Extensive and intensive margins and the choice of exchange rate regimes

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Abstract

This paper studies how the choice of fixed or flexible exchange rate regimes is affected by the existence of intensive and extensive margins. Using a contemporaneous entry model, only extensive margins vary under fixed exchange rate regimes following a demand shock. In such a case, the choice results from the balance between the lower average number, higher volatility in extensive margins and their stronger congruence with preferences. Fixed exchange rate regimes are thus preferred for high enough labor supply elasticities. In contrast, when entry is lagged because households smooth their consumption by lending/supplying their funds to firms, both intensive and extensive margins fluctuate. In such a case, extensive margins have a negative contribution to welfare through their lower average and higher volatility. Instead, the congruence between preferences and intensive margins brings a positive contribution to welfare. In this general setting, fixed exchange rate regimes are less likely to be supported for a larger set of parameters when product varieties are less alike (i.e. when consumers express a higher preference for product diversity).

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1 Introduction

Since Friedman (1953), the international and monetary economics literature has widely studied the transmission of international shocks within the production sector. Accordingly, the adoption of a pegging policy

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or a common currency union shifts most adjustments to the real side of the economy. Flexible exchange rates, on the other hand, correct relative price misalignments and absorb macroeconomic shocks. Adjustments on the real side of the economy consist not only of the expansion and contraction of existing firms, but also the entry and exit of firms. Thus shocks are transmitted through both intensive and extensive margins of production and trade. The choice of fixed or flexible exchange rate regimes is therefore likely to be affected by the way those margins respond to shocks. While previous literature has mainly focused on the role of intensive margins on this choice, the present paper aims to study the role of extensive margins.

This paper studies the costs and benefits of fixed and flexible exchange rate regimes in a general equilibrium model that includes two countries, money holdings, an elastic labor supply, stochastic exogenous demand shocks and endogenous intensive and extensive margins. Firms produce and sell differentiated products under monopolistic competition. They enter or exist by comparing operational profits with costs of entry. Wages are assumed to be sticky for one period. Money matters in the economy beyond a mere unit of account under such nominal rigidities. Thus, monetary policy may have an impact on both extensive as well as intensive margins. Without any state-contingent financial assets internationally held, the choice of exchange rate regimes becomes critical for welfare.

Accepting closed form solutions, the model allows us to discuss the effect of asymmetrical demand shocks on intensive and extensive margins and on the choice of exchange rate regimes. In particular, we compare two models. In the first, households can smooth their consumption only by money savings. In the second, they can additionally smooth their consumption by lending/supplying their funds to firms. The consumption smoothing behavior through future extensive margins in the latter case has a crucial implication under fixed exchange rate regimes: while investment and firm entry are aligned with demand shocks in the first case, firm entry lags behind demand shocks in the second case as a result of monetary reaction.

Our results are as follows. In general, the loss of the exchange rate instrument requires adjustment in extensive as well as intensive margins following a demand shock. When there is no possibility of consumption smoothing through new firm creation, the adjustment arises only in extensive margins. As a result, extensive margins are procyclical and have a lower average and higher variance under a fixed regime. Yet, such procyclical movements in extensive margins raise welfare because they are provided at the very moment of the shift of preference. The choice of a fixed exchange rate regime must therefore account for its costs (the lower mean level of and higher variability in extensive margins) and its benefits (the stronger congruence between preference and product diversity). These costs decrease with higher labor supply elasticities.

When households can smooth their consumption through the future number of firms, intensive margins fluctuate on impact of a demand shock while extensive margins arise with one period of lag. This is due
to the monetary reaction under fixed exchange rate regimes. Hence, on the one hand, we have a positive congruence effect under fixed regimes on intensive margins. On the other hand, because of this lag between investment and production of extensive margins, the entry of firms and new product varieties is not necessarily concomitant with consumers’ tastes. As a result, the welfare cost, arising from a smaller average of and higher volatility in extensive margins, remains. As in the first case, we show that fixed regimes are preferable under high enough labor supply elasticities. Importantly, we also show that fixed exchange rate regimes are supported for a smaller set of parameters when households are allowed to save and invest in the form of firm creation. This is further the case when product varieties are less alike, hence consumers express a higher preference for product diversity. The mean and variance effect in extensive margins has indeed a higher impact on households’ welfare when consumers attach a higher importance to product diversity.

In this paper, there is neither international borrowing and lending nor fiscal transfer.\(^1\) Risk sharing across countries is therefore imperfect and the flexible price allocation realized under flexible exchange rate regimes deviates from that obtained under complete asset markets.\(^2\) The Pareto efficient allocation would set the product supply or diversity according to changes in taste for each country’s products and to redistribute those margins appropriately across countries. However, although flexible exchange rates correct relative prices and realize higher average production and lower volatility, they fail to ensure that product supply and diversity align with consumer preferences. By contrast, fixed exchange rate regimes have the opposite properties.

Our contribution relates to the literature in the following way. First of all, our model and results can be compared with Devereux(2004)’s contribution qualifying the prevailing view that exchange rates are the most important shock absorbers. Discussing a static economy with two countries, two varieties, wage rigidities and constant returns to scale, Devereux shows that fixed exchange rates are more welfare improving than flexible one, when the elasticity of labor supply is sufficiently high. Our model extends this model to full dynamics, increasing returns to scale, monopolistic competition and the entry and exit of firms. The extension allows us to discuss the role of the balance between intensive and extensive margins in welfare ranking. Comparing results, it turns out that, when households cannot make investment choices for firm creation, the choice of an exchange rate regime is the same in Devereux (2004) as our model. The forces at play are, however, different since product substitutability and love of product diversity drive the welfare effects of extensive margins in this paper. Our results differ when households can invest by

\(^1\)This corresponds to situations where governments are unable to organize significant transfers between countries and where households and/or governments are unable to use the international credit market in the long run because of various borrowing constraints (e.g. credit constraints for households, Maastricht treaty for E.U. countries, I.M.F. constraints for developing countries). The 2011 Greece-EU crisis is a good illustration of the difficulties of organizing international transfers and access to credit markets.

\(^2\)The deviation is called a "demand imbalance" in the literature (Corsetti et al. 2010a, 2010b). See also Hamano (2009a, 2009b) for related topics of risk sharing and extensive margins.
holding shares of new firms. This paper also presents and discusses a formal decomposition of the welfare contribution of the two margins.

The present paper emphasizes the mechanisms through which intensive and extensive margins can be accountable for the choice of exchange rate regimes. Our approach contrasts with the classical literature, where firms’ entry is driven solely by “real factors” such as productivity and population shocks (Krugman 1980, 1991; Melitz 2003; Ghironi and Melitz 2005). In this instance, Naknoi (2008a, 2008b)’s contribution is very close to ours. Naknoi analyzes how different exchange rate regimes can impact extensive margins through endogenous tradability based on a Ricardian comparative advantage. While relocation of firms between tradable and non-tradable sectors arises in her model, our model accommodates free entry conditions and exporting by all firms. Because the sectoral relocation of firms is instantaneous in response to shocks in Naknoi’s model, there are none of the welfare costs that could arise from the mismatch between taste and extensive margins under fixed regimes. Baldwin and Nino (2006) and Bergin and Lin (2010) also look at the impact of a common currency on extensive margins. They, however, attach a special role to fixed costs and abstract away from monetary issues. Finally, our paper builds upon the so-called New Open Economy Macroeconomics (see, for instance, Obstfeld and Rogoff 1995, Corsetti et al. 2010b). While this literature has focused on optimal monetary policies under complete financial markets, it is new beginning to investigate those policies under incomplete financial markets, as we do in this paper. This approach is also followed by Ching (2003) and Picard and Worrall (2009), who consider monetary transfers within currency unions that correct for the incompleteness of financial markets.

The structure of the paper is as follows. Sections 2 and 3 present and discuss the model and equilibrium. Sections 4 and 5 discuss the choices of exchange rate regimes when entry is either contemporaneous or lagged with investment. Section 6 concludes.

2 Model

The present model discusses the welfare costs and benefits of exchange rate regimes between two countries, Home and Foreign. (Foreign variables are denoted with asterisks.) Each country is inhabited by a unit mass of households who are differentiated only in terms of their labor services. Wages are set by households one period in advance of production. There are no fiscal transfers and no borrowing and lending across countries. We describe the domestic country (Home). The same description holds for Foreign.

**Households** In every time period $t$, each household $i \in [0, 1]$ consumes goods in a domestic set $X_t$ and a foreign set $Z_t$ of differentiated varieties. It also holds a quantity of money $M_t(i)$ and supplies $l_t(i)$ labor units (worked hours). The household maximizes its expected intertemporal utility, $E_0 \sum_{t=0}^{\infty} \beta^t U_t$ where

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3Interaction between extensive margins and monetary policy has been investigated in a closed economy. See for instance, Bergin and Corsetti (2008), Bilbiie, Ghironi and Melitz (2007), Lewis (2009) and Bilbiie, Fujiwara and Ghironi (2011).
\( \beta \in (0, 1) \) is a common discount rate and where utility in period \( t \) is given by the following two-tier utility function:

\[
U_t(i) = \ln C_t(i) + \chi \ln \frac{M_t(i)}{P_t} - \kappa \frac{[l_t(i)]^{1+\psi}}{1+\psi}
\]

where

\[
C_t(i) = \left( \frac{X_t(i)}{\alpha_t} \right)^{\alpha_t} \left( \frac{Z_t(i)}{1-\alpha_t} \right)^{\alpha_t^*}
\]

and

\[
X_t(i) = \left( \int_{\omega \in \mathcal{X}_t} x_t(i, \omega) z_t(i, \omega) d\omega \right)^{\frac{1}{1-\sigma}} \quad \text{and} \quad Z_t(i) = \left( \int_{\upsilon \in \mathcal{Z}_t} z_t(i, \upsilon) d\upsilon \right)^{\frac{1}{1-\sigma}}
\]

In this definition, \( P_t \) is the consumer price index, and \( M_t(i)/P_t \) is household \( i \)'s real money holding. The parameter \( \psi \) measures the inverse of the (Frisch) elasticity of labor supply while the parameters \( \chi \) and \( \kappa \) measure the intensity of preferences towards real money holdings and individual labor supply (worked hours) respectively. We call \( C_t(i) \) the composite bundle and \( X_t(i) \) and \( Z_t(i) \) the consumption baskets of domestic and foreign product. While \( x_t(i, \omega) \) denotes its consumption of domestic varieties \( \omega \in \mathcal{X}_t \), \( z_t(i, \upsilon) \) denotes the consumption of foreign varieties \( \upsilon \in \mathcal{Z}_t \). Under the above preferences (Dixit and Stiglitz, 1977), the parameter \( \sigma > 1 \) measures the elasticity of substitution among varieties within the same consumption basket and also the (inverse of) love for product diversity within this basket. The parameters \((\alpha_t, \alpha_t^*)\) where \( \alpha_t^* = 1 - \alpha_t \in (0, 1) \) measures the preference between the two consumption baskets. Following Devereux (2004), we will consider that the economy is hit by demand shocks so that \( \alpha_t \) follows an i.i.d. stochastic process which is symmetrically distributed around a mean equal to \( 1/2 \).

A home household supplies its labor and earns an income \( w_t(i)l_t(i) \) where \( w_t(i) \) is its hourly wage. In addition to spending its income on the above product varieties and holding money, the household can invest in domestic firms. We consider two modes. In the first, firm investment and entry occur in the same time period so that \( m = 0 \). In the second, investment occurs one period before entry as a result of share holding choice by households - that is \( m = 1 \). The household can therefore invest in a firm that produces the variety \( \omega \in \mathcal{X}_{t+m} \) that become available either within the same period (\( m = 0 \)) or within the next period (\( m = 1 \)). The household budget constraint is then given by

\[
\int_{\omega \in \mathcal{X}_t} p_t(\omega) x_t(i, \omega) d\omega + \int_{\upsilon \in \mathcal{Z}_t} p_t(\upsilon) z_t(i, \upsilon) d\upsilon + \int_{\omega \in \mathcal{X}_{t+m}} s_t(i, \omega) q_t(\omega) d\omega + M_t(i) = w_t(i)l_t(i) + \int_{\omega \in \mathcal{X}_t} s_{t-m}(i, \omega) d_t(\omega) d\omega + M_{t-1}(i)
\]

where \( p_t(\omega) \) and \( p_t(\upsilon) \) are the (domestic) prices of Home and Foreign varieties \( \omega \in \mathcal{X}_t \) and \( \upsilon \in \mathcal{Z}_t \). In this expression \( q_t(\omega) \) denotes the price at date \( t \) for a share of a firm that enters at date \( t \) and produces variety \( \omega \in \mathcal{X}_{t+m} \) at date \( t + m \), while \( d_t(\omega) \) denotes the dividend paid by an incumbent producer \( \omega \in \mathcal{X}_t \) at period \( t \). Household \( i \) spends a share \( s_t(i, \omega) \) on the stock of an entering firm \( \omega \in \mathcal{X}_{t+m} \) and receives the share \( s_{t-m}(i, \omega) \) of the dividend paid by every incumbent producer \( \omega \in \mathcal{X}_t \).
Firms

Firms’ activities are described as follows. Consider a Home firm that produces a differentiated variety $\omega \in X_t$ under increasing returns to scale and sells its products under monopolistic competition at date $t$. We assume that to produce its output the firm must spend on ”establishment” activities (e.g. building a production plant) at the time period $t - m$ where $m = 0, 1$. Every firm employs a set of horizontally differentiated labor services. To make this more precise, we assume that each household $i \in [0, 1]$ offers a differentiated labor service and that every firm $\omega$ demands the quantities of labor services $\ell_t(i, \omega)$ and $e_t(i, \omega)$, for its production and setup activities, respectively. To produce $y_t(\omega)$ units of outputs, the firm uses the set of labor services given by

$$y_t(\omega) = \left( \int_0^1 \ell_t(i, \omega)^{1 - \frac{1}{\theta}} \, di \right)^{\frac{1}{1 - \theta}} \tag{1}$$

whereas it uses

$$f = \left( \int_0^1 e_{t-m}(i, \omega)^{1 - \frac{1}{\theta}} \, di \right)^{\frac{1}{1 - \theta}} \tag{2}$$

for its establishment activities at date $t - m$. In those expressions, $\theta > 1$ is the elasticity of substitution among the different labor services. The firm pays a dividend to its shareholders. This dividend is equal to the contemporaneous operational profit that includes sales and production cost:

$$d_t(\omega) = p_t(\omega) y_t(\omega) - \int_0^1 \ell_t(i, \omega) w_t(i) \, di \tag{3}$$

The setup cost is equal to $\int_0^1 e_{t-m}(i, \omega) w_{t-m}(i) \, di$.

Markets and governments

When product markets clear, each firm’s supply equals the demand for its variety by both domestic and foreign consumers:

$$y_t(\omega) = \int_0^1 x_t(\omega, i) \, di + \int_0^1 x^*_t(\omega, j) \, dj, \quad \omega \in X_t$$

where the superscript $*$ denotes foreign consumption and $j$ denotes each foreign household. Similarly, when labor markets clear, each household’s labor supply equals the demand by firms:

$$l_t(i) = \int_{\omega \in X_t} \ell_t(i, \omega) \, d\omega + \int_{\omega \in X_t, m} e_t(i, \omega) \, d\omega, \quad i \in [0, 1]$$

In equilibrium, trade must be balanced so that the value of domestic imports equates the value of exports. We get

$$\int_0^1 \int_{\nu \in Z_t} \varepsilon_t p^*(\nu) z_t(\nu, i) \, d\nu \, di = \int_0^1 \int_{\omega \in X_t} p(\omega) x^*_t(\omega, j) \, d\omega \, dj$$

where $\varepsilon_t$ is the exchange rate (namely, the price of one unit of foreign money in terms of the domestic currency), $p^*(\nu)$ is the price of foreign variety $\nu \in Z_t$ denominated in the foreign currency, and $x^*_t(\omega, j)$ is the foreign demand for the domestic variety $\omega \in X_t$.

Finally, the central bank supplies an amount of money $M_t$. When the money market clears, the money supply is equal to its demand so that

$$M_t = \int_0^1 M_t(i) \, di$$
Symmetric conditions hold for the foreign country.

Wages are sticky during one time period. We define the equilibrium as follows: (i) each household \( i \) chooses a plan of money holding \( \{M_t(i)\}_{t=0}^{\infty} \), consumption profiles \( \{x_t(i, \cdot), z_t(i, \cdot)\}_{t=0}^{\infty} \), stock market positions \( \{s(i, \cdot)\}_{t=0}^{\infty} \) and wages \( \{w_{t+1}(i)\}_{t=0}^{\infty} \) applying in the next period, that maximize its intertemporal utility subject to its per-period budget constraint, (ii) each firm \( \omega \in X_t \) chooses its product price \( p_t(\omega) \) and its labor demands \( \ell_t(i, \omega) \) and \( e_t(i, \omega) \) that maximizes its profit, (iii) the local stock market clears so that firms enter as long as they raise a stock price \( q_t(\omega) \) that meets future expected dividends and (iv) products, labor and money markets clear in every period. The money supply is set by each central bank with the objective of either a fixed or flexible exchange rate.

3 Equilibrium

We here describe the equilibrium choices by households and firms and determine the market equilibrium conditions for any exogenous monetary policy. Equilibrium conditions will be applied to the monetary policies of fixed and flexible exchange rate regimes in the next sections. For the sake of conciseness, we here discuss the cases of contemporaneous and lagged entry together, equilibrium conditions being identical or similar. We finish by discussing equilibrium welfare.

**Household choices** In period \( t \), the household \( i \) chooses its consumption profiles \( (x_t(i, \cdot), z_t(i, \cdot)) \), money holding \( M_t(i) \) and share holdings \( s_t(i) \). First, its optimal consumption of home and foreign varieties can be computed as

\[
x_t(i, \omega) = \left( \frac{p_t(\omega)}{P_{X,t}} \right)^{\frac{1}{1-\sigma}} X_t(i)
\]

and

\[
z_t(i, \nu) = \left( \frac{p_t(\nu)}{P_{Z,t}} \right)^{\frac{1}{1-\sigma}} Z_t(i)
\]

where

\[
X_t(i) = \alpha_t \frac{P_t C_t(i)}{P_{X,t}} \quad \text{and} \quad Z_t(i) = (1 - \alpha_t) \frac{P_t C_t(i)}{P_{Z,t}}
\]

are the chosen consumption baskets and

\[
P_{X,t} = \left( \int_{\omega \in X_t} p_t(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \quad \text{and} \quad P_{Z,t} = \left( \int_{\nu \in Z_t} p_t(\nu)^{1-\sigma} d\nu \right)^{\frac{1}{1-\sigma}}
\]

are the price indexes for those baskets. Finally, the consumer price index is given by

\[
P_t = P_{X,t}^{\alpha_t} P_{Z,t}^{1-\alpha_t}
\]

Second, the household’s optimal money holdings and share of stock are expressed as the following real money demand equation.

\[
\frac{M_t(i)}{P_t} = C_t(i) \frac{\chi}{1 - E_{t}AX_{t+1}(i)}
\]
for \( m = 0 \) and \( 1 \), where

\[
\Lambda_{t,t+1} (i) = \beta \frac{P_tC_t (i)}{P_{t+1}C_{t+1} (i)}
\]

denotes the endogenous discount rate between \( t \) and \( t + 1 \).

While the equilibrium share of stock should be such that stock prices equal dividends \( (q_t (\omega) = d_t (\omega)) \) when \( m = 0 \), the optimal share of stock is given by the following Euler equation when \( m = 1 \):

\[
q_t (\omega) = E_t \Lambda_{t,t+1} (i) d_{t+1} (\omega).
\]

**Firms’ decisions** Firms produce under monopolistic competition. Consider a firm \( \omega \in X_f \) that chooses its price and labor demand at date \( t \). It maximizes its dividend payment (3) with respect to \( p_t (\omega) \) and \( \ell_t (\cdot, \omega) \) subject to the production function (1). The cost-minimizing demand for labor services is then equal to \( \ell_t (i, \omega) = (w_t (i)/W_t) ^{-\theta} y_t (\omega) \forall i \) where

\[
W_t \equiv \left( \int_0^1 w_t (i) ^{1-\theta} di \right)^{\frac{1}{1-\theta}}
\]

is the wage index, common for all domestic firms. Considering the following iso-elastic demand for its variety

\[
y_t (\omega) = \int_0^1 x_t (i, \omega) di + \int_0^1 x_t^* (j, \omega) dj = \left( \frac{p_t (\omega)}{P_{X,t}} \right) ^{-\sigma} \left[ \int_0^1 X_t (i) di + \int_0^1 X_t^* (j) dj \right]
\]

the firm sets its optimal price

\[
p_t (\omega) = \frac{\sigma}{\sigma - 1} W_t
\]

which is the same for all domestic varieties. Taking into account the above conditions, the firm’s dividend is equal to \( d_t = p_t (\omega) y_t (\omega) / \sigma \).

Firm \( i \) also minimizes its setup cost \( \int_0^1 \epsilon_{t-m} (i, \omega) w_{t-m} (i) di \). Its cost-minimizing demand for labor services is equal to \( \epsilon_{t-m} (i, \omega) = (w_{t-m} (i)/W_{t-m}) ^{-\theta} f \), which results in a cost \( W_{t-m} f \).

**Wage setting** In this paper we consider wages that are sticky. The households set their wages one time period in advance. Accordingly, at date \( t \), a domestic household \( i \) sets the wage \( w_{t+1} (i) \) that maximizes its expected utility \( E_t U_{t+1} (i) \) subject to next period’s budget constraint and next period’s balance between labor supply and demand: \( \ell_{t+1} (i) = \int_{\omega \in X_{t+1}} \ell_{t+1} (i, \omega) dw + \int_{\omega \in X_{t+1} + \epsilon} \epsilon_{t+1} (i, \omega) dw \). In the latter expression, the first and second terms respectively represent the labor demands for production activities at date \( t+1 \) and for setup activities at date \( t+1 \) by the firms producing at date \( t+1+m \), \( m = 0, 1 \).

As seen above, this labor demand is a iso-elastic function of \( w_t (i) \). One can show that the household sets its wage such that its expected disutility of a marginal work effort, \( \kappa \theta w_{t+1} (i) ^{-1} E_t \ell_{t+1} (i) ^{1+\psi} \), equals its expected utility from the associated increase in consumption, \( (\theta - 1) E_t \left[ \ell_{t+1} (i) / (P_{t+1}C_{t+1} (i)) \right] \). Hence,

\[
w_{t+1} (i) = \frac{\kappa \theta}{\theta - 1} E_t \left[ \ell_{t+1} (i) ^{1+\psi} \right] / \left[ E_t \left[ \ell_{t+1} (i) / (P_{t+1}C_{t+1} (i)) \right] \right]
\]
**Local stock market equilibrium**  The equilibrium in the local stock market depends on investment timing. When a firm enters and establishes production in the same period as its sales \((m = 0)\), it asks a stock price of \(q_t(\omega) = W_t f\) and pays a dividend of \(d_t(\omega) = p_t(\omega) y_t(\omega) / \sigma\). Since \(q_t(\omega) = d_t(\omega)\) when \(m = 0\), the stock market clears when \(p_t(\omega) y_t(\omega) / \sigma = W_t f\).

By contrast, when a firm enters and invests in the period before its sales \((m = 1)\), it asks a stock price of \(q_t-1(\omega) = W_t-1 f\) and pays a dividend of \(d_t(\omega) = p_t(\omega) y_t(\omega) / \sigma\). The Euler equation (5) becomes

\[
W_{t-1} f = E_{t-1} \Lambda_{t-1,1} p_t(\omega) y_t(\omega) / \sigma
\]

**Product market equilibrium**  The above analysis shows that households make the same choices and firms the same decisions within each country. This symmetry allows us to dispense with household- and firm-specific notations. We can now drop the reference to \(i\) and \((\omega, \nu)\) without any ambiguity. The product market clear when trade is balanced. The balanced trade condition yields the exchange rate,

\[
\varepsilon_t = \frac{(1 - \alpha_t) P_t C_t}{\alpha_t P_t^* C_t^*},
\]

which simply compares the value of imports (numerator) to the value of exports (denominator). Given the above relationships, we can readily determine the dividends and wages as well as the extensive and intensive margins.

Let \(N_t\) be the mass of firms \(\omega \in \mathcal{X}_t^t\) that produce in the domestic country. With the above balanced trade condition, each domestic firm’s dividend is successively given by

\[
d_t = \frac{1}{\sigma} p_t y_t = \frac{1}{\sigma N_t} [P_t C_t + \varepsilon_t P_t^* C_t^*] = \frac{1}{\sigma} \frac{P_t C_t}{N_t}.
\]

**Wages, extensive and intensive margins**  On the one hand, consider a firm that enters and sets its production up in the same period as its sales \((m = 0)\). We know that \(d_t = p_t y_t / \sigma = W_t f\) so that the extensive and intensive margins are given by

\[
N_t = \frac{1}{\sigma W_t f} P_t C_t \quad \text{and} \quad y_t = \frac{\sigma W_t f}{p_t} = (\sigma - 1) f
\]

As usual in Dixit-Stiglitz models, the intensive margin is constant while the extensive margin absorbs all shock variability. The equilibrium labor supply is determined as follows. Consider the wage and labor services supplied in period \(t + 1\). The labor market clears if

\[
l_{t+1} = N_{t+1} l_{t+1} + N_{t+1} e_{t+1} = N_{t+1} y_{t+1} + N_{t+1} f = \frac{P_{t+1} C_{t+1}}{W_{t+1}}
\]

where we successively used (1), (2) and (9). Plugging this value of labor supply into the wage equation (6) yields the wage index

\[
W_{t+1} = \left(\kappa \frac{\theta}{\theta - 1}\right)^{\frac{1}{\psi}} \left[ E_t (P_{t+1} C_{t+1})^{1+\psi} \right]^{\frac{1}{1+\psi}}
\]
Ceteris paribus, the wage increases if the disutility from work increases (higher \( \kappa \)) or their labor services become weaker substitutes (lower \( \theta \)). In addition, it increases with a higher volatility of future nominal expenditures \( P_{t+1}C_{t+1} \) when \( \psi > 0 \).

On the other hand, consider a firm that enters and invests in the period before its sales \( (m = 1) \). Then, the dividend is given by \( d_t = p_t y_t / \sigma \) and the share price by the Euler equation (7). The latter condition yields the extensive margin as

\[
N_t = \frac{E_{t-1}N_{t-1}P_tC_t}{W_{t-1}f} = \frac{\beta P_{t-1}C_{t-1}}{\sigma W_{t-1}f},
\]

while the former yields the intensive margin

\[
y_t = \frac{\sigma - 1}{\sigma} P_tC_t \frac{1}{W_{t}}.
\]

Both margins respond to shocks. In particular the extensive margin responds to the previous period’s economic condition whereas the intensive margin responds to the current conditions. Note that the number of firms which appear as a consequence of households’ consumption smoothing across time falls with more impatient investors (smaller \( \beta \)).

When the labor market clears, the labor supply is equal to labor demand. So, using (1), (2), (10) and (11), we get the equilibrium labor supply as

\[
l_{t+1} = N_{t+1}\ell_{t+1} + N_{t+2}\ell_{t+1} = N_{t+1}y_{t+1} + N_{t+2}f = \frac{\sigma - 1 + \beta P_{t+1}C_{t+1}}{\sigma} \frac{1}{W_{t+1}}.
\]

Plugging this value of labor supply into the wage equation (6) yields the wage

\[
W_{t+1} = \left( \kappa \frac{\theta}{\theta - 1} \right)^{1+\psi} \left( \frac{\sigma - 1 + \beta}{\sigma} \right)^{1+\psi} \left[ E_t (P_{t+1}C_{t+1})^{1+\psi} \right]^{1+\psi}.
\]

As above, the wage increases with higher disutility from work (higher \( \kappa \)) and less substitutable labor services (lower \( \theta \)). In addition, the wage increases with higher impatience (lower \( \beta \)). This is because the extensive margin decreases, leading to a decrease in the remuneration from setup activities. At the limit, where \( \beta \to 1 \), the wage coincides to that under contemporaneous entry assumption \( (m = 0) \).

**Welfare** The household’s utility is the sum of its utility from consumption and disutility from working

\[
U_R = \ln C_t - \kappa k_m \frac{f_{t+1}}{1 + \psi},
\]

where \( k_m, m = 0, 1 \) are two constants\(^4\) and the utility from real money balance is

\[
U_M = \chi \ln \frac{P_t}{M_t}.
\]

\(^4\)For \( m = 0 \), \( k_0 = (1 + \psi)^{-1} (\theta - 1) \theta^{-1} \sigma^\psi (\sigma - 1 + \beta)^{-\psi} \) while for \( m = 1 \), \( k_1 = k_0 ((\sigma - 1 + \beta) / \sigma)^{1+\psi} \).
Following Obstfeld and Rogoff (1995), we assume that the latter utility can be neglected \((\chi \rightarrow 0)\). The household’s consumption can be computed as

\[ C_t = \alpha_t N_t^{\frac{\sigma_t}{\alpha_t}} N^*_{t}^{\frac{\sigma_t}{\alpha_t}} \left( \frac{y_t}{\alpha_t} \right)^{\alpha_t} \left( \frac{y^*_t}{\alpha^*_t} \right)^{\alpha^*_t}, \]

where \(\alpha^*_t = 1 - \alpha_t\). This consumption level has the same expression in the contemporaneous entry model.

Note that, using the equilibrium labor supply and taking the expectation of \(U^R_t\), the expected work disutility (second term in \(U^R_t\)) is found to be constant and identical across exchange rate regimes. What matters for welfare is the expected utility from consumption (the first term in \(U^R_t\)).

The intertemporal expected utility from consumption at period \(t = 0\) is equal to \(\sum_{t=0}^{\infty} \beta^t E_0 \ln C_t\)

\[ E_0 \ln C_t = E_0 \alpha_t \ln y_t + E_0 (1 - \alpha_t) \ln y^*_t + \frac{\sigma}{\sigma - 1} \left[ E_0 \alpha_t \ln N_t + E_0 (1 - \alpha_t) \ln N_t^* \right] + \text{cst} \]

In fact, the first and second and fourth and fifth terms in the above expression are the same because of the symmetry of \(\alpha_t\’s\) distribution. Thus developing the terms, the expected consumption will actually depend on

\[ E_0 \alpha_t E_0 \ln y_t + \text{cov}(\alpha_t, \ln y_t) + \frac{\sigma}{\sigma - 1} \left[ E_0 \alpha_t E_0 \ln N_t + \text{cov}(\alpha_t, \ln N_t) \right] \]

This expression presents sets of factors that affect domestic welfare. On the one hand, domestic welfare rises with the log of expected domestic production and diversity (first and third terms). The log of expected intensive and extensive margins can rise with their mean level and fall with their higher volatility.\(^5\) On the other hand, domestic welfare also depends on the congruence between domestic consumers’ preferences and the intensive and extensive margins (second and fourth terms in (13)). Specifically, the welfare effect of product diversity depends on \(\sigma\), that measures both the elasticity of substitution and the preference for variety.

4 Exchange rate systems and contemporaneous entry

As in Corsetti and Pesenti (2005, 2009), we define the monetary stance as

\[ \mu_t \equiv P_t C_t \]

The monetary stance is here derived from (4) as \(\mu_t = (M_t/\chi)(1 - E_t \Lambda_{t,t+1})\), which, after substituting for \(\Lambda_{t,t+1}\), yields the following recursive identity:

\[ \mu_t = \frac{M_t}{\chi} \left( 1 - \beta \mu_t E_t \mu_{t+1}^{-1} \right). \]

\(^5\)For instance, one can make the following approximation for small demand shocks: \(E_0 \ln y_t \simeq \ln \overline{y} - \frac{1}{2} \text{var} \left( \frac{y_t}{\overline{y}} \right)\) and \(E_0 \ln N_t \simeq \ln \overline{N} - \frac{1}{2} \text{var} \left( \frac{N_t}{\overline{N}} \right)\).
This identity solves to
\[ \mu_t = \frac{1}{\chi} \left( \frac{1}{M_t} \right) + \sum_{s=1}^{\infty} \beta^s E_t \left( \frac{1}{M_{t+s}} \right). \]
So, the current monetary stance is a function of the current and expected future money supply.

As a result, the exchange rate can be expressed in terms of the monetary stance as
\[ \varepsilon_t = \frac{\alpha^*_t \mu_t}{\alpha_t \mu^*_t}. \]
In the domestic country, the equilibrium wages, extensive and intensive margins are then computed as (see more variables in Appendix Table A1)
\[ W_t = \xi \left( E_{t-1}^{1+\psi} \right)^{\frac{1}{1+\psi}}, \quad N_t = \frac{1}{\sigma f} \frac{\mu_t}{W_t} \quad \text{and} \quad y_t = \sigma - 1 \frac{\mu_t}{W_t N_t} \]
where
\[ \xi = \left( \kappa \frac{\theta}{\theta - 1} \right)^{\frac{1}{1+\psi}} \]
is a constant.

We can make several comments from those expressions. First, wages are sticky and depend on the expectation of the monetary stance. At given wages, extensive margins increase proportionally with the monetary stance. An expansion of domestic money supply stimulates current expenditure on consumption goods and increases local firms’ profit, which triggers the entry of new product varieties. This effect is similar to the one discussed in Bergin and Corsetti (2008). Second, the expansion of the domestic money supply also stimulates production scales of incumbents but the latter are exactly cancelled out by the business stealing effect of new entrants. Indeed, given the above equalities, we get \( y_t = f (\sigma - 1) \). Third, it is shown below that \( \left( E_{t-1}^{1+\psi} \right)^{\frac{1}{1+\psi}} \) is an increasing function of \( \psi \) and the variance of \( \mu_t \). Therefore, wages increase with weaker labor supply elasticity \( \psi^{-1} \) and larger variance in monetary stance when \( \psi > 0 \). A larger variance in \( \mu_t \) amplifies the fluctuations in consumers’ product demands and therefore firms’ labor demands. This entices workers to claim higher wages in compensation for future wage uncertainty, hence increases firms’ costs. As a result of these higher wages, the number of firms falls. However, when the labor supply is infinitely elastic \( (\psi = 0) \), the variability of monetary stance does not matter in wage setting behavior.

In a flexible exchange rate regime, the domestic and foreign money supply \( (M_t, M_t^*) \) are constant for all time periods, so that the monetary stance is given as \( \mu_t = \mu_t^* = 2\mu_0 \) where \( \mu_0 \) is a constant. The exchange rate becomes \( \varepsilon_t = \alpha^*_t / \alpha_t \). Replacing \( P_t C_t \) with \( \mu_t = 2\mu_0 \) in the above expressions, we can compute the equilibrium wage, extensive and intensive margins as
\[ W_t = \mu_0 \xi, \quad N_t = \frac{1}{\sigma f \xi} \quad \text{and} \quad y_t = (\sigma - 1) f \]
In this regime, not only are wages but also extensive and intensive margins constant and independent of shock distributions. The exchange rate perfectly absorbs the effects of demand shocks on wages and margins. This is the allocation of production that would prevail in an economy without wage rigidities.
In a fixed exchange rate regime, the domestic and foreign money supply \((M_t, M_t^*)\) are set so that the exchange rate \(\varepsilon_t\) equals 1. This means that monetary authorities take procyclical monetary stances such as \(\mu_t = 2\mu_0 \alpha_t\) and \(\mu_t^* = 2\mu_0 \alpha_t^*\). Let

\[ A \equiv \left( E_{s-j} \alpha_s^{1+\psi} \right)^{1\over \psi} \quad \text{for} \quad s > j, \]

which is larger than \(E_t \alpha_t = 1/2\). \(A\) increases with increased variance in \(\alpha_t\). Replacing \(P_t C_t\) by \(\mu_t\) in the above expressions, we can compute wages, extensive and intensive margins as follows:

\[ W_t = 2\mu_0 \xi A, \quad N_t = \frac{1}{\sigma f \xi} A \quad \text{and} \quad y_t = (\sigma - 1) f \]

On the one hand, wages are constant but depend on shock distributions (through \(A\)). Wages coincide under flexible and fixed exchange rate regimes when labor supply is perfectly elastic \((\psi = 0 \iff A = 1/2)\). Since \(W_t\) rises with \(A\), wages increase with the shock variance and with the lower labor supply elasticity \((\text{larger } \psi)\). On the other hand, extensive margins respond to shocks. Higher demand for local goods triggers firm entry under procyclical monetary policy. Finally, extensive margins fall with \(A\). Indeed, a higher shock variance or a lower labor supply elasticity increases wages and therefore reduces firms’ incentives to enter in the market.

Compared to the flexible exchange rate regime, the domestic expected welfare at \(t = 0\) under fixed exchange rates differs only from its extensive margins \(N_t\). Hence, using (13), the condition with which a fixed regime is supported compared to a flexible regime is given by

\[ \frac{\sigma}{\sigma - 1} \left[ E_0 \alpha_t \ln \frac{\alpha_t}{A} + \text{cov}(\alpha_t, \ln \frac{\alpha_t}{A}) \right] > 0 \quad \text{(14)} \]

The first term in the bracket represents the welfare loss under fixed exchange rates. Provided a distribution of \(\alpha_t\), this term can be further negative with a lower labor supply elasticity \(\psi^{-1}\) through \(A\). As it has been mentioned, the lower labor supply elasticity and resulting higher wages reduce further the equilibrium level of extensive margins under fixed regime. The second term is strictly positive and measures the gains from the congruence between domestic preferences and domestic product diversity. Because \(A\) is a constant, this term is equal to \(\text{cov}(\alpha_t, \ln \alpha_t)\).

In expression (14), the choice of exchange rate regime is independent from the love for variety \(\sigma\). Only the elasticity of labor supply \(\psi^{-1}\) matters through the terms in \(A\). At \(\psi = 0\), we have that \(A = E_t \alpha_t\) so that expression (14) is proportional to \(\text{cov}(\alpha_t, \ln \alpha_t)\), which is strictly positive, hence supporting fixed exchange rate regimes. It is shown below that \(A\) increases in \(\psi\) so that the above expression falls below zero as \(\psi\) increases from zero to infinity. Therefore there exists a unique threshold \(\psi\) below which (14) is positive and above which it is negative.

**Proposition 1** In the contemporaneous entry model, there exists a labor elasticity threshold \(\psi_0^{-1}\) such that a fixed exchange rate system is preferred for labor elasticities \(\psi^{-1}\) larger than \(\psi_0^{-1}\).
Proof. We need to show that \( A \) is an increasing function of \( \psi \) and is negative for large \( \psi \). Let \( G : [1 - \bar{\sigma}, \bar{\sigma}] \to [0, 1] \) be the cumulative distribution of \( \alpha_t \) where \( \bar{\sigma} \in [1/2, 1] \) is the upper bound of the distribution. We first show that \( dA/d\psi > 0 \). Indeed, let \( f(\psi) \equiv A^{1+\psi} = \int \alpha^{1+\psi}dG(\alpha) \), which is lower than one because \( \alpha_t < 1 \) for all \( \alpha_t \) and is an increasing function as \( f'(\psi) = (1 + \psi) \int \alpha^\psi dG(\alpha) > 0 \). Then, we compute that \( d\ln A/d\psi = (d/d\psi) \left[ \ln f(\psi)^{1/(1+\psi)} \right] = (d/d\psi) \left[ (1 + \psi)^{-1} \ln f(\psi) \right] = -(1 + \psi)^{-2} \ln f(\psi) + (1 + \psi)^{-1} f'(\psi)/f(\psi) \), which is positive because each term is positive in the last expression.

Since \( d\ln A/d\psi = A^{-1} dA/d\psi \), it must be that \( dA/d\psi > 0 \). Second, expanding the covariance term, the bracket in expression (14) is equal to \( E_0 \alpha_t E_0 \ln \alpha_t - E_0 \alpha_t E_0 \ln A + E_0 \alpha_t E_0 \ln \alpha_t - E_0 \alpha_t E_0 \ln \alpha_t \), which simplifies to \( E_0 \alpha_t (\ln \alpha_t - \ln A) \). The latter expression is negative because \( \ln \alpha_t < \ln A = \ln \bar{\sigma} \) for \( \psi \to \infty \). Indeed, we successively get \( \lim_{\psi \to \infty} \ln A = \lim_{\psi \to \infty} \ln \left[ E_0 \alpha^{1+\psi} \right] \frac{1}{1+\psi} = \lim_{\psi \to \infty} \ln \left[ \int_{1-\bar{\sigma}}^{\bar{\sigma}} \alpha^{1+\psi} dG(\alpha_t) \right] \frac{1}{1+\psi} = \lim_{\psi \to \infty} \ln \left[ \int_{1-\bar{\sigma}}^{\bar{\sigma}} (\alpha/\bar{\sigma})^{1+\psi} \bar{\sigma}^{1+\psi} dG(\alpha_t) \right] \frac{1}{1+\psi} = \lim_{\psi \to \infty} \ln \bar{\sigma} + \lim_{\psi \to \infty} \left[ 1/ \left( 1 + \psi \right) \right] \ln g(\bar{\sigma}) = \ln \bar{\sigma} \).

Intuitively, on the one hand, fixed exchange rate regimes are definitely preferred because they realize an ideal composition of product diversity in the consumption basket following a shift in preferences. On the other hand, however, fixed regimes are less likely supported since they increase the variability of product diversity and reduce the equilibrium mean level of extensive margins. The latter negative effect on welfare, arising from higher equilibrium wages, becomes strong when the labor supply is less elastic.

This result has been highlighted in Devereux (2004) in a static economy with two differentiated goods where adjustments take place solely through intensive margins. Our result simply shows that the extensive margins have the same effect as intensive margins.

We now discuss the more realistic model where extensive and intensive margin coexist through households’ saving decision.

5 Exchange rate systems and lagged entry

We now consider the situation in which households save and invest one part of current wealth in the form of future product diversity in order to smooth their consumption.

As before, the exchange rate is a function of monetary stances, \( \varepsilon_t = (\alpha_t^* / \alpha_t)(\mu_t / \mu_t^*) \), which allows us to compute the following equilibrium wages, extensive and intensive margins (see other variables in Appendix Table A2)

\[
W_t = \xi \phi \left( E_{t-1} \mu_t^{1+\psi} \right) \frac{1}{1+\psi}, \quad N_{t+1} = \frac{\beta}{\sigma} \frac{\mu_t}{W_t} \quad \text{and} \quad y_t = \frac{\sigma - 1}{\sigma} \frac{\mu_t}{W_t N_t},
\]

where

\[
\phi \equiv \left( \frac{\sigma - 1 + \beta}{\sigma} \right)^{1/1+\psi}
\]

is a constant.
Given the saving and investment behavior of households, the expansion of current monetary stance boosts the number of firms in the next period. Higher nominal expenditure stimulated by an expansion of monetary stance increases firms’ discounted expected operational profits while entry costs remain unchanged because of wage stickiness. As a result, the entry of firms which produce with one period lag is induced. Since the number of firms behaves exactly as a state variable, the current production scale also expands with an expansion of the current monetary stance. Hence, there is no business stealing effect following current demand shocks.

In the flexible exchange rate regime, the domestic and foreign money supply are constant for all time periods so that the monetary stance is again 

\[ \mu_t = \mu^*_t = 2\mu_0 \]

and the exchange rate is equal to 

\[ \epsilon_t = \alpha^*_t / \alpha_t. \]

Replacing \( P_tC_t \) by \( \mu_t = 2\mu_0 \) in the above expressions, we get

\[ W_t = \mu_0 \xi \phi, \quad N_t = \frac{\beta \phi}{\sigma f \xi} \quad \text{and} \quad y_t = (\sigma - 1) \frac{f}{\beta} \cdot \]

As in the contemporaneous entry model, the exchange rate perfectly absorbs the effects of demand shocks on wages and margins so that the latter remain constant.

In the fixed exchange rate regime, the domestic and foreign money supplies are set to maintain a fixed exchange rate \( \epsilon_t = 1 \). Monetary stances are procyclical and then equal to \( \mu_t = 2\mu_0\alpha_t \) and \( \mu^*_t = 2\mu_0\alpha^*_t \). Replacing \( P_tC_t \) by \( \mu_t \) in the above expressions, the wages, the equilibrium extensive and intensive margins are computed as follows:

\[ W_t = 2\mu_0 \xi A, \quad N_{t+1} = \frac{\beta}{\sigma f \xi} \frac{\alpha_t}{A} \quad \text{and} \quad y_t = (\sigma - 1) \frac{f}{\beta} \frac{\alpha_t}{\alpha_{t-1}}. \]

As before, wages rise with the higher variance of the shocks (larger \( A \)) and lower elasticity of labor supply (larger \( \psi \)). Contrary to the contemporaneous entry model, both the extensive and intensive margins here respond to shocks under fixed exchange rate regimes. The future extensive margins vary with current period shocks due to the procyclical feature of monetary policy. The intensive margins adapt to both the current and previous period demand shocks which determine the current number of firms.

We can now compare welfare under the two regimes. Welfare differences stem from both extensive and intensive margins. Hence, using (13), the fixed exchange rate regime supporting condition is given by

\[
E_0\alpha_t E_0 \ln \frac{\alpha_t}{\alpha_{t-1}} + \text{cov}(\alpha_t, \ln \frac{\alpha_t}{\alpha_{t-1}}) + \frac{\sigma}{\sigma - 1} \left[ E_0\alpha_t E_0 \ln \frac{\alpha_{t-1}}{A} + \text{cov}(\alpha_t, \ln \frac{\alpha_{t-1}}{A}) \right] > 0.
\]

The first term reflects the impact of the mean and variance of intensive margins. Since demand shocks are i.i.d., this term is nil. In expectation, the mean and variance effect of current demand shocks is cancelled out by the mean and variance effect of extensive margins at the same period. The second term measures the congruence of present preferences and present supply of each domestic product. With i.i.d. shocks, this term simplifies to \( \text{cov}(\alpha_t, \ln \alpha_t) > 0 \), which reflects the benefit of a congruence between preferences and supplies of each product under fixed exchange rate regimes. The last square bracket
expresses the same trade-off in the cost and benefit of extensive margins. The first term is the mean and variance effect of extensive margins, negative under fixed exchange rate regimes. This cost can be, however, compensated by a higher labor supply elasticity (larger $\psi^{-1}$) through $A$. Since demand shocks are i.i.d., the second term in the bracket is nil. Past movements in firm entry and exit, hence the resulting fluctuations in product diversity, cannot be related to present preferences and do not bring any congruence benefit.

The above condition simplifies to

$$\text{cov}(\alpha_t, \ln \alpha_t) + \frac{\sigma}{\sigma - 1} E_0 \alpha_t E_0 \ln \frac{\alpha_{t-1}}{A} > 0$$

which decreases in $A$ and therefore in $\psi$ (lower labor supply elasticity). By the same argument as for Proposition 1, the expression (15) accepts a unique root $\psi_1$. Since expression (15) is smaller than (14) by the term $\text{cov}(\alpha_t, \ln \alpha_t)/(\sigma - 1)$ and increases in $\sigma$, this root $\psi_1$ is smaller than $\psi_0$ and that increases with larger $\sigma$.

**Proposition 2** In the lagged entry model there exists a labor elasticity threshold $\psi_1^{-1}$ such that a fixed exchange rate system is preferred for $\psi^{-1} > \psi_1^{-1}$. The labor elasticity threshold $\psi_1^{-1}$ is larger than $\psi_0^{-1}$ and falls as $\sigma$ rises.

In this proposition, the fixed exchange rate system is less likely supported for a given value of labor elasticity if consumers express a higher love for product diversity (higher $1/(\sigma - 1)$). While the ideal composition of intensive margins can be achieved under fixed exchange rate regimes, the cost arising from the lower mean level and higher variability of extensive margins has a larger impact on their welfare when households attach a higher importance to product diversity. The above point is summarized in Figure 1. When the value on the vertical axis exceeds the unity, a flexible system is supported. In the figure, the case where $\sigma = \infty$ coincides to the welfare ranking obtained with a contemporaneous entry economy and discussed in Devereux (2004).

### 6 Conclusion

This paper studies how the choice of fixed or flexible exchange rate regimes is affected by the existence of intensive and extensive margins. When there is no consumption smoothing though the future number of product varieties, intensive margins do not change following the shock and only extensive margins vary under fixed regimes. In such a case, the choice results from the balance between the lower average number of product varieties and higher volatility in extensive margins and their stronger congruence with preferences. Fixed exchange rate regimes are preferred for high enough labor supply elasticities.

In contrast, when entry is lagged as a result of households’ consumption smoothing by lending/supplying their funds to firms in addition to money savings, both intensive and extensive margins vary under fixed
Figure 1: Welfare comparison between fixed and flexible exchange rate system with different values of the elasticity of labor supply $\psi^{-1}$ and substitution $\sigma$. 

17
regimes. In such a case, *extensive margins* have a negative contribution to welfare through their lower average and higher volatility while the congruence between preference and *intensive margins* brings a positive contribution to welfare. In such a general setting, fixed exchange rate regimes are less likely to be supported for a larger set of parameters when product varieties are less alike and consumers express a higher preference for product variety.

References


[21] Naknoi, K., 2008a. The Benefit of Exchange Rate Flexibility, Trade Openness and Extensive Margin. Purdue University Economics Working Papers 1215. Purdue University, Department of Economics.


Appendix A: Optimal choice of households

The problem can be stated in terms of the following optimization of the Lagrangian function $L_0 (i)$:

$$E_0 \sum_{t=0}^{\infty} \beta^t \{ U_t (i) w_t (i) l_t (i) + \int_{\omega \in X_t} s_{t-m} (i, \omega) d_t (\omega) d\omega + M_{t-1} (i)$$

$$- P_t C_t (i) - \int_{\omega \in X_{t+m}} s_t (i, \omega) q_t (\omega) d\omega - M_t \}$$

with respect to $\{ x_t (i, \omega) , z_t (j, \nu) , M_t (i) , s_t (i, \omega) , w_t (i) \}_{t=0}^{\infty}$ where $\lambda_t (i)$ denotes the Lagrangian multiplier associated with the flow budget constraint at time $t$. Note that $P_t C_t (i) = \int_{\omega \in X_t} p_t (\omega) x_t (i, \omega) d\omega + \int_{v \in Z_t} p_t (v) z_t (i, v) dv$.

The first order condition with respect to $C_t (i)$ yields

$$\frac{1}{C_t (i)} - \lambda_t (i) P_t = 0 \tag{16}$$

So, $\lambda_t (i)$ represents the marginal utility stemming from one additional unit of nominal wealth. The above expression is identical for both models with $m = 0, 1$.

The first order condition with respect to $M_t (i)$ yields

$$\frac{\chi}{M_t (i)} - \lambda_t (i) + \beta E_t \lambda_{t+1} (i) = 0 \tag{17}$$

The first order condition with respect to $s_t (i, \omega)$ gives

$$- \lambda_t (i) q_t (\omega) + \beta E_t d_{t+1} (\omega) \lambda_{t+1} (i) = 0$$

The marginal utility of nominal wealth at $t$ is equal to the discounted marginal utility at $t + 1$. This condition is redundant when $m = 0$.

The household also sets the future wage $w_{t+1} (i)$ at $t$ knowing the demand function for her labor service $l_{t+1} (i)$. The first order condition with respect to $w_{t+1} (i)$ yields

$$k \theta \frac{E_t l_{t+1} (i) 1^{+\psi}}{w_{t+1} (i)} - (\theta - 1) E_t [\lambda_{t+1} (i) l_{t+1} (i)] = 0$$

Accordingly, the expected disutility of a marginal work effort is equal to the expected consumption utility of the associated marginal wage increase.

We can summarize the solutions for the contemporenous and lagged entry models $m = 0, 1$ for any exchange rate in the following tables:
Table A1: Solution in contemporaneous entry model \((m = 0)\)

<table>
<thead>
<tr>
<th>Home variables</th>
<th>Foreign variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_t = \alpha_t \Phi_t)</td>
<td>(C_t^* = (1 - \alpha_t) \Phi_t)</td>
</tr>
<tr>
<td>(\Phi_t \equiv N_t^{(1 + \frac{1}{\alpha_t})^\alpha_t} N_t^{*(1 + \frac{1}{\alpha_t})^{1-\alpha_t}} \left( \frac{y_t}{\mu_t} \right)^{\alpha_t} \left( \frac{\mu_t}{1-\alpha_t} \right)^{1-\alpha_t})</td>
<td>(\varepsilon_t = \frac{1 - \alpha_t}{\alpha_t} \frac{\mu_t}{\mu_t})</td>
</tr>
<tr>
<td>(l_t = \frac{\mu_t}{W_t})</td>
<td>(l_t^* = \frac{\mu_t^<em>}{W_t^</em>})</td>
</tr>
<tr>
<td>(N_t = \frac{1}{\sigma} \frac{\mu_t}{W_t})</td>
<td>(N_t^* = \frac{1}{\sigma} \frac{\mu_t^<em>}{W_t^</em>})</td>
</tr>
<tr>
<td>(y_t = \frac{\sigma - 1}{\sigma} \frac{\mu_t}{W_t N_t}) = ((\sigma - 1) f)</td>
<td>(y_t^* = \frac{\sigma - 1}{\sigma} \frac{\mu_t^<em>}{W_t^</em>} = (\sigma - 1) f^*)</td>
</tr>
<tr>
<td>(p_t = \frac{\sigma}{\sigma - 1} W_t)</td>
<td>(p_t^* = \frac{\sigma}{\sigma - 1} W_t^*)</td>
</tr>
<tr>
<td>(d_t = \frac{1}{\sigma} \frac{\mu_t}{N_t})</td>
<td>(d_t^* = \frac{1}{\sigma} \frac{\mu_t^<em>}{N_t^</em>})</td>
</tr>
<tr>
<td>(q_t = W_t^* f)</td>
<td>(q_t^* = W_t^* f^*)</td>
</tr>
<tr>
<td>(W_t = \xi \left( E_{t-1} \mu_t^{1 + \psi} \right)^{\frac{1}{1+\psi}})</td>
<td>(W_t^* = \xi \left( E_{t-1} \mu_t^{1 + \psi} \right)^{\frac{1}{1+\psi}})</td>
</tr>
<tr>
<td>(\mu_t = P_t C_t)</td>
<td>(\mu_t^* = P_t C_t^*)</td>
</tr>
</tbody>
</table>

Table A2: Solution in lagged entry model \((m = 1)\)

<table>
<thead>
<tr>
<th>Home variables</th>
<th>Foreign variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_t = \alpha_t \Phi_t)</td>
<td>(C_t^* = (1 - \alpha_t) \Phi_t)</td>
</tr>
<tr>
<td>(\Phi_t \equiv N_t^{(1 + \frac{1}{\alpha_t})^\alpha_t} N_t^{*(1 + \frac{1}{\alpha_t})^{1-\alpha_t}} \left( \frac{y_t}{\mu_t} \right)^{\alpha_t} \left( \frac{\mu_t}{1-\alpha_t} \right)^{1-\alpha_t})</td>
<td>(\varepsilon_t = \frac{1 - \alpha_t}{\alpha_t} \frac{\mu_t}{\mu_t})</td>
</tr>
<tr>
<td>(l_t = \frac{\mu_t}{W_t})</td>
<td>(l_t^* = \frac{\mu_t^<em>}{W_t^</em>})</td>
</tr>
<tr>
<td>(N_t = \frac{\beta}{\sigma} \frac{\mu_t}{W_t-1 f})</td>
<td>(N_t^* = \frac{\beta}{\sigma} \frac{\mu_t^<em>}{W_t-1 f^</em>})</td>
</tr>
<tr>
<td>(y_t = \frac{\sigma - 1}{\sigma} \frac{\mu_t}{W_t N_t})</td>
<td>(y_t^* = \frac{\sigma - 1}{\sigma} \frac{\mu_t^<em>}{W_t^</em> N_t^*})</td>
</tr>
<tr>
<td>(p_t = \frac{\sigma}{\sigma - 1} W_t)</td>
<td>(p_t^* = \frac{\sigma}{\sigma - 1} W_t^*)</td>
</tr>
<tr>
<td>(d_t = \frac{1}{\sigma} \frac{\mu_t}{N_t})</td>
<td>(d_t^* = \frac{1}{\sigma} \frac{\mu_t^<em>}{N_t^</em>})</td>
</tr>
<tr>
<td>(q_t = W_t^* f)</td>
<td>(q_t^* = W_t^* f^*)</td>
</tr>
<tr>
<td>(W_t = \xi \phi \left( E_{t-1} \mu_t^{1 + \psi} \right)^{\frac{1}{1+\psi}})</td>
<td>(W_t^* = \xi \phi \left( E_{t-1} \mu_t^{1 + \psi} \right)^{\frac{1}{1+\psi}})</td>
</tr>
<tr>
<td>(\mu_t = P_t C_t)</td>
<td>(\mu_t^* = P_t C_t^*)</td>
</tr>
</tbody>
</table>