Abstract

In a series of papers, Jonathan Eaton and Samuel Kortum (Eaton and Kortum, 2002, 2001 and 1999) have developed a new framework for analyses of international trade, productivity and growth. The modelling framework takes as point of departure a probabilistic distribution of productivity among countries, derived as a result of a search-theoretical model of technological change. By use of a specific probability distribution (the extreme value distribution of type II), implications for relative productivity, international trade and economic growth is derived. It is shown that some modifications of the Eaton-Kortum framework give rise to three regularities, known from empirical research: The first is the gravity equation for bilateral trade, the second is a ‘market potential’ expression for levels of GDP per capita and the third is a spatial lag model of economic growth.

* This paper has been prepared as a part of the project “Policy regimes, Efficiency and Distribution” sponsored by the NFR-program The Multilateral System in the Field of Development. I thank Leo Andreas Grünfeld, Arne Melchior and Hege Medin for valuable comments. Correspondence: The Norwegian Institute of International Affairs, P.O. Box 8159 Dep., N-0033 Oslo. E-mail: PerB.Maurseth@nupi.no.
1. Introduction

In empirical research on economic growth and international trade geography and geographical distance have been revealed as of being of great importance. Only recently, however, have theoretical advances incorporated geography into general equilibrium models of the international economy. In recent contributions, Jonathan Eaton and Samuel Kortum have developed a theoretical framework from which well-known empirical regularities are deducted. These are on relationships between international trade and productivity (Eaton and Kortum, 2002 and Eaton and Kortum, 2001). In Eaton and Kortum (1999) a search theoretical framework for innovation is developed. The point of departure is that innovation is assumed to be search among best alternatives. As a natural result, the results from innovation will be extreme value distributed. In their subsequent work (Eaton and Kortum, 2001 and 2002) this framework is used to analyse relative price levels for goods and and relative productivities between countries. Eaton and Kortum demonstrate that their modelling framework is in accordance with a set of empirical regularities. In Eaton and Kortum 2002 it is demonstrated that their model can be used to explain that i) trade diminishes with distance, ii) prices vary across locations, in contrast to the law of one price, iii) factor price equalisation does not occur and iv) countries’ productivities vary across countries. In Eaton and Kortum (2001) the same modelling framework is used to discuss specialisation in capital goods and its consequences for relative productivities. It is demonstrated that the framework is in accordance with i) concentration in equipment production in R&D intensive products, ii) countries that import most of their equipment are poorer than other countries, iii) trade in equipment goods displays a home bias, iv) in local prices equipment investments as a share of GDP shows little relation to income per habitant, v) relative prices of equipment decreases with development so that in combination with iv) poor countries have a lower real investment rate and vi) the relative price of equipment falls over time.

In this note I demonstrate that the framework developed by Eaton and Kortum can be used to deduce three further deductions that seem to fit well with results obtained in empirical research. The first is the well-known gravity modell of international trade.
First analysed by Linnemann (1966) this relationship has now become a workhorse in applied analyses of international trade. In short, this relationship predicts the bilateral volume trade between two countries, i and j, $X_{ij}$ as a positive function of the total income in both countries, $Y_i$ and $Y_j$, and a negative function of the distance between them, $d_{ij}$. The standard formulation for the gravity function is:

$$X_{ij} = Y_i^{\alpha_i} Y_j^{\alpha_j} d_{ij}^{-\alpha_i}$$

The $\alpha$’s in equation 1) are assumed to be positive and they are elasticities. In Evenett and Keller (2002) trade theory and the gravity model is discussed. They demonstrate that both versions of the Heckscher-Ohlin theory and increasing returns theory can account for the empirical success of the gravity model. However, these theories are at odds with the gravity equation when countries are completely specialised. In this note I demonstrate that the framework developed by Eaton and Kortum also gives support for the gravity equation. This framework is a purely Ricardian framework which is based on different specialisation between countries rather than different endowments or increasing returns. Furthermore, the framework used here does not predict complete specialisation when applied for empirical purposes.

The second empirical relationship is the positive relationship between market potential and productivity or GDP per capita in cross section samples of countries. In e.g. Redding and Venables (2002) a positive relationship between income and an inverse distance-weighted average of total GDP in countries is found an international dataset on countries. Similar results are found in Maurseth (2003) and in Maurseth (2001) for European regions. The relationship between income per habitant and market potential is well known. In previous research it has been interpreted as evidence for the importance of market access. In e.g. Redding and Venables this relationship is interpreted as an empirical support for increasing returns theories. In the result deduced from Eaton and Kortum’s framework, the relationship is about access to imports, not exports. Since countries will face access to cheaper capital goods the closer they are to the producers, these countries will have higher productivity. A standard formulation for the market access relationship is:
In equation 2) \( y_i \) denotes GDP per capita and the rest of the notation is as in equation 1). In empirical research there has been different approaches to whether country \( i \)’s income should be included in this country’s market potential or not. The best fit of the equation is obtained when own GDP is included in the equation.

The third empirical regularity is that growth rates of income per habitant are geographically clustered. This regularity has not received much attention in the recent surge of cross section growth regressions. The relationship is documented however, in Attfield et al. (2001) and in Maurseth (2003). This relationship is very often presumed in theoretical literature to be the result of geographically bounded technology spillovers. In Eaton and Kortum’s approach we demonstrate that technological progress will tend to benefit other countries, but that this benefit is geographically bounded. Countries will benefit from other countries’ technological progress because they get access to good in which the new technology is embedded. Because of the gravity relationship however, these benefits will tend to concentrate in neighbour countries. A formulation of this regularity is:

\[
3) \quad g_{yi} = \rho W g_y
\]

In equation 3) \( g_{yi} \) denotes the growth rate in GDP per habitant in country \( i \). \( \rho \) is a spatial lag coefficient, \( W \) is a distance matrix in which the elements decrease by the distance between all countries and \( g_y \) is the vector of growth rates in all other countries.

In the next section the modelling framework by Eaton and Kortum is presented. In section three it is demonstrated that small modifications in this framework results in the three empirical regularities. The most important modification is that we assume a country’s technological strength to be proportional to its total level GDP. This assumption is discussed in section 4 which also concludes.
2. The model

2.1 An introduction

There are two types of goods, capital goods, denoted by the subscript \( k \) and consumer goods, denoted by \( c \). There are transportation costs so that an exporter in country \( n \) must ship \( d_{ni} > 1 \) units of a good for one unit to arrive in country \( i \). Let \( L \) denote labour, \( K \) a CES aggregate of capital goods, \( A \) total factor productivity and \( Q \) production. Production of goods \( s \) in country \( i \) is assumed to be Coub-Douglas and is given by the production function in equation 4. There is intersectoral labour mobility but immobility of labour and capital between countries. Resource constraints are given by equations 5 and 6.

\[
\begin{align*}
4) \quad Q_{i,s} &= A_{s} K_{s}^{\alpha} L_{i}^{1-\alpha}, \quad s = k, c \\
5) \quad L_{i} &= L_{ci} + L_{ki} \\
6) \quad K_{i} &= K_{ci} + K_{ki}
\end{align*}
\]

The model focuses on technological change in the capital goods sector as a driving force for growth. Therefore \( A_{ci} \) is set equal to one. There is perfect competition so that effective costs determine prices. When \( A_{ks} \neq A_{ki} \) there is a case for international trade (no goods are assumed to be ‘intensive’ in any factors of production). Let \( P_{cn} \) denote prices on (homogenous) consumer goods in country \( n \) and \( P_{kn} \) be a price index for capital goods.

Now, for sake of simplicity, assume that there are two countries, \( s \) and \( n \) (this assumption will be relaxed below). Also assume that \( A_{ks} \) is so low that country \( s \) specialises completely in consumer goods (this assumption is discussed below). Normalise prices so that \( P_{cs} = 1 \). Since there is perfect competition we have that \( P_{cn} = d_{ns} P_{cs} = d_{ns} \). Furthermore, prices of capital goods in country \( n \) (the capital producing country) is given by: \( P_{kn} = P_{cn} / A_{kn} = d_{ns} / A_{kn} \). Since country \( s \) imports capital goods, prices of capital goods here will equal \( P_{ks} = d_{ns} P_{kn} = d_{ns}^2 / A_{kn} \).

In terms of prices of consumer goods in country \( s \), income levels of country \( s \) and \( n \) are given by:
7) \[ Y_s = K_{s}^a L_s^{1-a} = L_s \left( \frac{K_s}{L_s} \right)^a = L_s k_s^a \]

8) \[ Y_n = P_{nk} A_{nk} K_{nk}^a L_{nk}^{1-a} + P_{nc} K_{nc}^a L_{nc}^{1-a} = d_{ns} L_{nk} \left( \frac{K_{nk}}{L_{nk}} \right)^a + d_{ns} L_{nc} \left( \frac{K_{nc}}{L_{nc}} \right)^a = d_{ns} L_n^a k_n^a \]

Equation 7) is straightforward when \((K/L)\) is defined as \(k\). Equation 8) is obtained from inserting the prices and utilising the fact that the capital labour ratio equalises between sectors. Equations 7) and 8) correspond to national currency GDPs translated into a common currency. The expression for GDP per capita in terms of consumer prices, \(y_i = (Y_i/P_i^c L_i) = f(k_i)\), corresponds to real GDP per capita in international prices used in e.g. the Penn World tables.

Investments and depreciation govern the evolution of the capital stock according to equation 9). Expenditure on capital goods is \(P_{jk} I_j\). The driving force for growth is technological progress at the rate \(g\) in the capital goods producing sector. This rate of growth is assumed to be exogenous in the present model. The growth rate in the capital goods producing sector translates into price decrease of capital goods at the same rate. Denote the growth rate of national income \(g_y\). Because consumption and expenditures on capital goods sum to national income, in steady state these variables grow at the same rate, \(g_y\). Real investments correspondingly grow at the rate \(g + g_y\).

With the assumed production functions, we therefore have \(g_y = \alpha/(1-\alpha)g\). With constant savings rates, \(s\), the capital stock per worker grows according to equation 10). In that equation account is taken for price differences between consumer goods and capital goods.
We have two equations for the growth rate in physical capital. The first says this equals $g_y + g$. The second is derived from equation 9 giving equation 10). These two equations can be used to derive a steady state income level, as in equation 11). Equation 11) says that the steady state income level per person is an increasing function of the savings rate and a negative function of the relative price of capital goods.

Now, insert for prices in the above equation. Derive the relative income level per habitant. This produces the equation:

12) $\frac{y_a}{y_s} = \left( \frac{s_a}{s_s} \right)^{\frac{\alpha}{1-\alpha}}$

From this equation it is seen that a capital producing country is richer than a country that does not produce capital. These differences are, when savings rates are equal, determined by trade barriers. When trade barriers are reduced, differences decrease.

2.2 A multi country and multi good application

We now leave the assumptions that there are two countries and one homogenous capital good. Rather assume that there are N countries and that the capital stock in a country is a CES aggregate of differentiated capital goods:
Assume that country $i$ produces capital good $j$ with quality $z_i(j)$. Buying capital good $j$ from country $i$ therefore faces country $n$ with the cost $P_{kn}(j) = \frac{d_n}{z_i(j)}$. Country $n$ will actually buy good $j$ from country $i$ if it is indeed the cheapest among all countries, so $P_{kn}(j) = \min[P_{kn}(j)]$.

Assume that the qualities $z_i(j)$ are realisations of random variables drawn from the extreme value distribution $Pr(z_i \leq z) = \exp(-T_i z - \theta)$. In that expression, $T_i$ represents the stock of knowledge, or the knowledge base, in country $i$ and $\theta$ is a parameter governing (inversely) the variability. The extreme value distributions denote stochastic processes where the largest (or smallest) value is taken from samples generated by (independent) draws from statistical distributions. For our setting, where an underlying assumption is that innovation is searching for the best among many alternatives, the use of the extreme value distribution is natural. $Z_i$ has geometric mean $e^{\theta \log T_i}$ and its log has variance $\pi^2/6$.

Since $z_i(j)$ denotes the quality in which country $i$ can produce capital good $j$, the cost in country $n$ of buying good $j$ from country $i$ is $p_{n}(j) = \frac{c_i d_n}{z_i(j)}$.

Here we assume that the knowledge base grows at the exogenous rate $g$. By use of the assumed probability distribution for the $z$’s, the cost in country $n$ of buying good $j$ from country $i$ is drawn from $Pr(P_{kn} < p) = 1 - \exp(-T_i d_n - p \theta)$. The minimum across all countries is therefore:

$$
\text{Pr}[P_{kn} \leq p] = G_n(p) = 1 - \exp(-\Phi_n p^\theta)
$$

$$
\Phi_n = \sum_{i=1}^{N} T_i d_n^{-\theta}
$$

It is seen from equation 14 that the extreme value distribution keeps its functional form from aggregation over many sources. Under the above assumptions, it can be
shown that the fraction of capital goods that country \( n \) buys from country \( i \) is given by:

\[
\pi_{ni} = \frac{T_i d_{ni}^{-\theta \rho}}{\sum_{i=1}^{N} T_i d_{ni}^{-\theta \rho}} = \frac{T_i d_{ni}^{-\theta \rho}}{\Phi_n}, \quad \Phi_n = \sum_{i=1}^{N} T_i d_{ni}^{-\theta \rho}
\]

Now it can be shown that an exact price index for capital goods in country will be given by:

\[
P_{kn} = \gamma \Phi_n^{-\sigma/\theta} \gamma = \Gamma \left(1 - \frac{\sigma - 1}{\theta} \right)^{-\sigma}
\]

1 Eq. 15 is derived first by calculating the probability that country \( i \) offers country \( n \) a good at the cheapest possible price. This expressions is given by

\[
\pi_{ni} = \Pr\{P_{ni}(j) \leq \min\{P_{ni}(j); s \neq i\}\} = \prod_{s=1}^{N} [1 - G_{ni}(p)] G_{ni}(p)
\]

Solving the integral yields eq. 15. By the law of large numbers, this will also be fraction of goods that country \( n \) actually buys from country \( i \).

2 Eq. 16. is derived by calculating the moment generating function for the negative of the log prices. This function is

\[
E(e^{it}) = E(e^{-t \ln p_n}) = E(p_n^{-t})
\]

\[
= \int_0^{\infty} p^{-t} \Phi_n^{-\theta \rho} e^{-\Phi_n \rho \sigma} dp
\]

\[
= \Phi_n^{-\sigma/\theta} \Gamma \left(1 - \frac{1}{\theta} \right)
\]

Now insert for \( t = \sigma - 1 \) to obtain the price index for capital goods:

\[
P_{kn} = \Gamma \left(1 - \frac{\sigma - 1}{\theta} \right)^{-\sigma} \Phi_n^{-\sigma/\theta}
\]
Equation 15) denotes the share of capital goods that country \( n \) buys from country \( i \). If country \( n \) uses the share \( s_n \) of its income on capital goods, total expenditures on capital goods are:

\[
X_n^k = s_n Y_n
\]

Imports of capital goods from \( i \), as a share of GDP, are then

\[
17) \quad \frac{X_{ni}^k}{Y_n} = \pi_{ni} s_n = \frac{T_i c_i^{\theta} d_{ni}^{-\theta}}{\Phi_n} s_n
\]

2.3 Empirical implications

Now we introduce an important theoretical assumption. This is that country \( i \)'s technological level, \( T_i \), is identical to its total income level, \( Y_i \). This assumption is discussed in some further detail below.

The trade equation from above now becomes:

\[
18) \quad X_{ni}^k = \pi_{ni} s_n Y_n
= \frac{T_i c_i^{\theta} d_{ni}^{-\theta}}{\Phi_n} s_n = \frac{c_i^{\theta} s_n}{\Phi_n} d_{ni}^{-\theta} Y_i Y_n
\]

Equation 18) is a gravity equation for trade in capital goods which is what we wanted to find.

We had equation 11)
Also, remember the definition of the price index:

\[ P_{ln} = \gamma \Phi_{\phi}^{-\frac{1}{\phi}} = \gamma \left[ \sum_{i=1}^{N} T_i d_{ji}^{-\phi} \right]^{-\frac{1}{\phi}} \]

Now, take the logs of equation 11) to obtain:

\[ \ln y_j = \frac{\alpha}{\alpha - 1} \ln \left( \delta + \frac{g}{1 - \alpha} \right) + \frac{\alpha}{1 - \alpha} \ln P_{jc} + \frac{\alpha}{\theta(1 - \alpha)} \ln \left[ \gamma \sum_{i=1}^{N} T_i d_{ji}^{-\phi} \right] + \frac{\alpha}{1 - \alpha} \ln s_j \]

With the above definition of \( T_i \), we obtain:

\[ \ln y_j = \frac{\alpha}{\alpha - 1} \ln \left( \delta + \frac{g}{1 - \alpha} \right) + \frac{\alpha}{1 - \alpha} \ln P_{jc} + \frac{\alpha}{1 - \alpha} \ln s_j + \frac{\alpha}{\theta(1 - \alpha)} \ln \left[ \gamma \sum_{i=1}^{N} Y_i d_{ji}^{-\phi} \right] \]

Equation 20) corresponds to a positive relationship between market potential and income per habitant.

Now rewrite equation 11) as:

\[ y_j = B_j \left[ \sum_{i=1}^{N} T_i c_{ji}^{-\phi} d_{ji}^{-\phi} \right]^{-\frac{\alpha}{\theta(1 - \alpha)}} \]

In equation 21) \( B_j \) is a country-specific variable. Now derivate equation 21) with respect to time where only \( y_j \) and \( T_i \) are allowed to change. This amounts to the same
thing as perturbing the system of equations with technological shocks in the cross section of countries:

\[
\dot{y}_j = B \frac{\alpha}{\theta(1-\alpha)} \left[ \sum_{i=1}^{N} T_i c_i^{-\theta} d_{ni}^{-\theta} \right]^{\frac{\alpha}{\theta(1-\alpha)-1}} \sum_{i=1}^{N} \frac{\dot{T}_i}{T_i} T_i c_i^{-\theta} d_{ni}^{-\theta}
\]

\[
\rightarrow \frac{\dot{y}_j}{y_j} = \frac{\alpha}{\theta(1-\alpha)} \left[ \sum_{i=1}^{N} \frac{\dot{T}_i}{T_i} T_i c_i^{-\theta} d_{ni}^{-\theta} \right] = \frac{\alpha}{\theta(1-\alpha)} \sum_{i=1}^{N} g_i A_i
\]

where

\[A_i = T_i c_i^{-\theta} d_{ni}^{-\theta}\]

Now use the relationship from above that \(g_j = \alpha/\theta(1-\alpha)g\). From this we obtain:

\[
g_{sj} = \frac{\sum_{i=1}^{N} g_{sj} A_i}{\sum_{i=1}^{N} A_i}
\]

\[
\rightarrow g_{sj} \left( \frac{\sum_{i=1}^{N} A_i - A_j}{\sum_{i=1}^{N} A_i} \right) = \frac{\sum_{i \neq j} g_{sj} A_i}{\sum_{i=1}^{N} A_i}
\]

\[
22) \quad g_{sj} = \frac{\sum_{i \neq j} g_{sj} A_i}{\sum_{i \neq j} A_i}
\]

Equation 22) is a spatial lag model for economic growth in which the weights decrease with distance. The weights however, are also increase with the economic magnitude of other countries.
3. Discussion and conclusion

Our discussion of Eaton and Kortum’s modelling framework allowed us to draw three further conclusions. Their framework gives rise to a gravity model for international trade, a positive relationship between income per capita and market potential and a spatial lag model of economic growth. These are conclusions that Eaton and Kortum have not drawn in their papers. In order to reach that conclusion we introduced the assumption that a country’s technological strength is proportional with its total GDP. In growth literature, knowledge stocks are often defined as accumulated R&D or as GDP per capita level. For global data, accumulated R&D data are not available and if they were, they would probably not be very useful for poor countries. GDP per capita levels as proxy for knowledge stocks assume that small rich countries have the same technological level as large rich countries. This is a doubtful assumption, at least in the present context in which knowledge stocks in country i enter in the expression for the share of what country n buys from country i. Here the total level of GDP in a country is used as an approximation for knowledge stocks. This implies that a large poorer country might have the same knowledge stock as a small rich country. This assumption is in line with the growth models of Frankel (1962) and Romer (1986). With use of this definition of a country’s knowledge stock, GDP per capita in a country becomes a function of total GDP in all countries and the distance between the country in question and all other countries.

References


