Why do homogeneous firms export differently?
A density externality approach of trade.*

*preliminary and incomplete

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Abstract

While it is now well understood why the average exporting firm has a higher productivity than a pure domestic firm, the theoretical literature has remained silent on why firms do not enter foreign markets according to an exact hierarchy as predicted by models à la Mélitz. To this aim, this paper proposes a new model of international trade in which firms’ interactions are characterized by density externalities. This type of interactions are closely related to Mean Field Game. After showing that this type of interaction includes monopolistic competition, we plead for a different equilibrium concept allowing firms with identical characteristics to export to different destinations at equilibrium. Thereby we offer a rationale to the findings of Eaton Kortum and Kramarz (2011). Our results are shown to be robust to many different specifications of trade costs, consumer preferences and distribution of firms.

Keywords: Mean field game, dispersion in strategies, international trade, heterogeneous firms.

JEL Classification: F1

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1 Introduction

Over the last decade, the Melitz model (2003) has become the standard framework in international trade. Including heterogeneity in marginal costs in a monopolistic competition model à la Krugman (1979, 1980) with different fixed costs, he explains self selection between firms. Namely, he provides a rationale for a strong empirical evidence that is exporters tend to be more productive than firms that sell exclusively domestically. In this model, with foreign countries ranked by their attractiveness, Melitz predicts an exact hierarchy of trade meaning that if a firm exports to the $k - 1$st most attractive market then it should export to the $k$th most attractive as well. This feature is consistent with the data, on average (see Bernard and Jensen, 1995; Bernard and al., 2007; Mayer and Ottaviano, 2008; Berthou and Fontagné, 2008; among others). A corollary of this prediction is that two firms that have the same productivity should export to the same set of markets (since we admit homogeneity in terms of demand and fixed costs otherwise). However, recent empirical works (Eaton, Kortum and Kramarz, 2011; EKK hereafter) show that this model is not fully consistent with the data. While it is true that the average productivity is decreasing in the market access, firms do not enter markets according to an exact hierarchy. In particular, two identical firms in terms of their productivity can export to different markets.

In this paper, we try to reconcile both evidences without adding other degree of heterogeneity. We provide a simple and tractable model of export choice in which two identically productive firms can export in different locations. The body of the paper shows a model with homogeneous firms as our main purpose is to explain heterogeneity in strategies between identical players. By comparative statics, we show that our model remains consistent with the strong empirical fact that say that, on average, less attractive markets are served by the most productive firms. This paper provides a short run model of export choice that explains why homogeneous players can choose different strategies. This model consists in two stages where firms choose i) a unique destination to export ii) compete monopolistically in this destination with others exporters that have chosen the same destination. We show that, in the short run, the second stage provides local "density externalities" meaning that the profit of each firm is negatively affected by the entry of others. Namely, it simply means that profits in a given location is decreasing with the number of competitors in this location (here, the firms that choose the same destination). Solving the entire model by backward induction, this presence of "density externalities" allows us to solve the first stage following a Mean Field Game (MFG hereafter). With a unique
destination choice, firms face a trade-off. They want to export in close countries in order to avoid paying large transport costs, but at the same time they want to escape competition. By anticipating the equilibrium, firms know that closest destinations are precisely those where competition will be tougher. Hence, there is an incentive to export in remote countries in order to escape competition but at the same time, another incentive exists and forces firms to export in close countries. At the equilibrium, this trade-off leads to a dispersion of strategies without any dimension of heterogeneity. This dispersion is explained by the type of interactions that operates at the first stage.

Moreover, as we present the simplest version of the model with homogeneous firms, we proceed by comparative statics in order to show that the equilibrium generates an average self selection. By varying marginal cost, we show that the more productive are the firms, the further they export on average. This feature of the model is consistent with the empirical evidence showing that trade does not follow an exact hierarchy linking productivity of firms and attractiveness of export markets.

In the recent literature, two models (EKK, 2011 and Chaney, 2011) deal with the issue that we address. In both papers, homogeneous firms in terms of marginal cost can serve different sets of foreign markets. The common feature of these two papers is that they introduce at least a second dimension of heterogeneity between firms that helps to explain heterogeneous choices in markets served conditional on their productivity. In EKK model, the second dimension of heterogeneity that is added to the standard Melitz framework is market and firm-specific heterogeneity in entry costs and demand. They do so by the incorporation of Arkolakis’s (2008) formulation of market access. It implies that each firm draws a specific demand and specific fixed cost in each foreign market. At the end of the day, even if two firms are homogeneous in terms of marginal cost, they may differ from the demand addressed to them or from the fixed cost that they have to incur in each foreign market. This feature drives the fact that two identically productive firms take different choices regarding the decision to enter a given market. Only the one with the highest market-specific component of demand and/or lowest market-specific component of fixed cost enters.

Chaney also adds a second dimension of heterogeneity between firms. He argues that firms can meet trading partners in two ways. First by direct search which is modelled as a geographically biased random search in his model. Second, once a firm has acquired trading contacts in some foreign locations, she can develop a new network from these locations. So, in the Chaney’s model, firms differ in their
ability to develop a network of consumers in a given market. Firms of equal productivity can develop networks in different countries which explains heterogeneous set of countries in the export decision. This again implies that two identical firms in terms of productivity may take different decisions. Firms export in those markets in which they are able to develop their network, but primarily this ability is randomly distributed.

In both of these papers, heterogeneity in strategies is explained by an additional dimension of heterogeneity. Even if two firms are identical in terms of marginal costs, they do not share same market-specific demand and fixed cost (EKK) or same ability to develop their network of trading partners (Chaney). Unlike these recent contributions, our model provides an endogenous explanation, in the short run, of this recently discovered feature of international trade. Therefore, our model provides a rationale explaining why homogeneous players choose different strategies.

We propose a short run model of export choice. In a two steps model where firms choose first a unique destination to export and second the quantity to sell in this country, we show that firms have incentives to export differently in order to escape competition. First, the motivation of choosing a unique export destination in this short run model is based on recent evidences on sequential exporting. Defever, Heid and Larch (2010) and Albornoz, Calvo Pardo, Corcos and Ornelas (2012) pay attention on dynamics of exports or more generally expansion at the firm level. They show that export choice of firms features a sequential entry into foreign markets respectively with chinese and argentinian data. In our model, the choice of export destination is found following a mean field game. We show that with a second stage where firms compete monopolistically in quantities profits function in the short run display sufficient characteristics to play such a game in the first stage, as the resolution of the entire model is backward.

More precisely, we consider a world composed of a continuum of countries ordered in terms of attractiveness. On a Hotelling line, we consider a continuum of mass one of homogeneous firms, selling different varieties of a homogeneous good, all placed at 0. Therefore, all given $x \in [0; X]$ stands for a given country with 0 assumed to be the domestic country. In this context, $x$ captures distance with the domestic country and is an inverse measure of market potential or, more exactly a measure of multilateral resistance. Each firm $i$ plays to the following two stages game: first, she decides a unique destination $x$ to serve by export, second, she decides the quantity to export in the selected country knowing she engaged in a monopolistic competition with others exporters that have chosen the same destination. At the second stage,
profits in each country are positive and depends negatively on both distance of the destination (for any given positive transport cost) and the number or the density of others firms that choose the same destination, namely the number of competitors. So, the further the destination is, the higher the trade cost and the lower the profit. At the same time, the higher the density of firms in a given location, the tougher the competition in this market and then, the lower the expected profit. Hence, the form of the profit derived from stage two of the game displays local interactions meaning that each firm is negatively affected by the number of firms that have taken the same decision. This is the notion of "density externalities". This local interaction allows us to use a methodology closely related to MFG at the first stage. An interaction of mean field type is defined as situation where agents take their decision upon a given statistics (here the distribution of exporter according to their export choice) but each agent cannot influence by herself (but marginally) this statistics and then cannot influence by herself strategies of others players. However, each agent contributes marginally to the statistics used by agents to choose their strategy. As a consequence, each agent is atomized in the continuum and chooses a strategy that depends on the distribution of players' characteristics. MFG simplify interactions between agents. Instead of taking into account the huge set of possible interactions between agents (as done in oligopolies where interactions are strategic), players consider the influence of others as summed up by the distribution. So we apply this methodology to our export choice model. As already said, firms’ profits are negatively affected by both distance and density of others that choose the same destination. The model features an escape competition effect meaning that firms dislike to serve the same market as their competitor: this is the local interaction that we need for the resolution of the first stage. At the equilibrium, the latter leads to a dispersion in strategies: homogeneous firms serve different locations. At the same time, we endogenize the support of the distribution of locations. By doing so, we find a threshold country $x^*$ above which no firms exports. This threshold is an increasing function of productivity which means that, by comparative statics, our model is also consistent with the fact that, the most remote countries cannot be served by low productive firms. Moreover, the slope of the distribution is also increasing with productivity, meaning that, on average, more productive firms export further. Somehow, our model reveals an average self selection. We provide a rationale for EKK findings.

With this short-run model we argue that Melitz model fails to explain non-exact hierarchy observed in the data because it is derived in the long-run. With free entry condition, all identical firms play identically. Since profits are driven to zero, there is
no room for dispersion in strategies among homogeneous. We show that our model, derived at the long run leads to same conclusion as Melitz, namely trade follows an exact hierarchy between productivity and market access. Our proposition is such that what we observe in the data is a snapshot of the convergence to the long-run of the Melitz model.

The remainder is organized as follow: section 2 provides some clarification about MFG, section 3 presents the body of the model, section 4 presents and discusses the results at the short-run while section 5 discusses what would be the outcome of our model at the long-run.

2 Some words on Mean Field Game

There are two strands in the literature of MFG. The bulk of the theory focuses on dynamical games (see Larsy and Lions (2006, 2007), Lions (2006-2012), Gueant (2009, 2011), Lachapelle (2011)). However, a static game has been developed (see Lions (2008), Blanchet and al. (2011) and Blanchet and Carlier (2012)). Let us introduce this latter framework in few words. In a static context, MFG relies on the following assumptions

(i)- many homogeneous agents.

(ii)- weekly dependence: agents have infinitesimal influences.

(iii)- anonymity: agents do not know the identity of other agents.

(iv)- mean field interaction between agents:
the payoff function of agents depends on its action and on the density of other agents choosing the same action.\footnote{In this case, we say that the payoff function of agents exhibits density externalities (see Bruceckner and Larguey (2008) for empirical proof of this notion).}

More precisely, let us consider a continuum of homogeneous agents represented by a unit mass. Let be $X$ a common set of actions. $X$ is a compact subset of $\mathbb{R}^d$ with $d$ the number of dimensions. Agents are distributed according to a distribution $\mu : X \leftrightarrow \mathbb{R}_+$ with $\int_X \mu(x)dx = 1$. Let also be $\mathcal{M}(X)$ the set of absolutely continuous distributions on $X$ with respect to the Lebesgue measure. Agents are endowed with
the same continuous payoff function denoted by $\pi : X \to \mathbb{R}$. Agents has to choose an action in order to maximize their payoff function, that is

$$\max_{x \in X} \pi(x)$$

To be clearer, let us consider the following simple example: $X = [0, 1]$ and

$$\pi(x) = a - bx - c\mu(x)$$

with $a, b, c \in \mathbb{R}$.

Moreover, $\pi$ is assumed to be a differential of a potential functional $E : \mathcal{M}(X) \to \mathbb{R}$ in the following sense: $\forall \mu, \tilde{\mu} \in \mathcal{M}(X)$,

$$\lim_{\epsilon \to 0^+} \frac{E[\mu + \epsilon(\tilde{\mu} - \mu)] - E[\mu]}{\epsilon} = \int_X U(x)[\mu(x) - \tilde{\mu}(x)] dx$$

(3)

In our simple example, we have

$$E[\mu] = \int_X (a - bx)\mu(x)dx - \int_X \frac{c\mu(x)^2}{2}dx$$

(4)

In this context, we can define and characterize an equilibrium.

**Definition 1.** Lions (2008) A distribution of agents $\mu \in \mathcal{M}(X)$ is an equilibrium if

$$\text{Supp}(\mu) \subset \arg\max_{x \in X} \pi(x)$$

(5)

or equivalently, a distribution of agents $\mu \in \mathcal{M}(X)$ is an equilibrium if

$$\int_X \pi(x)[\mu(x) - \tilde{\mu}(x)] \geq 0$$

(6)

for all $\mu, \tilde{\mu} \in \mathcal{M}(X)$, or equivalently, a distribution of agents $\mu \in \mathcal{M}(X)$ is an equilibrium if there exists $\pi$ such that

$$\begin{cases} 
\pi(x) \leq \overline{\pi} & \text{for almost every } x \in X \\
\pi(x) = \overline{\pi} & \text{for almost every } x \in X \text{ and } \mu(x) > 0
\end{cases}$$

(7)

These three definitions are equivalent. The first definition says that an equilibrium is a distribution of actions or agents where each action that belongs to this distribution is a best response to others actions summarized by this distribution. This equilibrium is related to the notion of a generalized Nash equilibrium (see Debreu (1952)). The second definition shows that the equilibrium is a variational...
inequality. The third definition states that an equilibrium is a situation where \( \pi \) achieve its maximum value \( \pi \) on the support of \( \mu \). In other words, an equilibrium is a situation where agents reach the same utility function \( \pi \). In this case, unilateral deviations of agents are impossible because agents are indifferent.

**Proposition 1. Existence Lions (2008)** If \( \pi \) is a differential of \( E \) and if the potential functional \( E \) is concave, there exists an equilibrium \( \mu \).

**Proposition 2. Uniqueness Lions (2008)** If \( \pi \) is a differential of \( E \) and if the potential function \( E \) is strictly concave, the equilibrium is unique. Equivalently, if \( \pi \) is strictly decreasing in \( \mu(x) \) in the sense that \( \forall \mu, \tilde{\mu} \in \mathcal{M}(X) \)

\[
\int_X \left[ U(x) - \tilde{U}(x) \right] [\mu(x) - \tilde{\mu}(x)] < 0 \tag{8}
\]

the equilibrium is unique.

Finally, the equilibrium is determined solving the following system

\[
\begin{align*}
\pi(x) &= \pi \\
\int_X \mu(x) dx &= 1 \\
\mu(x) &\geq 0
\end{align*}
\tag{9}
\]

Thus, a equilibrium consists in finding a density of agent \( \mu(x) \) so that each agent obtain the same pay-off level \( \pi \). In our simple example, we obtain

\[
\mu(x) = \left( \frac{2b}{c} \right)^{\frac{1}{2}} - \left( \frac{b}{c} \right) x \tag{10}
\]

for all \( x \) in \( \left[ 0, \left( \frac{2b}{c} \right)^{\frac{1}{2}} \right] \).

We will see that we can apply this methodology in a model of trade. Our model displays a reduced form of profit which is a negative function of the number of firms that export in the same country (namely, profit decreases with the number of competitor in each destination). In a two stages game where the first consists in choosing a unique destination, we show that it appears tractable and realistic to introduce mean field interactions between firms. The second stage, which consists in setting quantity once destination is chosen is simply monopolistic competition.

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\(^2\)Notice that this method is similar to a method in labour economics to obtain wage dispersion (Burdett and Mortensen (1998), Mortensen (2003)) and in models of interacting social agents (Lemoy, Bertin and Jensen (2011)).
3 The Model

In what follows, we build the simplest model where homogeneous firms may export differently. We assume two goods in the economy. One homogeneous produced under perfect competition in all countries and one horizontally differentiated produced by firms that are assumed to be homogeneous in terms of marginal cost. Of course, the degree of differentiation between the varieties that they sell is a room for heterogeneity but here, as it is usual in trade models, we assume that each firm faces the same individual demand in each country. In this sense, we state that firms are homogeneous.

In this model, we assume that all the firms are located in 0 on a Hotelling line and each point on this line represents a country. All countries are assumed to be strictly identical in terms of demand and population. Countries differs in their attractiveness. To export in country \( x \), a firm has to pay a transport cost \( \tau x \) for each unity of good exported. As we assume no local firms in each \( x > 0 \), the attractiveness of the country is only captured by its distance to 0 where all firms that operate are located.\(^3\)

Firms play in two steps:

1. They choose a unique country \( x \) to export following a mean field game

2. They compete in quantity in the country \( x \) with all firms that export in this location.

Our model is solved in the short run. The fact that firms choose a unique destination forbids free entry and then profits are positive at the equilibrium.\(^4\) Notice that the model can be derived by assuming that each firm chooses a number \( n > 1 \) of destinations. Instead of choosing a scalar, firms would choose a vector of size \( n \) of countries. Results would be the same in nature and model would stay the same but adding intractability.

3.1 Firms

In 0, we assume a continuum of firms. Each of them produces a variety of an horizontally differentiated good. We denote respectively by \( q_i(x) \) and \( Q(x) \) quantities

\(^3\)More exactly, \( x \) captures the relative attractiveness of countries from the point of view of firms located in 0. Thus \( x \) captures more precisely the notion of multilateral trade resistance.

\(^4\)For an empirical motivation see Albornoz and al. on sequential exporting.
sold at the equilibrium in country $x$ for the variety $i$ produced by a monopolistically competitive firm and for the variety available in the country $x$. Hence, we have:

$$Q(x) = \int_s q_s(x) ds$$

As our main purpose is to explain why homogeneous firms can export differently, we assume that all firms are homogeneous in terms of marginal cost. We assume this marginal cost to be equal to $c$. Notice that one can introduce heterogeneity in marginal cost in this model but it adds complexity and does not bring others interesting implications. As there is a continuum of firms, interactions are not strategic. Thus, firms would take their decision with respect to average productivity. The result would simply be that more productive firms export further on average, which can be clearly seen by comparative statics in the context of homogeneous firms.

As already said, firms choose first a unique destination to serve and then the quantity to export in this destination. By denoting $\mu(x)$ the distribution of firms that choose to export in $x$, the entire program of the firm $i$ is:

$$\max_{x, q_i(x)} \pi_i(x, \mu(x), q_i(x))$$

The first stage of the model consists in choosing $x$ anticipating $\mu(x)$ and the second stage consists in choosing $q_i(x)$. In the present paper, we solve this program with quadratic utility function and linear transport cost but this model can support any specification of demand and of transport cost. The only restriction that is needed for the model to be solvable is to provide reduced forms that display profits made in $x$ as decreasing functions of $\mu(x)$ i.e. profits in $x$ as negative functions of the number of firms that sell in this country.

### 3.2 Demand

Let us consider a world composed of a continuum of countries indexed $x \in [0, X]$. 0 is assumed to be the home country while, by definition, the other countries are said to be foreign. Foreign countries differ in their distance to the home country and then, $x$ is an inverse measure of market potential. Countries are strictly homogeneous in terms of demand. Their population size is normalized to 1 and the representative consumer displays everywhere same preferences. The upper-tier utility function is quasi-linear:

$$U_x(z, q) = z + \mathcal{U}(q(x))$$
With $z$ the consumption of an Hicksian composite good produced under perfect competition and used as numeraire. We define $q(x)$ as the vector of consumption of varieties of the horizontally differentiated good: $q(x) = (q_s(x))^{S_x}_{s=0}$, with $S_x$ the number of varieties available in country $x$ which will be endogenously determined by MFG through $\mu(x)$.

The lower-tier utility of consuming the differentiated good is given by a continuum quadratic utility function:

$$U(q(x)) = \int_{S_x} (aq_s(x)ds - \frac{b}{2}q_s(x)^2)ds - \frac{\gamma}{2}(\int_{S_x} q_s(x)ds)^2$$

As parameter $a$ represents the consumer’s intrinsic evaluation of differentiated good we assume it to be the same for all varieties which implies that firms sell a good that is differentiated only horizontally. The parameter $b$ reflects substitutabily between varieties. It is assumed to be a constant and equal for each variety meaning that no variety is more or less substitutable to another. Finally, $\gamma$ captures demand linkage between varieties. A higher $\gamma$ simply means that varieties are less differentiated and the marginal utility of consuming an unit of variety $i$ decreases more rapidly with consumption of any variety $j \neq i$. We assume $a, b, \gamma$ to be the same for each variety. As a consequence, the demand function addressed to each single firm will be strictly the same in each country.

With $I$ the exogenous income of consumers, the budget constraint is the following:

$$z + \int_{0}^{S_x} p_s(x)q_s(x)ds \leq I$$

Knowing that with this kind of preferences, the marginal utility of income can be normalized to one (see Spence, 1976 and Neary 2009 for further explanations), each firm faces the same downward slopping demand function in each $x$:

$$p_i(x) = a - bq_i(x) - \gamma Q(x)$$

With $Q(x)$ the total quantity sold in $x$, taken as given by each firm. In this sense, firms compete monopolistically at the second stage of the model (when they choose their quantity sold at the equilibrium once destination has been chosen).
4 Results

The model is solved by backward induction. We show that monopolistic competition at the second stage between firms that export in $x$ provides reduced form of profits that display sufficient features for choosing a single destination following a mean field game. Namely, interactions are local in the sense that the profit of each firm that export in $x$ depends negatively on the number $\mu(x)$ of firms that export in this destination. Solving the first stage consists in finding $\mu(x)$.

4.1 Stage 2: Monopolistic Competition

The second stage of the model is usual. We assume monopolistic competition between firms in each country. Hence, as there are $\mu(x)$ firms in $x$, which has been found in stage 1, each firm take this number of firm as given. By doing so, they do not anticipate their impact on aggregates, thus there is no strategic interactions in this model.

The equilibrium in stage 2 simply consists in finding optimal quantities in $x$ for each firm that exports in this country. Then, the equilibrium is solved by the following program for each firm exporting in destination $x$:

$$\max_{q_i(x)} \pi(x) = (p_i(x) - c - \tau x)q_i(x)$$

Each firm take the aggregate $Q(x)$ as given. It leads to the maximizing quantities:

$$q_i(x) = \frac{a - c - \tau x - \gamma Q(x)}{2b}$$

As all firms are homogeneous, all firms that export in the same destination choose the same quantity to export at the equilibrium. Let this quantity be $q(x)$. Hence, the total quantity sold in each country is simply $\mu(x)q(x)$. Quantities exported in $x$ by each firm that has set $x$ as a destination are:

$$q(x) = \frac{a - c - \tau x}{2b + \gamma \mu(x)}$$

Not surprisingly quantities at equilibrium are decreasing in both $x$, namely the distance of the destination, and $\mu(x)$ the number of competitors that a firm faces in destination in which she exports.

With this quantity, the equilibrium profit made by a firm that exports in $x$ is simply:

$$\pi(x) = bq(x)^2$$
Clearly, the profit at the equilibrium in $x$ is a decreasing function of $\mu(x)$. Namely, the more numerous are firms that export in $x$, the lower the profit made by each firm that export in $x$. Of course, for strictly positive transport costs the further the destination, the lower the profit. Hence firms want to export in close destination in order to avoid to pay large transport cost, but at the same time, they want to escape competition, i.e. export in countries where few firms export. $\mu(x)$ is found by solving stage 1.

### 4.2 Stage 1: Mean Field Game

From the reduced form of profit given by the resolution of stage 2, it appears a trade-off from the point of view of the firm. The stage 1 is solved following a mean field game, meaning that firms anticipate and take as given $\mu(x)$ in each country. The trade-off consists in the will to export in close country and the will to escape competition. As transport costs are lower for close country, firms anticipate that more firms will export in these countries. Thus, to escape competition, firms have an incentive to export in more remote countries.

Stage 1 of the game consists both in finding a distribution $\mu(x)$ but also the size of the support of this distribution. Namely, we have to find a threshold $\bar{x}$ above which no firm that produces with the marginal cost $c$ export. This threshold immediately provides the support of the distribution which is $[0; \bar{x}]$.

The equilibrium of stage 1 is found by solving the following system:

$$\begin{align*}
\pi(x) &= \pi \\
\int_{0}^{\bar{x}} \mu(x) dx &= 1 \\
\mu(x) &\geq 0
\end{align*}$$

Where $\pi \in \mathbb{R}^+$. 

The first equation of the system means that an equilibrium in such a game is a situation where each firm receives the same total profit wherever she exports. The intuition is the following: at the equilibrium, each agent receives the same pay-off because two parts of a trade-off are compensated each other. As already said, the trade off appears here between the willingness to export in the most attractive markets (the closest ones) in order to avoid transport costs but at the same time, firms anticipate that these markets are coveted by competitors and then know that in these markets competition will be tougher. Then, firms also have an incentive to
export further away in order to escape competition. The equilibrium is the situation where both effects are such that all identical firms receive the same total profit even if they choose different strategies.

The second equation of this system ensures that all firms make a decision on the endogenous support. The third simply imposes the distribution to be positive everywhere.

With the first condition of the equilibrium we immediately find:

\[ \pi(x) = \bar{\pi} \iff \mu(x) = \frac{a - c - \tau x}{\gamma \sqrt{\frac{\pi}{b}}} - \frac{2b}{\gamma} \]

The size of the support defined by the threshold country \( \bar{x} \) further which no firm export is the first country for which this distribution is equal to zero, namely no firm export in this country. As \( \mu(x) \) as defined above is strictly decreasing in \( x \) for every positive transport cost, then \( \bar{x} \) is unique.

\[ \mu(\bar{x}) = 0 \iff \bar{x} = \frac{a - c - 2b \sqrt{\bar{\pi}}}{\tau} > 0 \]

Unsurprisingly, this threshold \( \bar{x} \) is decreasing in \( \tau \) meaning that a trade liberalization allows firm to export in further countries. \( \bar{x} \) is also decreasing in \( c \) which implies that, through comparative statics, in this model more productive firms are able to export further.

Now, we use the second equation of the system 11 which ensures the cumulative to be equal to one.

\[ \int_0^{\bar{x}} \mu(x) dx = 1 \iff \left( a - c - 2b \sqrt{\frac{\pi}{b}} \right)^2 = 2\tau \sqrt{\frac{\pi}{b}} \]

This equation admits a unique positive solution for \( \sqrt{\frac{\pi}{b}} \) which is:

\[ \sqrt{\frac{\pi}{b}} = \frac{[4(a - c)b + 2\tau]^2 + \sqrt{[4(a - c)b + 2\tau]^2 - 16(a - c)^2b^2}}{8b^2} \]  

This solution is a decreasing function of marginal cost. Meaning that the endogenous equilibrium total profit \( \bar{\pi} \) is increasing with the productivity. Reintegrating this solution in \( \bar{x} \) gives:

\[ \bar{x} = \frac{4(a - c)b - [4(a - c)b + 2\tau]^2 + \sqrt{[4(a - c)b + 2\tau]^2 - 16(a - c)^2b^2}}{4b\tau} \]  

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\( \bar{x} \) is itself a decreasing function of both \( c \) and \( \tau \). Meaning that the higher the productivity (low \( c \)) the longer the support of the distribution of \( \mu(x) \). Same comment applies for trade liberalization.

Finally we get the following form of the distribution of export choice of firms:

\[
\mu(x) = \frac{a - c - \tau x}{\gamma y} - \frac{2b}{\gamma} \tag{14}
\]

with \( y = \sqrt{\frac{\pi}{b}} \) as defined in 12 which is a function of parameters. Notice that as both \( q(x) \) and \( \mu(x) \), which capture respectively quantity sold by firms in \( x \) and the number of firms that export in \( x \) are decreasing functions of \( x \) both intensive and extensive margin decrease with distance that is consistent with the findings of Bernard, Jensen, Reading and Schott (2007). Moreover it means that per-unit profit is increasing in distance at the equilibrium. As an equilibrium in this model is defined as a situation where all firms obtain the same total profit and quantity is decreasing in distance thus per-unit profit must be higher for firms that export in more remote countries. This is a consequence of the decreasing shape of \( \mu(x) \).

With a distribution of export choice that is decreasing in distance then competition is lower in more remote countries. This leads to a higher price.

In figure 1 we show two distributions with two different marginal cost.\(^5\) The solid line depicts a distribution with a higher marginal cost than the dotted line.

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\(^5\)Values of parameters are set randomly. We show graphics in order to illustrate analytical results of equations 13 and 14. We just give values that ensure positive profit at the equilibrium.
Looking at the equation 14 and the figure 1, the first appealing result is that the distribution of firms with respect to their destination choice is not degenerated and far from a Dirac mass. This means that homogeneous firms in terms of productivity export differently in this short run. This feature of the result is explained by the presence of density externalities stemming from the monopolistic competition at stage 2. In this model, firms have incentive to export far and then to pay high transport costs in order to escape competition. At the same time, the distribution is a negative function of distance $x$ (which here, once again, is an inverse measure of the attractiveness of the country) because it remains a strong incentive for firms to export nearer in order to minimize transport costs. This is this common incentive that creates a negative competition effect lowering the quantity sold by each firm and thus profits when competitors are numerous. In other words, firms face the same trade-off: either export near to minimize transport costs but facing tougher competition or export further away to escape competition incurring higher transport costs. This trade-off explains the dispersion of strategies at the equilibrium. Thus, interestingly, when densities determine the outcome of agents, the dispersion is explained by the fact that agents are identical and not by heterogeneity.

We also see that $\bar{x}$ is a decreasing function of $c$, meaning that the higher the productivity the further firms export, not only on average. The average selection effect can seen through the slope of $\mu(x)$ which is decreasing in $x$ and lower for low $c$. In other words, the influence of distance is lower for high productivity. A decrease in $c$ leads to an increase in $y$. As $\mu(x)$ is negatively correlated to $x$ through $-\frac{x}{\gamma y}$, then the distribution is less negatively correlated to distance for low $c$. The same comment applies to $\bar{x}$ which is also negatively correlated to $c$. Hence, the model shows how firms can enter in markets according to a non exact hierarchy in the short run. The slope of the distribution is also negatively correlated to $\gamma$. A high $\gamma$ tends to make competition tougher as the negative influence of $\mu(x)$ profits at the equilibrium is higher for a high $\gamma$ and thus, the incentive to escape competition is higher which tends to make the slope more flat and diminishes the influence of distance. Notice that an increase in $a$ which can be interpreted as a higher intrinsic quality of goods leads to a lower influence of distance. Namely, an increase in $a$ has exactly the effect than a decrease in $c$, meaning that raising quality or productivity is equivalent. What matters is the mark-up captured by $a - c$.\textsuperscript{6}

Figure 2 displays two distribution with two different transport costs which allows us to analyse by comparative statics the effect of trade liberalization in this model.

\textsuperscript{6}See Di Comite and al. (2010) for discussion.
Unsurprisingly, firms go further with lower transport costs. We clearly see that \( \bar{x} \) is higher when transport costs are lower. We also see that firms tend to export further on average. \( \mu(x) \) is flatter which has a twofold consequence. First further countries are host more exporters but close countries see a lower number of export.

\[ W_x(\mu(x), q(x)) = \int_{\mu(x)}^{b} \frac{b}{2} q(x)^2 \]  

(15)

Figure 2: Firms distribution w.r.t. distance

The consequence of a trade liberalization can be analysed in terms of welfare. In every \( x \), welfare is defined as: \(^7\)

First of all, notice that the less attractive the country, the lower the surplus. Of course this result is entirely driven by the fact that we assume no local firms in all countries \( x \neq 0 \) but we can point out the fact that even with local firms, the surplus would be lower in less attractive markets because less firms would export in these countries and each firms that exports would sell lower quantities. Second, a trade liberalization has not the same impact on each country. Welfare is increasing in both \( \mu(x) \) taken as the number of available varieties in country \( x \) and \( q(x) \). For remote countries, a trade liberalization leads unambiguously to a higher welfare: it increases both \( \mu(x) \) and \( q(x) \). For close economies, things are more ambiguous. On one hand, it tends to raise \( q(x) \) which enhances welfare but on the other hand, it tends to decrease \( \mu(x) \). The intuition is the following: a fall in trade costs causes a shift in the distribution of firms according to the destination that they choose to

\(^7\)With no local firms, welfare is the consumers surplus.
export. The outcome becomes flatter meaning that some firms desert close countries for remote countries. At the extreme, when $\tau = 0$, the distribution is uniform, the quantity sold by each firm is the same and then the consumer surplus is strictly the same in all countries. In this context, trade liberalization tends to equalize consumer surplus across countries. It is harmful for the most attractive countries but beneficial for less attractive.

4.3 Comments

To be solvable the model needs only two characteristics: the second stage of the game must provide a reduced form of profits that is i) a negative function of distance and ii) a negative function of the density of players that choose the same strategy. Namely, transport costs must be positive and demand function must be downward slopping. These two characteristics lead to the trade-off that face firms in the model above: firms like both exporting close in order to avoid to pay transport costs and escape competition.

Summing up, the only conditions required are:

$$\pi(x) = \pi(x, \mu(x))$$

with $\frac{\partial \pi}{\partial x} < 0$ and $\frac{\partial \pi}{\partial \mu(x)} < 0$

Thus, our model is highly tractable and support any specifications of transport costs and demand that respect these two conditions which seem natural. These conditions do not only allow the model to be solvable but keep the nature of results unchanged. Since the trade-off exists dispersion in strategies between identical players is the outcome.

5 Discussion: Long Run Equilibrium

The model presented above is a short run where, with no free entry, profits are not driven to zero. This allows us to have positive profits at the equilibrium and to apply our methodology at the first stage. This model suggests that the Melitz model fails to explain non exact hierarchy because it is a long run model. We argue that what we observe in the data is a snapshot of the convergence to the equilibrium of the Melitz equilibrium. Our model, extend to long run, tends to have same conclusions as Melitz.
Suppose that we add periods to our model. At each period $t$, firms choose a unique destination and carry on to export in the destinations chosen in $t - n, n = [1, ..., t - 1]$. Since profits remain positive firms do not have incentive to quit markets in which they export before. At each period $t$, firm $i$ faces in each $x$ the following downward slopping demand function:

$$ p_{i,t}(x) = a - bq_{i,t}(x) - \gamma Q_t(x) $$

with $Q_t(x) = \left( \sum_{n=1}^{t-1} \mu_{t-n}(x) + \mu_t(x) \right) q_t(x)$

The definition of $Q_t(x)$ is the number of firms that export in $x$, namely those which exported before ($\sum_{n=1}^{t-1} \mu_{t-n}(x)$) and those which take the decision to export in $x$ at $t$ ($\mu_t(x)$). At the stage 2, the quantity sold in $x$ is, taken into account $Q(x)$. The stage 1 at each period $t$ consists in finding $\mu_t(x)$ taken $\sum_{n=1}^{t-1} \mu_{t-n}(x)$ as given.

The outcome of such a model at the long run (i.e. when $t \to \infty$) is the following: quantities exported in each country for each firm tend to decrease when we accumulate periods. Total profits decrease as time runs and the model tends to a long run equilibrium with zero profits. At the long run equilibrium, all firms export in all countries between 0 and $\bar{x}$ which, with a fixed cost is equivalent to Melitz. Moreover, $\bar{x}$ is a decreasing function of $c$. Hence, the more productive the firm, the further she export and she export in all countries $x < \bar{x}$. It is equivalent to an exact hierarchy of trade at the long run. An interesting prediction of this is that homogeneous firms tend to have similar strategies as time runs.

### 6 Conclusion

With a simple model of trade including mean field interactions in the choice of destination, we explain why identical firms may serve different markets at the short run. This result is not driven by an additional dimension of heterogeneity between firms as EKK (2011) and Chaney (2011). On average, less attractive markets remain served by more productive firms. A corollary to this is that trade does not follow an exact hierarchy, which is consistent with recent empirical findings.

In a two stages model where firms choose first a unique destination to serve and compete monopolistically in this destination we show that, at the short run, monopolistic competition exhibits density externalities. This presence of density externalities allows us to solve the first stage following a MFG. This model is highly tractable and requires, to be solvable and to keep the same nature of results, only two characteris-
tics: profits at the equilibrium must be decreasing functions of both distance (that captures here attractiveness) and of competition degree. Namely, the more numerous the players that choose the same strategy, the lower the pay-off obtained by playing this strategy. The outcome of this model is dispersion in strategies among homogeneous players. By doing so, we provide a rationale for EKK findings. Here, homogeneous firms may export endogenously in different locations. The dispersion of strategies is not driven by *ad hoc* additional dimension of heterogeneity.
References


