The rise and fall of regional inequalities with technological differences and knowledge spillovers.

Abstract
We extend the standard economic geography model by introducing regional differences in technology levels and by assuming that initial technological gaps may be closed only when the learning capabilities of the lagging region are sufficiently developed. Interregional knowledge spillovers take place only when the initial technological gap is not too wide, and when trade costs, taken as a proxy for the obstacles to interaction between firms of different regions, are sufficiently low. Hence, low trade costs may produce either the agglomeration or the dispersion of the modern sector, while high trade costs lead to its agglomeration in the leading region.

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1 Introduction

The “New Economic Geography” literature describes how the interactions of centripetal and centrifugal forces determine the locational decisions of firms and workers between two or more regions involved in trade. The interactions of these forces endogenously determine the size and the productivity of the regional economies. The market outcome is typically affected by the degree of integration among regions.

Increasing returns play a major role in these models that assume decreasing costs of production within each firm. Moreover, pecuniary externalities arise because of the assumptions of increasing returns at the firm level and trade costs in the manufacturing (or modern) sector. Pecuniary externalities induce mobile agents, workers or firms, to move towards regions where the size of the manufacturing sector is bigger. In this way, either consumers (if they demand goods produced in the modern sector) or firms (if they use these goods as production factors), may reduce the share of goods on which trade costs should be paid, if they did not move, and agents had to import them from other regions. However, each manufacturing firm (or consumer) that moves where pecuniary economies are larger, increases the incentive for its customer firms and workers to move in the same direction. These movements, in turn, increase the size of the region of destination and, therefore, the incentive for other firms and consumers to move towards the same region. Hence, concepts such as “backward and forward linkages” (Hirshman, 1958) or “cumulative causation” (Myrdal, 1957) turn out to be fundamental in this body of literature.

Centripetal forces, which favor cumulative causation and, therefore, a spatial concentration of the sector with increasing returns, are generated by three main factors: (1) workers’ mobility when the final sector exhibits increasing returns (Krugman, 1991b); (2) backward and forward linkages between firms producing intermediate and final goods, when intermediate goods are produced under increasing returns (Venables, 1996);1 (3) technological advantage of production in a

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1 Fujita and Thisse (2000a) point out that the assumption of the existence of an imperfectly competitive intermediate sector is sufficient to lead to a core-periphery structure.
particular region (Baldwin and Forslid, 2000). On the contrary, centrifugal forces are generated by: (1) immobile demand sources (such as that generated by immobile workers); (2) stronger competition for limited productive factors, and in good markets for firms that operate in core regions; (3) technological knowledge spillovers from regions with a more productive modern sector towards less developed regions. Whenever centripetal forces are stronger than centrifugal forces, the modern sector tends to be completely agglomerated in one region, while a uniform distribution of the economic activity emerges when centrifugal forces are stronger.

The interest in location and economic growth issues has led many economists to investigate how the main conclusions of the New Economic Geography literature can be affected when pecuniary externalities interact with the dynamic and external economies of scale introduced in the New Growth Theory by Romer (1990) and Grossman and Helpman (1991). Waltz (1996), Martin and Ottaviano (1999), Baldwin, Martin and Ottaviano (2001) and Fujita and Thisse (2002b), introduce dynamic economies of scale in New Economic Geography models by means of R&D activities, while Baldwin and Forslid (2000) introduce them by means of capital formation processes.\(^2\) The general result of this kind of model is that the dynamic economies of scale tend to strengthen centripetal forces.\(^3\) Nevertheless, as Fujita and Thisse (2002b) point out “even those that stay put in the periphery are better off than under dispersion provided that the growth effect triggered by the agglomeration is strong enough”.

Puga and Venables (1999, p. 292) observe that the “economic development may not be a gradual process of convergence by all countries, but instead involve countries moving sequentially from the group of poor countries to the group of rich countries”. They show that an exogenous

\(^2\) Moreover, Martin and Ottaviano (1996) analyze the effects of pecuniary externalities arising among firms producing manufacturing goods under increasing returns that are costly traded, and innovative firms that use manufacturing goods to produce new patents. New patents are then acquired by manufacturing firms with a fixed cost. Hence, Martin and Ottaviano (1996) show that growth is more sustained, and agglomeration is stronger when the market size is wider, the share of the differentiated good demanded by consumers is higher and when labor demand, the elasticity of substitution between any pair of varieties, trade costs, innovation costs and the subjective rate of discount are smaller.

\(^3\) A more complete summary of the attempts to link growth and geography models can be found in Baldwin, Forslid, Martin, Ottaviano and Robert-Nicoud (2002, ch. 7), which present also two models that summarize the global spillovers and local spillovers models introduced by Martin and Ottaviano (1999) and Baldwin, Martin and Ottaviano (2001).
productivity increase of all primary factors strengthens centripetal forces in developed countries by increasing immobile workers’ wages. In turn, this may lead some firms to start their production in a less developed country, where wages are lower. Besides, Puga and Venables assume that firms that start their production in newly industrialized countries may adopt the same technology as that used by firms in the leading countries. In other words, they do not focus on technological differences. By contrast, in this paper we want to stress that when there are technological differences, the lagging regions may not always be able to catch up with the leading ones, even though there are “potential” technological knowledge spillovers. In fact, we want to underline that some conditions must be satisfied before there can be a process of catching up, and therefore potential technological knowledge spillovers do not take place automatically towards firms in a lagging region. In this respect, we concur with Verspagen (1991, p. 361) when he claims:

“The basic (implicit) intuition behind the convergence hypothesis seems to be that international knowledge spill-overs take place automatically. In the (economic) literature dealing with the nature of technological change in more detail (e.g. Dosi, 1988) it is argued that this assumption is indeed a heroic one. Since the process of (international) technology spill-over is essentially a process of adoption of new techniques at the microeconomic (firm) level, the capabilities of the “receiving” country (firm) to “assimilate” (foreign) technological knowledge are critical to the success of diffusion. If countries (firms) do not have the relevant capabilities to assimilate new knowledge, spill-overs may not take place at all.”

The purpose of this paper is twofold. First, we want to extend the model of Puga (1999) in order to show how all the above-mentioned forces interact when regional levels of technological development of the modern sector may differ and change over time.4 Therefore, in our model we allow explicitly for differences in the regional levels of the technology.

Second, we want to account for the fact that the lagging regions are not always able to catch

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4 In particular, we will consider the case with labor migration across regions.
up completely with the leading regions. Here, a complete catch up can be achieved when (i) the technological gap between the two regions is not too wide and (ii) firms in the lagging region have enough opportunities to learn by interacting (e.g., watching) the technologies used by firms in the leading region. In particular, this may occur only if the lagging regions’ learning capabilities to assimilate potential technological spillovers are sufficiently large. The chances of benefiting from these spillovers depends on the opportunity to interact with firms operating in the leading regions. Since this opportunity is higher when regions are more integrated, our work stresses the relevance of trade costs levels - as a proxy for the difficulty of interacting - in allowing a successful process of catching up. More precisely, we represent trade costs by iceberg costs that are particularly suitable to describe the cost of the “distance” between any two regions, as well as the cost of all other natural and artificial barriers to trade. Therefore, while knowledge spillovers may take place from a leading region towards a neighbouring lagging region, they fail to occur if the lagging region is very far or if trade costs are too high, because its firms have less opportunities to interact with firms in the more developed region.

The model is presented in section 2, while section 3 deals with the necessary conditions for a complete process of catching up to occur. In section 4, the equilibria for given levels of the technology in the two regions are analyzed. More precisely, agglomeration equilibria are studied in 4.1, while the symmetric equilibrium is analyzed in 4.2. Using new dynamics based on technological spillovers, we discuss the different equilibria in section 5, where it is shown that the level of trade costs is critical in determining whether a catching up process can be successfully completed or not. Section 6 gives the conclusions.

2 The model

We consider a two-region economy \( r = n, s \) in which workers are indexed by \( h \). The case \( r = n \) corresponds to the “north” and \( r = s \) to the “south”. Workers have identical preferences and consume a homogenous (traditional or agricultural) good, and several varieties of a (modern or
manufactured) good. Varieties of the modern good are produced under increasing returns and sold in markets characterized by monopolistic competition. The traditional good is produced under constant returns using workers and arable land.

Let $\tilde{H}$ be the total number of workers, which can be employed in the manufacturing and in the agricultural sector, and which are perfectly mobile between the two regions. Hence, if they are employed in the two regions, we know from Krugman (1991b) that their regional real wages must be equal. Moreover, each region is endowed of the same units of arable land $K_r = \tilde{K}$. We notice that $K_r$ may also represent the endowment of unskilled workers of region $r$, which are intersectorally and interregionally immobile given their low levels of skills.

The production of any variety requires the use of all varieties of the manufactured good as intermediate inputs. Trading the manufactured good between the two regions is costly, while the traditional good is traded without cost. Finally, the efficiency of the technology available for producing the manufactured good may differ between the two regions. Furthermore, knowledge spillovers are present across regions.

2.1 Preferences and demand

A consumer in region $r$ maximizes a Cobb-Douglas utility function:

$$U(Q_{mr}, Q_{ar}) = Q_{mr}^{\mu_c}Q_{ar}^{1-\mu_c}$$

where $Q_{mr}$ is the quantity of the composite manufactured good and $Q_{ar}$ is the quantity of the agricultural good he/she consumes. The parameter $\mu_c$ is the share of consumers’ expenditure on manufactures with $0 < \mu_c < 1$.

The budget constraint is given by:

$$p_{mr}Q_{mr} + p_{ar}Q_{ar} = y_{hr}$$

where $y_{hr}$ is the income of a worker in region $r$. Workers’ income is given by the sum of their wage ($w_{hr}$) and of a share in the profits earned by manufacturing firms located in their region. This is
because the total number of firms has not yet reached a long run equilibrium value that ensures zero profits to all firms in the market. As will be seen below, the number of firms in each region reaches its equilibrium value more slowly than all other variables. Landowners, who are tied to their land, have the same preferences as workers and their income is given by rents.

Let \( p_{ar} \) be the price of the agricultural good in region \( r \), while \( p_{mr} \) is the price index of manufactured good in region \( r \). The composite \( Q_{mr} \) consumption good is obtained by aggregating the different varieties of the manufacturing good by means of a constant elasticity of substitution sub-utility function:

\[
Q_{mr} = \left( \sum_{i=1}^{n_n+n_s} Q^{\frac{1}{\sigma}}_{mir} \right)^{\frac{1}{\sigma}}
\]

where \( \sigma > 1 \) is the elasticity of substitution between any pair of varieties, whereas \( n_n \) and \( n_s \) are, respectively, the mass of firms in the north and in the south.

Because all manufacturing firms in a particular region are homogenous, the price index of the composite good in region \( r \) is:

\[
p_{mr} = \left[ n_r p_r^{1-\sigma} + n_v (p_v \tau)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}
\]

where \( r, v = n, s \) and \( v \neq r \).

Maximizing (1) under the budget constraint (2), we obtain the demand schedules for the agricultural good and for the composite manufactured good. Total consumers’ expenditure on the manufactured good in region \( r \) (\( E_{mcr} \)) is given by:

\[
E_{mcr} = \mu_c (w_{hr} H_r + n_r \pi_{ir} + r_r K_r)
\]

where \( H_r \) is the number of mobile workers employed in region \( r \), while \( \pi_{ir} \) are profits realized by each firm in the same region and, finally \( r_r \) denotes the maximized profit per unit of land in region \( r \). However, in order to determine the total expenditure on the manufactured good in region \( r \) (\( E_{mr} \)), we must also consider firms expenditure (\( E_{mmr} \)):

\[
E_{mr} = E_{mcr} + E_{mmr}
\]
2.2 Manufacturing sector

Each firm produces under increasing returns one variety of a differentiated good because consumers have a preference for variety, and because there are no scope economies. A firm’s output can be either directed towards local demand, or exported according to an iceberg trade cost. Firms in each region are homogenous and their mass $n_r$ is endogenously determined. Producing the quantity $Q_{mir}$ of each variety $i$ requires both a fixed and a variable amount of a composite productive input $I_{mir}$, which is defined below. While the fixed amount $\alpha$ is the same in the two regions, the variable amount $\beta/a_r$ may differ. Specifically, the more productive region is characterized by a higher value of $a_r$.

The production function of the firm supplying variety $i$ is given by:

$$Q_{mir} = \frac{a_r(I_{mir} - \alpha)}{\beta}$$

(7)

where $\alpha, \beta, a_r > 0$. The magnitude $a_r$ reflects the level of development of the technology in region $r$. For notational simplicity, we normalize the value of $a_r$ in region $n$ to 1:

$$a_n = 1$$

(8)

In the same spirit as Krugman’s and Venables’ (1995), we define the input $I_{mir}$ as a Cobb-Douglas composite of two production factors: one primary factor (workers, that is $H_{mir}$) and one intermediate factor (the manufactured composite good $D_{mir}$):

$$I_{mir} = \frac{H_{mir}^{1-\mu} D_{mir}^\mu}{(1-\mu)^{1-\mu} \mu^\mu}$$

(9)

where $0 < \mu < 1$.

For simplicity, we assume that the elasticity of substitution with which varieties enter consumers’ utility function is the same as that with which they enter firms’ production function. Therefore, the composite good $D_{mir}$ demanded by each firm $i$ has the same shares of the composite good $Q_{mir}$ demanded by consumers.
Finally, the minimization of the total cost of production under the constraint given by (7) and (9) yields the cost function of the firm supplying variety $i$:

$$\text{TC}_{mir} = \mu n_r w_{1-\mu}^\beta \left( \frac{\beta}{a_r} Q_{mir} + \alpha \right)$$ (10)

Since $\mu$ and $1-\mu$ respectively represent the shares of firms’ total production costs devoted to the composite good and to workers, firms’ total expenditure on manufactures in region $r$ ($E_{mmr}$) is given by:

$$E_{mmr} = \mu n_r \text{TC}_{mir}$$ (11)

### 2.3 Equilibrium

The traditional good is produced under constant returns, employing workers $H_{ar}$ and arable land, $K_r$:

$$Q_{ar} = g(H_{ar}, K_r)$$ (12)

Following Puga (1999), we introduce the restricted profit function, which gives maximized profits in each region as a function of the price of the traditional good, the wage and the land endowment in that region:

$$R(p_{ar}, w_{hr}, K_r) = \max_{Q_{ar}, H_{ar}} \{ p_{ar} Q_{ar} - w_{hr} H_{ar} | Q_{ar} \leq g(H_{ar}, K_r) \}$$ (13)

When the traditional good is produced in both regions, it is sold at the same price in the two regions. Moreover, we choose the traditional good as the numéraire: $p_{ar} = 1$. Maximized profits per unit of land are expressed by the rate of return function $r_r(w_{hr})$:

$$r_r(w_{hr}) = \frac{Q_{ar}}{K_r} - \frac{w_{hr} H_{ar}}{K_r}$$ (14)

Moreover, given the assumption of constant returns, we can write that:

$$R(1, w_{hr}, K_r) = r_r(w_{hr}) K_r$$ (15)

Differentiating the rate of return function with respect to $w_{hr}$ gives:

$$\frac{\partial r_r}{\partial w_{hr}} = -\frac{H_{ar}}{K_r}$$ (16)
particularly, in the following sections we assume a Cobb-Douglas production function for the
traditional sector when we examine agglomeration equilibria with:

\[ Q_{ar} = H_{ar}^\theta K_r^{1-\theta} \]  

(17)

where \(0 < \theta < 1\). In this case, the rate of return function is given by:

\[ r_r = \left( \frac{w_{hr}}{\theta} \right)^{\frac{\theta}{\theta-1}} (1 - \theta) \]  

(18)

Equation (16) together with the derivative of (18) gives the value of the wage in region \(r\)

\[ w_{hr} = \theta \left( \frac{H_{ar}}{K_r} \right)^{\theta-1} \]  

(19)

while profits per unit of land in region \(r\) are

\[ r_r = (1 - \theta) \left( \frac{H_{ar}}{K_r} \right)^\theta \]  

(20)

Consumers' and firms' demand functions in region \(r\) for varieties produced in the same region
or imported from region \(v\), are obtained by the minimization of the expenditure on manufacturing
goods under the constraint given by the aggregation of all varieties in the composite good. Let us
consider, for the moment, prices and quantities regardless of the location of production. So, we
must solve the following problem:

\[
\min_{Q_{mi}} \sum_{i=1}^{n} p_i Q_{mi} \text{di} \quad \text{s.t.} \quad Q_m = \left( \int Q_{mi}^\rho \text{di} \right)^{\frac{1}{\rho}}
\]  

(21)

where \(0 < \rho = \frac{\sigma-1}{\sigma} < 1\), \(p_i\) is the price of variety \(i\), and \(\rho\) represents the intensity of the preference
for variety in manufactured goods.

The solution of the constrained minimization problem yields the following demand function:

\[ Q_{mi} = \frac{p_i^{-\sigma}}{p_m^{-\sigma}} E_m \]  

(22)

where the total expenditure on manufacturing goods \((E_m)\) is:

\[ E_m = p_m Q_m \]  

(23)
However, we should notice that the existence of trade costs implies that the price of an imported variety is higher than the one paid in the region in which it is produced. More precisely, in order to import one unit of a variety, \( \tau > 1 \) units have to be shipped. Therefore, the demand function for the locally produced variety \( i \) in region \( r \) (\( Q_{mir}^{rd} \)) is given by:

\[
Q_{mir}^{rd} = \frac{p_{ir}^{-\sigma}}{p_{mr}^{-\sigma}}E_{mr}
\]

Similarly, the demand function for the imported variety \( i \) produced in region \( v \) (\( Q_{mir}^{rd} \)) is given by:

\[
Q_{mir}^{rd} = \frac{(\tau p_{iv})^{-\sigma}}{p_{mv}^{-\sigma}}E_{mr}
\]

where \( r, v = n, s \) and \( v \neq r \). Hence, the aggregate demand (\( Q_{mir}^{d} \)) for a firm producing variety \( i \) in region \( r \) is:

\[
Q_{mir}^{d} = Q_{mir}^{rd} + Q_{mir}^{vd} = p_{ir}^{-\sigma}\left( \frac{1}{p_{mr}^{-\sigma}}E_{mr} + \frac{\tau^{-\sigma}}{p_{mv}^{-\sigma}}E_{mv} \right)
\]

Each firm producing in region \( r \) maximizes profits (\( \pi_{ir} \)) under the aggregate demand function (26). It is readily verified that each firm sets a price with a constant mark-up over the marginal cost \( MC_{mir} \):

\[
p_{ir} = \frac{\sigma}{\sigma - 1}MC_{mir} = \left( \frac{\sigma \beta}{(\sigma - 1) a_r} \right) p_{mir}^{\mu}w_{hr}^{1-\mu}
\]

Thereafter we will drop the suffix \( i \) from the price of each variety produced in region \( r \) because all firms in a particular region are assumed to be equal.

Profits of a firm producing in region \( r \) are given by:

\[
\pi_{ir} = p_{mir}^{\mu}w_{hr}^{1-\mu}\left( \frac{\beta}{(\sigma - 1) a_r} Q_{mir} - \alpha \right)
\]

Following Puga (1999), without loss of generality, the values of \( \alpha \) and \( \beta \) can be chosen as follows:

\[
\alpha = \frac{1}{\sigma} \quad \text{and} \quad \beta = \frac{\sigma - 1}{\sigma}
\]

so that profits may be rewritten as follows:

\[
\pi_{ir} = p_{mir}^{\mu}w_{hr}^{1-\mu}\left( \frac{1}{a_r} Q_{mir} - 1 \right)
\]
The manufacturing sector is characterized by monopolistic competition. When there is free entry and exit, profits are equal to zero at the long run equilibrium. Therefore, the long run equilibrium price of each variety is equal to the average cost of production $AC_{mir}$:

$$p_r = \frac{TC_{mir}}{Q_{mir}} = AC_{mir} \quad (31)$$

Equation (27) together with equation (31) allows us to compute the long run equilibrium size of each firm in region $r$:

$$Q_{mir}^* = \frac{\alpha a_r (\sigma - 1)}{\beta} = a_r$$

Given the normalization above, profits are equal to zero for northern and southern firms when their production levels are respectively:

$$Q_{min}^* = 1 \quad \text{and} \quad Q_{mis}^* = a_s \quad (32)$$

The free entry and exit condition implies that the following expression must be satisfied at a long run equilibrium:

$$n_r \pi_{ir} = 0 \quad \pi_{ir} < 0 \quad n_r \geq 0 \quad (33)$$

When skilled workers are employed in both regions, we know from Krugman (1991b) that regional real wages of skilled workers are equal at the long run equilibrium:

$$\frac{w_{hn}}{p_{min}} = \frac{w_{hs}}{p_{mis}} \quad (34)$$

Moreover, the full employment of workers requires that:

$$\bar{H} = H_n + H_s \quad (35)$$

The labor market clearing condition for each region $r$ requires that total employment $H_r$ is given by the sum of workers employed in the manufacturing sector ($H_{mr}$) and in the agricultural sector ($H_{ar}$), that is $H_r = H_{mr} + H_{ar}$. By the application of Shephard’s lemma to (10) we obtain labor demand in manufacturing in region $r$:

$$H_{mr} = \frac{(1 - \mu)TC_{mir} n_r}{w_{hr}} \quad (36)$$
Labor demand in agriculture is obtained from (16). Therefore, the labor market clearing condition for region $r$ is:

$$H_r = \frac{(1 - \mu)TC_{mix} n_r}{w_{hr}} - \partial r_w \partial w_{hr} K_r$$

(37)

3 Technological evolution

In this paper we want to investigate what the interregional distribution of the economic activity becomes if interregional knowledge spillovers take place only when the initial technological gap is not too wide, and when trade costs, taken as a proxy for the obstacles to interaction between firms of different regions, are sufficiently low. Therefore, the critical force lies in the ability of firms located in the receiving regions to use the flow of additional knowledge.

In fact, Verspagen (1991, p. 362-363) points out that the learning abilities of a lagging region or country “depend both on an intrinsic capability, and on its technological distance from the leading country”. Furthermore, he maintains that the intrinsic learning capability “is determined by a mixture of social factors (Abramovitz, 1986), education of the workforce (Baumol et al., 1989), the level of the infrastructure, the level of capitalization (mechanization) of the economy, the correspondence of the sectorial mix of production in the leading and following country (Pasinetti, 1981), and other factors.”

However, we argue that learning through knowledge spillovers processes is also enhanced when firms in the less developed regions have more opportunities to observe and learn how all the different phases of production are conducted by firms active in the more developed regions. We believe that such an observation is more likely to occur when the level of integration is higher because natural and artificial barriers to trade are lower. Thus, the productivity of firms producing in a less developed region may be increased through a process of learning by interacting with firms that produce in the more developed region. Since knowledge spillovers do not take place automatically, we find it reasonable to assume that their chances to occur increase when trade costs are “small”, while knowledge spillovers fail to take place when trade costs are “high”. Therefore,
low trade costs act as a stabilizing force because they favor knowledge spillovers.\footnote{Also Baldwin and Forslid (2000) consider knowledge spill-overs as a stabilizing force, but they assume that their size can be determined by policy makers. In particular, they assume that knowledge spill-overs increase when integration takes place through a lowering of the cost of trading information, and that knowledge spill-overs are independent from trade costs viewed as the cost of trading goods. Hence, while in our model, high trade costs entail null knowledge spill-overs, and in this case act as a destabilizing force, Baldwin and Forslid show that high knowledge spill-overs may stabilize the symmetric equilibrium even if trade costs are high.}

In fact, recent empirical works, such as that by Coe and Helpman (1995) and Keller (2001a,b), illustrate the importance of trade as a mechanism of international knowledge spillovers.\footnote{The theoretical models on which are built these empirical works are the innovation-driven growth models by Grossman and Helpman (1991), Romer (1990) and Aghion and Howitt (1992). Moreover, Keller (2001a) refers also to the New Economic Geography contributions by Krugman (1991a,b) and Fujita, Krugman and Venables (1999).}

Particularly, Keller (2001a, p. 5) finds that, for manufacturing industries in the world’s seven major industrialized countries during the years between 1970 and 1995, “the scope for knowledge spillovers is severely limited by distance”. Furthermore, Keller finds (2001a, p. 1) that “trade patterns account for the majority of all differences in bilateral spillover flows, whereas foreign direct investments and communications flow differences account for circa 15% each”, and that “these three channels together account for almost the entire localization effect that would be otherwise attributed to geographic distance”.

In order to illustrate the fact that trade acts as a channel through which knowledge spillovers take place, we assume that learning capabilities $\psi$ depend upon trade costs. Specifically, we assume that when trade costs are above a certain threshold value $\bar{\tau}$, firms in the lagging region are unable to assimilate any of the potential knowledge spillovers from the leading region, so that the actual learning capabilities $\psi$ of this region are equal to zero. However, when trade costs are below $\bar{\tau}$, the region’s learning capabilities rise as trade costs fall. This leads us to assume that

$$
\psi(\tau) = \begin{cases} 
c(\bar{\tau} - \tau) & \text{if } \tau \leq \bar{\tau} \\
0 & \text{if } \tau > \bar{\tau}
\end{cases}
$$

(38)

where $c > 0$ is a parameter that represents the influence on learning abilities of all other above-mentioned factors. For simplicity, we assume that there are no interregional differences in these other factors so that $c$ is the same across regions.
In order to describe how the learning ability affects the production activity in the lagging regions, we assume that the technological level depends on the learning capabilities and on the technological gap between the two regions through the following dynamic equation:

\[ \dot{a}_s = \left[ (a_s - 1)^3 + \psi (1 - a_s) \right] \quad (39) \]

where \( a_s \geq 0 \).

This specification describes the fact that the technological advantage of a region tends to increase over time - following cumulative processes, that we consider exogenous in this paper - unless the technological gap between the two regions can be closed thanks to interregional learning capabilities, represented by positive values of \( \psi \).\(^7\) Furthermore, equation (39) takes into account the fact that when learning capabilities are small, even though they are positive, firms in the lagging region may recover their technological lag only when it is not too wide. In fact, when the technological gap is very wide, the amount of knowledge spillovers required by firms in the lagging region to catch up is very wide. And if this is not the case, because trade costs are high, the lagging region will definitively fall behind.

For the given normalization (8), the north (south) is the technological leader, while the south (north) is the lagging region, when \( a_s < 1 \) (\( a_s > 1 \)).

Three equilibrium values for \( a_s \), when \( \psi > 0 \), are:

\[ a_s = 1 \quad a_s = 1 - \sqrt[3]{\psi} \quad a_s = 1 + \sqrt[3]{\psi} \quad (40) \]

Thus, one of the possible equilibria for equation (39) is given by the symmetric equilibrium, which is characterized by identical regional levels of technology \( (a_s = a_n = 1) \).\(^8\) In Figure 1.a, we plot equation (39) when \( \psi = 0.5 \).

\(^{7}\) See Dosi (1988), for instance, that maintains that the technological advantage of a region tends to increase along a technological trajectory because leading regions tend to grow faster.

\(^{8}\) Equilibrium values can also be considered as steady state equilibrium values with positive and equal, exogenous growth rate of the technological level. In fact, function (39) expresses relative technological development since the normalization \( a_n = 1 \) has been adopted.
The intercept of the function (39) with the vertical axis is given by \( \psi - 1 \). Therefore, we may observe that \( a_s = 0 \) is a stable equilibrium with no firm producing the modern good in the south if learning capabilities of this region are very low (\( \psi < 1 \)). In this case, for a given value of \( c \), trade costs are too high (\( \tau > \bar{\tau} - 1/c \)) to allow firms in the south to assimilate technology spillovers from the north, which is the technological leader since \( a_s < 1 = a_n \), when its technological lag is sufficiently wide, that is when \( a_s < 1 - \sqrt{\psi} \). When this is so, the technological advantage of the north continuously increases over time.

The “symmetric” equilibrium characterized by \( a_s = a_n = 1 \) is stable if the slope of (39) is negative in a neighborhood of this point, that is, if \( \psi > 0 \). In this case, trade costs are low enough (\( \tau < \bar{\tau} \)) to allow firms in the receiving regions to assimilate technology spillovers.

Insert Figures 1.a-b about here

When we consider only the dynamic equation (39), the symmetric equilibrium is stable only if learning abilities in both regions are positive, that is, if \( \psi > 0 \). Moreover, when learning capabilities are high enough, namely when \( \psi > 1 \), the symmetric equilibrium is stable for any initial value of \( a_s < 1 \). Figure 1.b shows this case when \( \psi = 1.5 \).

However, when learning abilities are positive but not too high because trade costs are not low enough, the lagging region may benefit from interregional knowledge spillovers provided that its technological lag is not too wide. In fact, when the lagging region is the south (\( a_s < 1 \)), firms in this region may benefit from knowledge spillovers only when the level of the technology of this region is not too low, namely \( a_s > 1 - \sqrt{\psi} \). In other words, firms in the south may recover their lag only if the technological gap \((1-a_s)\) from the leading region is smaller than \( \sqrt{\psi} \). On the contrary, when the technological leading region is the south (\( a_s > 1 \)), firms in the northern region may recover their lag, thanks to knowledge spillovers from the south, only when the technological lag is not too wide for the given learning abilities, that is, if \( a_s - 1 < \sqrt{\psi} \). This two conditions taken together entail that the symmetric equilibrium \( a_s = a_n = 1 \) is a stable equilibrium for expression
(39), when $\psi > 0$, only if for a given initial value of the technology level, $a_s^0$, we have that:

$$1 - \sqrt{\psi} < a_s^0 < \sqrt{\psi} + 1$$

(41)

The width of the recoverable lag increases (decreases) when learning capabilities increase (decrease), namely when the economic integration between the two regions becomes higher (lower).

In short, when the south is the lagging region ($a_s < 1$), the following three cases may occur for respectively high, low or intermediate trade costs.

**Case 1** $\tau > \bar{\tau}$. When trade costs are too high, the symmetric equilibrium can never be reached because firms in the lagging region cannot benefit from technology spillovers from the leading region, given the low level of integration.

**Case 2** $\tau < \bar{\tau} - 1/c$. When trade costs are low, firms in the lagging region can successfully exploit potential technology spillovers from the leading region and the symmetric equilibrium is stable.

**Case 3** $\bar{\tau} - 1/c < \tau < \bar{\tau}$. For intermediate trade costs the process of catching up of the lagging region with the leading region may be completed because trade is sufficiently developed to allow firms in the lagging region to interact with the most productive firms in the leading region. However, this happens only when the technological gap between the two regions is not too wide for the given learning abilities. In other words, when: $1 - a_s^0 < \sqrt{\psi}$.

4 The full agglomeration and the symmetric equilibria

In this section, we analyze the full agglomeration and the symmetric equilibria when the regional levels of technology ($a_r$) are given. In other words, equation (39) is not considered here, but will be introduced in the following section.

4.1 Agglomeration

For given regional levels of the technology ($a_r$), agglomeration of the modern sector in region $v$ is a *sustainable equilibrium* when no firm finds it profitable to relocate or start its production in region $r$ (where $v, r = s, n$ and $v \neq r$). In other words, full agglomeration of the modern sector in region $v$ is a sustainable equilibrium if, and only if, with all firms located in region $v$, the sales

$^{9}$ There could be also a third type of equilibrium when $0 < \psi < 1$. In fact, there are two asymmetric equilibria characterized by technological level equal to $a_n = 1$ in the north and respectively $a_s = 1 - \sqrt{\psi}$ or $a_s = 1 + \sqrt{\psi}$ in the south. However, because these would never be stable equilibria for equation (39), we do not consider this type of equilibrium.
of a (potential) firm relocating to region $r$ ($Q_{mir}$) are less than the level required to break even ($Q_{mir}^*$):

$$Q_{mir} < Q_{mir}^*$$  \hfill (42)

Following Puga and Venables (1996) and Puga (1999), we compute in appendix A the conditions for the full agglomeration in region $v$ to be a sustainable equilibrium when the regional levels of technology $a_v$ and $a_r$ are given.

Agglomeration in region $v$ is a sustainable configuration when:

$$\frac{Q_{mir}}{Q_{mir}^*} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \tau^{\sigma(1-\mu)(1-\mu_c)-1} \left[\frac{(1 - \tau^{-2(\sigma-1)}) (1 - \mu_c) (1 - \mu)}{\tau^{\sigma+1}} + \tau^{2(1-\sigma)}\right] < 1$$  \hfill (43)

Expression (43) cannot be “easily turned into a closed-form solution for the range of trade costs for which agglomeration is sustainable”\textsuperscript{10}. However, following Puga (1999), we notice that the value of $Q_{mir}/Q_{mir}^*$ approaches $(\frac{a_v}{a_r})^{1-\sigma}$ when $\tau$ tends to 1, and that its derivative is negative for $\tau$ close to 1. Moreover, when $\tau$ becomes infinitely large so does $Q_{mir}/Q_{mir}^*$, provided that $(1 - \mu)(1 - \mu_c)\sigma - 1 - \mu_c \frac{\theta}{1-\theta} > 0$.

Let us define $\tau^*$ as the value of $\tau$, below which agglomeration becomes sustainable because $Q_{mir}/Q_{mir}^* < 1$.

The graphic in Figure 2 plots $Q_{mir}/Q_{mir}^*$ as a function of trade costs\textsuperscript{11}. When trade costs are higher than $\tau^*$, the full agglomeration of the manufacturing sector in region $v$ is not a sustainable configuration because a firm may start its production in region $r$ without suffering losses, given that $Q_{mir}/Q_{mir}^* \geq 1$. On the contrary, full agglomeration in region $v$ is a sustainable equilibrium if trade costs are smaller than $\tau^*$ because $Q_{mir}/Q_{mir}^* < 1$.


\textsuperscript{11} The graphic is obtained for the following parameter values: $\theta = 0.55$, $\mu = 0.2$, $\mu_c = 0.15$, $\sigma = 4$, $a_v = a_r = 1$.  

---

When the level of integration between the two regions is high, that is, when $\tau$ tends to 1, we
observe that
\[
\lim_{\tau \to 1} \frac{Q_{\text{mir}}}{Q^*_{\text{mir}}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \tag{44}
\]
Therefore, when the level of the technology of the core region \(v\) is higher than that of the periphery \(r\), that is when \(a_v > a_r\), and the two regions are highly integrated (\(\tau \to 1\)), full agglomeration of the modern sector in region \(v\) is more likely to occur because (44) is:
\[
\lim_{\tau \to 1} \frac{Q_{\text{mir}}}{Q^*_{\text{mir}}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} < 1 \quad \forall \sigma > 1 \tag{45}
\]
However, expression (43) implies that not only high trade costs, but also sufficiently low trade costs may lead some firms to locate their production in the periphery \(r\) provided that, for some reason, its level of technology is higher than in the core region \(v\). In this case, \(Q_{\text{mir}}/Q^*_{\text{mir}}\) may become higher than (or equal to) 1 for low trade costs. Therefore, the agglomeration of the modern sector in region \(v\) may be unsustainable not only for high, but also for low trade costs.

Let \(\tau^{**}\) be the value of trade costs at which agglomeration becomes unsustainable for \(\tau \leq \tau^{**}\) (with \(\tau^* < \tau^{**}\)). A necessary condition for \(\tau^{**}\) to exist is that \(Q_{\text{mir}}/Q^*_{\text{mir}} > 1\) when \(\tau = 1\), that is:
\[
\frac{Q_{\text{mir}}}{Q^*_{\text{mir}}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} > 1 \tag{46}
\]
Expression (46) is true when technological levels are such that
\[
a_r > a_v \tag{47}
\]
Figure 3 plots \(Q_{\text{mir}}/Q^*_{\text{mir}}\) when \(a_r > a_v\) and \(\tau^{**}\) exists.\(^{12}\)

Unused Figure 3 about here

When \(\tau^{**}\) exists, agglomeration of the manufacturing sector in region \(v\) is unsustainable for \(\tau \leq \tau^{**}\). While in Puga (1999) labor mobility implies a monotonic relationship between the

\(^{12}\) The graphic is obtained for the following parameter values: \(\theta = 0.55, \mu = 0.2, \mu_c = 0.15, \sigma = 4, a_v = 0.95, a_r = 1\).
sustainability of agglomeration and the levels of trade costs, in our work, the introduction of differences in the technology levels, allows us to show that the existence of technological differences may give rise to a non-monotonic relationship.\(^\text{13}\) In fact, we may come across the \(\cap\)-shaped relationship found by Venables (1996) when \(a_v > a_r\). When this is so, the higher level of technological development of region \(r\) leads to the dispersion of the economic activity for high level of integration, because firms find it profitable to produce in region \(r\).

Totally differentiating (43) we find that:

\[
\frac{\partial \tau^*}{\partial \mu} > 0 \quad \frac{\partial \tau^*}{\partial \mu_c} > 0 \quad \frac{\partial \tau^*}{\partial \theta} > 0 \quad \text{and} \quad \frac{\partial \tau^*}{\partial \sigma} < 0 \quad (48)
\]

The last result in (48) is obtained provided that the technological gap is not too wide, that is \(a_v\) is not too different from 1.\(^\text{14}\) Moreover, we may rewrite the sustainability conditions for agglomeration in region \(v\) (43) as:

\[
\frac{Q_{mi}}{Q^*_mi} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} k \quad (49)
\]

where \(k\) depends on the parameters. Since \(\sigma > 1\), we observe that when \(a_v/a_r\) increases, the function is shifted downward, and therefore \(\frac{\partial \tau^*}{\partial (a_v/a_r)} > 0\). A higher level of development of region \(v\) with respect to region \(r\) favors agglomeration in region \(v\), making agglomeration sustainable below a higher level of trade cost \(\tau\) because it increases the competitiveness of producing in region \(v\).

4.2 Stable Symmetric Equilibrium

The free entry and exit condition implies that the number of manufacturing firms in region \(r\) \((n_r)\) increases (decreases) when profits in the region are positive (negative).\(^\text{15}\) Therefore, the evolution of the mass of firms in region \(r\) is given by:

\[
\dot{n}_r = \delta \pi_{i'r} \quad (50)
\]

\(^{13}\) This is possible only if \(\tau^{**}\) exists.

\(^{14}\) We note that Puga (1999) finds the same results in (48) when \(a_v = a_r\).

\(^{15}\) See Puga (1999).
where $\delta$ is a positive constant.

Let us rewrite equation (50) as follows:

$$\dot{n} = \delta \pi_i$$  \hspace{1cm} (51)

where $\pi_i = \begin{bmatrix} \pi_{in} \\ \pi_{is} \end{bmatrix}$ and $n = \begin{bmatrix} n_n \\ n_s \end{bmatrix}$.

For given values of regional technological levels $(a_r)$, the economy can be considered at a short run or at a long run equilibrium. When the variables are at their short run equilibrium values, the number of firms in each region is not necessarily at its long run equilibrium value, because profits of firms may be positive or negative. Therefore, we may divide the variables of the models between what we call “slow” and “fast” variables. That is, the number of firms in a region ($n_n$ and $n_s$) is a slow variable, while the other variables of the model are referred to as fast variables.$^{16}$

This distinction underlines that, to carry out the stability analysis, we suppose that fast variables have already reached their short run equilibrium values (which depend on slow variables values) and move along them, while slow variables move towards their long run equilibrium values.$^{17}$

This distinction allows us to rewrite expression (51) in the neighborhood of a long run equilibrium as follows:

$$\dot{n} = \delta u(n) \equiv z(n)$$ \hspace{1cm} (52)

In fact, in appendix B we show that this is possible if profits in a neighborhood of a long run equilibrium can be expressed as a function of the number of firms $n$:

$$\pi_i = u(n)$$ \hspace{1cm} (53)

Differentiating $\dot{n} = z(n)$ and taking Taylor’s expansion of the first order evaluated at the equilibrium values for $n$ (denoted by $*$) yields:

$$\partial \hat{n} = \dot{n} = z(n^*) + \frac{\partial z}{\partial n}(n^*) (n - n^*)$$ \hspace{1cm} (54)

$^{16}$ With the exception of $a_n = 1$ and $a_s$ which, in this section, are considered given.

$^{17}$ See Boggio (1986, 1999). It should be noted that while we assume that fast variables are asymptotically stable. A more rigorous approach should prove it rather than assume it.
Given that \( z (n^*) = 0 \), this expression becomes:

\[
\dot{n} = \frac{\partial z}{\partial n} (n^*) (n - n^*)
\]  

(55)

where the matrix \( \frac{\partial z}{\partial n} (n^*) \) is the Jacobian matrix for equation (51) for given values of \( a_n \) and \( a_s \).

Let the Jacobian matrix evaluated at the equilibrium be:

\[
J_1^* = \frac{\partial z}{\partial n} (n^*) = \delta \frac{\partial u}{\partial n} (n^*) = \delta M
\]

(56)

where \( M = \frac{\partial u}{\partial n} (n^*) \).

It must be noted that the symmetric equilibrium is possible only if the levels of technology are the same in the two regions \( (a_n = a_s = 1) \).

Matrix \( J_1^* \) is symmetric when evaluated at the symmetric equilibrium and its eigenvalues are equal to the two eigenvalues \( \lambda_1 \) and \( \lambda_2 \) of matrix \( M \) multiplied by \( \delta \).\(^{18}\) Moreover, we observe that the eigenvalues of matrices \( J_1^* \) and \( M \) at the symmetric equilibrium are real numbers because the two matrices are symmetric.

Finally, we note that matrix \( M \) evaluated at the symmetric equilibrium with \( a_r = a_v = 1 \) can be computed as matrix \( \partial \pi / \partial n \) derived by Puga (1999, appendix A.1).\(^{19}\) Therefore, given that the two eigenvalues of matrix \( M \) depends on the values of trade costs and af parameters, we observe that different level of trade costs and parameters may affect the stability of the symmetric equilibrium. More precisely, the symmetric equilibrium is stable for values of trade costs and of parameters for which the two eigenvalues \( \lambda_1 \) and \( \lambda_2 \) are negative \( (\lambda_1 < 0 \) and \( \lambda_2 < 0) \), while it is unstable for values for which at least one of the two eigenvalues is positive.

Particularly, Puga (1999) finds that the symmetric equilibrium is a stable node for values of trade costs above the critical value \( \tau_1^* \) (with \( \lambda_1 < 0 \) and \( \lambda_2 < 0) \), while it is an unstable saddle

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\(^{18}\) Since \( \delta \) is a scalar different from zero and \( \delta \in C \), and \( J_1 = \delta M \), if the eigenvalues of \( M \) are \( \lambda_1 \) and \( \lambda_2 \), the eigenvalues of \( J_1 \) are \( \delta \lambda_1 \) and \( \delta \lambda_2 \). See Lütkepohl (1996).

\(^{19}\) Note that in Puga (1999) the share of consumers' expenditure on manufactures is denoted by the parameter \( \gamma \) instead of \( \mu_v \), and that we denote some variables with different letters (i.e.: \( p_{mr} \), \( \pi \), and \( H_r \) are respectively denoted by Puga \( q_r \), \( \pi \), and \( L_r \)). Moreover, we consider only two regions instead of a general number of regions (M in Puga).
point for values of trade costs below $\tau^*_s$ \(\text{with } \lambda_1 < 0 \text{ and } \lambda_2 > 0, \text{ or } \lambda_1 > 0 \text{ and } \lambda_2 < 0\).\(^{20}\) Puga (1999, p. 315) shows that the critical value $\tau^*_s$ “is higher (so agglomeration takes place earlier during a gradual process of regional integration) the lower is $\sigma$, and the higher are $\mu_c$, $\mu$ and $\eta$”, where $\eta$ denotes the elasticity of a region’s labor supply to the manufacturing sector with respect to local agricultural wages valued at the symmetric equilibrium.\(^{21}\)

5 Stability and technological evolution

In this section, the stability analysis of the two types of equilibrium considered above is extended to the case in which the regional levels of technology ($a_r$) may change over time according to equation (39).

5.1 Agglomeration in region $v$

When the relative regional levels of technology may change, agglomeration in region $v$ may be a sustainable equilibrium not only when (39) implies that the technological gap between the two regions gradually increases but also when it shrinks.

Let us first consider the case in which (39) entails a continuous increase in the technological gap ($a_v - a_r$) between the two regions. In this case, starting from a situation in which the level of the technology of firms in region $v$ is not smaller than that in region $r$ ($a_v \geq a_r$), agglomeration in region $v$ is a sustainable equilibrium in the following two subcases: (i) trade costs are so high ($\tau > \bar{\tau}$) that they do not allow firms in the lagging region $r$ to observe and assimilate potential knowledge spillovers from the more productive manufacturing firms in region $v$ and (ii) the lagging region’s capabilities to assimilate potential technology spillovers are too small because trade costs are too high for the given initial technological gap between the two regions ($a_v - a_r > \sqrt{\psi}$). In both cases, firms in the lagging region are unable to recover from their lagged position during the

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\(^{20}\) Provided that $\text{(1} - \mu_c)\text{[(1} - \mu)(\sigma - 1)] - \mu^2 \eta > 0$, where $\eta$ denotes the elasticity of a region’s labor supply to the manufacturing sector with respect to local agricultural wages valued at the symmetric equilibrium.

\(^{21}\) $\eta = \frac{whr}{H_{mr}} \frac{\partial H_{mr}}{\partial w_{hr}} = \frac{w_{hr}}{H_{mr}} \frac{\partial (H_{mr} - H_{ar})}{\partial w_{hr}}$

Moreover, given that $\frac{\partial a}{\partial w_{hr}} = -\frac{H_{mr}}{K_r}$, $\eta = \frac{w_{hr} K_r \frac{\partial^2 a}{\partial^2 w_{hr}} / \partial^2 w_{hr}}{H_r + K_r \frac{\partial a}{\partial w_{hr}}} = \frac{K_r \frac{\partial^2 a}{\partial^2 w_{hr}} / \partial^2 w_{hr}}{H_r + K_r \frac{\partial a}{\partial w_{hr}}}$
technological development process, and agglomeration of the manufacturing sector in the leading region is the only sustainable equilibrium. In fact, equation (39) implies that the technological advantage of region \( v \) continuously increases over time, and that \( a_v/a_r \) tends to infinity.

Let us consider the sustainability conditions for agglomeration in region \( v \) (43) rewritten as:

\[
\frac{Q_{mir}}{Q^{*}_{mir}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} k
\]

where \( k \) depends on the parameters. Since \( \sigma > 1 \), we observe that:

\[
\lim_{a_v \to \infty} \frac{Q_{mir}}{Q^{*}_{mir}} = \lim_{a_r \to 0} \frac{Q_{mir}}{Q^{*}_{mir}} = 0
\]

Hence, when the relative technological advantage of region \( v \) increases continually over time, as in the two above-mentioned cases, agglomeration in region \( v \) is the only sustainable equilibrium because \( Q_{mir}/Q^{*}_{mir} = 0 < 1 \) for all admissible values of \( \tau \). In fact, in these two cases the agglomeration forces are strengthened because firms in region \( v \) become more and more productive compared to those in region \( r \).

Figure 4 shows, for instance, what happens when the assimilation of interregional technology spillovers is impeded because (39) implies that the relative technological level \( a_v/a_r \) continues to increase over time (with \( \psi = 0 \)).22 Particularly, Figure 4 shows that if \( a_v/a_r \) increases from the initial value 1 to 1.2, and then to 1.4, the curve \( Q_{mir}/Q^{*}_{mir} \) is shifted downward strengthening centripetal forces in region \( v \).

Insert Figure 4 about here

Therefore, (39) and (58) imply that, when trade costs are higher than \( \bar{\tau} \), the symmetric equilibrium is unstable. An exogenous increase in the level of technology \( a_v \) would result in a continuous increase of the technological advantage of region \( v \), leading to the agglomeration of the modern sector in this region for any value of the trade costs.

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22 The graphics are obtained for the following parameter values: \( \theta = 0.55, \mu = 0.2, \mu_c = 0.15, \sigma = 5 \).
Finally, we must consider the case in which the technological gap between the two regions can be closed, because learning capabilities are high enough to allow technology spillovers to take place for the given initial technological gap \((\sqrt{\psi} > a_v - a_r)\). In this case, agglomeration in region \(v\) may be a sustainable equilibrium only if \(Q_{mir}/Q_{mir}^* < 1\) when the process of recovery of the technological lag is completed. Hence, in this case even though knowledge spillovers take place, pecuniary externalities are strong enough to foster the concentration of firms in region \(v\). On the contrary, if \(Q_{mir}/Q_{mir}^* \geq 1\) when the technological gap is closed, agglomeration of the modern sector is unsustainable, because centripetal forces are weaker than centrifugal forces.

5.2 Symmetric equilibrium

To study the stability of the symmetric equilibrium when technological change is possible, we must consider three differential equations. Two of them are given by expression (52), and the third is given by equation (39), which describes the change in the technology.

In this case, the Jacobian matrix is given by:

\[
J_2 = \begin{bmatrix}
\frac{\partial \dot{n}_n}{\partial n} & \frac{\partial \dot{n}_s}{\partial n} & \frac{\partial \dot{a}_s}{\partial n} \\
\frac{\partial \dot{n}_n}{\partial n} & \frac{\partial \dot{n}_s}{\partial n} & \frac{\partial \dot{a}_s}{\partial n} \\
\frac{\partial \dot{n}_n}{\partial n} & \frac{\partial \dot{n}_s}{\partial n} & \frac{\partial \dot{a}_s}{\partial n}
\end{bmatrix}
\]  \hspace{1cm} (59)

Since \(\frac{\partial \dot{a}_s}{\partial n} = \frac{\partial \dot{a}_s}{\partial n} = 0\), the Jacobian matrix (evaluated at the symmetric equilibrium denoted by \(\ast\)) is given by:

\[
J_2^\ast = \begin{bmatrix}
J_1^\ast & (\frac{\partial \dot{a}_s}{\partial n})^\ast \\
0 & \psi
\end{bmatrix}
\]  \hspace{1cm} (60)

Matrix \(J_2^\ast\) is a decomposable matrix and, therefore, two of its eigenvalues are given by the eigenvalues of matrix \(J_1^\ast\), that is, \(\delta \lambda_1\) and \(\delta \lambda_2\), and the third eigenvalue is given by:

\[-\psi = -c(\bar{\tau} - \tau)\]  \hspace{1cm} (61)

The symmetric equilibrium is stable when the three eigenvalues of matrix \(J_2^\ast\) are negative. As pointed out in section 4.2, the eigenvalues of matrix \(J_1^\ast\) are both negative - for a given level of
integration between the two regions - when $\tau > \tau^*_s$, that is when centrifugal forces for the given technology levels are stronger than centripetal forces.\(^{23}\) The third eigenvalue ($-\psi$), instead, is negative when trade costs are sufficiently low to allow for successful technology spillover processes, that is, when $\tau < \bar{\tau}$.

Table 1 reveals that the symmetric equilibrium is always unstable for low levels of economic integration, that is, for the highest range of trade costs values. More precisely, when $\tau > \bar{\tau}$, namely when the level of trade costs is higher than the value below which knowledge spillovers may be assimilated by firms in the lagging region, the symmetric equilibrium is unstable.

The fact that the symmetric equilibrium is unstable for the highest range of trade costs values is not in line with the general results of economic geography. While existing models predict that dispersion is always a stable equilibrium for high trade costs, the present work shows that this is not always the case. If trade costs are high enough, the lagging region cannot benefit from any potential knowledge spillovers process, and the initial technological gap increases over time leading to the agglomeration of the manufacturing sector in the leading region, which is a sustainable equilibrium.

By contrast, even when the two regions are sufficiently integrated ($\tau < \bar{\tau}$), and therefore the process of technological catching up may be implemented through learning by interacting processes because $\psi > 0$, the other centripetal forces may be strong enough to make the dispersion of the economic activity unstable. This is the case when at least one of the eigenvalues $\delta \lambda_1$ and $\delta \lambda_2$ of $J^*_2$ is positive.

Finally, if the economy is at the symmetric equilibrium, and this equilibrium is stable, technology may still evolve since this equilibrium is compatible with equal exogenous growth rates of $a$, for the two regions (steady state equilibrium).

\(^{23}\) The symmetric equilibrium technology levels are $a_s = a_n = 1$
To summarize, we observe that trade costs play a different role in the process of knowledge spillovers and in the process of interaction between the standard centripetal and centrifugal forces (for given technology development levels). More precisely, from the point of view of technology spillovers processes, a reduction in trade costs, when they are high, may enhance the recovery of the less developed region, but from the other point of view (that is, the interplay of centripetal and centrifugal forces evaluated at fixed technology levels), the reduction in trade costs may strengthen centripetal forces. As a consequence, in this case, there is a trade-off in the role played by trade costs in knowledge spillovers processes and in determining the result of the conflict between standard fixed-technology centripetal and centrifugal forces.

6 Conclusion

The model we develop in the first part of this paper can be considered as an extension of Puga (1999) that already generalizes the Core-Periphery models by Krugman (1991b), Krugman and Venables (1995). More precisely, to derive the results of Puga (1999) we have to assume that the regional technological levels are the same for the two regions \((a_s = a_n = 1)\). Moreover, Krugman’s model (1991b) corresponds to the case in which agriculture and manufacturing use different factors of production. Particularly, agriculture uses only unskilled interregionally immobile workers (or land in Puga (1999)) and manufacturing firms employ only skilled mobile workers.\(^{24}\) Krugman and Venables’ model (1995), instead, assume that: firms employ only intermediate manufacturing varieties and immobile workers, who correspond to unskilled workers in the present model; the labor supply from agriculture to manufacturing is perfectly elastic.\(^{25}\)

For given technological levels, we find that full agglomeration of the manufacturing sector in a region is unsustainable for high trade costs because centrifugal forces are stronger than centripetal ones. By contrast, full agglomeration may be an equilibrium for low trade costs. Moreover,

\(^{24}\)Krugman’s model (1991b) corresponds to the case in which \(I_{mir} = H_r, \theta = 0, \mu = 0\) and \(\eta = 0\).

\(^{25}\)Krugman and Venables’ model (1995) corresponds to the case in which \(I_{mir} = H_{mir} D_{mir} / [(1 - \mu)^{1 - \mu} \mu], \theta = 1\) and \(\eta = \infty\).
the introduction of differences in the technology levels, allows us to show that the existence of technological differences may give rise to a non-monotonic relationship between the sustainability of agglomeration and the levels of trade costs.

For given equal technological levels \( (a_x = a_n = 1) \), the traditional result of a stable symmetric equilibrium for high trade costs may holds. Moreover, we show that when the technological advantage of a region is very high with respect to the other region, full agglomeration of manufacturing in the leading region is sustainable even for the highest values of trade costs.

When we allow for technological change and potential knowledge spillovers, we enrich the analysis by considering new forces that may modify the above-mentioned results obtained with fixed-technology centripetal/centrifugal forces. More precisely, when obstacles to interacting, proxied by trade costs, are high, the symmetric equilibrium becomes unstable and centripetal forces induce the agglomeration of the manufacturing sector in the more developed region. Besides, low trade costs may yield either the agglomeration in the more productive region, or the dispersion of the modern sector.

In particular, the symmetric equilibrium can be attained only if the lagging region can complete a catching up process with the leading region. Hence, we show that the symmetric equilibrium is unstable when trade costs are too high, because firms in the lagging region cannot benefit from the potential knowledge spillovers from the leading region. In this case, firms in the less developed region do not have enough opportunities to interact with firms in the leading region and, therefore, they are unable to assimilate the more productive technologies used by the latter. As a result, the technological gap between the two regions increases, and the manufacturing sector ends up being completely concentrated in the leading region. By contrast, when trade costs are sufficiently low, firms in the lagging region can benefit from knowledge spillovers and the symmetric equilibrium may be stable if all the centripetal forces are weaker than the centrifugal ones.

Moreover, we find that, for intermediate trade costs values, the symmetric equilibrium can be stable, provided that the initial technological gap between the two regions is not too wide. In fact,
when it is very wide, firms in the lagging region are unable to assimilate the potential knowledge spillovers. When this is so, the agglomeration of the manufacturing sector in the leading region is the only sustainable equilibrium.

To sum up, there is a trade-off in the role played by trade costs in knowledge spillover processes, and in determining the result of the conflict among the other fixed-technology centripetal and centrifugal forces. The results of this trade-off depend on which of the effects produced by different trade costs levels prevail. Particularly, if trade costs are very high, manufacturing ends up being completely agglomerated in the region that has an initial technological advantage, because firms in the lagging regions are unable to benefit from the interregional potential knowledge spillovers. Thus, we like to stress that our results reverse the usual conclusion of New Economic Geography models that high trade costs favors economic dispersion by showing that high trade costs favor the agglomeration of firms in the more productive region.

Appendix A. Sustainability of agglomeration of the manufacturing sector in region $v$.

Agglomeration of the manufacturing sector in region $v$ is an equilibrium if sales of a (potential) deviant firm relocating in region $r$ are less than the level required to break even, that is if:

$$Q_{mir} < Q_{mir}^*$$  \hspace{1cm} (62)

Let us consider as given the regional levels of the technology $a_r$ and $a_v$. A manufacturing firm has positive (negative) profits if its production is higher (lower) than the amount required to break even, $Q_{mir}^*$, that is given by

$$Q_{mir}^* = a_r$$  \hspace{1cm} (63)

where $r = s, n$.

Let us consider the case in which the manufacturing sector is fully agglomerated in region $v$ and a firm that is a potential deviant in region $r$. This firm decides to start its production in
region $r$ if the demand that it faces by producing in region $r$ is higher than (or equal to) the amount $Q_{mir}^*$ required to break even by producing in this region. The relationship between the two regional break even quantities is:

$$Q_{mir}^* = \frac{a_r}{a_v} Q_{miv}^*$$

(64)

where $r, v = n, s$ and $v \neq r$.

When the manufacturing sector is fully agglomerated in region $v$, the composite good price indexes in region $v$ and in region $r$ are respectively:

$$p_{mv} = n_v^{1/\sigma} p_v \quad \text{and} \quad p_{mr} = n_v^{1/\sigma} \tau p_v$$

(65)

The demand for variety $i$ produced in region $v$ is:

$$Q_{miv} = \frac{E_{mv} + E_{mr}}{n_v p_v}$$

(66)

Moreover, given the free entry and exit hypothesis for manufacturing firms, each firm in region $v$ produces the quantity required to break even:

$$Q_{miv}^* = Q_{miv}$$

(67)

Hence, the zero profit level of output of the potential deviant firm in region $r$ is given by:

$$Q_{mir}^* = \frac{a_r}{a_v} \frac{(E_{mv} + E_{mr})}{n_v p_v}$$

(68)

The demand for variety $i$ produced by the potential deviant in $r$ is:

$$Q_{mir} = p_r^{-\sigma} \left( \frac{1}{p_{mr}} E_{mr} + \tau^{1-\sigma} \frac{1}{p_{mv}} E_{mv} \right)$$

(69)

Substituting the price indexes from (65) into (69), it is possible to express the deviant firm’s demand function as:

$$Q_{mir} = \left( \frac{p_r}{p_v} \right)^{-\sigma} \left( \tau^{\sigma} - 1 \frac{E_{mr}}{n_v p_v} + \tau^{1-\sigma} \frac{E_{mv}}{n_v p_v} \right)$$

(70)

From (27) we can derive relative prices ($p_r/p_v$):

$$\frac{p_r}{p_v} = \frac{a_r}{a_v} \left( \frac{p_{mr}}{p_{mv}} \right)^\mu \left( \frac{w_{hr}}{w_{hv}} \right)^{1-\mu}$$

(71)
From the regional price indexes of the composite good (65) we can derive the relative price index:

\[
\frac{p_{mv}}{p_{mr}} = \tau^{-1} \tag{72}
\]

In order to attract workers in region \( r \) we know that the deviant firm has to offer them at least the same real wage that they gain in region \( v \). Therefore, the following condition must hold:

\[
\frac{w_{hr}}{w_{hv}} = \left( \frac{p_{mv}}{p_{mr}} \right)^{-\mu_c} = \tau^{\mu_c} \tag{73}
\]

Therefore, we may rewrite the ratio of the price of the varieties produced in the two regions as:

\[
\frac{p_r}{p_v} = \frac{a_v}{a_r} \tau^{\mu_c + (1-\mu)\mu_c} \tag{74}
\]

Substituting (74) into (70) and eliminating \( n_v p_v \) yields the ratio between the demand for the potential deviant firm \((Q_{mir})\) and the break even amount for region \( r \) \((Q_{mir}^*)\):

\[
\frac{Q_{mir}}{Q_{mir}^*} = \left( \frac{a_v}{a_r} \right)^{1-\sigma} \tau^{1-\sigma(1+\mu_c)(1-\mu)\mu_c} \left( 1 + \frac{(\tau^{2(\sigma-1)} - 1) E_{mr}}{E_{mv} + E_{mr}} \right) \tag{75}
\]

Then we compute expenditures on manufactures in both regions, in order to substitute \( E_{mv} / (E_{mv} + E_{mr}) \) in the previous expression. When firms and skilled workers are fully agglomerated in the region \( v \), these are respectively:

\[
E_{mv} = \mu_c (w_{hv} H_v + r_v K) \tag{76}
\]

and

\[
E_{mr} = \mu_c (w_{hv} H_v + r_v K) + \mu n_v p_v Q_{miv} \tag{77}
\]

Unskilled wages and maximized profits for unit of land in region \( r \) are, respectively:

\[
w_{hr} = \theta \left( \frac{H_{ar}}{K} \right)^{\theta-1} \tag{78}
\]

and

\[
r_r = (1 - \theta) \left( \frac{H_{ar}}{K} \right)^{\theta} \tag{79}
\]

Moreover, given the free entry and exit condition, the wages of workers in region \( v \) employed in the manufacturing sector correspond to the share \((1 - \mu)\) of total revenues of firms in \( v \):

\[
w_{hv} H_{mv} = w_{hv}(H_v - H_{av}) = (1 - \mu) n_v p_v Q_{miv} \tag{80}
\]
Using (66), (76)-(79) we obtain

$$\frac{E_{mr}}{E_{mv} + E_{mr}} = \frac{(1 - \mu_v) (1 - \mu_c)}{\tau \mu_c + 1}$$

(81)

Substituting $E_{mr}/(E_{mv} + E_{mr})$ from (81) into (75), we obtain the following expression:

$$\frac{Q_{mir}}{Q_{mir}^*} = \left( \frac{\alpha_n}{\alpha_r} \right)^{1-\sigma} \left[ \tau^{2(1-\sigma)} \left( \frac{1 - \mu_v (1 - \mu_c)}{\tau \mu_c + 1} \right) \right]$$

(82)

Appendix B.

To prove that profits in a neighborhood of a long run equilibrium can be written as a function of the number of firms $n$

$$\pi_i = u(n)$$

(83)

it is necessary to determine the short run equilibrium, which is defined as a set of solutions to equations (84)-(87) below, once $n_n$ and $n_s$ are given. We express them in matrix form. To this end, variables without suffix $r$ define vectors, variables with superscript $\sim$ are 2x2 diagonal matrix with the i-th element of the corresponding vector in position (i,i) and zeros off the diagonal. Matrix $T$

is: $T = \begin{bmatrix} 1 & \tau^{1-\sigma} \\ \tau^{1-\sigma} & 1 \end{bmatrix}$.

Substituting manufacturing prices $p_r$ from (27), expenditures on the manufacturing good $E_{mr}$ from (5), (6) and (11), manufacturing quantities $Q_{mir}$ from (30), and production cost $TC_{mir}$ from (10), into (26), (4), (37), (34) and (35), for $r = n, s$, and given the normalizations of $\alpha$ and $\beta$, we obtain a system of 8 equations. The solutions of the system (84)-(87) after the substitution of $r = r(w_h)$ and $r_{w_h} = \begin{bmatrix} \frac{\partial r}{\partial w_n} \\ \frac{\partial r}{\partial w_s} \end{bmatrix}$, derived by the particular technology used in the agricultural sector, is given by the set of the 8 “fast” variables $(\pi_{in}, \pi_{is}, p_{mn}, p_{ms}, w_{hn}, w_{hs}, H_n, H_s)$ for given values of the slow variables $n_n$ and $n_s$.

In matrix form, the equilibrium is obtained by solving the following system, with $r = r(w_h)$ and $r_{w_h} = \begin{bmatrix} \frac{\partial r}{\partial w_n} \\ \frac{\partial r}{\partial w_s} \end{bmatrix}$:

31
a) two manufacturing good market short run equilibrium conditions:

$$\sigma \pi = a^{-1} \tilde{p}_m \sigma q w_h^{\sigma} \left(1 - \frac{1}{\sigma} \right) \left\{ T \tilde{p}_m^{-1} [\mu c (\tilde{H} w + \tilde{K} r) + \mu (\sigma - 1)] \tilde{n} \pi i + \mu \tilde{n} w_h^{1 - \sigma} \right\}$$

(84)

b) two composite good price indices:

$$0 = p_m^{1 - \sigma} - T a^{1 - \sigma} \tilde{p}_m^{(1 - \sigma)} w_h^{(1 - \sigma)}$$

(85)

c) two functions that express labor market equilibrium condition in the two regions:

$$0 = H + \tilde{K} r w_h - (1 - \mu) \tilde{n} [\sigma - 1] \tilde{n} \pi i + \tilde{p}_m w_h^{1 - \sigma}$$

(86)

d) and the condition of equal real wages for the two regions together with the condition that

$$H_n + H_s = \tilde{H}$$

Let:

1. $$x = (p_{mn}, p_{ns}, w_{hn}, w_{hs}, H_n, H_s)'$$, a column vector of six fast variables;

2. $$y = (\pi_i, x')'$$ the column vector of the eight fast variables;

3. $$G_k$$ be a function from $$R^{10}$$ to $$R$$ with continuous derivative in the neighborhood of a long run equilibrium (LRE), such that $$G_k(y, n) = 0$$, with $$k = 1, 2, ..., 8$$ are the four equations (84)-(87);

4. $$G(y, n) \equiv (G_k(y, n))$$.

If the $$det = \left[ \frac{\partial G_k}{\partial y_k} \right]_{y_i} \neq 0$$, where $$y_i$$ is a generic element of $$y$$ and $$*$$ means that the derivatives are evaluated at a LRE, equation $$G(y, n) = 0$$ allows us to define in a neighborhood of such LRE function $$u$$ from $$R^2$$ to $$R^2$$ with continuous derivative such that

$$\pi_i = u(n)$$
The Jacobian matrix of $u$ in a LRE is denoted by $\frac{\partial u}{\partial n}(n^*)$.

Finally, it should be noted that at the long run equilibrium values, that is, at long run equilibrium values of all fast and slow ($n_n$ and $n_s$) variables, profits should be equal to zero ($\pi_{in} = \pi_{is} = 0$).
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References


Figure 1.a and Figure 1.b
Figure 2
Figure 3
Figure 4
\[
\begin{array}{|c|c|c|}
\hline
\text{Condition} & \text{if } \tau < \bar{\tau} & \text{if } \tau > \bar{\tau} \\
\hline
\delta \lambda_1 < 0; \delta \lambda_2 < 0 & \text{Stable} & \text{Unstable} \\
\delta \lambda_1 < 0; \delta \lambda_2 > 0 & \text{Unstable} & \text{Unstable} \\
\delta \lambda_1 > 0; \delta \lambda_2 < 0 & \text{Unstable} & \text{Unstable} \\
\delta \lambda_1 > 0; \delta \lambda_2 > 0 & \text{Unstable} & \text{Unstable} \\
\hline
\end{array}
\]

Table 1. Symmetric equilibrium for different trade costs ($\tau$) values.