Comparative Advantage and Multi-product Firms

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Abstract

This paper examines the role of multi-product firms in resource reallocation and income redistribution during trade liberalization. We show that at the presence of multi-product firms who produce both in comparative advantage and comparative disadvantage industries, trade liberalization induces relatively more within industry resource reallocation in comparative disadvantage industries than in comparative advantage industries, which mitigates \textit{ex ante} comparative advantage. This result is in contrast with Bernard, Redding and Schott (2007). The existence of multi-product firms also mitigates the relative-wage losses of scarce factors after trade liberalization. At the firm-level, trade liberalization induces multi-product firms to adjust their product mix towards comparative advantage products in the domestic market, while in the export market, they adjust their product mix towards comparative disadvantage products.

\textit{Keywords}: comparative advantage, heterogeneous firms, multi-product firms, trade liberalization

\textit{JEL classification}: F11; F12; L11; J23

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1 Introduction

This paper examines the role of multi-product firms, who simultaneously produce products that have different factor intensity, in the process of resource reallocation and income redistribution during trade liberalization. We also investigate how these firms adjust their product mix and relative factor demand after trade liberalization. The important role played by multi-product firms both in the domestic and export markets has only been realized by trade economists very recently (e.g., Bernard et al., 2010; Mayer et al., 2010; Eckel and Neary, 2009). So far, the focus of this literature has been within firm across products differences in productivity and most of the models predict a reallocation of resources from low-productivity products to high-productivity products after trade liberalization, which produces a gain from trade not identified before. In addition to this focus in micro-level resource reallocation\(^1\), this literature has also ignored comparative advantage by considering only one factor. In this paper, we show that by bringing back comparative advantage to the model, the existence and omnipresent of multi-product firms will not only have implications for micro-level (within firm) resource reallocation, but also far reaching implications for macro-level resource reallocation as well as income redistribution during trade liberalization.

This paper is also related to the recently developed literature on heterogeneous firms. While neoclassical trade theory only focuses on country (factor endowment) and industry (factor intensity) characteristics, heterogeneous firm models (e.g., Melitz (2003)) emphasize within industry firm heterogeneity in productivity and trade liberalization will induce within industry reallocation from low productivity firms to high productivity firms. But so far, this literature has mostly ignored comparative advantage by considering only a single factor and industry. Bernard, Redding and Schott (2007) (BRS hereafter) is the first to combine the neoclassical Heckscher-Ohlin model with the heterogeneous firm model of Melitz (2003). Their main finding is that the within industry reallocation induced by trade liberalization is more pronounced in comparative advantage industry, which magnifies \textit{ex ante} comparative advantage. Our model is closely related to their work. Our model can also be regarded as introducing multi-product firms to the BRS model, by doing which we show that many of their results will be changed or even reversed.

Multi-product firms are omnipresent in the modern economy. Bernard, Redding and Schott (2006) find that 41% of US firms are multi-product firms and their output accounts for 91% of the total output. Using Belgian manufacturing firm-level data, Fuss and Zhu (2010) find that

\(^1\)One notable exception is that Bernard et al. (2010) also derives some implications on macro-level trade patterns from the existence and prevalence of multi-product firms.
about 30% of manufacturing firms produce in more than one 4-digit NACE code industries, and among these multi-industry firms, about one third of the variation in firm-product-level skill intensity is within firm variation rather than across firms. Further more, within 2-digit industries, about two thirds of the variation in product skill intensity takes place within multi-product firms. These findings highlight the fact that multi-product (-industry) firms play an important role in manufacturing and they often simultaneously produce products that have substantially different factor intensities. These findings motivate the key assumptions of our model.

We consider a world of two countries, two industries, and two factors. Countries differ in relative factor abundance and industries differ in factor intensity. There are a continuum of firms in the market who are heterogeneous in productivity. Firms have to pay a fixed entry cost to enter the domestic market (for entry into export market, firms have to pay extra entry cost, see section 2). After entry, firms learn their productivity and choose whether to stay in the market and (if they choose to stay) how many industries to produce in. There is fixed production cost to cover in each industry which means firms drawing a productivity level below some lower threshold choose to exit the market. Firms with productivity level higher than the threshold (but not high enough) choose to produce in only one industry. Only firms with high enough productivity level choose to produce in both industries. The key difference between our model and the BRS model is that we allow firms to produce in both industries (whose skill intensities are different) by paying entry cost only once, while in their model, firms have to pay separate entry cost for entering each industry. This change in assumption is to capture the idea that firms can learn from experience in one industry and this reduces their cost of entering new (possibly related) industries. For simplicity, we assume the extra entry cost firms have to pay to enter a second industry to be zero.

After trade liberalization, there will be Melitz-type within industry resource reallocation from low productivity firms to high productivity firms in both industries. Firms with lowest productivity will exit the market in both industries. Nevertheless, we find that such reallocation effect is higher in comparative disadvantage industry (CD industry hereafter) than in comparative advantage industry (CA industry hereafter) and induces higher growth in average productivity in CD industry, thus trade liberalization mitigates ex ante comparative advantage. This result is the opposite of the BRS model. The mechanism underlying the BRS result is that trade liberalization induces more entry into CA industry (because there is more export opportunity in CA industry) which intensifies competition and makes low productivity firms harder to survive in CA.

\footnote{Since in the BRS model there is no scope economy both in entry and production, there is no role for multi-product firms. In our model, there is scope economy in entry, i.e., multi-product firms can share their entry cost in both industries.}
industry. This induces more growth in average productivity in CA industry and thus magnifies \textit{ex ante} comparative advantage. But in our model, we allow firms to produce both in CA and CD industries after entry, thus more entry into CA industry also implies more entry into CD industry (since a portion of high productivity firms that enter CA industry will also enter CD industry). This, plus the fact that CD industry faces more import competition after trade liberalization, will make low-productivity firms harder to survive in CD industry and thus make CD industry more productive on average.

We also find that the existence of multi-product firms mitigates the relative-reward losses of the scarce factor after trade liberalization. As in the neoclassical trade model, both the BRS model and our model predict a decrease in the relative reward of the scarce factor after trade liberalization, but the decrease in our model is much more modest. The first reason for this is, as stated above, in our model trade liberalization mitigates \textit{ex ante} comparative advantage, while it is magnified in the BRS model. This implies that the income redistribution effect of trade liberalization will be less pronounced in our model. The second reason is that the relative number of firms that produce in CA industries increases much more modestly in our model after trade liberalization, which mitigates the increase in the relative demand for abundant factor and thus the increase in relative reward. This is because that in our model there are two sources of new entrants for each industry: one is new entrants from outside the market and the other is the incumbents in the other industry (who become multi-product firms after entry). Thus, an increase in the number of firms in one industry also implies an increase in the number of potential entrants for the other industry, which eventually results in increase in the number of firms in the other industry. This mechanism, which is induced by the existence of multi-product firms, makes any adjustment in the relative number of firms in CA and CD industries much more difficult in our model$^3$.

In addition to the above results (which are at macro-level), we also get some firm-level results which contribute to the recent literature on trade and multi-product firms (Bernard et al., 2010; Mayer et al., 2010; Eckel and Neary, 2009; etc.). We show that in equilibrium, other things being equal, low productivity firms will only produce CA products, while higher productivity firms will produce both CA and CD products. Within exporters, firms with relatively low productivity will only export CA products, while the more productive ones export both CA and CD products. The

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$^3$Although we find these dramatically different results, we consider our model to be more of an complement to the BRS model rather than a substitute, because at the more aggregated industry level, multi-product (or multi-industry) firms may be less of a concern, thus their model may be more suitable than our model for analysis at that level. Our predictions are more likely to hold in the less aggregated level where multi-product firms play a more important role.
intuition for this result is that since CD product is less competitive in the market, it requires higher productivity to profitably produce or export it.

After trade liberalization, multi-product firms will adjust their product mix towards comparative advantage products in the domestic market, while in the export market, they adjust their product mix towards comparative disadvantage products. The intuition for the first result is straightforward. The second result is because that the discrepancy in competitiveness in the export market between CD and CA products will be dampened by trade liberalization. These within firm product mix adjustments also have implications for changes in firm-level relative factor demand. Multi-product firms will experience more skill upgrading (here we assume skilled and unskilled labor to be the two factors) than single product firms during trade liberalization, while multi-product exporters will experience less skill upgrading than single product exporters (given that single product exporters are also multi-product firms in the domestic market). The first part of this result is simply because that, as just mentioned, multi-product firms adjust their product mix to favor CA product after trade liberalization. The second part of the result is caused by two reasons. First, as just explained, multi-product exporters will adjust their export product mix in favor of CD product which reduces the relative demand for skilled workers, while this adjustment does not happen to the single product exporters. Second, after trade liberalization, exporters will adjust their production from domestic market to the export market. Since single product exporters only produce skill intensive CA product for export market, this adjustment will increase their relative demand for skilled workers more than it does for multi-product exporters, since the later have similar skill intensity in both domestic and export markets. These results highlights the possibility that while trade liberalization may harm the scarce factor in general, the extent to which they are harmed may depend on the type of firms that are hiring them.

The remainder of this paper is organized as follows. Section 2 describes the model. Section 3 show the simulation results of the model and discusses them. Section 4 concludes and points out some possible extensions for future works.

2 The model

We consider a world of two countries, two industries (to fix mind, consider them to be in one sector), two factors, and a continuum of heterogeneous firms. Within each industry, there is only

4In the setting of our model, there is no labor market imperfection, so there is no unemployment and no within skill type wage differences across firms, thus it does not matter for workers that which type of firm is hiring them. So the results we state above will be more relevant when taking into account the labor market imperfection that exists in reality.
one product (thus we use 'product' and 'industry' synonymously in this paper), with each firm in that industry producing one unique variety of that product. Firms have to pay a fixed entry cost to enter the industry, and this cost is thereafter sunk. After paying the entry cost, firms learn their productivity in producing in the two industries and we assume that each firm has the same productivity in the two industries. After learning their productivity, firms have to decide whether to produce and if they do, how many products to produce. If they decide to produce in any industry, they have to pay a fixed production cost in each industry, as well as a variable cost, which decreases with productivity. High productivity firms may also choose to export to the foreign market, and if they do they have to pay fixed as well as variable trade cost for each product. We only consider the situation of costly trade in this paper. Throughout the model, we assume the parameters are selected so that there is selection into exports, i.e., not all firms export and only firms with high enough productivity export. There is consumers' love for variety and it is costless for firms to differentiate their products, so each firm will produce a unique variety of each product they produce (assuming there is no scope economy for adding varieties of the same product). The market structure for each product is of Dixit-Stiglitz type monopolistic competition.

Throughout the paper, we make the standard Heckscher-Ohlin assumption that countries are identical in terms of preferences and technologies but differ in terms of factor endowments. Factors of production can move between industries within countries but not across countries. Most of the settings of our model is the same as that in the BRS model, except that we allow firms to choose whether or not to produce both products after entry. We use $H$ to index the skill-abundant home country and $F$ to index the skill-scarce foreign country.

### 2.1 Consumption

The representative consumer's utility depends on consumption of the output of the two industries ($i$), each of which contains a mass of horizontally differentiated varieties ($\omega$) produced by heterogeneous firms (note that some firms may produce in both industries). We assume the upper tier of utility function takes Cobb-Douglas form, while the lower tier takes CES form,

$$ U = C_1^{\alpha_1} C_2^{\alpha_2}, \quad \alpha_1 + \alpha_2 = 1 $$

(1)

where for simplicity, we omit the country superscript except where important. $C_i$ is a consumption index defined over consumption of individual varieties, $q_i(\omega)$, with dual price index, $P_i$, defined
over prices of varieties, \( p_i(\omega) \):

\[
C_i = [\int_{\omega \in \Omega_i} q_i(\omega)^\rho d\omega]^{\frac{1}{\rho}}
\]

\[
P_i = [\int_{\omega \in \Omega_i} p_i(\omega)^{1-\sigma} d\omega]^{\frac{1}{1-\sigma}}
\]

where \( \sigma = \frac{1}{1-\rho} > 1 \) is the constant elasticity of substitution across varieties. For simplicity, we assume that the elasticity of substitution between varieties is the same in the two industries.

Consumers expenditure minimization yields the following expression for equilibrium expenditure on a variety,

\[
r_i(\omega) = R_i \left( \frac{p_i(\omega)}{P_i} \right)^{1-\sigma} = \alpha_i R \left( \frac{p_i(\omega)}{P_i} \right)^{1-\sigma}
\]

where \( R_i \) is the aggregate expenditure on industry \( i \), and \( R \) is the total expenditure of the economy.

### 2.2 Production

Production involves a fixed and variable cost each period, both of which use skilled and unskilled labor. The skill intensity of production is the same within industry across varieties, but varies across industries. All firms within one industry have the same fixed cost, but variable cost varies with firm productivity, \( \varphi \in (0, \infty) \). We assume the total cost function for producing product \( i \) takes the C-D form,

\[
\Gamma_i^H = \begin{cases} 
(f_i + \frac{q_i^H}{\varphi})(w_s^H)^{\beta_i}(w_i^H)^{(1-\beta_i)}, & \text{if it does not export} \\
(f_i + f_{xi} + \frac{q_i^H}{\varphi} + \frac{\tau_i q_i^H}{\varphi})(w_s^H)^{\beta_i}(w_i^H)^{(1-\beta_i)}, & \text{if it exports}
\end{cases}
\]

where \( w_s^H \) is the skilled wage and \( w_i^H \) the unskilled wage in the home market. \( f_i(w_s^H)^{\beta_i}(w_i^H)^{(1-\beta_i)} \) is the fixed production cost, \( f_{xi}(w_s^H)^{\beta_i}(w_i^H)^{(1-\beta_i)} \) is the fixed export cost, \( q_i^H \) is sales in the domestic market, and \( q_{ix}^H \) is sales in the foreign market. \( \tau_i \) is the ice-burg variable trade cost. We assume \( 1 > \beta_2 > \beta_1 > 0 \), which means product 2 is more skill-intensive. Thus, the total cost function for a firm with productivity \( \varphi \) is:

\[
\Gamma^H(\varphi) = \sum_{i=1}^{2} I_i \Gamma_i^H
\]
where $I_i$ is an indicator variable which equals to 1 if a firm produces product $i$ and zero otherwise. We assume there is no headquarter cost for multi-product firms.

Given the monopolistic competition market structure, profit-maximization implies the equilibrium price for any variety is a constant mark-up over marginal cost, with export prices higher due to the variable trade costs:

$$p_{ix}^H(\varphi) = \tau_i p_{id}^H(\varphi) = \frac{\tau_i (w_s^H)^{\beta_i} (w_l^H)^{(1-\beta_i)}}{\rho \varphi} \quad (7)$$

Hence the revenue from selling product $i$ is given by

$$r_i^H(\varphi) = \begin{cases} r_{id}^H(\varphi), & \text{if it does not export} \\ r_{id}^H(\varphi) + r_{ix}^H(\varphi), & \text{if it exports} \end{cases} \quad (8)$$

where the revenue of product $i$ from domestic market is (substituting equation 7 to equation 4)

$$r_{id}^H(\varphi) = \alpha_i R(\frac{P_i^H \rho \varphi}{(w_s^H)^{\beta_i} (w_l^H)^{(1-\beta_i)}})^{\sigma-1} \quad (9)$$

and that from foreign market is

$$r_{ix}^H(\varphi) = \tau_i^{1-\alpha} (\frac{P_i^F}{P_i^H})^{\sigma-1} (\frac{R_i^F}{R_i^H}) r_{id}^H(\varphi). \quad (10)$$

Thus a firm’s revenue is given by

$$r^H(\varphi) = \sum_{i=1}^{2} I_i r_i^H(\varphi) \quad (11)$$

where $I_i$ is an indicator variable which equals 1 if a firm produces product $i$ and zero otherwise. Then we have the profit function of producing product $i$ as given below:

$$\pi_i^H(\varphi) = \begin{cases} \pi_{id}^H(\varphi), & \text{if it does not export} \\ \pi_{id}^H(\varphi) + \pi_{ix}^H(\varphi), & \text{if it exports} \end{cases} \quad (12)$$

where

$$\pi_{id}^H(\varphi) = \frac{r_{id}^H(\varphi)}{\sigma} - f_i (w_s^H)^{\beta_i} (w_l^H)^{(1-\beta_i)}$$

$$\pi_{ix}^H(\varphi) = \frac{r_{ix}^H(\varphi)}{\sigma} - f_{ix} (w_s^H)^{\beta_i} (w_l^H)^{(1-\beta_i)} \quad (13)$$

Note that we apportion the entire fixed production cost to domestic profit and the fixed cost
of exporting to foreign profit, as in Melitz (2003). Therefore, a firm’s profit is given by the sum of profit from selling the two products.

\[ \pi^H(\varphi) = \sum_{i=1}^{2} I_i \pi^H_i(\varphi) \]  

(14)

2.3 Product choice and export decision

To enter the domestic market, a firm must pay a fixed entry cost, which is thereafter sunk. The entry cost also uses skilled and unskilled labor and we assume its skill intensity to be different from that of production:

\[ f_e(w^H_s)^{\beta_3}(w^H_l)^{(1-\beta_3)}, \quad 1 > \beta_3 > 0 \]  

(15)

After paying the entry cost, firm draws its productivity, \( \varphi \), from a distribution \( g(\varphi) \), with a CDF \( G(\varphi) \). Firm’s productivity is assumed fixed thereafter, and firms are subject to a constant exogenous probability of death each period, \( \delta \), which we interpret as force majeure events beyond manager’s control.

A firm drawing productivity \( \varphi \) will produce product \( i \) only when \( \varphi > \varphi^{*H}_i \), where \( \varphi^{*H}_i \) is the zero-profit productivity cut-off, which is determined by

\[ r^H_{id}(\varphi^{*H}_i) = \sigma f_i(w^H_s)^{\beta_1}(w^H_l)^{(1-\beta_1)} \]  

(16)

Since a firm needs to occur an amount of fixed cost to enter foreign market of each product, there are also a exporting productivity cut-off, \( \varphi^{*H}_{ix} (> \varphi^{*H}_i) \), for each product, above which firms produce for both the domestic and export markets:

\[ r^H_{ix}(\varphi^{*H}_{ix}) = \sigma f_{ix}(w^H_s)^{\beta_1}(w^H_l)^{(1-\beta_1)} \]  

(17)

Thus there will be four productivity cut-offs in the market, which are \( \varphi^{*H}_1, \varphi^{*H}_2, \varphi^{*H}_{1x} \) and \( \varphi^{*H}_{2x} \), respectively. A firm that draws a productivity lower than \( \min(\varphi^{*H}_1, \varphi^{*H}_2) \) will exit the market directly, while a firm draws a productivity that lies between \( \min(\varphi^{*H}_1, \varphi^{*H}_2) \) and \( \max(\varphi^{*H}_1, \varphi^{*H}_2) \) will produce only one product. For firms with productivity higher than \( \max(\varphi^{*H}_1, \varphi^{*H}_2) \), they become multi-product firms. Such sorting in activities also happen in the export market. According to firms’ productivity, there will be non-exporters, exporters that only export one product and exporters that export both products. We have the following relationship between the four cutoffs:
\[ \varphi_1^H = \frac{\varphi_2^H}{\Lambda_1^H} = \frac{\varphi_2^H}{\Lambda_2^H} = \frac{\varphi_1^H}{\Lambda_3^H} \]  

(18)

where

\[ \Lambda_1^H = \left( \frac{P_1^H}{P_2^H} \right) \left( \frac{\alpha_1 f_2}{\alpha_2 f_1} \right) \left( \frac{1}{\sigma - 1} \right) \left( \frac{w_i^H}{w_i^H} \right)^{\frac{\sigma (\sigma - 1)}{\sigma - 1}} \]

\[ \Lambda_2^H = \tau_2 \left( \frac{P_1^H}{P_2^F} \right) \left( \frac{R_H}{f_1} \right) \left( \frac{f_2}{f_1} \right) \left( \frac{\alpha_1}{\alpha_2} \right) \left( \frac{w_i^H}{w_i^H} \right)^{\frac{\sigma (\sigma - 1)}{\sigma - 1}} \]

\[ \Lambda_3^H = \tau_1 \left( \frac{P_1^H}{P_2^F} \right) \left( \frac{R_H}{f_1} \right) \left( \frac{f_1}{f_1} \right) \left( \frac{1}{\sigma - 1} \right) \]  

(19)

Regarding the relative position of the productivity cutoffs, the following proposition can be proved:

**Proposition 1** Other things being equal, the exporting productivity cut-off of comparative advantage industry is lower than that of the comparative disadvantage industry.

**Proof.** See Appendix □

Proposition 1 implies that, for exporters, firms with relatively low productivity will only export CA products, while the more productive ones export both CA and CD products. The intuition for this result is that since CD product is less competitive in the international market, it requires higher productivity to profitably export it.

### 2.4 Free Entry

The value of a firm with productivity \( \varphi \) is

\[ v(\varphi) = \max \left\{ 0, \frac{1}{\delta} \pi_1^H (\varphi), \frac{1}{\delta} \pi_2^H (\varphi), \frac{1}{\delta} \sum_{i=1}^{2} \pi_i^H (\varphi) \right\} \]  

(20)

while the *ex ante* probability of exporting product \( i \) conditional on producing product \( i \) is

\[ \pi_i^H = \frac{[1 - G(\varphi_{ix}^H)]}{[1 - G(\varphi_{x}^{H})]} \]  

(21)

There is an unbounded competitive fringe of potential entrants, in equilibrium we require the expected value of entry \( V^H \), which is the sum of expected profit from producing in industry 1 and expected profit from producing in industry 2, to equal the sunk entry cost:
\[ V^H = \frac{1 - G(\varphi_{i1}^H)}{\delta} [\pi_{id} + \varphi_{i1}^H \pi_{1x}] + \frac{1 - G(\varphi_{i2}^H)}{\delta} [\pi_{2d} + \varphi_{i2}^H \pi_{2x}] = f_e(w_s^H)^\beta_3 (w_l^H)^{1-\beta_3} \]  \hspace{1cm} (22)

where \( \pi_{id} = \pi_{id}(\varphi_{i1}^H) \), and \( \pi_{ix} = \pi_{ix}(\varphi_{ix}) \). Weighted average productivity of firms that produces product \( i \), \( \bar{\varphi}_{i}^H \), is determined by the zero-profit productivity cut-off, \( \varphi_{i1}^* \), and is defined as

\[ \bar{\varphi}_{i}^H(\varphi_{i1}^*) = \frac{1}{1 - G(\varphi_{i1}^H)} \int_{\varphi_{i1}^H}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \]  \hspace{1cm} (23)

and the weighted average productivity of firms that export product \( i \), \( \bar{\varphi}_{ix}^H \), is determined by the exporting productivity cut-off, \( \varphi_{ix1}^* \), and is defined as

\[ \bar{\varphi}_{ix}^H(\varphi_{ix1}^*) = \frac{1}{1 - G(\varphi_{ix1}^H)} \int_{\varphi_{ix1}^H}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \]  \hspace{1cm} (24)

Using the above expressions, free entry condition can further be expressed as a function of productivity cut-off and wages

\[ \sum_{i=1}^{2} \left\{ \frac{f_i}{\delta} \int_{\varphi_{i1}^H}^{\infty} \left[ \left( \frac{\varphi}{\varphi_{i1}^H} \right)^{\sigma-1} - 1 \right] g(\varphi) d\varphi + \frac{f_{ix}}{\delta} \int_{\varphi_{ix1}^H}^{\infty} \left[ \left( \frac{\varphi}{\varphi_{ix1}^H} \right)^{\sigma-1} - 1 \right] g(\varphi) d\varphi \right\} (w_s^H)^\beta_1 (w_l^H)^{1-\beta_1} = f_e(w_s^H)^\beta_3 (w_l^H)^{1-\beta_3} \]  \hspace{1cm} (25)

It is clear from the expression of the free entry condition, that firms’ entry decisions are affected by both expect profit from industry 1 and that from industry 2. This is one of the major difference between our model and the BRS model. In the BRS model, increase in the expected profit in one industry will only induce more entry into that industry, while in our model, it will induce the total entry into the whole market, which results in more new entrants for both industries. This ‘externality’ is induced by the existence of multi-product firms and that firms do not have to pay extra entry cost to enter the second industry. This feature of our model is the key to understand our results.

### 2.5 Goods market clearing

In the steady state, the mass of firms that enter, \( M_e^H \), and the mass of incumbent firms in industry \( i \), \( M_i^H \), must satisfy the following condition

\[ (1 - G(\varphi_{i1}^H)) M_e^H = \delta M_i^H \]  \hspace{1cm} (26)
where $M^H_i$ is the mass of home firms that produce product $i$, i.e., the mass of successful entrants for each industry must equal to the mass of firms that die in that industry in each period.

Equation 26 implies:

$$\frac{M^H_i}{M^H_2} = \frac{1 - G(\varphi^H_1)}{1 - G(\varphi^H_2)}$$  \hspace{1cm} (27)

i.e., the relative mass of firms producing in industry 1 and industry 2 is determined by the relative position of the zero-profit productivity cut-offs of the two industries. This property is also unique to our model and has important implications for the main results.

The product price indices can be expressed as

$$P^H_i = \left[ M^H_i \left( \frac{p^H_{id}(\varphi^H_i)}{p^H_i} \right)^{1-\sigma} + \chi^F_i M^F_i \left( \frac{\tau_i \varphi^F_{ix}}{p^F_i} \right)^{1-\sigma} \right]^{1/\sigma}$$  \hspace{1cm} (28)

The economy level revenue from producing product $i$ equals to the sum of domestic and foreign expenditure on domestic varieties

$$R^H_i = M^H_i \alpha_i R^H \left( \frac{p^H_{id}(\varphi^H_i)}{p^H_i} \right)^{1-\sigma} + \chi^H_i M^H_i \alpha_i R^F \left( \frac{\tau_i \varphi^F_{ix}}{p^F_i} \right)^{1-\sigma}$$  \hspace{1cm} (29)

### 2.6 Labor market clearing

For each country, full employment in the labor market implies

$$S_{p1} + S_{p2} + S_e = S$$

$$L_{p1} + L_{p2} + L_e = L$$

where $S$ denotes skilled workers and $L$, un-skilled workers, and subscript $p$ stands for production and $e$, for entry (investment).

Production labor market clearing implies the industry revenue equals the variable production cost times the mark-up:

$$R^H_i = \frac{1}{\rho} \left[ (w^H_i L^H_{pi} + w^H_S S^H_{pi}) - M^H_i f_i(w^H_S)^{\beta_i}(w^H_i)^{1-\beta_i} - \chi^H_i M^H_i f_{ix}(w^H_S)^{\beta_i}(w^H_i)^{1-\beta_i} \right]$$  \hspace{1cm} (31)

The market clearing condition for investment workers requires that the wage payment to the investment workers equals to the fixed entry costs paid by all the entrants:

$$w^H_i L^H_{ei} + w^H_S S^H_{ei} = M_e f_e(w^H_S)^{\beta_3}(w^H_i)^{1-\beta_3}$$  \hspace{1cm} (32)
3 Simulation results

As the model has no closed-form analytical solution, in this section, we turn to solve it numerically. These solutions enable us to compare the outcomes of our model to the outcome of the BRS model. We follow BRS to assume a Pareto distribution for \textit{ex ante} firm productivity:

\[ g(\varphi) = ak^a \varphi^{-(a+1)} \]  

(33)

where \( k > 0 \) is the minimum value for productivity (i.e., \( \varphi \geq k \)), and \( a > 0 \) is a shape parameter that determines the skewness of the Pareto distribution. We assume \( a > \sigma - 1 \) to ensure that the weighted average productivity is finite. To focus on comparative advantage, we assume that all product parameters except factor intensity (\( \beta_i \)) are the same across industries (for the factor intensity of entry investment, \( \beta_3 \), we assume it to be one half, which is between \( \beta_1 \) and \( \beta_2 \)). We consider symmetric differences in country factor endowments and symmetric differences in product factor intensities. We also consider symmetric preference of consumers for the two product, that is \( \alpha_1 = \alpha_2 \). A more detailed discussion of other parameter values is included in the Appendix. All the parameter values are set to be the same as in BRS whenever possible (except the initial endowments in the two countries, which we magnifies the difference to have clearer results).

3.1 Macro-level resource reallocation and income distribution

3.1.1 Cut-offs and productivity

Figure 1 shows the evolution of the various productivity cut-offs when variable trade cost changes. We can see that the comparative advantage industry’s zero-profit productivity cut-off is always lower than that of the comparative disadvantage industry (which is different from the result of the BRS model). Meanwhile, as stated in Proposition 1, the exporting productivity cut-off of the comparative advantage industry is lower than that of the comparative disadvantage industry. As in BRS, trade liberalization leads to a rise in the steady-state zero-profit productivity cut-offs and a decline in the exporting productivity cut-offs. While in contrast with their results, in our model, the zero-profit (exporting) productivity cut-off of the comparative \textit{disadvantage} industry increases (decreases) relatively faster than that of the comparative \textit{advantage} industry. As a result of the relatively faster increase in zero-profit productivity cut-off, the weighted average productivity of firms that produce in the comparative disadvantage industry also increases faster than that in the comparative advantage industry, as shown in Figure 2. Thus, trade liberalization mitigates
ex ante comparative advantage rather than magnifies it.

To better understand the difference between our model and the BRS model, we can interpret our model in a slightly different way. If we regard the entry cost as paid for entry into one industry, the assumption of our model implies that immediately after entry into one industry, firms also costlessly learn their productivity in producing in the other industry. We can call this as ‘learning-by-entry effect’. From this point of view, our model and the BRS model represent two extreme scenarios in terms of ‘learning-by-entry effect’, i.e., in our setting, firms can costlessly learn their productivity in the other industry after paying entry cost for one industry, while in the BRS model entering into one industry is not helpful at all for firms to enter the other industry. This difference is crucial in explaining most of the differences between the results of our model and that of BRS. However, as explained before, we consider our model as a complement rather than a substitute to the BRS model, in the sense that our model fits better the situation where the two industries are similar in terms of production and marketing, while their model is more suitable when the two industries are substantially different.

3.1.2 Mass of firms and domestic varieties

For the mass of firms that produce in each industry (which is also the mass of varieties of each industry), we obtain results as shown in Figure 3. With the reduction of variable trade cost, the mass of domestic varieties decreases monotonically in both industries (as domestic varieties are replaced by imported varieties). However, comparing with BRS, our model predicts a much more modest decline in the mass of varieties of comparative disadvantage product relative to that of comparative advantage product (see Figure 4). This is again because that we allow firms to produce in both industries in our model, which makes the mass of firms that produce each product interdependent. In BRS, trade liberalization induces relatively more entries into the comparative advantage product market, resulting in deeper specialization and relatively sharper decline in the comparative disadvantage product’s mass of varieties, while in our model, although trade liberalization still leads to more entries, it affects the two industries relatively equally since firms can choose to produce in both industries after entry. This result has important implications for relative factor demand, thus for income distribution. In accordance with the modest decline in relative mass of firms in comparative disadvantage industry, the relative demand, thus the relative wage of the scarce factor also declines much more modestly in our model, as we will see below.
3.1.3 Relative wages

With trade liberalization, the relative wage of unskilled workers with respect to skilled workers decreases monotonically (see Figure 5), which is similar as in the neoclassical trade model as well as the BRS model. Compared with the BRS model (see Figure 6), the decline in relative wage is more modest in our model. The first reason for this is, as shown in Figure 2, in our model trade liberalization mitigates *ex ante* comparative advantage, while it is magnified in the BRS model. This implies that the income redistribution effect of trade liberalization will be less pronounced in our model. The second reason is that, as shown in Figure 3 and Figure 4, the relative number of firms that produce in comparative advantage industries increases much more modestly in our model after trade liberalization, which mitigates the increase in the relative demand for abundant factor and thus the increase in relative reward.

3.2 Firm-level results

In this section, we first check the product mix adjustments within multi-product firms following trade liberalization, then we show the heterogeneity of skill intensity across different types of firms and the heterogeneous response in terms of skill upgrading (down-grading) of firms to trade liberalization. For this purpose, we first randomly generate 100 firms whose productivity follows *Pareto* distribution. Then we calculate the steady-state skill intensities for each surviving firm at each level of variable trade cost (i.e., \( \tau - 1 \), from 0.6 to 0). Our main findings are reported in Figure 7, Figure 8, Table 1 and Table 2.

3.2.1 Product mix

The current multi-product firm literature (Bernard et al., 2010; Mayer et al., 2010; Eckel and Neary, 2009 ) all predict that firms will adjust their product mix to concentrate on their 'core competence', which are products that they can produce most efficiently, after trade liberalization. In our model, since productivity is assumed to be identical across products within firm, product mix adjustment takes places only because of comparative advantage. Figure 7 shows the product mix adjustments taking place within a representative multi-product firm (similar adjustments will take place in other multi-product firms) when variable trade cost changes. After trade liberalization, multi-product firms will produce relatively more comparative advantage products in the domestic market, while exporting relatively more comparative disadvantage products to the foreign market. The intuition for the first result is straightforward. The second result is be-
cause that the discrepancy in competitiveness in the export market between CD and CA products will be dampened by trade liberalization. These within firm product mix adjustments also have implications for changes in firm-level relative factor demand, as will be shown below.

### 3.2.2 Firm’s relative factor demand

Figure 8 reports the skill intensities and their dynamics along the process of trade liberalization for four representative firms (one for each type of firms). As we can see, given variable trade cost, different types of firms have different skill intensity. Firms that produce only one product (single-non-exporter) have the highest skill intensity and firms that produce two products but do not export (multi-non-exporter) have the lowest skill intensity, while the skill intensity of exporters (single-exporter or multi-exporter) lies in between, with single-exporters (who only export the skill intensive product, but produce both products for the domestic market) having higher skill intensity\(^5\). It is straightforward that single-product firms and single-exporters are more skill intensive. The reason that multi-exporters are more skill intensive than multi-non-exporters is that the skill intensity of exports is higher than domestic sales for multi-exporters, which is because the skill intensive product is more competitive in the foreign market than in the domestic market comparing with the labor intensive product.

With trade liberalization, the skill intensity of each firm will change due to adjustment at two margins, i.e., intensive margin (changes in product mix) and extensive margin (changes in number of products for domestic sales or for exports). When the adjustments only happen at the intensive margin, the skill intensity of firms will only change continuously, while changes at extensive margin will induce discrete jumps in firms’ skill intensity, as shown in Figure 8. For the intensive margin adjustment, multi-product firms will experience more skill upgrading than single product firms when variable trade cost decreases, while multi-product exporters will experience less skill upgrading than single product exporters (given that single product exporters are also multi-product firms in the domestic market)\(^6\). The first result is because, for a single product firm, since trade liberalization induces the relative wage of skilled workers to increase, the relative demand for skilled workers will decrease. For multi-product firms, while they will also use relatively less skilled workers in producing each product due to the relative wage change, it will also produce relatively more skill intensive product in the domestic market (see Figure 7).

---

\(^5\)Using Belgian firm-level export data, Bernard, Van Beveren and Vandenbussche (2010) find that firm’s capital intensity tend to decrease when the number of products they export increases, which is consistent with the prediction of our model.

\(^6\)In this numerical exercise, the only type of firms that experience positive skill upgrading are the single-exporters. So more skill upgrading should be understood as less skill downgrading in some cases.
Thus the product mix adjustment counteracts the skill down-grading effect induced by the relative increase of skilled worker wages. The second result is because of two reasons. First, multi-product exporters will adjust their export product mix towards labor intensive product (also see Figure 7) which reduces the relative demand for skilled workers, while this adjustment does not happen to the single product exporters. Second, after trade liberalization, exporters will adjust their production from domestic market to the export market. Since from proposition 1, single product exporters only produce skill intensive product for export market, this adjustment will increase their relative demand for skilled workers more than it does for multi-product exporters, because the later have similar skill intensity in both domestic and export markets.

Table 1 and Table 2 report the summary statistics and regression results for all the 100 firms. From Table 1, we again confirm that single-product firms are more skill intensive than multi-product firms, and within multi-product firms, single product exporters are more skill intensive than multi-product exporters or multi-product non-exporters. Table 2 confirms the relative skill upgrading (down-grading) of different types of firms during trade liberalization, which we have just stated above, i.e., single-exporters experience the highest degree of skill upgrading, and multi-product firms experience more skill upgrading (less skill down-grading) than single-product firms. These results highlights the possibility that while trade liberalization may harm the scarce factor in general, the extent to which they are harmed may depend on the type of firms that are hiring them.

4 Conclusions and extensions

This paper examines the role of multi-product firms, who simultaneously produce products that have different factor intensity, in the process of resource reallocation and income redistribution during trade liberalization. We also investigate how these firms adjust their product mix after trade liberalization. By bringing back comparative advantage to the multi-product firm literature, we show that the existence and prevalence of multi-product firms will not only have implications for micro-level (within firm) resource reallocation, but also far reaching implications for macro-level resource reallocation as well as income redistribution during trade liberalization.

We show that, at the presence of multi-product firms, trade liberalization induces relatively more creative destruction in comparative disadvantage industries than in comparative advantage industries and mitigates *ex ante* comparative advantage, which is in contrast with Bernard, Redding and Schott (2007). The existence of multi-product firms also mitigates the increase in the
relative number of firms that produce in comparative advantage industries after trade liberaliza-
tion. These results lead to a much more modest relative-wage losses for scarce factors.

At the firm-level, we show that, other things being equal, low productivity firms will only
produce CA products, while higher productivity firms will produce both CA and CD products.
Within exporters, firms with relatively low productivity will only export CA products, while the
more productive ones export both CA and CD products. After trade liberalization, multi-product
firms will produce relatively more CA products in the domestic market, while multi-product
exporters will export relatively more CD products. These within firm product mix adjustments
also have implications for changes in firm-level relative factor demand. Multi-product firms will
experience more skill upgrading (assuming skilled and unskilled labor to be the two factors) than
single product firms during trade liberalization, while multi-product exporters will experience less
skill upgrading than single product exporters. These results highlights the possibility that while
trade liberalization may harm the scarce factor in general, the extent to which they are harmed
may depend on the type of firms that are hiring them.

For future works, one possible extension can be to introduce one more sector to the model,
while within each sector there are two industries as in the current model. Across sectors, factor
intensity differ in a higher magnitude, while within sector across industries, factor intensity differ
in a lower magnitude. Firms can only produce in one sector, as in the BRS model, but within each
sector, firms can produce in both industries, as in our current model. By doing so, we can have
a model that incorporates both the features of the BRS model and that of our current model, so
that we can test whether the BRS predictions can hold in the sector level, where multi-product
firms are not an issue, while our predictions may hold at the (within sector) industry-level, where
multi-product firms play important roles.

Another possible extension is to relax the assumption that firms only pay entry cost once and
do not have to pay extra entry cost to enter a second industry. We can assume that by entering
one industry; the entry cost for entering another industry will be reduced but not to zero. This
will make the assumption more realistic and further more, the BRS model and our current model
will be two special cases of the new model.

A third extension can be to relax the assumption that firms have the same productivity in
producing both products. In stead we can assume that the productivity of producing product
1 and product 2 are positively correlated, or like in Bernard, Redding and Schott (2006b) we
can assume there is a firm-level productivity as well as firm-product-level specialty. Finally, the
assumption that there are only two products can also be relaxed to allow n products or even a
continuum of products that differ in factor intensity.


References


Appendix: Proof of Proposition 1

Proof. From equations 18 and 19, we have

\[
\frac{\varphi_{2x}^H}{\varphi_{1x}^H} = \frac{\Lambda_2^H}{\Lambda_3^H} = \left(\frac{\tau_2}{\tau_1}\right) \left(\frac{f_{2x}^H}{f_{1x}^H}\right) \left(\frac{x_{2x}^H}{x_{1x}^H}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{w_{s}^H}{w_{l}^H}\right)^{\frac{\sigma(\beta_2-\beta_1)}{\sigma-1}} \left(\frac{\tau_1}{\tau_1}\right) \left(\frac{f_{1x}^H}{f_1^H}\right)^{\frac{1}{\sigma-1}}
\]

\[
= \frac{P_1^F}{P_2^F} \left(\frac{f_{2x}}{f_1}\right) \left(\frac{x_{2x}}{x_1}\right)^{\frac{1}{\sigma-1}} \left(\frac{w_{s}^H}{w_{l}^H}\right)^{\frac{\sigma(\beta_2-\beta_1)}{\sigma-1}} \left(\frac{f_{1x}}{f_1}\right)^{\frac{1}{\sigma-1}}
\]

\[
= \frac{P_1^F}{P_2^F} \left(\frac{w_{s}^H}{w_{l}^H}\right)^{\frac{\sigma(\beta_2-\beta_1)}{\sigma-1}} < 1
\]

The last equality is obtained by the assumptions that \(\tau_2 = \tau_1, f_1 = f_2, f_{1x} = f_{2x}, \alpha_1 = \alpha_2\). The last inequality is obtained from two facts: first, in foreign country (which is abundant in unskilled labor) product 1 (which is unskilled labor intensive) is relatively cheaper than product 2, i.e., \(\frac{P_1^F}{P_2^F} < 1\); second, the skilled labor is abundant in home country and has relative lower wage, i.e., \(\frac{w_s}{w_l} < 1\), which implies \(\left(\frac{w_s}{w_l}\right)^{\frac{\sigma(\beta_2-\beta_1)}{\sigma-1}} < 1\) since \(\beta_2 > \beta_1\). \(\blacksquare\)

Appendix: Parameter values in the simulation

We set the parameter values as following:

Entry cost: \(f_e = 4\)

Fixed cost: \(f_1 = f_{1x} = f_2 = f_{2x} = 0.1\).

Production function: \(\beta_1 = 0.4, \beta_2 = 0.6, \beta_3 = 0.5\).

Elasticity of substitution: \(\sigma = 3.8\).

Productivity distribution: Pareto distribution \(g(\varphi) = ak^a\varphi^{-(a+1)}\), \(a = 3.4, k = 0.2\).

Exogenous death rate: \(\delta = 0.025\)

Varieties preference share of expenditure: \(\alpha_1 = \alpha_2 = 0.5\).

Countries are asymmetric: \(\mathcal{S}^H = 1400, \mathcal{L}^H = 700, \mathcal{S}^F = 700, \mathcal{L}^F = 1400\).
Table 1: Skill intensity for different types of firms

<table>
<thead>
<tr>
<th></th>
<th>Single-product firm</th>
<th>multi-product firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-exporter</td>
<td>2.84</td>
<td>1.94</td>
</tr>
<tr>
<td>Single product exporter</td>
<td>2.80</td>
<td>2.14</td>
</tr>
<tr>
<td>Multi-product exporter</td>
<td></td>
<td>1.98</td>
</tr>
</tbody>
</table>

Table 2: Trade liberalization and heterogeneous skill upgrading

<table>
<thead>
<tr>
<th>Dependant variable</th>
<th>Skill intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.025</td>
</tr>
<tr>
<td>D_P1X1</td>
<td>0.118</td>
</tr>
<tr>
<td>D_P2X0</td>
<td>-0.725</td>
</tr>
<tr>
<td>D_P2X1</td>
<td>-0.225</td>
</tr>
<tr>
<td>D_P2X2</td>
<td>-0.681</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.158</td>
</tr>
<tr>
<td>$\times$D_P1X1</td>
<td>-0.104</td>
</tr>
<tr>
<td>$\times$D_P2X0</td>
<td>-0.132</td>
</tr>
<tr>
<td>$\times$D_P2X1</td>
<td>-0.363</td>
</tr>
<tr>
<td>$\times$D_P2X2</td>
<td>-0.133</td>
</tr>
</tbody>
</table>

Note: D_P1X1 is a dummy variable which equals to 1 if a firm produces one product and export one product. The other dummies are defined similarly. The omitted category refers to single product non-exporter.
Figure 1: Productivity cut-offs

Figure 2: Weighted average productivity
Figure 3: Mass of domestic varieties.

Figure 4: Mass of domestic varieties-BRS model (solid line for CA industry)
Figure 5: Relative wages

Figure 6: Relative wages-BRS model (solid line)
Figure 7: Product mix of multi-product firms.

Figure 8: Firm skill intensity.