International Migration of Brains, Educational Competition and National Interests: A Two-country, Game-Theoretic Approach

by

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August 2014

In Process

Abstract

A two-country, game-theoretic framework focuses on the nexus between human capital formation and international migration in an enlarged framework where distinctive brain drain and/or gain effects can arise from the option to study either at home, or abroad. Heterogeneous individuals choose between alternative university systems based, among other considerations, on their innate abilities, the quality of national educational systems, associated access costs, as well as employment prospects and anticipated wage earnings in both countries. The international welfare implications of non-cooperative and cooperative educational and labour-market strategies are examined. Paradoxically, scenarios arise where the impact of educational subsidies and/or improved educational standards in a given country can be immiserizing.

JEL classification codes:  F22, F15

Key words:  international migration, human capital formation, brain gain, brain drain, heterogeneous agents, sunk costs, educational systems, welfare
Acknowledgement: Prepared for presentation at the European Trade Study Group (ETSG) in Lausanne, September, 2010. Financial support from the CREM at the University of Caen has been much appreciated.

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PRELIMINARY VERSION (in process and incomplet): Comments are welcome.

Section I: Introduction

International human capital mobility has been at the heart of the brain drain literature, initiated in large part by Bhagwati and Hamada (1974), and subsequently elaborated to consider the counterbalancing effects of brain gain effects on economic welfare in source and recipient countries. In what Schiff (2006) has termed the “new brain drain literature”, Mountford (1997) and Stark, Helmenstein and Prskawetz (1997), have identified a potentially important source of brain gain, which is independent from return migration. Specifically, although migration can generate a loss of domestic talent, it can also prompt an upsurge in the overall educational level of a home country, as a result of higher propensities to invest in human capital. Attractive foreign labour market conditions offer heightened incentives for domestic workers to strive to attain higher qualification levels, whether or not they ultimately find jobs abroad, thereby fostering, ceteris paribus, increases in average productivity levels at home.
While certain existing approaches to modelling brain drain and brain gain effects entail macroeconomic frameworks with representative agents, as in Vidal (1998), many also consider microeconomic decisions at the level of individual agents, including choices regarding optimal investment levels in education. Stark, Helmenstein and Prskawetz (1997) have proposed a framework, which demonstrates how, given the opportunity to migrate, choices regarding educational attainment will determine an individual’s wage on the foreign labour market. In other modelling frameworks, as proposed by Stark, Helmenstein, and Prskawetz (1998), the potential migrant takes into account a probability of finding a job abroad, which is identical for all individuals, or, as in Stark (2004), constrained by a minimum threshold level of qualification. Mountford (1997) and Beine, Docquier, and Rapoport (2001, 2008) propose models where an individual’s decision is of a binary form – whether to undertake education or not, while the probability of finding foreign employment is exogenous. This does not allow a role for differences in individuals’ characteristics, so that migrants are randomly selected. In contrast, Chiswick (1999) provides for self-selection by migrants, since, assuming two categories of individuals, the rate of return to migration is greater for those with high-ability, relative to lower-ability persons. Nonetheless, the literature has principally focused on the links between incentives to invest in human capital at home and subsequent migration flows.

The evaluation of brain drain/brain gain effects is made in the literature by assessing the impact of migration on a variety of specific economic objectives, which, however, do not include an explicit social welfare per se. Notably, migration is shown to influence the growth rate of the home economy, as in Beine, Docquier, and Rapoport (2001), the average educational level, as highlighted by Stark et al. (1997, 1998) and Lien and Wang (2005), average productivity in Mountford (1997), as well as the wages of non-migrants in Stark (2004).

Although there is now a burgeoning number of empirical studies, assessing different dimensions of the potential impact of brain drain and gain, there remains a lack of consensus regarding the size of conjectured positive effects of migration upon levels of education, welfare and/or growth. Notably, Beine, Docquier, and Rapoport (2001, 2008) find that the proportion
of migrants must be low for such effects to be apparent. According to Schiff (2006), preliminary studies by the World Bank show no positive impact, while Groizard and Llull (2006) indicate a similar finding.

A recent critique by Rosenzweig (2006), which faults existing approaches to the analysis of brain drain and gain in two crucial respects, is particularly germane for motivating the modelling framework proposed in the current research. First, he contends that the potential impact of the “‘risk’ of emigrating” for “domestically-educated tertiary educated person(s)” is de facto quite minimal. Second, Rosenzweig goes on to suggest that “the literature ignores the endogeneity of the emigration probability”, while arguing that, in fact, “the choice of the location of tertiary education significantly affects the probability that the person can emigrate.”1 (p. 2-3) Critically, existing analytical research has paid relatively little attention to the question of whether distinctive brain drain and gain effects may arise, depending on the extent to which educational investments take place either in home and/or host countries. Nonetheless, the policy stakes of the international mobility of high-skilled workers are increasingly recognized as a source of substantial policy concern.2

This paper proposes an enlarged framework for analyzing the nexus between human capital formation and international migration. It extends the existing research of Franck and Owen (2009, 2010) to consider distinct categories of brain drain and brain gain effects, arising from the eventual decision to undertake further human capital formation, either at home, or abroad. Associated international welfare implications are explored in a two country, game-theoretic setting, where heterogeneous individuals face the option of eventually pursuing further studies, while choosing between the

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1 While the analytical framework proposed by Rosenzweig does not allow for differences in individual abilities, his empirical findings are consistent with a number of the modeling assumptions which are subsequently invoked here. Notably, he reports evidence that students are motivated by foreign studies in order to obtain employment in a host country and that quality differences in university systems also appear to trigger the decision to study abroad.

2 See, for example, Leipziger (2008) and Solimano (2008).
alternative university systems. These educational investment decisions are based, among other considerations, on the interrelation between differences in students’ innate abilities, quality and access costs of the educational systems, as well as subsequent employment prospects and anticipated wage earnings in both countries. Nonetheless, under certain conditions, certain individuals may opt to remain relatively less trained and, consequently, only be able to work in their country of origin at a lower wage rate. Accordingly, the analysis does not consider the additional complication of international migration by unskilled workers.

The organization of this paper is the following. The basic modelling analysis, proposed in Section II, starts with a general formulation of the heterogeneous individuals’ ex ante choices, regarding whether to undertake additional human capital formation at home or abroad, or remain less skilled. An individual’s underlying ability determines known productivity gains from studying in either university system, along with expected probabilities of subsequently obtaining foreign market employment at higher wages. These anticipated gains depend on the hypothetically realizable gains in productivity, which are, in turn, a function of individuals’ abilities across the heterogeneous population. A determination of the alternative evaluations of the net returns to additional educational investments depends on the specific costs borne by students in each university system. These net returns are, however, also compared with the lower wage for lower skilled workers. Individuals, who decide not to upgrade their skills, remain at a lower and uniform level of productivity. Unlike more educated workers, workers with less skills are understood to only have the option of working at home.

Section II then characterizes how alternative configurations of educational decisions are critically dependent on certain modelling parameters, including the heterogeneity of innate abilities, quality of university systems, educational costs, as well as employment prospects and anticipated wage earnings. Distinctive brain drain and brain gain depends on the size of the sub-populations of individuals who migrate permanently, as compared with those who return home with enhanced productivity, or upgrade their human capital by pursuing further studies at home.
Comparative static results highlight how changes in key model parameters impact international educational decisions and associated migration flows.

The analysis in Section III then starts by characterizing the determinants of economic welfare in the two countries. A key issue is again how the interrelation between a diverse set of modelling parameters, educational decisions and subsequent employment prospects, generates alternative regimes with distinctive brain drain and brain gain flows across the population of heterogeneous individuals, thereby defining the welfare calculations. The welfare implications of non-cooperative and cooperative national policies are then investigated in an initial shorter-term scenario where the quality of the university systems in the two countries is taken as given. However, governments can potentially impact individuals’ decisions to undertake further studies, along with the flows of students between countries, through their policies with regard to tuition payments and, in the case of the foreign country, the extent of labour market access. In the case of non-cooperative solution, the domestic (foreign) country seeks to maximize the gain in productivity resulting from a lowering of tuition fees in order to foster more human capital formation, but faces potential tradeoffs to the extent that brain drain dominates brain gain effects and due to the cost of public sector funding, when tuition fees fall short of the actual educational costs incurred by the universities. In the initial analysis presented here governments are assumed to have perfect information regarding students’ abilities. Yet, a potentially critical issue concerns how imperfect information can impact the determination of governments’ optimal policies. Accordingly, certain of the potential implications of alternative assumptions regarding the extent of a public authority’s knowledge of students’ underlying abilities are also identified, in line with the earlier work by Franck and Owen (2009, 2010).

A concluding section highlights the overall contribution of this research, while briefly summarizing key findings. The nature of the ongoing investigation of the proposed model is outlined, while certain directions for subsequent research are also suggested.
Section II: Basic Modeling Framework

A. General Overview

A two-country setting provides the basic framework for the subsequent game-theoretic analysis of the interrelation between international educational choices, migration flows and economic welfare at home and abroad. A central concern is how the international educational choices of a population of a heterogeneous population of individuals in a domestic country, in terms of their abilities, is impacted by quality and cost differences in university systems, as well as subsequent employment prospects and conditions in both countries.

A distinctive feature of the proposed analysis is the demonstration of how the option to invest in further human capital formation, either at home, or abroad, can generate distinct sets of brain drain and brain gain effects. Economic welfare in the two countries is thereby shown to be critically dependent on the quality of, and cost differences between, university systems, along with employment prospects and wage earnings in the domestic and foreign labor markets. More specifically, a representative individual, coming from a heterogeneous population of individuals in the domestic country, faces an ex ante educational choice as to whether to remain unskilled, or to upgrade his/her human capital by either undertaking further studies at home, or abroad in the foreign university system, which is assumed to be of higher quality. Of course, these educational decisions are also influenced by the portion of cost differentials between the two countries educational systems, which are borne by students. These, in turn, depend on countries’ educational pricing strategies, as well as eventual grant programs, aimed at promoting countries either country’s economic welfare non-cooperatively, or cooperatively.

B. Initial Elements of the Model

The point of departure for the more formal modeling is a characterization of the interrelation between the abilities of the heterogeneous individuals in the domestic country and attainable productivity levels, where
the latter depend on potential educational decisions. The domestic country is understood, then, to comprise a set of individuals with a range of abilities, where the capability of the representative \( k \)th agent, is designated as \( a_k \). These abilities are distributed across the population, such that \( a_k \in [a_1, a_2] \), where \( a_1 \) and \( a_2 \) indicate, respectively, the most, and least, capable persons in this heterogeneous set. An educational production function is understood to characterize how abilities, along with differences in the quality of national educational systems, co-determine an attainable productivity level, \( e_k \), for skilled individuals. However, individuals, who do not pursue further studies, remain relatively unskilled, and are assumed to remain at a lower productivity, \( e_0 \), which is the same regardless of innate abilities.

More formally, the educational production function, specified by \( e_k = f(a_k, Z) \), is an increasing function of its arguments and the cross-derivative, \( f_{12} \), is positive. Here, \( Z \) represents the quality of a particular country’s educational system, such that \( Z \in \{ z, z^* \} \). The latter symbols distinguishing the educational quality of, respectively, the domestic and foreign countries, where it will be assumed, in general, here that the foreign educational system is of higher quality, such that \( z^* \geq z \). Consequently, a distinction can be made between the higher level of productivity realized by the \( k \)th individual, \( e_k^* \), when studying abroad \( e_k^* = f(a_k, z^*) = e_k^*(a_k) \), relative to the level attainable through studies at home, \( e_k = f(a_k, z) = e_k(a_k) \). Furthermore, it is assumed that there is increased productivity gain for more capable individuals, when they are educated in a higher quality system, which in light of the foregoing discussion means that:

\[ \text{(H1a) } e_k \text{ and } e_k^* \text{ are both strictly increasing functions of } a_k, \text{ while } e_k^* > e_k \text{ for all } k; \text{ and} \]

\[ \text{(H1b) } e_k^* - e_k \text{ is a non-decreasing function of } a_k, \text{ again for all } k. \]

In light of the assumed superior quality of the foreign university system, the hypothetical educational options of pursuing further studies, either at home, or abroad, translate for the representative \( k \)th individual into an unique combination of productivity values \((e_k, e_k^*)\). The overall set of
attainable combinations of productivity levels can be represented for the heterogeneous population, as a whole, by a line segment in a graphical framework, where conceivable levels of domestic and foreign productivity are represented, respectively, on the horizontal and vertical axes. Such a line segment, which is referred to, here, as the talent-educational quality locus, represents the nexus of attainable productivity gains, determined by the interrelation between the distribution of individuals’ talents and the performance-enhancement generated by the quality of the two countries’ educational systems.

There are four major determinants of the characteristics of the talent-educational quality locus, and hence the position and shape of the associated line segment, which warrant further elaboration. First, the degree to which the foreign university system offers a superior opportunity to enhance certain individuals’ productivity levels is captured by the extent to which any part of the locus diverges away from a bisecting straight line, emanating from the origin, in the space of hypothetical realizable productivity values (e_k, e_k*) demarcated by the horizontal and vertical axes. Second, a related remark is that higher degrees of convexity of the upper part of the locus corresponds to scenarios where a higher quality of the foreign educational system offers a potential higher enhancement of the most talented individuals, since they would experience relatively greater relative productivity gains, as compared to those which could be realized through foreign studies by less able persons. Such an issue of heightened quality differentiation, according to students’ abilities, will be termed here as the relative degree of “elitism” of the foreign educational system. Third, for any given productivity scale, the initial point of departure of the talent-educational quality locus, relative to the origin, reflects the relative quality of the domestic country’s pre-university educational system.

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NB. A principal focus of the subsequent comparative analysis is to establish specific propositions on the basis of changes in variables which impact the relative positions of the talent-educational quality locus and the curves demarcating the three hypothetical educational regimes. More specifically, a
particular concern is how the balance between brain drain, grain and waste effects can depend on the hypothetical combinations of source and recipient countries.

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The initial position of the talent-educational quality locus is determined, in part, by the quality of pre-university educational system in the source country, where the latter can be regarded as a separate variable entering the educational production function. Thus, its endpoints are determined by the extreme values for individuals' talents, $a_1$ and $a_2$, and the quality of the domestic primary educational systems, its length by the degree of elitism in different educational systems and its slope will be steeper than a 45 degree line to the extent the foreign university educational system is superior to that of the domestic country.

In the proposed model, whereas skilled workers have the possibility of migrating abroad, it is postulated that unskilled individuals only can work at home at a fixed wage rate, $w_0$. In each labor market, the wages of skilled workers are understood to be an increasing function of workers' realized productivity levels, which, as noted, depend on both their abilities and educational choices. A necessary condition for the possibility of permanent international migration, driven by more favorable employment prospects abroad, is that, for a given level of individual productivity, the foreign salary is greater than that in the home country. Accordingly, for any given level of productivity, it is assumed that the corresponding salary in the foreign country, $w^* = w^*(e)$, is greater than that at home, $w = w(e)$, for all productivity levels and individuals. Furthermore, it will be postulated that the difference between these levels widens as productivity levels increase. In view of a lower productivity level, $e_0$, the wages attainable by unskilled

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1 However, as subsequently elaborated, temporary migration can occur in order to undertake studies abroad even when there is no prospect of foreign employment. A necessary condition is that the expected additional salary gain at home, resulting from enhanced productivity because of more favorable university conditions abroad, more than offsets any greater educational costs.
persons, denoted as \( w_0 = w(e_0) \), are always superior to those for skilled workers. In sum, the following holds:

(H2a) \( w \) and \( w^* \) are increasing functions of \( e \), such that \( w(e) < w^*(e) \) and \( w(e) > w_0 \) for all \( e > e_0 \),

(H2b) \( w^* - w \) increases with \( e \).

In order to facilitate the subsequent analysis of the critical interrelation between wages, productivity and educational choices, it is useful to introduce additional notation for the representative \( k \)th individual. More specifically, the wages earned on the home or foreign labor markets will differ depending on whether the individual is trained at home or abroad. The higher quality, foreign university system yields a greater productivity gain, which, in turn, yields relatively higher wages in the domestic and foreign job markets, denoted, respectively, as \( w(e^*_k) = w[e^*(a_k)] \) and \( w^*(e^*_k) = w^*[e^*(a_k)] \). Accordingly, the following inequalities summarize, then, the interrelation between wage earnings and the location of human capital formation: \( w(e_k) < w(e^*_k) \) and \( w^*(e_k) < w^*(e^*_k) \).

C. The Ex Ante Model of Human Capital Formation with Heterogeneous Individuals

The analysis now characterizes the interrelation between educational choices and both temporary and permanent migration flows by focusing on the individuals’ decisions of whether, or not, to invest in further human capital, either at home, or abroad. While this choice will be formulated for a representative \( k \)th person, it is essential to recognize that the specific choices can vary across the heterogeneous population as a function of differences in abilities. A variety of other factors can also critically impact the decision whether or/not to pursue further studies in one of the two countries’ university systems. These include the range of probabilities of gaining access to the foreign labor market, which as previously noted depend on the choice of university systems; as well as the interrelation between salary differentials and productivity levels for skilled workers in the two labor markets, as
compared to the fixed domestic wage for unskilled workers. Furthermore, both increases in productivity and associated gains in salaries differ across the population of heterogeneous individuals, according to abilities. Finally, the \textit{ex ante} human capital decisions are also potentially impacted by the tradeoff between expected higher financial returns from further education and the corresponding relative costs, either at home or abroad.

More specifically, a representative individual faces three conceivable choice options, designated as outcomes [0], [1] and [2]. These are, respectively, to: i. not undertake any further studies, ii. pursue further studies at home, or iii. undertake studies abroad. Those individuals, opting not to continue their studies, remain relatively unskilled, are unable to work abroad and face an exogenously given domestic wage equal to $w_0$. However, by undertaking further studies, a representative individual, $k$, can upgrade his/her level of productivity. This permits access to the foreign labor market with variable probabilities, depending on where the individual is educated and his/her ability. More specifically, if the individual were to study at home, or abroad, the corresponding probabilities of being hired in the foreign country are designated, respectively, as $p(e_k)$ and $p^*(e_k^*)$, where $p(e_k) \leq p^*(e_k^*)$. There are two distinct rationale for assuming that a foreign education can lead to enhanced prospects of being employed abroad. First, this may be due to the assumption that the foreign university system is of a higher quality. Second, there may be informational, network and host country labor market policies, which generate more favorable labor market access for host country trained students, even when they are equally qualified relative to those trained abroad. In light of the foregoing discussion, the following additional assumption applies:

\[(H3) \text{ } p \text{ and } p^* \text{ are increasing functions of both } e \text{ and } e^*, \text{ such that} \]

$$p^*(e_k^*) > p(e_k^*) > p(e_k).$$

\textsuperscript{4} While the latter inequality is not immediately relevant to an individual’s choice, it reflects the assumptions that the productivity level generated by a foreign education is higher than for a domestic one and that the probability of foreign employment is an increasing function of productivity.
The decision to pursue further studies potentially depends on a weighing of the expected salary gains in relation to the additional costs of further studies at home or abroad, where the latter are denoted, respectively, as I and I*. Note that such costs may include not just tuition costs, but also living and other expenses.\textsuperscript{5} The net cost differential for undertaking further studies in the two systems, is designated as i, such that, \( i = I^* - I \). The expected salary earnings from studying in either country can be represented by the following two general functional forms:

\[
(1) \quad g(e) = p(e) w^*(e) + (1-p(e)) w(e), \text{ and} \\
\quad g^*(e^*) = p^*(e^*) w^*(e^*) + (1-p^*(e^*)) w(e^*). 
\]

A series of technical propositions regarding the expected earnings functions \( g(e) \) and \( g^*(e^*) \) constitute the underpinnings for a subsequent analysis, characterizing how different modeling parameters determine the configuration of the three educational choice regimes – [0], [1] and [2]. In order to establish certain of these propositions, it is necessary to invoke the following two further assumptions:

- (H4) \( g(e) \) and \( g^*(e^*) \) are convex functions of \( e \) and \( e^* \).
- (H5) \( p^*(e) - p(e) \) is a non-decreasing function of \( e \).

Whereas an initial proposition follows directly from certain of the foregoing assumptions, the demonstrations of the subsequent lemmas are provided following their statement.

**Lemma 1**

Given assumptions (H1a), (H2a.b) and (H3), the expected earnings functions from further studies at home or abroad, \( g \) and \( g^* \), are increasing functions of the respective attainable productivity levels, \( e \) and \( e^* \), and, consequently of students’ abilities, \( a_k \), for all \( k \).\[5\]

\textsuperscript{5} While the analysis here abstracts from the potential heterogeneity of costs across the population. Such differences could arise because of locational, and other, individual specific factors, as well as government educational grants, which as analyzed by Franck and Owen (2009), can be tailored to promote social welfare in a given country.
**Lemma 2**

Given assumptions (H1b) and (H4), the expected incremental wage return, \( g(e^*) - g(e) \), from a foreign, instead of a domestic, education, when evaluated in terms of the domestic expected earnings function, \( g \), is an increasing function of students’ abilities, \( a_k \), for all \( k \).

**Demonstration:**

The derivative of the expression for the difference in the expected wages is given by:

\[
\frac{d}{da} [g(e^*) - g(e)] = \frac{dg}{de^*} \frac{de^*}{da} - \frac{dg}{de} \frac{de}{da}
\]

In light of assumption (H1b), \( \frac{de^*}{da} > \frac{de}{da} \), and given (H4), \( \frac{dg}{de^*} > \frac{dg}{de} \), so that, it follows that \( \frac{d}{da} [g(e^*) - g(e)] > 0 \).

**Lemma 3**

In light of the set of assumptions (H1) through (H5), the expected incremental wage return, \( g^*(e^*) - g(e) \), from a foreign education, evaluated in terms of the foreign expected earnings function, compared with a domestic education, evaluated in terms of the domestic earnings function, is an increasing function of students’ abilities, \( a_k \), for all \( k \).

**Demonstration:**

The difference between these two earnings functions, \( g^*(e^*) - g(e) \), can be equivalently expressed as: \( [g^*(e^*) - g(e^*)] + [g(e^*) - g(e)] \). In light of Lemma 2, the second term, \( g(e^*) - g(e) \), is an increasing function of individuals’ abilities, \( a_k \); whereas the first term, \( g^*(e^*) - g(e^*) \), is equal to \( [p^*(e^*) - p(e^*)][w^*(e^*) - w(e^*)] \). Finally, in light of assumptions (H2) and (H4), \( g^*(e^*) - g(e^*) \) is a non-decreasing function of \( e^* \) and, consequently, of \( a_k \).

The proposed formulation here of the decision determining individuals’ eventual investments in human capital is more general than in existing models of the brain drain, since it allows for both heterogeneous individuals and agent specific, human capital arbitrage decisions between two educational systems. The critical choice of each individual is whether to pursue further
studies and, if so, in which country; where the latter decision depends on the quality of the different educational systems and the relative prospects of access to the higher wage foreign labor market. Accordingly, three conditions characterize this educational decision for the representative $k$th individual. First, there is an incentive to continue his/her education at home, rather than remain unskilled, if the following condition, labeled (C1):

\[(2a)\quad p(e_k)w^*(e_k) + (1-p(e_k))w(e_k) - I > w_0,\]

If this condition holds, option [1] will be chosen in preference to [0], and it can be equivalently expressed as condition (C2):

\[(2b)\quad g(e_k) > I + w_0.\]

Second, studies abroad will be preferred to remaining unskilled, i.e. option [2] dominates [1], if condition (C12) holds:

\[(3a)\quad p^*(e_k^*)w^*(e_k^*) + (1-p^*(e_k^*))w(e_k^*) - I^* > w_0,\]

Or, alternatively,

\[(3b)\quad g^*(e_k^*) > I^* + w_0.\]

Conditions (2) and (3) are necessary for an individual to choose to undertake further studies, either at home, or abroad. Nonetheless, to determine a student’s final educational choice, it is also essential to consider an additional arbitrage condition, which compares the relative net returns from studying in the two university systems. The additional sufficiency condition for option [2] to prevail over [1] is:

\[(4a)\quad p^*(e_k^*) w^*(e_k^*) + (1-p^*(e_k^*))w(e_k^*) - i > p(e_k)w^*(e_k) + (1- p(e_k))w(e_k)\]

More simply, the latter can be expressed as:

\[(4b)\quad g^*(e_k^*) - g(e_k) > i.\]

Together, inequalities (3) and (4) constitute sufficient conditions for an individual to decide to study abroad. However, when the opposite inequality
to (4) holds, along with condition (2), an individual will instead elect to study at home, rather than either studying abroad, or remaining unskilled. Hence, option [1] then prevails.

Corresponding to each of the foregoing inequalities, (C1), (C2) and (C12), are equations consisting of equalities, which identify limiting values in a plane of productivity levels \((e, e^*)\); thereby demarcating zones, such that each of these conditions is met. More specifically, in the case of (C1) that equality can be expressed as \(e = g^{-1}[I + w_0]\), which represents a vertical line, as depicted in Figure 1. Analogously, the frontier, determining the set of productivity values such that condition (C2) is satisfied consists of a horizontal line, defined by \(e^* = g^{*^{-1}}[I^* + w_0]\). Ceteris paribus, an increase in the opportunity cost of studying in the domestic (foreign) university system leads to a rightward (upward) shift in the corresponding boundary line and, consequently, increases the threshold productivity level, \(e (e^*)\), necessary to undertake such further studies. The determination of nature of the boundary curve corresponding to Condition (C12) requires an evaluation of its slope, on the basis of the formula: 

\[
\frac{de^*}{de} = \frac{g'(e)}{g^{*'}(e^*)} = \frac{g'(e)}{g^{*'}(e^*)} \frac{g(e)}{g^{*}(e^*)}.
\]

Since \(g\) is a convex function of productivity, if \(e^* > e\), then \(g'(e^*) > g'(e)\), so that the first ratio of derivatives in the right-hand expression is less than one. Furthermore, given that \(g^*(e^*) - g(e^*)\) is a non-decreasing function of \(e^*\), \(g^{*'}(e^*) > g'(e^*)\). Hence, since all four derivatives are positive and both ratios are less than unity, it follows that \(de^*/de < 1\), so that the boundary associated with (C12) has a positive slope with a value less than one in the area of the plane where \(e^* > e\).

As depicted in Figure 1, a combination of two out of the three boundary lines, along with the associated inequality conditions, permits an identification of the three alternative, and mutually exclusive, choice regimes. Whereas in zone (0) there is no further investment in human capital, zones (1) and (2) correspond to a pursuit of further studies in, respectively, the home or foreign university systems.

Figure 1
Lemmas 1, 2 and 3 have established that the functions g(e), g*(e*) and g*(e*) – g(e) are all increasing functions of individuals’ abilities, a_k.
Furthermore, as already shown, the three conditions, (C1), (C2) and (C3) are each associated with equalities, which together identify the boundaries for each of the three regimes. Hence, each of these equations defines a unique hypothetical threshold value of abilities, such that the condition is satisfied (violated) above (below) such a critical value. Accordingly, these specific threshold levels, referred to as A01, A02 and A12, are determined, respectively, by the following equations:

(5a) \( g[e(A01)] = I + w_0 \)
(5b) \( g*[e*(A02)] = I* + w_0 \)
(5c) \( g*[e*(A12)] - g[e(A12)] = i \).
Crucially, by ascertaining the different conceivable orderings for the relative values of these three critical ability parameter values, it is possible to establish the position of all the possible talent-educational quality loci, in relation to the three zones, [0], [1] and [2].

**Lemma 4**

There are only two possible orderings for the three threshold levels of abilities, defined by equations (5). These are as follows: A01<A02<A12 and A12<A02<A01.

**Demonstration:**

If it is initially supposed that A01<A02, it can then be established that it is not possible to have A12<A01. For values of individuals’ abilities lying in the interval between A01 and A02, so that a ∈ [A01, A02], it must be the case that g[e(a)] > I + w0 and g*[e*(a)] < I* + w0. This in turn means that g*[e*(a)] - g[e(a)] < i, so that the set of abilities is such that a < A12. This is contrary to the initial hypothesis that A12 could be less than A01. Furthermore, it is also impossible that A01<A12<A02. If the latter inequalities were to hold, then it would have to be the case that g[e(A12)] > I + w0 and g*[e*(A12)] < I* + w0. However, by summing the latter inequalities to be valid, it follows that, g*[e*(A12)] - g[e(A12)] < i; but this violates the definition of A12 in equation (5c), so that the initial supposition cannot be valid. Consequently, together the foregoing demonstrations establish that A01<A02<A12.

Under the hypothesis that A02<A01, an analogous proof can be used to establish that the only feasible ordering of abilities is such that A12<A02<A01.

The actual educational choices over a hypothetical population of heterogeneous individuals requires an examination of the interrelation between, on the one hand, the previously identified factors determining the positions of the boundary conditions for the three regimes; and, on the other hand, the specific distributions of students’ abilities, such that a_k ∈ [a_1, a_2], as well as the quality differences in the two countries educational systems - both prior to and during university studies. The combined effects of these factors
translate into the determination of the position, length and slope of a line segment in \((e_k, e_k^*)\) space, which may, or may not, straddle more than one of the zones, \([0], [1] \) or \([2]\).

***Notably, such a line segment represents the nexus of attainable productivity gains, determined by the interrelation between the distribution of individuals’ talents and the performance-enhancement generated by the quality of the two countries’ educational systems. For expositional simplicity, this curve will be referred to, here, as the talent-educational quality locus. Its endpoints are determined by the extreme values for individuals’ talents, \(a_1\) and \(a_2\), and the quality of the domestic primary educational systems, its length by the degree of elitism in different educational systems and its slope will be steeper than a 45 degree line to the extent the foreign university educational system is superior to that of the domestic country.

The determinants of the relative position of the talent-educational quality locus, in relation to the human capital choice regimes, will now be examined in more detail.

The orderings established in Lemma 4, in conjunction with the relative positions of the regime boundary equations, represented by equations (5a), (5b) and (5c), uniquely determine the positions of all the conceivable talent-educational quality loci for a given population of individuals with heterogeneous abilities. The representative \(k\)th individual with ability \(a_k\) will realize a productivity level of \(e_k = e(a_k, z)\) if he/she undertakes further human capital investment at home, but will achieve a higher productivity, equaling \(e_k^* = e^*(a_k, z^*)\), as a result of foreign studies. By way of simplification, the relation between the two educational production functions can be expressed in terms of a function \(e_k^* = \mu(e_k)\), where in keeping with the assumptions \((H1a,b)\), \(\mu\) is an increasing function. For a given set of modeling parameter values, this functional form can be incorporated into the productivity plane \((e, e^*)\), depicted in Figure 1, as a specific curve having a slope greater than one.

*****The endpoints of such a curve, corresponding to the extreme values of abilities \(a_1\) and \(a_2\), may, or may not, straddle more than one educational choice
regime. Of course, when such a curve lies wholly within one of the three zones, all individuals choose the same human capital investment option. However, if such a talent-educational quality locus were to intersect at least one of the lines demarcating different regimes, the associated ordering for the critical boundary values of A01, A02 and A12 must be consistent with the ranking established in Lemma 4.

In this model, two distinct scenarios characterize the interrelation between the regime boundary conditions and the talent-educational quality locus. These different configurations depend on whether A01 is either interior, or superior to A02. First, as shown in Figure 2a, when the relevant ranking is A01<A02<A12, one conceivable outcome applies for the case when these boundary values lie between the limits of the postulated uniform distribution of attributes, \( a_1 \) and \( a_2 \). Then, whereas less capable individuals, whose abilities fall in the interval \( a_1 < a_k < A01 \), will not invest further in human capital (zone [0]), students with abilities lying in the interval, \( A01 < a_k < A12 \), will pursue an university education at home. Finally, the most capable persons with abilities such that \( A12 < a_k < a_2 \), will opt to continue their studies abroad. Second, when the relevant ranking is A12<A02<A01, a feasible result is that depicted in Figure 2b. This case is critically different from the previous outcome, since at most two human capital formation regimes will be manifest. While less capable individuals with abilities such that \( a_1 < a_k < A02 \), do not invest in human capital, more talented persons, with abilities in the interval \( A02 < a_k < a_2 \), will only study abroad.\(^6\) Crucially, given the configuration of abilities, quality of the potential university educational systems and the potential financial opportunity costs entailed by continued studies at home and abroad, there is no scope for the domestic country to maintain a viable system of higher education. Finally, another scenario, which also entails only two, out of the three, choices, is illustrated in Figure 2c. In this instance, the talent-educational quality locus does not contain one of the threshold values for the boundary conditions over the spectrum of individuals’ abilities, \( a_1 \) through \( a_2 \). and productivity gains,

\(^6\) Note that the value A12, corresponding to the point M12 in Figure 2b, lies in the interval of lower abilities, where \( a_1 < a_k < A01 \).
along the. Specifically, since $a_1 < A01 < a_2 < A12$, less talented individuals with abilities such $a_1 < a_k < A01$ will not undertake further human capital investment, while more capable persons for whom $A01 < a_k < a_2$, will receive a further education at home, so that no studies are undertake abroad.
Figure 2a

Scenario Where All Three Human Capital Regimes Are Chosen by Certain Subsets of Individuals Across Heterogeneous Population

To facilitate the illustration of certain choice outcomes in this, and certain of the subsequent, illustrations, the scales for the productivity levels are assumed to differ between the $e_k$ and $e_k^*$ axes. Consequently, the talent-education locus does not necessarily lie above a bisecting 45-degree line, as would otherwise be the case without such a distortion.

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7 To facilitate the illustration of certain choice outcomes in this, and certain of the subsequent, illustrations, the scales for the productivity levels are assumed to differ between the $e_k$ and $e_k^*$ axes. Consequently, the talent-education locus does not necessarily lie above a bisecting 45-degree line, as would otherwise be the case without such a distortion.
Figure 2b

Scenario Where Subsets of Heterogenous Individuals Choose Only Either to Remain Unskilled or to Study Abroad
**Figure 2c**

Scenario Where Subsets of Heterogenous Individuals Choose Only Either to Remain Unskilled or to Study at Home
D. A More Detailed Analysis of a Linearized Version of the Ex Ante Model

For the foregoing general formulation of the model, the associated expressions, characterizing both the populations of individuals’ choices and the associated consequences for economic welfare in the two countries, are highly non-linear. Accordingly, simplifying linearity assumptions are invoked here, in order to make the analysis more tractable. Specifically, the achievable levels of productivity, for the representative $k$th individual, will again depend on whether he/she is educated at home, or abroad. These are given, respectively, by:

(6a) $e_k = \lambda a_k + \beta$

(6b) $e_k^* = \lambda^* a_k + \beta^*$

The assumption of a higher quality foreign educational system can then be represented by:

(7) $e_k^* = \mu e_k$, 

\[ e_k^* = \lambda^* a_k + \beta^* \]
where $\mu > 1$.\(^8\) The extent to which $\mu$ deviates from 1 reflects, then the opportunity, in terms of a foregone productivity gain, for an individual studying abroad, rather than at home. Note, furthermore, that the applicable set of attainable productivity levels is truncated, since it is determined by the underlying set of individuals’ abilities, which have been assumed to have lower and upper bounds, such that $a_k \in [a_1, a_2]$.

Linear specifications for the interrelation between the wages of skilled workers and alternative values of levels of labor productivity, $e$, are for, respectively, the domestic and foreign countries given by:

\[(8b)\] $w_1(e) = \theta e$

\[(8b)\] $w^*(e) = \theta^* e$

Here, the foreign country is understood to offer higher wages, so that $\theta^* > \theta$, while the wage received by all unskilled workers is such that $w_0 = \theta e_0$. A final simplification of exogenous employment probabilities in the two countries is also made, in order to avoid certain of the multiplicative terms involving the individual productivity levels. Specifically, it is assumed that:

\[(9)\] $p(e_k^*) = p^*, p(e_k) = p < p^*, \forall k$

In light of the foregoing simplifications the expressions for the expected earnings from studying at home, or abroad, can be readily deduced, and are, shown as follows:

\[(10a)\] $g(e_k) = [p\theta + (1-p) \theta] e_k = \alpha e_k$

\[(10b)\] $g^*(e_k^*) = [p^*\theta^* + (1-p^*) \theta] e_k^* = \alpha^* e_k^*$

Since wages are higher in the foreign country, $\theta^* > \theta$, and the probability of being hired abroad is greater when educated abroad, $p^* > p$, it follows that $g^*(e_k^*) > g(e_k)$ for $\forall k$.

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\(^8\) This expression abstracts, without loss of generality, from the inclusion of an additional constant term.
More specific, but analytically equivalent, expressions for (5a), (5b) and (5c) capture individuals’ educational choices in this linearized version of the model. Accordingly, the conditions under which individuals are willing to pursue further studies, either at home, or abroad, are given, respectively, by:

\[(9) \quad \alpha e_k > I + w_0 \]
\[(10) \quad \alpha^* e_k^* > I^* + w_0 .\]

Consistent with expressions (5c), the additional condition determining whether the expected returns from being educated abroad are sufficiently large, to offset the higher educational costs of studying abroad is given by:

\[(11) \quad \alpha^* e_k^* - \alpha e_k > i \]

The foregoing three inequalities are represented in Figure 3, which is analytically equivalent to Figure 1, and identifies combinations of productivity levels \((e, e^*)\), corresponding to the three distinct educational regimes. In particular, (9) and (10) correspond, respectively, to vertical and horizontal lines, which demarcate those productivity levels, which are compatible with undertaking further studies, respectively, at home, or abroad. The additional condition, (11), determines whether it is more advantageous for individuals to study abroad, rather than at home. It reflects whether the difference in the expected earnings from being educated abroad, rather than at home, offset the additional costs of such further studies. This condition can be represented in the \((e, e^*)\) graphical framework by a portion of a line emanating from the vertical access, which has a slope less than one, since \(\alpha/\alpha^* < 1\). That line segment is truncated by conditions (9) and (10).
The interrelation between conceivable values of individuals’ productivity levels and threshold levels defining their educational choices are illustrated in Figure 3. When the attainable productivity values for individuals are relatively low, individuals will not pursue further studies, since the returns to educational investments in either country do not offset the associated cost. Such outcomes are reflected by the set of equilibria, represented by zone [0]. However, further studies are justified for productivity levels corresponding to regimes [1] and [2], which again represent, respectively, the decisions to study at home, or abroad. As specified by condition (11), When studies abroad generate relatively greater increases in productivity, than those at home, net of the difference in educational costs, condition (11) is satisfied so that studies in the foreign university system are preferred, corresponding to zone [2].

**Figure 3**

*Characterization of Boundary Conditions and Alternative Human Capital Regimes for the Linearized Version of the Model*
The feasible set of alternative talent-educational quality frontiers can now be incorporated into Figure 3, so as to identify different configurations of the conceivable educational choices across the heterogeneous population. For each individual, with an ability $a_k$ comprise between the limits of the distribution of abilities $a_1$ and $a_2$, there is an associated unique combination of productivity levels, corresponding to a point on the line $e^* = \mu$e. These values lie between two extreme values, which are labeled here as M1 and M2. Across the continuum of abilities, $a_k$, there are associated productivity values, $e_k$, which lie on the regime boundary conditions, specified by (9), (10) and (11). These are denoted as follows:

\[
\begin{align*}
(12a) \quad & \varepsilon_0 = \frac{I + w_0}{\alpha} ; \\
(12b) \quad & \varepsilon_2 = \frac{I^* + w_0}{\alpha^* \mu} ; \text{ and } \\
(12c) \quad & \varepsilon_1 = \frac{I^* - I}{\alpha^* \mu - \alpha}
\end{align*}
\]

It is straightforward to establish the following interrelation between these threshold values, which is $\varepsilon_2 = \lambda \varepsilon_0 + (1-\lambda)\varepsilon_1$, where $\lambda = \frac{\alpha}{\alpha^* \mu}$. Again, there are only two conceivable orderings for these threshold values. These are: $\varepsilon_0 < \varepsilon_2 < \varepsilon_1$ or $\varepsilon_1 < \varepsilon_2 < \varepsilon_0$. In the first instance, illustrated in Figure 4a, when the extreme values of productivity levels attainable from further studies in the domestic country, $e_1$ and $e_2$, encompass all three boundary values, so that $e_1 < \varepsilon_0 < \varepsilon_2 < \varepsilon_1 < e_2$, those individuals with abilities such that $e_k < \varepsilon_0$ will remain unskilled, while those for whom attainable productivity levels are $e_0 < e_k < \varepsilon_1$, while the sub-population with abilities such that $\varepsilon_1 < e_k$, will undertake studies abroad.

*****IN PROCESS*****

*Figure 4a*

Case Where All Three Educational Regimes Are Chosen by Specific Sub-populations of Heterogeneous Individuals such that $\varepsilon_0 < \varepsilon_2 < \varepsilon_1$
\[ e^* = \mu \epsilon \]

\[ e_1^* = \frac{\epsilon_0}{\alpha^*} \]

\[ e_2^* = \frac{(1 + w_0)/\alpha}{\alpha^*} \]

\[ e_1 = \frac{\epsilon}{\alpha^* \mu - \alpha} \]

\[ e_2 = \epsilon_1 \]
Figure 4b

Case Where Only Two Educational Regimes Are Chosen – Either No Studies or Abroadlations of Heterogeneous Individuals such that $\varepsilon_1 < \varepsilon_2 < \varepsilon_0$

$$e^* = \mu \cdot e$$

$$\varepsilon_2 = (I + w_0)/\mu \alpha^*$$

Figure 4c

Case Where All Three Educational Regimes Are Chosen by Specific Sub-populations of Heterogeneous Individuals such that $\varepsilon_0 < \varepsilon_2 < \varepsilon_1$
Alternative ways of demarcating how the different educational choice regimes potentially depend on certain key modeling, parameter values are depicted in Figures 5 and 6ab. The first of these shows how the configuration of the regimes is related to the threshold levels, \( \varepsilon_0 \) and \( \varepsilon_1 \), which are defined by equations .. and.. As a reminder, the first of these two critical values delimits the boundary between zones 0 and 1, thereby determining for any individual productivity level \( e_k \), whether a representative individual with ability \( a_k \) will remain unskilled or study at home. The second of these critical values determines whether such an agent would choose between zones 1 and 2, which corresponds to the decision of whether to study at home or abroad. .... The graphical representation of the alternative regimes in Figure 5 can, alternatively, be transformed into an equivalent framework presented in Figures 6ab, which show how the educational choice regimes depend on the values of educational opportunity costs, including the educational costs which are borne by students in the two countries, I and I*. Not surprisingly, there exist threshold levels for both countries’ fees, such that beyond those levels it will no longer be profitable for a representative individual to pursue further studies, as represented by the set of equilibria corresponding to zone 0. However, when one of the country’s level of educational costs is relatively high, while the other’s is relatively low, that will favor the choice of the lower-cost educational system.
Figure 5

Representation of Alternative Educational Regimes in Terms of Critical Values ($\varepsilon_0$ and $\varepsilon_1$)
Figure 6a

Representation of Alternative Educational Regimes in Terms of the Opportunity Costs of Continuing Studies in the Two Countries ($I + w_0$, $I^* + w_0$)
The subsequent game-theoretic framework will initially focus on the interrelation between each country’s choice of the level of educational fees can potentially impact individuals’ human capital decisions under alternative assumptions of non-cooperative and cooperative solutions.
Section III: A Game-Theoretic Analysis of International Economic Welfare as a Function of Migration Flows and Modeling Parameters

*****IN PROCESS*****

A. Characterization of National Economic Welfare in the Domestic and Foreign Country in Relation to Educational Costs Borne by Individuals
i. Alternative Specifications for National Economic Welfare
ii. The Welfare Implications of Alternative Distributions of Educational Costs
B. Game-Theoretic Analysis of Educational Grant and Pricing Policies

Section IV: Conclusion

*****IN PROCESS*****

References


Drain and National Welfare: Stakes and Paradoxes from a Microeconomic Perspective”, (Entitled in French: “Formation à l’étranger, fuite des cerveaux et bien-être national: Enjeux et paradoxes dans une perspective microéconomique”), Unpublished manuscript, CREM, University of Caen and LEMNA, Institute of Economics and Management of Nantes – I.A.E., University of Nantes


