International Strategic Trade Policies and Lobbying Contests

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This paper aims to provide an answer to the inefficiency problem arising from the introduction of a cost asymmetry in an oligopolistic model of Strategic Trade Policy. Through analysis, we conclude that the use of a lobbying contest in the heterogeneous market can succeed in revealing the highest efficiency firm. This in turn leads to the reduction of the Dixit oligopolistic model (1986) to the monopolistic one introduced by Brander and Spencer (1985).

The criticism concerning the shortcomings of strategic trade policy has been a major issue of debate amongst trade economists throughout the years. When the assumptions, upon which the original model by Brander and Spencer (1985) was based, are varied, then the results differ. In particular, when the firms that operate in the domestic market diverge in their production cost levels, and their number is higher than the one of the competing rival market, then the suggestion of Dixit in his model, concerning their taxation should no longer be valid. The inefficiency taxation tries to overcome would still exist in the form of the inability (due to the tax) of the lowest cost firm to alone compete in the international market and succeed in the profit shifting aim of strategic trade policy. As Montagna and Leahy (1997) suggest, the optimal policy would be the introduction of a firm specific subsidy that would promote the most efficient firm of the market (in terms of the production costs) and firm specific taxes for the rest of the domestic firms to ensure their lack of production for the international level. However, the setback in this model was that there existed no identification mechanism to determine the most efficient firm in the market, as the ability of each firm is private information. The particular suggestion of this paper is the use of a lobbying contest as an identification mechanism, to solve the problem described above. Using a contest model, where the optimal bidding of the firms depends on their particular production ability (the lower the marginal cost of production, the higher the efficiency) we derive the following results: the most

* I am grateful to my supervisors Dr Simon Clark and Dr Catia Montagna for their guidance. I thank Mr Apostolos Kasapis for his helpful remarks.
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efficient of the firms will be the one submitting the highest bid, and therefore will be the winner of the contest. This firm then will receive the award (subsidy) provided by the government whilst the rest of the firms will only incur the cost of entering the contest and will not be able to produce for the international level (this has the same function as the firm specific cost suggested by Montagna and Leah). Therefore we are able to provide a solution for the specific shortcoming.

1. Literature Review

In 1985, the paper by Brander and Spencer came out to contradict all the literature concerning international trade policies that had emerged until that moment. It stated that in the case of perfectly competitive markets, protectionism was not an optimal strategy. However, when the market was for any reason imperfect, then the most advantageous tactic for governments would be to safeguard their markets. This was shown through a Cournot Duopoly model between two monopolistic nations that were competing in a third market. The intervention of the government of one of these two nations, through the use of an export subsidy, as was shown, put its firm in a Stackelberg leader position, by creating a credible threat concerning the production level of its firm. The result was the shifting of profits (created in the third market) from the competing firm towards the protected one. Welfare was determined by the profits of the firm net of the subsidy bill, as the model assumes no domestic consumption and as a consequence, this shifting of profits lead to the increase of domestic welfare. Later in the same article they allowed for protectionism stemming from both competing nations, where the game ended in an equilibrium of subsides provided by the governments and an equilibrium in quantities as best responses to both the subsidies set by both policymakers, but also the quantity produced by the rival firm.

The more general case of oligopolistic rival markets was addressed by Dixit (1986). He used a reciprocal market model, where in the two competing markets the number of the symmetric operating firms was greater than one. The result that he arrived to was that in contrast to the outcome of the Brander and Spencer article, if the number of domestic firms is higher relative to the foreign ones, then a subsidisation policy might instead of improving the domestic welfare, worsen it. Then, the suggestion was
for the government to follow an export taxation policy, rather than subsidisation of
the producing firms. In order to exhibit more clearly the difference between the
policies suggested by these two models of strategic trade policy, a third market model
could be used, without loss of generality (Montagna, SGPE 2003/2004 lecture notes).
The results remain unchanged: if the number of domestic firms exceeds a critical
value, the optimal policy is taxation. This, because a subsidy has two contrasting
effects on domestic welfare: one stemming from the rent shifting (comparative
advantage result) and one from the domestic competition (excessive domestic
production). The first promotes domestic welfare, whilst the latter demotes it. The
higher the number of domestic firms relative to the foreign ones, therefore, the more
likely it is that the negative effect will dominate, and so the subsidy will cause the
decrease of domestic welfare.

The analysis of the above models is based, however on the assumption that the
countries are symmetric and that firms within each country are identical. In the real
world that would be rather unlikely to occur. There are two forms of cost
asymmetries to be taken into consideration: one where the asymmetry occurs between
the two monopolies and one where there exists a cost asymmetry among the many
firms in the oligopolistic markets.

In the first case, the literature provides us with the following findings: Neary (1994)
departs from the original Brander and Spencer model, introducing a cost asymmetry
among the two rival monopolies (i.e the cost of production differs between the
domestic and foreign firms). He then shows that the optimal subsidy to be provided is
a decreasing function of the domestic marginal cost and an increasing function of the
foreign cost. Therefore, the more cost efficient the domestic firm is, the higher the
incentive for the domestic government to provide a subsidy, and in this manner
improve the chances of profitable rent shifting in the expense of the foreign firm.
Consequently, the domestic government has the incentive to subsidise its firm more,
the lower the ratio of the domestic marginal cost of production over the foreign one.

The argument that the country with the lowest production cost in the monopolistic
framework will be providing the highest subsidisation level was analysed by de Meza
(1986). His finding goes as follows: “the lower a firm’s marginal cost, the greater the
benefit from being able to sell at the initial price”. This is why the low-cost country has the incentive to encourage its home firm to expand and therefore pays the highest subsidy.

Collie and de Meza (2003) also show that the absolute value of the policy measure (subsidy or tax) will be higher for a country that has a low cost firm, rather that a high cost one. “The general principal is that as the country with the low cost firm has more to gain from intervention, the absolute value of its policy will be greater than that of its higher cost rival”. The higher the subsidy, the higher the credible threat created.

Therefore, we could conclude that in the case of an oligopoly, where the government aims to pick the national champion, it has the incentive to choose the lowest cost firm, and provide the highest subsidy. This would lead to the increased probability of rent shifting in the domestic market.

In the second situation, if a cost asymmetry in the protectionist country was introduced along with an asymmetry in the market structure, incorporated in the difference between the number of domestic and foreign firms ($m > m^*$ i.e. domestic firms more than foreign ones and so the optimal policy is a tax, or $m < m^*$ with the opposite results), then we would depart slightly from the outcome to which Dixit came. In this case, according to N.V. Long and A. Subeyran (1997) the concentration of the firm, measured by the Herfindahl index\(^1\), along with the concavity or convexity of the demand curve will be defining the optimal protection policy. So, they come to the conclusion that in the cases where (i) the demand curve is convex ($P'' > 0$) and the index is high and (ii) the demand curve is concave ($P'' < 0$) and the index low, the subsidisation policy is beneficial as it increases domestic welfare. However, in the two opposite cases, where (i) the demand curve is concave and the index is high and

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\(^1\) The Herfindahl index of industry concentration measures industry concentration by imposing added weight to the bigger firms. The formula is: 
\[ H = \left(\%S_1\right)^2 + \left(\%S_2\right)^2 + \ldots + \left(\%S_m\right)^2, \]
where $S$ counts the share of the market owned by each of the company in the market. In the case where the costs are not identical, then some firms will have a higher share, and some lower, compared to the case where all firms are identical. The greater the difference of the shares captured by the different firms, the higher the index and therefore the higher the concentration of the market power in the “hands” of a small number of firms. Thus, the lower the competition within the market. The index takes its highest value of 10,000, in the case of a monopoly, where all the market power is concentrated on one firm. The larger the number of firms and the lower the dispersion of the costs the lower the index.
(ii) the curve is convex with a low index, then if the government were to impose a subsidy, it would lead to a reduction of welfare. Thus, in these two cases, the optimal policy would be to tax exports. The government will set a particular tax towards all firms, but that would be inefficient since it will be restricting the output of the most efficient firm, that in any other way would be the one managing to shift the highest amount of rents from the second competing country (if it were to compete on its own). The government would be able to increase domestic welfare if it could find a way to award only the most efficient firm (depending on the production costs, that at this point we assume are not known to the government, since it is not interested in the specific allocation of costs among the firms, until it sees that the subsidisation of all the firms would be harmful).

Leahy and Montagna (1997, 1998, 2001) suggest the following strategies that could help overcome this inefficiency. In the last of these papers (2001) they consider the case of the implementation of a firm-specific subsidisation policy, in the case where only the domestic government is policy active. They show that when the social cost of funding this policy is not too high, then the firms should receive subsidies according to their production abilities (i.e. production costs). Consequently, the higher the deviation of the marginal cost from the market average, the higher the subsidy they will receive, and so the higher their level of production in the international level. If on the other hand the social cost is high, then the optimal policy is a set of firm specific taxation measures, with the lowest cost firm (which has the highest production capability) required to pay the highest per unit tax, which will ensure the production of only up to the level the government allows. The policymaker, when the social cost is high is interested in minimising the subsidisation costs and no longer in the maximisation of the profit shifting, so the argument of rent-shifting that originally drives the activity of the government is weakened. The minimisation of costs is granted through the taxation of the firms with the larger share of the market, i.e. the cost-advantaged ones.

One of the main arguments in all of these models is the analysis concerning the social cost of public funds, since when this is unity, and equal weight is put on the government expenditure and the profits of the firms, then the most likely policy is subsidisation. However, in reality, it is rather unlikely that this should occur, and so,
different measures are to be adopted, especially in the case of a domestic oligopoly. In all cases, the analysis suggests that: “the government should select winning firms within industries”, or in other worlds they should try to select an industry champion.

This is done in Leahy & Montagna (1997) through the provision of a firm specific subsidy to the most efficient firm and firm specific taxes to all the remaining ones, instead of a unit subsidy towards the entire domestic market in order to succeed in the prevention of the effect of the excessive production. The tax will prohibit their export production and since only the most efficient of the firms will be competing in the third market, then the game reduces to the Brander and Spence framework and the effect the subsidy will have is only a rent shifting one. However, it is also mentioned that “one of the criticisms levelled against strategic trade policy concerns the ability of the government to identify the strategic sectors. The need to be able to identify the winning firms within an industry may be seen as casting further doubts on the feasibility of implementing strategic trade policies”. So the setback that exists in this model is the lack of government ability to identify the production cost and pick-the-winner.

The criticism therefore would lie on the capability of the government to make out which of all the domestic firms is the most efficient one. The fact that the government cannot positively identify the most efficient firm in the market implies that there is incomplete information on the part of the policymaker. The argument here would be that a mechanism is required to assist the identification of the most efficient firm i.e. the firm that produced with the lowest cost.

Signalling would serve as such (identification mechanism), as has been done in the past in international trade theory. Collie and Hviid (1993) under the assumption that the rival foreign firm is not informed of the production cost of the domestic firm, show that the domestic government has the incentive to use a subsidisation policy to signal the competitiveness of its firm. The foreign firm makes its production decision based on its knowledge concerning its own production cost, and its expectation of the domestic one. Then, the policymaker finds it to its benefit to influence these beliefs through the provision of a domestic subsidy, which would imply a low marginal cost. Then the foreign firm would produce a lower level of output to export in the third
market, allowing for the extraction of higher profits by the domestic one. The level of the subsidy in the case where it is used a signalling devise will be higher that it would be otherwise.

The correct signalling device must be sought that would serve the purpose in this case. In the past, output was used as such: Wright (1998) studies the situation where in the case of incomplete information concerning the domestic firm’s production cost, the level of output in the first period can serve as a signalling mechanism, both to the rival government and the domestic policymaker of the agent’s efficiency (in terms of its cost advantage) in the second one. The incentive that a low cost firm has to imitate a high cost one to succeed in receiving a higher subsidy and to induce the foreign firm to produce at a lower level, might actually lead to the reverse results. The government might provide a lower subsidy level than the one without a signal, as it understands this incentive. The same is the case with the output level of the rival firm. The result therefore is opposite than in the previously discussed model.²

Mailath (1989) uses prices as signals of the firms’ production costs. “All firms have private information about their costs. They simultaneously signal their information by their pricing decisions”.

In the case examined here however, neither would serve the purpose. Nevertheless, firms themselves might provide an answer, through their actions. The effort of agents to influence the decisions of the principal through the provision of contributions (monetary, informational) has been widely analysed in the general literature. So, this lobbying behaviour can be used by the policymaker in such a way, that the firms will at the end reveal their own costs. It has been argued, in the paper by de Figueiredo (2002) that “the more influential instrument in affecting policy outcomes”, is lobbying in the form of information, rather that monetary (campaign) contributions. “Information takes many forms: statistics, facts, arguments, forecasts, threats, signals or some combination of the aforementioned”. From the above, we understand that lobbying can serve as a signal in the case of asymmetric information, to assist the

² In Collie & Hviid signalling increases the policy measure, whilst in Wright it leads to its decrease.
policymaker in making a decision that would be in favour of the maximum level lobbying party, which will have he incentive to contribute the highest.

Esteban and Ray (2004) in their analysis concerning governments’ decisions about recourse allocation and the effect of lobbying, show that when the policymaker is uninformed about the productivity of the players, lobbying can serve as an information transmitting mechanism, as the higher productivity players will be the ones lobbying harder. The policymaker then will be able to make the decision concerning whom to favour, after observing the information sent by the agents.

The government can manipulate this to its benefit by setting up a lobbying contest, in which the firms will be asked to extend lobbying efforts, and in the end of the contest the winner will be receiving the award in the form of the subsidy. Through their contributions however, the firms will be providing a signal of their ability to the government, and as it turns out, the lowest cost firm will be the one extending the highest effort, and will be the winner.

It has been widely discussed in literature, that firms extend effort in the form of lobbying in order to affect the policy decision of the government. Pecorino (1999) provides an incentive for the existence and participation of lobbies in the setting of trade policies. In particular, subsidisation, which is an expenditure increasing measure for the policy maker, can only be implemented, once lobbying contributions from the interest groups are made to cover the cost of applying this course of action. When export subsidisation is treated as a revenue seeking process, then it is to the benefit of the industry, as it implies higher profits, to choose to provide the necessary means that will ensure the decision of the government in their favour. The higher the contribution level is, the higher the level of the measure shall be. Consequently, firms could be lead to extend lobbying effort (monetary in this case) in order to influence the decision of the policymaker.

The introduction of lobbying in the strategic trade literature in Moore and Suranovic (1993) shows that if the firms have the power to influence the implementation, the argument for strategic trade policy might be weakened. “Imperfectly competitive firms may use the excess funds to obtain higher subsidies granted in a strategic trade
programme”. The welfare function consisting of the profits of the domestic firm minus the subsidy bill in the original model, will now have to include the lobbying costs (as the firms profit function already incorporates the lobbying effort that the firm would extend). The inclusion of the lobbying cost in the objective function of the government could imply a reduction in national welfare, and so the optimal level of the subsidy would be decreased (if at all it is positive). “Even when a positive optimal subsidy exists, the government must lower the announced subsidy to compensate for the effects of lobbying”. Lobbying reduces the profits and so the rent shifting argument is weakened. Only if the government has the power to predict the effect of lobbying by including the costs in its objective function, can it provide a lower subsidy to compensate for the loss of welfare due to the existence of influential capability on the firm’s side. So, when a positive subsidy does exist, it will be lower than in the B&S model.

The idea that the higher ability firms in an oligopolistic environment will be able to lobby more in order to influence the policymaker’s decision, which is vital for our analysis, is introduced in Hillman, Long and Soubeyran (2001). Even though the measure used is a quota, the concept resembles the subsidy, in that they both increase, through their existence, the profits of the firm. It is then the higher ability firm, with the lowest cost and higher profits that will extend the highest lobbying action since the gain of the implementation of the measure is higher. The heterogeneity of the firms in abilities defines their “comparative advantage in lobbying for protection” activities.

In our analysis this is essential, as we require the highest ability player to extend the highest lobbying effort, in order to win in the contest and therefore the optimal national champion to emerge.

Tullock (1980) mentions that the effort that each of the agents will undertake will depend on their success probability, which in turn will depend on the efforts that the competing agents will extend. The heterogeneity in this model, which implies

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3 The quota, reduces the amount to be provided, and so increases the price, therefore profits rise. In the case of the subsidy, the cost of production is lowered and so again the difference of the revenue minus the cost for the firm increases. The incentive for lobbying activities is therefore optimal in both situations.
different abilities in the setting of the lobbying efforts, will influence the probabilities of success, inducing lower ability players lobby less.

Nti (2004) suggests that “if the players’ valuations display asymmetries, then, the low valuation players tend to put in less effort because they realise that their winning probabilities are less than average, which encourages high valuation players to attempt to claim the prize with reduced effort”.

Therefore, this paper’s model could be framed as a “pick-the-winner” one, where the device used for the emergence of the national champion will be the signal received through the lobbying effort undertaken by each domestic firm in its attempt to influence the policymaker’s decision concerning whom to award an export subsidy with.

The tactic used, would be the addition of one more stage to the original two-stage game introduced by Brander and Spencer. This stage would facilitate the selection of the most efficient firm as a national champion to compete in the third market.

So, if there are $n$ domestic firms, with a cost asymmetry amongst them, then the most efficient firm will have the incentive in some way to let the government know that it is the one with the lowest production cost, in order to avoid being asked to pay a tax (whether it is a uniform tax paid by all the firms in the market, or a firm specific tax, if it mistakenly taken for a high cost one). It will try to influence the decision of the policymaker, by signalling its efficiency with the use of lobbying contributions. As suggested by S. Bandyopadhyay, E. Park and H. Wall (1999) lobbying efforts expanded by the domestic firms create a link between domestic welfare and the cost heterogeneity. The former will define the level of subsidisation and the latter will determine the share of each of the firms in the domestic market, thus, also the lobbying contributions it will offer. Their argument is that when the market concentration is high, the low cost firms will have a higher incentive to lobby as they gain by both the fact that their return is high (the higher their effort, the higher the subsidy set by the government, as according to their argument “the subsidy level will be determined by total lobbying expenditure”) since they receive higher subsidies and that the rent shifting is bound to higher, the higher the policy implemented. They
argue that “in equilibrium, only the lowest cost domestic firm will expand lobbying effort”.

The main point in the lobbying literature is the theoretic description of it as an all-pay auction, since the players are bound to forfeit their bids (whether they are in the form of money contributions or effort extended).

As Cohen and Sela (2005) have shown, the value of the prize for each player in an asymmetric all pay auction is vital, as “the equilibrium strategies can not be explicitly calculated independently of the players’ values for the prizes”.

So, the set up of the model, which will be dealt with in this paper goes as follows: we assume that domestically we have an incompletely informed policymaker, the government and a number of domestic firms that exhibit cost heterogeneity. This translates to different market shares captured by each of the firms. The government is only aware of the existence of different cost levels for the firms, but is ignorant of their allocation among them. The policy that the government chooses to implement consists of the provision of a subsidy only towards the most efficient of the firms within the market. This is announced to the firms that have knowledge of their own cost level and are informed of the distribution function of the costs for the rest of the players. They will develop particular lobbying efforts to signal their efficiency, in order to be able to influence the decision of the policymaker. The firms will try to expand these to their highest affordable level so that they will ensure their distinction from the lowest ability contestants.\(^4\) So, the firms decide upon their lobbying effort levels, and signal these to the government. On the base of those, the government decides who the most efficient firm in the market is and awards that firm with the subsidy. After that, this national champion firm will be competing internationally in the third market, where it will capture a higher share due to the credible threat created by the subsidy provided by the domestic government concerning the production level of this firm.

\(^4\) They will expand as high an effort as they are capable of, since as is the case in most signalling models the high cost firms will try to pass themselves off as high efficiency ones, in order to cheat the government and get the subsidy, even though optimally, the most efficient one should be the one obtaining the award. This is avoided with the setting of particular incentive compatibility constraints that ensure the truthful revelation of the type of each player through their signal.
The basic aim of this paper is the creation of a contest model that will allow the
designer to sort the contestants according to their type, or more plainly we seek to
create a signalling contest model. What mostly interests us, consequently is that the
effort function proves to be a function of the specific ability that the players shall
have, in this heterogeneous framework. The correct design of this model will ensure
that the most efficient of the players will extent the highest effort and so, by
observing the effort, the designer can correctly allocate the award to the highest
ability contestant.

2. General Contest Model

The use of the contest by the designer closely relates the case of monopoly screening,
where the principal offers tasks to the players, and they chose according to their
types. However, in the case examined here, the designer chooses to use signalling
(from the point of the players) as a screening mechanism. This implies that the task he
offers is the participation in this contest and not specific assignments that would
distinguish the low from the high ability player (as in the monopoly screening
models). We could therefore consider the contest as a hybrid mechanism of signalling
and monopoly screening. The fact that the agents participate in this contest after the
request of the principal, implies that they voluntarily signal their types through the
setting of their effort level.

Each contestant has been allocated by nature with a specific type, denoted here with
an ability parameter. The lower the value of this parameter, the higher is the ability of
the player. The contestant, as required by the principal, has to signal this type through
the choice she will make in an action undertaken. The action here shall be the setting
of a bid, or, because of the specific design to be used, the setting of a bidding effort.
The principal therefore receives a signal in the form of the effort, and using that
information he makes his own choice, which is to allocate the award to the player that
has exerted the highest effort. Given the principal’s optimal strategy, the decision that the players have made on their effort level must maximise their utility.\(^5\)

The principal states that the award to be allocated will be defined as a function of the particular ability of the winner and so its real value is not specified in the beginning of the contest. As in most signalling games, the contestants reveal their type at the end of the game. Therefore, the exact level of the award shall depend on the ability of the player. Each player therefore, knowing their own type, can form private valuations of the level to be set. The higher the ability of the player, the higher the expectation, and so the higher the valuation. This implies that the winner shall be the highest ability player, that will have the highest valuation.

The contest design that most closely relates to a signalling framework, is that of a single price all-pay auction, where all the contestants are required to forfeit their bids, even if they do not win the award.

In order for the above signalling game to work, where the highest ability player is also the one with the highest valuation, we must combine two existing models in the contest literature.

Up until now, contests that use the framework of all-pay auctions, suggest that the winner will be either the player with the highest valuation of the award (Krishna and Morgan 1997, where the valuations are private information to the contestants) or the one with the highest ability (Moldovanu and Sela 2000). In the first case, the real value of the award is not known and the players form their own valuations, with the highest one being the valuation of the winning bid participant (it is assumed that the formation of the valuation is defined by a real valued signal that the player receives prior to the auction). In the latter case, the value of the award is known, but we still have the framework of a private information all-pay auction as the players are assumed to differ in ability (note that in the Moldovanu and Sela paper the contest

\[^5\text{At this point we must state on binding condition, in order for the weak players not to have the incentive to falsely exert a different effort that their optimal: the binding constraint shall be:} U(c_H, X_L) \leq U(c_H, X_H) \implies v(c_H) \leq c_H(X_L - X_H). \text{ This is more thoroughly discussed later on.}\]
will have a single award only if the cost function is linear or concave in effort. Here the function to be used is linear, and therefore the use of one award is optimal. The winner in this framework will be the player with the highest ability, since this enables her to expand the highest effort, therefore, the highest bid.

The following model attempts (using the structure of the paper by B. Moldovanu and A. Sela in the fact that each player is unique in an ability parameter that is privately observed and the assumption of the difference in the valuations of the award among players as in the Krishna & Morgan one) to combine the above two in the sense that each player has a different valuation of the award as a result of the different ability parameter. Therefore the ability allocated by nature to each of the contestants will be influencing the winning probabilities both through the cost function (as in M&S) and the valuation one (as in K&M), as they will define the effort to be extended.

Model Assumptions:

- $n$ risk neutral agents.

- Nature allocates each contestant with a different level of ability, which is given by an ability parameter $c_i > 0$ independently drawn from an interval $[c_L, c_H]$. This is private information to the player. Only its distribution function $F$, with $F' > 0$ (continuous density) is common knowledge. Note that the ability is a negative function of the parameter: the lower $c_i$ is, the higher the ability (e.g., the lower the production cost of a firm, the higher the production ability).

- One prize. The real value of this award is not defined, as the contest designer announces that the particular level will be set after the completion of the contest and will depend on the particular type (in terms of the ability parameter), since the types will be revealed after the end of the competition (as in most signalling games). Each contestant therefore can form expectations based on her private information concerning her type, so we shall have a particular value function $v_i: v_i = E_i[v(c_i)]$. The support for this function is $[v(c_H), v(c_L)]$. The private valuation of the award is
also based on the fact that the use of the reward is different for each contestant and shall be higher for that player that can use it in the most efficient way (eg, if we assume that the award is the licence for the use of some particular new technology, then the lower the cost with which the firm can use this, the higher the output from the use. So if the firm already has stuff that can use this technology with little or no new training, then the cost of using it will be far less than a firm that has to train their stuff. In this sense the lower cost firm is in the position to value the award more, as it will lead to higher profits, than those of the higher cost firm).

- Simultaneous submission of non-refundable bids. The bid or cost of participation in the contest for each opponent will be given by a common knowledge cost function that depends on the ability parameter and the effort undertaken. The general form it will obtain is given by: \( b_i(c_i, X_i) = w(c_i)\gamma(X_i) \). In this framework we shall examine the case of the cost function being linear in effort: \( \gamma(X_i) = X_i \) and non linear in the cost effort ability: \( w_i = w(c_i) \).

- The general ability parameter will define the cost ability for the bid that each of the contestants will have: \( w_i = w(c_i) \), with \( \frac{dw_i}{dc_i} > 0 \) i.e. a low ability parameter \( c_i \) means a low cost ability parameter \( w_i \). The interval from which these ability parameters are drawn is \([w_L, w_H] = [w(c_L), w(c_H)]\).

- Each participating contestant undertakes an “effort” that is observable, \( X_i \), in her attempt to win the single award. The player with highest bid \( b_i = w_i X_i \), or in other words the highest effort, wins the prize.

- Separability of the ability and the effort in the cost function (following the Moldovanu and Sela framework that use separability for simplification reasons)

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\( ^6 \) linear or concave as the cost ability is a positive function of the ability parameter \( c_i \)
• Use a deterministic rule of awarding the prize (once the outputs have been submitted, there is no uncertainty about who will be the winner). By using the deterministic rule, we eliminate the chances of loosing even if submitting the highest bid, which could be the case in a probabilistic framework.

\[
\text{Low } c_i \Rightarrow \text{high ability player } \Rightarrow \begin{cases} 
\text{low } w_i \\
\text{high } v_i 
\end{cases}
\]

The payoffs for contestant \( i \) with ability parameter \( c_i \) (i.e. valuation \( v_i = v(c_i) \) and cost ability parameter \( w_i = w(c_i) \)) are given by:

\[
\begin{cases}
    v_i - w_i X_i, X_i > \max_{i \neq j} X_j \\
    -w_i X_i, X_i < \max_{i \neq j} X_j
\end{cases}
\]

Therefore, all contestants will make their bid according to their valuation of the prize and the two ability parameters. They will choose \( X_i \) in order to maximise their objective function:

\[
\max_{X_i} [v_i - w_i X_i] \Pr(\text{winning}) + [-w_i X_i] \Pr(\text{not winning})
\]

The probability of winning is defined by the probability that player \( i \) expands the highest effort. This is given by:

\[
\Pr(X_i > X_j) = \Pr(v_i > v_j) = \Pr(w_i < w_j) = \Pr(c_i < c_j) = 1 - F(c_i)^7
\]

\text{7 The probability that } c_j < c_i \text{ is given by } F(c_i), \text{ therefore the probability that } c_i < c_j \text{ equals } [1 - F(c_i)].
In a symmetric equilibrium $X_i = X(c_i) \Rightarrow X^{-1}(X_i) = c_i$, so $[1-F(c_i)] = [1-F(X^{-1}(X_i))]$. The above implies that in the symmetric equilibrium, the effort will turn out to be a function of the ability parameter that is the most influential parameter in the model (observe that it defines both the valuation and the cost functions, the combination of which gives the payoff function, through the maximisation of which the optimal effort level emerges).

The probability of being the winner, i.e. having the highest effort among the $n$ players is given by the $(n-1)th$ order statistic: $[1 - F(X^{-1}(X_i))]^{n-1}$ i.e. all the rest of the $n-1$ players have higher ability parameters.

From the above it is clear that player $i$’s problem becomes:

$$\max_{X_i} [v_i - w_iX_i] (1 - F(c_i))^{n-1} + [-w_iX_i] (F(c_i))^{n-1}$$

Since $-w_iX_i[\Pr(\text{winning}) + \Pr(\text{not winning})] = -w_iX_i$, the above expression reduces to:

$$\max_{X_i} v_i [1 - F(c_i)]^{n-1} - w_iX_i$$

The effort to be expanded by player $i$ can be derived through the FOCs with respect to $X_i$ of the above problem:
\[ \frac{\partial v_i}{\partial X_i} [1 - F(X^{-1}(X_i))]^{n-1} - v_i(n-1)[1 - F(X^{-1}(X_i))]^{n-2} f(X^{-1}(X_i)) \frac{\partial X^{-1}(X_i)}{\partial X_i} = w_i = 0 \]

Also, as \( v_i = v(c_i) \), the derivative with respect to the effort takes the following form:

\[ \frac{\partial v_i}{\partial X_i} = \frac{\partial v_i}{\partial c_i} \frac{\partial c_i}{\partial X_i} = \frac{\partial v_i}{\partial c_i} \frac{\partial X^{-1}(X_i)}{\partial c_i} \]

So, the first order condition becomes:

\[ \left\{ \frac{\partial v_i}{\partial c_i} [1 - F(X^{-1}(X_i))]^{n-1} - v_i(n-1)[1 - F(X^{-1}(X_i))]^{n-2} f(X^{-1}(X_i)) \right\} \frac{\partial X^{-1}(X_i)}{\partial X_i} = w_i \Rightarrow \]

\[ \frac{\partial X^{-1}(X_i)}{\partial X_i} = \frac{w_i}{\left\{ \frac{\partial v_i}{\partial c_i} [1 - F(X^{-1}(X_i))]^{n-1} - v_i(n-1)[1 - F(X^{-1}(X_i))]^{n-2} f(X^{-1}(X_i)) \right\}} \]

As both the valuations and the cost ability parameters are defined by general ability one, then so will the effort. This implies that in equilibrium:

\[ \frac{\partial X^{-1}(X_i)}{\partial X_i} = \frac{1}{\frac{\partial X(c_i)}{\partial c_i}} \]

Thus the derivative that gives the effort will equal:

\[ \frac{\partial X_i(c_i)}{\partial c_i} = \left\{ \frac{\partial v_i}{\partial c_i} [1 - F(X^{-1}(X_i))]^{n-1} - v_i(n-1)[1 - F(X^{-1}(X_i))]^{n-2} f(X^{-1}(X_i)) \right\} \frac{1}{w_i} \]

In order to obtain the optimal bid, there is the need to integrate the above function. Firstly, we need to find the boundary conditions: the contestant with the lowest possible ability \( (c_i = c_{i_H}) \) can never win the prize, and so the optimal bid that she should expand is equal to zero: \( X(c_{i_H}) = 0 \).

So the solution to the bidding problem is given by:

---

Note here that the assumption of separability of the cost ability parameter and the bid allows for \( \frac{\partial w_i}{\partial X_i} = 0 \).

The symbol \( f(.) = F'(.) \)

The last part of the expression is clearer from the last equation in the page.
\[ X_i = \int_0^d dt = [t]_X^0 = [0 - X] = \left[ 0 - \int_{c_i}^{c_i} \frac{\partial X_j(c_i)}{\partial c_i} \right] = \frac{-\int_{c_i}^{c_i} \partial X_j(c_i)}{\partial c_i} \]

From the above, it is clear that the optimal bid to be places by each of the contestants is equal to:

\[ X_i = -\int_{c_i}^{c_i} \frac{\partial X_j(c_i)}{\partial c_i} = -\int_{c_i}^{c_i} \frac{\partial v_i}{\partial t} \left[ 1 - F(t) \right]^{n-1} + v_i(n-1) \left[ 1 - F(t) \right]^{n-2} f(t) dt \]

This is a general form of the optimal bidding function. It easily shown through the use of an example, that the highest effort is expanded by the player with the highest ability (i.e. lower \( c_i \)).

### 2.1 Comparative Statics

At this point of our analysis, we wish to address the issue of the effect the ability parameter has on the effort function. The aim of the model is to supply a way of sorting the players according to their abilities. We expect a decrease in the extended effort as the ability decreases (i.e the ability parameter increases in value). In order to
verify our expectations, we shall examine the derivative of the effort function with respect to the ability parameter:

\[
\frac{\partial X_i}{\partial c_i} = \frac{\partial v_i}{\partial c_i}[1 - F(c_i)]^{n-1}w_i^{-1} - v_i (n - 1)[1 - F(c_i)]^{n-2} f(c_i) w_i^{-1} < 0
\]

It is clear from the above that our anticipations are confirmed in this model. Therefore, the contest will provide the necessary signalling mechanism for the designer, as the winner will be the highest ability player. At the end of the contest, the player will be asked to reveal her particular ability and will be awarded accordingly.

### 3. Lobbying Contest

After analysing a general model of a contest where the ability of the player defines the valuation of the award, the cost of submitting a bid and therefore the probability of winning, we move to the application in the case of the lobbying contest that serves as a screening and signalling mechanism for the efficiency of a firm.

Through the use of this identification mechanism, one can be lead to the emergence of a national champion, in the market, which would be awarded with a subsidy along with a monopolistic licence. This would ensure that none of the loosing companies would produce in neither the domestic, nor the third market, ensuring that the outcome would be the competition described in Brander and Spencer. That would be the gaining of a Stackelberg leader position in the third market for the domestic national champion.

Again, the assumptions made in the general model, if applied in this particular framework, are the following:

- \( n \) domestic firms
- One award for the winner: the subsidy to be provided by the government along with a monopolistic licence. The exact value of the award is not specified. Instead the policymaker announces that the subsidy level will be defined at a later stage, after the winner of the contest has emerged. It is suggested that this will depend
on the particular efficiency of the national champion that will be competing at the international level in the third stage of the game. The positive value prize ensures that there will be no taxation in the market. According to the results derived in M&S, the existence of one prize supports the use of a linear function for the lobbying effort cost.

- The contestant that expands the highest bid, in the form of lobbying contributions will be the winner of the competition.
- Each of the domestic firms undertakes an observable (by the policymaker) “effort” denoted by $L_i$
- All firms simultaneously submit their bids, that are sunk as they are non retrievable. The cost of participation in this framework is given by the following function $w_i L_i$, where $w_i$ the lobbying cost ability parameter specific for each of the firms participating in the contest. This, as can be clearly observed by the function (which is common knowledge), is increasing in the effort undertaken.
- The general ability parameter to be used in this application, which determines the efficiency of the contestants, is the per unit production cost $c_i$ (i.e. fixed marginal cost of production which differ among the $n$ firms in the market). Nature has allocated the different cost levels, independent draws from a continuum $[c_L, c_H]$, where $c_L < .... < c_H$, among the $n$ domestic firms. The ability of each player is known only to her, however, its distribution function $G$ (with $G' > 0$) is common knowledge. The ability or in this case efficiency of the player is a negative function of the ability parameter. The lower $c_i$ is, the higher the efficiency of the firm (as it can produce the same amount as any other firm at a lower cost level).
- This general ability parameter defines the lobbying cost ability parameter. That is the per unit of lobbying effort cost, which again will be different for each of the firms (if the firms can produce at different cost levels, it is highly unlikely that the cost of lobbying will be the same for all of them). Therefore, the lobbying cost ability parameter $w_i$ will be a function of the production cost: $w_i = w(c_i)$, with $\frac{\partial w_i}{\partial c_i} > 0$.

The interval from which these parameters are drawn is $[w_L, w_H]$. Notice that $w_L$ corresponds to $c_L$, $w_H$ to $c_H$, etc.
The exact level of the subsidy is not known, therefore the player are required to form their own valuations concerning the real value of the award that will be defined after the lobbying contest has ended and the game moves to the second stage, where the government decides upon the optimal subsidisation level. This is given by the maximisation of the welfare function. At this point nevertheless, the firms can only form their valuations taking into account their own efficiency levels. The lower the lobbying cost, the higher the lobbying effort to be expanded. The firms in this way, through their lobbying effort signal their efficiency. They believe that the government will be willing to subsidise more, the more efficient the firm is. Therefore, their expectations concerning the real value of the award, hence their valuations will depend on the production cost. So, \( v_i = v(c_i) \), with \( \frac{\partial v_i}{\partial c_i} < 0 \) i.e. the lower the cost ability parameter, the higher the valuation of the award.

The government has knowledge of the contest mechanism and so it expects the higher effort to be undertaken by the highest efficiency firm.

Separability of the lobbying cost ability and the lobbying effort.

Again summarising the above, we obtain the following table:

| Low \( c_i \) ⇒ high efficiency firm ⇒ \( \begin{cases} \text{low } w_i \\ \text{high } v_i \end{cases} \) |

The payoffs for each of the firms entering the contest are given by:

\[
\Pi_i(L_i) = \begin{cases} 
P(Q)X_i + v_iX_i - C_i(X_i, c_i) - w_iL_i, L_i > \max_{j \neq i} L_j \\
- w_iL_i, L_i < \max_{j \neq i} L_j
\end{cases}
\]

The first line gives the payoffs-profits of the firm that wins the award and so proceeds to the international competition stage, whereas the second line provides us with the payoffs of the non-winning firm that only incurs the cost of entering the contest and does not produce for the international level.
All contestants will choose to place a bid that will maximise their payoffs, so the problem for each player takes the form:

$$\text{Max}_{L_i} \{P(Q)X_i + v_iX_i - C_i(X_i, c_i) - w_iL_i\} \Pr(\text{winning}) + \{-w_iL_i\} \Pr(\text{not winning})$$

Again as in the general model the probability of winning is defined by the probability of functioning with the lowest cost possible. This is given by:

$$\Pr(L_i > L_j) = \Pr(v_i > v_j) = \Pr(w_i < w_j) = \Pr(c_i < c_j) = 1 - G(c_i)^{10}$$

In a symmetric equilibrium, \( L_i = L(c_i) \) i.e. the lobbying effort is a function of the specific ability parameter. Hence, \( c_i = L^{-1}(L_i) \). The probability of winning will be given by the \((n - 1)th\) order condition as there are \( n \) players in the market (the probability of being the winner equals the probability of having the lowest general ability parameter-lowest per unit production cost among the rest of the \( n-1 \) players):

$$\text{prob}(\text{winning}) = [1 - G(L^{-1}(L_i))]^{n-1}$$

So the specific form of the maximization problem is given by:

$$\text{Max}_{L_i} \{P(Q)X_i + v_iX_i - C_i(X_i, c_i)\} \{1 - G(L^{-1}(L_i))\}^{n-1} + \{-w_iL_i\} G(L^{-1}(L_i))^{n-1}$$

Which reduces to

$$\text{Max}_{L_i} \{P(Q)X_i + v_iX_i - C_i(X_i, c_i)\} \{1 - G(L^{-1}(L_i))\}^{n-1} - w_iL_i,$$

as seen in the previous section (2).

In order to simplify the analysis, we can denote with \( \tilde{\Pi}_i = P(Q)X_i + v_iX_i - C_i(X_i, c_i) \), the payoffs of each player net of the lobbying cost in the case of winning the contest.

The solution to the problem will be provided by the first order condition with respect to \( L_i \) and its subsequent simplification$^{11}$:

---

$^{10}$ Note here that the cost of production defines the level of output \( X_i \), with \( \frac{\partial X_i}{\partial c_i} < 0 \) i.e. the lower the cost of production, the higher the production level. So we could at this point say that the following equation also holds:

$$\Pr(\text{ob}(X_i > X_j) = \Pr(\text{ob}(c_i < c_j) = 1 - G(c_i)$$

$^{11}$ A more analytical explanation is included in Appendix 1.
\[
L_i = -\int_{c_i}^{\bar{c}_i} \frac{\partial \tilde{\Pi}(t)}{\partial t} [1 - G(t)]^{-1} w(t)^{-1} dt + \int_{c_i}^{\bar{c}_i} \tilde{\Pi}(t) w(t)^{-1} (n - 1)[1 - G(t)]^{-2} g(t) dt
\]

### 3.1 Comparative Statics

As done in the previous section, we shall now examine the relationship between the ability parameter, here the production cost of each firm with lobbying effort function. We expect the effort to increase, as the ability parameter rises (i.e. the value of the ability parameter is reduced: firms produce at a lower cost level). Again, we inspect the derivative of the lobbying effort function with respect to the ability parameter:

\[
\frac{\partial L_i}{\partial c_i} = \frac{\partial \tilde{\Pi}}{\partial c_i} [1 - G(c_i)]^{-1} w^{-1} - \frac{\partial \tilde{\Pi}}{\partial c_i} (n - 1)[1 - G(c_i)]^{-2} g(c_i) w^{-1} < 0
\]

The first term has a negative sign due to the negative relationship between the profits and the production cost and the second term is also negative. The conclusion therefore is that the lower the cost level (ability parameter) the higher the lobbying effort each of the firms will expand. The end of the game will therefore find the highest ability player as the winner of the contest. The policymaker will then request the revelation of the ability parameter of the player and will set the award (subsidy) accordingly. The national champion then will be able to compete internationally in a Brander and Spencer framework.

### 4. Concluding Remarks:

In the above paper, we have provided a solution to the problem of the inability of the policymaker to identify the cost advantage firm in the domestic market, when he is interested in providing a strategic trade policy measure.

We have shown that through the use of a contest, the firms will expand efforts according to their abilities. Consequently, the highest effort will be that of the lowest

\[\frac{\partial \tilde{\Pi}}{\partial c_i} = -\frac{\partial C_i(X_i, c_i)}{\partial c_i} < 0\]
cost firm, and so the winner of the contest will be the firm that the government seeks to reward. The exertions of the participants in the contest therefore serve as signals of the particular ability of the players, and so the winner will be the highest ability one. This will be the most advantageous option for the government to set as the national champion that will compete in the international level against a single foreign firm.

The use of this model facilitates the reduction of the Dixit’s oligopolistic model with cost asymmetry to the original monopolistic framework introduced by Brander and Spencer.
REFERENCES


Appendix 1

\[ \frac{\partial \Pi_i}{\partial L_i} [1 - G(L^{-1}(L_i))]^{n-1} - \Pi_i (n-1)[1 - G(L^{-1}(L_i))]^{n-2} g(L^{-1}(L_i)) \frac{\partial L^{-1}(L_i)}{\partial L_i} - w_i = 0 \]

where \[ \frac{\partial \Pi_i}{\partial L_i} = \frac{\partial \Pi_i}{\partial c_i} \frac{\partial c_i}{\partial L_i} = \frac{\partial \Pi_i}{\partial c_i} \frac{\partial L^{-1}(L_i)}{\partial L_i} \]

This leads to the subsequent expression:

\[ \{ \frac{\partial \Pi_i}{\partial c_i} [1 - G(L^{-1}(L_i))]^{n-1} - \Pi_i (n-1)[1 - G(L^{-1}(L_i))]^{n-2} g(L^{-1}(L_i)) \} \frac{\partial L^{-1}(L_i)}{\partial L_i} = w_i \]

Solving and using the fact that in equilibrium \( \frac{\partial L^{-1}(L_i)}{\partial L_i} = \frac{1}{\partial L_i} \), the first derivative of the lobbying effort with respect to the general ability parameter equals:

\[ \frac{\partial L_i(c_i)}{\partial c_i} = \left\{ \frac{\partial \Pi_i}{\partial c_i} [1 - G(L^{-1}(L_i))]^{n-1} - \Pi_i (n-1)[1 - G(L^{-1}(L_i))]^{n-2} g(L^{-1}(L_i)) \right\} w_i \]

The lobbying effort function will be obtained through the integration of the above function. Initially however, it is necessary to attain the boundary conditions for the integral. Again, as in the description of the general contest model, the player with the lowest efficiency (i.e. with production cost \( c_{\ell} \)) has a probability zero of winning the prize and so her optimal decision concerning the level of the effort would be to undertake none: \( L(c_{\ell}) = 0 \)

Therefore the solution to the bidding problem is given by:

\[ L = \int_{L_{\ell}}^{0} dt = [t]_{L_{\ell}}^{0} = [0 - L_{\ell}] = [0 - \int_{c_{\ell}}^{c_{\ell}} \frac{\partial L_i(c_i)}{\partial c_i}] = - \int_{c_{\ell}}^{c_{\ell}} \frac{\partial L_i(c_i)}{\partial c_i} \]

14 The assumption of separability of the lobbying cost ability and the lobbying effort ensures that \( \frac{\partial w_i}{\partial L_i} = 0 \)

15 Since in equilibrium \( \frac{\partial L^{-1}(L_i)}{\partial L_i} = \frac{1}{\partial L_i} \frac{\partial L_i}{\partial c_i} \)
Substituting the expression found for the derivative of the lobbying effort with respect to the ability parameter we obtain the following:

\[
L_i = \int_{c_i}^{c_u} \frac{\partial \bar{\Pi}}{\partial c_i} \left[ -1 + \frac{1}{w_j} - \frac{1}{w_i} \right] \frac{\partial G(t)}{\partial t} dt + \int_{c_i}^{c_u} \frac{\partial \bar{\Pi}}{\partial c_i} \left[ 1 - G(t) \right]^{-1} w(t)^{-1} dt
\]

where \( \frac{\partial \bar{\Pi}(c_i)}{\partial c_i} = \frac{\partial C_i(X_i, c_i)}{\partial c_i} < 0 \)

**Appendix 2**

One of the main assumptions of auction theory is the existence of “honest” players. In the case of the first price sealed bid auction there exists the possibility of “lying”. Therefore there is the necessity of the imposition of specific Incentive Compatibility constraints to ensure the truthful revelation of the players’ types through the bidding setting.

**For the general case:**

We will allow for the type of the player to be depicted by \( i \). A lower ability player might attempt to misrepresent herself as the higher ability one or in an extreme case, the high ability one might try to pretend to be a low ability contestant. The existence of the incentive compatibility constraints ensures that situations like these do not occur.

The two general cases are:

- A low ability player (lower than \( c_L \)) would try to pass off as a high ability one. The constraint is given by:
  
  \[
  \text{Payoff of being truthful} > \text{Payoff of misrepresenting} \implies
  \]
\[ \pi(c_i, c_i) > \pi(c_L, c_L) \Rightarrow \]

where the first ability parameter in the brackets represents the type that the player is trying to pass off as, and the second term is the true type.

Probability of winning \( (c_i = c_i) \times \nu(c_i) - w(c_i)X(c_i) \)

\[ > \text{Probability of winning } (c_i = c_L) \times \nu(c_i) - w(c_i)X(c_L) \]

The probability of winning as already mentioned is given by: \( [1 - F(c_i)]^{n-1} \)

So, the expression above becomes:

\[ [1 - F(c_i)]^{n-1} \nu(c_i) - w(c_i)X(c_i) > [1 - F(c_L)]^{n-1} \nu(c_i) - w(c_i)X(c_L) \]

As already mentioned, \( [1 - F(c_L)]^{n-1} = 1 \), the above expression simplifies to:

\[ [1 - F(c_i)]^{n-1} \nu(c_i) - w(c_i)X(c_i) > \nu(c_i) - w(c_i)X(c_L) \Rightarrow \]

\[ \nu(c_i) \{1-[1 - F(c_i)]^{n-1}\} < w(c_i)[X(c_L) - X(c_i)] \]

Rearranging, the expression, we obtain the following:

\[ \frac{\nu(c_i)}{w(c_i)} < \frac{[X(c_L) - X(c_i)]}{[1-[1 - F(c_i)]^{n-1}]} \quad (1) \]

Following the exact same procedure, we obtain the constraint in the case where the high ability player tries to pass off a lower ability one:

Payoff of being truthful > Payoff of misrepresenting \( \Rightarrow \)

\[ \pi(c_L, c_L) > \pi(c_i, c_i) \Rightarrow \]

Probability of winning \( (c_i = c_L) \times \nu(c_L) - w(c_L)X(c_L) \)

\[ > \text{Probability of winning } (c_i = c_i) \times \nu(c_i) - w(c_i)X(c_i) \]

The probability of winning as already mentioned is given by: \( [1 - F(c_i)]^{n-1} \)

So, the expression above becomes:

\[ [1 - F(c_i)]^{n-1} \nu(c_L) - w(c_L)X(c_L) > [1 - F(c_i)]^{n-1} \nu(c_i) - w(c_i)X(c_i) \]

As already mentioned, \( [1 - F(c_i)]^{n-1} = 1 \), the above expression simplifies to:

\[ \nu(c_L) - w(c_L)X(c_L) > [1 - F(c_i)]^{n-1} \nu(c_i) - w(c_i)X(c_i) \Rightarrow \]

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\[ v(c_L) \{1-[1-F(c_i)]^{n-1}\} > w(c_L)[X(c_L) - X(c_i)] \]

Rearranging, the expression, we obtain the following:

\[
\frac{v(c_L)}{w(c_L)} > \frac{[X(c_L) - X(c_i)]}{\{1-[1-F(c_i)]^{n-1}\}} \quad (2)
\]

From the expressions 1 and 2 we obtain:

\[
\frac{v(c_L)}{w(c_L)} > \frac{v(c_i)}{w(c_i)},
\]

This expression holds since for any \( i: v(c_L) > v(c_i) \) and \( w(c_L) < w(c_i) \), which proves that the constraints hold.

In the most extreme case, the lowest ability player would try to pass off as a high ability one. Then the above condition would become:

\[
\frac{v(c_L)}{w(c_L)} > \frac{v(c_H)}{w(c_H)}
\]

This expression holds since: \( v(c_L) > v(c_H) \) and \( w(c_L) < w(c_H) \), which proves that again the constraints hold.

**For the lobbying contest case:**

The inducement for the various player types to imitate their co-contestants, as in the general case model, is avoided by the setting of binding constraints. Therefore, the two extreme cases are:

- The low type ability player tries to imitate the high one. This is avoided though the condition that the payoffs from been truthful are higher that the payoffs from deviating:

  Payoff truthful > payoff imitating

  \[ \pi(c_H, c_H) > \pi(c_L, c_H) \Rightarrow \]

  Probability of winning \((c_i = c_H) [P(Q)X_H + v_HX_H - C_H(X_H, c_H)] - w_H L(c_H) > \]

  Probability of winning \((c_i = c_L) [P(Q)X_H + v_HX_H - C_H(X_H, c_L)] - w_H L(c_L) \]

  The probabilities are given by \([1 - G(c_i)]\), so
\[
[P(Q)X_H + v_HX_H - C_H(X_H,c_H)][1 - G(c_H)]^{n-1} - w_HL(c_H) > \\
[P(Q)X_H + v_HX_H - C_H(X_H,c_H)][1 - G(c_L)]^{n-1} - w_HL(c_L)
\]

However, \([1 - G(c_H)] = 0\) and \([1 - G(c_L)] = 1\) ⇒
\[-w_HL(c_H) > [P(Q)X_H + v_H - C_H(X_H,c_L)] - w_HL(c_L)\]

We should note here, that the imitation in the contest only takes place at the lobbying effort setting, and not at the revenue, which is known only by the firm. That is, the firm will try to imitate being a high ability one, by lobbying as much, but when it comes to the production level, at the later stage, they will only be able to produce the low ability level, as their capability prevents them from producing any level above \(X_H\).

Rearranging the above expression, we obtain:
\[
w_H[L(c_L) - L(c_H)] > [P(Q)X_H + v_H - C_H(X_H,c_L)]
\]

Since, as has already been proven, \(L(c_H) = 0\), i.e. the lowest ability player expands no lobbying effort, so the expression becomes:
\[
\frac{L(c_L) > [P(Q)X_H + v_H - C_H(X_H,c_L)]}{w(c_H)}
\]

✓ On the other extreme hand we would have the case, where the high ability player might try to pass off as a low ability one. This is avoided through:

Payoff truthful > payoff imitating
\[
\pi(c_L,c_L) > \pi(c_H,c_L) \Rightarrow
\]

Probability of winning \((c_i = c_L)\) \[
[P(Q)X_L + v_LX_L - C_L(X_L,c_L)] - w_LL(c_L) > \\
\]

Probability of winning \((c_i = c_H)\) \[
[P(Q)X_L + v_LX_L - C_L(X_L,c_H)] - w_LL(c_H) > \\
\]

The probabilities are given by \([1 - G(c_i)]\), so
\[
[P(Q)X_L + v_LX_L - C_L(X_L,c_i)][1 - G(c_i)]^{n-1} - w_LL(c_i) > \\
[P(Q)X_L + v_LX_L - C_L(X_L,c_L)][1 - G(c_L)]^{n-1} - w_LL(c_L)
\]

As in the previous case, \([1 - G(c_H)] = 0\) and \([1 - G(c_L)] = 1\) ⇒
\[
[P(Q)X_L + v_LX_L - C_L(X_L,c_L)] - w_LL(c_L) > -w_LL(c_H)
\]

Rearranging, and using the condition \(L(c_H) = 0\) :
Combining the two constraints, we obtain:

\[
\frac{[P(Q)X_L + v_L X_L - C_L(X_L, c_L)]}{w(c_L)} > \frac{[P(Q)X_H + v_H - C_H(X_H, c_L)]}{w(c_H)}
\] or

\[
\frac{[P(Q)X_L + v_L X_L - C_L(X_L, c_L)]}{w(c_L)} > \frac{[P(Q)X_H + v_H X_H - C_H(X_H, c_L)]}{w(c_H)}
\]