Foreign Bidders Going Once, Going Twice... Government Procurement Auctions with Tariffs*

Matthew T. Cole‡
(Florida International University)

Ronald B. Davies
(University College Dublin)

Working Paper: Comments Welcome
August 16, 2013

Abstract

Until recently, government procurement bidding processes have generally favored domestic firms by awarding the contract to a domestic firm even if a foreign firm tenders a lower bid, so long as the difference between the two is sufficiently small. This has been replaced by an agreement abolishing this practice. However, the presence of other trade barriers, such as tariffs, can continue to disadvantage foreign firms. We analyze the bidding strategies in such a game and show that when domestic profits are valued, tariffs will be used to discriminate against foreign firms. Despite this, the optimal tariff is less protectionist than the optimal price preference.

JEL classification: F13, H57, F12

Keywords: Government Procurement; Tariffs; Price Preference

*The authors acknowledge that this paper is produced as part of the project “Globalization, Investment and Services Trade (GIST) Marie Curie Initial Training Network (ITN)” funded by the European Commission under its Seventh Framework Programme - Contract No. FP7-PEOPLE-ITN-2008-211429. All remaining errors are our own.

‡Corresponding author: Department of Economics, Florida International University, FL 33199, USA. Email: matcole@fiu.edu.
1 Introduction

Government procurement contracts are a significant part of many economies, often amounting to 15-20 percent of GDP (WTO, 2013). When seeking a provider for a government contract, it has been a long-standing tradition that the nature of the bidding favors domestic firms over foreign ones. For the most part, this has taken place via a system in which the contract is awarded to a foreign firm only if that firm’s bid is sufficiently lower than the lowest bid tendered by a domestic firm, known as a price preference. For example, under the European Community regulations, the contract was awarded to a member firm so long as its bid was no more than three percent higher than the lowest non-member bid (Branco, 1994). This preferential procurement policy has been attributed to a government which values domestic firm profits more than foreign firms (McAfee and McMillan, 1989). This notion was expanded upon by Branco (1994), who considers the optimal mechanism to implement this preference, and Miyagiwa (1991), who includes both public and private consumption. In 1996, this practice began to be dismantled by the Government Procurement Agreement, an international agreement in which signatories agree to non-discrimination, that is, a selection process by which foreign firms are treated no differently than their domestic competitors.\footnote{See WTO (2013) for a detailed description of this agreement.} This, however, ensures equal treatment under the bidding process but does not eliminate other mechanisms by which foreign firms are treated differently than domestic firms, most notably trade policy. Recently, concerns have been expressed that due to increasing demands for protectionism, that governments will resort to such methods to put foreigners at a disadvantage in procurement bidding. In particular, within the European Union, there has been mounting pressure to restrict bidding to foreign firms that would use a sufficiently high level of domestic content when fulfilling a government contract (Economist, 2012). Although there is a literature examining procurement processes with a price-premium offered to domestic firms, as yet, there is no analysis of competition under protection. This is the gap this paper fills.
We do so by considering an auction for a government contract in which two firms, one domestic and one foreign, tender bids to the domestic government. In contrast to the preferential procurement processes studied elsewhere, the contract is awarded to the firm with the lowest bid. A second difference is that we allow the domestic government to impose a tariff on the foreign firm. This tariff can also be thought of as the added cost to the foreign firm of meeting domestic content restrictions that would make it eligible to bid. We show that the imposition of a tariff does indeed impede the probability of the foreign firm winning the contract. Furthermore, it increases the range of bids that are submitted by the domestic firm.

When domestic profits are positively valued, the government will indeed impose a positive tariff in equilibrium. Nevertheless, this does not imply that the equilibrium is more protectionist than that under a price preference regime. In fact, when considering parameter values for which our model overlaps with Branco’s (1994), the equilibrium tariff regime is less protectionist, i.e. the foreign firm has a higher expected probability of winning and higher expected profits, than is the price preference regime. Furthermore, we show that when the weight the government places on domestic profits is low, that the equilibrium under the tariff regime is one of free trade and social welfare is maximized.

The paper proceeds as follows. In Section 2, we present the model and solve for the bidding strategies of firms. Section 3 describes the government’s optimal tariff. Section 4 concludes.

2 The Model

The model has three players, a domestic government, a domestic firm, and a foreign firm. The government has a project, to which it attaches value $V_G$, that it wishes to have completed.\footnote{We assume that this is sufficiently large so that, in equilibrium, the government’s value exceeds the winning equilibrium bid. We formalize this condition below.} To that end, it runs a first-price, sealed-bid auction in which the two firms simultaneously
tender bids (\(b_d\) for the domestic firm and \(b_f\) for the foreign firm) which are the firm’s price for which it will carry out the contract. The bidding results in the contract being awarded to the firm with the lowest bid. Note that this is in contrast to a policy with a price preference, in which the domestic firm can win the contract even if \(b_d > b_f\) so long as this difference is less than some level. In the event of equal bids, the contract is awarded to the domestic firm.

The timing of the model is as follows. In the first stage, the government sets a specific tariff \(\tau \geq 0\) which the foreign firm must pay if it wins the contract. This can be thought of as a tariff the foreign firm must pay on inputs brought in from its own country. Alternatively, instead of a tariff, this could represent the additional cost to the foreign firm of negotiating domestic content regulations (as might occur if it is forced to use a domestic supplier rather than its preferred supplier located in another country). In the second stage, the firms simultaneously submit bids. In the final stage, bids are opened, the contract is awarded, production takes place, and payoffs accrue. We solve the game via backwards induction.

### 2.1 Firms

Prior to the commencement of the game, each firm \(i = d, f\) obtains a cost \(c_i\) which is independently drawn from a uniform distribution with the support \([c_d, \overline{c}_d]\) for the domestic firm and \([c_f, \overline{c}_f]\) = \([c_d + \tau, \overline{c}_d + \tau]\) for the foreign firm. Thus, a priori the two firms differ only by the tariff. This assumption greatly simplifies the analysis, however we expect that the results generalize somewhat to asymmetric distributions even with a zero tariff. Note that there are limits to the potential asymmetry. In particular, if \(\tau_d \leq 2c_f - \tau_f\), the Nash equilibrium must have the domestic firm always winning by bidding \(c_f\), i.e. the foreign firm is unable to compete. As a consequence, we only consider cases wherein \(\tau < 2(\overline{c}_d - c_d)\) since for tariffs greater than this, the domestic firm always wins. While the distribution of costs

---

3 Further, if there are registration fees that the domestic firm, by virtue of its existence has already paid but the foreign firm has not, then \(\tau\) could represent those additional fees the foreign firm must pay after winning but before it can fulfill the contract. Under this latter interpretation, \(\tau\) would not be discriminatory against the foreign firm in the strictest sense.

4 This is discussed in detail for first-price auctions by Kaplan and Zamir (2012) and Kaplan and Wettstein (2000).
are public knowledge, each firm's cost realization is private information. Define $b_i(c_i)$ as the bid function and $c_i(b)$ to be the inverse bid function for firm $i$. As shown by Griesmer, et al. (1967) any non-trivial equilibrium (i.e. one in which both firms have a positive probability of winning) must be characterized by monotonic and differentiable bid functions and we therefore restrict our attention to this class of bid functions.

In the following analysis, we build on the results of Kaplan and Zamir (2012) who derive analytic solutions to an auction with uniform, but asymmetric, valuation distributions. We modify their analysis in order to fit it to the setting we consider. Before proceeding certain aspects of bidding behavior should be made clear. First, to rule out multiple equilibria, we assume that a firm with a zero probability of winning bids its cost. Second, in equilibrium, no bid greater than $\bar{c}_f$ will be tendered. The reason for this is that, should one firm do so, the other would be able to marginally undercut that bid and discretely increase its probability of winning while only marginally lowering its payoff. Thus, bids will be bounded from above.

We begin by considering the domestic firm. In the second stage of the game, firms take the tariff as given. The domestic firm's expected profit from tendering a bid of $b_d$ is

$$E[\pi_d] = (b_d - c_d) \Pr(\text{domestic firm wins} \mid c_d)$$

where the probability of it winning is the probability that it tenders the lower bid, i.e. that the foreign firm's cost leads it to tender a higher bid:

$$1 - H_C(c_f(b_d)) = 1 - \frac{c_f(b_d) - c_f}{\bar{c}_f - c_f} = \frac{\bar{c}_d + \tau - c_f(b_d)}{\bar{c}_d - c_d}.$$  

This results in expected domestic firm profits of:

$$E[\pi_d] = (b_d - c_d) \left[ \frac{\bar{c}_d + \tau - c_f(b_d)}{\bar{c}_d - c_d} \right]. \quad (1)$$

Maximizing expected profit, for an interior solution, the optimal inverse bidding strategy for
firm \( d \) solves the first order condition:

\[
\left[ \frac{\tau_d + \tau - c_f(b_d)}{\bar{c}_d - \underline{c}_d} \right] - \left( b_d - c_d(b_d) \right) \frac{c'_f(b_d)}{\bar{c}_d - \underline{c}_d} = 0.
\]

Defining the minimum and maximum bids which are tendered in equilibrium as \( \underline{b} \) and \( \bar{b} \) respectively, for any \( b_d \in [\underline{b}, \bar{b}] \), we must have

\[
[b - c_d(b)]c'_f(b) + c_f(b) = \bar{c}_d + \tau. \tag{2}
\]

Similarly for the foreign firm, the probability it has the winning bid, \( b_f \) is

\[
(1 - H_{C[c_d(b_f)]}) = 1 - \frac{c_d(b_f) - \underline{c}_d}{\bar{c}_d - \underline{c}_d} = \frac{\bar{c}_d - c_d(b_f)}{\bar{c}_d - \underline{c}_d}.
\]

Therefore, it has the following expected profit

\[
E[\pi_f] = (b_f - c_f) \left[ \frac{\bar{c}_d - c_d(b_f)}{\bar{c}_d - \underline{c}_d} \right]. \tag{3}
\]

Maximizing expected profit, the optimal inverse bidding strategy for the foreign firm solves the following first order condition:

\[
\left[ \frac{\bar{c}_d - c_d(b_f)}{\bar{c}_d - \underline{c}_d} \right] - \left( b_f - c_f(b_f) \right) \frac{c'_d(b_f)}{\bar{c}_d - \underline{c}_d} = 0.
\]

Therefore, in equilibrium, because this must hold for any \( b_f \in [\underline{b}, \bar{b}] \), we must have

\[
[b - c_f(b)]c'_d(b) + c_d(b) = \bar{c}_d. \tag{4}
\]
Thus, we have a system of two differential equations that define the equilibrium:

\[
\begin{align*}
[b - c_d(b)]c'_f(b) + c_f(b) &= \bar{c}_f, \\
[b - c_f(b)]c'_d(b) + c_d(b) &= \bar{c}_d.
\end{align*}
\] (5) (6)

From this, three results follow. First, a foreign firm with a cost \( c_f \) will submit a bid equal to this cost, i.e. \( c_f(\bar{c}_f) = \bar{c}_f \) which implies that \( \bar{b} \leq \bar{c}_f \).\footnote{Recall that firms with no chance of winning bid their costs.} Second, as the bidding functions are monotone, the lowest bid tendered by a domestic firm must be tendered by a firm with cost \( c_d \). The analogous result must hold for the foreign firm. Third, these minimum bids must be the same for each firm (and are therefore equal to \( \bar{b} \)). If this is not the case, for example if the domestic minimum bid is lower than the minimum foreign bid, then the domestic firm tendering such a bid can raise its bid and increase profits without decreasing its probability of winning, implying that such a bid could not have been an equilibrium bid. Thus, \( c_d(\bar{b}) = \bar{c}_d \) and \( c_f(\bar{b}) = \bar{c}_f \).

Adding equations (5) and (6) together and rearranging yields:

\[
c_d(b)c'_f(b) + c_f(b)c'_d(b) = c_f(b) + c_d(b) + c'_f(b)b + c'_d(b)b - (\bar{c}_f + \bar{c}_d)
\] (7)

or, recognizing that the right-hand side is the derivative of \((c_f(b) + c_d(b) - (\bar{c}_f + \bar{c}_d))b\) with respect to \( b \):

\[
c_d(b)c'_f(b) + c_f(b)c'_d(b) = \left[(c_f(b) + c_d(b) - (\bar{c}_f + \bar{c}_d))b\right]'
\] (8)

By integrating with respect to \( b \), we obtain:

\[
c_d(b) \cdot c_f(b) = [c_d(b) + c_f(b)] \cdot b - [\bar{c}_f + \bar{c}_d] \cdot b + \zeta
\] (9)

where \( \zeta \) is the constant of integration. In order to determine \( \zeta \), we require the maximum bid which is solved in our first lemma.
Lemma 1. The upper bound of the bid functions, $\bar{b}$, is given by

$$\bar{b} = \frac{\bar{c}_d + \bar{c}_f}{2}. \quad (10)$$

Proof. If the foreign firm has a cost greater than or equal to $c_f(\bar{b})$, it has no chance of winning and therefore bids its cost. Note that this implies that $c_f(\bar{b}) = \bar{b}$. This makes the probability of the domestic firm winning with a bid $\bar{b}$ equal to $\left[ \frac{\bar{c}_f - \bar{b}}{\bar{c}_d - \bar{c}_f} \right]$. By definition of the inverse bid function, the domestic firm with cost $c_d(\bar{b})$ does not benefit from bidding more than $\bar{b}$, meaning that expected profits must be such that:

$$(\bar{b} - c_d(\bar{b})) \left[ \frac{\bar{c}_f - \bar{b}}{\bar{c}_d - \bar{c}_f} \right] \geq (b - c_d(\bar{b})) \left[ \frac{\bar{c}_f - b}{\bar{c}_d - \bar{c}_f} \right], \quad \forall \ b \geq \bar{b}.$$ 

This can be rewritten as:

$$- \left( \bar{b} - \left( \frac{c_d(\bar{b}) + \bar{c}_f}{2} \right) \right)^2 \geq - \left( b - \left( \frac{c_d(\bar{b}) + \bar{c}_f}{2} \right) \right)^2 \quad (11)$$

Since this has to hold for all $b \geq \bar{b}$, this requires that $\bar{b} \geq \frac{c_d(\bar{b}) + \bar{c}_f}{2}$. Similarly, by definition of the maximum bid, the domestic firm with cost $c_d(\bar{b})$ cannot benefit from bidding below $\bar{b}$:

$$(\bar{b} - c_d(\bar{b})) \left[ \frac{\bar{c}_f - \bar{b}}{\bar{c}_d - \bar{c}_f} \right] \geq (b - c_d(\bar{b})) \left[ \frac{\bar{c}_f - c_f(b)}{\bar{c}_d - \bar{c}_f} \right], \quad \forall \ b \leq \bar{b}.$$ 

However since $c_f(b) \leq b$, we have

$$(\bar{b} - c_d(\bar{b})) \left[ \bar{c}_f - \bar{b} \right] \geq (b - c_d(\bar{b})) \left[ \bar{c}_f - b \right], \quad \forall \ b \leq \bar{b}.$$ 

This can happen only if $\bar{b} \leq \frac{c_d(\bar{b}) + \bar{c}_f}{2}$. Combining these then implies that:

$$\bar{b} = \frac{\bar{c}_d + \bar{c}_f}{2} \quad (12)$$
Note that this is the upper bound of the bid function, not the maximum bids tendered by the foreign firm. Because $\bar{b}$ is the average of the two upper limits of the cost distributions, for a positive tariff it is strictly less than the highest foreign firm cost. This implies that for a positive tariff, a high-cost foreign firm will bid its cost which exceeds this level. Recalling that for both firms to be able to compete that $\tau < 2(\bar{c}_d - \bar{c}_d)$, this maximum bid pins down the minimum value the government places on the project, implying that $V_G \geq \frac{\bar{c}_d + \bar{c}_f}{2}$ where $\bar{c}_f$ is the highest foreign cost inclusive of a tariff of $2(\bar{c}_d - \bar{c}_d)$. Intuitively, this valuation implies that for any tariff in which both firms compete, the government will choose to accept the winning bid in equilibrium.

Returning to (9), we can use the maximum bid to solve for the constant of integration. Recalling that $c_f(\bar{b}) = \bar{b}$, evaluating (9) at $\bar{b}$ reduces to:

$$\varsigma = \left(\frac{\bar{c}_d + \bar{c}_f}{2}\right)^2.$$  (13)

This allows us to rewrite (9) as:

$$c_d(b) \cdot c_f(b) = [c_d(b) + c_f(b)] \cdot b - [\bar{c}_f + \bar{c}_d] \cdot b + \left(\frac{\bar{c}_d + \bar{c}_f}{2}\right)^2.$$  (14)

With this in hand, we can find the lower bound of the bid function, $\underline{b}$.

**Lemma 2.** The lower bound of the bid function, $\underline{b}$ is given by

$$\underline{b} = \frac{\left(\frac{\bar{c}_d + \bar{c}_f}{2}\right)^2 - \bar{c}_d \cdot \bar{c}_f}{(\bar{c}_d - \bar{c}_d) + (\bar{c}_f - \bar{c}_f)}.$$  (15)

**Proof.** Recall that the lowest cost firms both submit bids of $\underline{b}$, i.e. $c_d(\underline{b}) = \bar{c}_d$ and $c_f(\underline{b}) = \bar{c}_f$. Using this and evaluating (14) at $\underline{b}$, the above result is found. 

Because bid functions are monotone and the costs are uniformly distributed, so too are
bids. Furthermore, Lemmas 1 and 2 give the limits of the distributions of bids. Specifically, domestic firm bids are distributed uniformly on $[b, \overline{b}]$ whereas foreign bids are distributed uniformly on $[b, \overline{c}_f]$.

### 2.1.1 Equilibrium Bid Functions

In order to solve for our bid functions, we need to reduce our two differential equations to one. Using (14), we can solve for $c_f(b)$ in terms of $c_d(b)$

$$c_f(b) = \frac{c_d(b)b - [\overline{c}_f + \overline{c}_d]b + \left(\frac{\overline{c}_d + \overline{c}_f}{2}\right)^2}{c_d(b) - b} \quad (16)$$

Plugging this into (11) yields:

$$-\left[\frac{\overline{c}_d + \overline{c}_f}{2} - b\right]^2 c_d'(b) = [\overline{c}_d - c_d(b)] [c_d(b) - b]. \quad (17)$$

Rearranging this, we are left with the single differential equation

$$\left([\overline{c}_d + \overline{c}_f] - 2b\right)^2 c_d'(b) = 4 \left[c_d(b) - \overline{c}_d\right] [c_d(b) - b] \quad (18)$$

Note that this solution not only satisfies the first order conditions but, as proven by Griesmer, et al. (1967), the second order conditions as well. We can now solve for the analytic solutions of the equilibrium bid functions.

**Proposition 1.** The equilibrium inverse bid functions are given by

$$c_d(b) = \overline{c}_d - \frac{(\overline{c}_f - \overline{c}_d)^2}{4(b - \overline{c}_f) + (2b - (\overline{c}_d + \overline{c}_f))\lambda_1 \exp\left(\frac{\overline{c}_f - \overline{c}_d}{\overline{c}_d + \overline{c}_f - 2b}\right)} \quad (19)$$

$$c_f(b) = \overline{c}_f - \frac{(\overline{c}_f - \overline{c}_d)^2}{4(b - \overline{c}_d) + (2b - (\overline{c}_d + \overline{c}_f))\lambda_2 \exp\left(\frac{\overline{c}_d - \overline{c}_f}{\overline{c}_d + \overline{c}_f - 2b}\right)} \quad (20)$$
where

\[\lambda_1 = -\exp\left(\frac{\tau_0 - \tau_1}{2(b - b)}\right) \left[ \frac{(\bar{\tau}_f - \bar{\tau}_d)^2}{\bar{\tau}_d - \bar{\tau}_d} + 4(\bar{\tau}_f - b) \right] < 0 \quad (21)\]

\[\lambda_2 = -\exp\left(\frac{\tau_0 - \tau_1}{2(b - b)}\right) \left[ \frac{(\bar{\tau}_f - \bar{\tau}_d)^2}{\bar{\tau}_f - \bar{\tau}_f} + 4(\bar{\tau}_d - b) \right] < 0. \quad (22)\]

**Proof.** First define \(\alpha \equiv (\bar{\tau}_d + \bar{\tau}_f) - 2\bar{\tau}_d = \bar{\tau}_f - \bar{\tau}_d, x \equiv b - \bar{\tau}_d, \) and \(D(x)\) such that

\[c_d(b) = \frac{\alpha^2}{D(x)} + \bar{\tau}_d \quad (23)\]

We then have \(c'_d(x) = -\frac{\alpha^2}{D(x)^2}D'(x)\), and equation (18) becomes

\[-\frac{\alpha^2}{D(x)^2}D'(x)(\alpha - 2x)^2 = 4 \left[ \frac{\alpha^2}{D(x)} + \bar{\tau}_d \right] \left[ \frac{\alpha^2}{D(x)} + \bar{\tau}_d - b \right],\]

\[-\frac{\alpha^2}{D(x)^2}D'(x)(\alpha - 2x)^2 = 4 \left[ \frac{\alpha^2}{D(x)} \right] \left[ \frac{\alpha^2}{D(x)} - x \right],\]

\[D'(x)(\alpha - 2x)^2 = 4 \left[ xD(x) - \alpha^2 \right],\]

\[D'(x)(\alpha - 2x)^2 = 4xD(x) - 16x(\alpha - x) - 4(\alpha - 2x)^2,\]

\[(D'(x) + 4)(\alpha - 2x)^2 = 4x[D(x) - 4(\alpha - x)],\]

Furthermore

\[\frac{D'(x) + 4}{D(x) - 4(\alpha - x)} = \frac{4x}{(\alpha - 2x)^2} = \frac{2\alpha}{(\alpha - 2x)^2} - \frac{2}{\alpha - 2x}\]

By integrating both sides, we obtain

\[\ln \left( D(x) - 4(\alpha - x) \right) = \frac{\alpha}{\alpha - 2x} + \ln(\alpha - 2x) + \ln \lambda_1,\]
and taking the exponent of both sides yields

\[ D(x) = (\alpha - 2x)\lambda_1 e^{\frac{\alpha}{\alpha - 2x}} + 4(\alpha - x) \]  \hspace{1cm} (24)

where \( \lambda_1 \) is a constant of integration. The lower boundary condition \( c_d(b) = c_d \) determines \( \lambda_1 \). When \( b = b \), we have \( x = x \equiv b - c_d \). From the definition, it follows that \( D(x) = \frac{\alpha^2}{c_d - \bar{c}_d} \).
Hence the boundary condition becomes

\[ \lambda_1 = \left[ \frac{e^{-\frac{\alpha}{\alpha - 2x}}}{\alpha - 2x} \right] \left[ \frac{\alpha^2}{c_d - \bar{c}_d} - 4(\alpha - x) \right] \]

which can be rewritten as (recall that \( \bar{b} = \frac{c_d + c_f}{2} \) and the definition of \( \alpha \))

\[ \lambda_1 = - \left[ \exp\left(\frac{c_d - c_f}{2(\bar{b} - b)}\right) \right] \left[ \frac{(c_f - c_d)^2}{c_d - \bar{c}_d} + 4(c_f - b) \right] < 0. \] \hspace{1cm} (25)

The analogous sequence of steps results in the foreign inverse bid function and \( \lambda_2 \).

Note that up to this point, we have not made use of the assumption that the firms’ cost distributions vary only in the tariff. If we impose this assumption, the inverse bid functions can be simplified to:

\[ c_d(b) = c_d - \frac{\tau^2}{4(b - c_d - \tau) + (2b - 2c_d - \tau)\lambda_1 \exp\left(\frac{\tau}{2c_d + \tau - 2\bar{b}}\right)} \] \hspace{1cm} (26)

\[ c_f(b) = c_d + \tau - \frac{\tau^2}{4(b - c_d) + (2b - 2c_d - \tau)\lambda_2 \exp\left(\frac{-\tau}{2c_d + \tau - 2\bar{b}}\right)} \] \hspace{1cm} (27)
where

\[
\lambda_1 = -\left[\frac{2(2(c - d) + \tau)}{2(c - d) - \tau}\right] \exp \left(\frac{-4\tau(c - d)}{4(c - d)^2 - \tau^2}\right),
\]

\[
\lambda_2 = -\left[\frac{2(2(c - d) - \tau)}{2(c - d) + \tau}\right] \exp \left(\frac{4\tau(c - d)}{4(c - d)^2 - \tau^2}\right),
\]

\[
\bar{b} = c + \frac{\tau}{2}, \quad \text{and}
\]

\[
b = \frac{(c + \frac{\tau}{2})^2 - c(c + \tau)}{2(c - d)}.
\]  

Note that in both of these bid functions, the fraction terms are non-negative (since costs are bounded from above by the maximum costs for each type of firm). In addition, by using the maximum bid, it must be that \(\eta \equiv 2b - 2c_d - \tau \leq 0\) with equality only at the maximum bid.

In Figure 1, we illustrate the bid functions corresponding to these inverse bid functions. We plot these as a function of firm costs where the foreign cost is inclusive of \(\tau > 0\). This demonstrates some of the features of the bidding functions discussed above. First, the lowest cost domestic and foreign firms both submit the minimum bid. Second, the highest cost domestic firm submits the maximum bid. Third, for foreign firms with costs in excess of \(\bar{b}\), they bid their cost and have a zero probability of winning. This is why the foreign bid function does not extend beyond \(\bar{b}\).

In addition, as \(\tau \to 0\), we have the symmetric case. Using L’Hôpital’s Rule, we have

\[
\lim_{\tau \to 0} c_d(b) = 2b - c_d
\]

\[
\lim_{\tau \to 0} c_f(b) = 2b - c_d.
\]  

These will be useful for evaluating welfare under free trade. Also, note that in this case, the maximum bid is \(\bar{c}_d\).

In order to derive the government’s optimal tariff, we need to derive the comparative stat-
ics of our inverse bid functions. In order to simplify the analysis, without loss of generality, we restrict the domestic firm’s cost distribution to the unit interval.\footnote{The only implication of this range is for the minimum value of $V_G$.} Note that this implies that the tariff is such that $0 \leq \tau < 2$ and, by plugging in the minimum and maximum bids, that $-\frac{(4-\tau^2)}{4} \leq \eta \equiv 2b - 2 - \tau \leq 0$ with strict equality at the minimum and maximum bids.

**Proposition 2.** The domestic firm’s bid is increasing in the tariff.

*Proof.* Begin by considering a domestic firm with a cost $c_d(b) < \bar{c}_d$. Taking the derivative of (20) with respect to the tariff yields:

$$\frac{dc_d(b)}{d\tau} = (c_d(b) - \bar{c}_d) \left[ \frac{2}{\tau} + \frac{4 + \eta \lambda_1 \exp \left( \frac{-\tau}{\eta} \right) \left[ \xi - \frac{\tau}{\eta} \right]}{4(b - \bar{c}_d - \tau) + \eta \lambda_1 \exp \left( \frac{-\tau}{\eta} \right)} \right]$$

(34)

where

$$\xi \equiv \left( \frac{8\tau^2(\tau_d - c_d)}{4(\tau_d - c_d)^2 - \tau^2} \right) = \left( \frac{1}{\eta^2} \right) \frac{\tau^2}{2}.$$

---

**Figure 1: Bid Functions**

![Bid Functions Graph](image-url)
This makes the second term positive and, since $c_d(b) < \tau_d$, the inverse cost function is weakly decreasing in the tariff. This means that as the tariff rises, the domestic firm cost associated with a given bid declines. Put differently, as the bid function is monotone in costs, the bid associated with a given cost increases. Further, because a domestic firm with the maximum cost $\bar{c}_d$ bids the maximum bid, $\bar{b} = \bar{c}_d + \frac{\tau}{2}$, such a firm’s bid also rises.

Thus, as the foreign firm’s is disadvantaged due to the tariff, the bid submitted by the domestic firm increases. Turning to the foreign firm’s inverse bid function, we are able to identify two properties of the relationship between it and the tariff. The first shows that, like the domestic firm, the foreign inverse bid function is non-increasing in the tariff when beginning from free trade.

**Lemma 3.** *When domestic costs are distributed uniform on the unit interval, at a zero tariff, the foreign inverse bid function is non-increasing in the tariff.*

\textbf{Proof.} For when $\bar{c}_d = 1$ and $\underline{c}_d = 0$,

$$\lim_{\tau \to 0} \frac{dc_f(b)}{d\tau} = -\left[\frac{1}{3} + \frac{8(b - 1)^3}{3}\right]$$

Recall that with a zero tariff, the foreign inverse bid function is $c_f(b) = 2b - \bar{c}_d$, thus bids will be between .5 and 1. Therefore this expression will range from 0 for the lowest cost foreign firm to $-\frac{1}{3}$ for the highest cost foreign firm.

Beginning from free trade and introducing a tariff, for a given bid this means that the associated cost inclusive of the tariff falls. Alternatively, a firm with a given cost increases its bid. It is important to note, however, that it increases its bid by no more than one. This implies that, although it increases its bid in an effort to cover the rise in costs, only the highest cost foreign firm fully passes the cost of the tariff through. For firms with less than this cost, they choose to absorb part of the tariff increase in order to better their chances of winning the contract. This introduces a tension for the foreign firm between its desire to
still win the contract (leading it to absorb the tariff) and its desire to maximize the profit of winning (leading it to pass through the tariff). Unlike the domestic firm where there is no such tradeoff, this tension prevents us from signing the change in the foreign inverse bid function with respect to a positive tariff for the full range of inverse costs. Nevertheless, we can show the following result.

**Lemma 4.** The foreign firm’s inverse bid function is decreasing in the tariff for some bid \( b \in [b, \bar{b}) \).

**Proof.** Focusing on the lower bound of the bid and cost space for the foreign firm, as the tariff rises there are two things that change in response to a tariff, \( b \) and \( c_f \) (which, recall, is inclusive of the tariff). Specifically,

\[
\frac{db}{d\tau} = 2 + \frac{\tau}{4} \quad \text{and} \quad \frac{dc_f}{d\tau} = 1.
\]

Recalling that the minimum bid is submitted by the lowest cost firm, these combine so that the inverse bid function moves such that:

\[
\frac{dc_f(b)}{d\tau} = \frac{\tau}{2 + \tau},
\]

which is between 0 and 1 for any positive tariff. Further, since \( \tau \leq 2 \), it follows that

\[
\frac{db}{d\tau} \leq \frac{dc_f}{d\tau}.
\]

By continuity and the fact that \( c_f(b) \) is monotonically increasing in \( b \), then for bids close to \( b \), the same holds.

Thus for at least some range of costs, the foreign firm increases its bid as the tariff rises even when the tariff is positive. This is because for firms with low costs, the probability of winning is sufficiently large that they are willing to tradeoff a lower chance of winning with a higher payoff if it does win.
3 The Government’s Optimal Tariff

The government sets the tariff to maximize expected welfare, which is the sum of the value of the project, the expected payoff conditional on the domestic firm winning, and the expected payoff conditional on the foreign firm winning. In the first of these, from the perspective of the government, we weight the domestic firm’s profits by $\theta \in [0, 1]$. This is comparable to McAfee and McMillan (1989). In other words, if $\theta = 0$, the profit of the domestic firm has no effect on the government’s payoff and if $\theta = 1$, the domestic firm’s profit fully enters the government’s payoff (as it does in Branco (1994)). For a given bid $b$ by the domestic firm, welfare is then:

$$W(b, \tau) = V_G + \frac{c_f - c_f(b)}{c_f - c_f(b)} [-b + \theta (b - c_d(b))] + \frac{c_f(b) - c_f}{c_f - c_f} \left[ \tau - \frac{b + \bar{b}}{2} \right].$$  \hspace{1cm} (35)

To find expected welfare, it is then necessary to integrate across $b$, making expected welfare as a function of the tariff:

$$W(\tau) = \frac{1}{\bar{b} - b} \int_{\frac{b}{2}}^{\bar{b}} W(b, \tau) db.$$  \hspace{1cm} (36)

We can now state our next proposition.

**Proposition 3.** The optimal tariff is positive whenever $\theta > 0$, i.e. when domestic profits are positively valued.

**Proof.** This is shown in Figure 2 by simulating expected welfare. Analytical proofs are forthcoming.

Thus, when domestic profits are valued, the government will impose a positive tariff. However, even when the optimal tariff is positive, it will not be prohibitive, i.e. it will not be so high that low cost foreign firms cannot win the contract.

Thus, even when domestic profits are as valuable as government surplus, there is an incentive for the home government to allow low cost foreigners to successfully compete.
Figure 2: Government Welfare

Figure 2 illustrates this for a variety of $\theta$s. Note that, unsurprisingly, as the value placed on domestic firm profits falls, so too does the optimal tariff. To illustrate this in an alternative fashion, Figure 3 shows the optimal tariff as a function of $\theta$.

Note that in this analysis, we have only considered non-negative tariff values to simplify discussion. If we were to permit negative tariffs, however, in equilibrium they would never be used. There are two points needed to recognize this. First, there is a discontinuity in the expected government welfare at a zero tariff which guarantees that only non-negative tariffs will be used. This is because when $\tau < 0$, it is no longer the case that the domestic firm has a (weak) cost advantage. Therefore the ranking of the two cost distributions reverse and the bid functions switch, i.e. the home firm bids according to (20) and the foreign firm bids according to (19). After making that adjustment, however, because the home government places no weight on the foreign firm’s profits (similar to a $\theta = 0$), it has no incentive to subsidize the foreign firm, particularly as this comes with the cost of a negative subsidy. Therefore, for values of $\theta$ below this cutoff, the optimal tariff is a corner solution and equal
As an alternative to the government welfare above where foreign profits are not valued, we could instead consider a social planner who seeks to maximize expected total surplus, that is the (unweighted) sum of expected government surplus, expected domestic profits, and expected foreign profits. It is straightforward to show that this is maximized by a zero tariff. This is because with a zero tariff, firms use the same bidding function, ensuring that the contract is awarded to the lowest cost firm. As the social planner is indifferent between surplus accruing to the government or either firm, minimizing the expected cost maximizes expected total surplus.

**Proposition 4.** When $\theta = 0$ the equilibrium maximizes expected total surplus.
4 Relative Protectionism

A natural question is how the equilibrium under the new system, that is, without a price premium but with a tariff, compares to previous one where a price preference is used but there was no tariff as is analyzed by Branco (1994). In this alternative auction, the winning firm is paid its bid, however, the winner need not be the lowest bid firm, since the domestic firm wins so long as the difference in bids does not exceed the price preference. In his paper, Branco derives, among other things, the optimal price preference under a sealed-bid first price auction when firm profits are valued the same as government surplus and the costs of both the domestic and foreign firm are distributed uniformly on the unit interval. As shown in his paper, at the government’s optimal price preference, equilibrium bids are:

\[ b_d(c_d) = \frac{3 - c_d^2}{2(2 - c_d)} \]  

(37)

and

\[ b_f(c_f) = \max \left\{ c_f, \frac{1 + 2c_f}{4} \right\} \]  

(38)

with the domestic firm winning whenever

\[ \frac{c_D(b_D) - c_F(b_F)}{c_F(b_f)} < 1. \]  

(39)

Plotting the probability of the domestic firm winning for our model with no price preference but the optimal tariff (calculated for \( \theta = 1 \) to correspond to Branco’s model) and that in Branco’s model with no tariff but the optimal price preference yields Figure 4.

As can be seen, excepting when the domestic firm has the lowest cost, in which case both policies award the contract to the domestic firm with certainty, the tariff policy has a lower probability that the domestic firm wins the contract than does the price preference.

\(^7\)Among the other aspects of his model is that he allows for a deadweight loss to raising government revenues, something we do not consider. Thus, to draw the comparisons between his results and ours, set \( \lambda = 0 \) in his notation and \( \theta = 1 \) in ours.
policy. Alternatively, this means that the foreign firm’s probability of winning is at least as large under the tariff system. Furthermore, as illustrated in Figure 4, even though a winning foreign firm has to pay a positive tariff, this greater probability of winning results in greater expected profits for all but the highest cost foreign firms. This indicates that the tariff system is less protectionist than the price preference system because the foreign firm The rationale for this lower level of protectionism is that, with the tariff system, when the foreign firm wins the government still collects tariff revenues. This added benefit to a rewarding the contract to the foreign firm means that the government has less incentive to increase the chance that contract ends up in the domestic firm’s hands. Thus, a shift from the price preference system to a tariff system reduces the level of protectionism even if it does not fully level the playing field across firms. Furthermore, if domestic profits are only weakly valued, the optimal tariff is zero, there is no protectionism, and expected total surplus is maximized.
Although these results are driven by our assumptions, not the least of which are those for the cost distributions, it does suggest that the recent shift in the approach towards foreign firms bidding for government contracts can be a step towards efficiency.

Figure 5: Expected Foreign Profit

5 Conclusion

The purpose of this paper has been to consider the role of trade barriers such as tariffs, on competition for government contracts. Comparable to the prior regime in which price preferences are used, tariffs act as a barrier for foreign firms and inhibit their ability to successfully bid for contracts. Further, similar to the results of McAfee and McMillan (1989) and Branco (1994), the domestic government will choose erect a barrier against foreign firms when domestic firm profits are positively valued. Despite this, however, these policies are not identical. In particular, we show that protectionism under the tariff is lower than that
under the price preference precisely because the tariff increases the payoff from a winning foreign bid. Furthermore, when domestic firm profits are not overly valuable to the home government, there is no protectionism in equilibrium which maximizes expected total surplus. Therefore although moving from the previous regime with price preferences need not result in the desired goal of an equal playing field for all firms, it can represent a step in that direction.

References


