Credit constraints, endogenous innovations, and price setting in international trade*

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Abstract

We introduce credit frictions in a general equilibrium model of international trade with two dimensions of heterogeneity and endogenous investments. Firms’ competitiveness consists of capabilities to conduct process and quality innovations at low costs, whereas investment outlays have to be financed by external investors. Motivated by Holmstrom & Tirole (1997), moral hazard induces credit rationing in equilibrium. Only the most capable producers have access to finance and export, whereas low capability firms fail to raise external capital and exit. Consistent with recent empirical evidence, we show that increases in trade and credit costs are associated with higher (lower) firm-level prices if the scope for vertical product differentiation is low (high). These effects are driven by within-firm adjustments. Faced with cost shocks, producers re-optimize innovation choices resulting in opposing quality and cost effects on marginal production costs and prices. In general equilibrium, both adjustments along intensive and extensive margins contribute to the negative welfare effect of credit tightening.

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1 Introduction

A growing empirical literature stresses the negative impact of credit constraints on firms’ export behavior. Exporting usually requires additional upfront costs for investments in marketing, capacity, product customization or distribution networks. Thereby, transportation leads to longer time lags between investment outlays and profit realization. Consequently, empirical studies using firm-level data show that credit rationing decreases firm-level exports and reduces the probability of serving foreign markets (Minetti & Zhu, 2011; Berman & Héricourt, 2010). Negative effects of credit constraints on both intensive and extensive margins of international trade are rationalized by recent theoretical trade models that combine fixed upfront costs and firm heterogeneity as in Melitz (2003) with capital market imperfections. If firms require external finance for fixed trade costs, Manova (2013) shows that credit frictions restrict export market entry of low productivity firms, whereas variable cost financing by outside capital distorts firm-level exports as well.

Besides intensified productivity sorting, credit constraints and leverage negatively affect exporters’ choice of product quality (Fan et al., 2013a; Bernini et al., 2013). Recent firm-level studies document competition through quality on foreign markets and point to positive relations of prices with firm size as well as positive correlations between trade costs and average FOB prices (Baldwin & Harrigan, 2011; Johnson, 2012; Kugler & Verhoogen, 2012). Accordingly, successful exporters sell higher-quality products at higher prices in more distant markets. In contrast, cost-based productivity sorting à la Melitz (2003) suggests a negative relationship between prices and firm size (Roberts & Supina, 1996; Foster et al., 2008) as well as a negative correlation between average unit values and market distance.

The purpose of this paper is to analyze the effects of credit frictions on endogenous innovation choices, price setting and selection into exporting when both cost-based productivity and product quality determine the competitiveness of a producer. Therefore, we develop a general equilibrium model of international trade with credit constraints, two sources of firm heterogeneity and endogenous sunk costs. To allow for both cost-based and quality-based sorting, producers differ in capabilities to conduct process and quality innovations. Investments are associated with endogenous sunk costs that decrease in firm-specific capabilities. In contrast to standard models with monopolistic competition, innovation choices endogenously determine marginal production costs. Depending on their capabilities, firms choose different investment levels and prices. Process innovations decrease marginal costs and hence increase the cost-based productivity of a firm for any given quality level. Whereas

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this channel is closely related to productivity sorting in Melitz (2003), the second type of investment is motivated by the recent quality and trade literature. Quality innovations shift demand up for any given price at the expense of higher marginal production costs. The two innovation types are complements as they increase price-adjusted quality and thus firm profits. Firms have to raise external capital for fixed and endogenous investment costs, whereas variable production costs can be financed by internal funds. Based on Holmstrom & Tirole (1997), we motivate capital market frictions by ex-post moral hazard that leads to credit rationing beyond profitability requirements. In equilibrium, only the most capable firms overcome financial frictions and become exporters, whereas some low capability producers with profitable investment projects fail to borrow external capital and exit the market.

Consistent with recent empirical evidence, our model rationalizes positive as well as negative relations of firm-level FOB prices with trade as well as credit costs, depending on the role of quality differentiation in a sector. For Italian firm-level data, Secchi et al. (2013) show that credit constraints and prices are positively related, whereas this relationship disappears in sectors with high vertical product differentiation.² Fan et al. (2013b) arrive at similar conclusions for trade liberalization. Accordingly, tariff reductions induce quality upgrading that is associated with higher prices in highly differentiated sectors, but with lower prices if the scope for vertical product differentiation is small. Furthermore, Fan et al. (2013a) find evidence for negative effects of financial frictions on firms’ quality choice. In our model, tighter credit conditions negatively affect both types of innovation and trigger quality as well as cost effects that influence marginal production costs and thus optimal prices in opposite directions. Lower process innovations push up marginal costs (cost effect), whereas reduced product quality decreases prices (quality effect). Whether quality or cost effects dominate depends on the relative scope for vertical product differentiation in a sector, measured as the ratio of expenditures associated with product upgrades relative to investment outlays for processes. Theoretically, this ratio is independent of firm capabilities and determined by exogenous technology parameters. Empirically, the measure is closely related to sectoral proxies of vertical differentiation used in firm-level studies.³ If the scope for product differentiation is high (low), the quality (cost) effect dominates and tighter credit conditions are associated with lower (higher) firm-level prices.

In the general setup with two dimensions of firm heterogeneity both opposing effects are present. Additionally, our model nests pure cost-based sorting à la Melitz (2003) and pure quality-based sorting as special cases, when relative investment costs for one type of

²Comparable to our theoretical analysis, Secchi et al. (2013) follow Kugler & Verhoogen (2012) and measure the scope for vertical product differentiation as the ratio of advertising and R&D expenditures to total sales in U.S. industries.
innovation are prohibitively high. In a Melitz-type economy only cost effects occur resulting in a positive relationship between credit costs and prices, whereas the opposite is true if only quality differentiation matters.

Besides within-firm adjustments, selection of producers into exporting negatively depends on trade as well as credit costs and on access barriers due to imperfect capital markets. Consistent with previous empirical and theoretical work, our model suggests that capital frictions impede access to foreign markets (e.g. Manova, 2013). More serious moral hazard problems and higher credit costs negatively affect the extensive margin of international trade. In general equilibrium, both within-firm adjustments and selection effects along the extensive margin contribute to the negative welfare response to credit tightening.

The paper is organized as follows. The next section reviews related theoretical literature. Section 3 sets up the model and derives optimal firm behavior. In chapter 4, we analyze within-firm adjustments triggered by credit cost shocks. The following two sections present the general equilibrium and discuss welfare implications of adjustments along intensive and extensive margins. Part 7 analyzes the effects of trade liberalization and section 8 concludes.

2 Related theoretical literature

To rationalize positive as well as negative correlations of firm-level prices with trade costs and credit frictions, Fan et al. (2013b) extend Melitz (2003) by quality upgrading, whereas Fan et al. (2013a) build on Arkolakis (2010) as well as Manova (2013) and differentiate between exogenous and endogenous quality. If quality is exogenous, the model predicts a positive relationship between credit constraints and prices, but a negative relation in case of endogenous quality choice. In contrast, our model considers two types of endogenous investments and explains both negative and positive correlations of credit and trade costs with firm-level prices through reoptimization of process and quality innovations inducing opposing cost and quality effects. Most closely related to our theoretical setup with two dimensions of heterogeneity, Hallak & Sivadasan (2013) and Sutton (2007) develop two-attribute firm models of international trade with endogenous sunk costs. Besides Melitz-type productivity, Hallak & Sivadasan (2013) allow producers to differ in their ability to develop high-quality products at low fixed outlays. We additionally consider endogenous process investments and introduce imperfect capital markets. Whereas our framework is based on monopolistic competition, Sutton (2007) considers Cournot competition and non-CES preferences and thus allows only for vertical product differentiation but neglects horizontal differentiation. Similar to both models, cost-based and quality-based capabilities jointly determine the competitiveness of a firm and are summarized in a one-dimensional productivity measure related to Melitz (2003).
Additionally, this work is related to papers that incorporate investment decisions in heterogeneous firm models. Bustos (2011), Lileeva & Trefler (2010) as well as Yeaple (2005) allow for process innovations that reduce marginal production costs. Consistent with our framework, these models predict that trade liberalization increases the incentive of technology upgrading. With respect to quality innovations, we build on recent efforts to extend international trade models by quality sorting (Baldwin & Harrigan, 2011; Johnson, 2012) as well as endogenous quality and input choices (Ferguson, 2012; Kugler & Verhoogen, 2012).

Furthermore, this paper is related to a growing literature on financial frictions and international trade with heterogeneous firms. These models are mainly based on productivity sorting à la Melitz (2003) and focus on financial constraints of exporters. In contrast, we assume that domestic as well as international sellers face credit frictions concerning endogenous innovation choices in processes and quality. Manova (2013) considers external financing of fixed and variable export costs and motivates credit constraints by imperfect financial contractibility. By introducing liquidity as a second source of heterogeneity, Chaney (2013) and Suwantaradon (2012) break up the the one-to-one relationship between productivity and firm success in the presence of credit constraints. Accordingly, low liquidity prevents firms from exporting even if they are highly productive. While we assume that fixed production costs and endogenous innovations have to be financed by external capital, these approaches stress the role of internal funds for financing of fixed export costs (Chaney, 2013) and capital inputs (Suwantaradon, 2012). Feenstra et al. (2011) introduce financial frictions by information asymmetry between firms and a monopolistic bank, whereas the latter cannot observe the productivity of the former. Instead, we assume perfect competition in the financial sector and symmetric information, but ex-post moral hazard motivated by Holmstrom & Tirole (1997) introduces financial frictions.

3 Setup of the model

Our model departs from Melitz (2003) by allowing for two sources of firm heterogeneity. Producers differ in their capabilities to conduct both process and quality innovations. Firm-specific capabilities determine the endogenous sunk costs associated with investments, such that higher capability producers are able to innovate at lower costs. These endogenous investment choices affect marginal production costs and hence prices in opposite directions. Process innovations increase productivity for any given quality level, whereas quality innovations raise demand and marginal costs. To analyze the impact of credit costs on innovation and optimal price setting, we assume that endogenous investments and fixed production outlays have to be financed by outside capital, whereby variable production costs can be financed by internal funds. Motivated by Holmstrom & Tirole (1997), the relationship between borrowers and investors is characterized by ex-post moral hazard which leads to credit rationing in equilibrium. Firms endogenously choose the optimal levels of process and quality innovations dependent on their individual capabilities and credit costs. Changes in borrowing costs lead to within-firm adjustments, as producers reoptimize both process and quality innovations. The following subsections discuss the demand and production side of the model.

3.1 Consumers

There are two symmetric countries with population of size $L$ trading in differentiated varieties. Labor is the only factor of production and the nominal wage is normalized to one. Preferences of a representative consumer in each country are characterized by a CES utility function over a continuum of goods indexed by $i \in \Omega$:

$$Q = \left[ \int_{i \in \Omega} (\lambda_i q_i)^{\frac{\sigma-1}{\sigma}} \, di \right]^{\frac{\sigma}{\sigma-1}},$$

(1)

where $\sigma > 1$ is the elasticity of substitution and $\lambda_i$ denotes the quality of a product. The quality-adjusted price index is defined as:

$$P = \left[ \int_{i \in \Omega} \left( \frac{p_i}{\lambda_i} \right)^{1-\sigma} \, di \right]^{\frac{1}{1-\sigma}}.$$

(2)

From the consumer’s maximization problem follows that demand for one differentiated variety $i$ increases in quality and decreases in the price:

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$^4$We denote expenditures financed by outside investors as capital. Note, however, that costs of firms associated with production and innovation choices are measured in labor units.
\[ q_i = \lambda_i^{\sigma-1} Q \left( \frac{P_i}{P} \right)^{-\sigma}. \]  

(3)

By introducing a quality component in the utility function of the representative consumer (1), we follow the recent quality and trade literature.\(^5\) Product quality \(\lambda_i\) is chosen endogenously by producers and shifts demand outwards for any given price. Besides that, firms decide on the optimal level of process innovations. The next section describes production and investment choices under credit constraints.

### 3.2 Production and investment under credit constraints

The production sector of the economy is characterized by monopolistic competition. Each firm manufactures one differentiated variety \(i\) and decides on process and quality innovations that both are associated with endogenous sunk costs increasing in investment levels: \(^6\)

\[ f(\lambda) = \frac{1}{\kappa} \lambda^a; \quad h(e) = \frac{1}{\varphi} e^c. \]  

(4)

Parameters \(a\) and \(c\) determine the convexity of the investment cost functions and are exogenously given for producers. Hence, \(\frac{1}{a}\) and \(\frac{1}{c}\) reflect the elasticities of quality and processes to innovation outlays. Low values of \(a\) and \(c\) imply that one additional unit of investment spending is very effective.\(^7\) Firms, however, differ in their capabilities to invest in process innovations \(\varphi\) and quality upgrades \(\kappa\). Higher values of these firm-specific draws scale down investment costs and hence increase incentives to innovate. Process and quality investments affect marginal costs in opposite directions. The benefit of process innovations \(\varphi\) is a reduction of marginal production costs \(mc\), whereas quality innovations \(\lambda\) increase demand for one variety (3) as well as marginal costs:

\[ mc(\lambda, e) = \frac{\lambda^\theta}{e} \text{ with } 0 < \theta < 1, \]  

(5)

where \(\theta\) describes the sensitivity of marginal costs to changes in quality. Hence, the level of process innovations is closely related to the productivity draw in Melitz (2003). However, cost-based productivity is an endogenous outcome of optimal investment decisions in our model. The notion that quality and marginal production costs are positively related, is motivated by the recent quality and trade literature. Baldwin & Harrigan (2011) introduce


\(^6\)For notational simplicity we drop the firm’s index \(i\) in what follows.

\(^7\)The elasticities of quality and processes with respect to innovation expenditures are given by: \(\frac{\partial \lambda}{\partial f(\lambda)} \cdot f(\lambda) = \frac{1}{a}\); \(\frac{\partial e}{\partial h(e)} \cdot h(e) = \frac{1}{c}\). See Sutton (2012), section 1.10, for a comparable specification of quality outlays. In section 3.3, we impose a convexity assumption for technology parameters \(a\) and \(c\).
a reduced-form relationship between firm’s productivity and quality to explain the positive correlation between distance and export prices found in empirical firm-level studies. As in our model, other papers endogenize firm’s quality choice and assume a positive relationship between quality and marginal production costs according to (5).\textsuperscript{8} Whereas Melitz (2003) suggests productivity sorting such that firms with lowest marginal costs and hence lowest prices overcome fixed and variable trade costs to serve foreign markets, this approach stresses that successful exporters use higher-quality inputs and sell higher-quality varieties at higher prices. By allowing both for cost-based and quality-based sorting with endogenous sunk costs, our model is closely related to Sutton (2007, 2012) and Hallak & Sivadasan (2013). The latter allow for endogenous quality and introduce quality-based capability besides the common Melitz-type productivity. We additionally endogenize investment decisions for processes and analyze the effect of credit conditions on optimal firm behavior. Therefore, we assume that firms have to raise outside capital to cover fixed production costs and expenditures associated with endogenous innovations (4) before revenues are realized, whereas variable production costs can be financed internally. The decision problem of a single firm consists of four stages:

1. Entry stage. A potential producer of a single differentiated variety decides to enter the market and pays a fixed entry cost $f_e$. After entry, the entrepreneur draws both investment capabilities $\varphi$ and $\kappa$ from a joint probability distribution $g(\varphi, \kappa)$.

2. Financial contracting and investment. Producers choose the optimal levels of process and quality innovations and sign a contract with an outside investor to cover fixed production and investment costs. Optimal prices are set.

3. Ex-post moral hazard. After financial contracting, the agent in the firm can choose to conduct the project diligently or to misbehave and reap a non-verifiable private benefit.

4. Production and profit realization. If the entrepreneur shirks, no profit is realized and the outside investor receives no loan repayment. If the project is conducted well, the firm produces, profits are realized and the loan is repaid to the lender.

Stages 2 and 3 introduce financial frictions and endogenous investment choices. Based on Holmstrom & Tirole (1997), the borrower-investor relationship is characterized by ex-post moral hazard. The optimal contract between a firm and an outside investor specifies the loan size $d_l > 0$ at a gross interest rate $\beta > 1$ and the credit repayment $k_l$, whereas the index $l \in d, x$ denotes domestically active firms ($d$) and exporters ($x$). We solve the model

\textsuperscript{8}See Kugler & Verhoogen (2012) or Johnson (2012), among others.
by backward induction. The next section describes firm’s maximization problem at stages 2 and 3, after the firm has already entered and drawn the two capabilities $\varphi$ and $\kappa$.

### 3.3 Optimal firm behavior

After entry, firms know their cost-based ($\varphi$) and quality-based capabilities ($\kappa$), decide on optimal innovations in processes and quality and set optimal prices. Therefore, firms maximize profits that consist of revenues net of variable production costs and loan repayment $k_l$:

$$\max_{p_l, e_l, \lambda_l} \pi_l = r_d + I_x r_x - mc(\lambda_l, e_l) (q_d + I_x \tau q_x) - k_l,$$

where the variable $I_x$ takes a value of 1 if the firm exports and zero otherwise. Revenues are defined as $r_l = p_l q_l$, whereas demand is given by equation (3). Marginal costs depend on endogenous choices of quality and process innovations as shown by (5) and can be financed internally. Exporting firms can additionally sell their products to consumers in an identical foreign country. Assuming symmetry implies that aggregate variables are the same for both regions. Exporting involves additional fixed costs $f_x$ and iceberg-type transportation costs such that $\tau > 1$ units of a good have to be shipped for 1 unit to arrive. Hence, international activity leads to additional revenues as well as higher variable production costs. Firms maximize (6) to solve for optimal levels of process ($e_l$) and quality innovations ($\lambda_l$) as well as optimal price setting. Investments are non-separable between the two markets. Hence, if a firm exports, the benefits of decreases in marginal costs and increases in product quality are spread across sales in both destinations. However, international firms set different prices in export markets due to transportation costs. Depending on their export status $l \in d, x$, firms face the following constraints:

$$d_l \geq f_d + I_x f_x + \frac{1}{\varphi} e_l^p + \frac{1}{\kappa} \lambda_l^p,$$  \hspace{1cm} (7)

$$k_l \geq \beta d_l,$$  \hspace{1cm} (8)

$$\pi_l \geq 0.$$  \hspace{1cm} (9)

The budget constraint (7) states that the received credit amount has to be sufficiently high to cover fixed production and endogenous investment costs. Internationally active firms have to raise additional external capital for export fixed costs. Investors only participate in a contract if loan repayments at least compensate for credit provision (8). Additionally, the firm will only be active in the market if profits are non-negative (9).
Motivated by Holmstrom & Tirole (1997), we introduce ex-post moral hazard which induces credit rationing in equilibrium. After financial contracting and loan provision, the agent in the firm has discretionary power regarding the use of external funds. There are no information asymmetries with respect to firm characteristics, but the success of investment projects depends on the agent’s behavior that is non-verifiable and hence non-contractible for external investors.\(^9\) On the one hand, the agent can decide to behave diligently and conduct the project properly. In this case, borrowed funds are used for production and investment, profits are realized according to (6) and the loan is repaid to the investor as specified in the contract. On the other hand, if the agent chooses to misbehave, no profits and loan repayment occur, but the borrower can reap non-verifiable private benefits \(b > 0\), that are assumed to be proportional to fixed production costs. Thus, the optimal contract has to satisfy an additional incentive compatibility constraint:\(^{10}\)

\[
\pi_{u,t} \geq b (f_d + I_x f_x).
\]

According to (10), net profits of conducting the project properly have to be at least as high as private benefits in case of misbehavior. This non-verifiable project choice might reflect shirking problems or managerial incentives to divert resources for own perks at the expense of project’s success. Hence, the private benefit \(b\) in condition (10) can be seen as share of the loan size, that the agent can enjoy in case of shirking or by diverting resources. For analytical simplicity, we assume that private benefits \(b\) are only related to fixed production costs and not to the total credit amount \(d_t\), including endogenous investments. The model can be easily extended by assuming that total investment costs are prone to moral hazard, which would be more consistent with the idea of diversion problems.\(^{11}\) This alternative specification reinforces restrictions imposed by incentive compatibility, without changing the qualitative implications of the model. Both specifications follow the idea that moral hazard is proportional to loan size.\(^{12}\) For international firms, private benefits are related to the sum of domestic and export fixed costs. Hence, exporters face a trade-off between additional profits from international activity in case of diligent behavior and the prospect of higher perks in case of misbehavior. To solve for optimal investment and pricing decisions under credit constraints, firms maximize (6) by taking into account conditions (7)-(9) as well

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\(^9\)Note that Holmstrom & Tirole (1997) consider differences in wealth, whereas in our model innovation capabilities determine access to external capital. Hence, we neglect the role of internal liquidity to overcome credit frictions as analyzed by Chaney (2013). See the discussion on firm selection in section 3.4.

\(^{10}\)Holmstrom & Tirole (1997) assume that misbehaviour reduces projects’ success probability. For simplicity, we assume that success probabilities equal 1 in case of diligent behavior and are 0 in case of shirking.

\(^{11}\)Appendix 9.2 discusses the model’s implications if the private benefit is proportional to total loan size.

\(^{12}\)The assumption that private benefits of entrepreneurs are proportional to investment is common in the corporate finance literature. See Holmstrom and Tirole (1997) as well as Tirole (2006), among others.
as incentive compatibility (10). In equilibrium, the firm’s budget constraint (7) as well as investor’s participation condition (8) have to bind. Optimal behavior of firms is characterized in two steps. Initially, investment choices and price setting are derived. Thereby, we neglect the zero-profit condition (9) and incentive compatibility (10), that determine access to finance and the selection pattern of firms in a second step. Solving firm’s maximization problem conditional on access to finance leads to the following optimal choices of process and quality innovations:\[13\]

\[e_l(\varphi, \kappa) = \left(\frac{A_l}{\beta}\right)^{\frac{a}{\gamma}} \left(\frac{1 - \theta}{a} \kappa\right)^{\frac{(\sigma - 1)(1 - \theta)}{\gamma}} \left(\frac{\varphi}{c}\right)^{\frac{\sigma + (1 - \theta)(1 - \sigma)}{\gamma}}, \tag{11}\]

\[\lambda_l(\varphi, \kappa) = \left(\frac{A_l}{\beta}\right)^{\frac{c}{\gamma}} \left(\frac{1 - \theta}{a} \kappa\right)^{\frac{\sigma + 1 - \alpha}{\gamma}} \left(\frac{\varphi}{c}\right)^{\frac{\alpha - 1}{\gamma}}, \tag{12}\]

whereby \(\gamma = ac + (1 - \sigma) [a + (1 - \theta) c]\) and \(A_d = QP^\sigma (\frac{\sigma - 1}{\sigma})^{\sigma}, A_x = (1 + \tau^{1-\sigma})A_d\) are measures of market size for domestic sellers and exporters respectively. Consistent with theoretical and empirical work on investment activity and international trade, our model suggests a positive relationship between innovation and market size.\[14\] As exporters spread investment costs across both markets, they face larger incentives to engage in quality and process innovations, \((A_x > A_d)\), whereas iceberg transportation costs \(\tau\) enter negatively. Furthermore, equations (11) and (12) show a negative impact of credit costs on investment choices. Faced with higher borrowing rates \(\beta\), producers scale down innovation activities.

We assume that investment costs are sufficiently convex: \(a, c > (\sigma - 1)(2 - \theta)\), such that \(\gamma > 0\). The convexity assumption implies that quality and process innovations are complements and increase in both capabilities \(\varphi\) and \(\kappa\). This complementary structure relates to the literature on simultaneous process and product R&D choices.\[15\] A higher capability draw for process or quality innovations has a direct positive impact on the corresponding investment level, since endogenous sunk costs are lower. Additionally, the complementary relationship between both innovations implies that a higher capability, e.g. for quality improvements, induces a higher incentive to invest in the other type of innovation as well, e.g. processes. The rationale behind this complementary effect is that both types of innovation affect marginal production costs and increase the success of a firm in the market, either through higher product quality for any given price or lower marginal production costs for any given quality.

\[13\]See Appendix 9.1 for a detailed derivation of firm’s maximization problem.

\[14\]See Bustos (2011) as well as Kugler & Verhoogen (2012), among others.

\[15\]Theoretical studies discuss complementarities between product and process innovations under different modes of market competition (Rosenkranz, 2003; Lin & Saggi, 2002; Athey & Schmutzler, 1995) as well as over the product life cycle (Klepper, 1996; Lamberti & Mantovani, 2010).
level. Hence, increases in one type of investment raise the marginal benefit of both innovation decisions. Consequently, firms will always engage in both types of innovation, whereas the relative investment in processes compared to quality improvements is given by:

$$
\frac{e_l(\varphi, \kappa)}{\lambda_l(\varphi, \kappa)} = \frac{\lambda_l}{\beta} \left( 1 - \frac{\theta}{\alpha} \right)^{\frac{(\sigma-1)(2-\theta)-c}{\gamma}} \left( \frac{\varphi}{c} \right)^{\frac{\sigma + (2-\theta)(1-\sigma)}{\gamma}},
$$

(13)

While investment levels increase in both capabilities, the convexity assumption regarding endogenous sunk costs implies further that investments in process innovations relative to quality upgrades increase in the cost-based capability and decrease in the quality-based capability:

$$
\frac{\partial(e)}{\partial \varphi} > 0, \quad \frac{\partial(e)}{\partial \kappa} < 0.
$$

Additionally, exogenous technology parameters affect relative investment behavior. If investment costs become more convex, captured by larger parameters $\alpha$ and $c$, firms scale down the relative level of the corresponding investment type as marginal costs of innovations increase. Higher sensitivity of marginal production costs with respect to quality (larger $\theta$) reduces the marginal benefit of vertical product differentiation and increases the relative investment in processes. In the extreme case, if $\theta = 1$, higher quality leads to a one-to-one increase in marginal costs (5), such that marginal benefits of product upgrades and thus innovation choices (11) and (12) are driven down to zero.

As common in models with monopolistic competition and CES demand structure, firms set the optimal price as a constant markup over marginal costs. In contrast to Melitz (2003), marginal production costs are determined endogenously by the two innovation choices, whereas $p_l$ denotes domestic prices of firms with export status $l \in d, x$ and $p_{x*}$ stands for the export price of internationally active producers:

$$
p_l(\varphi, \kappa) = \frac{\sigma}{\sigma - 1} e_l^{\theta} \lambda_{l,l}, \quad p_{x*}(\varphi, \kappa) = \tau p_x(\varphi, \kappa).
$$

(14)

The pricing rule captures two opposing effects of investment behavior. A higher level of process innovations enhances production efficiency, whereas quality innovations increase marginal costs according to (5). Consequently, the optimal price decreases in the cost-based capability $\varphi$, but increases in the quality-based capability $\kappa$.

Hence, the setup with two innovation choices captures both a negative relation between prices and firm size based on cost-based sorting à la Melitz (2003) and a positive correlation between prices and firm size as suggested by the recent quality and trade literature (e.g. Kugler & Verhoogen, 2012). The success of a producer in the market results from the ability to invest in processes as well as product quality at low costs. Therefore, we define firm’s overall efficiency as a combination

\[\text{Elasticities of prices with respect to capabilities are given by: } \frac{\partial p_l}{\partial \varphi} \frac{\varphi}{p_l} = \frac{\sigma - 1 - a}{\gamma} < 0 \text{ and } \frac{\partial p_{x*}}{\partial \kappa} \frac{\kappa}{p_{x*}} = \frac{\theta - \gamma + 1}{\gamma} > 0, \text{ if } c > \frac{\gamma - 1}{\gamma}. \text{ Note that this condition for the technology parameter } c \text{ is more restrictive than the convexity assumption discussed earlier in this section.}\]
of both capabilities: \( z = \varphi^a \kappa^c(1-\theta) \). Figure 1 depicts an example for an iso-efficiency curve in the two-dimensional space, whereas the vertical axis shows the quality-based capability \( \kappa \) and the horizontal axis shows the cost-based capability \( \varphi \). The curve represents a non-linear trade-off between the two attributes: \( \frac{\partial \kappa}{\partial \varphi} < 0 \) and \( \frac{\partial^2 \kappa}{\partial \varphi^2} > 0 \). If a firm possesses a low ability to invest in processes (low \( \varphi \)), it requires a relatively high quality-based capability \( \kappa \) to achieve the same overall efficiency level. Firms located along a particular iso-efficiency curve earn the same revenues and profits, since the latter can be expressed as monotone and increasing functions of efficiency \( z \):

\[
\begin{align*}
\pi_l(z) &= \frac{(\sigma - 1) v}{\sigma} r_l(z) - (f_d + I_x f_x) \beta,
\end{align*}
\]

where\( v = \frac{1}{\sigma - 1} - \left( \frac{1}{c} + \frac{1-\theta}{a} \right) > 0 \). Comparable to single-attribute firm models, efficiency \( z \) is a one-dimensional measure of profits and firm size. However, producers with the same size or efficiency \( z \) choose different levels of quality and process innovations and thus set different prices, depending on their firm-specific capabilities. Innovation activity is positively related to revenues and profits. Hence, the latter depend positively on market size \( A_l \), but negatively on borrowing rates \( \beta \) and investment cost parameters \( a \) and \( c \). Equations (11)-(16) characterize optimal behavior of firms that have access to external finance. So far, the analysis has neglected the zero profit condition (9) and incentive compatibility (10). The latter constraint determines the access to external capital. Thus, in a second step, we show how producers self-select into domestic production and exporting according to efficiency.

### 3.4 Selection of firms

Moral hazard, as described in section 3.3, induces credit rationing in the economy. Financial constraints solely arise from the agency-problem between firms and outside investors and no aggregate restriction of capital supply is required for this outcome. Even if investors are endowed with an infinite amount of liquidity, they will not be willing to provide loans to all firms with profitable projects. Firms that meet incentive compatibility (10) are able to sign a contract with an outside investor and raise external capital. Since profits (16) are a function of efficiency \( z \), the binding incentive constraint (10) determines a minimum efficiency level that is necessary to obtain outside capital:
\[ z_l = \left( \frac{(f_d + I_x f_x)(\beta + b)}{v} \right)^{\frac{1}{\alpha + c(1-\theta)}} \beta^{a+c(1-\theta)} c^{a \left( \frac{a}{1-\theta} \right)} A_l^{\frac{-a}{\alpha}}. \] 

(17)

Depending on export status \( l \in d, x \), all firms with efficiencies \( z \geq z_{j,l} \) obtain credit from outside investors, while firms with efficiencies \( z < z_{j,l} \) have no access to external capital. If private benefits \( b \) increase, moral hazard problems become more severe and incentive compatibility (10) requires higher efficiency levels. Furthermore, access to finance becomes more difficult if the borrowing rate \( \beta \), technology cost parameters \( a \) and \( c \), or fixed investment costs increase. In contrast, larger market size decreases the cutoff efficiency level.

If firms cannot generate sufficiently high project returns, agents have an incentive to misbehave and reap private benefits. In this case, investors are not willing to provide credit since they anticipate that they will never receive their loan back. Therefore, introducing imperfect capital markets motivated by Holmstrom & Tirole (1997) leads to credit frictions in the model. The comparison of the zero-profit condition (9) with incentive compatibility (10) shows immediately that the latter constraint is more restrictive as long as private benefits are positive: \( b > 0 \). Some low efficiency firms meet the zero-profit condition (9) and hence would find it profitable to be active in the market. However, moral hazard problems prevent access to finance and a range of profitable projects is not conducted in the presence of financial frictions. The negative impact of credit market imperfections on the entry decision of firms is consistent with other heterogeneous firm models that allow for credit constraints, e.g. Manova (2013). Graphically, condition (17) specifies the location of a marginal-access curve in the two-dimensional space \((\varphi, \kappa)\), denoted by \( z_{ICC} \) in Figure 2. All firms above this curve have access to external capital and are active in the market, whereas firms below that curve have too low combinations of capabilities to meet incentive compatibility and exit, even when they satisfy the zero-profit condition \( z_{ZPC} \). Independent of the export status, the difference between the two cutoffs is an increasing function of the financial market imperfection, namely the private benefit \( b \):

\[ \frac{z_{ICC}}{z_{ZPC}} = \left( \frac{\beta + b}{\beta} \right)^{\frac{a}{\alpha - 1}}. \] 

(18)

We compare cutoff efficiency levels (17) dependent on export status \( l \in d, x \), and derive a condition under which only the most capable firms select into exporting:

\[ z_x > z_d \text{ if } \frac{f_d + f_x}{f_d} \left( 1 + \tau^{1-a} \right)^{\frac{-\alpha}{\alpha}} > 1. \]

(19)

Analogous to Melitz (2003), this condition states that fixed \( f_x \) and variable trade costs \( \tau \) have to be sufficiently high, such that access to export markets is more difficult compared...
to domestic sales. However, the restriction on trade costs (19) differs from Melitz (2003) because exporters spread expenditures associated with endogenous investments among sales in both markets. Hence, changes in variable trade costs affect optimal levels of process and quality innovations that are non-separable between countries.

**Proposition 1** If Condition (19) holds, the most efficient firms with \( z \geq z_x \) export. Producers in the middle range of the efficiency distribution \( z_d \leq z < z_x \) sell only domestically, while the least efficient firms \( z < z_{m,d} \) have no access to external finance and exit.

Figure 3 depicts the selection pattern of firms under Proposition 1, whereby the marginal-access curve for exporting lies above the one for domestic activity. Marginal firms, characterized by cutoff efficiencies \( z_d \) and \( z_x \), just meet incentive compatibility and are indifferent between diligent behavior or shirking. Hence, their profits are equal to private benefits: \( \pi(z_l) = b(f_d + I_x f_x) \). Additionally, revenues and investment expenditures of marginal producers are independent of capabilities and depend on fixed parameters only: 18

\[
\sigma \left( z_l \right) = \frac{\sigma}{(\sigma - 1) \nu} \left( f_d + I_x f_x \right) (\beta + b),
\]

\[
\frac{1}{\varphi} \epsilon_l(z_l) = \frac{1}{c \nu} \frac{\beta + b}{\beta} \left( f_d + I_x f_x \right) ; \frac{1}{\kappa} \lambda_l(z_l) = \frac{1 - \theta}{a \nu} \frac{\beta + b}{\beta} \left( f_d + I_x f_x \right).
\]

If private benefits increase, incentive compatibility (10) becomes more restrictive and the cutoff efficiency raises (17). Low capability firms at the margin lose access to finance since they are not able to overcome moral hazard problems anymore. Graphically, marginal access curves in Figure 3 shift upwards. Due to the exit of low efficiency firms, revenues and investment expenditures of marginal producers increase. Similar selection effects occur if fixed production costs or credit costs increase. Whereas private benefits impose entry barriers to finance and affect only the extensive margin, changes in credit shocks induce additional within-firm adjustments. As discussed in section 3.3, optimal innovation choices (11) and (12) depend negatively on the borrowing rate \( \beta \). The role of credit conditions on both within-firm adjustments and market participation can be interpreted in a slightly different way: capital market imperfections impose minimum quality requirements. To see this, we follow Sutton (2012) and derive the quality-price ratio that reflects the effective competitiveness of a firm: 19

---

17 In Melitz (2003), a similar condition requires that \( \frac{f_d}{I_x} \tau^{\sigma-1} > 1 \).

18 Expressions for marginal firms are obtained by combining optimal innovation choices (11) and (12) with the minimum efficiency levels (17).

19 Compare Sutton (2012), chapter 1.6.
\[ \frac{\lambda_t}{p_t} = \frac{\sigma - 1}{\sigma} \left( \left( \frac{A_t}{\tilde{\beta}} \right)^{a+c(1-\theta)} e^{-a} \left( \frac{1 - \theta}{\alpha} \right)^{c(1-\theta)} \right)^{\frac{1}{\gamma}}. \] (22)

Like revenues and profits, the quality-price ratio is an increasing function of both innovation choices and thus of firm’s efficiency \( z \), as depicted in Figure 4. Whereas process innovations decrease prices for any given quality, product upgrades increase quality for any given price. Faced with higher borrowing rates, firms scale down both types of innovation resulting in a lower quality-price ratio. Graphically, within-firm adjustments correspond to a downward shift of the quality-price profile depicted in Figure 4 for two different borrowing rates: \( \beta_1 < \beta_2 \). While this effect negatively influences the intensive margin of international trade, selection effects reduce the extensive margin. The horizontal lines represent minimum quality requirements that are necessary to obtain external capital. These thresholds are derived by inserting the cutoff efficiency levels (17) in equation (22). Under Condition (19), the entry barrier is relatively higher for exporters \( (z_x > z_d) \), which implies a higher quality requirement. Only firms with sufficiently large price-adjusted quality are able to raise outside capital for international activity. Higher borrowing costs increase the efficiency cutoff and hence the minimum quality requirement reflected in an upward shift of horizontal lines in Figure 4. Changes in private benefits lead to adjustments along the extensive margin only, whereas within-firm adjustments and hence changes in the individual price-adjusted quality are not present. The remainder of the paper discusses the implications of within-firm adjustments and selection effects in more detail. Consistent with recent empirical evidence, the following section shows that reoptimizations of innovation choices can explain positive as well as negative correlations of credit costs with export prices, depending on the scope for vertical product differentiation. In chapters 5 and 6, we analyze the general equilibrium and welfare implications of adjustments along intensive and extensive margins.

4 Quality and cost effects of credit shocks

This part analyzes how firms optimally adjust innovation choices and price setting in face of worsening credit conditions. Since our model allows for two sources of firm heterogeneity, credit shocks influence export behavior through quality and cost effects. Facing higher borrowing rates \( \beta \), firms scale down both process innovations (11) and quality investments (12), as the following elasticities show:

\[ \frac{\partial e_t(\varphi, \kappa)}{\partial \beta} \frac{\beta}{e_t(\varphi, \kappa)} = -\frac{a}{\gamma} < 0 ; \quad \frac{\partial \lambda_t(\varphi, \kappa)}{\partial \beta} \frac{\beta}{\lambda_t(\varphi, \kappa)} = -\frac{c}{\gamma} < 0. \] (23)
Reductions in both types of investment influence marginal costs (5) and hence optimal price setting in opposite ways. On the one hand, firms scale down process innovations resulting in lower production efficiency and increased marginal costs. As (14) shows, this cost effect pushes optimal prices up. On the other hand, producers reduce product investments which leads to an opposing quality effect and dampens prices. The overall impact depends on the scope for vertical product differentiation in the production sector. To analyze the relative importance of quality and cost effects triggered by credit shocks, we proceed in three steps. First, we follow the quality and trade literature and introduce a measure of vertical product differentiation which will be decisive to determine the relative importance of quality and cost effects in our model. In a second step, we consider two special cases for which only the cost or quality channel is active. The third subsection allows for both quality and cost sorting of firms and derives more general predictions.

4.1 The scope for vertical product differentiation

The relative importance of quality and cost effects depends on the scope for vertical product differentiation. Following Sutton (2001) as well as Kugler & Verhoogen (2012), we define this measure as the ratio of expenditures for quality innovations relative to firm revenues:

\[
\frac{\frac{1}{\kappa} \lambda_{i}^a(\varphi, \kappa)}{r_{i}(\varphi, \kappa)} = \left(\frac{\sigma - 1}{\sigma}\right) \frac{1 - \theta}{a\beta}. \tag{24}
\]

As equation (24) shows, the scope for product differentiation is independent of firm-specific capabilities, but increases in the elasticity of substitution \(\sigma\) and decreases in the borrowing rate \(\beta\). Furthermore, quality differentiation is lower if investment costs become more convex (higher \(a\)) and the sensitivity of marginal costs to quality increases (higher \(\theta\)). A similar measure expresses the scope for process innovations relative to firm size:

\[
\frac{\frac{1}{\varphi} \epsilon_{i}^c(\varphi, \kappa)}{r_{i}(\varphi, \kappa)} = \left(\frac{\sigma - 1}{\sigma}\right) \frac{1}{c\beta}. \tag{25}
\]

Increased product market competition (higher \(\sigma\)) has a positive effect on process intensity, whereas the borrowing rate \(\beta\) as well as convexity of investment costs \(c\) lower innovation expenditures relative to firm size. The combination of (24) and (25) describes the relative scope for vertical product differentiation, compared to process innovations, as a constant ratio of technology parameters:

\[
\frac{\frac{1}{\kappa} \lambda_{i}^a(\varphi, \kappa)}{\frac{1}{\varphi} \epsilon_{i}^c(\varphi, \kappa)} = \frac{(1 - \theta) c}{a}. \tag{26}
\]
This measure for quality differentiation is only affected by changes in the relative investment cost structure. Increases in $a$ and $\theta$ make quality innovations less effective and reduce the relative expenditures for this investment type. Conversely, the ratio increases in $c$, which changes investment in favor of product upgrades. Hence, the ratio (26) reflects the relative effectiveness of quality upgrades compared to process innovations and is closely related to the estimation of quality ladders proposed by Khandelwal (2010). In sectors with higher relative effectiveness, firms engage more in vertical product differentiation resulting in a larger demand shifter $\lambda$ for any given price. Following Khandelwal (2010) higher consumer’s valuation for quality, conditional on prices, translates into larger market volumes and represents a proxy for market’s quality ladder.

### 4.2 Two special cases: the cost and quality channel

Faced with changes in borrowing costs, firms reoptimize both product quality and process innovation. Here, we consider two special cases, summarized in Table 1, in which the effectiveness for one innovation type becomes prohibitively low. Consequently, our framework collapses to a one-dimensional model and only one channel of adjustment is active.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
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<tr>
<td>Parameter restriction</td>
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<tr>
<td>Efficiency $z = \varphi^a \kappa^{c(1-\theta)}$</td>
<td>$z_1 = \kappa^{c(1-\theta)}$</td>
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<td>Sorting of firms</td>
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<td>Vertical product differentiation</td>
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Table 1: Credit shocks and special cases of firm selection

**Case 1: Quality-based sorting** If $a \to 0$, firms’ success is solely determined by the quality-based capability, since the efficiency $z$ collapses to a one-dimensional measure of competiveness: $z_1 = \kappa^{c(1-\theta)}$. In this scenario, the equilibrium is characterized by quality sorting only and the relative scope for vertical product differentiation (26) approaches infinity. As Figure 5 shows, marginal-access curves are horizontal and access to finance is solely determined by a minimum requirement on quality-based capabilities. All firms with $\kappa \geq \overline{\kappa}$ are active in the market, whereas firms with $\kappa < \overline{\kappa}$ exit. In this first special case, our model

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20Note that the slope of the marginal-access curves is exactly the negative inverse of the relative scope for product differentiation: $\frac{d \ln \kappa}{d \ln \varphi} = -\frac{a}{c(1-\theta)}$. Hence, if $c \to \infty$, marginal access curves are horizontal as well.
is closely related to single-attribute frameworks that focus on quality sorting, e.g. Kugler & Verhoogen (2012) or Baldwin & Harrigan (2011). Consistent with recent empirical evidence, prices and firm size are positively correlated if the scope for vertical product differentiation is high (Hallak & Sivadasan, 2013; Kugler & Verhoogen, 2012; Manova & Zhang, 2012). Larger firms with higher quality-based capability $\kappa$ invest more in quality upgrades resulting in higher prices: $\frac{\partial p_{j,t}}{\partial \kappa}_{p,j,t} = \frac{\sigma - \sigma + 1}{\gamma} > 0$.\footnote{See the pricing rule (14) and Footnote 16.} Faced with interest rate shocks, firms adjust only through the quality channel, as the elasticities (23) show. If borrowing becomes more costly, firms reduce quality innovations, which lowers marginal costs and optimal prices. Hence, in case of pure quality-sorting, there is a negative relationship between credit costs and prices.

**Case 2: Cost-based sorting** If $c \to 0$, the model collapses to a Melitz (2003) - type economy. The only decisive dimension is the cost-based capability of firms: $z_2 = \varphi^a$. This one-dimensional measure is comparable to productivity in Melitz (2003). However, marginal costs are determined by endogenous innovation choices in our model. Instead of quality, the competitiveness of firms is determined by their ability to conduct process innovations at low costs. Therefore, this second case captures productivity sorting, whereas the relative scope for product differentiation (26) tends to zero. With decreasing $c$, the importance of quality-based capabilities diminishes such that marginal access curves become steeper and are vertical in the limit case ($c \to 0$), as depicted in Figure 6. Hence, as in Melitz (2003), selection of firms is determined by a cost-based productivity threshold. All firms with $\varphi \geq \overline{\varphi}$ produce, whereas firms with higher marginal production costs exit. In sectors with low scope for vertical product differentiation, empirical studies point to a negative relation of firm size and productivity with prices (Roberts & Supina, 1996; Foster et al., 2008). Accordingly, larger firms with higher cost-based capability $\varphi$ invest more in process innovations that reduce marginal costs and prices: $\frac{\partial p_{j,t}}{\partial \varphi}_{p,j,t} = \frac{\sigma - 1 - a}{\gamma} < 0$. In this case, credit shocks induce cost effects only. If the interest rate for external capital increases, firms scale down process innovations, whereas quality changes play no or a negligible role. Consequently, a Melitz-type world suggests a positive relationship between credit costs and optimal prices as lower process innovations push up marginal costs. The two special cases show that the relation of credit costs and prices depends on the relative scope for vertical product differentiation. This reasoning is generalized in the following subsection.
4.3 The relative importance of cost and quality effects

If the two special cases do not apply, the scope for vertical product differentiation is finite and positive, such that both cost and quality effects coexist in equilibrium. Higher credit costs lead to reductions in quality as well as process innovations affecting marginal costs and prices in opposite directions. Changes in relative investment and prices follow immediately from equations (14) and (23):

\[
\frac{\partial e_l(\varphi, \kappa)}{\partial \beta} \frac{\beta \lambda_l(\varphi, \kappa)}{e_l(\varphi, \kappa)} = \frac{c - a}{\gamma}; \quad \frac{\partial p_l(\varphi, \kappa)}{\partial \beta} \frac{\beta}{p_l(\varphi, \kappa)} = \frac{a - c\theta}{\gamma}.
\] (27)

The reasoning derived in the previous subsection applies to the more general case as well. The relative importance of counteracting cost and quality effects depends on the scope for vertical product differentiation that is determined by the investment cost parameters, \(a\) and \(c\), as well as the sensitivity of marginal costs with respect to quality \(\theta\).

**Proposition 2** If the scope for vertical product differentiation is relatively high (low) and hence \(a < (>) c\), firms respond to higher credit costs by decreasing (increasing) the quality of their products relative to cost-based productivity and set lower (higher) prices.

In face of higher borrowing costs, firms adjust proportionally more the type of investment which is used with higher intensity. If the degree of product differentiation is relatively high, firms react to worsening credit conditions by reducing the investment in quality and by setting lower prices. Contrarily, firms with a low degree of quality differentiation, mainly reoptimize process innovations and the associated cost effect tends to push up prices. Consistent with Proposition 2, Secchi et al. (2013) exploit Italian firm-level data and find that constrained exporters charge higher prices than unconstrained firms within the same product-destination market. This positive relationship between credit frictions and prices points to cost effects, but disappears for product categories with high quality differentiation. Following Kugler & Verhoogen (2012), Secchi et al. (2013) use the ratio of advertising and R&D expenditures to total sales in U.S. industries as a proxy for vertical product differentiation. Hence, the measure is comparable to expression (24) in our theoretical model. Closely related, Fan et al. (2013a) analyze Chinese firm-level data and find evidence for a negative relationship between credit frictions and prices. The authors rationalize this result by a partial equilibrium model based on Arkolakis (2010) and Manova (2013) and differentiate between exogenous and endogenous quality. Fan et al. (2013a) show that constrained firms sell at higher prices when quality is exogenous, whereas the opposite holds in case of endogenous quality choice. In contrast, our model explains the prevalence of quality and cost effects when firms endogenously choose two innovation types that affect marginal production costs.
in opposite ways. Therefore, we reconcile recent empirical evidence and stress the role of vertical product differentiation for counteracting cost and quality effects on prices. Besides within-firm adjustments, credit frictions affect the extensive margin of international trade. Both margins of adjustment influence welfare per worker in general equilibrium. The following two sections analyze the equilibrium in the open economy as well as selection and welfare effects of financial shocks.

5 Equilibrium in the open economy

To close the model, we consider the first stage of firm’s decision problem and discuss how potential producers decide to enter the market before knowing their capability draws. At the entry stage, firms draw both investment capabilities $\varphi$ and $\kappa$ from a joint probability distribution $g(\varphi, \kappa)$ with positive support over $[\varphi, \overline{\varphi}] \times [\kappa, \overline{\kappa}]$. As described in section 3.3, we summarize these two capabilities in a single measure of firm’s efficiency: $z = \varphi^a \kappa^{c(1-\theta)}$.

The marginal-access cutoff levels (17) define regions in the two-dimensional capability space $(\varphi, \kappa)$, as depicted in Figure 3:

\[ D = \left\{ (\varphi, \kappa) \in [\varphi, \overline{\varphi}] \times [\kappa, \overline{\kappa}] : z \geq z_d \right\}, \]
\[ D_d = \left\{ (\varphi, \kappa) \in [\varphi, \overline{\varphi}] \times [\kappa, \overline{\kappa}] : z_d \leq z < z_x \right\}, \]
\[ D_x = \left\{ (\varphi, \kappa) \in [\varphi, \overline{\varphi}] \times [\kappa, \overline{\kappa}] : z \geq z_x \right\}, \]

where $D$ is the set of all active firms in equilibrium and $D_l$, with $l \in \{d, x\}$, denotes regions of domestic producers and exporters respectively. Ex-ante probabilities of being active in one particular region $\chi_l$ as well as the probability of success $\chi_s$ are defined as:

\[ \chi_l = \int \int g(\varphi, \kappa) d\varphi d\kappa \quad \chi_s = \int \int g(\varphi, \kappa) d\varphi d\kappa, \]

and corresponding conditional probabilities are given by $\mu_s(\varphi, \kappa) = \frac{g(\varphi, \kappa)}{\chi_s}$ and $\mu_t(\varphi, \kappa) = \frac{g(\varphi, \kappa)}{\chi_t}$. For aggregation purposes we define the average efficiency within each group:

\[ (\overline{z}_l)^{\frac{a-1}{a}} = \int \int z^{\frac{a-1}{a}} \mu_l(\varphi, \kappa) d\varphi d\kappa. \]

Denoting the mass of active firms within one region by $M_l$, aggregate revenues and expected profits by group are:
\[ R_t = M_t \int_{(\varphi,\kappa)\in D_l} r_l(\varphi, \kappa) \mu_l(\varphi, \kappa) d\varphi d\kappa ; \quad E\pi_t = \int_{(\varphi,\kappa)\in D_l} \pi_l(\varphi, \kappa) \mu_s(\varphi, \kappa) d\varphi d\kappa. \] (33)

Analogous to Melitz (2003), revenues of a particular firm with efficiency \( z \) can be expressed relative to the marginal producer or exporter, characterized by efficiency \( z_l \):

\[ r_l(z) = \left( \frac{z}{z_l} \right)^{\frac{1}{\gamma}} r_l(z_l). \] (34)

As discussed in section 3.4, revenues of marginal firms depend only on fixed parameters of the model: \( r_l(z_l) = \frac{\sigma}{(\sigma - 1) \nu} \left( f_d + I_x f_x \right) (\beta + b) \).\(^{22}\) By taking into account the definition of average efficiency (32), this allows to write expected revenues and profits as follows:

\[ \tilde{r}_l = s_l \frac{\sigma}{(\sigma - 1) \nu} (b + \beta) \left( f_d + I_x f_x \right) \left( \frac{z_l}{z_i} \right)^{\frac{1}{\gamma}} ; \quad E\pi_l = \frac{\sigma - 1}{\sigma} \nu \tilde{r}_l - \beta \left( f_d + s_x f_x \right), \] (35)

whereas the share of producers in one group is defined as \( s_l = \frac{s_i}{x_s} \). The equilibrium is characterized by (35) and a free entry condition to ensure that fixed entry costs \( f_e \) are equal to expected profits before firms know their combination of capability draws:

\[ E\pi = \frac{1}{\lambda_s} \delta f_e, \] (36)

whereas \( \delta \) is the exogenous probability of a death shock and \( E\pi = \sum_t E\pi_t \). Equations (35) and (36) determine the minimum efficiency of marginal firms, \( z_d \), that are just able to produce for the domestic market. Aggregate investment expenditures for processes and quality upgrades are functions of average revenues and the number of firms in the market:

\[ M_t \int_{(\varphi,\kappa)\in D_l} \frac{1}{\varphi} e_l^r(\kappa, \varphi) \mu_l(\varphi, \kappa) d\varphi d\kappa = \left( \frac{\sigma - 1}{\sigma} \right) \frac{1}{\beta c} M_t \tilde{r}_l, \] (37)

\[ M_t \int_{(\varphi,\kappa)\in D_l} \frac{1}{\kappa} \lambda_l^a(\kappa, \varphi) \mu_l(\varphi, \kappa) d\varphi d\kappa = \left( \frac{\sigma - 1}{\sigma} \right) \frac{1 - \theta}{\alpha \beta} M_t \tilde{r}_l. \] (38)

The ratio of aggregate investment expenditures leads to the scope for vertical product differentiation (26) that is independent of firm capabilities, as discussed in section 4.1. To close the model, labor market clearing has to be satisfied. Labor requirements of single firms

\(^{22}\)Compare equation (20).
consist of variable production costs that are financed internally, as well as credit costs for fixed and endogenous investments that have to be covered by external capital:

\[
mc_l(\varphi, \kappa)q_l(\varphi, \kappa) + \beta d_l(\varphi, \kappa) = \frac{\sigma - 1}{\sigma} r_l(z) \left( 1 + \frac{1}{c} + \frac{1 - \vartheta}{a} \right) + \beta (f_d + I_x f_x). \tag{39}
\]

As equation (39) shows, firm’s labor demand can be expressed as a function of revenues, borrowing rates and technology parameters. Producers with higher efficiency \(z\) employ more labor due to increased investment expenditures and larger sales. More convex investment costs (higher \(a\) and \(c\)) scale down process and quality innovations which leads to lower labor requirements. Increasing borrowing rates make finance for fixed outlays more costly, but decrease investment expenditures and revenues. In equilibrium, the inelastic labor supply \(L\) has to be equal to labor demands in the entry sector \((L_e = M_e f_e)\) and in the two groups of active producers: \(L = L_e + \sum_l L_l\). Analogous to Melitz (2003), aggregation of single labor requirements pins down the mass of active firms \(M\) in one country:

\[
M = \frac{L}{\bar{r}}, \tag{40}
\]

where \(\bar{r}\) denotes average revenues in the total economy. This relationship is obtained by imposing aggregate stability such that the mass of successful entrants is equal to the mass of firms that are forced to exit due to the exogenous death shock: \(\chi_s M_e = \delta M\). In the following section, we exploit general equilibrium properties to derive welfare implications.

6 Selection and welfare effects of financial shocks

Besides within-firm adjustments, credit conditions affect the extensive margin of international trade. This section analyzes the effects of changes in private benefits \(b\) and borrowing rates \(\beta\) on welfare in general equilibrium. Since we consider two symmetric countries, welfare implications hold for variations of financial conditions in both regions. By imposing symmetry, we implicitly assume that capital markets are completely integrated. Hence, our welfare analysis neglects implications of bilateral differences in financial development or in credit costs. In contrast, another strand of literature examines how national differences in financial characteristics affect cross-border trade and capital flows. According to these models, financial development serves as comparative advantage, whereas goods and capital mobility are complements under imperfect financial institutions (see Furusawa & Yanagawa, 2010; Antràs & Caballero, 2009, among others). Welfare per worker in the open economy is given by the real wage, whereas the nominal wage is normalized to one:
\[ W = P^{-1} = \left(\frac{\sigma - 1}{\sigma}\right)^{\gamma/a} c^{-1/a} \left(1 - \theta\right)^{1-\theta/a} \left(1 + c(1-\theta)/\alpha\right) \left(v/\beta + b\right) \left(f_d/\gamma\right) \left(L^\gamma z_d^{\gamma/\gamma}\right) \quad (41) \]

Analogous to Melitz (2003), welfare is a positive function of the cutoff efficiency level \( z_d \). To derive explicit solutions for this entry threshold, we assume that capabilities \( \varphi \) and \( \kappa \) are independently Pareto distributed with positive support over \([1, \bar{\varphi}] \times [1, \infty] \) and \( \bar{\varphi} > 1 \). The probability of drawing a particular combination of \( \varphi \) and \( \kappa \) is then given by:

\[ g(\varphi, \kappa) = g_\varphi(\varphi) g_\kappa(\kappa) \text{ with } g_\kappa(\kappa) = \xi \kappa^{-\xi-1} \text{ and } g_\varphi(\varphi) = g_\varphi(\varphi) = \vartheta \frac{\varphi^{\vartheta-1}}{1-\vartheta} \text{, where } \xi \text{ and } \vartheta \text{ are the shape parameters of the Pareto distributions.} \]

### 6.1 Changes in private benefits

Higher private benefits \( b \) aggravate moral hazard problems and incentive compatibility (10) becomes more restrictive which is associated with an increase in the cutoff efficiency (17).

As discussed in section 3.4, stronger capital market frictions shift marginal-access curves upwards (Figure 3) and impose higher minimum quality requirements (Figure 4). Therefore, higher private benefits negatively affect the number of firms in equilibrium: \( M \)

\[ \frac{\partial M}{\partial b} b = -\frac{b}{\beta + b} < 0 \quad (42) \]

Changes in private benefits do not induce adjustments within firms such that the only reaction occurs along the extensive margin. The exit of least efficient firms, in case of higher \( b \), leads to two opposing effects on welfare (41). On the one hand, welfare decreases due to a lower number of varieties. On the other hand, higher private benefits induce an increase of average efficiency \( (\frac{\partial z_d}{\partial b} > 0) \) and hence higher average price-adjusted quality offered in the economy:

\[ \frac{\partial W}{\partial b} b = -\frac{b}{\alpha c} \left[ \frac{\gamma}{\sigma - 1} \frac{1}{\beta + b} - \frac{\partial z_d}{\partial b} \frac{1}{z_d} \right] < (>) 0 \quad (43) \]

**Proposition 3** There is a unique threshold \( b^* \), such that the welfare effect of higher private benefits is negative (positive) if \( b > (<) b^* \).

**Proof.** See Appendix 9.5. □

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23 See Appendix 9.3 for a derivation of the welfare function.

24 For technical reasons, we assume that \( \xi > \frac{c(1-\theta)(\sigma-1)}{\gamma} \) and \( \vartheta > \frac{\alpha \xi}{\gamma(1-\vartheta)} \). Appendix 9.4 derives the cutoff efficiency \( z_d \) explicitly under the assumption of Pareto distributed capabilities.

25 See Appendix 9.4 for an explicit derivation for the number of firms in one country.
Whether the negative variety effect or the positive efficiency effect dominates, depends on the level of financial development, as Proposition 3 shows. If capital market imperfections are very low, increases in the access barrier to finance, driven by higher private benefits, lead to a strong efficiency gain that outweighs the negative welfare effect due to lower variety for consumers. With increasing private benefits, the number of firms decreases and potential efficiency gains diminish relative to the exit of producers. Hence, strong capital market frictions reinforce the negative welfare implications of financial shocks. Furthermore, the negative variety effect is more likely to dominate if the Pareto shape parameters $\xi$ and $\vartheta$ are higher. In this case, the distribution of firms in the efficiency space is more dispersed. Since changes in the entry barrier to external capital force the least efficient firms to exit, decreases in the product range are more pronounced resulting in higher welfare losses.

Whereas changes in private benefits $b$ affect only market participation, interest rate shocks additionally induce within-firm adjustments, as discussed in section 4. Hence, welfare effects of variations in $\beta$ can be separated into reactions along intensive and extensive margins.

### 6.2 Changes in interest rates

Comparable to changes in private benefits, higher interest rates $\beta$ induce negative reactions on the number of active firms. If borrowing becomes more costly, marginal firms are not able to meet incentive compatibility anymore and have to exit the market:

$$ \frac{\partial M}{\partial \beta} \frac{\beta}{M} = -\frac{\beta}{\beta + b} < 0. $$

 besides reactions along the extensive margin, interest rate changes induce within-firm adjustments. As analyzed in section 4, firms reoptimize innovation choices for processes and product quality. These adjustments directly influence revenues and profits at the firm level and lead to an additional negative welfare effect beyond selection effects. The welfare change of interest rate shocks can thus be written as:

$$ \frac{\partial W}{\partial \beta} \frac{\beta}{W} = \frac{a + c (1 - \theta)}{ac} - \frac{1}{ac} \left( \frac{\gamma \beta}{\sigma - 1 - \beta + b} + \frac{\partial z_d}{\partial \beta} \frac{\beta}{z_d} \right) < 0. $$

Besides reactions along the intensive margin, adjustments along the extensive margin consist of a negative welfare effect due to decreases in the available number of varieties and a counteracting positive efficiency effect, since the entry cutoff level increases ($\frac{\partial z_d}{\partial \beta} > 0$). In contrast to the previous case, Proposition 4 shows that adjustments along the extensive margin lead always to negative welfare changes.
Proposition 4  For changes in borrowing rate $\beta$, the variety effect always outweighs the efficiency effect, such that $\frac{\partial W}{\partial \beta} < 0$.

Proof. See Appendix 9.5. ■

Compared to changes in private benefits, there are two important differences regarding welfare effects of variations in credit costs. First, within-firm adjustments reflect an additional channel through which welfare per worker is affected negatively. Even without changes along the extensive margin, existing firms in the market reduce innovation activity resulting in a higher quality-adjusted price on average. As the first part of elasticity (45) shows, negative welfare implications of within-firm adjustments become stronger with increasing quality differentiation and process intensity, when technology parameters $a, c$ and $\theta$ are low. Hence, higher measures for quality- and cost-based differentiation (24) and (25) imply that firms respond stronger to variations in credit shocks resulting in larger welfare changes. Second, Proposition 4 shows that selection effects differ as well. Compared to changes in private benefits, higher credit costs do not only increase access barriers to finance, but in addition affect negatively the average efficiency in the economy. Hence, the negative variety effect caused by firm exit always outweighs average efficiency gains. Therefore, our model predicts unambiguously negative welfare effects of interest rate increases, driven by negative reactions of the extensive as well as the intensive margin.

7 Trade liberalization

The preceding analysis has focused on adjustments along intensive and extensive margins triggered by financial shocks. This section shows that trade liberalization induces within-firm adjustments comparable to variations in credit costs. Therefore, we consider a change in iceberg-transportation costs $\tau$. Lower variable trade costs induce exporters to invest more both in process and quality innovations shown by the following elasticities:

\[
\frac{\partial e_x}{\partial \tau} < 0 ; \quad \frac{\partial \lambda_x}{\partial \tau} < 0.
\]

(46)

As for credit shocks, adjustments regarding both types of innovation induce opposing quality and cost effects. If iceberg-transportation costs $\tau$ decrease, firms increase investments in processes. This cost effect reduces marginal production costs and lowers optimal prices. Furthermore, an opposing quality effect tends to push up marginal costs and prices as firms respond to trade liberalization with increased quality differentiation. Analogous to section 4,

\[26\]The derivatives follow immediately from equations (11) and (12).
the relative scope for vertical product differentiation is decisive to determine the adjustment of relative investment and hence the direction of price changes:²⁷

\[ \frac{\partial}{\partial \tau} \left( \frac{e_x}{\lambda_x} \right) = \frac{(a - c)(1 - \sigma)}{\gamma} \frac{\tau^{1-\sigma}}{1 + \tau^{1-\sigma}}; \quad \frac{\partial p_x}{\partial \tau} p_x = \frac{(a - c\beta)(\sigma - 1)}{\gamma} \frac{\tau^{1-\sigma}}{1 + \tau^{1-\sigma}}. \] (47)

**Proposition 5** If the scope for vertical product differentiation is relatively high (low), such that \( a < (>) c \), firms respond to lower variable trade costs by increasing (decreasing) product quality compared to production efficiency and charging higher (lower) prices: \( \frac{\partial (e_x)}{\partial \tau} > (\tau) \frac{\partial p_x}{\partial \tau} < (\tau) 0 \).

Proposition 5 follows immediately from elasticities (46) and the relative scope for vertical product differentiation (26). Trade liberalization increases incentives to engage in both types of innovation and triggers opposing quality and cost effects. Depending on the scope for vertical product differentiation, relative investment in processes compared to quality and hence prices may go in both directions. If the degree of quality differentiation is high, exporters tend to adjust mainly through the quality channel. Product quality increases more than cost-based productivity leading to upward pressure on marginal costs and prices. Conversely, if the industry is characterized by low product differentiation, increases in process innovations and thus the cost reducing effect dominate and lead to negative price reactions.

Predictions of Proposition 5 are consistent with recent empirical evidence. For Chinese firm-level data, Fan et al. (2013b) show that tariff reductions induce quality upgrading of exporters resulting in positive or negative price reactions, depending on whether the degree of vertical product differentiation is high or low. To rationalize this result, the authors extend a Melitz-type partial equilibrium model by endogenous quality choice. Thereby, the scope for vertical product differentiation is measured by exogenous parameters reflecting the effectiveness of quality investments. Depending on these parameters, firms readjust their quality choice by solving a trade-off between increases in demand due to higher quality and decreases in sales due to higher prices. In contrast, our model captures readjustments of both quality and process innovations. Changes in relative investment and thus price reactions are driven by a trade-off between net benefits of both innovations.

Hence, compared to recent heterogeneous firm models that allow for product quality, our framework shows that trade and credit costs influence prices at the firm level through endogenous adjustments of innovation choices. As discussed in the context of credit cost shocks in section 4, our model with two sources of firm heterogeneity allows for quality sorting as well as cost-based productivity sorting, depending on the scope for vertical product differentiation.

²⁷Compare the expression for relative investment (13) and optimal price setting (14).
differentiation. One important motive for the recent quality and trade literature is the positive relationship between distance and FOB prices found in empirical firm-level studies, e.g. Baldwin & Harrigan (2011). Heterogeneous firm models with CES demand based on Melitz (2003) predict no effect of iceberg-transport costs on FOB prices at the firm level, but a negative relation between distance and average unit values. The latter effect is driven by selection of the most productive firms in more distant markets which reduces average prices in destinations associated with higher trade costs. To rationalize the positive relationship between unit values and distance, recent heterogeneous firm models allow for quality sorting of firms, e.g. Kugler & Verhoogen (2012). In contrast to Melitz (2003), successful exporters to distant markets are not characterized by low marginal cost and prices, but rather offer high-quality products at high prices. Whereas standard heterogeneous firm models stress selection effects, our framework with two sources of firm heterogeneity and endogenous investments additionally links trade and credit shocks to firm-level prices. Variations in production or technology costs translate into adjustments of innovations and affect marginal production costs and thus optimal price setting, as stated in Propositions 2 and 5. This additional channel is closely related to recent empirical and theoretical work stressing that exporters adjust to trade and cost shocks within the boundaries of the firm.28

8 Conclusion

This paper has analyzed the effects of credit frictions on within-firm adjustments and selection into exporting in a two-dimensional heterogeneous firm model with endogenous innovation choices. Whereas existing trade models with financial frictions are mainly based on Melitz (2003), three elements are crucial for our theoretical analysis. First, we combine Melitz-type cost sorting and vertical product differentiation as suggested by the recent quality and trade literature and allow for two sources of firm heterogeneity. As in single-attribute models, firm’s competitiveness and hence profits are determined by a one-dimensional productivity measure. The latter, however, can be separated along two dimensions, namely the cost-based and the quality-based capability of a producer. Second, we allow for innovation in quality and processes associated with endogenous sunk costs decreasing in capabilities. As a third component, we assume that fixed and endogenous investment costs have to be financed by external investors and introduce imperfect capital markets motivated by ex-post moral hazard as in Holmstrom & Tirole (1997). Moral hazard problems induce credit rationing in equilibrium such that only the most capable firms overcome financial frictions and export.

These three elements allow us to analyze the effects of trade and credit cost shocks on

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28See Bustos (2011) or Lileeva & Trefler (2010), among others.
adjustments within the boundaries of a firm. Compared to standard heterogeneous firm models, marginal production costs are determined by endogenous investment choices. Whereas process innovations decrease marginal production costs for any given quality level, product upgrades shift demand outwards at the expense of higher marginal costs. Hence, the levels of innovation are positive functions of firm-specific capabilities and depend negatively on trade as well as capital costs and exogenous technology parameters. Faced with variations in credit and trade costs, firms readjust investment activities resulting in opposing quality and cost effects on marginal production costs and prices. Consistent with recent empirical evidence, these within-firm adjustments explain positive as well as negative correlations between credit and trade costs with firm-level prices, depending on the scope for vertical product differentiation. This measure relates the expenditures of quality upgrades to firm size and depends only on exogenous investment cost parameters. Additionally, our model nests pure cost-based sorting à la Melitz (2003) and pure quality-based sorting as special cases, when investment costs for one type of innovation become prohibitively high. Besides within-firm adjustments, credit frictions negatively affect the number of active firms. In general equilibrium, adjustments both along intensive and extensive margins contribute to negative welfare effects.

Our theoretical analysis could be extended in several directions. First, we do not allow for market-specific investments. Both process innovations and quality upgrades are spread across domestic and foreign markets, whereas recent empirical evidence points to quality-based market segmentation of exporters (Manova & Zhang, 2012; Bastos & Silva, 2010; Flach, 2014). Second, suppliers rely on one source of external capital to finance total investment costs. This allows us to focus on within-firm adjustments, whereas selection effects between different sources of external finance might play an important role as well (see Hou & Zhang, 2012). Third, whereas our analysis focuses on a CES demand structure, credit frictions may influence price-cost markups. Egger & Seidel (2012) analyze credit frictions in the framework of Melitz & Ottaviano (2008) with varying markups. Lastly, we concentrate on ex-post moral hazard to introduce credit rationing. Empirical and theoretical literature suggests other channels through which financial market imperfections may influence export behavior like higher default risk, information asymmetries regarding firm attributes or imperfect financial contractibility (see Manova, 2013; Feenstra et al., 2012; among others).
9 Mathematical Appendix

9.1 Maximization problem of firm

This section derives optimal investment and pricing behavior of a firm with export status \( l \in d, x \), conditional on access to finance, whereas \( I_x = 1 \) if the firm is an exporter and \( I_x = 0 \) otherwise. Firms maximize profits (6) which can be written as:

\[
\pi_l = QP^\sigma \lambda_l^{\sigma-1} \left[ p_l^{1-\sigma} + I_x p_x^{1-\sigma} - \frac{\lambda_l^\theta}{e_l} \left( p_l^{-\sigma} + I_x \tau p_x^{-\sigma} \right) \right] - k_l,
\]  

(48)

subject to the constraints (7) and (8). If incentive compatibility (10) is satisfied, the first order conditions for optimal domestic prices \( p_l \) and export prices \( p_x^* \), as well as investment levels \( e_l \) and \( \lambda_l \) are given by:

\[
QP^\sigma \lambda_l^{\sigma-1} \left[ (1 - \sigma) p_l^{-\sigma} + \sigma \frac{\lambda_l^\theta}{e_l} p_l^{-\sigma-1} \right] = 0,
\]  

(49)

\[
QP^\sigma \lambda_x^{\sigma-1} \left[ (1 - \sigma) p_x^{-\sigma} + \sigma \frac{\lambda_x^\theta}{e_x} p_x^{-\sigma-1} \right] = 0,
\]  

(50)

\[
QP^\sigma \lambda_l^{\sigma-1} \frac{\lambda_l^\theta}{e_l^2} \left( p_l^{-\sigma} + I_x \tau p_x^{-\sigma} \right) - \mu_1 \frac{c}{\varphi} e_l^{c-1} = 0,
\]  

(51)

\[
QP^\sigma \left[ \left( p_l^{1-\sigma} + I_x p_x^{1-\sigma} \right) (\sigma - 1) \lambda_l^{\sigma-2} - (\theta + \sigma - 1) \frac{\lambda_d^{\theta+\sigma-2}}{e_d} \left( p_l^{-\sigma} + I_x \tau p_x^{-\sigma} \right) \right] - \mu_1 \frac{a}{\kappa} \lambda_d^{a-1} = 0.
\]  

(52)

Optimality conditions with respect to loan repayment \( k_l \) and credit amount \( d_l \) are:

\[-1 + \mu_2 = 0, \]

(53)

\[
\mu_1 - \beta = 0,
\]  

(54)

whereas \( \mu_1 \) and \( \mu_2 \) are the Lagrange multipliers of the constraints (7) and (8). Equations (53) and (54) imply that \( \mu_1 = \beta > 1 \) and \( \mu_2 = 1 \). The optimal prices (14) follow immediately from (49) and (50). Combining the optimal pricing rules with the first-order conditions for quality (51) and process innovations (52), leads to:
\[ e_l = \left[ \frac{\varphi}{c\beta} A_l \right]^{\frac{1}{c+1-\sigma}} \lambda_l^{\frac{(\sigma-1)(1-\theta)}{c+1-\sigma}}, \quad (55) \]

\[ \lambda_l = \left[ \frac{1 - \theta}{a\beta} \kappa A_l \right]^{\frac{1}{\sigma+(1-\theta)(1-\sigma)}} e_l^{\frac{\sigma-1}{\sigma+(1-\theta)(1-\sigma)}}, \quad (56) \]

where market size for domestic producers and exporters is defined as: \( A_d = QP^\sigma \left( \frac{\sigma-1}{\sigma} \right)^\sigma \), \( A_x = (1 + \tau^{1-\sigma})A_d \). Equations (55) and (56) show the complementary structure of process and quality innovations, as discussed in section 3.3. Combining the two expressions leads to the optimal investment choices described by equations (11) and (12).

### 9.2 Alternative modelling of private benefits

This section outlines an alternative specification of private benefits that are assumed to be proportional to the total credit amount \( d_l \), whereas index \( l \in d, x \) denotes the export status of firms. Whereas the maximization problem of unconstrained firms is still described by equations (7)-(9), incentive compatibility (10) is now given by:

\[ \pi_l \geq bd_l, \quad (57) \]

Analogous to equations (20) and (21), by just meeting incentive compatibility, revenues and investment costs of marginal firms are independent of capabilities and can be expressed as functions of exogenous parameters only:

\[ r_l(z_l) = \frac{\sigma}{\sigma - 1} \frac{f_d + I_x f_x}{\Gamma}, \quad (58) \]

\[ \frac{1}{\varphi} e_l^c(z_l) = \frac{1}{c} \frac{f_d + I_x f_x}{\Gamma} ; \quad \frac{1}{\kappa} \lambda_l^\tau (z_l) = \frac{1 - \theta}{\kappa} \frac{f_d + I_x f_x}{a \Gamma \beta}, \quad (59) \]

whereas \( \Gamma = \frac{1}{\sigma - 1} + b - \frac{1}{\beta} \left( \frac{1}{c} + \frac{1 - \theta}{a} \right) > 0 \). Analogous to section 3.4, selection of firms is determined by cutoff efficiencies derived from incentive compatibility (57):

\[ z_l = \left( \frac{f_d + I_x f_x}{\Gamma \beta} \right)^{\frac{\sigma}{\sigma - 1}} \left( \frac{\beta}{A_l} \right)^{\frac{\alpha}{\sigma}} e^\alpha \left( \frac{a}{1 - \theta} \right)^{\sigma(1-\theta)}, \quad (60) \]

Compared to the specification in section 3.4, not only fixed costs, but rather total investment is prone to moral hazard. Marginal firms have to take into account agency costs for endogenous investment choices as well. Hence, incentive compatibility becomes more restrictive which implies that access cutoffs are even higher compared to the baseline model.
The comparison of cutoff levels (17) and (60) shows that the latter are always higher as long as \( b > 0 \). Hence, private benefits that are proportional to total investment reinforce financial frictions in the model, whereas the qualitative results regarding selection patterns and the effects of financial and trade shocks do not change.

### 9.3 Derivation of welfare

To derive the welfare function (41), we aggregate the price index (2) by taking into account firm-specific quality-price ratios (22) and using the relationship that aggregate revenues equal total labor supply \((L = R)\) in equilibrium:

\[
W = \left( \frac{\sigma - 1}{\sigma} \right) \frac{a(c + 1) + c(1 - \theta)}{ac} c^{-\frac{1}{\gamma}} \left( \frac{1 - \theta}{\alpha} \right) \left( \frac{L}{\gamma} \right) \left( \frac{M_d \tilde{z}_d^\gamma}{\alpha c (\sigma - 1)} + (1 + \tau^{1-\sigma}) \frac{a c M_x \tilde{z}_x^\gamma}{\alpha (\sigma - 1)} \right).
\]

Analogous to Melitz (2003), we solve for the average efficiencies of domestic producers and exporters, \( \tilde{z}_d \) and \( \tilde{z}_x \), by using the relationship \( \frac{r_l(z)}{r_l(z)} = \frac{\xi \kappa - \xi^{-1}}{\varphi} \) and the expression for cutoff efficiency \( z_l \) (17). Furthermore, we exploit that the ratio of cutoff levels is given by: \( \frac{z_x}{z_d} = \left( \frac{f_d + f_x}{f_d} \right) \tilde{z}_d^{-\frac{1}{\gamma}} \left( 1 + \tau^{1-\sigma} \right)^{-\frac{\sigma - 1}{\sigma - 1}} \). This allows to write welfare per worker as a function of the cutoff efficiency \( z_d \), as specified in equation (41).

### 9.4 Solution with Pareto distributed capabilities

To obtain an explicit solution for the cutoff efficiency \( z_d \), we assume that firm specific capabilities \( \varphi \) and \( \kappa \) are independently Pareto distributed with positive support over \([1, \varphi] \times [1, \infty]\) and \( \varphi > 1 \). The probability of drawing a particular combination of \( \varphi \) and \( \kappa \) is then given by: \( g(\varphi, \kappa) = g_\varphi(\varphi)g_\kappa(\kappa) \) with \( g_\kappa(\kappa) = \xi \kappa^{-\xi-1} \) and \( g_\varphi(\varphi) = \theta \varphi^{-\theta-1} \), where \( \xi \) and \( \theta \) are the shape parameters of the Pareto distributions. Probabilities of success \( \chi_s \) and of belonging to the groups of domestically active producers and exporters respectively \( \chi_l \), as defined by equation (31), can be expressed as functions of cutoff efficiency levels \( z_l \), for \( l \in d, x \):

\[
\chi_s = \frac{1}{\Psi} z_d^{-\frac{\xi}{\gamma}}; \quad \chi_d = \frac{1}{\Psi} \left( z_d^{-\frac{\xi}{\gamma}} - \frac{z_x^{-\frac{\xi}{\gamma}}}{z_d^{-\frac{\xi}{\gamma}} - \frac{z_x^{-\frac{\xi}{\gamma}}}{z_d^{-\frac{\xi}{\gamma}}} \right); \quad \chi_x = \frac{1}{\Psi} z_x^{-\frac{\xi}{\gamma}};
\]

whereby \( \Psi = \frac{\frac{\partial^2 c}{\partial c(1-\theta)} \cdot \frac{1 - \varphi^{-\theta}}{\varphi^{-\theta} \cdot \varphi^{1-\theta} - 1}}{a \xi (1-\theta) + \xi c(1-\theta)} \). The shares of exporters and domestic firms, \( s_l = \chi_l / \chi_s \), are then given by:

\[
s_x = \left( \frac{z_d}{z_x} \right)^{\frac{\xi}{\xi(1-\theta)}}; \quad s_d = 1 - \left( \frac{z_d}{z_x} \right)^{\frac{\xi}{\xi(1-\theta)}}.
\]
The components of expected profits in equation (35) can be expressed as:

\[ s_d \left( \frac{z_d}{z_d} \right)^{\frac{\sigma-1}{\gamma}} = \Omega \left( 1 - \left( \frac{z_d}{z_x} \right)^{\frac{\xi \gamma - c(1-\theta)(\sigma-1)}{\gamma c(1-\theta)}} \right), \]  

(61)

\[ s_x \left( \frac{z_x}{z_x} \right)^{\frac{\sigma-1}{\gamma}} = \Omega \left( \frac{z_d}{z_x} \right)^{\frac{\xi}{c(1-\theta)}}, \]  

(62)

where \( \Omega = \frac{\xi \gamma}{\xi \gamma - c(1-\theta)(\sigma-1)} \). The free entry condition, as defined by (36), is an increasing function of the cutoff efficiency \( z_d \):

\[ E\pi = \delta f_E \Psi z_d^{(1-\theta)}. \]

For technical reasons, we assume that the Pareto shape parameters are sufficiently large, \( \xi > \frac{c(1-\theta)(\sigma-1)}{\gamma} \) and \( \theta > \frac{\alpha c}{c(1-\theta)} \), such that \( \Omega, \Psi > 0 \). For the further analysis, we define a measure for average efficiency \( \Delta_z \) and average fixed costs \( \bar{f} \) in the economy:

\[ \Delta_z = 1 + \left( \frac{f_d}{f_d + f_x} \right)^{\Lambda_1 - 1} (1 + \tau^{1-\sigma})^{\Lambda_2} \left( 1 - (1 + \tau^{1-\sigma})^{\frac{\alpha c}{\gamma}} \right), \]

\[ \bar{f} = \left( f_d + \left( \frac{f_d}{f_d + f_x} \right)^{\Lambda_1} (1 + \tau^{1-\sigma})^{\Lambda_2} f_x \right). \]

whereas \( \Lambda_1 = \frac{\xi \gamma}{c(1-\theta)(\sigma-1)} \), \( \Lambda_2 = \frac{\xi \alpha c}{c(1-\theta)(\sigma-1)} \). In an autarkic situation, if trade costs are prohibitively high, the share of exporters tends to zero and the efficiency measure approaches one: \( \lim_{\tau \to \infty} \Delta_z = 1 \) and \( \lim_{f_x \to \infty} \Delta_z = 1 \). Furthermore, average fixed production costs are only determined by domestic expenditures: \( \lim_{f_x \to \infty} \bar{f} = f_d \). Combining expected profits and the free entry condition, leads to an explicit solution for the cutoff efficiency level \( z_d \):

\[ z_d = \left( \frac{\Omega (b + \beta) f_d \Delta_z - \beta \bar{f}}{\delta f_E \Psi} \right)^{\frac{c(1-\theta)}{\xi}}. \]

(63)

**Number of active firms**  
As shown by equation (40), the number of active firms in one country is a function of labor supply \( L \) and average revenues as defined by (35). To solve for the number of firms explicitly, we use the expressions for expected efficiencies of domestic firms and exporters (61) and (62). With Pareto distributed capabilities, average revenues can be expressed as:

\[ \bar{r} = \frac{\sigma}{(\sigma-1)\gamma} (b + \beta) \Omega \left[ f_d \left( 1 - \left( \frac{z_d}{z_x} \right)^{\frac{(\Lambda_1 - 1)(\sigma-1)}{\gamma}} \right) + (f_d + f_x) \left( \frac{z_d}{z_x} \right)^{\frac{\xi}{c(1-\theta)}} \right]. \]
Using the relation of domestic and export cutoff efficiency levels:

\[ \frac{z_d}{z_x} = \left( \frac{f_d}{f_d + f_x} \right)^{\frac{\gamma}{\sigma}} (1 + \tau^{1 - \sigma})^{\frac{\delta_x}{\sigma - 1}}, \]

leads to the following solution for the number of active firms in one country:

\[ M = \frac{(\sigma - 1) \nu}{\sigma \Omega (\beta + b) f_d} L \Delta_r^{-1}, \tag{64} \]

whereas \( \Delta_r = 1 + \left( \frac{f_d}{f_d + f_x} \right)^{\Lambda_1 - 1} (1 + \tau^{1 - \sigma})^{\Lambda_2} \left( 1 - (1 + \tau^{1 - \sigma})^{\frac{\gamma}{\sigma}} \right), \) and the number of total varieties in one economy is defined as: \( M_x = (1 + s_x) M. \) Reactions of extensive margins with respect to changes in private benefits \( b \) and interest rates \( \beta, \) as shown in equations (42) and (44), follow immediately from derivations of (64).

9.5 Proofs

Proof of Proposition 3. Deriving the cutoff efficiency \( z_d \) (63) with respect to private benefits, leads to the positive efficiency effect in equation (43):

\[ \frac{\partial z_d}{\partial b} \frac{1}{z_d} = \frac{c (1 - \theta)}{\xi} \frac{\Omega f_d \Delta_x}{\Omega (\beta + b) f_d \Delta_x - \beta f} > 0. \]

Hence, \( \frac{\partial W}{\partial b} \frac{b}{W} > ( < ) \) 0 if \( \frac{\partial z_d}{\partial b} \frac{1}{z_d} > ( < ) \frac{\gamma}{\sigma - 1} \frac{1}{\beta + b}, \) which leads to the following condition, whereas \( \Lambda_1 > 1. \)

\[ \frac{\partial W}{\partial b} \frac{b}{W} > ( < ) \) 0 if \( \frac{1}{\Lambda_1} \frac{\Omega (\beta + b) f_d \Delta_x}{\Omega (\beta + b) f_d \Delta_x - \beta f} > ( < ) \) 1.

Since the LHS decreases in \( b, \) it becomes more likely that the negative welfare effect dominates if financial frictions are high.

Proof of Proposition 4. Analogous to variations in private benefits, we derive the efficiency effect of changes in borrowing rate \( \beta: \)

\[ \frac{\partial z_d}{\partial \beta} \frac{\beta}{z_d} = \frac{c (1 - \theta)}{\xi} \frac{\Omega \beta f_d \Delta_x - \beta \tilde{f}}{\Omega (\beta + b) f_d \Delta_x - \beta f} > 0. \]

The negative variety effect dominates the positive efficiency effect as \( \Lambda_1 \frac{\Omega (\beta + b) f_d \Delta_x - \beta \tilde{f}}{\Omega (\beta + b) f_d \Delta_x - (\beta + b) \tilde{f}} > 1. \) Hence, the adjustment along the extensive margin leads to an unambiguously negative welfare effect, as stated in Proposition 5.
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Figure 1: Iso-efficiency curve in two-dimensional capability space

Figure 2: Zero-profit condition ($z_{ZPC}$) and marginal-access curve ($z_{ICC}$)
Figure 3: Selection pattern in open economy

Figure 4: Financial frictions and quality sorting
Figure 5: Special case 1 - Quality-based sorting

Figure 6: Special case 2 - Cost-based sorting