EFFECTS OF INTENSIFYING COMPETITION IN THE SINGLE EUROPEAN MARKET (theoretical approach)

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In this paper we demonstrate that a creation of a united market, e.g. Single European Market results in intensification of competition and in welfare gains of the member states. We consider effects of competition’s intensification on welfare of producers’ and consumers’ countries constituting the integrated area. The remainder of the paper is organised as follows.

In section I we provide the analytical framework of the model and analyze a partial equilibrium model a monopoly initially supplying the domestic markets of two countries. In section II we replace the monopoly with an oligopoly (duopoly) created after unification of both markets. We don’t analyze the process of creating a new enterprise assuming that it happens e.g. because of possible capital mobility. Finally, section III reports the resulting changes in welfare and provides conclusions.

I. Open economies with monopoly supplying both national markets

We assume linear demand in the domestic markets of two countries constituting the integrated area (we mark them with subscripts 1 and 2). We differentiate markets of both countries by different characteristics of their linear demand functions:

\[ P_1(q_1) = a_1 - b_1 \cdot q_1 \]

\[ P_2(q_2) = a_2 - b_2 \cdot q_2 \]

with \( a_1, a_2, b_1, b_2 > 0 \).

We assume that \( b_1 > b_2 \) (the demand in market 1 is generally less elastic than the one in market 2; the \( P_1(q_1) \) line is steeper than the \( P_2(q_2) \)) and \( a_1 > a_2 \) (consumers in country 1 are generally ready to accept a higher price than consumers in country 2; prohibition price for country 1 is higher than for country 2).

Technology of production is described with a constant marginal cost (MC = \( c > 0 \)) which is the only cost to cover. By assumption: \( a_2 > c \), what is a technical condition necessary for positive quantities supplied to both markets.

The monopoly is not allowed to differentiate prices. Though, we can aggregate both national demand functions and get an unified demand function \( Q(P) \):

\[ b_1 \cdot q_1 = a_1 - P_1 \Rightarrow q_1 = \frac{a_1}{b_1} - \frac{P_1}{b_1} \]
\[ b_2 \cdot q_2 = a_2 - P_2 \Rightarrow q_2 = \frac{a_2}{b_2} - \frac{P_2}{b_2} \]

with an unique price in both markets \((P_1 = P_2 = P)\):

\[
Q(P) = q_1 + q_2 = \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{b_1 \cdot b_2} - \left( \frac{P \cdot b_2 + P \cdot b_1}{b_1 \cdot b_2} \right)
\]

\[
Q(P) = \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{b_1 \cdot b_2} - \frac{P \cdot (b_1 + b_2)}{b_1 \cdot b_2}.
\]

(1)

We reverse the demand function from equation (1):

\[
P(Q) = \frac{-b_1 \cdot b_2 \cdot Q}{b_1 + b_2} + \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{b_1 + b_2}
\]

(2)

Now we define a monopoly profit as a total revenue subtracted a total cost:

\[
\Pi(Q) = P(Q) \cdot Q - c \cdot Q
\]

(3)

Before calculating the maximal profit we substitute \(P(Q)\) in the profit function of the monopoly with the reversed demand function from the equation (2):

\[
\Pi(Q) = \left( \frac{-b_1 \cdot b_2 \cdot Q}{b_1 + b_2} + \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{b_1 + b_2} \right) \cdot Q - cQ = \frac{a_1 \cdot b_2}{b_1 + b_2} \cdot Q + \frac{a_2 \cdot b_1}{b_1 + b_2} \cdot Q - \frac{b_1 \cdot b_2 \cdot Q^2}{b_1 + b_2} - cQ
\]

(4)

Then we use the first order condition for profit maximisation:

\[
\frac{\partial \Pi}{\partial Q} = \frac{a_1 \cdot b_2}{b_1 + b_2} + \frac{a_2 \cdot b_1}{b_1 + b_2} - \frac{2b_1 \cdot b_2 \cdot Q}{b_1 + b_2} - c = 0
\]

and calculate the profit maximising quantity supplied to the common market \((Q_0)\):

\[
Q_0 = \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - c(b_1 + b_2)}{2b_1 \cdot b_2},
\]

(5)

and the equilibrium price \((P(Q_0))\), unique in both national markets:

\[
P(Q_0) = \frac{-b_1 \cdot b_2 \cdot Q_0}{b_1 + b_2} + \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{b_1 + b_2} = \frac{-b_1 \cdot b_2}{b_1 + b_2} \cdot \left( \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - c(b_1 + b_2)}{2b_1 \cdot b_2} \right) + \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{b_1 + b_2}
\]
\[ \frac{a_1 \cdot b_2 + a_2 \cdot b_1 + c(b_1 + b_2)}{2(b_1 + b_2)} = \frac{a_1 \cdot b_2 + a_2 \cdot b_1 + c}{2} \]

(6)

Note that it holds:

\[ P(Q_0) < \frac{a_1 + c}{2} < a_1 \]

and

\[ P(Q_0) > \frac{a_2 + c}{2} > c. \]

If we additionally assume that

\[ a_2 > \frac{a_1 + c}{2} \]

(7)

then we get:

\[ P(Q_0) < \frac{a_1 + c}{2} < a_2. \]

(8)

what is a necessary condition for a monopoly to supply both national markets after introducing a single market regulation (and – as a consequence – a unique price in both markets). The maximal monopoly profit in the united market is equal to:

\[ \Pi^* = P(Q) \cdot Q - c \cdot Q = (P(Q) - c) \cdot Q = \]

\[ \left( \frac{a_1 b_2 + a_2 b_1 + c(b_1 + b_2)}{2(b_1 + b_2)} - \frac{a_2 b_2 + a_2 b_1 - c(b_1 + b_2)}{2(b_1 + b_2)} \right) \cdot \frac{a_1 b_2 + a_2 b_1 - c(b_1 + b_2)}{2(b_1 + b_2)} \cdot \frac{a_2 b_2 + a_2 b_1 - c(b_1 + b_2)}{2(b_1 + b_2)} \]

\[ = \frac{(a b_2 + a_2 b_1 - c(b_1 + b_2))^2}{4b_2(b_1 + b_2)} > 0 \]

(9)

\[ \Pi^* \] in formula (9) is obviously positive.

Now we calculate a consumer surplus in both countries under unique equilibrium price \( P(Q_0) \). We present this result also in graph 1. We first calculate quantity \( q_0 \) visualized in the graph 1. We use the demand function of country 1 with the price \( P_1(q_0) \) equal to \( a_2 \):

\[ q_0 = \frac{a_1 - a_2}{b_1}. \]

The equilibrium quantity \( Q_0 \) is bigger than \( q_0 \):
\[ Q_0 = \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - c(b_1 + b_2)}{2b_1 \cdot b_2} > \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - (2a_2 - a_1)(b_1 + b_2)}{2b_1 \cdot b_2} = \]
\[ \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - 2a_2 \cdot b_1 + a_1 \cdot b_1 - 2a_2 \cdot b_2 + a_1 \cdot b_2}{2b_1 \cdot b_2} = \frac{(a_1 - a_2)b_1 + 2b_2(a_1 - a_2)}{2b_1 \cdot b_2} = \]
\[ \frac{(a_1 - a_2)(b_1 + 2b_2)}{2b_1 \cdot b_2} > \frac{(a_1 - a_2)2b_2}{2b_1 \cdot b_2} = \frac{a_1 - a_2}{b_1} = q_0. \]

(10)

**Fig. 1: Consumers’ surpluses in both countries under monopoly**

Source: own concept.

Before we calculate the social welfare of both countries under the single market equilibrium with monopoly we find the consumer surplus as a sum of the areas A, B and C in graph 1:

\[ A = \frac{1}{2} q_0 \cdot a_1 = \frac{a_1 \cdot (a_1 - a_2)}{2b_1} \]

As – by assumption – \( a_1 > a_2 \), the area A in formula (11) is positive.

\[ B + C = \frac{1}{2} (q_0 + Q_0) \cdot (a_2 - P(Q_0)) = \frac{1}{2} \left( \frac{(a_1 - a_2)}{b_1} + \right. \]
\[ \left. \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - c(b_1 + b_2)}{2b_1 \cdot b_2}, \left\{ a_2 - \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{2(b_1 + b_2)} - \frac{c}{2} \right\} \right) \]
We can prove that sum of areas B and C calculated in the formula (12) is positive. We do it after transforming the equation (12) as follows:

\[
B + C = \frac{[(a_2 - c)b_1 + (a_1 - c)b_2 + 2(a_1 - a_2)b_2]}{8b_1b_2(b_1 + b_2)} \cdot \frac{[(a_2 - c)b_1 + (a_1 - c)b_2 - 2(a_1 - a_2)b_2]}{8b_1b_2(b_1 + b_2)}
\]

which is equal to:

\[
B + C = \frac{[(a_2 - c)b_1 + (a_1 - c)b_2 + 2(a_1 - a_2)b_2]}{8b_1b_2(b_1 + b_2)} \cdot \frac{[(a_2 - c)b_1 + b_2(a_1 - c - 2a_1 + 2a_2)]}{8b_1b_2(b_1 + b_2)}
\]

(12a)

In equation (12a) the denominator is positive, as \(b_1\) and \(b_2\) are the positive parameters of the demand functions. All elements in the first set of brackets in the numerator and in the first multiplication in the second set of brackets are positive, because of the assumed relation between parameters \(a_1\), \(a_2\) and \(c\). The last multiplication in the second set of brackets is positive because:
\[ a_1 - c - 2a_1 + 2a_2 = 2a_2 - a_1 - c, \]

where \((2a_2 - a_1)\) is bigger than \(c\) due to assumption (7). It means that also the last part of the multiplication in the numerator is positive and the total fraction in the formula (12a) is positive. Adding up the areas calculated in formulas (11) and (12) we get the consumers’ surplus in both countries:

\[ A + B + C = \frac{a_1(a_1 - a_2)}{2b_1} + \frac{[(a_2 - c)b_1 + (a_1 - c)b_2]^2 - 4(a_1 - a_2)^2b_2^2}{8b_1b_2(b_1 + b_2)} \]

\[ = \frac{4a_1(a_1 - a_2)b_2(b_1 + b_2) + [(a_2 - c)b_1 + (a_1 - c)b_2]^2 - 4(a_1 - a_2)^2b_2^2}{8b_1b_2(b_1 + b_2)} \]

\[ = \frac{[(a_2 - c)b_1 + (a_1 - c)b_2]^2 + 4(a_1 - a_2)b_2[a_1(b_1 + b_2) - (a_1 - a_2)b_2]}{8b_1b_2(b_1 + b_2)} \]

\[ = \frac{[a_1b_2 + a_2b_1 - c(b_1 + b_2)]^2 + 4(a_1 - a_2)b_2(a_1b_1 + a_2b_2)}{8b_1b_2(b_1 + b_2)} \]

(13)

We know that the profit maximising monopoly is supplying both national markets, because we have ensured it with the assumption (7).

The social welfare of both countries under single market equilibrium with monopoly (without price differentiation) is a sum of the monopoly profit \(\Pi^*\) (formula (9)) and the consumers’ surplus from the formula (13)):

\[ W = \Pi^* + A + B + C = \frac{[a_1b_2 + a_2b_1 - c(b_1 + b_2)]^2}{4b_1b_2(b_1 + b_2)} + \frac{[a_1b_2 + a_2b_1 - c(b_1 + b_2)]^2}{4b_1b_2(b_1 + b_2)} + 4(a_1 - a_2)b_2(a_1b_1 + a_2b_2) \]

\[ = \frac{3[a_1b_2 + a_2b_1 - c(b_1 + b_2)]^2 + 4b_2(a_1 - a_2)(a_1b_1 + a_2b_2)}{8b_1b_2(b_1 + b_2)} \]

(14)

II. United oligopolistic market

If - instead – the united market would be supplied by two enterprises one of which comes from one country, each of them would get a following profit:

\[ \Pi_1(q_1) = P(q_1 + q_2) \cdot q_1 - c \cdot q_1 = \frac{-b_1 \cdot b_2}{b_1 + b_2} \cdot (q_1^2 + q_1 \cdot q_2) + \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{b_1 + b_2} \cdot q_1 - c \cdot q_1 \]

(15)

\[ \Pi_2(q_2) = P(q_1 + q_2) \cdot q_2 - c \cdot q_2 = \frac{-b_1 \cdot b_2}{b_1 + b_2} \cdot (q_2^2 + q_1 \cdot q_2) + \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{b_1 + b_2} \cdot q_2 - c \cdot q_2 \]
(16)

where equation (15) is the profit function of the enterprise from country 1 and (16) – the profit function of the firm from country 2. For aggregated demand for product supplied by both firms we use the formula (2).

From the first order conditions (differentiation of the formulas (15) and (16) by – respectively – \( q_1 \) and \( q_2 \)) we get optimal quantities sold on the united market by both firms:

\[
q_1^* = q_2^* = \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - c \cdot (b_1 + b_2)}{3b_1b_2}.
\]

(17)

Now we calculate new oligopolistic price for the united market:

\[
p(q_1^* + q_2^*) = \frac{-b_1 \cdot b_2 
\frac{(q_1^* + q_2^*) + a_1 \cdot b_2 + a_2 \cdot b_1}{b_1 + b_2} \cdot \frac{2[a_1 \cdot b_2 + a_2 \cdot b_1 - c \cdot (b_1 + b_2)]}{3b_1b_2} + \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{b_1 + b_2} = 
-2a_1 \cdot b_2 - 2a_2 \cdot b_1 + 2c \cdot (b_1 + b_2) + 3a_1 \cdot b_2 + 3a_2 \cdot b_1
\frac{3b_1 + b_2}{3b_1 + b_2} = \frac{a_1 \cdot b_2 + a_2 \cdot b_1 + 2c \cdot (b_1 + b_2)}{3b_1 + b_2} + \frac{2c}{3}.
\]

(18)

The oligopolistic price in formula (18) is lower than the profit maximising price of the monopoly calculated in the equation (9). It is easy visualised after we reformulate both equations:

\[
p(q_1^* + q_2^*) = \frac{2(a_1 \cdot b_2 + a_2 \cdot b_1) + c \cdot b_1 + c \cdot b_2 + 3c \cdot (b_1 + b_2)}{6(b_1 + b_2)} < \frac{3(a_1 \cdot b_2 + a_2 \cdot b_1) + 3c \cdot (b_1 + b_2)}{6(b_1 + b_2)} = p(Q_0),
\]

(19)

Inequation (19) holds by assumptions: \( c < a_1 \) and \( c < a_2 \).

Moreover, the profit maximising oligopoly price is bigger than the marginal cost \( c \), therefore both oligopolists obtain positive profits producing optimal quantities \( q_1^* \) and \( q_2^* \) (respectively):

\[
p(q_1^* + q_2^*) = \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{3(b_1 + b_2)} + \frac{2c}{3} < \frac{c \cdot (b_1 + b_2)}{3(b_1 + b_2)} + \frac{2c}{3} = c.
\]

(20)

\[
p(q_1^* + q_2^*) = \frac{a_1 \cdot b_2 + a_2 \cdot b_1 + 2c}{3(b_1 + b_2)} < \frac{(2a_2 - c) \cdot b_2 + a_2 \cdot b_1}{3(b_1 + b_2)} = \frac{2a_2 \cdot b_2 - c \cdot b_2 + a_2 \cdot b_1 + 2c \cdot (b_1 + b_2)}{3(b_1 + b_2)} = 
\frac{2a_2 \cdot b_2 + a_2 \cdot b_1 + 2c \cdot b_1 + c \cdot b_2}{3(b_1 + b_2)} < \frac{2a_2 \cdot b_2 + a_2 \cdot b_1 + 2a_2 \cdot b_1 + 2a_2 \cdot b_1 + a_2 \cdot b_2}{3(b_1 + b_2)} = 
\frac{3a_2 \cdot b_2 + 3a_2 \cdot b_1}{3(b_1 + b_2)} = \frac{3a_2 \cdot (b_1 + b_2)}{3(b_1 + b_2)} = a_2
\]

(21)
Inequation (21) proves ultimatively that both markets are supplied.

Now we calculate the maximal profit of both oligopolists. We use the general formula (15) for profit of the oligopoly from country 1 (the oligopoly from country 2 obtains exactly the same profit as the firm from country 1):

\[
\Pi_1(q_1^*) = \left( \frac{a_1 \cdot b_1 + a_2 \cdot b_1}{3(b_1 + b_2)} + \frac{2c}{3} - c \right) \cdot \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - c(b_1 + b_2)}{3b_1 \cdot b_2} = \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - c(b_1 + b_2)}{3(b_1 + b_2)} \cdot \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - c(b_1 + b_2)}{3b_1 \cdot b_2} = \frac{(a_1 \cdot b_2 + a_2 \cdot b_1 - c \cdot b_1 - c \cdot b_2)^2}{9b_1 \cdot b_2 (b_1 + b_2)} > 0.
\]

(22)

III. The welfare gains from competition’s intensification

Fig. 2: Consumers’ surpluses in the single oligopolistic market

![Graph showing consumers' surpluses in the single oligopolistic market](image)

*Source: own concept.*

Now we calculate the aggregated consumer surplus as a sum of the areas A’, B’ and C’ in graph 2, where the area A’ is exactly the same as A in formula (11):

\[
A' = \frac{1}{2} q_0 \cdot a_1 = \frac{a_1 \cdot (a_1 - a_2)}{2b_1}
\]
and \( B' \) and \( C' \) can be calculated as follows:

\[
B' + C' = \frac{1}{2}(q_0 + q_1^* + q_2^*) \cdot (a_2 - P(q_1^* + q_2^*)) = \frac{1}{2} \left( \frac{a_1 - a_2}{b_1} \right) + \\
\frac{2a_1 \cdot b_2 + 2a_2 \cdot b_1 - 2c(b_1 + b_2)}{3b_1 \cdot b_2} \cdot \left( a_2 \cdot b_2 + a_2 \cdot b_1 - \frac{2c}{3} \right)
\]

(23)

The consumer surplus is the sum of areas \( A' \), \( B' \) and \( C' \):

\[
A' + B' + C' = \frac{1}{2} \left( a_1 - a_2 \right) \cdot a_1 + \\
\frac{1}{2} \left[ \frac{3a_2 \cdot b_1 + b_2 - a_1 \cdot b_2 - a_2 \cdot b_1 - 2c(b_1 + b_2)}{3b_1 \cdot b_2} \right] = \\
\frac{1}{2b_1} \left( a_1 - a_2 \right) \cdot a_1 + \\
\frac{1}{18b_1 \cdot b_2 (b_1 + b_2)} \cdot \left[ 2b_1 \cdot (a_2 - c) + b_2 \cdot (5a_1 - 3a_2 - 2c) \right] \cdot \left[ 2b_1 \cdot (a_2 - c) + b_2 \cdot (3a_2 - a_1 - 2c) \right]
\]

(24)

As all the multiplications in the numerator of the formula (24) are positive and its denominator is positive, the calculated consumers’ surplus is positive.

Now we rearrange the equation (24):

\[
A' + B' + C' = \frac{1}{2b_1} \left( a_1 - a_2 \right) \cdot a_1 + \\
\frac{4(a_2 - c)^2 b_1^2 + 2(a_2 - c) \cdot b_1 \cdot b_2 \cdot (5a_1 - 3a_2 - 2c + 3a_2 - a_1 - 2c) + (5a_1 - 3a_2 - 2c) \cdot (3a_2 - a_1 - 2c) b_2^2}{18b_1 b_2 (b_1 + b_2)}
\]

\[
A' + B' + C' = \frac{2[a_1 b_2 + a_2 b_1 - c(b_1 + b_2)]^2}{9b_1 b_2 (b_1 + b_2)} + \frac{(a_1 - a_2) \cdot (a_1 \cdot b_1 - a_2 b_2)}{2b_1 (b_1 + b_2)}
\]

(24a)

Now we can calculate the social welfare as a sum of the aggregate consumers’ surplus (equation 24a) and the profits of both oligopolists (equation (22) multiplied by 2):
The social welfare under oligopoly calculated in equation (25) is bigger than in the initial situation when both national markets were supplied by one firm ($W$ in the equation (13)):

$$W' = \Pi_1(q_1^*) + \Pi_2(q_2^*) + A'B'C' = \frac{2[a_1 \cdot b_2 + a_2 \cdot b_1 - c(b_1 + b_2)]^2}{9b_1 \cdot b_2 \cdot (b_1 + b_2)} + \frac{2[a_1 \cdot b_2 + a_2 \cdot b_1 - c(b_1 + b_2)]^2}{9b_1 \cdot b_2 \cdot (b_1 + b_2)} + \frac{(a_1 - a_2) \cdot (a_1 \cdot b_1 + a_2 \cdot b_2)}{2b_1(b_1 + b_2)}$$

(25)

In the inequation (26) we prove that the intensifying competition is welfare increasing for the integrated area.