Occupational Choice and Compensation for Losers from International Trade*

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Abstract

Trade liberalization creates winners and losers in the domestic economy. Nonetheless, current compensation schemes for losers are deemed unsatisfactory. Why is there both insufficient compensation coverage and overcompensation for some groups of individuals at the same time? This paper introduces a general equilibrium model of occupational choice in which agents are endowed with multidimensional skills and may change jobs based on their individual comparative advantages. When the government cannot set taxes based on agents’ unused traits, it is impossible to design a program that ensures Pareto improvement from trade liberalization without overcompensating some groups.

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One of the problems with free trade is we never compensate the losers. We always say that there are more winners than losers, and that’s true. But there are losers, and we’re not helping them. [Clyde Prestowitz, President of the Economic Strategy Institute.]

When academic economists teach their students about the benefits of free trade, the focus is on gains in aggregate efficiency. It is widely known that international trade, a kind of production process that transforms exports into imports, expands the set of feasible allocations available in the economy. However, international economists point out that trade liberalization typically affects the distribution of individual incomes. (Dani Rodrik 1997, p.30) Thus, a change in the terms of trade favors some groups of individuals over others. This is an area in which protectionists have the upper hand. Some economists argue that compensating losers could solve the problem. There is a larger pie to share, and losers can be fully compensated even if the beneficiaries from trade are better off than under autarky. In practice, however, many are dissatisfied with current compensation schemes.

While this section’s epigraph, by Clyde Prestowitz, implies that compensation for losers is either absent or insufficient, the opposing view appears in The Washington Post. This is that the present compensation scheme, in the form of the Trade Adjustment Assistance (TAA) program, is far too magnanimous and could put a huge strain on the federal budget. Assuming that the recent expansion of the TAA program will be approved by the Senate in exchange for the president’s fast-track “trade promotion authority” bill, the Post goes on to note that

conservative critics are dismayed at the concessions they were forced to make, and they are hoping that budget constraints will prevent the establishment of a large new entitlement program.

“Socialist governments all over the planet are trying to stop doing this kind of thing, and now we’re doing it,” said Sen. Phil Gramm (R-Tex.), referring to government largess for the unemployed.

All of this reflects a growing sentiment among conservatives that protectionist compensation schemes tend to cost so much money that (some) losers are overcompensated. The model of this paper seeks to explain the difficulty of ensuring Pareto gains from trade when individuals are heterogeneous and can freely move between

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2For a classic example, see Stolper and Samuelson (1941).

3This is known as the compensation principle. However, the compensation criterion requires only a “potential” Pareto improvement. The improvement does not depend on compensation actually taking place.

different sectors. It concludes that no government can attain Pareto improvement unless it makes inefficient (from the point of views of the government) and excessive transfers.

The paper proposes a model of occupational choice that captures the difficulties of identifying gainers and losers from trade. It does so by incorporating agents who differ in their relative and absolute abilities in undertaking different occupations. The model predicts aggregate gains from trade even when individuals are allowed to switch jobs (or equivalently, when there are temporally displaced workers). To incorporate displaced individuals effectively within a general equilibrium (full employment)\textsuperscript{5} framework, I assume that each individual faces an occupational choice.\textsuperscript{6}

Changes in the terms of trade may improve the best outcome available to some agents while worsening the best outcome for others. While job stayers are stuck in their industries, job switchers change their occupations following changes in the economic environment. The distribution of fortune and misfortune spreads across the whole population, affecting both job stayers and job switchers. In Heckscher-Ohlin models and specific-factors trade models, it is not difficult to identify gainers and losers from trade.\textsuperscript{7} In a model with occupational choice, it is difficult to identify gainers and losers among those who switch their occupations. Consequently, it is difficult to design a redistribution program that targets losers from trade.

The primary reason for this difficulty is the asymmetric information between the government and individual agents. The gains and losses from trade depend on the amounts of talent actually used by individuals relative to their hidden (unused) talents. While one can easily observe talents that individuals use, one cannot easily observe their hidden latent talents.\textsuperscript{8} Even if the government can condition its taxation scheme on those variables that represent the actual use of factors, the (nonfeasible) first-best compensation scheme must also base its tax rates on the latent talents of job switchers. It is not difficult to show that there are individuals who are identical in terms of the talents they currently use, but who are either gainers or losers because of different hidden talents. Given typical taxation and subsidy schemes, the government has no mechanism for inducing individuals to reveal their latent talents. This means that if the government seeks a Pareto improvement on autarky, it cannot avoid the overcompensation problem, and hence, it may fail to balance its budget.

\textsuperscript{5}For an analysis of structural unemployment and the gains from trade, see Richard A. Brecher and Ehsan U. Choudhri (1994).

\textsuperscript{6}A justification of this full-employment assumption (with occupational choice) has been provided by Daniel T. Griswold, associate director of The Center for Trade Policy Studies at The Cato Institute, a libertarian research group: “trade had little long-term impact on the overall number of jobs, because the American economy tended to create jobs in more sophisticated industries to replace those that are lost.” in The New York Times: Tuesday October 29, 2002. Page 11, “TRADE WINDS; Global Trade in Elmwood Park: Familiar Saga With a Twist.”

\textsuperscript{7}For results based on the Heckscher-Ohlin model, see Wolfgang F. Stolper and Paul A. Samuelson (1941), and for the specific-factors model, see Ronald W. Jones (1971) and Samuelson (1971).

\textsuperscript{8}The phrase “easily observe” is used casually; it refers to whether a variable is observed for tax purposes.


**Heterogeneity of Agents in this Paper**

In the model of this paper, I assume that individual agents are doubly heterogeneous. That is, they differ in both the absolute and relative magnitudes of their capabilities in their different occupations. We explain heterogeneity by using an example.

Suppose that every individual could, in principle, work either as an opera singer or as an economics professor. Naturally, every individual differs in his or her abilities to sing operatic arias and to serve as an economics professor. These differences are referred to as heterogeneity in *absolute advantages*. No individual is going to be equally good at both tasks. Individuals’ *relative* strengths vary widely. These variances are referred to as heterogeneity in *comparative advantages*. Some will be good at singing but not at economics, and vice versa, and others will be good at both. A way of capturing these differences in absolute and comparative advantages is to assume that, for every individual, \( j \in J \), there is a vector, \((\theta^j, \tau^j)\), of abilities. The element \( \theta \) (of the vector) measures how much “effective economics-professorial service” the individual can produce in a given period, while the element \( \tau \) measures how much “effective opera singing” the individual can produce over the same period. These elements will inevitably differ between individuals, and they represent heterogeneity in absolute advantages. In addition, the ratio of the elements of the ability vector, \( \theta / \tau \), reflects the comparative advantage an individual has in being an economics professor.\(^9\) A relatively low \( \theta / \tau \) indicates a comparative advantage in opera singing. Note that my model shares many features of the Roy model from labor economics.\(^10\) (A.D. Roy 1950)

Furthermore, every individual \( j \) faces an occupational choice. This decision is a discrete one: whether to work as an opera singer or as an economics professor. The decision depends on economic variables such as relative output prices. Given a particular economic environment, an individual might choose to be an opera singer but switch to being an economist following a change in the relative price. Note that each element of the ability vector is indivisible and nontransferable.\(^11\)

\(^9\)This framework is similar to the model of interpersonal comparative advantages in Roy Ruffin (1988, 2001). While Ruffin’s model allows for the multi-sectoral use of the same factors of production, I model this comparative advantage as a source of occupational choice. In addition, my model allows for a continuum of varieties of individual heterogeneity, whereas Ruffin introduced a finite set of groups of individuals. Ruffin’s model is not consistent with my model’s assumption of atomless agents.

\(^10\)I was not aware of the labor literature until I had completed my analysis of a model similar to Roy’s. I thank Sujata Visaria for bringing my attention to the literature on labor economics. I also thank Professor Elhanan Helpman for referring me to the unpublished thesis of Kevin Murphy and Professor Motoehide Itoh for referring me to the related works by Sherwin Rosen.

\(^11\)The economy described in this model is somewhat similar to the Ricardian economy with a large number of commodities in Dornbusch, Fischer and Samuelson (1977). Whereas the Dornbusch–Fischer–Samuelson model focuses on comparative advantages across different categories of outputs, my model emphasizes both the absolute and the comparative advantages of individuals’ talents. Another difference is that my model focuses primarily on the welfare change of individual agents but also examines
The remainder of this paper is divided into eight sections. In the next section, I present a non-technical overview of the model. In section 2, I develop a simple general equilibrium trade model that has two final outputs. This model comprises a large number of heterogeneous agents who possess both generic-mobile and individual-specific occupational factors. I examine the Walrasian (trading) equilibrium of the model and show that there are aggregate gains from trade. In section 3, I present the key result for some there to be gainers among displaced individuals. In section 4, I introduce pertinent definitions and properties of compensation schemes. In section 5, I propose a compensation scheme, not anticipated by individuals, based on taxes and subsidies relating to currently observable variables. In section 6, I analyze the case in which individual agents learn about the compensation scheme and examine disincentives by manipulating the mechanism. The final section concludes the paper and suggests extensions.

1 Overview of the Model

I provide a non-technical overview of this paper's analytical approach, before developing the model formally in the next section. Parts of the model's structure closely resemble the independently discovered framework first proposed in A.D. Roy (1951) and elaborated on by Sherwin Rosen (1978) and Michael Mussa (1982, pp.131-134).

Consider a small open economy that faces exogenously given international output prices. Output markets are assumed to be competitive, both internationally and domestically. Ignoring distributional concerns, in the aggregate, free trade is more efficient than restricted trade for such an economy because there are no terms-of-trade externalities and hence no scope for optimal tariffs. The economy comprises a continuum of individual agents who own two types of factor endowment: generic factors and occupation-specific talents.

The generic-type factors are homogeneous factors of production, the property rights of which are well defined and traded competitively on domestic markets. Examples of these generic factors are unskilled wage labor, capital goods that are easily sold for money or exchanged for other capital goods, and all kinds of homogeneous inputs used in the production of outputs.

Occupation-specific talents (or occupational abilities) characterize the heterogeneity of individual agents compensation schemes that aim for Pareto improvements.

\footnote{Having completed my analysis, I discovered these classic works by Roy (1951), Rosen (1978) and Mussa (1982), which use a similar framework to that of my model. The Roy model has been used to analyze the inequality of individual earnings but not international trade. Rosen applied Roy's model to the case of two workers and many jobs. Mussa introduces a similar model to support the assumption of a convex input transformation curve. Despite the similarity of the set up to my model, however, Mussa's analysis differs from that of this paper.}

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in this economy. Agents vary in both their absolute and relative abilities in different occupations. The occupation-specific talents are specific to the individual and to the industry (or chosen occupation). This may mean that human capital is sector specific, but agents can still have multiple talents in different sectors to different degrees. In addition to the talents being specific, their other important characteristic is their intangibility.¹³ (Kevin Murphy 1986) Unlike the generic factors mentioned in the previous paragraph, the property rights of the specific talents (or occupational abilities) are embodied in each individual. In other words, the occupational talents are intangible and noncontractible.¹⁴ Given that these talents belong to utility-maximizing economic agents, I postulate that the occupational abilities are indivisible.¹⁵ I also assume that each individual agent is a residual claimant to his or her own specific talent that is in use.

Furthermore, I assume that each individual undertakes only one occupation at a time. The decision is discrete; I do not allow for the existence of individual agents who are employed in multiple sectors.¹⁶ Usually, in this type of nonconvex decision space, the equilibrium is difficult to verify. For this, I depend on Werner Hildenbrand (1974), who showed that nonconvexity can be overcome by having a continuum of atomless individual agents.¹⁷ (For an analysis of a large economy with nonconvexities, see Mas-Colell, Whinston and Green (1995, Section 17.1, p.627).)

Each individual agent is a residual claimant who collects the residual profits after paying the cost of production that is incurred for any generic factor of production.¹⁸ Note that individuals are residual claimants of the “actual use” of their talents. Although they may have “latent” (unused) talents, they have no claim

¹³As Kevin Murphy states, “human capitals are embodied in each individual”.

¹⁴This intangibility implies that individual talents are nonverifiable and nontransferable. In this context, I assume that property rights are not well defined; therefore, insurance and stock markets for individuals’ talents do not exist.

¹⁵In other words, I assume that individuals expend all their efforts. Thus, the returns to their occupational talents take the form of residual profits rather than market prices multiplied by the number of efficiency units. This is because the use of talents is not included in the utility functions of individual agents. When the cost (disutility) of effort is zero, agents will maximize their effort levels so that they can consume as much as possible. For an analysis of effort choices when individuals experience disutility from making effort, see David Spector (2001).

¹⁶In this context, Robert C. Feenstra and Tracy R. Lewis (1994) allow one factor to be supplied to multiple industries. They do not, however, allow for the existence of perfectly mobile generic factors, as I do in this paper.

¹⁷“Atomless” means that no point measure has a positive Lebesgue measure.

¹⁸The notion of residual-claimant property should not be interpreted too literally. It is an attempt to capture the specificity of a certain factor of production and reflects the difficulty of verifying the amount of the factor being used. All workers have both generic factors and human-specific and industry-specific talents. Residual-claimant property could be interpreted as self-employment. However, the notion is restricted to the self-employment interpretation. Employed individuals retain complete bargaining power over the residuals from production, because they have the option of being self-employed. Thus, it is reasonable to assume that individuals are residual claimants of their talents being used in their current jobs.
on these. In other words, a person chooses to produce a good by hiring as many inputs as necessary from the competitive market and then earns residual profits from this activity. However good the agent may be at other jobs, he or she has no claim on the residual profits from activities in which he or she is not actually engaged. By choosing one job over another, a person forgoes other opportunities. The opportunity cost is the return from the second-best job, given the relative price of outputs.\(^{19}\) The difference between actual and second-best returns differ between individuals. Hence, the ranking of the alternative jobs may also change. However, the idea that a person is the residual claimant to his or her best talent, given the environment, is reasonable.

Now that I have described the nature of the factor endowments held by individual agents, I present a simple general-equilibrium model, in which there are two outputs and, therefore, two occupations and one generic factor.\(^{20}\) Let \(X\) (respectively, \(Y\)) denote the output good that is an export (respectively, import) good for the home country, and that is produced by using the occupational ability, \(\theta\) (respectively, \(\tau\)). Let \(K\) denote the total amount of the generic factor endowed in the economy. Individual \(j \in J\) can be characterized by an occupational ability vector, \((\theta^j, \tau^j)\), and by an endowment of the generic factor, \(K^j\). Let \(P_X\) and \(P_Y\) denote the output prices of \(X\) and \(Y\), respectively. Let \(r\) denote the market price for the generic factor. Given the endowment of ability and the generic factor, each individual calculates the potential residual returns from every occupational choice (of which there are two in this model).

Let \(\pi_X\) and \(\pi_Y\) denote the residual returns from the two sectors. Agents can freely trade (domestically) generic factors at the market price, \(r\), in order to maximize their best available occupational returns. Since agents are price-takers in both the output and the generic-factor markets, they compare the expected residual returns between different occupations. Agents choose the sector in which to produce output by comparing \(\pi_X \gtrless \pi_Y\). The distribution of the ability vectors is given by \((\theta^j, \tau^j) \sim F(\theta, \tau)\), where \(F(\theta, \tau)\) represents the joint cumulative distribution function. I assume that \(F(\theta, \tau)\) has a full support over a compact and convex set, and that its shape is common knowledge. Its density function, \(f(\theta, \tau)\), is bounded and continuously differentiable. I also assume that the available technology (production functions for both \(X\) and \(Y\)) is common knowledge. The technology is characterized by constant returns to scale. Its production function is increasing in every input and is twice continuously differentiable, strictly concave, and satisfies the Inada conditions. The tastes of the consumers are assumed to be identical and homothetic. Therefore, I focus on agents’ heterogeneity with respect to their factor incomes.

The terms of trade, the price of \(X\) relative to \(Y\) (represented by \(P_X/P_Y\)), is the key decision variable for each individual. To explain why, I use diagrams familiar from specific-factor models of production. (See Fig.

\(^{19}\)In this sense, opportunity costs and factor returns change when there is a change in terms of trade after trade liberalization.

\(^{20}\)Much of this analysis applies to the basic diagrams used in the specific-factors model of trade.
1.) Given both the specification of the production functions and the individual talents, \((\theta^i, \tau^i)\), I draw curves representing the value of the marginal product of the generic factor for both occupations. Let \(VMPK_X\) and \(VMPK_Y\) denote these curves. The vertical axis represents the monetary value of the marginal product of the generic factor given the occupational talents of the individuals. The horizontal axis represents the quantity of generic factors being employed in the production of each output.

Let the lower-case letter \(k\) denote the employment (use) rather than the endowment, \(K\), of generic factors. Since both curves are downward sloping in \(k\), the generic factors have diminishing marginal products.

Both elements of the ability vector (\(\theta\) and \(\tau\)) and the relative output price (\(P_X/P_Y\)) are shift parameters for the \(VMPK_X\) and \(VMPK_Y\) curves. A higher \(\theta\) shifts the \(VMPK_X\) curve up. Similarly, a higher \(\tau\) shifts up \(VMPK_Y\). Higher talent shifts the corresponding value-marginal-product curve up. An increase in the price of \(X\) relative to \(Y\) shifts the \(VMPK_X\) curve up and the \(VMPK_Y\) curve down. A decrease in the price of \(X\) induces opposite movements. When individuals calculate their residual profits, they take the generic factor price, \(r\), as given, even though the equilibrium value of \(r\) depends on the relative price, \(P_X/P_Y\).\(^2\)

The area below the \(VMPK\) curves and above the horizontal line at \(r\) represents the residual reward (or profit), \(\pi\), derived from the corresponding occupational talent. Given the relative price, \(P_X/P_Y\), an individual with \((\theta^i, \tau^i)\) produces \(X\) if \(\pi_X(\theta^i) > \pi_Y(\tau^i)\), and produces \(Y\) if \(\pi_X(\theta^i) < \pi_Y(\tau^i)\). This individual is indifferent between producing \(X\) and \(Y\) if \(\pi_X(\theta^i) = \pi_Y(\tau^i)\). (Given the assumption of atomless agents, indifference does

\(^2\)The equilibrium value of \(r\) also depends on the shape of the distribution, \(F[\theta, \tau]\) of individuals’ talents.
Figure 2: Individual occupational rewards, given output prices.

Fig. 2 shows the occupational rewards (profits), $\pi_X(\theta^i)$ and $\pi_Y(\tau^i)$, for a given individual, $(\theta^i, \tau^i)$, for all possible relative prices, $P_X/P_Y \equiv P$. The vertical axis represents the monetary value of occupational rewards, given the talent of the individual. The horizontal axis represents the relative price of output. (Note that in Fig. 2, the height corresponds to the area of the previous graph, Fig. 1.) Let $P$ denote $P_X/P_Y$. Let the intersection of the two occupational-reward curves occur at $P^* = (P_X/P_Y)^*$. The individual produces $Y$ when the relative output price is $P < P^*$. When $P = P^*$, the individual is indifferent between producing $X$ and $Y$. The individual produces $X$ if $P > P^*$. Note that, for any degree of trade liberalization, the shifts in the terms of trade occur in a discrete manner. Then, for some positive discrete change, $\Delta > 0$, in the relative price $P$, the ex ante price is $P^0$ and the ex post price is $P^1 = P^0 + \Delta$. When $P^0 < P^1 < P^*$, the individual is a producer of $Y$ in both periods. (The individual is stuck in $Y$ production.) Such an individual loses out because of an increase in the relative price. When $P^* < P^0 < P^1$, the individual produces $X$ in both periods. This individual benefits from the positive price change. The individual depicted in Fig. 2 changes occupation when the relative price passes the $P^*$ point. For job switchers, $P^0 < P^* < P^1$, the welfare change is ambiguous. Note that, so far, my argument has not depended on the assumption of a specific distribution of talents, $F(\theta, \tau)$.

To simplify the exposition, assume that the ability vector, $(\theta, \tau)$, is distributed over the support of a unit square, $[0, 1] \times [0, 1]$. (The support of a unit square is not central to the results of this section. It is used only
for graphical convenience. Also assume that the production and utility functions, together with the joint distribution, are such that the division of labor under autarky occurs at a 45-degree line on the unit square. This line divides the unit square into two partitions: one represents the $X$ producers and the other represents the $Y$ producers. \textsuperscript{22} See Fig. 3.

Figure 4: A unit square subdivided according to occupational choice.

\textsuperscript{22}The assumption of a symmetric autarky division of agents, while not central to the results, facilitates the exposition because results need not be classified into many different cases.
Let \( P^A = (P_X/P_Y)^A \) denote the relative autarky price. In Fig. 4, the 45-degree line, OA, corresponds to the relative autarky price, \( P^A \). Consider an economy opening up to free trade. Let \( P^W \) denote the world (international) relative price. Then, because \( X \) is assumed to be a natural export good for the home country, it follows that \( P^W > P^A \). Given the world price, \( P^W \), some individual agents may decide to switch their occupations, having compared their present occupational rewards with those expected from the other sector under the new price, \( P^W \). Thus, in Fig. 4, the new ray from the origin, OW, has a steeper slope than OA. While OA corresponds to the autarky division of occupational choice, OW represents the free-trade division of occupational choice. Now, I partition the unit square into three sections. \( C_{X\rightarrow X} \) denotes the partition that includes all the job-staying individuals who produced \( X \) under autarky and produce \( X \) under free trade. \( C_{Y\rightarrow X} \) denotes the partition of job stayers in the sector \( Y \). The partition \( C_{Y\rightarrow X} \) represents all the individuals who have switched occupations, such as those who produced \( Y \) under autarky but produce \( X \) under free trade. Given the change in output prices, no one switches jobs in the opposite direction.

Figure 5: A comparison of three types of individual.

For the same size of relative price change

\[
P^0 = P^0 + \Delta
\]

Job-stayers in sector X

Job-switchers from sector X to Y

Job-stayers in sector Y
There is a one-to-one correspondence between Figures 2 and 4. Each individual has a different job-switching value, \( P^* \). The location of this trigger value depends only on the agent’s comparative advantage and, hence, on the relative amount of talent: \( \theta/\tau \). Note also that in Fig. 4, there is a one-to-one mapping between relative levels of talent and the slope of the ray from the origin to the point that represents the individual’s endowment. The higher the value of \( \theta/\tau \), the higher the comparative advantage the agent has in producing \( X \), and therefore the flatter the slope of the ray from the origin at which the agent is located in the unit square. This is readily apparent because, given the equation representing the ray from the origin, \( \tau = \gamma \theta \), the slope, \( \gamma \), is the inverse of \( \theta/\tau \).

For a given relative price change, different individuals face different decisions about their occupation choices. Fig. 5 compares the residual rewards for representative agents from three groups of individuals with different comparative advantages. Note that Fig. 5 has the same features as Fig. 2; i.e., three different agents have different trigger values, \( P^* \). The different agents are one from group \( C_{X^{-}} \) (a job stayer in sector \( X \)), one from group \( C_{Y^{-}} \) (a job stayer in sector \( Y \)), and one from group \( C_{Y^{-}} \) (a job switcher from sector \( Y \) to sector \( X \)). The agent from group \( C_{X^{-}} \) has a low value of \( P^* \), the agent from group \( C_{Y^{-}} \) has a medium value, and the agent from group \( C_{Y^{-}} \) has a high value. Only job switchers are affected by the relative price change from \( P^0 \) to \( P^1 = P^0 \ + \Delta \), which passes the trigger value, \( P^* \), where \( \Delta > 0 \). Hence, a rise in the relative price of \( X \) favors job stayers in industry \( X \) and disfavor job switchers in industry \( Y \). The third graph, however, provides ambiguous results for job switchers from \( Y \) to \( X \). In fact, those who switch occupations include winners as well as losers.

Figure 6 shows that what distinguishes winners from losers among job switchers are the comparative advantages of individuals. The left-hand panel of Fig. 6 provides a finer division of the group of agents from \( C_{Y^{-}} \) (job switchers from \( Y \) to \( X \)) into winners and losers. The right-hand panel of Fig. 6 shows the profit functions for the corresponding agents (winners and losers) given a discrete price change. Each job switcher has a unique amount of relative talent, \( \theta/\tau \) and, hence, a unique job-switching trigger value for the relative price, \( P^* \). For a given increase in the relative price of \( X \), winners tend to be agents with a higher \( \theta/\tau \). Fig. 6 illustrates an example of two types of agent: a job switcher who is a loser (in the upper graph of the right-hand panel) and a job switcher who is a winner (in the lower graph of the right-hand panel).

Given that there are winners and losers among job-switching individuals, I now explain a government’s difficulty in implementing a fully Pareto-improving compensation scheme without overcompensating job switchers. Assume that the government combines taxes and subsidies on outputs and inputs, including the residual returns to the talents of individuals. Assume also that the tax (subsidy) base for the government is restricted to currently observable variables. Thus, a wage-insurance scheme based on information about the jobs that
Figure 6: Gainers and losers among job switchers.

Note: There exist both gainers and losers among job switchers.

individuals had before switching is not allowed.

A Pareto-improving compensation scheme for job-staying individuals is easily designed. The direction and size of the gains and losses are calculated similarly to the way that gains and losses for specific (immobile) factors are obtained in the context of specific-factors models. The percentage gains and losses for job-staying individuals are the same for all stayers, irrespective of their talents and of whether they use them.

For job switchers, the creation of a Pareto-improving compensation scheme inevitably causes complications. This is because the size and direction of individuals' gains or losses are not necessarily correlated with currently observable variables. Note in particular that the percentage change in occupational residual profits is the same for all individuals on the same ray from the origin. However, the government cannot distinguish job-switching winners from job-switching losers if they earn the same residual profits from their current production activities. Thus, while in Fig. 7 the iso-percentage-gain-or-loss lines are the rays from the origin, the iso-profit lines based on current production are the vertical lines representing \textit{ex post} producers from sector $X$. (The horizontal lines represent \textit{ex post} producers from sector $Y$.) Thus, Fig. 7 illustrates this complex case of job-switching individuals from sector $Y$ to sector $X$.

The left-hand panel of Fig. 7 depicts the iso-percentage-gain-or-loss lines, while the right-hand panel illustrates the iso-current-profit lines for \textit{ex-post} producers of the sector $X$ outputs. The iso-percentage-gain-
Iso-percentage gain or loss line  

Iso-profit line for ex-post $X$ producer

Note: The left-hand panel depicts the iso-percentage gain-or-loss lines, while the right-hand panel depicts the iso-current-profit lines for \textit{ex post} producers of $X$.

or-loss lines are the rays from the origin, while the iso-current-profit lines for $X$ producers are the parallel vertical lines. The closer the iso-percentage-gain-or-loss lines are to the $OA$ line (and hence the flatter the slope of the rays from the origin), the larger are the gains (and the smaller are the losses). (Of those rays from the origin depicted in Fig. 7, it is the $OZ$ line that represents a gain or loss of zero for job-switching individuals.) The further to the right of the panel are the iso-current-profit lines, the higher are current profits. While the sizes and directions of individuals’ gains and losses depend on the iso-percentage-gain-or-loss lines, the government only observes the information based on the iso-current-profit lines. For example, the points $q$ and $r$ in both figures are consistent with the same value of $\theta$ but are consistent with different values of $\tau$. The individual at point $q$ has a larger $\tau$ than does the individual at point $r$. The difference in the values of $\tau$ is sufficiently large for the individual at point $q$ to be a loser from trade while the individual at point $r$ gains. However, since they are making the same current profits, the government treats them the same for tax purposes. In other words, although $q$ and $r$ are on the same iso-current-profit line, they are on different iso-percentage-gain-or-loss lines.

The analysis of the preceding paragraph has made it clear that the government cannot both attain a Pareto-improving compensation and avoid excessive compensation. This is because the government must give
the same amount of subsidy to \( r \) and \( q \), even though the individual at point \( r \) gains from trade. Moreover, the government must provide the same subsidy or impose the same tax on individuals on the same iso-current-profit line, whatever their gains or losses. To achieve a Pareto improvement, the government must provide the same amount of subsidy to all as is provided to the biggest losers, who are on the upper side of the square in Fig. 7. Hence, the government inevitably overcompensates job-switching individuals, except those on the line on the upper side of the square.

In this section, I provided an intuitive, diagrammatic, explanation of why job switchers gain from trade following a change in the terms of trade. In addition, I explained the impossibility of implementing a compensation scheme that achieves a Pareto improvement without overcompensating some job switchers. To make these points more rigorously, a formal model is developed in the following section.

2 The Formal Model

Consider a continuum of agents, \( j \in J \), each of whom is endowed with an individual-specific occupational ability vector, \((\theta^j, \tau^j) \sim F(\theta, \tau)\) and a generic factor, \( K^j \geq 0 \).\(^{23}\) Let \( f(\theta, \tau) \geq 0 \) denote the joint density function for \( F(\theta, \tau) \), and assume that \( f \) is integrable over any partition of the ability space, \( \Theta \). Agents are price takers in the output and the generic-factor markets. An economy-wide endowment of generic factors is inelastically supplied at \( K = \int_j K^j \). Agents trade their generic factors freely on the competitive market. The factor price is denoted by \( r > 0 \). Each element of the ability vector, \((\theta^j, \tau^j)\), represents an occupational talent; their magnitudes measure the innate capabilities of agent \( j \) in the production of \( X \) and \( Y \).

An agent decides whether to produce \( X \) by using \( \theta \) or to produce \( Y \) by using \( \tau \). Every element of the ability vector, \((\theta^j, \tau^j)\), is indivisible and nontransferable. In the context of self-employed agents, this element can be considered a managerial talent of the owner of the business. An ability vector, \((\theta^j, \tau^j) \in \Theta\), is unobservable to the government, but its aggregate distribution is publicly known. \( \Theta \subset \mathbb{R}^2 \) represents the space of individual characteristics. \( \Theta \) is assumed to be a compact and convex set.

Having stipulated the individual characteristics, I describe the technological side of the economy. Technology is a nonrival good, and every individual has access to the best available production techniques. Thus, individuals differ only in factor endowments. The timing of decision making and market clearing is described below.

1. The world market determines the output prices, \( P_X \) and \( P_Y \). Under free trade, the home market takes

\(^{23}\)The distribution of \( K^j \) can be quite general, since there is a competitive market for it. Therefore, I do not specify its distribution function but instead simply assume that the total mass is represented by \( K \).
prices as given. In analyzing domestic equilibrium, I determine relative prices endogenously. However, because there is an infinite number of agents, they take the equilibrium prices as given.

2. Individual agents observe their own vectors, \((\theta^i, \tau^i) \in \Theta\).

3. The agent forms a conjecture about the market price of the generic factor, \(r\), predicts the profit-maximizing level of generic-factor employment, and calculates the associated occupational rewards or residual profits, \(\pi_X^j(P_X, r, \theta^j)\) and \(\pi_Y^j(P_Y, r, \tau^j)\), to be gained from both occupational choices.

4. Agents decide (based on the expected rewards) in which sector to produce, and hire from the factor market the profit-maximizing level of the generic factor. They choose to produce either \(X\) or \(Y\) (not both or a convex combination of the two) by using \(\theta\) or \(\tau\). This process determines the aggregate employment of specific factors.

5. The generic-factor market clears. The equilibrium factor price, \(r\), is consistent with the conjectures of agents.\(^{24}\)

6. Given domestic production and domestic demand, the home country engages in trade with the rest of the world.

Both outputs are assumed to be produced according to symmetrical production functions that are twice continuously differentiable, strictly increasing, strictly concave, and homogeneous of degree one.\(^{25}\)

In particular, assume for simplicity the following Cobb–Douglas specification:

\[
\begin{align*}
    x^j(k_X^j, \theta^j) &= (k_X^j)^a(\theta^j)^{1-a} \\
    y^j(k_Y^j, \tau^j) &= (k_Y^j)^a(\tau^j)^{1-a}
\end{align*}
\]

where \(x^j\) and \(y^j\) are individual-level outputs, in which \(k_X^j\) and \(k_Y^j\) represent the individual-level uses of the generic factor, and \(\theta^j\) and \(\tau^j\) represent the occupational talents. Note that use of the generic factor is not constrained by the individual endowment, \(K^j\), because there is a perfectly competitive market for some of their endowments.\(^{26}\)

\(^{24}\)These conjectures could be considered rational expectations. However, any disequilibrium adjustment process, such as that represented by the Walrasian auctioneer, will suffice.

\(^{25}\)Symmetry of the production functions is not essential to my results. Incorporating all the heterogeneity in the endowments simplifies the algebra.

\(^{26}\)The size of the endowment, \(K^j\), matters only to the calculation of individual factor incomes. Agents can buy more than they are endowed with, because I implicitly assume the existence of a perfect capital market in which people can freely borrow money to pay for additional generic factors.
Given the output prices, \( P_X \) and \( P_Y \), and the factor price, \( r \), individuals compare the expected rewards from different occupations (net of payments to the employed generic factors), namely \( \pi^i_X \) and \( \pi^j_Y \). Given standard profit maximization, the following two equations represent such a comparison.

\[
\begin{align*}
\pi^i_X(P_X, r, \theta^i) &= \max_{k_X} P_X \cdot x^i(k_X, \theta^i) - r \cdot k_X \\
\pi^j_Y(P_Y, r, \tau^j) &= \max_{k_Y} P_Y \cdot y^j(k_Y, \tau^j) - r \cdot k_Y
\end{align*}
\]

(2)

Calculating the hypothetical employment levels of optimized generic factors yields

\[
\begin{align*}
k^i_X(P_X, r, \theta^i) &= \left( \frac{a P_X}{r} \right)^{\frac{1}{\gamma+1}} \cdot \theta^i, \text{ or } \\
k^j_Y(P_Y, r, \tau^j) &= \left( \frac{a P_Y}{r} \right)^{\frac{1}{\gamma+1}} \cdot \tau^j.
\end{align*}
\]

(3)

The occupational decision is based on the relative size of the postoptimization level of the occupation rewards; thus, \( \pi^i_X(P_X, r, \theta^i) \geq \pi^j_Y(P_Y, r, \tau^j) \). The postoptimization level of the rewards are

\[
\begin{align*}
\pi^i_X(P_X, r, \theta^i) &= \left( P_X \right)^{\gamma+\frac{1}{\gamma+1}} \left( \frac{1}{r} \right)^{\frac{1}{\gamma+1}} \left( a \frac{\gamma}{\gamma+1} - a \frac{1}{\gamma} \right) \cdot \theta^i, \text{ or } \\
\pi^j_Y(P_Y, r, \tau^j) &= \left( P_Y \right)^{\gamma+\frac{1}{\gamma+1}} \left( \frac{1}{r} \right)^{\frac{1}{\gamma+1}} \left( a \frac{\gamma}{\gamma+1} - a \frac{1}{\gamma} \right) \cdot \tau^j.
\end{align*}
\]

(4)

The occupational rewards increase with agents’ abilities and with their own output prices. Note that, in equation (4), the symmetry of the production functions from (1) neutralizes the effect of the generic-factor price. (Roy Ruffin and Ronald W. Jones 1977)

I partition the ability space, \( \Theta \), by self-selection of individuals. Using \( P \) to denote \( P_X/P_Y \) yields

\[
\begin{align*}
R &= \left\{ (\theta^i, \tau^j) \in \Theta : \tau^j < P^{\frac{1}{\gamma+1}} \cdot \theta^i \right\} \\
S &= \left\{ (\theta^i, \tau^j) \in \Theta : \tau^j > P^{\frac{1}{\gamma+1}} \cdot \theta^i \right\},
\end{align*}
\]

(5)

where the partition \( R \) represents individuals who produce \( X \), and the partition \( S \) represents producers of \( Y \) given \( P \). Note that the ray from the origin, which can be expressed as \( \tau^j = \gamma \cdot \theta^i \), where \( \gamma \) is constant, is the dividing line between the two partitions.27

So far, I have described how individuals choose their occupations and get involved in a production process. In (5), the partition of individual agents represents an endogenous determination of the allocation of specific factors available in the economy. I now examine the economy-wide allocation of specific factors.

Let \( V^R_\theta \) (respectively, \( V^S_\tau \)) denote the volume integral with respect to the variable \( \theta \) (respectively, \( \tau \)) on the region \( R \) (respectively, \( S \)). This volume integral represents the aggregate employment of each specific factor.

\[
\begin{align*}
V^R_\theta &= \iint_R \theta \cdot f(\theta, \tau)d\tau d\theta \\
V^S_\tau &= \iint_S \tau \cdot f(\theta, \tau)d\theta d\tau.
\end{align*}
\]

(6)

27 I use a strict inequality for both partitions because the measure of the line, \( \tau^j = (P_X)^{\frac{1}{\gamma+1}} \cdot \theta^i \), is zero.
When the joint density function, \( f(\theta, \tau) \), has a full support and is continuous, it is not difficult to show that \( V_\theta^R \) is strictly increasing in \( P \) and that \( V_\tau^S \) is strictly decreasing in \( P \).

Given the self-selection condition for individual occupational choice represented by equation (5), the generic-factor market clears. The corresponding full-employment condition is expressed by the following equation.

\[
\int_{(\phi^\prime, \tau^\prime) \in R} k^1_X(P_X, r, \theta^j)f(\theta, \tau)d\theta d\tau + \int_{(\phi^\prime, \tau^\prime) \in S} k^1_Y(P_Y, r, \tau^j)f(\theta, \tau)d\theta d\tau = K. \tag{7}
\]

Factor-market demand, as represented by the left-hand side of equation (7), is an aggregation of individual factor demands over each partition, \( R \) and \( S \).

By substituting the optimized values from equation (3) for the employment-level generic-factors into (7), and by using the notation in (6), equation (7) can be rewritten as

\[
\left( \frac{aP_X}{r} \right)^{\frac{1}{\gamma_X}} V_\theta^R + \left( \frac{aP_Y}{r} \right)^{\frac{1}{\gamma_D}} V_\tau^S = K. \tag{8}
\]

Thus, equation (8) implicitly shows that \( r \), the reward for the generic factor, is a function of output prices, with \( K \) and \( a \) being parameters.\(^{28}\) I assume that the solution of (8) for \( r \) is unique and that it can be written as

\[
r = r(P_X, P_Y). \tag{9}
\]

To derive simply the properties of the reward function (9) for the generic factor, I postulate a specific functional form for the demand side of the economy.

### 2.1 Demand Side

Generally, each consumer \( j \)’s problem can be described by

\[
\max_{c_X^j, c_Y^j} u(c^j_X, c^j_Y) \quad \text{s.t.} \quad P_X \cdot c^j_X + P_Y \cdot c^j_Y \leq I^j,
\]

where \((c^j_X, c^j_Y)\) represents the consumption bundle for individual \( j \). This individual’s income is

\[
I^j = r \cdot K^j + \max\{\pi_X^j(P_X, r, \theta^j), \pi_Y^j(P_Y, r, \tau^j)\}.
\]

In general, the utility function must be twice continuously differentiable, strictly quasi-concave, homothetic, and strictly increasing. For ease of exposition, I assume the following Cobb–Douglas form. (Note that the constant term has been added to simplify the Walrasian-demand function and the indirect-utility function.)

\[
u(c^j_X, c^j_Y) = 2\sqrt{c^j_X c^j_Y} \tag{10}
\]

\(^{28}\)Note that both \( V_\theta^R \) and \( V_\tau^S \) depend on the relative output price, \( P \).
I use the price normalization, $P_X = p$ and $P_Y = 1/p$. Note that $P = P_X/P_Y = p^2$. Given the price normalization, the indirect utility function can be normalized to the income of the individual (in terms of the parameter $p$).

$$v(P_X, P_Y, I^j) = \frac{I^j}{\sqrt{P_X I_Y}} = I^j(p)$$

(11)

Note that the last equality takes into account the dependence of income on relative prices.

Given the above normalization of the price parameter $p$, the following equation expresses the equilibrium level, $r$.

$$r(p) = a \cdot K^{-(1-a)} \left[p^{\frac{1}{1-a}} \cdot V^R_\theta (p) + p^{\frac{1}{1-a}} \cdot V^S_\tau (p) \right]^{1-a}$$

(12)

Note that aggregate employment of the specific factors, $V^R_\theta (p)$ and $V^S_\tau (p)$, depends on the parameter for relative output prices, $p$.

Equilibrium national income can also be expressed as a function of relative output prices, $p$.

$$I(p) \equiv \int_{(\theta, \tau) \in \Theta} I^j(p) = r(p) \cdot K + \int_R \int_S \pi^X_\theta \cdot f(\theta, \tau) d\theta d\tau + \int_S \pi^Y_\tau \cdot f(\theta, \tau) d\theta d\tau$$

(13)

I now present an intermediate result relating to the relationship between national income and generic-factor income.

**Lemma 1** Generic-factor income is proportional to national income, as expressed by the following equation.

$$r(p) \cdot K = a \cdot I(p)$$

(14)

This follows directly from equations (4), (12) and (13). This proportional relationship in (14) arises because the production functions for the two sectors are Cobb–Douglas and symmetric. The proof is in the Appendix.

Note also that national factor income is equal to the gross national product.

$$I(p) = P_X \cdot \int_R x^j(p, r, \theta^j) \cdot f(\theta, \tau) d\theta d\tau + P_Y \cdot \int_S y^j(p, r, \tau^j) \cdot f(\theta, \tau) d\theta d\tau$$

(15)

The relationship in (14) is consistent with (15).

2.2 Goods-market Equilibrium

In this section, I analyze goods-market equilibrium. There are two equilibria: one for autarky and one for free trade. I derive the goods-market-clearing conditions for the autarky equilibrium and investigate trade volumes for the trading equilibrium.

A trading equilibrium is represented by a net import vector, $m(p)$, for a given relative price, $p$:

$$m(p) \equiv (ED_X(p), ED_Y(p)) = (C_X(p) - X(p), C_Y(p) - Y(p)),$$
where $ED_X(p)$ and $ED_Y(p)$ are the excess demand functions for sectors X and Y, respectively, and $C_X(p)$ and $C_Y(p)$ represent aggregate demand for X and Y:

$$C_X(p) \equiv \int_{\Theta} c^X_j dF(\theta, \tau) \text{ and } C_Y(p) \equiv \int_{\Theta} c^Y_j dF(\theta, \tau).$$

In addition, $X(p)$ and $Y(p)$ represent aggregate supply for X and Y:

$$X(p) \equiv \int_R x^i dF(\theta, \tau) \text{ and } Y(p) \equiv \int_S y^i dF(\theta, \tau).$$

Autarky is a special case in which the autarky price, $p^A$, makes $m(p^A) = 0$. I now derive the conditions for the autarky equilibrium. Given the utility function (10), the aggregated Walrasian-demand functions for goods X and Y can be written respectively as

$$\begin{cases}
  c^X_j(p, I^j) = \frac{I^j}{p} \\
  c^Y_j(p, I^j) = \frac{I^j}{p} 
\end{cases} \quad \Rightarrow \quad
\begin{cases}
  C_X(p) = \frac{I(p)}{x} \\
  C_Y(p) = \frac{I(p)}{y} 
\end{cases}$$

where the left panel shows the individual demand functions and the right panel shows the market demand functions. By using the previous results (derived by substituting (3) into (1)), I express the aggregate production in terms of $p$, as follows.

$$\begin{cases}
  x^j(k^X_j, \theta^j) = \left( \frac{a}{p^2} \right)^{\frac{1}{2}} \cdot \theta^j \\
  y^j(k^Y_j, \tau^j) = \left( \frac{a}{p^2} \right)^{\frac{1}{2}} \cdot \tau^j 
\end{cases} \quad \Rightarrow \quad
\begin{cases}
  X(p) = \left( \frac{a}{p^2} \right)^{\frac{1}{2}} \cdot V^R_{\tau}(p) \\
  Y(p) = \left( \frac{a}{p^2} \right)^{\frac{1}{2}} \cdot V^S_{\tau}(p) 
\end{cases}$$

Thus, given the result in (14), when $p = p^A$, the following equations hold.

$$\begin{cases}
  V^R_{\tau}(p) = \frac{K}{2} \cdot \left( \frac{a}{p^2} \right)^{\frac{1}{2}} \\
  V^S_{\tau}(p) = \frac{K}{2} \cdot \left( \frac{a}{p^2} \right)^{\frac{1}{2}} 
\end{cases} \quad (16)$$

Substituting the equilibrium generic-factor return (12) into (16) yields the following autarky condition for aggregate employment of the specific occupational factors.

$$p^{\frac{1}{2}} \cdot V^R_{\tau}(p) = p^{\frac{1}{2}} \cdot V^S_{\tau}(p) \bigg|_{p = p^A} \quad (17)$$

Under autarky, $p = p^A$, as expressed in equation (17).

Now, I present an intermediate result relating to the return for the generic factor, $K$.

**Lemma 2** Let $p^A$ be the autarky-level price parameter. The factor price, $r(p)$, is a function of $p$, which represents relative output prices. The function is U-shaped around $p = p^A$; i.e., it is increasing in $p$ when $p > p^A$, decreasing in $p$ when $p < p^A$, and has a zero slope at $p = p^A$.

The proof is in the Appendix. This lemma, in conjunction with the condition given by (14), implies another important result relating to aggregate gains from trade.
Proposition 1 The model implies that there are aggregate gains from international trade. That is, real national income, \( I(p) \), is U-shaped around \( p = p^A \). In other words, any deviation from the autarky price raises real national income.

Proof. This follows from Lemmas 1 and 2. ■

The model predicts gains from trade at the aggregate level even if there is a large number of heterogeneous multitalented individuals who are able to change their occupations.\(^{29}\)

I have demonstrated the equilibrium property of this model, which is characterized by heterogeneous agents who face occupational choices. I also have shown that there are aggregate production gains from trade in this economy. Now I focus on the welfare effects on different groups in the economy.

3 Welfare Effects on Individual Groups

Thus far, I have analyzed the equilibrium properties of the model by focusing on comparative-static effects on aggregate variables. Now, I focus on individuals in the economy. More specifically, I compare the well-being of various groups (of individuals) when there is a discrete change in output prices. The first result concerns the welfare for job stayers.

Proposition 2 Job stayers gain from an increase in the relative prices of their own outputs (those produced by applying their talents). Job stayers lose from a decrease in the relative prices of their own outputs.

These results are the same as those for specific-factor owners in the specific-factor model of international trade. In Fig. 8, the change in relative prices following a shift from autarky to free trade—a change from \( p^A \) to \( p^W \)—is represented by a shift in the division-of-labor line from \( OA \) to \( OW \). Partitions \( C_{X,X} \) and \( C_{Y,Y} \) each illustrate a group of job-staying individuals. Because the price change is favorable to the exporting sector, individuals who stay in sector \( X \) gain while those who stay in sector \( Y \) lose. A formal proof is in the Appendix. Note that this proposition is consistent with the monotonicity of the reward values shown in Fig. 2. When the relative price \( p \) of \( X \) increases, the reward from \( Y \) declines in \( P \) and the reward from \( X \) increases.

The second result concerns the well-being of job-switching individuals.

Proposition 3 Among those who change their occupations, there are both gainers and losers from trade, in the absence of compensation. When there is a change in relative prices, whether a job-switching individual gains depends on the ratio of used talent to latent talent.

\(^{29}\) Although such a nonconvex decision space for an individual agent is usually problematic, it is not a problem in the context of this model.
Contrary to popular belief, there are gainers among those who are “forced” to change their occupations. I sketch a proof below.

Proof. The proof is established in three steps. First, I show that there are individuals who are indifferent between sector X and sector Y under autarky; in the context of Fig. 8, these individuals are to the right, on the OA line. Under autarky, these individuals receive equal occupational returns from sectors X and Y. Thus, on the basis of Proposition 2, if they begin in sector Y and then switch to sector X, they inevitably gain from the price change.

Second, I show that there are individuals who are indifferent between switching to sector X and staying in sector Y under free trade; these are the individuals on the OW line. Under free trade, these individuals must have equal occupational returns in sectors X and Y. Therefore, whether they switch jobs or not, they lose out equally, being job stayers following trade liberalization. (This is derived from the result in Proposition 2.)

Third, I show that there are individuals who are neither gainers nor losers from trade liberalization; these individuals are on the OZ line. To establish this, I express the gain or loss as a function of $\tau/\theta$, the parameter of comparative advantage. Then, I show that the function is continuous across its domain and use the intermediate-value theorem.

The gain or loss for a job switcher can be expressed as $\Delta \pi(p^A, p^W, \tau^j, \theta^j) \equiv \pi_X^j(p^W, \tau^j, \theta^j) - \pi_Y^j(p^A, \tau^j)$, where

$$\pi_X^j(p^W, r, \theta^j) = \left[ p^W \frac{1}{r(p^W)} \left( \frac{1}{\theta^j} + \frac{\theta^j}{a} - \frac{1}{a} \right) \right] \cdot \theta^j \equiv g(p^W) \cdot \theta^j$$
and
\[ \pi^j_Y(p^A, r, \tau^j) = \left[ p^A \left( \frac{1}{r(p^A)} \right)^{\frac{r}{p^A}} \left\{ a^{\frac{r}{p^A}} - a^{\frac{r}{p^A}} \right\} \right] \cdot \tau^j \equiv g(p^A) \cdot \tau^j. \]

For given values of $K, p^A, p^W$, the terms in the square brackets, which can be denoted by $g(p)$, are constant. Therefore, the gain-loss function can be written as $\Delta \pi = g(p^W) \cdot \theta^j - g(p^A) \cdot \tau^j$, where $g(p^A)$ and $g(p^W)$ are the corresponding constant given prices. The percentage change in the gain or loss is
\[ \% \Delta \pi \left( \frac{\theta^j}{\tau^j} \right) = \frac{\Delta \pi}{\pi^j_Y} = \frac{g(p^W) \cdot \theta^j - g(p^A) \cdot \tau^j}{g(p^A) \cdot \tau^j} = \frac{\theta^j}{g(p^A)} - 1 = \frac{g(p^W)}{g(p^A)} \cdot \frac{\theta^j}{\tau^j} - 1. \]  

Equation (18) is a continuous function of $\tau/\theta$. The value of the percentage change in the gain-loss function, $\% \Delta \pi (\theta^j/\tau^j)$, is positive $\tau/\theta$ equals the slope of the $OA$ line, but is negative when $\tau/\theta$ equals the slope of the $OW$ line. Since the function is continuous, there is a value of $\tau/\theta$ that implies $\% \Delta \pi = 0$. This value equals the slope $OZ$ in Fig. 8. The size of the gain or loss is determined by the amount of used talent relative to latent talent.

While the gains and losses for job stayers have the same properties as those for specific-factor owners, the gains and losses for job switchers depend on the level of used talent relative to latent talent. Therefore, I can state the following result, which relates to limits on government policy.

**Corollary 1** When the government only observes current (not past) profits, the calculation of gains and losses for job stayers is straightforward. However, the calculation of gains and losses for job switchers is difficult.

Calculation of the gains or losses for job-staying individuals is based on Proposition 2. The difficulty of calculating the gains and losses for job switchers is illustrated by Fig. 8. In Fig. 8, consider the two points, $q$ and $r$. The individual $q$, as a producer in sector $Y$, has a higher ability level than does the individual $r$. However, as producers in sector $X$, these two are equivalent. Nevertheless, individual $q$ is among the losers, while individual $r$ is a winner. While the government is able to observe the current profit of $X$, it cannot distinguish between $q$ and $r$ because they differ only with respect to their latent talents. Who gains and who loses depends on the relative strengths of their used and unused talents. (The iso-percentage-gain lines would be the rays from origin, while the iso-current-profit lines would be vertical.)

Since latent talent is unobservable, the government cannot distinguish gainers from losers. This makes the implementation of Pareto-improving compensating redistribution schemes difficult.

4 The Design of Compensation Schemes

The results of the preceding analysis have shown that there are both winners and losers among job switchers. Having analyzed the effect of a terms-of-trade change without compensation, next, I consider a government
redistribution policy that aims to achieve a Pareto improvement (from trade) and a balanced budget (by avoiding overcompensation).

I begin by comparing two situations: autarky (prohibitive tariffs) and free trade. There is not necessarily free trade \textit{ex post}. Although restricted trade is possible, for simplicity, I compare autarky and free trade.\textsuperscript{30} The initial equilibrium is the autarky one. The uncompensated free-trade equilibrium was analyzed in sections 2 and 3. When the policymaker implements a compensation scheme, the free-trade equilibrium becomes a compensated free-trade equilibrium.

In choosing the instruments of the compensation scheme, I follow the literature in ignoring lump-sum compensation because of the associated informational requirements.\textsuperscript{31} Therefore, I examine a compensation scheme that is based on factor taxes and commodity taxes.\textsuperscript{32} (Anthony B. Atkinson and Joseph E. Stiglitz 1980, p. 20) I now formally define the compensation scheme.

\textbf{Definition 1} The \textit{compensation scheme}, $\sigma$, is a combination of taxes and subsidies levied on the following variables: (1) output prices, (2) generic-factor prices, and (3) occupational rewards. Tax-subsidy rates can be linear or nonlinear.

The taxes (or subsidies) on output prices are commodity taxes, and the taxes on both generic-factor prices and occupational rewards are factor taxes.\textsuperscript{33} Following Avinash Dixit and Victor Norman (1980, 1986) and Robert C. Feenstra and Tracy R. Lewis (1994), I consider a two-stage compensation procedure. Because both Dixit and Norman (1986) and Feenstra and Lewis (1994) analyze Pareto-improving compensation schemes, the first stage of their analysis focuses on making everyone in the economy as well off as they would be under autarky. To achieve this, a policymaker must use both commodity taxes and factor taxes and, according to Feenstra and Lewis, relocation subsidies. Both Dixit and Norman and Feenstra and Lewis show that not only are government revenues from such first-stage schemes nonnegative, but also they return to individuals in the second stage.

\textbf{Definition 2} The \textit{compensation scheme}, $\sigma$, can be implemented in two stages. In the \textbf{first stage}, the government tries to minimize the rents that accrue to individual agents; in other words, it seeks to capture all these rents in the form of positive revenue. I refer to the results from this stage as the $\sigma 1$ equilibrium. In the

\textsuperscript{30} Similarly, there could be restricted trade \textit{ex ante}.


\textsuperscript{32} Negative taxes are the same as subsidies. This notion of factor taxes and commodity taxes has been adopted from the standard public economics textbook of Atkinson and Stiglitz (1990).

\textsuperscript{33} In Dixit and Norman, “commodity taxes” embrace both commodity and factor taxes, simply because they use a general approach that does not distinguish outputs from inputs.
**second stage**, the government returns this positive revenue to individual agents in the form of either a poll subsidy or lower commodity taxes. The result of this second stage is referred to as the \( \sigma 2 \) equilibrium. This \( \sigma 2 \) equilibrium could be referred to as a \( \sigma \) equilibrium, since the result of the second stage is also the result of the overall compensation scheme.

The purpose of the first stage is to ensure Pareto gains from trade by matching, as closely as possible, an equilibrium in which all individual agents are as well off as they are under autarky. The first stage may generate nonnegative government revenue (or positive revenue if there are strict production gains from trade). The second stage returns this nonnegative government surplus to individual agents in the form of either a poll subsidy or lower consumption taxes (higher factor subsidies). Since the technical requirements for the second-stage redistribution, including the Weymark conditions, (John A. Weymark 1979) have been closely examined by Dixit and Norman (1986), I take these results as given. I focus on the first-stage equilibrium.

First, I note the desirable and undesirable properties of the compensation scheme. Its single most important property is related to the concept of *ex post* Pareto efficiency.

**Definition 3** The compensation scheme, \( \sigma \), is **weakly Pareto improving** if every individual is at least as well off as he or she was under autarky.

Formally, the requirement for weak Pareto improvement is based on a comparison of individual welfare levels, \( W \):

\[
(W^j)^\sigma \geq (W^j)^A, \forall j \in J,
\]

where the superscript \( \sigma \) denotes individual welfare under the compensation scheme, \( \sigma ' \), and the superscript \( A \) denotes individual welfare under autarky. In both cases, \( W \), the welfare measure, indicates the real income of each individual; in this model, real income represents individual indirect utility. (See equation (11).)

Another important property of the first-stage equilibrium is that of rent neutrality. A positive rent arises if a policy change or change in the environment raises individual welfare. The gain is a premium or windfall profit, in the sense of a Marshallian rent. For example, if the inequality

\[
(W^j)^\sigma > (W^j)^A
\]

is satisfied for agent \( j \), that agent derives a strictly positive rent, with a value of \( (W^j)^\sigma - (W^j)^A \), from a policy shift from autarky to free trade under the compensation scheme, \( \sigma \). Two-stage compensation schemes are common in the existing literature because of the economist’s preference for discussing efficiency without addressing equity issues. Indeed, rent-neutral economic policy is desirable. That is, policy-induced arbitrary wealth redistribution, when the objective is a Pareto improvement from trade, should be avoided.
I say little about the second-stage redistribution of positive government revenues. I simply reiterate that rent neutrality is a desirable feature of the first stage of a compensation scheme. Evidence for this is that the first-stage equilibria of both Dixit and Norman and Feenstra and Lewis are consistent with rent neutrality. Next, I codify my definition of rent neutrality.

**Definition 4** The first-stage compensation equilibrium, $\sigma_1$, is **rent neutral** if all consumers have the same utility levels as under autarky. In other words, positive rents should become government revenues.

Dixit and Norman’s original first-stage equilibrium is rent neutral. This is because all the consumers are in the same situation as they were in under autarky in the first stage. Dixit and Norman generate this result by equating both output and input prices to their levels under autarky. Fixing input prices at the autarky level guarantees autarky-level incomes for consumers. If the policymaker were to fix output prices at the autarky level, consumers would be in the same utility-maximizing situation as they were under autarky, given that only income and output prices affect the consumer’s problem. The same is true of the Feenstra and Lewis scheme. The only difference is that, in their paper, relocation subsidies are given to some consumers to compensate for the loss of income arising from positive adjustment costs associated with moving factors from one industry to another. Under the assumptions made by Feenstra and Lewis (1994), the government offers the smallest relocation subsidy consistent with some consumers being indifferent between moving and not moving to a new industry. Hence, the first-stage equilibrium in Feenstra and Lewis’s scheme is also rent neutral.

As I show subsequently, in this paper, the government cannot achieve a rent-neutral first-stage equilibrium. To achieve a Pareto improvement relative to autarky, the government must provide positive rents to some groups of individual agents. I refer to this undesirable property as textit{overcompensation}.

**Definition 5** A scheme **overcompensates** a group of individuals if some within that group receive positive rents in the first-stage compensation equilibrium, $\sigma_1$.

Note that my definitions of overcompensation and rent neutrality represent two sides of the same coin. When the scheme is rent neutral, it does not overcompensate any group of consumers, and by same token, if the scheme is overcompensating some group, it is not rent neutral. However, I can identify the groups that receive positive rents, based on the definition of overcompensation.

The other important property of the compensation scheme concerns the budget of the government.

**Definition 6** The compensation scheme, $\sigma$, is **self-financing** if it achieves nonnegative government revenue in the first-stage equilibrium, $\sigma_1$:

$$B^{\sigma_1} \geq 0,$$  \hspace{1cm} (21)

26
where $B^{\sigma_1}$ is the net government balance from the first-stage equilibrium of the scheme; i.e., the revenue from taxes minus the cost of subsidies.

This definition of a self-financing scheme is adapted from the definition of self-financing tariffs introduced by Michihiro Ohyama (1972, p.49). A compensation scheme based on taxes and subsidies on economic variables is self-financing if the government can balance its budget solely from the net revenue earned from the scheme. The reason why equation (21) does not have a strict equality sign is that any positive revenue can be returned to individuals in the second stage.

The procedure that I adopt to design a compensation scheme is similar to those considered in Dixit and Norman (1986) and Feenstra and Lewis (1994). There are two important features. (1) The subsidies and taxes leave every consumer in the same situation as he or she was in under autarky. This policy may generate positive government revenues. (2) If there are positive revenues, the government returns these to individuals in the form of either poll subsidies or adjustments to taxes and/or subsidies. The latter is an option because, in this model, the Weymark condition of Dixit and Norman (1986) is automatically satisfied.\(^{34}\) In discussing the compensation scheme, I focus on the first step, which is to design a system of taxes and subsidies that leaves all consumers at least as well off as they were under autarky. This is because (the issues of) the second step is exhaustively discussed in the existing literature.

Another important property of any compensation scheme is its feasibility. Despite the fact that much of the literature (on mechanism design) discusses the concept of “feasibility” in terms of nonnegativity of governmental budgets (self-financing), this paper separates the governmental budget issues (discussed above) from the issues associated with the feasibility of a compensation scheme. In this paper, a scheme is feasible when the policy instruments of the government are based on observable variables.

**Definition 7** A scheme, $\sigma$, is **informationally feasible** if it is based solely on currently observable variables. Or if it is based on the variables that are regularly considered as tax-base.

This definition of informational feasibility is based on the observability of variables by the government. (Here, the phrase observable should not be interpreted literally. The observability relates to the concept of taxability. Therefore, I claim here that the variables are observable if the policymaker can use the variable as a tax base.) What are observable variables? Which characteristics of individuals are observable to policymakers? I propose the following three reasonable assumptions about observability. (1) The government records

\(^{34}\)The Weymark condition states that there is one good for which some consumers are net buyers and no consumer is a net seller. In traditional trade models, in which consumers are net sellers of factors of production and net buyers of consumer goods, this condition is automatically satisfied.
information on aggregate variables. (2) Therefore, it has information on aggregate variables under autarky. (3) Only current data on individuals are observed at no cost.

These assumptions make sense, because while most aggregate data is available in various forms, it is difficult to find past data that are specific to an individual. For example, the income tax rate is primarily determined by current income and does not usually depend on income from previous years. Thus, individual data for the autarky period are presumed to be costly to verify in the free-trade period.

Suppose that the government can observe (and use as tax-base) the following variables:

Y1. output prices, \( P_X, P_Y \) (both at the autarky and free-trade levels);

Y2. generic-factor prices, \( r \) (both at the autarky and the free-trade levels); and

Y3. residual returns (profits) from the individual’s current (free-trade) occupation.

In addition, I suppose that the government is able to observe the following two characteristics of individuals:

Y4. which industry the individual is currently working in; and

Y5. whether the individual has changed his or her occupation.

I further suppose that the government cannot observe the following variables:

N1. individual consumption vectors;

N2. individual generic-factor endowments;

N3. individual occupational-ability vectors; and

N4. residual returns (profits) from the individual’s previous (autarky) occupation.

Most of the above assumptions about observability are standard in the literature. (See, e.g., Roger Guesnerie (1995).) Given the assumption about the observability of profits, the following result can be used in subsequent analysis.

**Result 1** Given the production set-up of the model, and given that the government can observe the residual profits of individuals, a profits tax does not distort individual behavior. In other words, individuals maximize their profits truthfully, given that the elasticity of the after-tax (subsidy) share, with respect to profit, is larger than \(-1\). Formally, they do so whenever

\[
\varepsilon = \frac{\partial T/T}{\partial \pi/\pi} > -1,
\]

(22)

\(^{35}\)This relates to the absence of a cumulative-profit tax system of which the late William Vickrey of Columbia University had been a proponent ever since the 1940s.
where $T(\pi) = 1 - t(\pi)$, with $\pi$ being residual profit and $t(\pi)$ being an ad valorem tax rate (or, if $t(\pi)$ is negative, a subsidy rate).

See the Appendix for a proof. Note also that the linear tax has an elasticity of $\varepsilon = 0$ and thus satisfies condition (22). In addition, given that individual agents are assumed to be acting truthfully, I can conclude that the policymaker can observe agent's currently-used talents.

**Remark 1** Given the previous observation in Result 1 about the truthfully maximized current levels of individuals' residual returns, the government can recalculate $\theta$ for $X$-producers and $\tau$ for $Y$-producers. The planner can infer the amount of talent being used, as opposed to an agent's endowment of latent talent.

This is straightforward. If policymakers can condition their policy on current profits, then either

$$
\begin{align*}
\pi_X^i(P_X, r, \theta^i) &= \left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{a^{\frac{1}{\gamma}} - a^{\frac{\gamma}{\gamma}}}{\frac{a^{\frac{1}{\gamma}} - a^{\frac{\gamma}{\gamma}}}}\right) \cdot \theta^i, \\
\pi_Y^j(P_Y, r, \tau^j) &= \left(\frac{1}{1+\gamma}\right)^{\frac{1}{\gamma}} \left(\frac{a^{\frac{1}{\gamma}} - a^{\frac{\gamma}{\gamma}}}{\frac{a^{\frac{1}{\gamma}} - a^{\frac{\gamma}{\gamma}}}}\right) \cdot \tau^j.
\end{align*}
$$

Given the observability of aggregate variables such as the output prices, $P_X$ and $P_Y$, and the generic-factor return, $r$, the inversion of profit to type is a simple calculation. One might also say that the profit is a strictly increasing function of the size of the type, in which case, any tax-subsidy rate that is proportional to the observed profit could be used. Hence, it is almost as if the government observes the type.

Now that I have defined all the necessary properties of the compensation scheme and examined the relevant results, I examine the results of possible compensation schemes. I investigate two distinctive cases with respect to the timing of implementation. In the first case, an unanticipated compensation scheme, free trade occurs before the government announces that it will compensate the losers from trade. In the second case, an anticipated compensation scheme, individual agents expect the compensation scheme to be implemented by the government once there is free trade. In the following section, I investigate the first case.

## 5 An Unanticipated Compensation Scheme

Despite the tradition of lump-sum compensation being introduced before trade (or the opening the markets) (Mas-Colell et al. 1995, p.328), a more plausible and realistic policy is a “post-trade compensation scheme” (Murray C. Kemp and Henry Y. Wan 1986, p.99), in which the government first opens up to trade, then creates the compensation scheme to help losers from trade. Arguably, this unanticipated compensation scheme was used in the 1960s. In response to the Kennedy round of GATT multilateral tariff reductions, the United States government introduced the first TAA (trade adjustment assistance) program to accommodate workers displaced by the tariff reduction.
In this section, I explore a possible unanticipated posttrade compensation policy, given the informational restrictions described. In the next section, I examine the case in which individuals anticipate the existence of the compensation scheme and analyze how this anticipation affects individual incentives.

When designing the optimal compensating redistributing scheme, it is important to consider Pareto improvements over autarky. To design such a scheme, the policymakers must be aware of the informational feasibility constraint, given the limited observability of the unused talents of individual agents. When the scheme comprises two stages, the policymakers tries to accrue all the rents in the form of governmental revenues in the first stage. Thus, the ideal first-stage equilibrium is rent neutral. Because of the informational feasibility constraint, however, my model does not posit rent neutrality of the first-stage equilibrium. Nevertheless, I explore the process of creating a compensating scheme.

For analytic convenience, I focus on the case in which the price change occurs in one direction (the other case being completely symmetric). More specifically, this is the case in which the postrade price is \( p > p^4 \), and therefore, there are job switchers from sector \( Y \) to sector \( X \). Given the set-up of the model, described in Section 2, I consider five cases (Case I. - Case V.) relating to the gains and losses of different groups of individuals, as follows.

Case I. Generic-factor owners all gain, since \( r(p) > r(p^4) \). Specifically, the gain for those who own \( K^j \) is given by

\[
(r(p) - r(p^4)) \cdot K^j = a \cdot K^{-(1-a)} \cdot \left\{ [s(p)]^{1-a} - [s(p^4)]^{1-a} \right\} \cdot K^j > 0,
\]

where

\[
s(p) = p^{\frac{1-a}{1-a}} \cdot V^R_\theta(p) + p^{-\frac{1-a}{1-a}} \cdot V^S_\tau(p).
\]

Note that this group’s gain from trade is proportional to the agent’s endowment of the generic factor, \( K^j \). The multiplier component,

\[
a \cdot K^{-(1-a)} \cdot \left\{ [s(p)]^{1-a} - [s(p^4)]^{1-a} \right\},
\]

is the same for all agents. Both \( a \) and \( K \) are parameters of the model. Given the relative price change, \( p^4 \rightarrow p \), the values for both \( s(p^4) \) and \( s(p) \) are determined in the aggregate equilibrium. Because the policymaker knows the joint distribution of the talent vector, \((\theta, \tau)\), he or she also knows the values of \( V_\theta^R(p) \) and \( V_\tau^S(p) \) and, hence, of \( s(p) \) and \( s(p^4) \). Thus, by imposing on the market for generic factors an \textit{ad valorem} tax rate of

\[
t_{r(p)} = \frac{[s(p)]^{1-a} - [s(p^4)]^{1-a}}{[s(p)]^{1-a}},
\]

30
the policymaker can make the status of all owners of generic factors the same as that under autarky in the first-stage equilibrium.

Case II. Job stayers in sector $X$—those who are in the area $\tau < (p^A)^{\frac{a}{1-a}}\theta$—all gain, since $\pi^i_X(p) > \pi^i_X(p^A)$ when $p > p^A$. Specifically, the gain for those who have talent of $\theta^i$ is given by

$$\pi^i_X(p) - \pi^i_X(p^A) = K^a(1-a) \cdot \left( p^{\frac{a}{1-a}} [s(p)]^{-a} - p^A^{\frac{a}{1-a}} [s(p^A)]^{-a} \right) \cdot \theta^i > 0,$$  \hspace{1cm} (26)

where the definition of $s(p)$ is the same as in equation (24). Similarly to Case I, the gain from trade for job stayers in sector $X$ is proportional to agents’ endowments of used talent, $\theta^i$. The multiplier component,

$$K^a(1-a) \cdot \left( p^{\frac{a}{1-a}} [s(p)]^{-a} - p^A^{\frac{a}{1-a}} [s(p^A)]^{-a} \right),$$

is the same for all these agents. Thus, by imposing on the returns from talent of job stayers in sector $X$ an "ad valorem" tax rate of

$$t_{\pi X} = \frac{p^{\frac{a}{1-a}} [s(p)]^{-a} - p^A^{\frac{a}{1-a}} [s(p^A)]^{-a}}{p^{\frac{a}{1-a}} [s(p)]^{-a}},$$ \hspace{1cm} (27)

the policymaker can make the status of these individuals the same as that under autarky in the first-stage equilibrium.

Case III. Among job-switching individuals—those who are in the area $(p^A)^{\frac{a}{1-a}}\theta^i < \tau^i < \frac{g(p^W)}{g(p^A)}\cdot \theta^i$—all gain, since $\pi^i_X(p) > \pi^i_Y(p^A)$ when $p > p^A$. Specifically, the gain for those who have the talent vector, $(\theta^i, \tau^i)$, is given by

$$\pi^i_X(p) - \pi^i_Y(p^A) = g(p^W) \cdot \theta^i - g(p^A) \cdot \tau^i > 0,$$ \hspace{1cm} (28)

where

$$g(p^W) = p^{\frac{a}{1-a}} \left( \frac{1}{r(p)} \right)^{\frac{a}{1-a}} \left( a^{\frac{a}{1-a}} - a^{\frac{a}{1-a}} \right)$$

and where

$$g(p^A) = p^A^{\frac{a}{1-a}} \left( \frac{1}{r(p^A)} \right)^{\frac{a}{1-a}} \left( a^{\frac{a}{1-a}} - a^{\frac{a}{1-a}} \right).$$

Unlike in Cases I and II, the gain for job-switching individuals is not proportional to their endowments of used talent, $\theta^i$. Although $g(p^W)$ and $g(p^A)$ are the same for all these individuals and the policymaker can calculate $g(p^W)$ and $g(p^A)$, the gain, $g(p^W) \cdot \theta^i - g(p^A) \cdot \tau^i$, depends on both elements of the talent vector, $(\theta^i, \tau^i)$, which is not observed by the policymaker. The policymaker could recalculate the value of used talent, $\theta^i$ based on the profits from the
production of $X$. However, the value of $\tau^j$, is not known by the policymaker. To understand this, suppose that the policymaker would like to impose an \textit{ad valorem} tax rate of

$$
t_{\pi X - Y} = \frac{g(p^W) \cdot \theta^j - g(p^A) \cdot \tau^j}{g(p^W) \cdot \theta^j} = 1 - \frac{g(p^A) \cdot \tau^j}{g(p^W) \cdot \theta^j}, \quad (29)$$


to make these Case III individuals as well off as they were under autarky. However, the tax rate to be imposed by the policymaker should be of the form $t_{\pi X - Y}(\pi_X(\theta))$. That is, it should be based only on the currently observable $\pi_X(\theta)$, which depends on the current use of talent, $\theta$.

Case IV. Other job-switching individuals—those in the area $\frac{g(p^W)}{g(p^A)} \cdot \theta^j < \tau^j < p^{\frac{a}{1-a}} \theta^j$—all lose since $\pi_{Y1}^i(p) < \pi_{Y0}^i(p^A)$ when $p > p^A$. Specifically, the loss for those who have talent of $(\theta^j, \tau^j)$ is given by

$$- \left( \pi_{Y1}^i(p) - \pi_{Y0}^i(p^A) \right) = g(p^A) \cdot \tau^j - g(p^W) \cdot \theta^j > 0. \quad (30)$$


This case is quite similar to Case III, when it comes to the loss for each individual and the subsidy rate. The infeasible subsidy rate that the policymaker would like to impose on this group is

$$s_{\pi X - Y} = \frac{g(p^A) \cdot \tau^j - g(p^W) \cdot \theta^j}{g(p^W) \cdot \theta^j} = \frac{g(p^A) \cdot \tau^j}{g(p^W) \cdot \theta^j} - 1, \quad (31)$$


whereas the feasible subsidy rate must be of the form $s_{\pi X - Y}(\pi_X(\theta))$.

Case V. Job-staying individuals in sector $Y$—those who are in the area $p^{\frac{a}{1-a}} \theta < \tau$—all lose, since $\pi_{Y1}^i(p) < \pi_{Y0}^i(p^A)$ when $p > p^A$. More specifically, the loss for those who have talent of $\tau^j$ is given by

$$- \left( \pi_{Y1}^i(p) - \pi_{Y0}^i(p^A) \right) = K^a(1-a) \cdot \left(p^{\frac{a}{1-a}} [s(p^A)]^{-a} - p^{\frac{a}{1-a}} [s(p)]^{-a}\right) \cdot \tau^j > 0. \quad (32)$$


Similarly to Cases I and II, the gain from trade for job stayers in sector $Y$ is proportional to their endowments of used talent, $\tau^j$. The multiplier component,

$$K^a(1-a) \cdot \left(p^{\frac{a}{1-a}} [s(p^A)]^{-a} - p^{\frac{a}{1-a}} [s(p)]^{-a}\right),$$


is the same for all these agents. Thus, by imposing on the returns from talent of job stayers in sector $Y$ an \textit{ad valorem} subsidy rate of

$$s_{\pi Y} = \frac{p^{\frac{a}{1-a}} [s(p^A)]^{-a} - p^{\frac{a}{1-a}} [s(p)]^{-a}}{p^{\frac{a}{1-a}} [s(p)]^{-a}}, \quad (33)$$


the policymaker can make the status of all the job-staying individuals in sector $Y$ the same as it was under autarky in the first-stage equilibrium.
It is instructive to look at a first-best case, even if in reality it is impossible to achieve. Consider the following first-best scheme.

**Scheme 1** As a first-stage equilibrium, tax the winning groups (Cases I, II and III) and subsidize the losing groups (Cases IV and V) in amounts equal to their gains and losses, so that every individual is in the same situation as he or she was in under autarky. Such tax and subsidy rates are represented by the equations (25), (27), (29), (31), and (33).

This hypothetical first-best scheme would be rent neutral. However, while the taxation and subsidy schemes for Cases I, II and V are feasible, the determination of the tax and subsidy rates for the job switchers, Cases III and IV, must be based on a combination of observable and unobservable variables. The government cannot distinguish between the groups in Cases III and IV because it cannot observe the relative value of \((\theta^i, \tau^i)\) for each individual. The policymaker can observe only the profit from current production and thus can observe, when \(p > p^4\), only the profit from production in sector \(X\). The policymaker cannot observe (or condition the taxation scheme on) the counterfactual profit from sector \(Y\) that is proportional to the agent’s unused latent talent, \(\tau\). In terms of Fig. 8, for instance, this means that the government cannot distinguish between points \(q\) and \(r\), because in equilibrium, the individuals at these points earn the same profit and produce the same amount of product \(X\). This leads to the following result.

**Proposition 4** Given the set-up of the model, if the government is aiming to achieve a Pareto improvement over autarky, there is no informationally feasible first-stage compensated equilibrium that is rent neutral.

By consulting the equations (23), (26), and (32), which represent the gains and losses for the various groups of individuals, I establish the taxation and subsidy rates for the following three groups of individuals and make them as well off as they were under autarky: (a) owners of the generic-factor \(K\), at the rate (25); (b) job stayers in sector \(X\), at the rate (27); and (c) job stayers in sector \(Y\), at the rate (33). I can do this because these individuals’ gains and losses are proportional to their factor returns (in terms of both residual profits and generic-factor returns), and thus also proportional to their employed talents (or factor endowments). In this case, a linear tax or subsidy system applies. (Recall, from Result 1 in section 4, that any linear tax-subsidy system is incentive compatible.)

I now focus on job-switching individuals. From equations (28) and (30), individual gains or loss depend on relative amounts of used talent, \(\theta\) and unused latent talent, \(\tau\). Because the policymaker does not have data on each individual—past profits and losses—the policymaker can base a taxation-subsidy scheme only on currently observable variables. In this case, the current profit from sector-\(X\) production is observable. In effect, the policymaker can observe \(\theta\) but not \(\tau\). (The policymaker observes the profits of the individual agents.
If profit is reported truthfully, the policymaker can infer the amount of talent being used. See Remark 1 in section 4.) Thus, the policymaker cannot make all job-switching individuals as well off as they were under autarky, except in a case that I examine subsequently. Hence, I conclude as follows.

**Proposition 5** Given the set-up of the model, if the government is aiming to achieve a Pareto improvement over autarky, an informationally feasible posttrade compensation scheme must overcompensate job-switching individuals in its first-stage equilibrium.

If the policymaker’s most pressing concern is to ensure a Pareto improvement over autarky, then the informationally feasible scheme must overcompensate job-switching individuals. The preceding analysis shows that the policymaker can tax and subsidize job stayers in a rent-neutral manner but cannot do so for job switchers simply because the policymakers can only observe their \( \theta \), not \( \tau \).

I return temporarily to Fig. 7, which has a unit-square support for the joint distribution of talents. The left-hand side of the figure contains lines that represent the same percentage change in the gain or loss from trade. The right-hand side contains lines indicating that those individuals are making the same amount of residual profit. The iso-percentage-gain-or-loss lines are rays from the origin, and the iso-current-profit lines for X producers are parallel vertical lines.

While this first-best scheme requires a linear taxation-subsidy system to be imposed along the iso-percentage-gain-or-loss lines, the policymaker only observes the differences between individuals along the iso-current-profit lines. This is because job-switching individuals appear the same when they are earning the same profit, and hence are represented by the same iso-current-profit line.

Of those who earn the same profit, it is individuals on the upper bound of the iso-current profit line who gain least (lose most) from trade. Since the policymaker cannot distinguish between the individuals on the same iso-profit line, the policymaker must compensate all the individuals on the same profit line at the same level as the least fortunate individual, who is on the upper bound of that line. However, apart from the least fortunate individual, individuals receiving the same amount of compensation from the policymaker have positive rents because their iso-percentage-gain-or-loss lines are higher than the individual on the upper bound.

Review the two points \( q \) and \( r \) in Fig. 7, which are on the same iso-current-profit line. Thus, although they appear the same to the policymaker, \( q \) represents a loser while \( r \) is a winner. Nevertheless, compensation must be the same for both. Even if the individual at \( r \) is a winner, he or she receives the same amount of subsidy (as opposed to paying a tax) as the individual at point \( q \). Hence, a government aiming for a Pareto improvement inevitably overcompensates job-switching individuals.

To explore this more thoroughly, I define the iso-current-profit set, \( I^{CP}(\theta^*) \).
Definition 8 The iso-current-profit set, $I^{CP}(\theta^*)$, is the set of all those job-switching individuals who have the same talent, $\theta^*$:

$$I^{CP}(\theta^*) \equiv \{(\theta^j, \tau^j) \in C_{Y-X} : \theta^j = \theta^*\},$$

where $C_{Y-X}$ is a partition of job switchers; i.e.,

$$C_{Y-X} \equiv \left\{(\theta^j, \tau^j) \in \Theta : (p^A)^{\frac{\tau^j}{\theta^*}} \theta < \tau^j < (p^A)^{\frac{\tau^*}{\theta^*}} \theta\right\}.$$ Note that $I^{CP}(\theta^*)$ is a linear, one-dimensional subspace of $\mathbb{R}^2$. Let $\tau(\theta^*)$ be the lower bound for the value of the element $\tau$ in a set $I^{CP}(\theta^*)$, and let $\overline{\tau(\theta^*)}$ be the upper bound for the same subspace. Note that $\overline{\tau(\theta^*)}$ is equal to $(p^A)^{\frac{\tau^*}{\theta^*}} \theta^*$, whereas $\overline{\tau(\theta^*)}$ depends on the value of $\theta^*$. In particular,

$$\overline{\tau(\theta^*)} = \sup \left\{ (p^A)^{\frac{\tau^*}{\theta^*}} \theta^*, \overline{\tau(\theta^*)}\right\},$$

where $\overline{\tau(\theta^*)}$ is an upper bound for the element $\tau$ in the whole $\Theta$ space when $\theta^j = \theta^*$. In the case of a unit-square support for the joint distribution, $\overline{\tau(\theta^*)} = 1$.

Because all individuals in the set $I^{CP}(\theta^*)$ are job switchers from sector $Y$ to sector $X$, they are currently producing output $X$. Since all members of the set $I^{CP}(\theta^*)$ have the same talent, $\theta^*$, their profit is the same: $\pi^j_X(p, r(p), \theta^*)$. Their individual gains or losses, however, differ because they have different latent talents, $\tau$. Given (28) and (30), the individual gains or losses can be expressed as $|g(p^W) \cdot \theta^* - g(p^A) \cdot \tau^j|$. Whether individual $j$ (who has the talent $\theta^*$) gains or loses, and what the gain or loss is, depends on the value of $\tau^j$. Among those who belong to the set $I^{CP}(\theta^*)$, there are many individuals who have the latent talent $\tau$ in the interval $[\tau^{\theta^*}, \overline{\tau(\theta^*)}]$. The policymaker, however, cannot distinguish between them.

A policymaker who wants to ensure Pareto gains from trade must be sure to make the least well-off individual as well off as he or she was under autarky. Note also that this least well-off individual must have had the most talent in the previous sector, $Y$, and hence must have been the one with the most latent talent, $\overline{\tau(\theta^*)}$. Therefore, for all individuals, $(\theta^*, \tau) \in I^{CP}(\theta^*)$, the subsidy or tax must be $|g(p^W) \cdot \theta^* - g(p^A) \cdot \overline{\tau(\theta^*)}|$.

The ad valorem rate for any individual with the profit $\pi(\theta^*)$ is

$$t_{\pi X-Y}(\pi(\theta^*)) = \left| \frac{g(p^W) \cdot \theta^* - g(p^A) \cdot \overline{\tau(\theta^*)}}{g(p^W) \cdot \theta^*} \right|. \quad (34)$$

If $g(p^W) \cdot \theta^* - g(p^A) \cdot \overline{\tau(\theta^*)} > 0$, equation (34) represents a tax rate. If $g(p^W) \cdot \theta^* - g(p^A) \cdot \overline{\tau(\theta^*)} < 0$, it represents a subsidy rate. With the exception of the individual at the point $(\theta^*, \overline{\tau(\theta^*)})$, which represents zero, all individuals in the set $I^{CP}(\theta^*)$ are overcompensated, since the inequality

$$g(p^W) \cdot \theta^* - g(p^A) \cdot \overline{\tau(\theta^*)} < g(p^W) \cdot \theta^* - g(p^A) \cdot \tau \quad (35)$$

must hold for all those with the latent talent, $\tau \in [\tau^{\theta^*}, \overline{\tau(\theta^*)}]$. 35
From (35), it follows that
\[
\int_{\tau(\theta^*)}^{\tau(\theta^*)} \left\{ g(p^W) \cdot \theta^* - g(p^A) \cdot \tau(\theta^*) \right\} f(\theta^*, \tau)d\tau < \int_{\tau(\theta^*)}^{\tau(\theta^*)} \left\{ g(p^W) \cdot \theta^* - g(p^A) \cdot \tau \right\} f(\theta^*, \tau)d\tau.
\]
Integrating over all job-switching individuals yields
\[
\int_{C_{Y-X}} \int_{\tau(\theta^*)}^{\tau(\theta^*)} \left\{ g(p^W) \cdot \theta^* - g(p^A) \cdot \tau(\theta^*) \right\} f(\theta, \tau)d\theta d\tau < \int_{C_{Y-X}} \int_{\tau(\theta^*)}^{\tau(\theta^*)} \left\{ g(p^W) \cdot \theta^* - g(p^A) \cdot \tau \right\} f(\theta, \tau)d\theta d\tau,
\]
with the integration being over $\theta^*$ for all job-switching individuals. The difference between the right- and left-hand sides of the inequality (36) relates to the total amount of overcompensation for job-switching individuals.

These overcompensation results lead to the following proposition.

**Proposition 6** An *informationally feasible* posttrade compensation policy that achieves weak Pareto improvement may or may not be self-financing, depending on the joint distribution of individual talents.

According to Ohyama (1972), a Pareto-improving compensation scheme is self-financing if the set of aggregate consumption possibilities is larger than that under autarky, if there is a lump-sum transfer. In this model, however, when I impose the informational feasibility condition, a compensation scheme without a lump-sum transfer may or may not be self-financing. This is because overcompensating job-switching individuals may cancel out the positive aggregate rents arising from trade. Whether the amount of overcompensation is large depends on the shape of the joint distribution of talents. In particular, if the total mass of job-switching individuals is large, then the total amount of overcompensation is high. Some parameter values then imply that the total compensation scheme is not self-financing.

I now consider an example in which the support of the joint distribution is a unit square. Figure 9 illustrates the scheme for this case. For this unit-square case, I introduce a finer separation of the partition $C_{Y-X}$ into two groups: a group of absolute gainers and a group of gainers and losers—based only on the observable variables. For clarification, consider the following:

(i) generic-factor owners: same as Case I;

(ii) all individuals in partition $C_{X-X}$: same as Case II;

(iii) those individuals in partition $C_{Y-X}$ who meet the condition $\theta > \frac{\tilde{g}(p^A)}{\tilde{g}(p^W)}$;

(iv) those individuals in partition $C_{Y-X}$ who meet the condition $\theta < \frac{\tilde{g}(p^A)}{\tilde{g}(p^W)}$; and

(v) all individuals in partition $C_{Y-Y}$: same as Case V.
Figure 9: The informationally feasible posttrade compensation scheme.

Note that in Fig. 9, the dotted line, OZ, denotes the zero-gain line: \( \theta = \frac{g(p^*)}{g(p^w)} \cdot \tau \). This categorization uses only observable variables because the distinction between partition (iii) and partition (iv) is based solely on \( \theta \), which can be inferred from individuals’ current profits. Given this new categorization, I propose a revised posttrade compensation scheme.

**Scheme 2** As a first-stage equilibrium, tax (i), (ii) and (iii) and subsidize (iv) and (v). Note in particular that the tax and subsidy rates are represented by the following equations: (25) for (i); (27) for (ii); (34) for groups (iii) and (iv); and (33) for (v).

Since this scheme is based on observable variables, it is feasible. However, it is second best because groups (iii) and (iv) are overcompensated. This is inevitable given that winners and losers in this category are indistinguishable.

To find the appropriate tax-subsidy rates, I obtain the minimum subsidy rate and the maximum tax rate for each group that satisfies the weak-Pareto-improvement requirement shown in (19). Because my model uses a price normalization that ensures that nominal income is equal to real income, it is easy to find the tax-subsidy rates for all groups at which everyone is as well off as they were under autarky. Note that the tax-subsidy rates must be based on observable variables or variables that are easily calculated. Thus, the features of the tax-subsidy rates for each group are:

(i) (linear) factor (commodity) tax on generic factors;
(ii) (linear) profits tax on the occupation rewards for job-staying producers of output $X$;

(iii) (nonlinear) profits tax on the occupation rewards for job-switching producers of output $X$;

(iv) (nonlinear) profits subsidy on the occupation rewards for job-switching producers of output $X$; and

(v) (linear) profits subsidy on the occupation rewards for job-staying producers of output $Y$.

The linear factor tax for generic-factor owners is the same as that in the first-best case. Now I focus on the individual heterogeneity of talents. Given the above categorization, I denote the partitions of the ability vector space more finely, as follows.

1. $\mathcal{C}_{X-X} \equiv \left\{ (\theta^i, \tau^j) \in \Theta : \tau^j < (p^A)^{\frac{1}{p^A}} \theta^i \right\}$

2. $H = \mathcal{C}_{Y-Y}^H \equiv \left\{ (\theta^i, \tau^j) \in \Theta : p^{\frac{1}{p}} \theta > \tau^j > (p^A)^{\frac{1}{p^A}} \theta^i \text{ and } 1 > \frac{g(p^W)}{g(p^A)} \cdot \theta^i \right\}$

3. $M = \mathcal{C}_{Y-Y}^M \equiv \left\{ (\theta^i, \tau^j) \in \Theta : p^{\frac{1}{p}} \theta > \tau^j > (p^A)^{\frac{1}{p^A}} \theta^i \text{ and } 1/(p^{\frac{1}{p^A}}) < \theta^i < \frac{g(p^w)}{g(p^A)} \right\}$

4. $L = \mathcal{C}_{Y-Y}^L \equiv \left\{ (\theta^i, \tau^j) \in \Theta : p^{\frac{1}{p}} \theta > \tau^j > (p^A)^{\frac{1}{p^A}} \theta^i \text{ and } 0 < \theta^i < 1/(p^{\frac{1}{p^A}}) \right\}$

5. $\mathcal{C}_{Y-Y} \equiv \left\{ (\theta^i, \tau^j) \in \Theta : p^{\frac{1}{p}} \theta < \tau^j \right\}$

The groups of job stayers, $\mathcal{C}_{X-X}$ and $\mathcal{C}_{Y-Y}$, face the same linear tax-subsidy scheme as in the first-best case. Thus, I focus on the job switchers, $H$, $M$ and $L$, all of whom are currently producing the output $X$. Because the government cannot distinguish between those earning the same profit from their production of $X$, the policymaker must take from (give to) each individual the same tax (subsidy) as that taken from (given to) the individual who gains the least (loses the most) among those earning the same profit. For a given profit, those who gain the least are those who have the most latent ability to produce $Y$. For the groups $H$ and $M$, those who gain the least (lose the most) are the individuals with $\tau(\theta^*) = 1$. For group $L$, they are $\tau(\theta^*) = p^{\frac{1}{p}} \theta^*$.

Next, I check the optimal tax rate for those who have an ability vector $(\theta^*, 1)$, where $1 \geq \theta^* > 1/(p^{\frac{1}{p^A}})$, and the optimal tax rate for those with a vector $(\theta^*, p^{\frac{1}{p}} \theta^*)$, where $0 < \theta^* < 1/(p^{\frac{1}{p^A}})$. Thus, the individuals in group $H$ who earn $\pi(\theta^*)$ have imposed on them a tax rate of

$$t_H(\pi(\theta^*)) = \frac{g(p^W) \cdot \theta^* - g(p^A)}{g(p^W) \cdot \theta^*} - \delta(\theta^*),$$

while the individuals in group $M$ who earn $\pi(\theta^*)$ are given a subsidy of

$$s_M(\pi(\theta^*)) = \frac{g(p^W) - g(p^A) \cdot \theta^*}{g(p^W) \cdot \theta^*} + \delta(\theta^*),$$

38
where \( \delta(\theta^*) > 0 \) represents an arbitrarily small number for which \( \delta'(\theta^*) > 0 \). The purpose of this additional term is to avoid breaching the condition \( \varepsilon = \frac{\partial \varepsilon}{\partial \theta^*} > -1 \), arrived at in Result 1 of the previous section. Without this term, \( \delta(\theta^*) \), the condition is \( \varepsilon = -1 \). (For a formal proof, see the Appendix.) The group-\( L \) individuals have the linear subsidy

\[
s_L = \frac{g(p^A) \cdot p^{-\pi \theta^*} - g(p^W) \cdot \theta^*}{g(p^W) \cdot \theta^*} = \frac{g(p^A) \cdot p^{1 - \pi \theta^*} - g(p^W)}{g(p^W)}
\]

This completes the description of the tax-subsidy scheme for the first-stage equilibrium in the unit-square case.

6 An Anticipated Compensation Scheme

In the previous section, a compensation program was implemented after the economy opened to trade. The introduction of the program is assumed to have been a surprise. This may have been the case in the 1960s, but may not describe more recent situations. Once a compensation scheme is in place, individual agents take its existence into account. They change their behavior because the program affects their incentives.\(^3\) In this section, I analyze an anticipated compensation scheme.

To begin, I consider the situation in which individual agents expect the compensation program to exist and they behave accordingly. In the previous section, some agents switched occupations before knowing whether there would be a compensation scheme. In this section, I posit that some individual agents who had changed their jobs under that scenario (without compensation) may not switch their occupations if they expect compensation only if they remain in a declining industry. This is inevitable, since any compensation scheme must specify the tax and subsidy rates not just for job switchers but also for job stayers. When job stayers stay in their own industry, policymakers cannot tell if they are counterfactual job switchers. Indeed, one can only tell which job stayers have changed their jobs because of the compensation scheme. With this difficulty in mind, I analyze an anticipated compensation scheme.

I use the same approach as before. In the first-stage equilibrium, the policymaker tries to make agents at least as well off as they were under autarky.\(^3\) Any nonnegative revenues that accrue to the government can

\(^3\)The argument is analogous to the Friedman-Phelps hypothesis about the natural rate of unemployment. Policymakers who try to take advantage of the Phillips curve by choosing higher inflation to reduce unemployment only succeed in reducing unemployment temporarily. High inflation shifts the augmented Phillips curve upwards because expected inflation at the natural rate of unemployment rises. Thus, policymakers must wait for a long time before they can take advantage of surprise inflation. By a similar logic, the policymaker cannot take advantage of an unanticipated compensation scheme for long.

\(^3\)It may be necessary to provide some positive surplus for informational reasons.
be returned to agents in the second stage. I consider the following tax scheme for the producers of $X$ under autarky.

1. For those who stay in industry $X$, there is a linear tax rate of

$$t_{ant} = \frac{\pi^j_{X1} - \pi^j_{X0}}{\pi^j_{X1}} = \frac{p^{1-a} [s(p)]^{-a} - p^{1-a} [s(p^A)]^{-a}}{p^{1-a} [s(p)]^{-a}}.$$  

This tax rate can make job stayers in $X$ indifferent between compensation and autarky.

2. For those who switch from industry $X$ to industry $Y$, there is a linear tax rate of

$$t^*_{ant} > \frac{\pi^j_{X1} - \pi^j_{Y0}}{\pi^j_{X1}} = \frac{p^{1-a} [s(p)]^{-a} - p^{1-a} [s(p^A)]^{-a}}{p^{1-a} [s(p)]^{-a}}.$$  

In practice, there are no job switchers in this direction, given the change in the terms of trade.

Thus, all members of the $C_{X-X}$ group stay in industry $X$, and all must pay the amount of tax that makes them indifferent between compensation and autarky. No agent switches from $X$ to $Y$, since paying tax at the rate $t^*_{ant}$ makes no sense.

Now, to ensure that those in group $C_{Y-Y}$ are at least as well off as they were under the autarky situation, I consider the following subsidy scheme for the producers of $Y$ under autarky.

3. Any producer of $Y$ under autarky who chooses to stay in industry $Y$ under free trade is granted a positive subsidy, which is proportional to his or her occupational return in producing $Y$. The linear subsidy rate is

$$s_{ant} = \frac{\pi^j_{Y0} - \pi^j_{Y1}}{\pi^j_{Y1}} = \frac{p^{1-a} [s(p^A)]^{-a} - p^{1-a} [s(p)]^{-a}}{p^{1-a} [s(p)]^{-a}}.$$  

This offer by the government guarantees that no one is made worse off by trade, because the autarky producers of $Y$ now have the option of staying in the same industry and earning the same return as before.

The government specifies the tax-subsidy scheme for those who switch from sector $Y$ to sector $X$—namely, the group $C_{Y-X}$. For a more rigorous analysis, I consider Fig. 10, in which there is a unit-square support.

I divide the unit square into five partitions. As well as natural job stayers—the groups $C_{X-X}$ and $C_{Y-Y}$—there are three new groups of counterfactual job switchers. These are: (1) $D$, comprising individuals who were job switchers under free trade but who remain in industry $Y$; (2) $L$, comprising winning job switchers under free trade but whose current profits are indistinguishable from those of losing job switchers; and (3) $H$, comprising winning job switchers under free trade whose current profits exceed those of losing job switchers.
With respect to group $D$, the government cannot do better than to implement the above subsidy scheme, targeting those who stay in industry $Y$. If the latter decide to stay in sector $Y$, they are indistinguishable from natural stayers in that sector. Therefore, the tax scheme targets two groups primarily: $L$ and $H$. This entails the following.

4. Tax Exemption for group $L$. Those who are in this group are natural gainers from trade. Therefore, despite the subsidy for job stayers in sector $Y$, the agents find it profitable to switch occupations, conditional on the tax exemption in the new sector.

5. Group $H$ is taxed at the same rate as in the posttrade unanticipated scheme:

$$t^{**}(\pi(\theta^*)) = \frac{\pi^j_{X1} - \pi^j_{X0}}{\pi^j_{X1}} \frac{g(p^W) \cdot \theta^* - g(p^A)}{g(p^W) \cdot \theta^*} - \delta(\theta^*).$$

Then, all except those who have $\tau = 1$ gain a positive rent. Thus, this tax rate is incentive compatible for those who are in group $H$. The term $\delta(\theta^*)$ has the same property as in the previous section.

This scheme satisfies all three conditions: it has informational feasibility, it delivers weak Pareto improvement, and it is self-financing. It is informationally feasible since all tax and subsidy rates are incentive compatible. It is weakly Pareto improving since every agent is at least as well off as under autarky. If there are aggregate gains from trade, the tax revenues from this scheme exceed the costs of subsidy. The net government revenues brought in by the job-staying individuals in both sectors $X$ and $Y$ are likely to be positive.

With respect to the job switchers, who created an overcompensation problem in the unanticipated case, this
scheme either taxes some or exempts some from tax; hence, the policymaker generates strictly positive tax revenue. Although there are some positive rents, and hence overcompensation in the form of smaller taxes for group $H$, this overcompensation does not negatively affect the government budget since it takes the form of a smaller-than-ideal tax rate.

Nevertheless, the allocation achieved in this scheme is not without costs. Although the scheme satisfies informational feasibility, delivers weak Pareto improvement, and is self-financing, it generates aggregate-level inefficiency in the form of a smaller aggregate consumption possibility set when evaluated at the world price. Smaller aggregate gains arise because there are fewer job switchers.

**Proposition 7** There is an anticipated (ex ante) compensation program that is informationally feasible, weakly Pareto improving and self-financing. The aggregate consumption possibilities set is smaller than that of the unanticipated (ex post) scheme.

Furthermore, in the context of the current TAA program, I find a striking result. Noting that my model does not have frictional costs for occupation switching, I propose taxing at a positive rate or exempting from tax those who switch occupations. This contradicts the results in Feenstra and Lewis (1994), which propose a relocation subsidy for job switchers. My optimal scheme suggests to the contrary that the policymaker should give no subsidy to job switchers. I propose that the subsidy be given only to job stayers who remain in a declining industry. Given that the model has no frictional moving (between sectors) costs, it is no surprise to obtain this negative result relating to the current TAA, which provides a poll subsidy to occupation switchers.

**Proposition 8** The poll subsidy for those who have changed industries creates a disincentive. It induces an inefficient allocation of individuals.

Given the set-up of the model in this paper, the minimum subsidy for job-switching individuals must be nonpositive; i.e., it must contain a tax exemption for group $L$ and a positive tax for group $H$. By giving a positive subsidy to job-switching individuals, some job stayers in sector $Y$ (particularly those closer to the zero-gain line, $OZ$) may find it profitable to move to sector $X$. However, while this positive subsidy is successful in inducing some counterfactual job switchers to move to a more efficient sector (in the posttrade world), it also creates a huge side effect. Because the policymaker cannot distinguish between counterfactual job switchers and natural (winning) job switchers, a positive subsidy overcompensates job switchers who are on the same iso-current-profit lines. The policymaker must offer the same tax-subsidy rates that apply in the unanticipated posttrade compensation scheme if the government is maximize the number of job switchers and thereby maximize aggregate production gains. This subsidy generates overcompensation and makes self-financing questionable.
When the policymaker wants to balance the budget, taxing job switchers may be a policy option.\textsuperscript{38} Taxing job switchers, but not too heavily, may induce some natural job switchers to change their occupations. Since these job switchers pay tax, this policy helps to balance the budget problem but may induce fewer individuals to switch to an efficient industry. More individuals will remain in a declining industry. Thus, the trade-off between the government budget and aggregate gains remains.

The preceding analysis has shown that, in the case of an anticipated compensation scheme in which the government aims to attain a Pareto improvement over autarky, there is a trade-off between the aggregate production gains from trade and the amount of overcompensation.

7 Conclusion

In this paper, I have developed a model that predicts aggregate production gains from trade. I have attempted to model a realistic situation in which individual agents often find themselves. I assume that individual agents must choose one job at a time and that they are endowed with multi-valued talents in various sectors. Productivity is assumed to differ between agents. This set-up creates winners and losers from trade, but the gains and losses are based on the talents that agents use relative to their hidden latent talents. If the government chooses to impose a realistic tax-subsidy scheme on current factor prices and profits, policymakers face a trade-off between Pareto improvement and overcompensation. In other words, if policymakers do achieve a Pareto improvement, the compensation scheme necessarily overcompensates job-switching individuals. If, on the other hand, policymakers rigorously avoid overcompensation because they care about a balanced budget, the compensation program is not Pareto improving.

In addition to this trade-off, when a compensation scheme is anticipated by individual agents, there is another trade-off, which is between overcompensation and aggregate production gains. Although most policymakers are aware of these trade-offs, few studies of the issue exist. Thus, in this paper, I have developed a theoretical framework to explain the trade-offs that governments face when trying to implement compensating redistribution schemes.

In this paper, I have also provided an explanation of the difficulty in distinguishing winners from losers when an economy opens to trade. Such distinctions have been made in the context of basic trade models, such as the Heckscher–Ohlin model and the specific-factors model. Feenstra and Lewis (1994) noted the difficulty of identification in their imperfectly mobile factors model, which they developed to investigate heterogeneous adjustment costs. While Feenstra and Lewis assumed positive adjustment costs for their imperfectly mobile

\textsuperscript{38}I thank Professor Eichi Miyagawa for pointing out the possibility of this type of policy.
factors, my model reveals cases in which the adjustment costs for some job-switching agents may be negative and, hence, there are gainers. Thus, the poll subsidy for job-switching individuals (proposed by Feenstra and Lewis) may not be desirable in the context of my model. Furthermore, any observation of current profits does not reflect actual gains or losses from opening to trade. This makes it difficult for any government to implement a reliably Pareto-improving compensation scheme that bases taxes and subsidies on current variables.

This paper has provided a model of individuals’ occupational-choices and welfare changes when the economy faces a change in the terms of trade, and in particular, a change from autarky to free trade. I found that there are both winners and losers among job switchers. However, although this paper’s analysis can explain individuals’ long-run gains and losses from moving to a new sector, the model does not take into account short-run costs of labor adjustment. (I implicitly assumed that frictional unemployment costs are zero.) Therefore, the paper’s chief theoretical result—that no positive subsidy should be given to job-switching individuals in a self-financing compensation scheme—should not be taken too literally. Indeed, the compensation provided by the United States Department of Labor through its trade adjustment assistance (TAA) program involves a relocation subsidy for those who move to a new location when switching jobs due to trade. Such a program may be justified to the extent that there are short-run frictional costs associated with job switching.

A simplifying assumption made in this paper is that occupational talents are exogenously given for each individual. In reality, people may invest much of their time in expanding their skills. I have omitted the possibility of such dynamic development of individual talents through human-capital investment. Gene M. Grossman and Carl Shapiro (1982) analyzed the determinants of individual talent training when the individual agents are identical \textit{ex ante}. An interesting extension of this paper’s model would be to incorporate a dynamic formation of specific factors, by allowing for investment in individual occupational talents. This is a promising avenue for future research.

References


A  Proofs

Proof of Lemma 1. From (12),

\[ \left[ p \frac{1}{\alpha} \cdot V^R_0(p) + p^{-\frac{1}{\alpha \tau}} \cdot V^S_\tau(p) \right] = K \cdot \left( \frac{r(p)}{a} \right)^{\frac{1}{\alpha \tau}}. \]

Then, by substituting (4) into (13) I get

\[ I(p) = r(p) \cdot K + \left( \frac{1}{r(p)} \right)^{\frac{1}{\alpha \tau}} \left( a \frac{1}{\alpha} - a \frac{1}{\alpha \tau} \right) \left[ p \frac{1}{\alpha} \cdot V^R_0(p) + p^{-\frac{1}{\alpha \tau}} \cdot V^S_\tau(p) \right]. \]

Combining the above two equations yields

\[ I(p) = r(p) \cdot K \cdot \left( 1 + \left( a \frac{1}{\alpha} - a \frac{1}{\alpha \tau} \right) \cdot a \frac{1}{\alpha \tau} \right). \]

Simplifying yields

\[ I(p) = \frac{r(p) \cdot K}{a}, \]

which is precisely equivalent to the condition in (14).
Some mathematical derivations for the Proof of Lemma 2

The changes with respect to each specific factor’s economy-wide employment have opposite signs; i.e.,

\[ \text{sign} \left( \frac{dV^R_s}{dp} \right) = -\text{sign} \left( \frac{dV^S_s}{dp} \right) \text{ for some } dp. \]

Then, by taking the total derivative of the autarky condition in (17) with respect to \( p \), and after adjusting the sign, I arrive at

\[
\frac{1}{p(1 - a)} \cdot \left[ p^{\frac{a}{1-a}} \cdot V^R_S(p) - p^{-\frac{a}{1-a}} \cdot V^S_S(p) \right] + \left[ p^{\frac{a}{1-a}} \cdot \frac{dV^R_s}{dp} + p^{-\frac{a}{1-a}} \cdot \frac{dV^S_s}{dp} \right] = 0, \tag{37}
\]

when \( p = p^A \).

When \( p > p^A \), the home country exports good \( X \). Therefore, the excess demand for \( X \) is negative—i.e., \( ED_X(p) < 0 \)—while the excess demand for \( Y \) is positive: \( ED_Y(p) > 0 \). This relationship can be expressed as

\[ X(p) > C_X(p) \iff p^{\frac{a}{1-a}} \cdot V^R_S(p) > p^{-\frac{a}{1-a}} \cdot V^S_S(p) \mid_{p > p^A}. \tag{38} \]

Similarly, it follows that

\[ p^{\frac{a}{1-a}} \cdot V^R_S(p) < p^{-\frac{a}{1-a}} \cdot V^S_S(p) \mid_{p < p^A}. \tag{39} \]

**Proof of Lemma 2.** I first look at equation (12). Since \( a \cdot K^{-(1-a)} > 0 \), regardless of the value of \( p \), I evaluate the derivative of

\[ \left[ p^{\frac{a}{1-a}} \cdot V^R_S(p) + p^{-\frac{a}{1-a}} \cdot V^S_S(p) \right]^{1-a} \]

(40)

with respect to \( p \). Let \( s(p) \equiv p^{\frac{a}{1-a}} \cdot V^R_S(p) + p^{-\frac{a}{1-a}} \cdot V^S_S(p) \). The derivative of equation (40) can then be expressed as

\[ (1 - a) [s(p)]^{-a} \frac{ds(p)}{dp}. \]

Since \( (1 - a)[s(p)]^{-a} > 0 \), I need to check the signs of \( s'(p) = \frac{ds(p)}{dp} \).

\[ s'(p) = \frac{1}{p(1 - a)} \cdot \left[ p^{\frac{a}{1-a}} \cdot V^R_S(p) - p^{-\frac{a}{1-a}} \cdot V^S_S(p) \right] + \left[ p^{\frac{a}{1-a}} \cdot \frac{dV^R_s}{dp} + p^{-\frac{a}{1-a}} \cdot \frac{dV^S_s}{dp} \right]. \tag{41} \]

From the autarky condition (37), \( s'(p) = 0 \) when \( p = p^A \). By using the conditions (38) and (39), and by noting that the second term in (41) is small relative to the first term, it follows that \( s'(p) < 0 \) when \( p > p^A \) and that \( s'(p) > 0 \) when \( p < p^A \). This concludes the proof. ■

**Proof of Proposition 2.** I express the occupational return for \( X \) producers as follows.

\[ \pi^i_X(p, r, \theta^j) = \left[ p^{\frac{a}{1-a}} \left( \frac{1}{r(p)} \right)^{\frac{a}{1-a}} \left( a^{\frac{a}{1-a}} - a^{-\frac{a}{1-a}} \right) \right] \cdot \theta^j. \tag{42} \]

By substituting equation (12) into (42), I obtain an expression for occupational rewards in terms of the output price.

\[
\pi^i_X(p, \theta^j) = \left[ a^{\frac{a}{1-a}} \left( a^{\frac{a}{1-a}} - a^{-\frac{a}{1-a}} \right) K^a \right] \cdot p^{\frac{a}{1-a}} \cdot \left[ p^{\frac{a}{1-a}} \cdot V^R_S(p) + p^{-\frac{a}{1-a}} \cdot V^S_S(p) \right]^{-a} \cdot \theta^j
\]

\[
= K^a (1 - a) \cdot p^{\frac{a}{1-a}} [s(p)]^{-a} \cdot \theta^j
\]

47
Since the constant term, $K^\alpha(1 - a)$, is positive and $\theta^j$ is nonnegative by assumption, the derivative of $p^{\frac{1}{1-a}} [s(p)]^{-a}$ has the same sign as the derivative of $\pi_X^j(p, \theta^j)$ with respect to $p$. Therefore, showing that

$$
\frac{d}{dp} \left( p^{\frac{1}{1-a}} [s(p)]^{-a} \right) > 0
$$

is equivalent to applying the validity of the above proposition to that relating to job stayers in sector $X$.

$$
\frac{d}{dp} \left( p^{\frac{1}{1-a}} [s(p)]^{-a} \right) = s^{-a} \cdot p^{\frac{1}{1-a}} \cdot a \cdot \left( \frac{1}{a(1-a)} - \frac{p \cdot s'(p)}{s(p)} \right)
$$

Given that $0 < a < 1$ and $p > 0$, it is clear that

$$
s^{-a} \cdot p^{\frac{1}{1-a}} \cdot a > 0 \text{ and } \frac{1}{a(1-a)} > 0.
$$

Given, from $p > p^A$, that $s'(p) < 0$, it follows that

$$
\left( \frac{1}{a(1-a)} - \frac{p \cdot s'(p)}{s(p)} \right) > 0.
$$

Hence, I have shown that (43) holds. Since a similar analysis could be carried out of the occupational rewards for $Y$, the proof is omitted. ■

B A Profit-Tax System

Assume that the production function is

$$
x = X(k, \theta),
$$

where $x$ is the quantity of output, $k$ is the amount of the generic factor employed by the firm, and $\theta$ is the specific occupational factor that is indivisible and embodied in the individual agent. Let $X(k, \theta)$ be increasing in both arguments, strictly concave, and infinitely continuously differentiable, and let it have constant returns to scale.

Let $p$ be the output price of $x$. Let $r$ be the market price for the generic factor, $k$. The agent’s profit-maximization program is

$$
\max_k \pi(k, \theta; p, r) = p \cdot X(k, \theta) - r \cdot k.
$$

(45)

Note that the only choice variable for the agent is $k$, because $\theta$ is embodied and indivisible. The regular first-order condition is

$$
\frac{\partial \pi}{\partial k} = 0 \iff p \cdot \frac{\partial X}{\partial k} = r.
$$

(46)

Strict concavity of the production function, $X(\cdot, \cdot)$, guarantees that the second-order condition for the regular problem (45) holds with strict inequality.

$$
\frac{\partial^2 \pi}{\partial k^2} < 0
$$

(47)
Now, consider a profits tax on the profits of the agent, given equation (45). If the \textit{ad valorem} tax rate is $t$, then the profit-maximization program is

$$
\max_k (1 - t) \left\{ p \cdot X(k, \theta) - r \cdot k \right\}.
$$

(48)

When $t$ does not depend on $k$ or $\theta$, the profit-maximization problem faced by an individual is unchanged. Hence, the first-order condition is (46).

**B.1 A Tax Rate Proportional to Profit**

Now let $1 - t = T(\pi)$ be the profit-tax schedule. The rate of tax depends on the observed profit of the individual. The program is now

$$
\max_k \{T(\pi) \cdot \pi\} = T(\pi) \left\{ p \cdot X(k, \theta) - r \cdot k \right\}.
$$

(49)

The first-order condition for (49) is

$$
\frac{\partial T}{\partial \pi} \cdot \frac{\partial \pi}{\partial k} \cdot \pi + T \cdot \frac{\partial \pi}{\partial k} = \frac{\partial T}{\partial \pi} \cdot \left\{ \frac{\partial T}{\partial \pi} \cdot \pi + T \right\} = 0.
$$

(50)

Condition (50) implies that $\frac{\partial T}{\partial \pi} = 0$, except when

$$
\frac{\partial T}{\partial \pi} \cdot \pi + T = T \left( 1 + \frac{\partial T}{\partial \pi} \cdot \frac{\pi}{T} \right) = T (1 + \varepsilon) = 0,
$$

with $\varepsilon \equiv \frac{\partial T/T}{\partial \pi/\pi}$ being the elasticity of the tax rate with respect to profit. Thus, unless $\varepsilon = -1$, the first-order condition (50) implies the same condition as (46).

The second-order condition for the profit-maximization is

$$
\frac{\partial^2 \pi}{\partial k^2} \left\{ \frac{\partial T}{\partial \pi} \cdot \pi + T \right\} + \frac{\partial \pi}{\partial k} \cdot \frac{\partial \pi}{\partial \pi} \left\{ \frac{\partial T}{\partial \pi} \cdot \pi + T \right\} \equiv SOC < 0.
$$

(51)

The second term of $SOC$ is

$$
\frac{\partial \pi}{\partial k} \cdot \left\{ \frac{\partial^2 T}{\partial \pi^2} \cdot \frac{\partial \pi}{\partial \pi} \cdot \pi + 2 \left( \frac{\partial T}{\partial \pi} \cdot \frac{\partial \pi}{\partial k} \right) \right\}.
$$

This is evaluated around the optimum point, where $\frac{\partial \pi}{\partial k} = 0$. Thus, given (47), it follows that the relevant condition for the program’s second-order condition is

$$
\frac{\partial T}{\partial \pi} \cdot \pi + T = T (1 + \varepsilon) > 0.
$$

Given that $T > 0$, the condition can also be written as

$$
\varepsilon = \frac{\partial T/T}{\partial \pi/\pi} > -1.
$$

(52)

So, unless the profit-tax rate decreases by more than 1% as the profit simultaneously increases by 1%, the agent maximizes profit even after profit has been taxed.
B.2 Tax Rate Proportional to Output

Now let $1 - t = T(x)$ be a new profit-tax schedule. The rate of tax depends on the observed output of the individual. The program is now

$$\max_{k} \{ T(x) \cdot \pi \} = T(x) \{ p \cdot X(k, \theta) - r \cdot k \}. \quad (53)$$

The first-order condition is

$$\frac{\partial T}{\partial x} \cdot \frac{\partial X}{\partial k} \cdot \pi + T \cdot \left\{ p \cdot \frac{\partial X}{\partial k} - r \right\} = \frac{\partial X}{\partial k} \cdot \left\{ \frac{\partial T}{\partial x} \cdot \pi + pT \right\} - rT = 0. \quad (54)$$

Note that the optimal level of $k$ is smaller than the no-tax case (45), because

$$\frac{\partial T}{\partial x} \cdot \frac{\partial X}{\partial k} \cdot \{ p \cdot X(k, \theta) - r \cdot k \} < 0,$$

together with $r > 0$ and $T > 0$ implies that

$$\left\{ p \cdot \frac{\partial X}{\partial k} - r \right\} > 0.$$

Thus, the profit-tax system based on observed output is inevitably distortionary.