Innovation and the size of exporting firms

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Abstract

This paper contributes to the literature explaining firm-level heterogeneity in the extensive margin of trade, defined as the number of products exported by each firm. More specifically, we develop a model describing the dynamics of both the intensive and the extensive margin of firm exports. The former is assumed to depend on an exogenous preference shock on the demand side, whereas firms must invest in R&D to maintain and increase their portfolio of goods. This mechanism governs the extensive margin of trade. Our model predicts a lognormal distribution for the intensive margin of trade and a Pareto distribution for the number of products exported by each firm. This second result is obtained from the dynamics of firm entry and exit and of the process of product innovation by new and incumbent firms, whereby the probability to capture new products is a function of the number of varieties already exported. Finally, we show that our model is consistent with a number of empirical findings recently emerged in the empirical literature, and provides evidence based on a large dataset of French firms.

Keywords: International Trade, Extensive Margin, Innovation, Dynamic Growth Model, Preferential Attachment.

JEL classification: F14, F43, L11, O3

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1 Introduction

Increasing availability of firm-level data has taught us that firm engagement in international markets differ widely. Empirical evidence suggests that the cross-sectional heterogeneity is primarily explained by the extensive margin, i.e. the difference in the number of products exported and/or destinations served by either countries or firms (Bernard et al, 2009).

In a recent contribution, Chaney (2011) discusses how existing trade models featuring heterogeneous firms (such as Melitz, 2003; Bernard et al, 2003; Chaney, 2008) are unable to make any prediction on the cross-sectional distribution of the extensive margin. Focusing on the number of destinations served by each exporter, he proposes a model based on social network theory that accurately matches the empirical features of the data.

This paper contributes to the literature on the extensive margin of export (see Arkolakis and Muendler, 2010, for another example) by looking at the other component of the extensive margin of trade, i.e. the number of products exported by each firm. In addition, we provide a simple explanation for the very skewed distribution that characterizes also the intensive margin of trade and, by combining the two, we end up with a simple explanation for the size distribution of exporting firms. More specifically, we develop a model in the spirit of Simon (1955); Klette and Kortum (2004a); Luttmer (2007) that describes the dynamics of both the intensive and the extensive margin of firm exports. The former is assumed to depend on a exogenous preference shock on the demand side, whereas firms must invest in R&D to maintain and increase their portfolio of goods. This mechanism governs the extensive margin of trade.

Our setup predicts a lognormal distribution for the intensive margin of trade and a Pareto distribution for the number of products exported by each firm. This second result is obtained from the dynamics of firm entry and exit and of the process of product innovation by new and incumbent firms, whereby the probability to capture new products is a function of the number of varieties already exported. Finally, we show that our model is consistent with a number of empirical findings recently emerged in the empirical literature, and provides evidence based on a large dataset of French firms.

The paper is organized as follows: next section provides a quick glance at the data as a motivation of the paper. Section 3 presents the model, while Section 4 provides some formal test of its main predictions. Section 5 concludes.

2 On the skewed distributions of the intensive and extensive margins

Figure 2 shows the distribution of the extensive margin based on a large sample of French exporting firms (that covers more than 100,000 firms). Irrespective of whether we look at the number of products exported (left panel) or at the combination of product-destination pairs (right panel), the distributions appear very skewed, and a power-law fit provides a good approximation of the data.\footnote{The power-law fit is obtained using the methodology described in Clauset et al (2009).}

Table 1 provides summary statistics that further characterize the very large heterogeneity in the data. While the number of products exported (according to the 8-digit Combined Nomenclature) ranges between 1 and 4,140, 39.24% of firms export a single product, the median value is 2 and less than 15% of firms export more than 10 products.

Finally, Figure 2 shows the distribution of export sales (in logarithms) of products by different firms for 2003: the distribution resembles a truncated normal.

The empirical regularities presented here appear robust to the specific year analyzed and to the level of aggregation chosen, both in terms of digits of the specific classification employed and the way export
flows are identified (product vs product-destination pair).

3 The Model

Firms are distributed over a finite set of $C$ identical countries and engage in costly trade (iceberg type). At any time $t$, each country is populated by a continuum of identical consumers of measure $H_t = H e^{\eta t}$, where $\eta \geq 0$ is the growth rate of the population. Time is continuum and denoted by $t$, with initial time $t = 0$. At each point in time, the representative consumer is endowed with one unit of labor and has the following utility function

$$U_t = E_t \left[ \int \ln(X_t) e^{-\rho t} dt \right]$$

where $\rho > 0$ is the discount factor and $X_t$ is a composite good. The differentiated good $X_t$ is produced with a continuum of varieties

$$X_t = \left( \int_{\omega \in \Omega_t} a_t(\omega) x_t(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$$

where $x_t(\omega)$ is consumption of variety $\omega$, $a_t(\omega)$ is a preference shock distributed lognormal which hits variety $\omega$ at time $t$, $\sigma > 1$ is the elasticity of substitution across varieties, and $\Omega_t$ is total mass of varieties at time $t$. In each country, the composite good sector consists of a large group of monopolistically

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Lognormally distributed preference shocks would result from a process of “impeded growth” à la Kalecki (1945) whereby a Gibrat diffusion process is stabilized by relaxing the original assumption that shocks are independent of size and assuming instead that there is a negative correlation between size and growth: $a_t(\omega) = a_{t-1}(\omega)^{(1-\beta)} \forall t$. A possible interpretation for this process is a product life-cycle type of dynamic: young varieties are subject to higher shocks, i.e. are more volatile,
competitive producers. At any point in time, each variety $\omega$ is produced by one and only one firm.

Each variety is produced according to the following production technology

$$x_t = z_t l_t$$

where $z$ is labor productivity which is common across varieties and firms. We assume that $z$ evolves exogenously over time according to $z_t = z e^{\theta t}$.

### 3.1 Households

The representative household maximizes utility subject to a standard budget constraint. The corresponding first order conditions are

$$\frac{\dot{Y}}{Y} = r - \rho$$

$$x_t(\omega) = a_t(\omega) \left( \frac{p_t(\omega)}{P_t} \right)^{-\sigma} Y_t / P_t$$

where $r$ is the interest rate and $P_t = \left( \int_{\omega \in \Omega_t} a_t(w)p_t(\omega)^{1-\sigma} d\omega \right)^{1/(1-\sigma)}$ is the price index. Total household expenditure on the composite good $X$ is

$$\int_{\omega \in \Omega_t} x_t(\omega)p_t(\omega)d\omega = Y_t$$

Equation (4) is the standard Euler equation and (5) is the demand for variety $\omega$. We let total expenditure $Y_t$ be the numeraire and set it to a constant over every period implying $r = \rho$ in the balanced growth path.

while as products get older they reach maturity and their volatility decreases.
3.2 Production and Innovation

Trade costs are symmetric and of the standard iceberg type: \( \tau > 1 \) units shipped result in 1 unit arriving. Since there are no entry costs in the export market, active firms sell both to the domestic and to the foreign markets. At each point in time, the representative firm maximizes profits subject to the demand function (5) coming from the domestic and from the foreign markets. The pricing rule of the firm in the domestic market and in a foreign market are

\[ p_{d,t}(\omega) = \left( \frac{\sigma w_t}{(\sigma - 1)P_{d,t}} \right)^{1-\sigma} \quad Y_{d,t} \]

\[ p_{f,t}(\omega) = \left( \frac{\sigma w_t}{(\sigma - 1)P_{f,t}} \right)^{1-\sigma} \quad Y_{f,t} \]

As \( r_{f,t}(\omega) = r_{d,t}(\omega)^{\tau_{1-\sigma}} \), total revenues from domestic and foreign sales of variety \( \omega \) are \( r_{f,t}(\omega) = r_{d,t}(\omega) + (C - 1)r_{d,t}(\omega)^{\tau_{1-\sigma}} \). Profits from the domestic market and a foreign market are

\[ \pi_{d,t}(\omega) = a_t(\omega) \left( \frac{\sigma w_t}{(\sigma - 1)P_{d,t}} \right)^{1-\sigma} \frac{Y_{d,t}}{\sigma \tau_{1-\sigma}} \]

\[ \pi_{f,t}(\omega) = a_t(\omega) \left( \frac{\sigma w_t}{(\sigma - 1)P_{f,t}} \right)^{1-\sigma} \frac{Y_{f,t}}{\sigma \tau_{1-\sigma}} \]

Total profits from the domestic market and the foreign markets are \( \pi_{T,t}(\omega) = \pi_{d,t}(\omega) + (C - 1)\pi_{d,t}(\omega)^{\tau_{1-\sigma}} \).

### 3.2.1 Innovation by incumbents

To increase its portfolio of goods, a firm must invest in R&D activities. We assume that new innovations arrive following an exponential distributed waiting time with mean

\[ \lambda_t = f(i_t) \]  

(6)

where \( i_t \) is labor used in the replication process. We assume that \( f \) is increasing and exhibits strictly decreasing returns to scale. Each firm faces also the probability that some firm will innovate over a good it is currently producing. When this event occurs, the incumbent producer loses the good from its portfolio. An existing variety is lost with an exponentially distributed waiting time with mean

\[ \mu_t = g(j_t) \]  

(7)

where \( j_t \) is labor used to maintain existing products and \( g \) is (i) strictly decreasing and convex. We can rewrite the profits as

\[ \frac{\sigma w_t}{(\sigma - 1)P_{d,t}} \left[ \frac{1}{\sigma - 1} \phi - (i + j) \right] + \lambda n \left[ V_t(n + 1) - V_t(n) \right] + \mu n \left[ V_t(n - 1) - V_t(n) \right] \]

(8)

The expectation operator is conditional on information at time \( t \). As firms only learn about the preference shock over a new variety after they succeed in the innovation process, they expect to receive
the same flow of profits from each innovation. Moreover, as each innovation guarantees the same expected
flow of profits, all firms choose the same innovation rate \( \lambda_t \) and maintenance rate \( \mu_t \).

As in Klette and Kortum (2004b), the unique solution to (8) must be proportional to the number
of products \( V(n) = vn \). The optimal levels of investment in new varieties and of maintenance of existing
varieties are determined by

\[
\lambda_t = f(i_t) \quad \mu_t = g(j_t) \quad v_t f'(i_t) = -v_t g'(j_t) = w_t
\]  

Intuitively, the innovation process described here suggests that firms base their investment policy only
on the number of products they already export, independently of whether these products rank high or
low in consumers’ preferences (i.e. independently of the \( a_t(\omega) \)). This implies a form of compensation
across different products exported by the same firm: profits generated by "best sellers" make up for the
lower-than-average profits generated by products characterized by low \( a_t(\omega) \). Such a result can easily be
accommodated for firms exporting a large number of products \( n_L \), as total profits are likely to differ
only slightly from those obtained from exporting \( n_L \) times the “average” product (i.e. a product yielding
the average profit). On the other hand, it is possible that “unlucky” firms selling only a few products
\( n_S \) characterized by low \( a_t(\omega) \) may be unable to invest as if they were getting “average” profits. When
this happens, then we could observe some departures from the equilibrium distribution of the number
of products exported by firms (whose shape is discussed in Section 3.4 below) in the lower tail of the
distribution.

3.2.2 Innovation by entrants

New varieties can also be produced from scratch by agents acting as entrepreneurs. At each point in
time, agents are endowed with one unit of effort that can be allocated between two tasks: supplying labor or
producing a new variety. Following Luttmer (2011), we assume that each agent has a skill vector \((x,y)\),
where \( x \) corresponds to the rate at which agents generate a new variety and \( y \) is the amount of labor
supplied per unit of time. Agents with skill vectors that satisfy \( v_t x > w_t y \) will become entrepreneurs,
while agents with skills vectors that satisfy \( v_t x < w_t y \) will supply labor to existing firms.

Let \( T \) be a time-invariant talent distribution defined over the set of all possible skill vectors with finite
mean and density \( \psi \). The resulting per capita supply of entrepreneurial effort is

\[
E(v_t/w_t) = \int_{v_t x > w_t y} x dT(x, y)
\]  

for \( \pi \in \Pi \). The per capita supply of labor is

\[
L(v_t/w_t) = \int_{v_t x < w_t y} y dT(x, y)
\]

Given a per capita stock of entrepreneurial activities \( E(v_t, w_t) \) and a stock of varieties \( N_t \), the rate \( \nu_t \)
at which new entrepreneurs add a new variety is determined by

\[
\nu_t N_t = H_t E(v_t, w_t)
\]

Labor market clearing requires

\[
N_t(l_t + i_t + j_t) = H_t L(v_t, w_t)
\]
3.3 Balanced growth

Along the balanced growth path, the measure of varieties grows at the rate $\eta$. The allocation of labor is constant $(i, j, l)$. From the consumer’s problem, aggregate variables $w_t$ and $C_t$ grow at a rate $k = \theta + \frac{\eta}{(\sigma - 1)}$ with a rate that is larger when goods are less substitutable. The implied interest rate is $r = \rho + k$. The Bellman equation (8) implies that wages and the values of firm $s$ must satisfy

$$\frac{v}{w} = \frac{\frac{1}{\sigma - 1} \phi - [i + j]}{r - k - [\lambda - \mu]}$$

(14)

where $(i, j)$ and $(\lambda, \mu)$ satisfy (9).

Holding $l$ fixed, these conditions imply that $v/w$ is equal to the maximum subject to $[\lambda, \mu] = [f(i), g(j)]$ of the right-hand side of equation (14), as long as it is finite. As the total number of varieties grows at rate $\eta$, new entrepreneurs must contribute at the non-negative rate $\eta - [\lambda - \mu]$. If $E(v, w)$ is positive, from (13) we obtain the entrepreneurial steady-state supply of varieties

$$\frac{N}{H} = \frac{E(v, w)}{\eta - [\lambda - \mu]}.$$  

(15)

Alternatively, $E(v, w) = 0$ and $\eta = \lambda - \mu$. Along the balanced growth path, the market clearing condition becomes

$$\frac{N(l + i + j)}{H} = L(v, w)$$

(16)

Luttmer (2011) shows that if $\rho + k > k + \eta$ and $\eta > f(0) - g(0)$, for a positive $E(v, w)$, then equations (9), (14), (15) and (16) define the unique balanced growth path and $\eta > \lambda - \mu$. A balanced growth path can arise with $E(v, w) = 0$ if the talent distribution has bounded support. In this case, new varieties are only produced by existing firms.

3.4 The distribution of the number of products exported

In absence of fixed export costs, all active firms sell both to the domestic and to the foreign markets. A new variety can then be thought as a new trade link. Firms form new trade links by creating new commodities and lose trade links when some firms innovate over a good they are currently producing. It follows that one can identify the growth process of the number of products for an individual firm with the distribution of its links. Let us define $M_{n,t}$ the mass of a firm with $n$ products at time $t$. The aggregate measure of products is

$$N_t = \sum_{n=1}^{\infty} n M_{n,t}.$$  

(17)

The change in the number of firms with one commodity over time is

$$\dot{M}_{1,t} = \mu 2 M_{2,t} + \nu N_t - (\mu + \lambda) M_{1,t}$$

(18)

where $\lambda$, $\mu$ and $\nu = \eta - [\lambda - \mu]$ are constant along the balanced growth path. The number of firms with one commodity increases because firms with two commodities loses one or because of entry. The number decreases because firms with one commodity gain or lose one.

The number of firms with more than one commodity evolves according to

$$\dot{M}_{n,t} = \lambda(n - 1) M_{n-1,t} + \mu(n + 1) M_{n+1,t} - (\mu + \lambda)n M_{n,t}$$

(19)

for all $n - 1 \in N$. A stationary distribution for a firm exists if (18) and (19) have a solution for which $rac{M_{n,t}}{N_t}$ is constant over time. Since along the balanced growth path $N$ grows at rate $\eta$, $\dot{M}_t = \eta M_{n,t}$ for all
Given that \( N \) and \( M_n \) grow at the same rate \( \eta \), we can define

\[
P_n = \frac{M_{n,t}}{\sum_{n=1}^{\infty} M_{n,t}}
\]

(20)

for all \( n \in N \). Equation (20) gives the fraction of firms with \( n \) commodities. We can also define the fraction of all commodities produced by firms of size \( n \) as

\[
Q_n = \frac{n M_{n,t}}{\sum_{n=1}^{\infty} n M_{n,t}}
\]

(21)

for all \( n \in N \). Using these definitions we can rewrite (18) and (19) as

\[
\eta Q_1 = \mu Q_2 + v - (\lambda + \mu) Q_1
\]

(22)

\[
\frac{1}{n} \eta Q_n = \lambda Q_{n-1} + \mu Q_{n+1} - (\lambda + \mu) Q_n
\]

(23)

Luttmer (2011) provides a solution for (22)-(23) in Appendix A. Luttmer (2011) shows that if \( \lambda, \mu, \eta \) and \( \nu = \eta - (\lambda - \mu) \) are positive, the sequence \( \{\beta_n\}_{n=0}^{\infty} \) defined by the recursion

\[
\beta_n = \frac{1}{1 - \left(\frac{\lambda \beta_n}{\mu}\right)} \frac{(n+1) \prod_{m=n}^{n+k} \beta_m}{\mu}
\]

and the initial condition \( \beta_0 = 0 \) is monotone and converges to \( \min\{1, \mu/\lambda\} \). The only non-negative and summable solution to equations (22)–(23) is given by

\[
Q_n = \sum_{k=0}^{\infty} \frac{1}{\lambda} \frac{1}{\beta_{n+k}} \prod_{m=n}^{n+k} \beta_m \prod_{m=n}^{n+k} \frac{\lambda \beta_m}{\mu}
\]

(24)

For large \( n \) and \( \lambda \neq \mu \) the distribution satisfies

\[
Q_n \sim \frac{\nu}{|\lambda - \mu|} \prod_{m=1}^{n-1} \frac{\lambda \beta_m}{\mu}
\]

(25)

If \( \nu = 0 \), the only non-negative and summable solution to equations (22)–(23) is identically zero, implying that there does not exist a stationary distribution in this case. If \( \nu > 0 \), equation (24) adds up to 1 by construction and defines a stationary distribution \( \{P_n\}_{n=1}^{\infty} \) via \( P_n \propto \frac{Q_n}{n} \). The mean number of links of a firm can be written as \( \frac{1}{\sum_{n=1}^{\infty} Q_n/n} \) which is finite by construction.

If \( \lambda < \mu \), \( Q_n \) is bounded above by a multiple of the geometrically declining sequence \((\lambda/\mu)^n \). When \( \lambda > \mu \) then \( (\lambda \beta_0/\mu)^n \) approaches 1 and (25) declines at a rate that is slower than any given geometric rate.

Luttmer (2011) shows that under some parameter restrictions the connectivity distribution features a Pareto tail with a shape parameter greater than unity. If \( \eta > 0 \), \( \lambda > \mu \) and \( \eta > \lambda - \mu \), then the right tail probabilities

\[
R_n = \sum_{k=n}^{\infty} P_k
\]

of the stationary connectivity distribution satisfy

\[
limit_{n \to \infty} \left( 1 - \frac{R_{n+1}}{R_n} \right) = \xi
\]

(26)

where \( \xi = \frac{\eta}{(\lambda - \mu)} \). That is, \( R_n \) is a regularly varying sequence with index \(-\xi\) and \( \xi > 1 \).

### 3.5 The size distribution of exporting firms

#### 3.6 Implications of the model

Our model provides clear predictions for the evolution of both the intensive and the extensive margin of trade. The assumed evolution of preference shocks gives rise to a lognormal distribution of export

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3When the rate \( \nu = \eta - (\lambda - \mu) \) goes to zero, the limiting tail index \( \xi = 1 \) associated with Zipf’s law arises.
sales for products Kalecki (1945). For what concerns the extensive margin, i.e. the number of products exported by each firm, the dynamics of firm entry and exit and of the process of product innovation by new and incumbent firms give rise to a Pareto distribution.

The assumption of lognormality for the distribution of product sales is consistent with the empirical findings recently emerged in the empirical literature. In section 2, we show that the lognormal distribution is a good fit for the distribution of exports values by products in a database of French firms (Figure 2).

A crucial assumption of the model is the absence of fixed export costs. This implies that each innovation leads to a new trade link. As a result, the evolution of the number of goods produced by a firm describes the dynamics of the number of trade links or the connectivity distribution of that firm. The growth in the number of varieties (due to population growth) and the dynamics of firm entry and exit ensures that the right tail of the connectivity distribution follows a Pareto with a tail index greater than unity. This prediction is consistent with the empirical finding emerged from the data of a shape parameter for the Pareto tail of the distribution of the extensive margin (i.e. number of products by firm) greater than 1 (between 1.73 and 1.83).

A further implication of the model has to do with the size of exporting firm, and results from the combination of the above results. Growiec et al (2008) show that the size distribution of business firms depend on the shape of the distribution its components, namely the number of products sold ($n$) and the value of sales of each product. In particular, when the former is Pareto and the latter is lognormal, the distribution of firm sales is a lognormal distribution multiplied by a stretching factor which increases with $n$: when $n$ is small, the stretching factor is negligible and the distribution is close to a lognormal; on the contrary, for large $n$ the size distribution shows a departure that leads to the emergence of a Pareto upper tail.

### 3.7 Extensions

An important extension of the model entails the inclusion of heterogeneous fixed entry costs in foreign markets. Under the assumption that firms need to pay a country-specific entry cost, and retaining the idea that preference shocks are product-specific, we would have two novel implications: (i) firms that have captured only (few) low-selling products may be prevented from exporting as their profits are insufficient to cover the fixed exporting costs; (ii) there is hierarchy among products whereby top-selling items are more likely to be shipped to many destinations (see Arkolakis and Muenchler, 2010, for evidence along these lines).

Suppose there are heterogeneous fixed entry cost into foreign markets ($F_j$, with $j = 1, \ldots, C$ indexing destinations) and that we can order rank countries on the basis of these fixed entry cost such that $F_1 < F_2 < \cdots < F_C$.

[...]
### Table 2: Test for “impeded growth” hypothesis

<table>
<thead>
<tr>
<th></th>
<th>size ((t - 1))</th>
<th>constant</th>
<th>Observations</th>
<th>Firms</th>
<th>R-squared</th>
<th>F-test: (\beta = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.846***</td>
<td>-0.634***</td>
<td>430,494</td>
<td>67,661</td>
<td>0.708</td>
<td>p-value: 0.000</td>
</tr>
</tbody>
</table>

*clustered standard errors (by firm) in parentheses*

*** significant at 1%

### Table 3: Test for power-law upper-tail behavior in export values. Data for 2003 at different levels of aggregation (by firm-product and by firm)

<table>
<thead>
<tr>
<th></th>
<th>Total export by</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>firm-product</td>
<td>firm</td>
<td></td>
</tr>
<tr>
<td>UMPU</td>
<td>650 (0.08%)</td>
<td>400 (0.35%)</td>
<td>43.17%</td>
</tr>
<tr>
<td></td>
<td>850 (0.10%)</td>
<td>800 (0.70%)</td>
<td>46.70%</td>
</tr>
<tr>
<td>ME</td>
<td>1230 (0.15%)</td>
<td>590 (0.51%)</td>
<td>51.71%</td>
</tr>
</tbody>
</table>

*First row: number of observations in power-law tail*

*Second row: percentage of observations in power-law tail*

*Third row: share of total trade in power-law tail*

### Table 4: Test for power-law upper-tail behavior in extensive margin (data for 2003)

<table>
<thead>
<tr>
<th></th>
<th>N. products by firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>UMPU</td>
<td>2100 (1.83%)</td>
</tr>
<tr>
<td></td>
<td>29.76%</td>
</tr>
<tr>
<td>ME</td>
<td>2300 (2.01%)</td>
</tr>
<tr>
<td></td>
<td>31.03%</td>
</tr>
<tr>
<td>GI</td>
<td>2680 (2.34%)</td>
</tr>
<tr>
<td></td>
<td>33.24%</td>
</tr>
</tbody>
</table>

*First row: number of observations in power-law tail*

*Second row: percentage of observations in power-law tail*

*Third row: share of total num. of products in power-law tail*
5 Conclusion

We develop a dynamic model of innovation by new and incumbent firms in the spirit of Klette and Kortum (2004a) and Luttmer (2007) where firms must invest in order to capture new products and maintain their existing portfolio. This process gives rise to a cumulative dynamic whereby large firms tend to invest more and therefore grow even larger. The power-law distribution of the number of products exported by each firm well approximates the data, so that the model provides a novel explanation for the large heterogeneity in the extensive margin among exporting firms.

Moreover, the model predicts a lognormal distribution of export sales of each product and the emergence of a power-law tail in the size distribution of exporting firms. Both these predictions are well matched by data on a large sample of French exporting firms.

[To be completed]
References


Appendices

A Household’s problem

The representative household maximizes the following intertemporal utility function subject to an intertemporal budget constraint

\[ U_t = E_t \left[ \int_t^{\infty} \ln(X_t) e^{-\rho t} dt \right] \]

\[ A_t = r_t A_t + w_t - X_t P_t \]
where $X_t$ is a composite good, $P_t$ is the price of the composite good, $w_t$ is the wage rate, and $A_t$ is the value of the household’s asset holdings. At any period $t$, the representative consumer is endowed with one unit of labor. Total spending in final good at $t$ is $Y_t = P_t X_t$.

The consumer’s problem is solved in two steps.

**A.1 First step: dynamic consumption problem**

The current value Hamiltonian is

$$H(C_t, A_t, v_t) = \ln(C_t) + v_t [r_t A_t + w_t - X_t P_t]$$

The first order conditions are

$$X_t : \quad v_t = \frac{1}{X_t P_t} = \frac{1}{Y_t} \quad (27)$$

$$A_t : \quad \frac{\dot{v}_t}{v_t} = \rho - r_t \quad (28)$$

Taking the time derivative of (28), we get

$$\frac{\dot{v}_t}{v_t} = -\frac{Y_t}{Y_t}$$

using (29), we get the standard Euler equation

$$\frac{Y_t}{Y_t} = r_t - \rho$$

We set $Y_t$ to a constant over every period implying $r = \rho$ in the balanced growth path.

**A.2 Second step: static choice across varieties**

The representative consumer chooses the optimal bundle of varieties to consume given its budget constraint

$$X_t = \left( \int_{\omega \in \Omega_t} a_t(\omega) x_t(\omega) \frac{\sigma - 1}{\sigma} d\omega \right)^{-\frac{1}{\sigma - 1}}$$

$$\int_{\omega \in \Omega_t} x_t(\omega) p_t(\omega) d\omega = Y_t$$

The corresponding first order condition gives the demand function for variety $\omega$

$$x_t(\omega) = a_t(\omega) \left( \frac{p_t(\omega)}{P_t} \right)^{-\sigma} Y_t$$

**B The problem of the firm**

Let $p_t^{DD}$ and $x_t^{DD}$ be price and quantity for the domestic market and $p_t^{DF}, x_t^{DF}$ prices and quantities for the foreign market.

The representative firm maximizes the following profit function:

$$\pi_t = \left( p_t^{DD} x_t^{DD} - \frac{w_t X_t}{z_t} \right) + \sum_{c=1}^{C-1} \left( p_t^{DF_c} x_t^{DF_c} - \frac{w_t x_t}{z_t} \right)$$
subject to

\[ x_t^{DD} = a_t \left( \frac{p_t^{DD}}{p_t^{ll}} \right)^{-\sigma} \frac{Y_t^D}{P_t^D} \]

\[ \sum_{c=1}^{C-1} x_t^{DFc} = a_t \left( \frac{p_t^{DFc}}{P_t^{Fc}} \right)^{-\sigma} \frac{Y_t^{Fc}}{P_t^{Fc}} \]

The resulting first order conditions give the price for the domestic and the foreign market respectively

\[ p_t^{DD} = \frac{\sigma}{\sigma - 1} \frac{w_t}{z_t} \]

\[ p_t^{DFc} = \frac{\sigma}{\sigma - 1} \frac{w_t}{z_t} \]