between the countries, there is a disparity in wages between them, even if $E = 0$. The immigration authority behaves as a budget-maximizing government bureau (see Davila et al. (1999)).\(^4\)

Both host country and foreign firms have CES production functions, respectively:

$$X = \left[\delta \cdot K^{-\rho} + (1 - \delta) \cdot L^{-\rho}\right]^{-1/\rho}, \quad (2.1)$$

and

$$X^* = A \cdot \left[\delta^* \cdot K^{*-\rho^*} + (1 - \delta^*) \cdot L^{*-\rho^*}\right]^{-1/\rho^*}, \quad (2.2)$$

where $X$ is the level of host-firm output; $K$ is the level of host-country real capital; $L$ denotes Host's employment of both domestic labor and illegal foreign labor, $\rho$ and $\delta$ are positive constants with $-1 < \rho < \infty$ and $0 < \delta < 1$. The label “*” denotes foreign country variables. Finally, $A$ is the foreign technology level defined over the interval $(0,1)$ and is the source of the difference in production technology between the two countries.

The first order conditions for cost-minimization by domestic and foreign firms (provided in Appendix 1) can be solved for wage rates ($w, w^*$) and rental prices ($r, r^*$) in terms of illegal immigration, $I$. In equilibrium, the number of illegal immigrants employed by host-country firms is determined such that the

\(^3\) See Yoshida (2004) for more discussion on equation (1).
\(^4\) Bandyopadhyay and Bandyopadhyay (1998) assume that enforcement expenditure, $E$, is a function of the percentage of illegal immigrants in the labor force of the host country. This implies that higher numbers of illegal immigrants result in higher levels of enforcement. However, since the aim of this paper is to re-examine the impacts of enforcement, we follow Bond and Chen and assume that equation (1) holds.
cost of employing a domestic worker is equal to that of employing an illegal foreign worker.\textsuperscript{5} As such, and following Lorenz (pp. 248-255, 1993), we consider the following immigration adjustment procedure:

\[
\dot{I} = \alpha \left( w - w^* - p \cdot z \right),
\]

(3)

where \( \dot{I} \) is a time derivative, \( \frac{dI}{dt} \), \( \alpha(>0) \) is the speed of adjustment and \( z \) is the penalty paid by a host-country firm for each detected illegal foreign worker. The number of illegal immigrants who are not apprehended increases over time when the host-country firm can hire an immigrant at a lower cost than a domestic worker, i.e. \( \dot{I} > 0 \) when \( w > w^* + p \cdot z \).

\textbf{Proposition 1: Given CES production, capital immobility and the adjustment procedure defined in (3), there exists a unique equilibrium in which a positive number of immigrants are employed illegally by host-country firms, i.e. \( \dot{I} > 0 \).}

\textbf{Proof:} See Appendix 1.

The dynamic adjustment of the number of illegal immigrants can be depicted in a phase diagram as in Figure 1.

\[\text{[Figure 1 is depicted here.]}\]

\textsuperscript{5} Note that illegal domestic workers earn the same wage as they would in the foreign country since the level of border patrol is assumed to be fixed throughout (see Bond and Chen (1987, p. 317) and Yoshida (2000, p. 37) for details).
Figure 1 confirms the existence of a stable equilibrium at $E_1$ where $I = 0$. We assume that $I = I(>0)$ where $\hat{I} = 0$. That is, the equilibrium number of illegal immigrants, $\hat{I}$, is assumed to be an interior solution to equation (3). Having determined the level of illegal immigration, the equilibrium host and foreign country wages and rental prices can be obtained. These depend on $E$, the level of employer sanctions in the host nation.

The unique equilibrium identified in Proposition 1 corresponds to that discussed by Bond and Chen (1987). The reader can easily verify that higher levels of enforcement in Host result in reduced illegal immigration; host country labor benefits and foreign labor loses. In particular, note that Proposition 1 suggests that under immobile capital, the nature of equilibrium and the impact of employer sanctions on both the host and foreign country are not sensitive to the value of the elasticity of production substitution between labor and capital. As will be shown subsequently, this robustness does not survive in a world of mobile capital.

### 3. Capital Mobility in a Static Framework

In this section capital is permitted to be internationally mobile within a static framework. All countries now possess identical production technology. The first order conditions for cost minimization by host-country and foreign firms are provided in Appendix 2. They can be solved for factor prices in terms of illegal immigration, $I$, and Host capital exports to Foreign, $K_F$.

With no barriers to capital mobility, host-country capital shifts to Foreign if the return there is greater. This equalizes capital returns and wages in the two locations – factor price equalization. Consequently, in a world characterized by perfect capital mobility, there would be no illegal immigration. Here, however, it is assumed that capital migrates to the foreign country until the return ($r^*$) net of taxes

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*Although the actual impact of stricter internal enforcement is believed to be very small, these results can be reproduced in a model in which border patrol rather than employer sanctions is the policy instrument of choice.*
(t \in [0,1])$, equals the return to capital employed in the host country $(r)$. Thus, in equilibrium, the following condition must hold:

\[ r(I, K_F) - (1-t)r^*(I, K_F) = 0 \]  \hfill (4)

Equation (4) implies that capital earnings cannot be identical between the two countries, provided that there is a non-zero tax rate. This ensures that equilibrium will be characterized both by capital and (illegal) labor flows. As with immobile capital, it must also be the case in equilibrium that the cost of employing a legal domestic worker in the host country is equal to the cost of illegally employing a foreign worker, i.e.

\[ w(I, K_F) - w^*(I, K_F) = p(E)z . \]  \hfill (5)

From conditions (4) and (5), it is possible to show that, under capital mobility, the uniqueness and stability of equilibrium depends on the elasticities of (production) substitution in Host and Foreign. Proposition 2 states this result.

**Proposition 2:** In a static framework characterized by CES production and capital mobility, when $\sigma > \sigma^*$, there is a unique, stable equilibrium. When $\sigma < \sigma^*$, however, there are multiple equilibria, one of which is stable and another unstable.

**Proof:** See Appendixes 2 and 3.

Figures 2 and 3 illustrate Proposition 2 for the case $\sigma > \sigma^*$ and $\sigma < \sigma^*$ respectively. In each
The $II$ curves, meanwhile, describe the locus of $(I, K_F)$ pairs for which equilibrium condition (5) holds. As shown in Appendix 2, the slope of the $II$ curve is positive when $\sigma > \sigma^*$ and negative when $\sigma < \sigma^*$. On the other hand, the $K_F K_F$ curve is always negatively sloped.

[Figures 2 and 3 are depicted here.]

It is clear from Figure 2 (where $\sigma > \sigma^*$) that point $S$ is a stable, unique equilibrium – the slope of the $II$ locus is positive, while the slope of the $K_F K_F$ curve is negative. In Figure 3 (where $\sigma < \sigma^*$) there are two equilibria. The equilibrium at point $S$ is stable – the $II$ curve is steeper than the $K_F K_F$ curve. The equilibrium at $U$, however, is unstable – the slope of the $II$ locus at this point is less than that of the $K_F K_F$ curve.

To determine the effect of an increase in the level of internal inspections, consider Figures 4.1 and 4.2.

[Figures 4.1 and 4.2 are inserted here]

Figures 4.1 and 4.2 demonstrate that an expansion of internal inspections in the host country leads to a decline in illegal immigration and a rise in the volume of Host capital outflows. The $II$ curve shifts to the left from $II$ to $II'$ and the initial equilibrium $S$ shifts to a new equilibrium $S'$. Hence, within a static framework, Bond and Chen’s (1987) results are robust to different assumptions about the elasticity of substitution in production in the host and foreign countries. As will be demonstrated in the next section, however, in a dynamic framework this is no longer the case.
4. Capital Mobility within a Dynamic Framework

We next consider a dynamic framework of two-way factor movements, using the following dynamic adjustment procedure:

\[ I = \beta_1 \cdot \{w - w^* - p(E) \cdot z\}, \quad (6.a) \]

\[ K_F = \beta_2 \cdot \{(1-t) \cdot r^* - r\}, \quad (6.b) \]

where \( \beta_i (> 0), \ i = 1,2 \) are the speeds of adjustment. Equation (6.a) is analogous to equation (3).

We know from equation (6.b) that outflows of host-country capital increase over time, \( \dot{K}_F > 0 \), when earnings net of tax for a unit of capital in Foreign are larger than in Host, i.e. \((1-t)r^* - r > 0\).

Linearizing equation system (6) yields the following proposition:

**Proposition 3:** In a dynamic framework, assuming CES production and internationally mobile capital, there are numerous potential equilibria, including: a stable node, a saddle point, a stable focus, an unstable focus and center dynamics.

**Proof:** See Appendix 3.

In the case of a stable node it can be shown that when employer sanctions are increased in the host country, the level of illegal immigration decreases and capital exports increase. To see this, consider the phase diagram presented in Figure 5 from which the global stability of the point \( E_1, (I, K_F) \), can be seen.

[Figure 5 is inserted here.]
Figure 5, illustrates the global change in the level of illegal immigration and capital exports resulting from a large increase in the host country’s level of enforcement – the equilibrium shifts from $E_1$ to $E_2$. Stronger enforcement causes the $I = 0$ curve to shift to $I' = 0$. Hence, illegal immigration decreases and, consequently, the labor-force in the foreign country increases. The increase in the foreign labor-capital ratio causes the foreign rental price of capital to rise, which results in Host capital moving to the foreign country. This results in the $K_F = 0$ curve moving to $K_F' = 0$ in Figure 5.

However, we obtain the opposite effects on illegal immigration and capital outflows when it is assumed that the equilibrium is saddle point stable (see Appendix 3). In particular, the following proposition holds:

**Proposition 4**: Assuming CES production, when equilibrium with capital mobility is saddle point stable, tighter employer sanctions induce more illegal immigration and fewer capital exports.

**Proof**: See Appendix 3.

Proposition 4 is illustrated in the phase diagram in Figure 6.

As shown in Figure 6, an initial steady state is represented by the point $E_1$, and a new steady state is given by the point $E_3$. To interpret the figure, note first that an increase in expenditures on employer
sanctions, in the host country, results in a higher number of arrests of illegal immigrants, who are then deported. This causes the \([I = 0]\) locus to shift inward. The repatriation of illegal migrants decreases the foreign wage rate and increases host-country wages. This encourages more foreign workers to migrate. Therefore, in terms of equation (6.a), the level of illegal immigration must be increased to uphold the equality \(w = w^* + p \cdot z\). As a result, the \([I = 0]\) locus moves outward again to the \([I' = 0]\) locus.

Similarly, higher levels of enforcement result in a decrease in illegal immigration and an increase in the outflow of host-country capital to Foreign. This makes the \([K_F = 0]\) locus shift outward, Host's rental price rises while Foreign's rental price falls. Considering equation (6.b), \(K_F < 0\), and hence the \([K_F = 0]\) locus shifts inward, reverting to \([K_F' = 0]\).

Since an increase in the level of internal enforcement in the host country expedites the apprehension and repatriation of illegal migrant-workers, this creates a gap between Host and Foreign wages, \(w - w^* > 0\). Hence, attempted illegal migration increases at once, producing a rise in the number of illegal migrants who can work in the host country (a shift from \(E_1\) to \(E_2\) in Figure 6). This leads to the host-country wage and foreign rental price decreasing, and the foreign wage and Host's rental price rising.

In turn, therefore, illegal immigration and capital outflows decrease simultaneously (see the new saddle path \(SS\) in Figure 6). Hence, equilibrium moves from the initial point \(E_1\) to \(E_3\) via \(E_2\). The equilibrium at \(E_3\) is associated with more illegal immigration and less capital outflow owing to tougher employer sanctions in the host country. Note also that the number of arrested and deported illegal migrants increases over time due to strengthened internal inspections.

In summary, when the host country’s government intensifies internal interdiction, illegal immigration immediately jumps to point \(E_2\) on the new saddle-path \(SS\) and then decreases somewhat at the same
time as capital outflow falls (see Shone (1997) for a saddle-path).

In the traditional literature on illegal immigration, such as Ethier (1986), Bond and Chen (1987), and Bucci and Tenorio (1996), it is well accepted that tightening internal enforcement reduces illegal immigration. Given internationally mobile capital, this relationship is confirmed when the equilibrium is a stable node. However, the relationship is reversed when the equilibrium is saddle point stable.

While this counter-intuitive saddle point equilibrium is theoretically appealing, the jump in illegal immigration from \( E_1 \) to \( E_2 \) in Figure 6 is not unknown in applied contexts. As was pointed out in the introduction, increased illegal immigration following the introduction of employer sanctions is commonly observed. This result is usually ascribed to the increased use of fraudulent work authorization documentation by illegal aliens. In the model presented in this paper, however, the legality of worker documentation is not modeled. In fact, it has been implicitly assumed that workers do not lie about their legal status to prospective employers. It is our conjecture that relaxing this assumption would only serve to make the positive relationship between employer sanctions and illegal immigration more pronounced.

The sudden increase in migrants implied by the move from \( E_1 \) to \( E_2 \) in Figure 6 is also not uncommon. China, for example, has recently witnessed significant and rapid changes in the volume and pattern of inter-regional migration flows as a result of increasingly significant wage differentials between urban and rural regions.

[Tables 1 and 2 are inserted here.]

Table 1 shows that over the period 1995-2000 total migration more than doubled to 28 million people (2.35% of the population) when compared with the period 1990-1995. Over the same period, Table 2 reveals that the proportion of these migrants moving from (high-wage) inland to (low-wage) coastal areas almost doubled from 32% to 60%.
5. Factor Price Comparative Statics

In this section, we show that the relationship between employer sanctions and factor prices is, in fact, not robust to different specifications of production in the host and foreign country. As an initial step, note that it is not possible to determine the signs of $\frac{dw}{dE}$, $\frac{dr}{dE}$, $\frac{dw^*}{dE}$ and $\frac{dr^*}{dE}$ without first identifying the effects of tougher employer sanctions on the volume of illegal immigration and capital exports. In other words, the factor-price-enforcement relationship depends crucially on $\frac{dI}{dE}$ and $\frac{dK_F}{dE}$ (see Appendix 4). The following proposition uses this fact to relate the intensity of internal enforcement, $E$, to factor prices.

**Proposition 5:** In a stable equilibrium characterized by capital mobility and non-Cobb Douglas production in at least one country (i.e. either $\rho \neq 0$ or $\rho^* \neq 0$ or both):

i. If $|\frac{dI}{dE}| < |\frac{dK_F}{dE}|$, then host-country labor and foreign capital lose while foreign labor and Host capital gain.

ii. If $|\frac{dI}{dE}| > |\frac{dK_F}{dE}|$, then host-country labor and foreign capital gain while foreign labor and Host capital lose.

**Proof:** See Appendix 4.

Appendix 4 decomposes the impact of a change in the intensity of internal enforcement on the host and foreign labour-capital ratios. It is shown that when $|\frac{dI}{dE}| < |\frac{dK_F}{dE}|$, stricter (weaker) employer sanctions in the host country increase (decrease) Host’s labor-capital ratio, $L/K$, and decrease (increase) Foreign’s labor-capital ratio, $L^*/K^*$. As such, host-country wages, $w$, and the foreign rental price, $r^*$, fall (rise) while the foreign wage, $w^*$, and Host’s rental price, $r$, rise (fall). When $|\frac{dI}{dE}| > |\frac{dK_F}{dE}|$, however, the opposite occurs. For example, tighter internal inspection reduces
**L/K** while increasing **L*/K*. Consequently, \( w^* \) and \( r^* \) rise while \( w^* \) and \( r \) decline.

Proposition 5 contrasts with the findings of Bond and Chen (1987). They argue that in the presence of capital mobility, stricter internal enforcement causes host-country and foreign wages to increase while, simultaneously, the rental price of capital falls in both countries. On the other hand, our model confirms Bond and Chen’s results for the special case of Cobb-Douglas production (see Yoshida (2001), Yoshida and Woodland (Chapter 7, 2005)). This demonstrates that the impact of more intensive internal inspection on factor prices is not robust to different specifications of the elasticity of substitution in production.

Finally, note that Bond and Chen (1987) also state that a necessary condition for the equilibrium to be stationary is \((L^*/K^*) - (L/K) > 0\). However, as has been argued above, the magnitudes of \( L^*/K^* \) and \( L/K \) vary with \( dI/dE \) and \( dK_F/dE \), and hence with changes in the intensity of internal enforcement, \( E \).

**5. Conclusions**

In this paper, we have implemented the Bond and Chen (1987) two-country model of illegal immigration, using a CES production function. We have analyzed the existence and nature of equilibrium, both when capital is internationally mobile and when it is not. With immobile capital, a unique stable equilibrium is found which displays the usual characteristics.

Capital mobility, however, complicates the outcome. Within a static framework, when the elasticity of production substitution between labor and capital of the host country is larger than that of the foreign country, there is a unique and stable equilibrium. However, when this relation is reversed, there are multiple equilibria, some of which are unstable. The latter outcome only exists when the production function assumes the CES form.

Considering capital mobility within a dynamic framework yields counter-intuitive results. In this case, six types of equilibrium may arise which are, in turn, stable node, saddle point, stable focus, unstable...
focus and center dynamics. These outcomes were derived using a dynamic adjustment procedure, which has not been analyzed in any preceding research on illegal immigration. For the saddle point stable equilibrium under capital mobility, stricter internal enforcement results in an increase in illegal immigration and a decrease in capital outflows. This outcome runs contrary to the existing literature in which an increase in enforcement not only causes a decrease in illegal immigration but also leads to an expansion of capital exports.

Furthermore, Proposition 5 stated that when the equilibrium is stable, strengthening internal inspections increases the price of domestic labor and foreign capital and decreases the price of domestic capital and foreign labor, in accordance with the size of illegal immigration and capital exports. These results differ from those of Bond and Chen (1987), in which tougher employer sanctions result in increases in both domestic and foreign wages and decreases in the capital rental price in both countries.

This paper has shown that, given mobile capital, the equilibrium relationship between internal enforcement and illegal immigration is not robust to different assumptions about the form of the production function. In particular, introducing the CES production function into the Bond and Chen framework has allowed us to decompose the effect of enforcement on each country’s factor prices into an effect on illegal immigration and an effect on capital inflows, provided the elasticity of substitution is not unity. The Cobb-Douglas case on the other hand reproduces the Bond and Chen results.
Appendix 1:

The first order conditions for cost minimization by home and foreign firms may be written as:

\[ w = (1 - \delta) \cdot (X / (\bar{L} + I))^{1+\rho}, \]  
(A.1a)

\[ r = \delta \cdot (X / \bar{K})^{1+\rho}, \]  
(A.1b)

\[ w^* = A^{-\rho} \cdot (1 - \delta^*) \cdot (X^* / (\bar{L}^* - I))^{1+\rho^*}, \]  
(A.1c)

\[ r^* = A^{-\rho^*} \cdot \delta^* \cdot (X^* / \bar{K}^*)^{1+\rho^*}, \]  
(A.1d)

where \((\bar{L}, \bar{K})\) and \((\bar{L}^*, \bar{K}^*)\) are the initial endowments of the host and foreign country respectively; \(w\) and \(w^*\) are the domestic and foreign labor wage rates; \(r\) and \(r^*\) denote the rental price of domestic and foreign capital; and the successful number of illegal immigrants is given by \(I\).

Proposition 1 in the text can thus be proved as follows:

Proof:

For the line arising around the steady state (see equation (3) in the text), we obtain:

\[ \dot{I} = \alpha(\mu_1 + \mu_2)dl, \]  
(A.2)

where,
$$\mu_i = (1 - \delta) \cdot (1 + \rho) \cdot (X / (L + I))^\rho \cdot (1 / (L + I)) \cdot \{w - (X / (L + I))\} < 0,$$

$$\mu_2 = (1 - \delta^*) \cdot (1 + \rho^*) \cdot A^{-\rho^*} \cdot (X^* / (L^* - I))^\rho^* \cdot (1 / (L^* - I)) \cdot \{w^* - (X^* / (L^* - I))\} < 0.$$

Due to the concavity of CES production functions, \( \mu_i < 0, \ i = 1, 2 \) and, hence, \( (\mu_1 + \mu_2) < 0 \). Note that the marginal productivity of labor in each country is smaller than the average productivity of labor. Therefore, the gradient of \( \dot{I} \) is negative in \( \dot{I} - \dot{I} \) space. This ensures the existence of a unique equilibrium value, \( \dot{I} \) (see Figure 1).

\[\square\]

**Appendix 2:**

With capital mobility, the factor market equilibrium conditions in the host and foreign countries are given by:

$$w = (1 - \delta) \cdot (X / (L + I))^{1 + \rho}, \quad \text{(A.3a)}$$

$$r = \delta \cdot (X / (K - K_F))^{1 + \rho}, \quad \text{(A.3b)}$$

$$w^* = (1 - \delta^*) \cdot (X^* / (L^* - I))^{1 + \rho^*}, \quad \text{(A.3c)}$$

$$r^* = \delta^* \cdot (X^* / (K^* + K_F))^{1 + \rho^*}, \quad \text{(A.3d)}$$

where production technology is identical in both countries, and \( K_F \) is the level of capital exports to the
foreign country.

Equations (A.3a) – (A.3d) show that factor prices are functions of \((I, K_F)\) given \(E\). As explained in the text, in equilibrium, the following two conditions must hold:

\[
w(I, K_F) - w^*(I, K_F) = p(E)z, \quad (A.4a)
\]
\[
r(I, K_F) - (1 - t)r^*(I, K_F) = 0. \quad (A.4b)
\]

Plotted in \((I, K_F)\) space, the locus of points that satisfy these conditions can be referred to, respectively, as the illegal immigration curve, \(II\), and the host-country capital exports curve, \(KK\). These curves are illustrated in Figures 2 and 3 in the text. Given \(E\) and \(t\), and totally differentiating (A.4a) and (A.4b), yields:

\[
\eta_1 dl + \eta_2 dK_F = 0, \quad (A.5a)
\]
\[
-\eta_3 dl - \eta_4 dK_F = 0, \quad (A.5b)
\]

where

\[
\eta_1 = (1 + \rho)w_L + (1 + \rho^*)w^* - \frac{w - (X / (L + I))}{X(L + I)} \frac{w^*}{X^* - I} < 0. \quad (A.6a)
\]
\[
\eta_2 = -\frac{1}{1/(1 + \rho) X} - \frac{1}{1/(1 + \rho^*) X^*} \geq 0. \quad (A.6b)
\]
\[
\eta_3 = (1 + \rho)rw / X + (1 - t)(1 + \rho^*)r^*w^*/X^* > 0. \quad (A.6c)
\]
\[
\eta_4 = -r(1 + \rho)(K - K_F) - \frac{r - (X / (K - K_F))}{K / (K - K_F)} > 0.
\]
\[-r^*(1+\rho^*)(1-t)(\bar{K}^*+K_F)\left\{\frac{r^*-(X^*/(\bar{K}^*+K_F))}{X^*(\bar{K}^*+K_F)}\right\}>0. \tag{A.6d}\]

The marginal productivity of labor in each country is always smaller than the average productivity of labor owing to the assumption of CES production. Hence, in equation (A.6a), \(\frac{w^*-(X^*/(\bar{L}+I))}{X(\bar{L}+I)}<0\) and \(\frac{w^*-(X^*/(\bar{L}^*-I))}{X^*(\bar{L}^*-I)}<0\). This ensures that \(\eta_1<0\).

The sign of \(\eta_2\) is negative (positive) only if \(\sigma=1/(1+\rho)<(>)1/(1+\rho^*)=\sigma^*\). That is, if the elasticity of substitution in host-country production, is smaller or larger than its foreign counterpart.

As with \(\eta_1\), the concavity of CES production implies that the marginal productivity of capital in each market is less than the average. Therefore, \(\eta_4\) is positive.

Equations (A.5a), (A.6a) and (A.6b) imply that the slope of the II curve is positive when \(\sigma>\sigma^*\) and negative when \(\sigma<\sigma^*\). From equations (A.5b), (A.6c) and (A.6d), it is clear that the \(K_FK_F\) curve is always negatively sloped. The net result of these observations is that when \(\sigma>\sigma^*\) a unique equilibrium exists (see Figure 2 in the text). However, when \(\sigma<\sigma^*\), multiple equilibria exist (see Figure 3 in the text). Note also that when \(\sigma=\sigma^*\), the II curve is a vertical line. This case necessarily produces a unique and stable equilibrium. So when we assume a Cobb-Douglas production function, \(\sigma=\sigma^*=1\), we confirm the equilibrium (see Yoshida (2001), Yoshida and Woodland (Chapter 7, 2005)). However, the CES production function excludes this case and hence yields multiple equilibria, when \(\sigma<\sigma^*\).

Appendix 3

Following Lorenz (pp. 248-255, 1993), we next consider a dynamic adjustment processes for both
illegal immigration and home capital exports in order to investigate the stability properties of the equilibria identified above. Let

\[
\dot{I} = \beta_i \cdot \{w - w^* - p(E) \cdot z\}, \quad \text{(A.7a)}
\]

and

\[
\dot{K}_F = \beta_2 \cdot \{(1-t) \cdot r^* - r\}, \quad \text{(A.7b)}
\]

where \( \beta_i (>0), \ i=1,2 \) represent the speed of adjustment. (A.7a) is identical to equation (3) in the text. We know from (A.7b) that outflows of host capital increase over time, i.e. \( \dot{K}_F > 0 \), when the return on a unit of capital (net of tax) in the foreign country is larger than the return in the host country itself, i.e., \((1-t)r^* - r > 0\).

Total differentiating (A.3a) – (A.3d) and linearizing (A.7a) and (A.7b) around the solutions, \( I \) and \( K_F \), yields in matrix form:

\[
\begin{bmatrix}
\dot{I} \\
\dot{K}_F
\end{bmatrix} = \begin{bmatrix}
\beta_1 \eta_1 & \beta_1 \eta_2 \\
-\beta_2 \eta_3 & -\beta_2 \eta_4
\end{bmatrix} \begin{bmatrix}
dI \\
dK_F
\end{bmatrix}, \quad \text{(A.8a)}
\]

where

\[
J = \begin{bmatrix}
\beta_1 \cdot \eta_1 & \beta_1 \cdot \eta_2 \\
-\beta_2 \cdot \eta_3 & -\beta_2 \cdot \eta_4
\end{bmatrix} \quad \text{(A.8b)}
\]

If \( J \) is a totally stable matrix, then it is a Hicksian matrix. Such a matrix implies that: (1) every principal minor of \( J \) of even order is positive and (2) every principal minor of \( J \) of odd order is
negative (see Quirk and Saposnik (1968)). Let $J_i$ denote the $i$th principal minor. We then have

$$J_1 = \eta_i < 0,$$

and

$$J_2 = -\beta_1\beta_2(\eta_4\eta_3 - \eta_2\eta_1).$$

Therefore, if an equilibrium exists, it is stable provided that $J_2 > 0$. In fact, $J_2 > 0$ when $\sigma > \sigma^*$. This situation corresponds to the unique static equilibrium illustrated in Figure 2 in the text. However, when $\sigma < \sigma^*$, $J_2 > 0$ requires that $|\eta_1\eta_4| > |\eta_2\eta_3|$ (corresponding to the static stable point $S$ in Figure 3), otherwise $J_2 < 0$ (which corresponds to the unstable equilibrium point $U$ in Figure 3).

Considering this continuous-time dynamic system (A.7), the roots of the characteristic equation are given by

$$\lambda_{1,2} = \frac{trJ \pm \sqrt{(trJ)^2 - 4\det J}}{2},$$

where $\lambda_{1,2}$ are eigenvalues (see also Appendix 2). Referring to Lorenz (1993), we show various equilibria in the model because we cannot determine the signs of $\eta_i$, $i = 1, 2, 4$ without any sufficient conditions, which are constituents of the Jacobian matrix $J$. Note also that the sign of $\eta_3$ is always positive.

We show equilibria in turn that occurs in our model:
i) \textit{Real roots:}

(i.a) These eigenvalues determine the dynamic behavior of the system (6). The eigenvalues \( \lambda_{1,2} \) in (A.9) are real when the discriminant, \( \Delta_1 = (trJ)^2 - 4 \det J \), is positive or equal to zero. A positive determinant, \( \det J > 0 \), implies that both eigenvalues have the same sign. If the trace of \( J \) is negative, both eigenvalues are negative, and the trajectory of the system monotonically approaches a finite point \((I, K_F)\). As a result, we can confirm that there are other interior solutions for \( \hat{w}, \hat{r}, \hat{w}^* \) and \( \hat{r}^* \) from formulas (2) and (A.3a) – (A.3d). We should note that if we assume \( \rho = \rho^* = 0 \), then \( \eta_1 < 0 \), \( \eta_2 > 0 \) and \( \eta_4 > 0 \), the CES production function reduces to a Cobb-Douglas production function, and the case of (i.a) results.

(i.b) A negative sign for \( \det J \) can arise if \( (\eta_1 \cdot \eta_4 - \eta_2 \cdot \eta_3) > 0 \), and only if \( \eta_1 < 0 \), \( \eta_2 < 0 \) and \( \eta_4 > 0 \). It is therefore possible in the present system (A.7) for the sign of discriminant \( \Delta_1 \) to be positive, in which case the equilibrium is saddle point stable. This produces contrary effects on illegal immigration and capital mobility to those in case (i.a) in which the equilibrium is a stable node.

ii) \textit{Complex roots:}

(ii.a) The sign of \( \det J \) is positive when the symbol for \( \eta_1 \) is the reverse of that for \( \eta_4 \), and \( \eta_2 > 0 \), which brings about \( \Delta_1 < 0 \), only if \( (trJ)^2 < 4 \det J \). Thus, we are aware of the existence of a stable or unstable focus in the system (A.7) when \( \text{Re} \ \lambda_i \) is negative or positive.

(ii.b) We can also consider the case when \( \text{Re} \ \lambda \) is zero in (A.9), which is called the \textit{center dynamics} or \textit{neutrally stable}. 

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We have shown that there are all sorts of equilibria that have not been previously investigated, as in cases (i.a) – (ii.b).

**Appendix 4:**

Differentiation of equilibrium conditions (4), (5) and (A.3a) – (A.3d) with respect to $E$ produces the following effects on $I$, $K_F$, $w$, $r$, $w^*$ and $r^*$:

\[
\frac{dI}{dE} = \frac{\{z / (R \cdot \beta) \cdot (E / R)^{(1 - \beta) / \beta} \} \cdot (\eta_4 / \Delta_2)}{1 / (1 + I)} \tag{A.10}
\]

\[
\frac{dK_F}{dE} = -\{z / (R \cdot \beta) \cdot (E / R)^{(1 - \beta) / \beta} \} \cdot (\eta_3 / \Delta_2) \tag{A.11}
\]

\[
\frac{dw}{dE} = (1 - \delta) \cdot (1 + \rho) \cdot (X / (\bar{L} + I)) \cdot (1 / (\bar{L} + I))
\]

\[
\times \left[\{w - (X / (\bar{L} + I))\} \cdot (dI / dE) - r \cdot (dK_F / dE)\right] \tag{A.12}
\]

\[
\frac{dr}{dE} = \delta \cdot (1 + \rho) \cdot (X / (\bar{K} - K_F)) \cdot (1 / (\bar{K} - K_F))
\]

\[
\times [(1 - \delta) \cdot (X / (\bar{L} + I))^{1+\rho} \cdot (dI / dE)
\]

\[
+ \{X / (\bar{K} - K_F)\} - r^* \cdot (dK_F / dE)\right] \tag{A.13}
\]

\[
\frac{dw^*}{dE} = (1 - \delta^*) \cdot (1 + \rho^*) \cdot (X^* / (\bar{L}^* - I))^{\rho^*} \cdot (1 / (\bar{L}^* - I))
\]
where \( \Delta_2 = \eta_1 \cdot \eta_4 - \eta_2 \cdot \eta_3 < 0 \).

From (A.10) – (A.15), it is clear that the impact of stricter employer sanctions on factor prices are determined according to sizes of effects on illegal immigration, \( dE / dE \), and capital outflows, \( dK_F / dE \).

However, when we use the Cobb-Douglas production function, i.e., \( \sigma = \sigma^* =1 \), severe employer sanctions cause both wages to increase, and both rental prices to decrease.

We should notice that the sign of \( \Delta_2 \) is negative as long as the equilibria with capital mobility in the static framework examined in section 3 are stable. So we do not consider \( \Delta_2 \) is positive when the equilibria are unstable (see Appendixes 2 and 3).
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Figures and Tables

Figure 1

Figure 2
Figure 4.2
Figure 5
Figure 6
### Table 1: Migration Rates in China, 1985-90 and 1995-2000.

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Total migration (million)*</td>
<td>11.83</td>
<td>27.53</td>
</tr>
<tr>
<td>Total migration rate (% of population)*</td>
<td>1.11</td>
<td>2.35</td>
</tr>
<tr>
<td>Migration of working-age population (million) †</td>
<td>10.47</td>
<td>25.78</td>
</tr>
<tr>
<td>Rate of migration of working-age population (%) †</td>
<td>1.33</td>
<td>3.01</td>
</tr>
<tr>
<td>Migration of employed population (million) ‡</td>
<td>8.13</td>
<td>21.24</td>
</tr>
<tr>
<td>Rate of migration of employed population (%) ‡</td>
<td>1.23</td>
<td>3.13</td>
</tr>
</tbody>
</table>

Source: Lin, Wang and Zhao (2004). Data calculated from 1% of the population census in 1990 and 0.95% of the population census in 2000. Migration rates are measured as the total number of migrants divided by the population of relevance. Tibet is excluded. Migration numbers for 2000 are adjusted to conform to definitions in the 1990 census.

* Population refers to all people older than 5 years at census time.
† Working age is defined as between the ages of 15 and 64 at census time.
‡ A working age person is employed if they were employed the week before the census was conducted.

### Table 2: Composition of Inter-provincial Migration of the Working-age Population, 1985-90 and 1995-2000 (%).

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Coast to inland</td>
<td>14.0</td>
<td>6.1</td>
</tr>
<tr>
<td>Inland to coast</td>
<td>32.5</td>
<td>60.1</td>
</tr>
<tr>
<td>Within coast</td>
<td>27.1</td>
<td>18.6</td>
</tr>
<tr>
<td>Within Inland</td>
<td>26.3</td>
<td>15.2</td>
</tr>
</tbody>
</table>

Source: Lin, Wang and Zhao (2004). Data calculated from 1% of the population census in 1990 and 0.95% of the population census in 2000.