The Factor Content of Trade and Relative Wages in US:

A Dual Approach

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Abstract

In this paper a General Equilibrium model is used in order to measure the effect of international trade on relative wages between skilled and unskilled workers in the US economy. The dual representation of technology is used for the US economy, a revenue function, instead of the usual primal cost function following Kohli (1991) and (1993). After estimating the revenue function for the US economy the “Equivalent Autarkic Equilibrium” approach, Deardorff and Staiger (1988), is implemented in order to calculate the effect of international trade on the relative wages between skilled and unskilled workers in US from 1968 to 1991.

JEL classification: F11, F16 and J31.

Keywords: international trade, relative wages, Factor Content of Trade, revenue function.

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1. Introduction

The possible relationship between international trade and wage inequality in developed countries has been a very important and interesting topic for both academics and politicians. The reason lies on two facts; first, unskilled workers in many developed countries and especially in US have seen a significant decline in their relative wages. At the same time international trade was named as one of the most likely candidates to explain this decline. Economists have approached from two different angles this question of whether international trade is the main and solely reason explaining the increased wage inequality experienced by the US and other developed economies over the last thirty years.

The first is based on the traditional Stolper-Samuelson theorem where changes in prices cause changes in factor rewards. This approach was followed by Leamer (1997). Leamer represented technology in the standard neoclassical trade tradition that is cost functions and showed that there is an effect of declining prices of unskilled labour intensive products due to international trade that can explain wage inequality. Also Harrigan and Balaban (1999) are using the same Stolper-Samuelson approach, in the form of elasticities, to assess the same question with the difference that they use revenue function instead, in order to describe the technological parameters of US economy. They found that relative price and relative factor supply changes explain wage inequality, but they mainly attributed these changes to technological change and not to international trade.

The second approach, followed by Borjas et al (1992), Katz and Murphy (1992), Wood (1994) and Baldwin and Cain (1997), has focused on the Factor Content of Trade theorem, Vanek (1968), where the volume of net exports is transformed into
input quantities with the use of an input requirement matrix. This approach is calculating the amount of net exports of input quantities and then by using a partial equilibrium model tries to calculate the effect of these changes of input quantities on relative factor rewards. The technology representation in all of these studies has been the primal cost function and they conclude, with the exception of Wood, that trade has a very small impact on explaining wage inequality, since they argue that the main reason is technological change.

But this second approach has also generated a debate on the question whether the Factor Content approach is the appropriate tool to assess the effect of international trade on relative wages from a theoretical point of view, Deardorff (2000), Panagariya (2000), Krugman (2000) and Leamer (2000). FCT studies have been heavily criticised on the ground that they lack a solid theoretical foundation and especially that FCT is not related with factor prices. The main argument, Panagariya (2000) and Leamer (2000), was that FCT calculates quantities of indirectly exported and imported factors via international trade but according to the Stolper-Samuelson theorem, it is product prices, not factor quantities, which are related with factor prices. But that criticism was invalid according to Deardorff (2000) and Krugman (2000), since Deardorff and Staiger (1988) had already provided a theoretical framework using the concept of the ‘‘Equivalent Autarkic Equilibrium’’.

In addition, the FCT allows for a decomposition of the overall increase in relative wage inequality between international trade and technical changes as Leamer (2000) shows. However the focus of this paper is away from the trade or technology debate. In particular, this paper points out the necessary restrictions for the validity of the ‘‘Equivalent Autarkic Equilibrium’’ when a revenue function is used and also proposes a different way for measuring the Factor Content of Trade. The dual
representation of technology is implemented, revenue (GNP) function, for the reason that treats product prices and endowments as exogenous which is in agreement with the standard neoclassical trade theory. Then by following the Equivalent Autarky Equilibrium approach calculates the effect of international trade on relative wages between skilled and unskilled workers in the US economy between 1968 and 1991.

2. The Model

A General Equilibrium model is implemented to assess the effect of international trade on relative wages. Both sides of the economy, the supply and the demand, are initially considered in order to derive the relationships that characterize the environment within the economy is operating. In the supply side, technology is represented by a revenue function, instead of the usual primal cost functions. The use of duality and more specifically the implementation of revenue function is preferred because it complies with the standard assumptions made in international trade theory that product prices and endowments are given while factor prices and outputs are the endogenous variables to be determined.

In the supply side of the economy, assume a non-joint and linearly homogeneous technology, producing $y = (y_1, \ldots, y_n)$ goods with the use of the same number of $v = (v_1, \ldots, v_m)$ inputs $(n = m)$ in a perfectly competitive environment. Then, at given international prices $p = (p_1, \ldots, p_n)$ and domestic inputs $v$, there exists a competitive production equilibrium. In such equilibrium we can think of the economy as one that maximises the value of total output subject to the technological and endowment constraints. In other words there is a revenue or GDP function such that:
where \( y_i = f^i(v_{\mu}) \) is the production function of the \( i \)th good and \( v_{\mu} \) is the amount of input \( j \) used in the production of the \( i \)th good. Under the above assumptions, the revenue function \( R(p,v) \) is increasing, linearly homogeneous and concave in \( v \) and non-decreasing, linearly homogeneous and convex in \( p \). In addition if \( R(p,v) \) is differentiable then equilibrium output and factor rewards are given by:

\[
y(p,v) = R_p(p,v) \tag{2}
\]

\[
w(p,v) = R_v(p,v) \tag{3}
\]

where \( R_p(p,v) \) is the first partial derivative of the revenue function with respect to the product prices \( p \) and \( R_v(p,v) \) is the first partial derivative of the revenue function with respect to inputs \( v \).

On the demand side, the economy’s preferences defined over the \( n \) goods could be represented by an expenditure function, which is continuous and once differentiable on prices:

\[
E(p,u) = \min_{c} \left\{ pc : g(c) \geq u \right\} \tag{4}
\]
where \( u \) is the level of utility and \( c = (c_1, \ldots, c_n) \) is the consumption bundle. By differentiating \( E(p,u) \) with respect to \( p \) we get the consumption vector of the economy:

\[
c(p,u) = E_p(p,u) \tag{5}
\]

where \( E_p(p,u) \) is the first partial derivative of the expenditure function with respect to product prices.

2.1 The Trading Equilibrium

Assume now the case where the economy is in a trading equilibrium and at the same time trade is balanced, then the following should be true:

\[
R(p, v^\circ) = E(p, u) \tag{6}
\]

that is the total value of production should be equal to the total expenditure for the economy. The difference between production and consumption should give the economy’s vector of net exports:

\[
t(p, v, u) = R_p(p, v^\circ) - E_p(p, u^\circ) \tag{7}
\]

\(^1\) A zero superscript denotes initial values
where \( t(p,v,u) \) is the net exports vector. From (7) it is obvious that net exports depend on product prices, endowments and the level of utility.

### 2.2 Equivalent Autarky Equilibrium

Deardorff and Staiger 1988, proved that under the same assumptions we made above there exists a new equilibrium (Equivalent Autarky Equilibrium) with the same product and consequently factor prices, so long as factor rewards are determined solely from product prices, where production equals consumption. The economy can reach this new equilibrium by subtracting from its initial endowments the Factor Content of net exports. The Factor Content of net exports is defined as the product of the input requirement matrix \( A \) with the net exports vector.

\[
FCT = A \times t(p,v)
\]  

(8)

In other words it is nothing more but the amount of inputs necessary to produce the vector of net exports. In Equivalent Autarky Equilibrium the economy is producing what it desires to consume, having no incentive to trade with other countries. Hence, the vector of net exports is going to be a vector of zeroes, which means that production and consumption are equalised for every good produced, which is shown in (9).

\[
R_p(p,v^i) = E_p(p,u^o)
\]  

(9)
where \( v^1 \) is the vector of the new endowments that resulted after subtracting the Factor Content of net exports from the initial endowments vector \( v^0 \).

In the Equivalent Autarky Equilibrium, we assume that preferences’ technology remains the same and because product prices are also unchanged the vector of consumption is unaltered. Under the assumption of balanced trade, the value of GDP should be equal to the economy’s total expenditure (same as in the trading equilibrium). Hence, again the value of production should equal the value of consumption in this new equilibrium

\[
R(p, v^1) = E(p, u^o) \tag{6'}
\]

From (6) and (6’) we get that:

\[
R(p, v^o) = R(p, v^1) \tag{10}
\]

which implies that the total value of production has not changed between the two equilibria, despite the different endowment vectors. Substituting (9) into (7) we get a new relationship for the net exports vector in the trading equilibrium:

\[
\tau(p, v) = R_p(p, v^o) - R_p(p, v^1) \tag{11}
\]

which states the core result of the Heckscher-Ohlin theory. Trade depends on international prices and differences on factor endowments only. By solving (11) for \( v^1 \) we can find the amount of endowments that the economy should have had, if it had
no trade at the existing international prices \( p \). In other words we are able to find the Factor Content of net exports, because the new endowments vector is nothing more but the difference between the original endowments vector and the vector of the Factor Content of net exports. Alternatively, multiplying both sides of (11) by the inverse of the matrix of the partial derivatives of outputs with respect to inputs \( \left[ R_{pv}(p) \right]^{-1} \) and rearranging we get:

\[
\left[ R_{pv}(p) \right]^{-1} t(p,v) = \left[ R_{pv}(p) \right]^{-1} R_p(p,v^0) - \left[ R_{pv}(p) \right]^{-1} R_p(p,v^1) \Rightarrow
\]

\[
FCT = v^0 - v^1 \Rightarrow v^1 = v^0 - FCT
\]

(12)

For (12) to hold the assumptions of linear homogeneity in inputs \( v \) and non-jointness in output for the revenue function have to be satisfied, which implies that it takes the following form as Kohli (1983) shows:

\[
R(p,v) = R(p) = \sum_{j=1}^{M} \theta^j(p)v_j
\]

(13)

where \( \theta^j(p) \) is nonnegative, non-decreasing, linearly homogeneous and convex in \( p \). In that case \( R_{pv}(p,v) \) is locally independent of inputs \( v \) and its inverse matrix \( \left[ R_{pv}(p) \right]^{-1} \) is identical to an input requirement matrix as Harrigan (2002) points out.

This approach firstly makes the analysis more simple and tractable, since the selection of endogenous and exogenous variables complies with the neoclassical trade theory. It also verifies the validity of the ‘‘Equivalent Autarkic Equilibrium’’ and the necessary restrictions when a revenue function is used to describe the technological
parameters for the production side of the economy. And finally suggests a new way of calculating the Factor Content of Trade.

2.3 Autarky

A real autarky equilibrium can be reached by performing the opposite thought experiment than before. The vector of the Factor Content of net exports is now added to the new endowments $v^1$ leaving the economy with its initial endowments of inputs. The difference is that by altering the factor endowments in the Equivalent Autarky Equilibrium the production vector will change and since prices are the same the consumption vector will be the same. Markets will not clear and for that reason prices have to change in order to ensure an autarkic equilibrium, where domestic consumption equals domestic production. These changes in prices have to cause a change in factor rewards. Hence, the alteration of endowments, equal to the Factor Content of net exports, generates the change in factor rewards described above and because the FCT is related to the volume of trade (8) a relationship between the volume of trade and factor rewards is established. This is the ‘’strong’’ as it called version of the Equivalent Autarky Equilibrium approach and requires Cobb-Douglas technology in both the supply and demand side of the economy.

A more clear presentation of the relationship between FCT and factor rewards requires assuming two different trading equilibria, where only product prices and the volume of trade differ, while technology and factor endowments remain the same. Deardorff and Staiger (1988) established the relationship between factor rewards and the Factor Content of Trade in the ‘’strong’’ version of the Equivalent Autarky
Equilibrium after normalising prices in both equilibria to guarantee that expenditure is equal to one as:

\[
\frac{w_j^2 - w_j^1}{w_j^1} = \frac{FCT_j^2 - FCT_j^1}{v_j^0 - FCT_j^2}
\]  

(14)

where \(w_j^t\) indicates the reward of factor \(j\) in equilibrium \(t = 1, 2\), \(FCT_j^t\) indicates the Factor Content of Trade of factor \(j\) in equilibrium \(t\) and \(v_j^0\) indicates the common endowment in both equilibria of the \(jth\) factor. (14) states that the percentage change in the reward for the \(jth\) factor between the two equilibria is positively related to its Factor Content of Trade of the latter equilibrium and negatively related to its Factor Content of Trade of the earliest equilibrium. From the above equation it is clear what change in the reward of the \(jth\) factor is attributed to trade if we move back from the second equilibrium to the first. In other words it tells by how much the reward of the \(jth\) factor would have changed if we restricted the volume of trade of the second equilibrium to be equal to the one in the first assuming technology and endowments have remained unchanged.

3. Econometric Specification

The functional form of the revenue (GDP) function that is used, discussed in Kohli (1991) and Kohli (1993), is the Symmetric Normalise Quadratic revenue function augmented by time:

\[
R(p, v, t) = \frac{1}{2} \left( \sum_{j=1}^{M} \beta_j v_j \right) \left( \sum_{i=1}^{N} \sum_{h=1}^{N} a_{ih} p_i p_h \right) \left( \sum_{i=1}^{N} \alpha_i p_i \right)^{-1} + \sum_{i=1}^{N} \sum_{j=1}^{M} c_{ij} p_i v_j + \]
where $t$ represents time. There are $N(N-1)+M(M-1)+N \times M + 2$ unknown parameters $a_{ih}$, $b_{jk}$, $c_{ij}$, $d_i$, $e_j$, $h_i$ and $h_n$, where $i, h = 1, \ldots, N$ and $j, k = 1, \ldots, M$. There are also $N + M$ predetermined parameters $\alpha_i$ and $\beta_j$. In particular, $\alpha_i$ and $\beta_j$ can be set equal to the share value of each product and input respectively at the base year. Symmetry conditions are imposed $a_{ih} = a_{hi}$; $b_{ij} = b_{ji}$ and the assumptions of linear homogeneity in $p$ and $v$ require some additional restrictions:

\[
\sum_{i=1}^{N} \alpha_i = \sum_{j=1}^{M} \beta_j = 1, \text{ and } \sum_{i=1}^{N} a_{ih} = \sum_{h=1}^{N} b_{jk} = \sum_{i=1}^{N} d_i = \sum_{j=1}^{M} e_j = 0 \quad (16)
\]

Then by differentiating the revenue function with respect to the factor endowments we get the reward of the $j$th factor:

\[
w_j = \frac{1}{2} \beta \left( \sum_{i=1}^{N} \sum_{h=1}^{N} a_{ih} p_i p_h \left( \sum_{i=1}^{N} \alpha_i p_i \right)^{-1} + \sum_{i=1}^{N} \alpha_i p_i \left( \sum_{h=1}^{M} b_{jk} v_k \left( \sum_{h=1}^{N} b_{jk} v_k \right)^{-1} \right) \right)
\]

\[
- \frac{1}{2} \beta \left( \sum_{i=1}^{N} \alpha_i p_i \left( \sum_{j=1}^{M} \sum_{k=1}^{M} b_{jk} v_k \left( \sum_{j=1}^{M} \beta_j v_j \right)^{-2} \right) + \sum_{i=1}^{N} c_{ij} p_i + \beta_j \left( \sum_{i=1}^{N} d_i p_i \right) \right) t +
\]
\[ e_j \left( \sum_{i=1}^{N} \alpha_i p_i \right) t + \beta_j \left( \sum_{i=1}^{N} \alpha_i p_i \right) h_j t + \frac{1}{2} \beta_j \left( \sum_{i=1}^{N} \alpha_i p_i \right) h_j t^2 \]  

(17)

Similarly, by differentiating the revenue function with respect to the product prices we get the output of the \( i \text{th} \) good:

\[ y_i = \frac{1}{2} \alpha_i \left( \sum_{j=1}^{M} \sum_{k=1}^{M} b_{jk} v_j v_k \right) \left( \sum_{j=1}^{M} \beta_j v_j \right)^{-1} + \left( \sum_{j=1}^{M} \beta_j v_j \right) \left( \sum_{k=1}^{N} a_{ik} p_k \right) \left( \sum_{i=1}^{N} \alpha_i p_i \right)^{-1} \]

\[ -\frac{1}{2} \alpha_i \left( \sum_{j=1}^{M} \beta_j v_j \right) \left( \sum_{k=1}^{N} a_{ik} p_k \right) \left( \sum_{i=1}^{N} \alpha_i p_i \right)^{-2} + \sum_{j=1}^{M} c_{ij} v_j + d_i \left( \sum_{j=1}^{M} \beta_j v_j \right) t + \]

\[ \alpha_i \left( \sum_{j=1}^{M} e_j v_j \right) t + \alpha_i \left( \sum_{j=1}^{M} \beta_j v_j \right) h_j t + \frac{1}{2} \alpha_i \left( \sum_{j=1}^{M} \beta_j v_j \right) h_j t^2 \]  

(18)

All the parameters of the revenue function (15) are included in (17) and (18), hence by estimating (17) and (18) we can obtain an estimate of (15). The factor rewards and output supplies equations will be estimated by adding a random disturbance in each of them, with the usual classical assumptions. (17) has M and (18) has N equations all of which are linearly independent. Hence, a system of M+N equations is going to be estimated with the above restrictions using the iterative version of Seemingly Unrelated Regression Estimator method. The likely issue of endogeneity in product prices can be tackled by reestimating the above model using Iterative Three Stage Least Squares and possible instruments can be a constant and the lagged value of product prices.
The revenue function must satisfy the conditions for concavity on inputs and convexity on product prices. The necessary and sufficient condition for global concavity in inputs is that matrix $B = [b_{jk}]$ be negative semi-definite and for global convexity that matrix $A = [a_{jk}]$ be positive semi-definite. If these are not satisfied then they are imposed following Diewert and Wales (1987) without removing the flexibility properties of the revenue function.

By obtaining an estimate of the revenue function, the inverse of the matrix of the partial derivatives of outputs with respect to inputs $[R_{pv}(p)]^{-1}$ can be estimated and then from (12) the Factor Content of Trade can be calculated. Then with the use of (14) we can find the effect of trade on the relative wages of unskilled workers under the assumptions of the ‘’strong’’ version of the Equivalent Autarky Equilibrium approach.

4. Data Construction and Description

Data are collected for the 2-digit SIC87 classification for US manufacturing for the period 1968 to 1991. The estimated model consists of three aggregate products and three factors of production. The aggregate products are exportable, importable and non-tradable and the factors are capital, skilled labour and unskilled labour. Initially the products are divided between tradables and non-tradables. A product is tradable if the ratio of the value of its exports plus imports divided by its GDP is above 10%, otherwise is treated as non-tradable. Then the tradable products are grouped to exportables and importables depending on whether their net exports are
positive or negative respectively. Trade data at the 2-digit SIC87 level were obtained online from the Centre for International Data at the University of California Davis.\(^2\)

Data for the value and price of capital and labour, at a 2-digit SIC87 analysis are obtained from Dale Jorgerson’s database. The value added is constructed for these three aggregate goods. Data for output deflator are obtained from the Bureau of Economic Analysis. Because these are available from 1977 onwards, the values of output deflators for years before 1977 are obtained by interpolation assuming a constant growth rate equal to the growth rate between 1977 and 1978.

From the Current Population Surveys: March Individual-Level Extracts, 1968-1992 (ICPSR 6171) we get data on the educational level of labour, wages per hour, status and days worked for full time and part workers in 2-digit SIC industries. With this information it is possible to separate workers into skilled and unskilled. Following Beaudry and Green (2003) a worker is treated as skilled if he or she spent more than nine years in education. Then we calculate the total number of days worked for both occupational groups and by assuming that a working day consists of eight hours and that a full time worker works for 2000 hours per year we calculate the number of equivalent full time workers in the sample. Then we sum the value of wages for each occupational group for a year and we divide by the number of equivalent full time workers times 2000 in order to calculate the full time equivalent hourly wage for each group respectively. After that we calculate the share of full time equivalent skilled and unskilled workers relative to the total number of full time equivalent workers. The same is applied for the calculation of the shares of wages of both occupational groups in the sample. These shares are multiplied with the total

\(^2\) http://data.econ.ucdavis.edu/international/index.html
\(^3\) http://post.economics.harvard.edu/faculty/jorgenson/data/35klem.html
\(^4\) www.bea.doc.gov
\(^5\) Following Harrigan and Balaban (1999), self-employed workers are excluded since they tend to give inaccurate information about their income and wages.
number of all workers and total wages obtained from Jorgerson’s dataset in order to get the number and wages for skilled and unskilled workers in US manufacturing\textsuperscript{6}.

The aggregation for goods is achieved in three stages\textsuperscript{7}. First, the value added for each aggregate good is found by summing the value added of all goods belonging in this aggregate group. An aggregate price is constructed for each aggregate good by calculating its weighted average price, using as weights the share of value of each 2-digit SIC good relative to the total value of each aggregate good. Finally, the aggregate quantity of output is calculated by dividing the value added of each aggregate good by its aggregate price. The same method applies for calculating aggregate price and quantity for capital, skilled and unskilled labour, since the data from Jorgenson’s database are in a 2-digit SIC\textsuperscript{87} analysis.

5. Estimation and Conclusion

The estimation of the model is incomplete and for this reason is not discussed.

\textsuperscript{6} These calculations are based on the assumption that the information obtained from the sample provided from CPS is representative for the whole US manufacturing.

\textsuperscript{7} Table 1 shows the SIC categories that are included in each aggregate good.
<table>
<thead>
<tr>
<th>Aggregate Good</th>
<th>SIC Code Category</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exportable</strong></td>
<td>Food &amp; Kindred Products (SIC 20)</td>
</tr>
<tr>
<td></td>
<td>Chemicals &amp; Allied Products (SIC 28)</td>
</tr>
<tr>
<td></td>
<td>Industrial &amp; Commer Machinery &amp; Computer Equip. (SIC 35)</td>
</tr>
<tr>
<td></td>
<td>Electronic &amp; Other Electric. Equip. (SIC 36)</td>
</tr>
<tr>
<td></td>
<td>Transportation Equipment (SIC 37)</td>
</tr>
<tr>
<td></td>
<td>Instruments, Photographic, Medical &amp; Optical Goods (SIC 38)</td>
</tr>
<tr>
<td></td>
<td>Textile Mill Products (SIC 22)</td>
</tr>
<tr>
<td></td>
<td>Apparel &amp; Other Finished Products (SIC 23)</td>
</tr>
<tr>
<td></td>
<td>Lumber &amp; Wood Products (SIC 24)</td>
</tr>
<tr>
<td><strong>Importable</strong></td>
<td>Paper &amp; Allied Products (SIC 26)</td>
</tr>
<tr>
<td></td>
<td>Petroleum Refining &amp; Related Industries (SIC 29)</td>
</tr>
<tr>
<td></td>
<td>Leather &amp; Leather Products (SIC 31)</td>
</tr>
<tr>
<td></td>
<td>Primary Metal Industries (SIC 33)</td>
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<tr>
<td></td>
<td>Misc. Manufacturing Industries (SIC 39)</td>
</tr>
<tr>
<td></td>
<td>Tobacco Products (SIC 21)</td>
</tr>
<tr>
<td></td>
<td>Furniture &amp; Fixtures (SIC 25)</td>
</tr>
<tr>
<td><strong>Non-tradable</strong></td>
<td>Printing, Publishing &amp; Allied Industries (SIC 27)</td>
</tr>
<tr>
<td></td>
<td>Rubber &amp; Misc. Plastic Products (SIC 30)</td>
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<tr>
<td></td>
<td>Stone, Clay, Glass &amp; Concrete Products (SIC 32)</td>
</tr>
<tr>
<td></td>
<td>Fabricated Metal Products, Except Machinery (SIC 34)</td>
</tr>
</tbody>
</table>

References


Centre for International Data at UC Davis


