Unionization, Information Asymmetry and the De-location of Firms*

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Abstract

We analyze the effects of unionization on the location decision of firms. We consider a model in which home and foreign workers are perfect substitutes and firms have an informational advantage concerning their productivity. The union offers wage-employment contracts to induce truth-telling. Although the firm has an incentive to over- and understate its productivity (countervailing incentives), we find that, under fairly mild assumptions on the distribution of firms’ productivity, the overstating incentive always dominates. The equilibrium contract offered by the union is then characterized by overemployment. Moreover, higher productivity firms are less valuable to the union (compared to the full-information economy) such that firm exclusion increases. This finding is backed by OECD data which suggest a positive correlation between foreign direct investment (measuring de-location) and unionization.

Keywords: trade unions, information asymmetry, open economy, countervailing incentives, de-location

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1 Introduction

1.1 Unions and firm de-location

What is the effect of unionization (or the threat of it) for the decision of a firm where to locate production? In a globalized economy, too high a wage demand by unions may trigger a de-location process of firms. Firms can escape the union which then leads to a moderation of wage demands.

In addition to the wage damping effect, the unions’ wage setting is restricted since the union can only capture (parts of) the quasi-rent defined as the difference between profit when producing at home and abroad. The marginal firm, i.e. the firm that is just sitting on the fence when it comes to the de-location decision, does not earn any quasi-rents. Therefore, a union facing the marginal firm cannot set higher wages (as compared to the competitive case). Consequently, the de-location decision of the marginal firm is not affected by unionization. Based on this argument, we would expect that the degree of unionization has no effect on the amount of firms de-locating their production facility. Using data from OECD countries where we employ the stock of foreign direct investment (FDI) as a proxy for de-location, we find, however, that unionization and FDI are positively correlated, see Figure 1. A fixed effect panel regression for 33 OECD countries shows that an increase in unionization by 10%ig points increases FDI per 1000 employees by 40 million US$.

How can we explain this finding given the argument above that called for a no-effect of unionization on de-location? The crucial point is that this argument is based on world without any frictions. For instance, we have implicitly assumed that the firm’s productivity (and hence the size of the quasi-rents) is common knowledge both to the union and the firm. As pointed out by e.g. Ruiz-Verdú (2007), however, this assumption is not assailable, in particular if the effects of unionization is analyzed.\(^1\) Taking up this point and assuming that the firm has superior information regarding its productivity, the line of arguments regarding the effect of unionization on the de-location of firms changes.

In a world with information asymmetries, the union has to take the self-selection constraint of the firm into account when constructing a wage-employment contract. This leads to a concession of information rent payments from the union to the firm in order to ensure self-selection. Since this information rent is a function of the different productivity types that

\(^1\)Ruiz-Verdú (2007) presents a game-theoretic argument that (endogenous) unionization will be more likely under asymmetric information. Thus, analyzing union behavior under asymmetric information adds an important aspect to the general picture of effect of unions.
can be included in the contract, the union faces a trade-off when excluding firms from contracting. Exclusion results in savings on rent payments on the one hand, but also in a loss of quasi-rents that could have been captured by wage hikes on the other. If the former effect outweighs the latter, the union excludes firms and forces them to produce abroad which under information symmetry or competitive labor markets would have produced at home.

We develop this argument within a stylized economy. Consider a monopolist who is endowed with some production process and serves a world market, employing labor as the only factor of production. The monopolist can either produce at home facing a union, or de-locate production technology and produce abroad. The opportunity costs of labor are lower abroad than at home. As such, every firm, independent of its productivity, would prefer producing abroad over producing at home. De-location, however, requires fixed costs which are independent of the firms’ productivity. If labor markets are perfectly competitive, only firms which are equipped with a production process whose productivity is larger than some threshold produce abroad due to fixed cost depression.
Introducing unionized labor markets in this setting implies that the union could capture part of the quasi-rent which is made up of the operating profit difference plus the fixed costs. The quasi-rent is positive for a firm that is characterized by a low productivity production process and decreasing with for higher productivities.\(^2\) The union constructs wage-employment contracts in advance of meeting a firm with a specific productivity. If the union could observe this productivity, it could capture the entire quasi-rent such that the marginal firm and hence its de-location decision is unaffected by unionization.

With asymmetric information, however, the firm has to construct a wage-employment contract that can only be conditioned on self reported productivity of the firm. This optimal self reporting of the firm is driven by two countervailing incentives. The firm would like to understate its true productivity to get a more favorable contract. Low productivity, however, indicates high quasi-rents. The resulting wage hike provokes an incentive to overstate its productivity.

Without knowing the productivity of the firm, the union proposes wage-employment contracts such that the firm reports its true productivity and participates in the contract. We solve the union’s problem (applying dynamic optimization techniques) and show that under a fairly mild condition on the distribution function of firm productivity, the overstating incentive dominates. The union offers contracts that imply larger employment than would be realized under information symmetry. The intuition for this overemployment result is that the requirement to employ a large workforce decreases the incentive to overstate productivity. Low-productivity firms would find it hard to support a large labor force. Hence, the information rent that induces truth-telling can be reduced.

As already argued before, the union will not include every firm into the contract. This is also true for symmetric information.\(^3\) But with asymmetric information, the value of a firm is smaller due to inefficient employment and there is a gain of excluding high-productive firms because this narrows the possibility of overstating productivity and hence saves on information rents. As a result, more firms are expected to de-locate (respectively being forced to de-locate) their production process abroad. As a corollary from this result, we find first that unionization decreases expected productivity and second that the effect on (expected) employment remains ambiguous. Less firms are expected to produce at home which, however, employ a larger work-force.

\(^2\)This is because the (value) of the operating profit difference becomes larger and hence the fixed de-location costs less relevant.

\(^3\)Also in the case of competitive labor markets some firms will choose to de-locate production as argued above.
1.2 Related literature

Our paper is concerned with the effect of unionization on the firm’s choice of its location of production. Hence, we focus on a peculiar variety of FDI in an economy without trade (there exists only one world market which can be accessed without costs) in which home and foreign workers are perfect substitutes. The usual notion of FDI in the literature is more complex, see e.g. Helpman (2014) and the literature cited therein. Firms either de-locate production facilities in order to save on trade costs when accessing a foreign market (horizontal FDI) or they de-locate (parts of) their supply chain in order to profit from lower production costs abroad (vertical FDI). Consequently, papers which study the effect of unionization on FDI usually integrate union wage setting/bargaining into these type of FDI models.

Eckel and Egger (2009) is a study in this spirit. They analyze the effects of unionization in a situation in which the domestic firm can serve the foreign market either through trade or through a foreign subsidiary. Unionization affects the trade-off between the trade and the the de-location decision, favoring the latter. FDI then not only saves on trade costs (when serving the foreign market) but also imposes a threat to cross-haul (i.e. serving the home market from abroad). This lowers the bargained wage and increases the attractiveness of FDI. Additionally, they show that the wage dampening effect of FDI increases employment at home.

The effect of unionization on vertical FDI is analyzed by Koskela and Stenbacka (2009). In their model, output is produced using home and foreign labor and is sold in the home market. Home and foreign employment are (imperfect) substitutes, but only home labor is represented by union bargaining the wage. The ability of the firm to substitute home by foreign labor makes home labor demand more elastic. As a result, bargained wages decrease (which is a standard result in the literature on unionized labor markets). However, FDI generates a hold-up (because the firm decides ex-ante on the amount of foreign labor) which has a wage increasing effect. If the wage dampening effect dominates (which is the case for high union bargaining power), FDI implies moderate wages, home employment increases and unionization hence lead to more FDI.4

Our approach in modeling FDI is deliberately simpler than the one pursued in the literature. We do not consider trade, have no hold-up effects,

\footnote{A somehow related albeit earlier study is Zhao (1995). This paper assumes home and foreign labor to be perfect substitutes, but that both home and foreign labor are represented by a (national) union. The hold-up effect of Koskela and Stenbacka (2009) is not present. Consequently, the wage dampening effect dominates and employment increases.}
no strategic effects on the product market and no imperfect substitutability between home and foreign labor. The reason is that we want to focus as clearly as possible on a mechanism of unionization on FDI (and vice versa) that is important, but has not received very much attention in the literature: information asymmetries between unions and firms, where firms are assumed to have superior information concerning their productivities.\footnote{Information asymmetries between unions and firms have been analyzed before (see e.g. Oswald (1986)), however, in a closed economy setting and with a different focus.}

The combination of unionization, FDI and information frictions adds an important twist to the aforementioned analyses. As argued above, the firm’s incentive structure is then characterized by countervailing incentives as e.g. analyzed by Lewis and Sappington (1989) or Maggi and Rodriguez-Clare (1995). Thus, the union’s effect on firm-level employment, on (expected) employment as well as on FDI when information are asymmetrically distributed between the firm and the union remains to be analyzed. This is the point of departure of our paper.

The remainder of our paper is structured as follows. Section 2 presents the model and ensuing properties of union’s and firm’s decisions. Section 3.2 solves for the benchmark contract under information symmetry (which is trivial but nevertheless informative). Finally, section 3.3, the heart of our analysis, presents the equilibrium contract under asymmetric information and discusses equilibrium properties. Moreover, we put forward a numerical calibration to gain some insight into the potential quantitative effects of unionization on de-location/FDI and (expected) employment. Eventually, section 4 concludes and shows some avenues for further research.

2 The Model

2.1 The Firm

We consider a monopolistic firm that sells output $x$ facing the (world market) inverse demand function

$$p = x^{-\alpha}, \quad (1)$$

where $p$ denotes the price and $\alpha$ the value of the reciprocal price elasticity of demand with $0 < \alpha < 1$. Output is produced using labor input $l$ only. The production function is given by

$$x = \theta l, \quad (2)$$

where $\theta$ denotes the (exogenously) given productivity of labor input. We assume that nature endows the firm with this level of technology, but that
the firm can decide whether to employ this technology at home or move the technology abroad (and hence de-locate production). Moving the technology, however, comes at a fixed cost of \( K > 0. \)

The profit of the firm that produces at home \( \pi \) is given by
\[
\pi = (\theta l)^{1-\alpha} - wl, \tag{3}
\]
where \( w \) denotes the wage level at home, whereas the profit of moving the technology \( \pi^F \) is given by
\[
\pi^F = (\theta l^F)^{1-\alpha} - w^F l^F - K, \tag{4}
\]
where the superscript \( F \) indicates foreign variables.

### 2.2 Workers/The Union

At home, a mass of \( l \) workers infinitely elastic supply labor at the reservation wage \( b \) (i.e. the opportunity costs of working). Labor supply is hence given by
\[
w = b. \tag{5}
\]
Likewise abroad, a mass of \( l^F \) workers infinitely elastic supply labor at the reservation wage \( b^F \) which results in foreign labor supply
\[
w^F = b^F. \tag{6}
\]
We assume that the foreign reservation wage falls short of that at home, i.e. \( b > b^F \), because e.g. the system of social protection or the unemployment insurance is less generous in the foreign country than at home.

Workers at home are allowed to form a union. The union then gains the right to set wage-employment contracts on behalf of their members \( l \). The union is utilitarian such that the union utility is given by
\[
U_{\text{Union}} = lw + (l - l) b = l(w - b) + lb. \tag{7}
\]
For notational convenience we focus in the following exclusively on the rent maximization part of the union’s utility
\[
V_{\text{Union}} := U_{\text{Union}} - lb. \tag{8}
\]

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6Thus, we assume that moving a low-productivity (i.e. low-technology) production process is as costly/complicated as moving a high-productivity (i.e. high-technology) one. In reality, it seems more likely that moving costs depend on the technological. However, ex-ante the sign of the dependence seems to be unclear. Therefore, we stick to the independence assumption.

7It is clear that without this labor cost advantage there will be no de-location.
Foreign workers are not allowed/able to form a union. We hence abstract from competition between international unions, because we want to exclusively focus on the 'pure' effects of unionization in the home country.

3 Equilibrium

3.1 Competitive labor markets – Benchmark

In the benchmark case of perfectly competitive labor markets, firms first decide where to start production and then decide how much to produce. Turning first to the labor demand decision. Profit maximization leads to

\[ (1 - \alpha)(\theta l)^{-\alpha} \theta = w \]  
(9)

\[ (1 - \alpha)(\theta l^F)^{-\alpha} \theta = w^F \]  
(10)

for home and abroad, respectively. Combining this with the labor supply situation at home and abroad determines equilibrium employment \( \hat{l} \) and \( \hat{l}^F \). Under the assumption \( b > b^F \), we find that \( \hat{l} < \hat{l}^F \), i.e. a firm with a given production process employ more labor abroad than at home. Note that we only consider situations such that the resource constraint of the economy never becomes binding (there will be always unemployment), i.e. \( \hat{l} < \bar{l} \) and \( \hat{l}^F < \bar{l}^F \).

Equilibrium profits at home and abroad are then given by, respectively

\[ \hat{\pi} = \alpha(\theta \hat{l})^{1-\alpha} \]  
(11)

\[ \hat{\pi}^F = \alpha(\theta \hat{l}^F)^{1-\alpha} - K. \]  
(12)

The firm chooses to de-locate production if

\[ \alpha(\theta \hat{l})^{1-\alpha} < \alpha(\theta \hat{l}^F)^{1-\alpha} - K. \]  
(13)

**Proposition 1** There exists some threshold productivity \( \hat{\theta} \) for which a firm is indifferent between producing at home and de-locating production. A firm characterized by a productivity \( \theta > (\leq) \hat{\theta} \) de-locates its production technology abroad (produces at home).

**Proof 1** The operating profit difference \( \delta := \alpha \left( (\theta \hat{l}^F)^{1-\alpha} - (\theta \hat{l})^{1-\alpha} \right) \) is increasing in \( \theta \) because

\[ \frac{d\delta}{d\theta} = (1 - \alpha)(\theta)^{-\alpha} \left( (\hat{l}^F)^{1-\alpha} - (\hat{l})^{1-\alpha} \right) > 0 \]  
(14)

where we used the fact that the labor demand elasticity is independent of the location of production.
Firms with low productivities produce less output such that the average costs of de-location are high. With increasing productivity we have a regression of fixed de-location costs due to increased production. Thus, for high productivity firms, fixed costs become more and more irrelevant when deciding on de-location which makes it the more attractive to produce abroad.

In the following, we focus on a situation in which \( \hat{\pi}(\theta = 1) > \hat{F}(\theta = 1) \) which implies that \( \theta > 1 \). This ensures that when the nature equips a firm with productivity levels \( \theta \in [1, \bar{\theta}] \), the probability of de-locating production is not one.\(^8\)

### 3.2 Unionization – Information symmetry

Consider now a situation in which workers form a union (before nature endows the firm with productivity \( \theta \)). The union is then capable of offering wage-employment contracts for different types of firms. After the firm is endowed with its productivity, it will become common knowledge. Hence, it is known to both the union and the firm. Thus, the union can ex-ante condition the contract on the true productivity. We assume that nature draws firm’s productivity \( \theta \) from the density function \( g(\theta) \) whose support is given by \( \theta \in [1, \bar{\theta}] \).

The objective of the union is hence

\[
E(V^{Union}) = \int_1^{\bar{\theta}} g(\theta) l(w - b)d\theta. \tag{15}
\]

The union chooses \( l \) and \( w \) subject to the participation constraint of the firm which is given by \( \pi \geq \hat{F}. \)\(^9\) By the definition of the profit \( \pi \) we can rewrite union’s utility in terms of employment \( l \) and profit \( \pi \)

\[
E(V^{Union}) = \int_1^{\bar{\theta}} g(\theta)((\theta l)^{1-\alpha} - \pi - lb)d\theta, \tag{16}
\]

which implies that the union can likewise choose employment and allows the level of profit for the firm. The Lagrangean for this problem is

\[
\mathcal{L} = \int_1^{\bar{\theta}} g(\theta)((\theta l)^{1-\alpha} - \pi - lb)d\theta + \mu(\pi - \hat{F}), \tag{17}
\]

\(^8\)Without having this assumption, the solution to the problem is trivial because then every firm independent of its productivity level would choose to produce abroad.

\(^9\)Due to the monopsonistic behavior of the union, the firm is only offered its outside option to ensure participation.
which results in first-order conditions given by

\begin{align*}
g(\theta)((1 - \alpha)(\theta l)^{-\alpha} \theta - b) &= 0, \tag{18} \\
-g(\theta) + \mu &= 0, \tag{19} \\
\mu(\pi - \hat{\pi}^F) &= 0. \tag{20}
\end{align*}

From (18) we find that the union sets employment efficiently, i.e. equilibrium employment will be the same as under competitive labor markets \( l^{IS} = \hat{l} \) (where the superscript \( IS \) denotes the equilibrium under information symmetry). Moreover, (19) and (20) reveal that the equilibrium profit is then given by \( \pi^{IS} = \hat{\pi}^F \), i.e. equal to the foreign profit. The equilibrium wage is then directly determined by \( \pi^{IS} \) and \( l^{IS} \) and is given by

\begin{equation}
w^{IS} = \left(\frac{\theta \hat{l}}{l}\right)^{1-\alpha} - \hat{\pi}^F = b + \frac{\hat{\pi} - \hat{\pi}^F}{\hat{l}}. \tag{21}
\end{equation}

Note, however, that this implies a ‘subsidies’ from the union to firms whose productivity would exceed \( \theta \) in the form of \( w^{IS} < b \). In these states of the world, workers would be better off to take up their outside option. Thus, besides choosing the path of \( l \) and \( \pi \) (or \( w \) for that case) over \( \theta \), the union also chooses with which firm to strike an agreement and which to exclude (i.e. offering the null-contract). Those firms would then have to de-locate and produce abroad.

The marginal gain of including a firm with some productivity \( \theta \) is

\begin{equation}
g(\theta)((\theta l)^{1-\alpha} - \pi^{IS} - l^{IS} b), \tag{22}
\end{equation}

which reflects the production value net of the outside options (=opportunity costs) of both the union and the firm. With efficient employment, this gain is given by the difference between \( \hat{\pi}(\theta) \) and \( \hat{\pi}^F(\theta) \). Using the assumption concerning this difference for \( \theta = 1 \) (see before) and proposition 1, we arrive at the following proposition:

**Proposition 2** There exists some (unique) threshold productivity \( \theta^{IS} \), for which the marginal gain of including a firm is zero. Lower (Higher) productivity firms provide a positive (negative) marginal gain and hence are included in (excluded from) the contract. Moreover, this de-location threshold is the same as as under competitive labor markets, i.e. \( \theta^{IS} = \hat{\theta} \).

**Proof 2** For the first part note that the marginal gain can be rewritten to yield \( g(\theta)(K - \delta(\theta)) \). By assumption this is positive for \( \theta = 1 \) and decreasing in \( \theta \). For the second part note that the condition for \( \theta^{IS} \) is \( g(\theta^{IS})(K - \delta(\theta^{IS})) = 0 \) which is also true for \( \hat{\theta} \).
Summarizing we find that under information symmetry, unionization (in the form of offering wage-employment contracts to firms) has no allocative effect in the sense that employment and de-location remains efficient. Unionization solely increases the wage and shifts profits from the firm to workers, i.e. only has distributional effects. In this setting, unionization does not affect the fraction of firms that de-locate production.

3.3 Unionization – Information asymmetry

3.3.1 The union’s problem

Let us now refrain from the assumption that the union can perfectly observe the firm’s productivity after the contract has been set-up. Consider instead a situation in which this information is private to the firm. When constructing the wage-employment contract, the union anticipates the information friction and hence only constructs (deterministic) contracts which result in self-selection of the firm. The set of viable contracts is then smaller than before, because the union not only has to take participation into account, but also incentive compatibility. As derived in appendix A.1, the incentive compatibility constraint requires a specific form of the profit and of the employment path. Denoting the rent for the firm which is the difference between the profit of the firms and its outside option, i.e. the foreign profit of the firm, by $\Delta$, we have

$$
\frac{d\Delta}{d\theta} := \frac{\pi}{d\theta} - \frac{\pi^F}{d\theta} = (1 - \alpha)(\theta l)^{-\alpha} l - (1 - \alpha)(\theta l^F)^{-\alpha} l^F
$$

(23)

$$
\frac{dl}{d\theta} \geq 0.
$$

(24)

As already foreshadowed in the appendix, the model at hands is characterized by countervailing incentives (on the side of the firm). It is not clear whether a firm which is endowed with a certain productivity has an incentive to over- or understate its true productivity when being asked by the union. Consequently, when designing the optimal contract, the union does not know which firm to pay a rent (if any) to prevent it from not telling the truth.

The objective of the union is very similar to that under information symmetry. However, besides allowing for the participation constraint $\Delta \geq 0$, the union also needs to take the incentive compatibility constraint (23) into account. This restricts the path of $\Delta$ and for that case the path of $\pi$. Thus, when constructing a contract for some type of firm with productivity $\theta$, the union has to take the effect on the self-selection for all other firms into account as well.
Moreover, as was already foreshadowed by the information symmetry case, the union excludes firms from trade. The problem here, however, is that as long as we do not know the path of the information rent \( \Delta \) in equilibrium, we do not know the gain of including a firm into the contract. To circumvent these problems, we apply a two-step procedure. First, we solve for the equilibrium employment and information rent path as if all firms were included into the contract. Second, we derive which firms will be excluded from the contract. This approach is legitimate because exclusion is decided conditional on the optimality of employment and the information rent.

Finding these equilibrium paths requires the union to solve

\[
\mathbb{E}(V_{\text{Union}}) = \int_{1}^{\bar{\theta}} g(\theta)((\theta l)^{1-\alpha} - \Delta - \pi^F - lb) d\theta
\]

subject to \( \frac{d\Delta}{d\theta} \) and \( \Delta \geq 0 \). Note that we used the definition of \( \pi \) (as before) and that of \( \Delta \) to rewrite the objective of the union. We solve for the equilibrium a.) ignoring for the moment the constraint on form of the control path \( l(\theta) \) (the monotonicity constraint, see appendix A.1), verifying it ex-post and b.) employing dynamic optimization techniques identifying \( \Delta \) as the state and \( l \) as the control variable.

### 3.3.2 Solution (I): Contract

The Hamilton-Lagrange function for the full problem (i.e. including all firms) reads

\[
\mathcal{H} = g(\theta)((\theta l)^{1-\alpha} - \Delta - lb - \pi^F) + \lambda \left( (1 - \alpha)(\theta l)^{-\alpha} l - (1 - \alpha)(\theta l^F)^{-\alpha} \right)
\]

\[\mathcal{L} = \mathcal{H} + \mu \Delta.\]

where \( \lambda \) is the ‘inter-typical’ shadow value and \( \mu \) is the shadow value of the participation constraint (in the case it is binding). The first-order conditions for this problem are given by

\[
g(\theta)((1 - \alpha)(\theta l^{I\text{AS}})^{-\alpha} \theta - b) + \lambda^{I\text{AS}}(1 - \alpha)^2(\theta l^{I\text{AS}})^{-\alpha} = 0
\]

\[
\frac{d\lambda^{I\text{AS}}}{d\theta} = g(\theta) - \mu^{I\text{AS}}
\]

\[
\mu^{I\text{AS}} \Delta^{I\text{AS}} = 0
\]

where the superscript \( I\text{AS} \) denotes the values of the endogenous variables along the equilibrium path under information asymmetry. Moreover, the
problem is characterized by the transversality conditions that \( \lambda(1) = \lambda(\hat{\theta}) = 0 \) (which is implied by the fact that the union is free to choose the rent at the ‘corners’ of the productivity support).\(^{10}\)

We propose the following equilibrium (see appendix A.2 for details of the derivation). Let us define \( \hat{\theta} \) as the solution to

\[
g(\hat{\theta})\hat{\theta} \frac{b - b^F}{(1 - \alpha)b^F} = G(\hat{\theta}). \tag{31}
\]

Suppose that \( \hat{\theta} \notin [1, \bar{\theta}] \), the participation constraint is not binding and the equilibrium is then given by

\[
l^{\text{IAS}} = \left( \frac{g(\theta)(1 - \alpha)\theta^{1-\alpha} + G(\theta)(1 - \alpha)^2 \theta^{-\alpha}}{g(\theta)b} \right)^{\frac{1}{\alpha}} \Delta^{\text{IAS}} = \Delta^{\text{IAS}}(1) + \int_1^{\hat{\theta}} (1 - \alpha)(\hat{\theta}l^{\text{IAS}})^{-\alpha}l^{\text{IAS}} d\hat{\theta} - (\hat{\pi}^F(\theta) - \hat{\pi}^F(1))
\]

\[
\Delta^{\text{IAS}} = \frac{(\theta l^{\text{IAS}})^{1-\alpha} - \Delta^{\text{IAS}} - \hat{\pi}^F}{l^{\text{IAS}}} \mu^{\text{IAS}} = 0 \quad \lambda^{\text{IAS}} = G(\theta).
\]

Up to now we have ignored the monotonicity constraint \( \frac{d}{d\theta} \geq 0 \) such that truth-telling leads to firm’s profit maximum. To ensure that this constraint is fulfilled, we focus in the following on productivity distributions such that the ‘realization elasticity’ of the probability function \( \eta(\theta) := g(\theta)\frac{\theta}{G(\theta)} \) is decreasing in \( \theta \).\(^{11}\)

**Proposition 3** The informational friction results in a deviation of employment from its efficient level. The economy is characterized by overemployment. This is, however, not true for the least productive firm. Employment there is efficient.

**Proof 3** We can rewrite the expression for the equilibrium employment path and get

\[
l^{\text{IAS}} = \left( \hat{l}^\alpha + G(\theta)(1 - \alpha)\theta^{-\alpha}g(\theta)b \right)^{\frac{1}{\alpha}} \tag{32}
\]

which implies that \( l^{\text{IAS}} > \hat{l} \) for \( \theta > 1 \) and with \( G(1) = 0 \) that \( l^{\text{IAS}} = \hat{l} \).

\(^{10}\)Note that for any interior interval, these transversality conditions would be given by the continuity of the state variable \( \Delta \).

\(^{11}\)This restriction is in the same spirit as the monotone hazard assumption (i.e. that \( \frac{G(\theta)}{g(\theta)} \) is strictly de-/increasing depending on the type of model) that is usually put forward in standard textbook models of adverse selection.
The intuition for the overemployment result is that the requirement to employ a large workforce decreases the incentive to overstate productivity. Hence, the union can reduce information rent payments by setting a contract with overemployment. For the least productive firm, the incentive to overstate productivity is strongest. Thus, having this firm to tell the truth is most valuable which results into the highest information rent, but at the same time (to make up for this high payment) to efficient employment (i.e. the maximum size of the pie).

An important property of the equilibrium (which will become important when determining the threshold productivity for exclusion) is the fact that the wage $w^\text{IAS}$ falls short of $w^\text{IS}$ except again for the lowest productivity firm. Rewriting gives

$$w^\text{IAS} = b + \frac{\theta^\text{IAS} - b^\text{IAS} - \Delta^\text{IAS} - \tilde{\pi}^F}{l^\text{IAS}}$$

which is smaller than $w^\text{IS}$ due to two reasons. First of all (which is the obvious effect), the union has to pay an information rent, thus the firm accrues a larger part of the production value, leaving less to the union. Second, employment is inefficient, thus the size of the pie is smaller than under information symmetry. Bluntly speaking, the consequence of the information friction is that the union can only capture a smaller piece of a smaller pie by its wage demands.

So far we have discussed the equilibrium in which the participation constraint is not binding. In the situation in which a binding interval exists (i.e. a situation in which the contract offered by the union makes some firms indifferent between accepting the contract and de-locating production), the equilibrium over this interval $(\tilde{\theta}, \hat{\theta}]$ is characterized by\footnote{The equilibrium over the non-binding interval is obviously identical to the one that has been discussed above.}

$$l^\text{IAS} = \bar{F}$$
$$\Delta^\text{IAS} = 0$$
$$w^\text{IAS} = \bar{F} + \frac{K}{\bar{F}}$$
$$\chi^\text{IAS} = g(\theta)\theta \frac{b - b^F}{(1 - \alpha)b^F}$$
$$\mu^\text{IAS} = g(\theta) - \frac{d(g(\theta)\theta)}{d\theta} \frac{b - b^F}{(1 - \alpha)b^F}.$$  

Note that also over the binding interval, the contract wage $w^\text{IAS}$ is smaller than $w^\text{IS}$ (see, appendix A.3). The intuition is basically the same as that in the non-binding case except for that there is no information rent to be paid. Nevertheless, employment remains inefficient resulting in a smaller pie.
3.3.3 Solution (II): Exclusion

Having characterized the equilibrium contract conditional on the fact that all firms will be offered a contract, we can now turn to the exclusion decision of the union. The union offers only (non-null) contracts to those firms whose marginal gain of inclusion is positive. The marginal gain of including a firm (which has not been included in the contract previously) is given by (see, e.g. Seierstad and Sydsaeter (1987))

$$
g(\theta)(l(\theta)^{IAS}(w(\theta)^{IAS} - b)) + \lambda(\theta)^{IAS}\frac{d\Delta(\theta)^{IAS}}{d\theta}. \quad (34)
$$

The marginal gain is made up of two things. First, the direct gain is the production value minus the payment to the specific firm such that it participates and tells the truth (which is $w^{IAS}$) exceeds the opportunity costs of working. Second, there is an indirect effect because including a firm of some specific productivity $\theta$ also (by the incentive compatibility constraint) increases the payment that has to be made for all other firms under the contract. This argument leads to the following proposition:

**Proposition 4** The per-employee marginal value of including a firm is larger under symmetric information than under asymmetric information. Hence, the per-employee marginal value of including a firm is decreasing with $\theta$.

**Proof 4** Concerning the first part of the proposition, note that $w^{IS}$ is larger than $w^{IAS}$. Moreover, the effect on the incentive compatibility constraint is negative, because $\lambda^{IAS}\Delta^{IAS}(\theta) \leq 0$. For the second part of the proposition, note that $\frac{K - b(\theta)}{l(\theta)}$, which is the per-employee marginal gain of a firm under information symmetry, is decreasing.

The intuition for this proposition is that with asymmetric information, the union has to compensate the firm not only for giving up the opportunity of producing abroad but also for telling the truth. In addition, due to the dominating overstating incentive, including more productive firms into the contract makes the self-selection constraint more severe.

Proposition 4 is important because it sheds light on the effect of unionization on the de-location choice of firms which are summarized in the following corollary:

**Corollary 1** First, if $g(1)(l^{IAS}(1)(w^{IAS}(\theta) - b)) < 0$, the union excludes all firms from the contract. Then, the product is exclusively produced abroad. Second, for parameter vectors for which there is production at home, the productivity of the marginal firm that is just included $\theta^{IAS}$ (which is determined by $g(\theta^{IAS})(l^{IAS}(\theta^{IAS})(w^{IAS}(\theta^{IAS} - b)) + \lambda^{IAS}\Delta^{IAS}(\theta^{IAS} = 0)$ is smaller than $\hat{\theta}$. 

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In the situation in which de-location is only driven at the extensive margin (e.g. because domestic and foreign labor are perfect substitutes), unionization only affects the de-location threshold as long as we have asymmetric information. With symmetric information, the union would want to include any firm into the contract with positive quasi-rents (that can be captured). But these are positive for all firms that would choose to produce at home under competitive labor markets. Hence, the union has not effect along the extensive margin. Under information asymmetry, however, parts of the quasi-rents are protected from being captured by the union. Including firms is then less valuable. As a result, the union offers null-contracts to a larger fraction of firms, thus making de-location more likely.

3.3.4 A numerical example

What are the quantitative effects of the analysis provided so far? In other words, how much of the FDI can we explain by unionization, i.e. how large is the fraction of moving firms and what are the effects of employment at home? To answer these questions, we calibrate our model using data and information from the literature.

When it comes to the numerical solution of the model, we have to take a stand concerning the form and parameters of the distribution function of firms’ productivity. For robustness and to gain an insight how different assumption on this distribution impact the equilibrium, we pursue two different specifications. In the first one we employ insights from the structural estimation of a model with firm heterogeneity as put forward in Balistreri et al. (2011), who employ a Pareto distribution for their estimation of a Melitz type trade model. Applying these insights to our context (with an upper truncation point) we hence have

$$G(\theta) = 1 - \frac{1 - \theta^{-c}}{1 - (\theta)^{-c}},$$  \hspace{1cm} (35)

where the shape parameter is assumed to be $c = 4.5$ in accordance with the estimation of Balistreri et al. (2011). Given our equilibrium specification, the upper bound $\bar{\theta}$ has no allocation effects (i.e. the equilibrium remains unaffected). Since the choice of $\bar{\theta}$ only has quantitative effects, we are free to choose a value and arbitrarily set it to 4.

The second specification is based on insights that the size of firms (in terms of employment) in the US is Zipf distributed (i.e. Pareto distributed with shape parameter 1), see Axtell (2001). Arguing that the US economy is basically characterized by competitive labor markets, our model allows us to infer the form of the productivity distribution based on the employment

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distribution (for which we have data). Using equation (9) and the assumption that employment is Zipf distributed, we conclude that productivity is Pareto distributed\(^\text{13}\) (as in (35)) with shape parameter \(c = \frac{1 - \alpha}{\alpha}\) and \(\tilde{\theta} = 10^6 \pi_n\).\(^\text{14}\)

The parameter \(\alpha\) (i.e. the value of the inverse of the price elasticity of demand) measures the competitiveness of the industry under consideration and determines the size of the mark-up (over unit costs that the firm sets). This mark-up is given by \((1 - \alpha)^{-1}\). There is some variation in the literature concerning the size and the behavior of this mark-up (depending on data and the underlying production technology, see e.g. Rotemberg and Woodford (1999)). In the following, we assume a value of \(\alpha = 0.26\) which corresponds to a mark-up of 35%. The reason for this choice is a.) that it is reasonably close to what is assumed in the macroeconomic literature and b.) that \((1 - \alpha) = 0.74\) would depict labor's share if we had combined a linear technology with a convex production technology.

The cost of setting up a firm abroad \(K\) are specified following the quantitative analysis in Coşar et al. (2010) who find that the costs for setting up a firm are in the order of 25 times the (annual, competitive) wage which is the service sector wage in their case. Since we do not consider different industries or occupations, we assume this outside wage to be given by \(b_F\), hence we have \(K = 25b_F\).

The final assumptions concern the values for the opportunity costs of working at home and abroad, \(b\) and \(b_F\), respectively. To specify these values, we apply the following normalization approach. Concerning \(b_F\), we focus exclusively on a situation in which the lowest productivity firm just makes zero profits when de-locating its production technology abroad. Using (12) and the assumptions concerning \(K\) and \(\alpha\) then specifies the according value \(b_F\). In the theoretical model, we have assumed that all firms can earn positive profits when moving abroad. The reason for this assumption was that we did not want to interact the outside option of moving the technology with the outside option of stopping producing altogether (which would be the reasonable threat of a firm that has negative profit opportunities abroad). Conditional on this assumption, the specified \(b_F\) is then the upper bound of foreign opportunity costs, i.e. allowing for a strong effect of unionization. The opportunity costs of working at home \(b\) are just assumed to be a multiple of \(b_F\) where we consider some alternative values. The chosen parameter vector for the two specifications is summarized in Table 1.

\(^{13}\)See Casella and Berger (2002), Theorem 2.1.2 p. 51 for the argument that the distribution of productivity mirrors that of the distribution of employment.

\(^{14}\)The assumption concerning the truncation point of the Pareto distribution is based on the observation that the in US data the distance between the smallest and largest firm is of the order of \(10^6\). The assumption concerning \(\tilde{\theta}\) then generates this observed distance.
<table>
<thead>
<tr>
<th></th>
<th>Specification 1</th>
<th>Specification 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$ (Shape Parameter)</td>
<td>4.5</td>
<td>$\frac{1}{a}$=2.8</td>
</tr>
<tr>
<td>$\theta$ (Maximum productivity)</td>
<td>4</td>
<td>$10^{\frac{1}{\alpha}}=78.76$</td>
</tr>
<tr>
<td>$\alpha$ (Value of the inverse price elasticity)</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>$b^F$ (Opportunity costs of working – abroad)</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>$K$</td>
<td>$25 b^F=6$</td>
<td>$25 b^F=6$</td>
</tr>
<tr>
<td>$b$ (Opportunity costs of working – home)</td>
<td>$1.1 b^F$</td>
<td>$1.1 b^F$</td>
</tr>
<tr>
<td></td>
<td>$1.3 b^F$</td>
<td>$1.3 b^F$</td>
</tr>
<tr>
<td></td>
<td>$1.5 b^F$</td>
<td>$1.5 b^F$</td>
</tr>
</tbody>
</table>

Table 1: Parameter values used in calibration

The results shown in Table 2 depict the effect of unionization and information asymmetry on the fraction of firms producing at home and on aggregate employment in the competitive case and with unionization (denoted by $L$ and $L^{IAS}$, respectively) for both specifications. As already shown above, unionization decreases the fraction of firms that produce at home. The effects range from around 3% (for a very low opportunity cost advantage abroad) to as a high a value as 46% in the case in which unionization shuts down the industry. These magnitudes in the same ballpark range between these two specification. We, hence, can conclude the results are robust against changes in the (form of) the productivity distribution function. The important point here is that shutting down the industry will occur (at least in our specifications) even for very modest differences between the opportunity costs of working at home and abroad (in our case of only 50%). Moreover, this forced de-location is inefficient because production at home would have been occurred with competitive labor markets.

The second important point is that the aggregate employment effects of unionization are relatively modest (except obviously for the case in which the union completely shuts down the industry). For very small opportunity cost advantages, it even turns out that the firm-level overemployment effect even survives at the aggregate level resulting in too high employment. Thus, when considering the impact of unionization, focusing on employment gives a biased picture on the allocation effect because the effects on de-location (and hence de-industrialization) has to be taken into account, too.
4 Conclusion

In this paper, we analyze the effects of unionization on the de-location of firms when information frictions are present. To model this friction, we assume that firms have superior information regarding their productivities. When constructing a wage-employment contract, the union then has to ensure the self-selection of firms into ‘their right’ contract, i.e. firms truthfully signal their productivity to the union. In doing so, the union must pay an information rent to the firm if it decides to include this type of firm into the contract. However, due to the open economy framework in which firms have the outside option to shift their production facilities abroad, there are countervailing incentives for the firms when deciding about their productivity announcement. On the one hand, firms would like to understate their true productivity to get a more favorable contract. On the other hand, firms have an incentive to overstate their true productivity since this signals a higher outside option.

In the equilibrium, we can show that a.) the overstating incentive dominates. Hence, low-productivity firms receive the highest information rent payment from the union. b.) We also find that employment is inefficient large with the exception of the firm with the lowest productivity (no distortion at the bottom). Intuitively, the union can save information rent payments since the requirement to employ a large workforce decreases the incentive to overstate productivity. c.) As our main finding, we show that the union excludes firms from the contract and forces them to de-locate production which would produce at home under information symmetry or perfect competitive labor markets. Hence, unionization leads to a higher share of
FDI measured by de-location. The reason for this is that excluding firms narrows the possibility of overstating productivities for the remaining firms; the union can thereby save information rent payments.

In future research, two extensions of the model presented here are important to show how robust the aforementioned results are. First, the open economy setting with information frictions between firms and unions should be incorporated in a general equilibrium approach, e.g. in the spirit of Melitz (2003). Second, it should also be looked at the case where unions have to compete for firms – similar to the tax competition literature.

A Appendix

A.1 Incentive compatibility

Consider the problem of the firm reporting its productivity to the union. True productivity is \( \theta \) and announced productivity is \( \theta' \). The union has (at the first stage) designed a contract that is conditioned on the productivity announcement of the firm. When choosing \( \theta' \), the objective of the firm is to maximize the rent (i.e. the difference between the profit under the contract and the foreign profit). This is given by

\[
\Delta(\theta, \theta') := \pi(\theta, \theta') - \tilde{\pi}^F(\theta) = (\theta l(\theta'))^{1-\alpha} - w(\theta')l(\theta') - \tilde{\pi}^F(\theta).
\]  

(36)

In order to understand the incentives of the firm, consider for the moment a naive union that offers a contract as if it could observe the productivity (see section 3.2). The rent can be written as

\[
\Delta(\theta, \theta') := \pi(\theta, \theta') - \tilde{\pi}^F(\theta) = (\theta l(\theta'))^{1-\alpha} - (\theta' l(\theta'))^{1-\alpha} + \tilde{\pi}^F(\theta) - \tilde{\pi}^F(\theta').
\]  

(37)

If the firm tells the truth (under the naive contract) the rent obviously will be zero. It is, however, not clear whether the firm would then have an incentive to overstate \( (\theta' > \theta) \) or understate \( (\theta' < \theta) \) its productivity. This is a variant of the classic Lewis and Sappington (1989) case. In our example, the countervailing incentives are driven by the fact that overstating its productivity, the firm is faced by a more favorable contract because of the better outside option. If the profit when de-locating production was not a function of the firms productivity (i.e. if \( \tilde{\pi}^F(\theta') - \tilde{\pi}^F(\theta) = \tilde{\pi}^F \)), then the firm obviously would have a (generic) incentive to understate its productivity because the union offered for this case a more attractive contract.

Turning now to the restriction the union contract have to obey to ensure truth-telling by the firm. The optimal productivity announcement is
implicitly given by the first-order condition

\[ (1 - \alpha)(\theta l(\theta'))^{-\alpha}\theta \frac{dl(\theta')}{d\theta'} - w(\theta') \frac{dl(\theta')}{d\theta'} - \frac{dw(\theta')}{d\theta'} l(\theta') = 0 \]  

(38)

which gives \( \theta' \) as a function of \( \theta \) where this relation \( \theta'(\theta) \) is shaped by the form of the contract. Let us consider contracts only such that telling the truth is optimal (i.e. self selection occurs). Thus, we only consider contracts such that \( \theta'(\theta) = \theta \). (38) restricts the form of the wage-employment contract. Differentiating this with respect to \( \theta \) (under truth telling) we find that

\[ soc + (1 - \alpha)^2 (\theta l(\theta'))^{-\alpha} \frac{dl(\theta')}{d\theta'} = 0 \]  

(39)

where \( soc \) denotes the second order condition for the problem. With the optimal \( \theta' \) resulting in a maximum, it must be true that the optimal contract is such that \( \frac{dl(\theta')}{d\theta'} \geq 0 \) which is the monotonicity constraint.

Moreover, in a truthtelling equilibrium the change in the rent over the different productivities is restricted to be

\[ \frac{d\Delta}{d\theta} = (1 - \alpha)(\theta l)^{-\alpha} l - (1 - \alpha)(\theta l F)^{-\alpha} l F. \]  

(40)

\( \Delta \) paths that obey this 'slope' restriction imply that firms truthfully reveal their type and hence self-select into 'their' contracts. This is the second restriction that the union has to take into account when designing the contract.

A.2 Equilibrium derivation under information asymmetry

Finding the equilibrium under countervailing incentives is a bit tricky, because without knowing the equilibrium path of the information rent (which is in standard problems unambiguously determined by the incentive compatibility constraint) it is ad-hoc unclear where (and if) the participation constraint is binding.\(^{15}\) In finding the equilibrium, we rely on an approach suggested by Maggi and Rodriguez-Clare (1995). The idea here is very simple. First, assume that the participation constraint was binding over the complete support of \( \theta \). In our case, this implies that

\[ \Delta = 0 \Rightarrow \frac{d\Delta}{d\theta} = 0 \Rightarrow l^{tAS} = l^{F}. \]  

(41)

\(^{15}\)In standard problems, this constraint is only binding at one corner of the support of \( \theta \).
Over any binding interval, equilibrium contract employment will be identical to (equilibrium) employment when de-locating the firm. Using this the resulting 'intertypal' shadow value (over a binding interval is given by)

\[ \bar{\lambda}^{I_{A_S}}(\theta) = -g(\theta) \frac{b^F - b}{(1 - \alpha)\theta^{-1}b^F} = g(\theta)\theta^b - b^F > 0 \]  

(42)

where we used the fact that over the binding interval it is true that \((1 - \alpha)\theta b^F = b^F\) (by the optimality condition under de-location). Note that in general along an optimal employment path it must be true that (rewriting (28))

\[ t^{I_{A_S}} = \left( \frac{g(\theta)(1 - \alpha)\theta^{1-\alpha} + \lambda^{I_{A_S}}(1 - \alpha)^2\theta^{-\alpha}}{g(\theta)b} \right)^{\frac{1}{\alpha}} \]  

(43)

which implies that equilibrium employment is increasing in \(\lambda^{I_{A_S}}\). Thus, for all paths \(\lambda^{I_{A_S}} > (\lambda^{I_{A_S}})\) it is true that employment must be larger (smaller) than \(t^F\). Because over an interval over which the participation constraint is binding it must be true that \(\frac{d\Delta}{dt} = 0\) and that \(\frac{d^2\Delta}{dt^2} > 0\), we find that those \(\lambda\) paths imply \(\frac{d\Delta}{dt} > (\lambda^{I_{A_S}})\).

Consider the case in which the participation constraint was not binding and \(\frac{d\Delta}{dt} < 0\) throughout the support of \(\theta\). Using the transversality conditions, this implies that \(\lambda(1) = 0\) such that by (29) the path for \(\lambda^{I_{A_S}}\) is given by

\[ \lambda^{I_{A_S}}(\theta) = G(\theta) \geq 0 \]  

(44)

where \(G(\theta)\) denotes the distribution function associated with the density \(g(\theta)\). To get some more structure, let us impose the following assumption:

**Assumption 1** Over the support of \(\theta\) we focus on situations in which \(\frac{d\lambda^{I_{A_S}}}{d\theta} := \frac{d\lambda^{I_{A_S}}}{d\theta} b - b^F}{(1 - \alpha)\theta^\alpha} \leq (\geq) g(\theta) := \frac{d\lambda^{I_{A_S}}}{d\theta} .\)

Using this assumption we can prove that

**Lemma 1** The intercept between \(\bar{\lambda}^{I_{A_S}}(\theta)\) and \(\lambda^{I_{A_S}}(\theta)\) exists and is unique or does not exist over the support of \(\theta\). A necessary condition for \(\hat{\theta} \in [1, \hat{\theta}]\) is that \(\frac{d\lambda^{I_{A_S}}}{d\theta} b - b^F}{(1 - \alpha)\theta^\alpha} \leq g(\theta)\). Moreover, with \(\hat{\theta} \in [1, \hat{\theta}]\), we have that \(\bar{\lambda}^{I_{A_S}}(\theta) > (\lambda^{I_{A_S}}(\theta)\) if \(\theta < (\hat{\theta})\).

**Proof 5** At \(\theta = 1\), we have \(\bar{\lambda}^{I_{A_S}}(1) > \lambda^{I_{A_S}}(1) = 0\). Combining this with assumption 1 proves the lemma.
This implies that under assumption 1 either no binding interval (i.e. \( \mu = 0 \)) or a binding interval over the upper part of the support of \( \theta \) exists. The assumption ensures that \( \mu \) is non-negative. By (29), we have that \( \mu = g(\theta) - \frac{d\lambda_{\text{IAS}}}{d\theta} \).

The equilibrium path of \( \lambda \) (which then determines the equilibrium paths of \( l \) and \( \Delta \)) is given by

\[
\lambda_{\text{IAS}}^I(\theta) = \frac{G(\theta)}{\theta} \quad \theta \in [\tilde{\theta}, \hat{\theta}]
\]

\[
\lambda_{\text{IAS}}^A(\theta) = \begin{cases} 
G(\theta) & \theta \in [\tilde{\theta}, \hat{\theta}] \\
g(\theta)\theta \frac{b-l^F}{(1-\alpha)b} & \theta \in (\hat{\theta}, \tilde{\theta}].
\end{cases}
\]

### A.3 \( w_{\text{IAS}} \) over the binding interval and \( w_{\text{IS}} \)

Using the expression for the wage over the binding interval, we can write

\[
w_{\text{IAS}} = \frac{bF \tilde{l}^F + K}{\tilde{l}^F} \quad (47)
\]

\[
\Leftrightarrow w_{\text{IAS}} = \frac{bF \tilde{l}^F + K - (\theta \tilde{l}^F)^\alpha}{\tilde{l}^F} + \frac{(\theta \tilde{l}^F)^\alpha}{\tilde{l}^F} \quad (48)
\]

\[
\Leftrightarrow w_{\text{IAS}} = \frac{\tilde{\pi} - \tilde{\pi}^F}{\tilde{l}^F} + \frac{(\theta \tilde{l}^F)^\alpha}{\tilde{l}^F} - \frac{\tilde{\pi}}{\tilde{l}^F} \quad (49)
\]

\[
\Leftrightarrow w_{\text{IAS}} = \frac{\tilde{\pi} - \tilde{\pi}^F}{\tilde{l}^F} + \frac{(\theta \tilde{l}^F)^\alpha}{\tilde{l}^F} - \frac{\tilde{\pi}}{\tilde{l}^F} + w_{\text{IS}} - \frac{\tilde{\pi} - \tilde{\pi}^F}{\tilde{l}} - b \quad (50)
\]

\[
\Leftrightarrow w_{\text{IAS}} = w_{\text{IS}} + \frac{\tilde{\pi} - \tilde{\pi}^F}{\tilde{l}^F} - \frac{\tilde{\pi} - \tilde{\pi}^F}{\tilde{l}} + \frac{(\theta \tilde{l}^F)^\alpha - b\tilde{l}^F - \tilde{\pi}}{\tilde{l}^F} \quad (51)
\]

which implies that because \( l < \tilde{l}^F \) and the profit \( \tilde{\pi} \) is a maximum, that \( w_{\text{IAS}} < w_{\text{IS}} \) over the binding interval.
References


