Trade in quality and income distribution: an analysis of the enlarged EU market

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Abstract

This paper contributes to the understanding of the determinants of country-level comparative advantages in terms of quality. More precisely, while the literature has mainly focused so far on supply-side determinants of such comparative advantages, we investigate both theoretically and empirically the role played by income distribution (average income and level of inequalities) of a country on the quality of its exports. Doing so, we provide new insights on the existence of demand-based determinants of the quality content of a country’s exports, in line with the Linder (1961) hypothesis, claiming that firms produce and export goods suited to the specific tastes of their local consumers. We build a model with economies of scale where non-homothetic preferences and within-country income differences determine the quality composition of production and exports. Having neutralized any supply-side comparative advantage, we show that richer countries produce and export higher quality goods, while the level of inequalities has an heterogenous impact, positively affecting the quality content of exports for rich enough countries only. We then corroborate our theoretical predictions on bilateral trade data for the enlarged European Union (EU), an integrated market displaying significant heterogeneity in terms of both average income and within-country inequalities of its members.

Keywords: Product quality, Income distribution, Trade, Economies of scale, European Union.

JEL classification: F12, L15, O15.

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1 Introduction

The existence and importance of vertical intra-industrial trade flows, also known as trade in quality, has been acknowledged in the past few years (Hummels and Klenow, 2005; Fontagné et al., 2008). The position of a country’s production and exports on the quality ladder is a topic of interest, since it has implications on many dimensions of country-level performance and social conditions. Hausmann et al. (2007) show that countries exporting more sophisticated products grow faster, while Verhoogen (2008) uses Mexican data to show that quality upgrading of production and export is associated with increased wage inequalities. It is thus important to understand the determinants of vertical specialization.

While most papers have focused so far on the supply-based determinants of the quality content of a country’s exports, we investigate the influence of different dimensions of income distribution on a country’s vertical comparative advantage. More precisely, average income might hide strong differences in terms of underlying income distribution, and we show that the way income is distributed around average income does matter for the quality content of a country’s exports.

Classical models of trade explain inter-industrial trade flows between countries differing in terms of technologies (Ricardo) or in terms of production factors relative endowments (Hecksher-Ohlin). Krugman (1979) introduces preferences exhibiting love for variety in order to explain the existence of intra-industrial trade between similar countries: while in classical models, inter-industrial patterns are explained by supply-side determinants, intra-industrial trade is hence generated by specific demand features. However, this analysis only applies for horizontal intra-industrial trade, while a growing body of evidence shows that two-way trade flows also encompass exchanges of different qualities of the same good. In other words, vertical intra-industrial trade also needs to be accounted for. The seminal model of intra-industrial trade in quality-differentiated products has been provided by Flam and Helpman (1987), who design a framework where non-homothetic preferences and differences in human capital endowment between the North and the South account for vertical differentiation and quality-based product life cycles. Consequently, assuming a comparative advantage of the North in high-quality products, Flam and Helpman (1987) conjecture a supply-side theoretical explanation for patterns of production, empirically confirmed by Schott (2004). In the same vein, Verhoogen (2008) emphasizes the role of firm heterogeneity within a country to explain the quality produced and exported. Finally, Fieler (2011a,b), introducing non-homothetic preferences in a Ricardian model, also relies on supply side arguments to explain why richer countries export higher quality goods towards richer partners.

Regarding demand-based determinants, income distribution has been widely accepted as an important determinant of the quality of imports. Indeed, Hallak (2006) shows that richer countries tend to import higher quality goods, while Choi et al. (2009) show that countries displaying similar income distributions tend to exhibit similar distribution of
import prices. On the other hand, the impact of income distribution on the quality of exports has been recently brought back into light by Fajgelbaum et al. (2009), in line with the Linder (1961) hypothesis claiming that firms produce and export goods suited to the specific tastes of their local consumers. They indeed provide a theoretical framework where patterns of aggregate demand translate into patterns of specialization and trade. In their nested-logit demand structure framework, the fraction of consumers buying high-quality goods increases along income level. Through a home-market effect, an increase in average income, keeping the size of the country and the level of inequalities unchanged, unambiguously shifts upwards the quality produced and exported. The role of inequalities is less clear-cut. They obtain predictions when the fraction of consumers buying the high-quality brands is lower than 50% at all income level, in which case an increase in inequality within population is unambiguously associated with an increase in the quality content of the country’s exports.

We contribute to this literature on two dimensions. From a theoretical point of view, we build a model with economies of scale and rich and poor consumers displaying non homothetic preferences. More precisely, consumers can simultaneously purchase horizontally-differentiated varieties of two qualities of the same good, entering as two CES-bundles nested in a Stone-Geary-type utility function. Within this framework, we neutralize the possible supply-side determinants of vertical comparative advantage by studying patterns of trade between two countries with similar technologies and homogeneous labor. As Fajgelbaum et al. (2009), we find that countries with higher average income export higher quality varieties of a given good. Our model however provides a different perspective on the role of income distribution beyond average income. We show that the effect of inequalities on the quality content of exports depends on the level of average income: a mean-preserving spread of income distribution increases quality for rich enough countries only. Inequalities hence have a heterogeneous impact, depending on the average income level.

The second contribution of our paper is empirical. We test the predictions of our model on bilateral trade flows within the enlarged European Union (EU25). Indeed, in 2004 the EU integrated 10 more countries from Eastern and Southern Europe. With this enlargement, EU25 is a unique example of a free trade area with strong variations across countries, both in terms of average income and inequalities. It is thus a perfect ground to investigate the influence of income distribution on the vertical comparative advantage of countries. To this end, we use the CEPII dataset on bilateral export and import flows at the 6-digit product level over the 1995-2007 period and merge it with income distribution data from Eurostat. Consistently with our model, we show that having controlled for average income, no clear impact of inequalities (as measured by interquintile ratio or Gini index) is detected on unit values, while the interaction between average income and inequalities has a positive and significant effect. The positive impact of this interaction term and of average income

\footnote{The list of these countries is: Cyprus, Czech Republic, Estonia, Hungary, Latvia, Lituania, Malta, Poland, Slovakia, Slovenia. Romania and Bulgaria follow in 2007.}
on the quality of exports of a country is robust to the inclusion of controls for supply-side determinants of prices and vertical comparative advantage (wages, education). Beyond the validation of our theoretical predictions, we hence provide an empirical confirmation of the role of demand-based determinants of a country’s exports quality.

The rest of our paper proceeds as follows. Section 2 develops our model in a closed economy, while section 3 introduces trade. We present our data in section 4 and our empirical results in section 5. Section 6 concludes.

2 The model in closed economy

We first consider an economy at autarky featuring one constant return to scale industry A producing an homogeneous good and two increasing returns to scale industries L and H producing low-quality and high-quality varieties of the same good.

2.1 Preferences and quality differentiation

We consider a fixed number of consumers \(N\), which are assumed to differ with respect to their endowment in effective labor supply. We model a two-class society, with consumers belonging either to the poor (P) or the rich (R) class. The share of poor consumers within the population is denoted by \(\beta\). The extent of inequalities within the economy is determined by this share, as well as by the repartition between rich and poor of the aggregate amount \(L\) of effective labor supply available within the economy. \(d \in (0,1)\) is defined as the ratio of a poor consumer’s labor supply \(l_P\) relative to the average per-capita labor supply \(L/N\): 

\[ d = \frac{l_P}{L/N}. \]

As \(d\) gets closer to 1, the level of inequalities within the economy diminishes. Given \(d\), it is possible to compute the labor supply of respectively a poor and a rich consumer as 

\[ l_P = d \frac{L}{N} \quad \text{and} \quad l_R = \frac{1-\beta d}{1-\beta} \frac{L}{N}. \]

Within such a framework, a mean-preserving increase in the level of inequalities corresponds to a decrease in \(d\), while an increase in the average level of income leaving the level of inequalities unchanged corresponds to an increase in \(L\) or a decrease in \(N\).

The utility of a consumer belonging to group \(i\) (\(i = R, P\)) is assumed to be of the form:

\[ U_i = \left(\frac{C_i^1 - \mu_i}{(\gamma + C_i H)}\right)^\theta A^{1-\theta}. \]

We hence have a Cobb-Douglas utility specification, where the consumer allocates consumption between an homogenous good and varieties of a differentiated good. The homogenous good is priced competitively and produced with unit efficient labor requirement, therefore serving as numeraire. Preferences for the differentiated good are described by an embedded Stone-Geary specification, where \(C_{ij}\) \((j = H, L)\) is an index of consumption of the varieties of quality \(j\). With \(n_j\) being the number of varieties of quality \(j\) produced, we
define $C_{ij}$ as

$$C_{ij} = \left[ \int_0^{n_j} c_{ij}(k)^{\frac{\sigma_j}{1-\sigma_j}} dk \right]^\frac{1}{\sigma_j}, \sigma_j \in (1, +\infty)$$  \hspace{1cm} (2)$$

with $c_{ij}(k)$ being the consumption of a variety $k$ of quality $j$, and $\sigma_j$ being the elasticity of substitution between any two varieties of quality $j$.

The budget constraint of a consumer belonging to group $i$ is of the form:

$$A + P_L C_{iL} + P_H C_{iH} = l_i$$  \hspace{1cm} (3)$$

$P_j$ is the price index of the differentiated good, given by

$$P_j = \left[ \int_0^{n_j} p_j(k)^{(1-\sigma_j)} dk \right]^{\frac{1}{1-\sigma_j}}$$  \hspace{1cm} (4)$$

with $p_j(k)$ being the price of variety $k$ of quality $j$. The demand of a consumer belonging to group $i$ for a variety $k$ of quality $j$ can be expressed as:

$$c_{ij}(k) = \left( \frac{p_j(k)}{P_j} \right)^{-\sigma} C_{ij}$$  \hspace{1cm} (5)$$

The Cobb-Douglas form of our utility specification guarantees that the income $l_i$ of a type $i$ consumer will be spread between consumption of the homogenous good and consumption of the differentiated good along constant shares $1 - \theta$ and $\theta$. We are however left with determining the optimal allocation between the two available quality bundles of the total expenditure $\theta l_i$ dedicated to the differentiated good. Setting $\gamma = 0$, our Stone-Geary specification would take a familiar Cobb-Douglas form, with constant expenditure shares $\mu$ and $1 - \mu$. We however assume $\gamma > 0$, which implies that goods of quality H are not consumed (nor produced) when the income level $l_i$ is below a certain threshold. More precisely, the demand functions of a consumer belonging to group $i$ are given by:

$$C_{iL} = \min\{\frac{(1-\mu)(\theta l_i + \gamma P_H)}{P_L}, \theta l_i\}, \quad C_{iH} = \max\{0, \frac{\mu \theta l_i - (1-\mu)\gamma P_H}{P_H}\}$$  \hspace{1cm} (6)$$

with $C_{iH} > 0$ provided the income $l_i$ is above a threshold $l^* = \frac{(1-\mu)\gamma P_H}{\mu \theta l_i}$. Defining the expenditure shares of a consumer belonging to group $i$ for each bundle as $m_{ij} = \frac{C_{ij} P_j}{\theta l_i}$, we have:

$$m_{iL} = \min\{(1-\mu)(1 + \frac{\gamma P_H}{\theta l_i}), 1\}, \quad m_{iH} = \max\{0, \mu - \frac{(1-\mu)\gamma P_H}{\theta l_i}\}$$  \hspace{1cm} (7)$$

Hence, the share of expenditures accruing to L goods (resp. H goods) is decreasing
(resp. increasing) along income. This property of the Stone-Geary utility function is usually used in models of structural change, where growing income induces a demand shift from first necessity to luxury goods (Murata, 2008). We however claim that this type of non-homothetic preferences can also capture heterogeneous consumption patterns of high and low quality varieties of the same good by consumers differing in income.

Our modeling strategy implies the possibility of simultaneous consumption of low and high quality varieties by certain consumers. This differs from the discrete consumption choice framework usually adopted in the microeconomic literature on quality differentiation. In this literature, heterogenous consumers in terms of income only buy the quality that, given its price, offers them the highest utility. This is obtained through unit consumption of a good available at different quality levels and strategic pricing of firms in a situation of natural oligopoly (Shaked and Sutton, 1982). However, we will rely in our empirical part on semi-aggregated data that do not enable us to account for the specificity of this market structure. Fajgelbaum et al. (2009) model discrete quality choices within a monopolistic competition framework. However, these discrete choice models do not account for the “joint-purchase” feature (i.e. consumers with a higher income buying varieties of both low and high quality) that Gabszewicz and Wauthy (2003) emphasize as relevant for quality-differentiated markets. Indeed, they argue that “the mere improvement of living standards through the population allows many households to be equipped with several quality-differentiated variants of the same indivisible product. It is indeed far from seldom to observe households equipped with two or three different cars, or several TV-sets or P.C.’s”. Finally, in addition to its empirical relevance, this utility function will prove quite tractable to provide new testable predictions concerning the effects of different dimensions of income distribution, and is thus worth of investigation.

2.2 Firms

Firms compete monopolistically. In the quality segment $j$, producing a quantity $x_j(k)$ of variety $k$ requires $f_j + a_jx_j(k)$ units of labor, with $f_j$ and $a_j$ being respectively the fixed and marginal labor requirements in sector $j$. We assume free entry in each sector. A firm $k$ producing a variety with quality $j$ chooses its price in order to maximize its profit:

$$\pi_j(k) = (p_j(k) - a_j)d_j(k) - f_j$$

with $d_j(k)$ being the total demand (in real terms) for a variety $k$ of quality $j$. Defining as $D_j$ the total demand (in real terms) for all varieties of good $j$, we have:

2The nested logit demand function used by Fajgelbaum et al. (2009) furthermore yields at the macro level predictions that are closely similar to those obtained with a Stone-Geary utility function. Indeed, in their model the individuals have idiosyncratic evaluation of the attributes of every variety, regardless of their quality. The demand for low-quality goods is then positive for every income class while the market share of those goods decreases along income, two predictions that are reminiscent of the ones obtained with our specification.
\[ d_j(k) = \left( \frac{p_j(k)}{P_j} \right)^{-\sigma} D_j \]  

(9)

The producers of different varieties of quality \( j \) set prices to maximize profits taking the aggregate price indices \( P_j \) as given, as is common in models of monopolistic competition. The price maximizing the profits of a firm producing any variety \( k \) with quality \( j \) is then:

\[ p_j = \frac{\sigma_j}{\sigma_j - 1} a_j \]  

(10)

From equations (10) and (4), it is possible to obtain:

\[ P_j = n_j^{\frac{1}{\sigma_j}} \frac{\sigma_j}{\sigma_j - 1} a_j \]  

(11)

Entry at each quality level proceeds until the next entrant fails to cover its fixed costs. Using expressions (8) and (10), the zero-profit condition yields the following equilibrium output for a firm \( k \) operating in sector \( j \):

\[ d_j = \frac{f_j(\sigma_j - 1)}{a_j} \]  

(12)

2.3 Equilibrium analysis

Substituting (10) and (11) into (9), we obtain:

\[ d_j = n_j^{\frac{\sigma_j}{1-\sigma_j}} D_j \]  

(13)

Using (13), it is then possible to determine that in our closed economy, the number of firms within each sector \( j \) is a couple \( (n_L, n_H) \) characterized by the two following conditions:

\[ \pi_L = a_L \left( \frac{\sigma_L}{\sigma_L - 1} - 1 \right) n_L^{\frac{\sigma_L}{\sigma_L - 1}} D_L - f_L \leq 0, \quad n_L \pi_L(n_H, n_L) = 0 \]  

(14)

\[ \pi_H = a_H \left( \frac{\sigma_H}{\sigma_H - 1} - 1 \right) n_H^{\frac{\sigma_H}{\sigma_H - 1}} D_H - f_H \leq 0, \quad n_H \pi_H(n_H, n_L) = 0 \]  

(15)

In a classic model of horizontally-differentiated goods and monopolistic competition, the demand for all varieties of a given good is then simply deduced from the constant expenditure shares accruing to each bundle. In our model however, the precise specification of the demand \( D_j \) for a given quality depends on the wealth distribution within our economy. Indeed, as stated further up, individual consumption \( C_{iH} \) of the high-quality bundle within group \( i \) is strictly positive if and only if the income \( l_i \) of consumers belonging to this group is above the threshold \( \bar{l} = \beta P_H \).

Given our discrete distribution of consumers among two distinct income groups, we then need to distinguish for 3 different parametric cases:
1. Case C1: \( l_P \leq l^* \) and \( l_R \leq l^* \). Neither rich nor poor consumers can afford the high-quality good, i.e. \( C_{PH} \) and \( C_{RH} \) are both null. We then have:

\[
D_L = \frac{\theta L}{P_L} \\
D_H = 0
\]

We immediately obtain that the equilibrium number of producers for each quality is \( n_L = \frac{\theta L}{f_L \sigma_L} \) and \( n_H = 0 \).

2. Case C2: \( l_P \leq l^* \) and \( l_R > l^* \). Only rich consumers display a positive demand for the high-quality good, i.e. \( C_{PH} = 0 \). Using (6), we then obtain the following expressions for \( D_L \) and \( D_H \):

\[
D_L = \frac{\beta d \theta L}{P_L} + \left( \frac{(1 - \mu)((1 - \beta d)\theta L + \gamma(1 - \beta)NP_H)}{P_L} \right) \\
D_H = \frac{\mu(1 - \beta d)\theta L - (1 - \beta)N(1 - \mu)\gamma P_H}{P_H}
\]

Substituting for those expressions into the system (14)-(15) and using (11), we then get the two following equilibrium equations:

\[
f_H \sigma_H n_H = \mu(1 - \beta d)\theta L - (1 - \beta)N(1 - \mu)\gamma n_H \left( \frac{\sigma}{\sigma - 1} \right) a_H \tag{16} \\
f_L \sigma_L n_L + f_H \sigma_H n_H = \theta L \tag{17}
\]

3. Case C3: \( l_P > l^* \) and \( l_R > l^* \). Both types of consumers consume both low- and high-quality goods. Using (6), we then obtain the following expressions for \( D_L \) and \( D_H \):

\[
D_L = \frac{(1 - \mu)(\theta L + N \gamma P_H)}{P_L} \\
D_H = \frac{\mu \theta L - N(1 - \mu)\gamma P_H}{P_H}
\]

Substituting for those expressions into the system (14)-(15) and using (11), we get the two following equilibrium equations:

\[
f_H \sigma_H n_H = \mu \theta L - N(1 - \mu)\gamma n_H \left( \frac{\sigma_H}{\sigma_H - 1} \right) a_H \tag{18} \\
f_L \sigma_L n_L + f_H \sigma_H n_H = \theta L \tag{19}
\]

Concerning Cases C2 and C3, we notice that \( n_L \) only appears in equations (17) and (19), which correspond to the labor market equilibrium condition in both possible parametric cases. The number of producers active in the low-quality sector \( n_L \) will hence be determined residually, once the number of active producers of high-quality varieties \( n_H \) has been obtained from the zero-profit condition in the high-quality sector, i.e. (16) in Case C2 or (18) in Case C3. In order to determine the firm distribution across sectors, we hence
focus on (16) and (18), where the left-hand side (LHS) describes the costs incurred by all the active firms within the sector (i.e. wages distributed, since labor is the only input), while the right-hand side (RHS) is equal to the overall benefits, i.e. total consumers expenditure for the high-quality bundle.

**Proposition 1 (Existence and uniqueness of the equilibrium):** For given income distribution parameters $\beta$ and $d$ and sufficiently high fixed costs in the high-quality sector $f_H$, there exists a unique positive solution to the system of two equations defined by (14)-(15), defining the number of active firms in the high- and low-quality segments of the markets, $n_H$ and $n_L$.

**Proof.** See Appendix A.

We will now focus on the influence of the income structure on firm distribution across the two qualities H and L. As already stated previously, a mean-preserving increase in the level of inequalities within our framework corresponds to a decrease in $d$, while an increase in the average level of income leaving the level of inequalities unchanged corresponds to an increase in $L$ or a decrease in $N$. From now on, we fix the number $N$ of consumers in the economy, and model variations in average income as variations of $L$. Relevant comparative statics concerning the influence of income distribution on equilibrium allocation of firms across sectors and hence on the quality-mix being produced are summarized as follows:

**Proposition 2 (Impact of the average income and the level of inequalities on the quality bundle):**

1. If the economy is in Case 2 at equilibrium, we have the following comparative statics for $n_H$: $\frac{\partial n_H}{\partial d} < 0$, $\frac{\partial n_H}{\partial L} > 0$.
2. If the economy is in Case 3 at equilibrium, we have the following comparative statics for $n_H$: $\frac{\partial n_H}{\partial L} > 0$

**Proof.** See Appendix A.

As we have already previously commented, the number of active producers of low-quality varieties $n_L$ is residually determined through the labor equilibrium equation once $n_H$ has been obtained through the zero-profit condition in the high-quality sector. Hence, an increase in $n_H$ is only possible through a decrease in $n_L$, and can hence be interpreted as a shift of the production mix towards high quality at the equilibrium. Indeed, we can define the “average quality” of the overall production within the considered economy at the equilibrium as $q^e = \frac{n_H d_H p_H + n_L d_L p_L}{n_H d_H + n_L d_L}$. Substituting for the values of $d_H$, $d_L$, $p_H$ and $p_L$, and
using (17)/(19) to substitute for \( n_L \), \( q^e \) translates into:

\[
q^e = \frac{\theta L}{f_L \sigma_L} + n_H f_H \left( \frac{(\sigma_H-1)}{a_H \sigma_H a_L} - \frac{\sigma_H(\sigma_L-1)}{f_L \sigma_L a_L} \right)
\]

(20)

For \( a_H \) sufficiently big and \( f_L \) sufficiently small,\(^3\) it is indeed easy to demonstrate that \( \frac{\partial q^e}{\partial n_H} > 0 \). Clearly, economies in Cases C2 and C3 hence exhibit higher \( q^e \) than economies in Case C1, since low quality varieties only are produced in the latter.

Part (1) of Proposition 2 hence states that in the case where only consumers belonging to the rich group \( R \) consume the high-quality bundle, both an increase in the level of inequalities (i.e. \( \Delta d < 0 \)) and an increase in the level of average income (i.e. \( \frac{\Delta L}{N} > 0 \)) lead to an increase in the average quality \( q^e \) produced by the country at the equilibrium. This result can be simply interpreted, since \( \Delta d < 0 \) and \( \frac{\Delta L}{N} > 0 \) both lead to an increase in the size of the market for high quality goods: indeed, the two shocks increase the share of the overall income accruing to the rich group \( R \), which is the only one to have a positive demand for high quality varieties in Case 2. Such a demand shift toward higher quality raises the relative profitability of high-quality varieties, leaving the possibility for a higher number of firms to enter the market.

Part (2) of Proposition 2 states that a mean-preserving variation in the level of inequalities does not have any impact on the distribution of firms once both groups have a positive demand for the high-quality bundle, while an increase in the average income of the economy still has a positive impact on the quality-mix produced at the equilibrium.

The relationship between the quality content of production and the different dimensions of income distribution within the economy hence depends on which case corresponds to the equilibrium. Clear predictions concerning the effects of inequalities and average income on the production quality mix of a country are thus only possible once the equilibrium subcase is identified. The occurrence of a given case is however itself strongly dependent on the income structure of the economy. The income threshold \( l^* \) depends on \( n_H \) and \( n_L \), for which it is impossible to obtain analytical forms. We hence carry out simulations to determine the equilibrium subcase for a given level of inequalities and average income.

We proceed to simulations for \( d \) varying from 0.2 to 0.6 and \( \frac{L}{N} \) varying from 2000 to 12000. In line with our research question and our empirical exercise, those values are calibrated in order to replicate the extent of variations in inequalities and average income within the EU25. We also proceed to a sensitivity analysis along different values of \( \sigma_H \), \( \mu \) and \( \gamma \). We notice the following regularities:

- **Numerical finding 1**: For high enough values of \( \gamma \), Case 3 disappears
- **Numerical finding 2**: Higher income increases the probability to be in Case C2 rather

\(^3\) These two conditions on the parameters simply amount to imposing for the price of a low-quality variety \( p_L \) to be lower than the price \( p_H \) of a high-quality variety.
than in Case C1

- **Numerical finding 3**: Higher inequality levels are a minor determinant of this probability, having both a much lower marginal impact and lower explanatory power.

As already stated higher, our specification of preferences tends to become a Cobb-Douglas as $\gamma$ approaches zero, i.e. preferences come close to being homothetic. Indeed, the lower $\gamma$ the more similar the consumption baskets of the rich and the poor consumers, as it can be seen from expression (7). Hence, our first numerical finding establishes that the occurrence of Case C3, i.e. consumption of the high quality bundle by both the rich and the poor consumers, implies a very low degree of distortion toward high quality goods of the rich consumers’ consumption basket when compared to consumption patterns of the poor consumers. As a consequence, we deem those parametric cases less relevant, and focus on cases C1 and C2.

We then use numerical findings 2 and 3 to analyze the role of average income and inequalities on the quality mix being produced at the equilibrium. A growing level of average income leads to an increased probability to be in C2 rather than C1, whatever the distribution of income around this average level (Numerical findings 2 and 3). On the other hand, inequalities do not play such a role: for low levels of average income, even high levels of inequalities will not be sufficient for Case C2 to emerge as an equilibrium (Numerical finding 3). Moreover, inequalities positively impact on the quality mix of production only in case C2. Put together, these results point to a heterogeneous impact of inequalities along the average income dimension: a mean-preserving spread of income distribution increases quality for rich enough countries only.

3 Equilibrium with trade

3.1 Preferences, technology and profits

Within the same framework, we now allow for the possibility of international trade between two countries, D and F, totally similar in terms of production technology. We hence only allow for demand-based differences, i.e. countries D and F may only differ in their distribution of the overall amount of efficient labor ($d_D \neq d_F$ and $L_D \neq L_F$).

We model transport costs of the “iceberg” type: in order to export to country $r$ one unit of quality $j$’s output manufactured in country $s$, a firm must ship $\tau_j \geq 1$ units. We assume that the homogenous good is freely traded, i.e. that $\tau_A = 1$, which will ensure that the wage of a unit of effective labor is the same for the two countries.\footnote{We assume that labor supply in both countries is sufficient with respect to the equilibrium labor demand of the producers of differentiated varieties, so that at equilibrium some labor is devoted to the production of the homogeneous good in both countries.} On the other hand, we impose strictly positive and similar transport costs for low and high quality varieties of the differentiated good, i.e. $\tau_H = \tau_L = \tau > 1$. Firms fully pass on their shipping costs to
their foreign customers. Hence, one unit of variety $k$ of quality $j$ manufactured in country $s$ is sold to consumers of country $r$ at price $p_{sj}^r(k) = \tau p_{sj}$ where $p_{sj}$ is the mill price.

The price index in country $r$ for quality $j$ is hence of the form:

$$P_rj = \left[ \int_0^{n_j^r} p_{rj}(k)(1-\sigma)dk + \tau^{1-\sigma} \int_0^{n_j^s} p_{sj}(k)(1-\sigma)dk \right]^{\frac{1}{1-\sigma}}$$

(21)

where $n_j^r$ and $n_j^s$ are the number of firms producing varieties of quality $j$ in country $r$ and country $s$ respectively.

Consumers have the same preferences in each country, and have access to domestically- and foreign-produced goods. Defining as $D_{rj}$ the total demand in country $r$ for all varieties of quality $j$ (both domestically- and foreign produced), total demand for a variety $k$ of quality $j$ produced in country $r$ is of the form:

$$d_{rj}^r(k) = p_{rj}^{-\sigma} (P_{rj}^\sigma D_{rj} + \tau^{1-\sigma} P_{sj}^\sigma D_{sj})$$

(22)

As one can see from equation (22), a producer located in country $r$ now sells to both domestic and foreign consumers. However, such a producer is a less effective competitors on the foreign market $s$, because of transport costs being fully passed on the price charged to the foreign consumers. Hence, within each country, demand for a foreign variety is discounted by $\tau^{1-\sigma} < 1$.

The profit function of a firm $k$ producing quality $j$ in country $r$ is:

$$\pi_{rj}^r(k) = (p_{rj}(k) - a_j)d_{rj}^r(k) - f_j$$

(23)

Profit maximisation yields the following optimal price:

$$p_{rj} = \frac{\sigma}{\sigma - 1} a_j$$

(24)

From equations (24) and (21), the price index in country $r$ for quality $j$ can then be re-expressed as:

$$P_{rj} = (n_j^r + \tau^{1-\sigma} n_j^s)^{\frac{1}{\sigma - 1}} \frac{1}{\sigma - 1} a_j$$

(25)

This expression can be compared to equation (11), defining the price index $P_j$ for quality $j$ in a closed economy as a function of the number of active competitors $n_j$. Here, the price index $P_{rj}$ is similarly function of $n_j^r$ and $n_j^s$, i.e. the number of both local and foreign producers of quality $j$ varieties. The number of foreign competitors $n_j^s$ is however discounted by a factor $\tau^{1-\sigma}$, expressing the fact that those foreign producers are less competitive on the local market because of the existence of transport costs fully impacting the price charged to the consumers. Similarly to Fajgelbaum et al. (2009), we then define the number of “effective competitors” of quality $j$ present on the domestic market $r$ as
\[ \tilde{n}_j^r = n_j^r + \tau^{1-\sigma} n_j^s. \]

Substituting (24) and (25) into (22), we then get the following expression for \( d_j^r \):

\[ d_j^r = (\tilde{n}_j^r)^{1-\sigma} D_{rj} + \tau^{1-\sigma} (\tilde{n}_j^s)^{1-\sigma} D_{sj} \tag{26} \]

### 3.2 Equilibrium distribution of firms

The equilibrium distribution of firms across countries and sectors \((n_D^D, n_H^D, n_L^F, n_F^F)\) is determined by the following zero-profit conditions:

\[
\pi_j^D \leq 0, \quad n_j^D \pi_j^D = 0, \quad j = H, L \tag{27}
\]

\[
\pi_j^F \leq 0, \quad n_j^F \pi_j^F = 0, \quad j = H, L \tag{28}
\]

Once again, because of the discrete allocation of consumers among two different income groups in each one of the countries, different cases need to be considered for the overall demands \(D_{rj} (j = H, L\) and \(r = D, F)\) that enter equations (27)-(28). Since there are now two trade partners, 9 different parametric cases are possible, as opposed to only 3 in the closed economy set-up. Considering the symmetry of some of those cases, we can bring them down to 6 possible configuration types:

1. **Case O1.** \(l_{PD} < l_{D}^r, l_{RD} < l_{D}^r, l_{PF} < l_{F}^s, l_{RF} < l_{F}^s\): the demand for the high-quality bundle is null in both countries. Only low quality varieties are produced and traded.

2. **Case O2.** \(l_{Pr} < l_{r}^r, l_{Rr} > l_{D}^r, l_{Ps} < l_{F}^s, l_{Rs} < l_{F}^s\): the high-quality bundle is consumed by the rich consumers only in country \(r (r = D, F)\) and by none in country \(s\). Only low quality varieties are traded.

3. **Case O3.** \(l_{Pr} > l_{r}^r, l_{Rr} > l_{D}^r, l_{Ps} < l_{F}^s, l_{Rs} < l_{F}^s\): the high-quality bundle is consumed by both the poor and the rich consumers in country \(r (r = D, F)\) and by none in country \(s\). Only low quality varieties are traded.

4. **Case O4.** \(l_{PD} < l_{D}^r, l_{RD} > l_{D}^r, l_{PF} < l_{F}^s, l_{RF} > l_{F}^s\): the high-quality bundle is consumed only by the rich consumers in both countries. Both high and low quality varieties are traded.

5. **Case O5.** \(l_{Pr} < l_{r}^r, l_{Rr} > l_{D}^r, l_{Ps} > l_{F}^s, l_{Rs} > l_{F}^s\): the high-quality bundle is consumed by the rich consumers only in country \(r (r = D, F)\) and by both rich and poor consumers in country \(s\). Both high and low quality varieties are traded.

6. **Case O6.** \(l_{PD} > l_{D}^r, l_{RD} > l_{D}^r, l_{PF} > l_{F}^s, l_{RF} > l_{F}^s\): the high-quality bundle is consumed by both the rich and poor consumers in both countries. Both high and low quality varieties are traded.

\(^5\)A country does not produce a quality that is not domestically consumed. Indeed, as it will be shown later on, the domestic demand faced by a producer of a quality \(j\) variety has to be the same in both countries at equilibrium if this quality is traded.
For the following analysis, we restrict ourselves to the parametric cases where both countries produce at least one of the two qualities, i.e. to the cases where trade occurs with multiple exporters of the same good. As commented later on and already demonstrated by Fajgelbaum et al. (2009) in a comparable set-up, this is not necessarily the case, but occurs for high enough trade costs \( \tau \) and low enough asymmetries in terms of absolute size \( N_r \) between the two countries. Indeed, when countries are highly asymmetric, for low enough trade costs, both qualities are entirely produced and exported by the big country. We exclude this parametric case because of the empirical strategy implemented when testing the theoretical predictions of our model. Indeed, we then compare across EU25 countries the weighted average unit value of their exports to other EU25 members for a given product. In our two-country model, this amounts to comparing the quality mix of the exports of the two trading partners for a given good, which is not possible if one of the trading partners captures the total production of both qualities \( (n_H^r = n_L^r = 0) \).

The equations defining the equilibrium distribution of firms for all the possible parametric cases are detailed in Appendix B. In order to get the main intuitions, we will here only present as an example the equilibrium equations for case O4, which is one of the 3 cases where high quality varieties are traded.

Using once again (6), we obtain the following expressions for \( D_{Lr} \) and \( D_{Hr} \) in case O4:

\[
D_{Lr} = \frac{\beta d_r \theta L_r}{P_{Lr}} + \left(\frac{(1 - \mu)((1 - \beta d_r)\theta L_r + \gamma(1 - \beta)N P_{Hr})}{P_{Lr}}\right)
\]

\[
D_{Hr} = \frac{\mu(1 - \beta d_r)\theta L_r - (1 - \beta)N(1 - \mu)\gamma P_{Hr}}{P_{Hr}}
\]

Substituting for \( D_{Hr} \) into the system (27)-(28) for \( j = H \) and using (26), we get the following zero-profit condition for producers of high-quality varieties in country \( r \) \( (r = D, F) \):\(^{6}\)

\[
f_H\sigma = (\tilde{n}_H^r)^{-1}(\mu(1 - \beta d_r)\theta L_r - (1 - \beta)N(1 - \mu)\gamma(\frac{\sigma}{\sigma - 1})a_H(\tilde{n}_H^r)^{-\frac{1}{\sigma - 1}})
\]

\[
= A_H
\]

\[
+ \tau^{1-\sigma}(\tilde{n}_H^r)^{-1}(\mu(1 - \beta d_s)\theta L_s - (1 - \beta)N(1 - \mu)\gamma(\frac{\sigma}{\sigma - 1})a_H(\tilde{n}_H^r)^{-\frac{1}{\sigma - 1}})
\]

\[
= B_H
\]

For this equation to hold for both \( r = D, F \) we necessarily have that \( A_H = B_H \), i.e. the domestic demand faced by a producer of a high quality variety is the same in both

\(^{6}\)Once again, \( n_D^L \) and \( n_F^L \) do not appear in those two equations, and will be determined residually.
countries.\textsuperscript{7} The condition (29) then boils down to the following expression (\(r = D, F\)):

\[
f_H \sigma \tilde{n}_H^r = \mu (1 - \beta d_r) \theta L_r - (1 - \beta)N (1 - \mu) \gamma (\tilde{n}_H^r) \left( \frac{\sigma}{\sigma - 1} \right) a_H
\]

(30)

Substituting for the expression of \(D_{Lr}\) into the system (27)-(28) for \(j = L\) and using (25) as well as (30), the zero-profit condition for producers of low-quality varieties in country \(r\) (\(r = D, H\)) is of the form:

\[
f_L \sigma = (\tilde{n}_L^r)^{-1} (\theta L_r - f_H \sigma \tilde{n}_H^r) + \tau^{1 - \sigma} (\tilde{n}_L^r)^{-1} (\theta L_s - f_L \sigma \tilde{n}_L^s)
\]

(31)

Again, for this equation to hold for both \(r = D, F\) it must be that \(A_L = B_L\), i.e. producers of low-quality varieties in each country must achieve the same volume of domestic sales. Equation (31) then boils down to the following expression (\(r = D, F\)):

\[
f_H \sigma \tilde{n}_H^r + f_L \sigma \tilde{n}_L^r = (1 + \tau^{1 - \sigma}) L_r
\]

(32)

Conditions (30) and (32) define two independent systems of two equations: two relationships jointly determining \(\tilde{n}_H^D\) and \(\tilde{n}_L^D\), i.e. the number of effective competitors within each quality segment in country \(D\), and two relationships similarly determining \(\tilde{n}_H^F\) and \(\tilde{n}_L^F\). Each one of those systems is exactly similar to the equilibrium conditions (18)-(19) that defined the distribution of firms across qualities in a closed economy, except that the number of producers of a given quality \(n_j\) has been replaced by \(\tilde{n}_j^r\), i.e. the number of effective competitors within country \(r\) on the quality segment \(j\). Using Proposition 1, we then argue that for sufficiently high fixed costs in the high-quality sector \(f_H\), there exists a unique positive solution \((\tilde{n}_L^D, \tilde{n}_H^D, \tilde{n}_L^F, \tilde{n}_H^F)\) to the system of four equations defined by (30)-(32).

Two kinds of equilibria are then possible: both countries produce and export the two qualities, but in different relative quantities (partial specialization equilibrium), or each country specializes in the production of one quality (full specialization equilibrium).

We have the following expression for \(n_j^r\):

\[
n_j^r = \frac{\tilde{n}_j^r - \tau^{1 - \sigma} \tilde{n}_s^r}{1 - \tau^{2(1 - \sigma)}} \quad r \neq s, \; j = H, L, \; r = D, F
\]

(33)

which entails the following condition for \(n_j^r\) to be positive, i.e. to have partial specialization of both countries:

\[
\tau^{1 - \sigma} < \frac{\tilde{n}_j^F}{\tilde{n}_j^s} < \frac{1}{\tau^{1 - \sigma}}
\]

(34)

Condition (34) is scarcely respected for low levels of transport costs, i.e. \(\tau\) very close to 1, but always met for high enough values of \(\tau\).\textsuperscript{8} Full specialization of countries can

\textsuperscript{7}This property of our model directly stems from the fact that we have imposed for the two countries to be strictly similar in terms of production technology. Entry at each quality level and within each country proceeds until the next entrant fails to cover its fixed costs, which leads to a similar equilibrium output across countries, \(d_j^r = d_s^r = \frac{L_j (\sigma_j - 1)}{a_j}\).\textsuperscript{8}For low values of \(\tau\), condition (34) is respected when countries \(D\) and \(F\) are relatively similar in terms of average income \(\frac{L_d}{N_d}\) and absolute size \(N_r\).
be seen as an extreme case of the partial specialization: each quality will be entirely produced in the country where domestic demand for this quality is the highest. This case is not qualitatively different from the partial specialization case, and we thus focus our predictions on this latter case.

For high enough transport costs $\tau$, we obtain the following proposition:

**Proposition 3 (Existence and uniqueness of the equilibrium with trade):** For given income distribution parameters $\beta$, $L_r$ and $d_r$ ($r = D, F$), sufficiently high fixed costs in the high-quality sector $f_H$ and sufficiently high transport costs $\tau$, there exists a unique positive solution to the system of four equations defined by (27)-(28), defining the distribution of firms across country and sectors ($n^D_L, n^D_H, n^F_L, n^F_H$).

Given our research question (the impact of different dimensions of income distribution on the quality mix being produced and exported by a country), we now focus on the cases O4 and O6, in which both qualities are produced by both countries and traded at the equilibrium.\(^9\)

As in closed economy, the number of effective competitors $\tilde{n}^r_j$ in the low-quality market segment of a given country $r$ is residually determined through (32) once the number of effective competitors in the high-quality market segment $\tilde{n}^r_H$ has been obtained from (30). Hence, an increase in $\tilde{n}^r_H$ is only possible through a decrease in $\tilde{n}^r_L$. Furthermore, we have that:

$$\frac{\partial n^r_j}{\partial \tilde{n}^r_j} > 0, \quad \frac{\partial n^r_j}{\partial \tilde{n}^r_s} < 0, \quad r \neq s, \quad j = H, L, \quad r = D, F$$ (35)

Those comparative statics imply that, provided that we are in an equilibrium with partial trade specialization (i.e. for high enough values of $\tau$), an increase in the number $\tilde{n}^r_j$ of "effective" producers of a given quality $j$ in country $r$ increases the number $n^r_j$ of domestic producers of this quality. We can hence directly interpret an increase in $\tilde{n}^r_H$ as an increase in $n^r_H$, and a decrease in $\tilde{n}^r_L$ as a decrease in $n^r_L$. In other words, an increase in $\tilde{n}^r_H$ leads to a shift of the production- and export-mix of country $r$ towards high quality at the equilibrium. Using the results presented in Proposition 2, we obtain the following comparative statics in an equilibrium with partial trade specialization:

**Proposition 4 (Impact of the average income and the level of inequalities on the quality bundle of exports in an equilibrium with partial trade specialization):**

1. If the economy is in Case O4 at equilibrium, we have the following comparative statics for $n^r_H$ ($r = D, F$): $\frac{\partial n^r_H}{\partial d^r} < 0$, $\frac{\partial n^r_H}{\partial L^r} > 0$.
2. If the economy is in Case O6 at equilibrium, we have the following comparative statics for $n^r_H$ ($r = D, F$): $\frac{\partial n^r_H}{\partial L^r} > 0$.

---

\(^9\)Case O5 also features production and exports of the two qualities by both countries, but is redundant with respect to the results obtained for cases O4 and O6.
The results featured in Proposition 4 in the case of an equilibrium with partial trade specialization are strongly reminiscent of the results presented in Proposition 2 in the case of a closed economy. Keeping characteristics of country F constant, an increase in average income of country D will distort the quality mix of production and exports from D to F towards high quality, whether we are in case O4 or case O6. On the other hand, a mean-preserving variation in the level of inequalities in country D only has an impact on the quality mix of exports from D to F in case O4.

The intuitions underlying those results are totally similar to the ones in the closed economy case, except that production now involves export. Indeed, in Case O4, both an increase in \( d_D \) and in \( L_D \) lead to an increase in the size of the market for high quality goods within country D, which will itself lead to an increase in the number of “effective competitors” within the high-quality segment \( \tilde{n}_{DH} \), itself implying an increase in the number of domestic producers (and a decrease in the number of foreign producers) of high quality varieties \( n_{DH} \). The quality mix of exports is hence shifted towards a higher average quality level. In Case O6, only an increase in \( L_D \) results in such an expansion of the high-quality good market, since both rich and poor consumers have a strictly positive demand for high quality goods.

The similarity of the results obtained in Proposition 2 (closed economy) and Proposition 4 (trade with partial specialization) enables us to predict a heterogeneous impact of inequalities on the quality mix of exports along the average income dimension: a mean-preserving spread of income distribution increases the average quality being exported for rich enough countries only. We now move to testing those predictions.

4 Data and database construction

We test in the empirical part of this paper the predictions of our model about the heterogeneous impact of inequalities on the quality of exports within the enlarged EU. We first discuss our measure of quality and the variables, as well as the data we use.

4.1 How to measure quality?

In this paper, we use unit values as a proxy for quality of exports. Within a given product category (defined at the 6-digit level of the Harmonized Commodity System), more expensive varieties are assumed to be of higher quality (a product/exporting country couple \( p_x \) being identified as a variety of product \( p \)).

This measure of quality has some limitations, emphasized in recent papers developing alternative quality measures. In particular, Hallak and Schott (2008) and Khandelwal (2010) point at the fact that differences in unit values might capture other elements than quality. For example, exogenous differences in factor prices or exchanges rates misalignments might impact on unit values of exports, without being directly linked to the quality of exported products.
To circumvent this drawback of unit values, Hallak and Schott (2008) infer quality of US imports from prices and trade balances of source countries. Their intuition is that for a given export price, a country with a higher trade balance vis-à-vis the world produces and exports a higher quality. In other words, if for a given price, a country sells more to the rest of the world than its competitors, this means that it produces a better quality. However, to derive their index, they assume CES-preferences over vertically differentiated varieties, nested into a Cobb-Douglas utility function. This specification yields homothetic preferences, and does not allow to capture income distribution effects on the quality mix of production and exports. Khandelwal (2010) implements a different methodology, based on a similar intuition. He estimates a nested logit demand system for US imports that embeds preferences for both horizontal and vertical attributes. For a given product, controlling for export price, countries that sell more to US consumers are considered to produce a higher quality. However, quality is a complex notion, and no consensus exists for the moment on what its exact definition should be. For example, di Comite et al. (2011) show that in a model with quadratic preferences, consumers’ willingness to pay for a given variety increases with quality, as well as the quantity they want to purchase. In equilibrium, higher quality varieties exhibit higher prices and higher quantities. On the other hand, keeping quality constant, a higher “horizontal” taste for a given variety will translate into a higher quantity purchased, but not into a higher price.

If we give credit to this interpretation, this means that the measure of quality proposed by Khandelwal (2010) partly captures differences in horizontal tastes of consumers. This could be less of an issue when one focuses on the comparison of import prices on a single market, as Khandelwal (2010) does. Indeed, one can argue that quality should not necessarily be conceived as something related to the technological content of a given variety. This however becomes more problematic if one is interested, as we are, in exporter-level determinants of the quality of exported varieties on different markets. The ordering of exporting countries on each market could change, but not the exporter-level variables used to explain quality.

In the end, we prefer to rely on unit values as a proxy for quality. We are however very concerned by the fact that different prices might be explained by differences in labour costs or other supply-determinants. Since we work on countries of the enlarged EU, we can control for these other determinants thanks to harmonized data from Eurostat on labour costs and education. Moreover, Khandelwal (2010) shows that the quality index he

\[\text{If Germany produces red cars and France produces blue cars, and if Italian consumers massively prefer red cars, Italian consumers will buy higher quantities from Germany than from France at equal price. In that case, German cars can be conceived as being of higher quality than French cars for the Italian market. This is acceptable in a conception of quality not only based on technological characteristics. However, assume that British consumers prefer blue cars to red cars. At equal price, France will sell higher quantities than Germany on the British market. It will thus be said to produce a higher quality than Germany, yielding different rankings of France and Germany on different export markers, even though each country exports in reality the same cars on both markets. It is consequently difficult to exploit repeated cross-sections including both the Italian and the British markets to explain differences in quality between French and German exports. Indeed, the ordering of exporting countries on each market will change, but not the exporter-level variables used to explain quality.}^{10}\]
develops is more correlated to unit values in industries with a longer quality ladder (i.e. a larger difference between the maximum and the minimum values of the index within the considered industry). In line with this result, we show that our conclusions are strengthened when we focus on products for which the coefficient of variation of export unit values is higher.

4.2 Dependent and explanatory variables

We use the BACI database to construct unit values that we use as proxies for quality. BACI has been developed by CEPII based on COMTRADE data. It records all bilateral trade flows at the HS6-product level, in value (dollars) and in volume (tons). We use data from 2005 to 2007, corresponding to the post-Enlargement period.

We conserve in the sample bilateral export flows involving EU25 countries only as exporters. As commonly done in the literature (see for example Choi et al. (2009)), we clean the data and consider trade flows for which the quantity shipped is at least equal to one kilogram and for which the unit value is higher than 10% of the median and lower than 10 times the median unit-value observed for that commodity among EU25 exporters. This amounts to dropping 5-6% of non missing observations per year. We restrict our analysis to manufacturing industries.

We then calculate for each country the following weighted average unit value of its exports to other EU25 members:

\[ \bar{uv}_{xpt} = \sum_{m \in M} s_{x\text{mpt}}uv_{x\text{mpt}} \]  

where M is the set of EU25 importers, \( s_{x\text{mpt}} = \frac{q_{x\text{mpt}}}{\sum_{i \in M} q_{x\text{ipt}}} \) is the share of country m in the total volume of exports within EU25 of product p by country x at time t, and \( uv_{x\text{mpt}} \) is the unit value of exports of product p by country x to country m at time t. We obtain in the end a database with one observation per exporting country, HS6 product and year.

For each year, we also need information on exporters characteristics. We use the Eurostat database, which provides data for all EU25 members from 2005 to 2007. Average income, in PPP and in current value, and total population are directly available. We also have information on the shares of the first and the last quintiles in total income. Thanks to these information, we can compute the ratio of average income in the first and the last quintiles (hereafter interquintile ratio), in PPP and in current value.

As shown by Schott (2004) on US imports data for the period 1972-1994, high income

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11 See Gaulier and Zignago (2010).
12 They correspond in our dataset to industries HS28 to HS97.
13 Note that each HS6 product category is divided into several HS8 or HS10 product lines. We do not have bilateral export data at the HS8 or HS10 level, since classifications are not harmonized across countries at such a detailed level. Consequently, higher unit values at the HS6 level must be interpreted as higher quality varieties for each HS8 or HS10 sub-product composing the considered HS6 headline. Alternatively, higher unit values indicate a mix of exports within the HS6 category distorted in favour of more expensive HS8 or HS10 products. Given the high degree of disaggregation, these HS8 or HS10 products can also be seen as better quality varieties of the considered HS6 product category.
countries tend to export more expensive varieties of a given product. This positive correlation between unit values and income is partly explained by higher capital per labour ratio and higher share of population attaining secondary or higher education in richer countries. To be sure that income variables capture in our regressions demand-based determinants of export quality, we include controls for potential supply-based determinants of vertical comparative advantage. In particular, we use the share of population with tertiary education as a proxy for skills. In order to check that differences in unit values do not reflect differences in labour cost, we also include the level of hourly earnings in each country provided by Eurostat for year 2002.

Note that in the BACI database, Belgium and Luxembourg are considered as a single entity. We thus calculate Belgium-Luxembourg characteristics as a weighted average of variables for each country, using as weights their respective share in the total population of both countries.

All the variables considered so far are country-specific, while our dependent variable is product-country specific. They consequently do not control for the specific ability of a country in a particular product that could affect unit values. This could make the estimation of the impact of other country-specific determinants noisy. In particular, Bernard et al. (2007) develop a model featuring firms with heterogeneous productivity, countries with different relative factor abundance and industries with different factor intensity. In this framework, they show that trade liberalization induces more firm-level selection in the comparative advantage industry, and consequently magnifies ex ante comparative advantage. Since, for a given level of quality, higher productivity firms charge lower prices, we should observe a negative relationship between unit value and comparative advantage. We do not have direct information on product-level comparative advantages of countries but we can construct an index of revealed comparative advantage. We use the Balassa index $B_{xpt}$ defined as follows:

$$B_{xpt} = \frac{X_{xpt}/X_{xt}}{X_{EU25pt}/X_{EU25t}}$$

(37)

where $X$ denotes exports in volume. This index measures the share of product $p$ in exports of country $x$, as compared to the share of product $p$ in total exports of EU25 countries. The higher is $B_{xpt}$, the higher the comparative advantage of country $x$ for product $p$, as compared to its EU25 competitors. We compute the Balassa index using the BACI database and focusing on trade flows among EU25 members.

4.3 Estimated equation

The two main predictions of our model we want to test can be stated as follows:

**Prediction 1:** Within a given product category, richer countries specialize in high quality varieties, and thus exhibit higher unit values of exports than their partners.

**Prediction 2:** Within a given product category, within-country inequalities have a heterogeneous impact on vertical specialization. Inequalities increase specialization in high-quality
varieties for rich countries only.

To test empirically these predictions, we bring the following baseline relationship to data:

\[
\bar{u}v_{xpt} = \alpha \text{pop}_{xt} + \beta \text{avg} \_ \text{inc}_{xt} + \delta \text{ineq}_{xt} + \gamma \text{bal}_{xpt} + \mu_p + \epsilon_{xpt}
\]  

(38)

where, after log-linearization, \( \bar{u}v_{xpt} \) is the average unit value of exports of product \( p \) by country \( x \) to its EU25 partners in year \( t \), \( \text{avg} \_ \text{inc}_{xt} \) is the average income in PPP of country \( x \), \( \text{ineq}_{xt} \) is the interquintile ratio of income in country \( x \) and \( \text{bal}_{xpt} \) is the Balassa index for country \( x \) and product \( p \) at time \( t \), taking EU25 countries as partners and reference. In our model, for a given product, the quality content of exports of a country is determined by the size of the market for the high quality within this country. The quality of exports is higher when demand for the high quality within this country is higher. Three variables determine the size of the market for the high quality: population, average income and inequality. We introduce the three variables separately to disentangle the different channels at play.

Our dependent variable (exporter-product-year specific) is more disaggregated than our independent variables of interest (exporter-year specific). According to Moulton (1990), standard-errors of the coefficients on exporter-year characteristics might consequently be downward-biased. To correct for this, we cluster all regressions at the exporter-year level\(^{14}\).

Following Bernard et al. (2007), we expect \( \gamma \) to be negative in this regression. Indeed, a higher revealed comparative advantage should be associated with a better efficiency and lower prices. In light of our model, \( \beta \) should on the opposite be positive. For a given size of the population and a given level of inequalities, higher average income increases the demand for high quality varieties, and thus increases the average quality of the export basket. According to our model, the sign and significance of \( \delta \) is undetermined. Indeed, the regression does not distinguish rich countries, for which inequalities have a positive impact on vertical specialization, and poor countries, for which inequalities have no impact. Finally, the coefficient \( \alpha \) on the size of the population is also undetermined. Indeed, for a given level of average income and inequalities, increasing the size of population increases the size of the market for both the low and the high quality varieties.

To test directly prediction 2 on the heterogeneous impact of inequalities, we implement in a second step the following regression:

\[
\bar{u}v_{xpt} = \alpha \text{pop}_{xt} + \beta \text{avg} \_ \text{inc}_{xt} + \delta \text{ineq}_{xt} + \eta \text{rich} + \kappa \text{rich}_t \times \text{ineq}_{xt} + \gamma \text{bal}_{xpt} + \mu_p + \epsilon_{xpt}
\]  

(39)

where \( \text{rich} \) is a dummy identifying rich countries. We define a country as being a rich country if its average PPP income is bigger than 16000 euros, which allows to split the data into two equal-sized samples in terms of number of countries. Rich countries are

\(^{14}\)From a computational point of view, the dimension of the fixed effect being different from the dimension of the cluster, we first calculate demeaned variables in the product-year dimension, and we then run the regressions using OLS estimations and clustering standard-errors at the exporter-year level.
Italy, Sweden, Finland, France, Denmark, Germany, Belgium-Luxembourg, Netherlands, Cyprus, Ireland, Austria and UK. In this regression, following the prediction of our model, we now expect $\delta$ to be insignificant and $\kappa$ to be positive and significant. This corresponds to a discretization of our heterogenous impact, in line with our theoretical model. We will however show that our results are robust to a more continuous approach where average income and inequalities are directly interacted.

We are interested in demand-based determinants of vertical comparative advantage. We must consequently rule out the possibility that omitted supply-side determinants of prices and quality drive our results. To this purpose, we introduce in the regression hourly wages to control for differences in labour costs, and the share of population with a tertiary degree to control for skills of the workforce.

This first empirical approach compares, for a given product-year, average unit values of exports realized by EU25 countries within the enlarged EU market. For a given exporter, this amounts to assigning a single average quality to all its export flows within EU25. However, it might be the case that a given country produces different qualities that it exports in different relative quantities, depending on the demand in the destination country. This is why we also exploit the bilateral dimension of our data and explain, for a given importer-product-year, variations in the unit value of imports by differences in income and income distribution in the source country.

We estimate the following equation:

\[
\bar{u}v_{xmpl} = \nu \text{dist}_{xm} + \alpha \text{pop}_{xt} + \beta \text{avg}_-\text{inc}_{xt} + \delta \text{ineq}_{xt} + \eta \text{rich} + \kappa \text{rich} \times \text{ineq}_{xt} + \gamma \text{bal}_{xpt} + \mu_{mpt} + \epsilon_{xmpl}
\]

(40)

As in equation 39, the estimation of our effects is based on repeated cross-sections. However, we introduce now an importer-product-year fixed effect $\mu_{mpt}$. As Schott (2004) does when he studies unit values of US imports, this amounts to exploiting variations in the quality of imports for a given importing country and a given product. The only difference is that we consider several markets since our sample includes all EU25 importers.

We also introduce in the regression the bilateral distance between the exporting and the importing country. Indeed, several recent papers on firm-level data have shown bilateral distance to be positively correlated with unit values of exports, either due to strategic pricing-to-market or to spatial sorting of exported qualities along distance (see Martin (2010) and Bastos and Silva (2010)). We thus expect $\nu$ to be positive.
5 Results

5.1 Descriptive statistics

In this section, we give an overview of average income and inequalities, as measured by interquintile ratio, in countries of the enlarged EU in 2005. We distinguish “rich” and “poor” countries, rich countries having an average income higher than 16000 euros over the period. This threshold splits countries of the enlarged EU into two equal-sized samples. Results presented in Table 1 show that rich countries are almost twice as rich as poor countries in terms of average PPP income, whatever the income quintile we consider. In 2005, “poor” countries seem to be slightly more unequal than “rich” countries: the interquintile ratio is equal to 4.88 for the former and to 4.28 for the latter. Poor countries are more homogeneous along all the dimensions described in the table, the standard-deviation of the different variables being lower for this group of countries.

Moreover, Graph 1 shows that both average income and inequalities vary strongly across countries of the enlarged EU, whatever the sample of countries we consider. Indeed, we observe in the data rich and unequal countries (Great-Britain, Germany), rich and equal countries (Sweden, Denmark), but also poor and unequal countries (Latvia, Lithuania) and poor and equal countries (Hungary, Slovakia). No clear correlation exists between average income and inequalities in the different samples of countries. This confirms that the enlarged EU is a perfect geographic zone to investigate the role of income distribution on the specialization of countries in terms of quality.

Table 1: Average income and inequalities in the enlarged EU in 2005 - PPP

<table>
<thead>
<tr>
<th></th>
<th>Poor countries</th>
<th>Rich countries</th>
<th>All countries</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Sd</td>
<td>Mean</td>
</tr>
<tr>
<td>Average income</td>
<td>9900</td>
<td>3376</td>
<td>18970</td>
</tr>
<tr>
<td>1st quintile Average</td>
<td>4084</td>
<td>1593</td>
<td>8311</td>
</tr>
<tr>
<td>5th quintile Average</td>
<td>19005</td>
<td>6489</td>
<td>35233</td>
</tr>
<tr>
<td>Interquintile ratio</td>
<td>4.88</td>
<td>1.23</td>
<td>4.28</td>
</tr>
</tbody>
</table>

Finally, we investigate the correlation between the average income in the first quintile, in the last quintile and in the whole population. As emphasized in Table 2, in poor countries, average income is highly correlated with both the average income in the 1st quintile and in the last quintile. For rich countries, average income is more correlated with the 5th quintile than with the 1st quintile. In line with this observation, the average income of the first and the last quintiles are much more correlated in poor countries than in rich countries (0.77 vs 0.19). These two stylized facts are consistent with an important intuition of our model. When average income is low, even when inequalities are high, the support of income in the economy is narrow and income of the rich remains quite low. This implies a quite strong correlation between the average income of the first and the last quintiles. When average income is high, the support of income in the economy is larger, and different income distributions are compatible with the measured average income. This reduces the
Table 2: Average income and inequalities in the enlarged EU in 2005 - PPP

<table>
<thead>
<tr>
<th>Poor countries</th>
<th>Rich countries</th>
<th>All countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st quint.</td>
<td>1st quint.</td>
<td>1st quint.</td>
</tr>
<tr>
<td>0.91 1</td>
<td>0.53 1</td>
<td>0.95 1</td>
</tr>
<tr>
<td>0.96 0.77</td>
<td>0.92 0.19</td>
<td>0.98 0.88</td>
</tr>
</tbody>
</table>

CZE ESP EST GRC HUN LTU LVA MLT POL PRT SVK SVN SWE

PPP Avg Income
3 4 5 6 7

PPP Interquintile Ratio

Figure 1: Average income and inequalities in 2005

correlation observed between the average income of the rich and the poor. Inequalities become in that case crucial to determine the size of the market represented by the demand of rich people.

5.2 Average export prices

We now present our regression results, focusing first on average unit values of exports realized by EU25 exporters within the enlarged EU market.

5.2.1 Interquintile ratio as a proxy for inequalities

We use as a first measure of inequalities the interquintile income ratio. As predicted by our model, results in column (1) of Table 3 show that for a given product, unit values of exports within the enlarged EU are positively correlated with average income (in PPP) of the exporting country. This positive correlation is even stronger when we consider products
that are more vertically differentiated\textsuperscript{15} (see column (4)). Inequalities have no significant impact. This is in line with our model which shows that the role of inequalities is \textit{a priori} ambiguous. We can also note that the estimated coefficient for the size of population is not significant and very close to zero. In trade models with imperfect competition such as ours, population size impacts on quantities produced and exported through a home market effect. However, there is no direct intuition for an impact of population on the quality content of exports. What our model predicts is that in case a strong size asymmetry between trade partners, the small country could, all else equal, in equilibrium, be importer of both the high and the low quality varieties. However, our regressions are carried out on observed export flows. Our interpretations are valid conditioning on the fact that a country exports some varieties of a given good. In that case, once average income and inequalities are controlled for, a bigger population increases the size of the market for both the high and low quality varieties. According to our model, conditioning on the fact that the country exports the good we consider, the impact of an increase in population size on the quality content of exports is ambiguous or neutral. As expected, the Balassa index of revealed comparative advantage at the exporter-product level is affected by a negative and coefficient significant: when a country has a revealed comparative advantage for a product, it tends to export varieties of this good at a cheaper price.

In column (2), we test directly for the existence of a heterogeneous impact of inequalities along the average income dimension. We thus add a dummy identifying rich countries, and an interaction term between this dummy and our measure of inequalities. In line with our model, this interaction term has a positive and significant impact. As predicted by our model, inequalities affect positively the quality mix of exports \textit{if} the population of the exporting country is rich enough. Indeed, an increase in inequalities is associated with a significant increase in the demand for high quality varieties in rich enough countries only. Again, the magnitude of the coefficients is reinforced in column (4), where we focus on more vertically differentiated products.

Our regressions do not account so far for supply-side determinants of the quality-content and prices of exports. It is not obvious how this could affect our results on inequalities. However, it is clear that richer countries might also have higher labour costs due to more developed social security systems. They might also be more capital-intensive or better endowed in skilled workers, which could determine their comparative advantage in terms of quality. We thus include in our baseline regressions the level of hourly wages in 2002, taken from Eurostat, which proxies for labour costs. We also include the share of population with tertiary education attainment to proxy for skills. The level of hourly wages is of course highly correlated to average income in the country. Its introduction consequently reduces the level of the coefficient on average income. Quite surprisingly, the share of population with tertiary education is affected by a negative coefficient. However, this result should not be interpreted as a negative impact of the level of education on the quality of exports. Indeed, this regression also includes average income and the level of wages. It thus shows

\textsuperscript{15}Defined as the top 50\% of products in terms of coefficient of variation of unit values of exports.
that after having controlled for average income and wages, countries with a higher share of people with tertiary education tend to export, in the enlarged EU, products with a lower unit value. This is largely related to the fact that Eastern countries have a high share of their population holding a tertiary degree, as a result of strong education policies in these countries under the Communist era. However, results in columns (3) and (6) show that the inclusion of such variables do not affect our results about the heterogeneous impact of inequalities on the quality of exports.

One might also be concerned by the direction of causality. Indeed, Verhoogen (2008) shows that quality upgrading is associated with increased demand for qualified workers and thus increased wage inequalities, while we focus on the reverse relationship. However, note first that we use a measure of income inequalities, and not of wage inequalities. Both measures are not necessarily collinear, since differences in wealth are not uniquely determined by labour revenues. Moreover, the explanation provided by Verhoogen (2008) does not account at all for the heterogeneous correlation between quality and inequalities we highlight.

Consequently, we can conclude from the results presented in Table 3 that beyond supply-side determinants of vertical comparative advantage, income distribution impacts on the quality of varieties produced and exported by countries. For a given good, richer countries export higher quality varieties while inequalities have a heterogeneous impact: they positively impact on the quality of exports for rich countries only.

Table 3: Average export prices and exporter characteristics

<table>
<thead>
<tr>
<th>Model:</th>
<th>Dependent Variable: Ln uv_{xpt}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All manuf. ind.</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Ln Pop</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
</tr>
<tr>
<td>Ln Avg PPP Income</td>
<td>0.249</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
</tr>
<tr>
<td>Ln PPP Interquintile ratio</td>
<td>-0.034</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
</tr>
<tr>
<td>Rich countries</td>
<td>-0.292</td>
</tr>
<tr>
<td></td>
<td>(0.117)</td>
</tr>
<tr>
<td>Ln PPP Interquintile ratio × Rich countries</td>
<td>0.238</td>
</tr>
<tr>
<td></td>
<td>(0.074)</td>
</tr>
<tr>
<td>Ln Balassa ind. vol.</td>
<td>-0.112</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
</tr>
<tr>
<td>Ln Share. pop. tert. educ.</td>
<td>-0.090</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
</tr>
<tr>
<td>Ln Individual wage</td>
<td>0.115</td>
</tr>
<tr>
<td></td>
<td>(0.042)</td>
</tr>
<tr>
<td>N</td>
<td>237477</td>
</tr>
<tr>
<td>R²</td>
<td>0.141</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses *, **, and *** respectively denoting significance at the 1%, 5% and 10% levels. HS6 product fixed effects in all regressions. Standard errors are clustered at the exporting country level.
5.2.2 Continuous measures of inequalities and heterogeneity

The previous specification was very close to our model, considering only two classes in the economy, rich and poor (i.e. the last and the first income quintiles). However, the distribution of income is in reality much more continuous. One way to approach inequalities along the entire distribution of income is to use the Gini index. Indeed, the Gini index measures, at all income levels, the difference between the observed distribution and a uniform distribution. We check in this section that our results hold when using this inequality index.

Results presented in Table 4 are qualitatively exactly the same. For a given product, average unit values, and thus quality of exports, are positively related to average income. On the whole sample of countries, inequalities, as measured by the Gini index, have no significant impact. This non significant coefficient is in reality due to a heterogeneous impact of inequalities: the Gini index is positively correlated with unit values of exports in rich enough countries only. These results hold when controlling for supply-side determinants of vertical comparative advantage, and are reinforced when we consider manufactured goods with high vertical differentiation.

Table 4: Average export prices and exporter characteristics - Gini index

<table>
<thead>
<tr>
<th>Model :</th>
<th>Dependent Variable: ( \ln u_{xpt} )</th>
<th>All manuf. ind.</th>
<th>Vert. diff. manuf. ind.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td>Ln Pop</td>
<td>0.003</td>
<td>-0.014*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.008)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ln Avg PPS Income</td>
<td>0.248a</td>
<td>0.082</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.021)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ln Gini Index</td>
<td>-0.075</td>
<td>-0.109*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.055)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rich countries</td>
<td>-1.061a</td>
<td>-1.123b</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.376)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rich countries x Ln Gini Index</td>
<td>-0.112a</td>
<td>-0.114a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ln Balassa ind. vol.</td>
<td>-0.095a</td>
<td>-0.102a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ln Share. pop. tert. educ.</td>
<td>-0.095a</td>
<td>-0.102a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.046)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ln Individual wage</td>
<td>0.095a</td>
<td>0.063</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.025)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>237477</td>
<td></td>
</tr>
<tr>
<td></td>
<td>R(^2)</td>
<td>0.141</td>
<td>0.143</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses \(^a\), \(^b\) and \(^c\) respectively denoting significance at the 1%, 5% and 10% levels. HS6 product fixed effects in all regressions. Standard errors are clustered at the exporting country level.

The two specifications so far measure a “discrete” heterogeneity in the impact of inequalities. They allow for a different coefficient on the inequality index for rich and poor countries, both categories being defined according to a pre-determined income threshold. However, it might be the case that the change in the impact of inequalities along average income is continuous, and not dichotomic. In Table 5, we investigate such a continuous heterogeneity by interacting directly average income and the interquintile ratio. Again,
results are qualitatively very much the same. According to regression (2), the average income from which inequalities start having a positive impact is 12 855 euros\textsuperscript{16}.

Table 5: Average export prices and exporter characteristics - Continuous heterogeneity

<table>
<thead>
<tr>
<th>Model</th>
<th>Dependent Variable: ( \ln \bar{w}_{vpt} )</th>
<th>All manuf. ind.</th>
<th>Vert. diff. manuf. ind.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>( \ln \text{Pop} )</td>
<td>0.003</td>
<td>-0.001</td>
<td>-0.013</td>
</tr>
<tr>
<td></td>
<td>( (0.008) )</td>
<td>( (0.009) )</td>
<td>( (0.012) )</td>
</tr>
<tr>
<td>( \ln \text{Avg PPP Income} )</td>
<td>0.249\textsuperscript{b} -0.011 -0.044</td>
<td>0.305\textsuperscript{c} -0.121 -0.067</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( (0.022) )</td>
<td>( (0.129) )</td>
<td>( (0.029) )</td>
</tr>
<tr>
<td>( \ln \text{PPP Interquintile ratio} )</td>
<td>-0.034 -1.599\textsuperscript{b} -1.141\textsuperscript{c}</td>
<td>-0.014 -2.578\textsuperscript{a} -1.943\textsuperscript{b}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( (0.035) )</td>
<td>( (0.630) )</td>
<td>( (0.046) )</td>
</tr>
<tr>
<td>( \ln \text{Avg PPP Income} \times \ln \text{PPP Interquintile ratio} )</td>
<td>0.169\textsuperscript{b} 0.124\textsuperscript{c}</td>
<td>0.276\textsuperscript{a} 0.211\textsuperscript{b}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( (0.076) )</td>
<td>( (0.069) )</td>
<td>( (0.095) )</td>
</tr>
<tr>
<td>( \ln \text{Balassa ind. vol.} )</td>
<td>-0.112\textsuperscript{a} -0.112\textsuperscript{a} -0.114\textsuperscript{a}</td>
<td>-0.141\textsuperscript{a} -0.142\textsuperscript{a} -0.143\textsuperscript{a}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( (0.002) )</td>
<td>( (0.002) )</td>
<td>( (0.003) )</td>
</tr>
<tr>
<td>( \ln \text{Share. pop. tert. educ.} )</td>
<td>-0.085\textsuperscript{a}</td>
<td>-0.087\textsuperscript{b}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( (0.032) )</td>
<td>( (0.041) )</td>
<td>( (0.041) )</td>
</tr>
<tr>
<td>( \ln \text{Individual wage} )</td>
<td>0.089\textsuperscript{b}</td>
<td>0.054</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( (0.039) )</td>
<td>( (0.052) )</td>
<td>( (0.052) )</td>
</tr>
<tr>
<td>( N )</td>
<td>237477</td>
<td>237477</td>
<td>237477</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.141</td>
<td>0.142</td>
<td>0.144</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses \textsuperscript{a}, \textsuperscript{b} and \textsuperscript{c} respectively denoting significance at the 1%, 5% and 10% levels. HS6 product fixed effects in all regressions. Standard errors are clustered at the exporting country level.

5.3 Bilateral export prices

In the preceding section, we had re-aggregated the bilateral data of BACI at the exporter-product level. However, our model is originally bilateral and not unilateral. Moreover, it might be the case that some countries produce and export different qualities of the same HS6-product, adapting their export mix to the destination country. By aggregating the data, we consequently lose some information. In this section, we exploit the bilateral dimension of our data. For a given importer-product, we investigate how differences in the unit values of imports can be explained by income distribution in the source country.

5.3.1 Interquintile ratio as a proxy for inequalities

Consistently with our unilateral approach, results in Table 6 show that an importer tends to import more expensive varieties of a given good from richer exporters. On the opposite, the varieties this importer sources from countries having a revealed comparative advantage in the good we consider tend to be cheaper. Finally, again, inequalities have an ambiguous impact. While they seem to have on average a negative impact (see column (1)), the interaction term between the interquintile ratio and the dummy identifying rich countries is positive and significant, pointing at a heterogeneous impact of inequalities. However, note that this heterogeneity is to a certain extent different from the one measured in unilateral regressions. When considering bilateral export flows, inequalities appear to

\textsuperscript{16}We obtain this result from the following calculation: \( 1.599 \div 0.169 = 12855 \)
be negatively correlated with unit values for poor countries, and on average not correlated for rich countries. This might be however a feature of our inequality index.

Table 6: Bilateral export prices and exporter characteristics

<table>
<thead>
<tr>
<th>Model</th>
<th>Dependent Variable: Ln uv_{sympt}</th>
<th>All manuf. ind.</th>
<th>Vert. diff. manuf. ind.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Ln Pop</td>
<td>-0.013b</td>
<td>-0.021a</td>
<td>-0.021b</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Ln Avg PPS Income</td>
<td>0.225b</td>
<td>0.136a</td>
<td>0.146b</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.023)</td>
<td>(0.074)</td>
</tr>
<tr>
<td>Ln PPS Interquintile ratio</td>
<td>-0.084b</td>
<td>-0.157a</td>
<td>-0.152a</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.025)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>Rich countries</td>
<td>-0.241a</td>
<td>-0.218</td>
<td>-0.310a</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
<td>(0.156)</td>
<td>(0.119)</td>
</tr>
<tr>
<td>Ln PPS Interquintile ratio × Rich countries</td>
<td>0.225b</td>
<td>0.210b</td>
<td>0.272a</td>
</tr>
<tr>
<td></td>
<td>(0.057)</td>
<td>(0.098)</td>
<td>(0.084)</td>
</tr>
<tr>
<td>Ln Balassa ind. vol.</td>
<td>-0.084a</td>
<td>-0.085a</td>
<td>-0.085a</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Ln Distance</td>
<td>0.126a</td>
<td>0.135a</td>
<td>0.135a</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.007)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Ln Share. pop. tert. educ.</td>
<td>-0.008</td>
<td>0.024</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.032)</td>
<td></td>
</tr>
<tr>
<td>Ln Individual wage</td>
<td>-0.004</td>
<td>-0.091c</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.047)</td>
<td>(0.053)</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>2421833</td>
<td>2421833</td>
<td>2421833</td>
</tr>
<tr>
<td>R²</td>
<td>0.052</td>
<td>0.054</td>
<td>0.054</td>
</tr>
<tr>
<td></td>
<td>1039186</td>
<td>1039186</td>
<td>1039186</td>
</tr>
<tr>
<td></td>
<td>0.061</td>
<td>0.062</td>
<td>0.063</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses a, b and c respectively denoting significance at the 1%, 5% and 10% levels. HS6 product-importing country fixed effects in all regressions. Standard errors are clustered at the exporting country level.

5.3.2 Continuous measures of inequalities and heterogeneity

Indeed, we now use the Gini index as an alternate measure of inequalities. When introduced alone, inequalities measured by the Gini index are again affected by a negative coefficient. However, in column (2) of Table 7, the net effect of inequalities in rich countries appears to be positive. Nothing changes regarding the other control variables. Results obtained when introducing the interaction between average income and inequalities also confirm our previous results. Calculations based on column (2) of Table 8 indicate that the estimated income threshold from which inequalities start impacting positively on the quality content of export is now equal to 19670 euros, and is thus higher than the one calculated in the previous section.

6 Conclusion

We have developed a model with non-homothetic preferences and increasing returns to scale, in which vertical comparative advantage of countries is generated by demand-based determinants. We show that richer countries produce and export higher quality varieties of a given good. More interestingly, we provide new predictions on the role of inequalities.
Table 7: Bilateral export prices and exporter characteristics - Gini index

<table>
<thead>
<tr>
<th>Model :</th>
<th>Dependent Variable: Ln u_vxmpt</th>
<th>All manuf. ind.</th>
<th>Vert. diff. manuf. ind.</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ln Pop</td>
<td>-0.013^c</td>
<td>-0.021^a</td>
<td>-0.021^b</td>
<td>-0.018^c</td>
<td>-0.026^a</td>
<td>-0.020^a</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Ln Avg PPS Income</td>
<td>0.226^a 0.136^b 0.153^b</td>
<td>(0.008) (0.007) (0.008)</td>
<td>(0.009) (0.009) (0.010)</td>
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</tr>
<tr>
<td>LnGini Index</td>
<td>-0.136^b -0.290^a -0.280^a</td>
<td>(0.060) (0.047) (0.054)</td>
<td>(0.081) (0.053) (0.067)</td>
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<tr>
<td>Rich countries</td>
<td>-1.217^b -1.099^b -1.102^b</td>
<td>(0.284) (0.478) (0.402)</td>
<td>(0.564)</td>
<td></td>
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</tr>
<tr>
<td>Rich countries×LnGini Index</td>
<td>0.390^a 0.356^b</td>
<td>(0.085) (0.141)</td>
<td>(0.122) (0.166)</td>
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</tr>
<tr>
<td>Ln Balassa ind. vol.</td>
<td>-0.084^a -0.085^a -0.085^a</td>
<td>(0.003) (0.003) (0.002)</td>
<td>(0.003) (0.003) (0.003)</td>
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</tr>
<tr>
<td>Ln Distance</td>
<td>0.126^a 0.135^b 0.136^b</td>
<td>(0.008) (0.008) (0.008)</td>
<td>(0.009) (0.009) (0.009)</td>
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</tr>
<tr>
<td>Ln Share. pop. tert. educ.</td>
<td>-0.012</td>
<td>(0.027)</td>
<td>(0.032)</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Ln Individual wage</td>
<td>-0.010</td>
<td>(0.046)</td>
<td>(0.050)</td>
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<tr>
<td>N</td>
<td>2421833 2421833 2421833 1039186 1039186 1039186</td>
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<tr>
<td>R^2</td>
<td>0.052 0.055 0.055 0.061 0.063 0.063</td>
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</tbody>
</table>

Note: Standard errors in parentheses ^a, ^b and ^c respectively denoting significance at the 1%, 5% and 10% levels. HS6 product-importing country fixed effects in all regressions. Standard errors are clustered at the exporting country level.

Table 8: Bilateral export prices and exporter characteristics - Continuous heterogeneity

<table>
<thead>
<tr>
<th>Model :</th>
<th>Dependent Variable: Ln u_vxmpt</th>
<th>All manuf. ind.</th>
<th>Vert. diff. manuf. ind.</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ln Pop</td>
<td>-0.013^b -0.016^b -0.019^b</td>
<td>(0.007) (0.008) (0.009)</td>
<td>(0.009) (0.010) (0.011)</td>
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<tr>
<td>Ln Avg PPS Income</td>
<td>0.225^c -0.035 -0.057</td>
<td>(0.026) (0.124) (0.176)</td>
<td>(0.030) (0.140) (0.207)</td>
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<tr>
<td>Ln PPS Interquintile ratio</td>
<td>-0.084^a -1.661^b -1.641^c</td>
<td>(0.037) (0.717) (0.854)</td>
<td>(0.049) (0.837) (0.995)</td>
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<tr>
<td>Ln Avg PPS Income × Ln PPS Interquintile ratio</td>
<td>0.168^b 0.167^c 0.215^b 0.185^c</td>
<td>(0.078) (0.093) (0.091) (0.108)</td>
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</tr>
<tr>
<td>Ln Balassa ind. vol.</td>
<td>-0.084^a -0.085^a -0.086^a</td>
<td>(0.003) (0.003) (0.003)</td>
<td>(0.003) (0.003) (0.003)</td>
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<td></td>
</tr>
<tr>
<td>Ln Distance</td>
<td>0.126^a 0.125^b 0.124^b</td>
<td>(0.008) (0.008) (0.007)</td>
<td>(0.009) (0.008) (0.009)</td>
<td></td>
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</tr>
<tr>
<td>Ln Share. pop. tert. educ.</td>
<td>-0.020</td>
<td>(0.028)</td>
<td>(0.032)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Ln Individual wage</td>
<td>0.145^a 0.144^a 0.145^a</td>
<td>(0.073) (0.073) (0.073)</td>
<td>(0.073) (0.073) (0.073)</td>
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<tr>
<td>N</td>
<td>2421833 2421833 2421833 1039186 1039186 1039186</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>R^2</td>
<td>0.052 0.055 0.055 0.061 0.061 0.061</td>
<td></td>
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</tbody>
</table>

Note: Standard errors in parentheses ^a, ^b and ^c respectively denoting significance at the 1%, 5% and 10% levels. HS6 product-importing country fixed effects in all regressions. Standard errors are clustered at the exporting country level.
We show that the way income is distributed around its average within a country impacts on the quality produced and exported for rich enough countries only. Indeed, when average income is low, even for high levels of inequalities, the local demand remains predominantly oriented toward the low quality varieties. On the opposite, for higher levels of income, rich people consume the high-quality varieties, and any increase in inequalities will reinforce the demand, and thus the size of the local market, for the high-quality.

We moreover provide strong empirical support for the role of demand-based determinants of the quality content of exports. We focus on the EU25, which as a free-trade area of highly heterogeneous countries in terms of income and inequalities, provides a unique playground for our analysis. In line with our model, we show that having controlled for average income, no clear impact of inequalities (as measured by interquintile ratio or Gini index) is detected on unit values, while the interaction between average income and inequalities has a positive and significant effect. The positive impact of this interaction term and of average income on the quality of exports of a country is robust to the inclusion of controls for supply-side determinants of prices and vertical comparative advantage (wages, education).

We believe that our results have several interesting implications. Showing that the income distribution of a country is a significant determinant of its export specialization patterns suggests that there might be an impact of redistributive policies on export performance of a country. In a more dynamic view, we can conjecture that high quality varieties are more likely to generate externalities or to induce technology adoption. Some interesting new results could thus certainly be obtained on the link between income distribution and growth through this quality channel. Finally, we show that income distribution impacts on vertical specialization of countries. But it has also been shown that quality upgrading is also associated with changes in income distribution. There consequently might be feedbacks or magnification effects of the two mechanisms that an integrated framework would help to understand. These could be interesting avenues for future research.

References


Appendix A. The equilibrium distribution of firms and comparative statics in closed economy.

(a) Proof of proposition 1.

We first prove that within each possible parametric case, there exists a unique equilibrium determining the number of active firms in the high- and low-quality segments of the markets, \( n_H \) and \( n_L \).

1. Case C1. For income distribution parameters \( \beta \) and \( d \) such that we are in Case 1, we have already established that there exists a unique positive solution to the system of two equations defined by (14)-(15), such that \( n_L = \frac{L}{f_L} \) and \( n_H = 0 \).

2. Case C2. Searching for possible solution(s) to equation (16) amounts to solving for roots of the following function:

\[
F(n_H) = \mu(1 - \beta d)L - (1 - \beta)N(1 - \mu)\gamma a_H^{\frac{1}{\sigma - 1}} \left( \frac{\sigma}{\sigma - 1} \right) a_H - f_H n_H \sigma
\]

The function \( F(.) \) reaches a unique extremum for:

\[
\frac{\partial F}{\partial n_H} = \frac{\sigma}{(\sigma - 1)^2} (1 - \beta)N(1 - \mu)\gamma a_H^{\frac{1}{\sigma - 1}} - \sigma f_H = 0 \iff n_H^* = \left[ \frac{(1 - \beta)N(1 - \mu)\gamma a_H}{(\sigma - 1)^2 f_H} \right]^{\frac{\sigma - 1}{\sigma}}.
\]

We furthermore have \( \frac{\partial^2 F}{\partial n_H^2} < 0 \), implying that \( F(n_H^*) \) is a maximum. Hence, \( F(.) \) has zero, one or two roots, depending on the sign of \( F(n_H^*) \). We have \( F(n_H^*) > 0 \) under the following condition on the parameters:

\[
\mu(1 - \beta d)L > [(1 - \beta)N(1 - \mu)\gamma a_H]^{\frac{\sigma - 1}{\sigma}} \left[ (\sigma - 1)^2 f_H \right]^{\frac{\sigma}{\sigma - 1}} \left[ 1 - f_H^{\frac{\sigma - \sigma}{\sigma - 1}}(\sigma - 1)^{3 - 2\sigma} \right]
\]

For big enough values of \( f_H \), i.e. for high fixed entry costs in the high-quality sector, the right-hand side of condition (41) becomes negative, therefore ensuring the existence of two distinct roots \( n_{H1} \) and \( n_{H2} \) of function \( F(.) \), with \( n_{H1} < n_{H2} \). However, only one of those two roots corresponds to a stable equilibrium for the described economy. Indeed, when the number of firms in the high quality segment is equal to \( n_{H1} \), firm-level profits are indeed null, but the entry of one more firm entails positive profits: there is thus an incentive for new entrants to start producing and \( n_{H1} \) is consequently unstable. When the number of firms is equal to \( n_{H2} \), profits are also null. However, if one more firm enters the market, profits become negative, while the exit of one of the active firms entails positive profits, generating an incentive for the
entry of new firms. \( n_{H2} \) is consequently the unique stable equilibrium value for the number of active firms in the high-quality segment, entailing through equation (17) a unique positive equilibrium value of the number of active firms in the low-quality segment, \( n_{L} \).

Hence, for an economy with a high level of fixed entry costs \( f_{H} \) in the high-quality sector and income distribution parameters \( \beta \) and \( d \) such that we are in Case 2, there exists a unique positive solution to the system of two equations defined by (14)-(15), defining the number of active firms in the high- and low-quality sectors, \( n_{H} \) and \( n_{L} \).

3. Case C3. Searching for possible solution(s) to equation (18) amounts to solving for roots of the following function:

\[
G(n_{H}) = \mu L - N(1 - \mu)\gamma n_{H}^{\frac{1}{\sigma}}(\frac{\sigma}{\sigma - 1})a_{H} - f_{H}n_{H}\sigma
\]

The function \( G(.) \) reaches a unique extremum for:

\[
\frac{\partial G}{\partial n_{H}} = 0 \iff n_{H}^{**} = \left[ \frac{N(1 - \mu)\gamma a_{H}}{(\sigma - 1)^{2}f_{H}} \right]^{\frac{\sigma - 1}{\sigma}}
\] (42)

We furthermore have \( \frac{\partial^{2}G}{\partial^{2}n_{H}} < 0 \), implying that \( G(n_{H}^{**}) \) is a maximum. Hence, \( G(.) \) has zero, one or two roots, depending on the sign of \( G(n_{H}^{**}) \). We have \( G(n_{H}^{**}) > 0 \) under the following condition on the parameters:

\[
\mu L > \left[ \frac{N(1 - \mu)\gamma a_{H}}{(\sigma - 1)^{2}f_{H}} \right]^{\frac{\sigma - 1}{\sigma}} \left[ (\sigma - 1)^{2}f_{H} \right] \left( \frac{\sigma}{\sigma - 1} \right) \left[ 1 - f_{H}^{3-\sigma}(\sigma - 1)^{3-2\sigma} \right]
\] (43)

Again, for big enough values of \( f_{H} \), i.e. for high fixed entry costs in the high-quality sector, the right-hand side of condition (43) becomes negative, therefore ensuring the existence of two distinct roots \( n_{H1} \) and \( n_{H2} \) of function \( G(.) \), with \( n_{H1} < n_{H2} \). Following the same kind of economic reasoning than in Case 2, \( n_{H2} \) is the unique stable equilibrium value for the number of active firms in the high-quality segment, entailing through equation (19) a unique positive equilibrium value of the number of active firms in the low-quality segment, \( n_{L} \).

Hence, for an economy with a high level of fixed entry costs \( f_{H} \) in the high-quality sector and income distribution parameters \( \beta \) and \( d \) such that we are in Case 3, there exists a unique positive solution to the system of two equations defined by (14)-(15), defining the number of active firms in the high- and low-quality sectors, \( n_{H} \) and \( n_{L} \).

(b) Proof of proposition 2.

1. Case 2. As it can be seen from (41), \( n_{H1}^{*} \) does neither depend on \( d \) nor on \( L \). However, \( F(n_{H1}^{*}) \) depends on both variables:

\[
F(n_{H1}^{*}) = \mu(1 - \beta d)L - (1 - \beta)N(1 - \mu)\gamma \frac{\sigma}{\sigma - 1}a_{H}(n_{H1}^{*})^{\frac{1}{\sigma}} - \sigma f_{H}n_{H1}^{*}
\] (44)
It is clear from equation (44) that the higher the level of inequalities (the lower \( d \)) and the higher the average income (the higher \( L \)), the more likely \( F(n_H^*) \) is positive and the higher the value of this maximum. Equation (41) furthermore shows that the slope of the curve \( F(.) \) is not influenced by \( d \) or \( L \). Hence, a shock on one of those two variables will simply shift \( F(.) \) upwards or downwards. This necessarily means that when \( F(n_H^*) \) is higher, due to stronger inequalities and/or higher average income, \( n_{H2} \) is bigger: the number of firms in the high quality segment increases with inequalities and average income. We hence have the following comparative statics: \( \frac{\partial n_H}{\partial d} < 0, \frac{\partial n_H}{\partial L} > 0. \)

2. Case 3. As it can be seen from equations (18) and (19), the level of inequalities \( d \) does not have any influence on the equilibrium in Case 3. We will hence focus on variations of the overall effective labor supply \( L \). (42) shows that \( n_H^{**} \) does not depend on \( L \). However, \( G(n_H^*) \) is of the form:

\[
G(n_H^*) = \mu L - N(1-\mu)\frac{\sigma}{\sigma - 1} a_H(n_H^*) \gamma a_H(n_H^*)^{1-\gamma} - \sigma f_H n_H^*
\]

(45)

It is clear from equation (45) that the higher the average income (the higher \( L \)), the more likely \( G(n_H^*) \) is positive and the higher the value of this maximum. Equation (42) furthermore shows that the slope of the curve \( G(.) \) is not influenced by \( L \). Hence, a shock \( L \) will simply shift \( G(.) \) upwards or downwards. This necessarily means that when \( G(n_H^*) \) is higher, due higher average income, \( n_{H2} \) is bigger: the number of firms in the high quality segment increases with inequalities and average income. We hence have the following comparative statics: \( \frac{\partial n_H}{\partial L} > 0. \)

Appendix B. Equilibrium equations in open economy.

1. Case O1. \( l_{PD} < l_{P}^*, l_{RD} < l_{D}^*, l_{PF} < l_{F}^*, l_{RF} < l_{F}^* \): the demand for the high-quality bundle is null in both countries, only low quality varieties are traded. We then have \( \tilde{n}_{L}^D = \frac{(1+r^{1-\sigma})a_L D_L}{f_L D_L} \) and \( \tilde{n}_{L}^F = \frac{(1+r^{1-\sigma})a_L F_L}{f_L F_L} \).

2. Case O2. \( l_{P} < l_{P}^*, l_{R} > l_{D}^*, l_{PS} < l_{F}^*, l_{RS} < l_{F}^* \): the high-quality bundle is only consumed by the rich consumers in country \( r \) (\( r = D, F \)) and by none in country \( s \). Only low quality varieties are traded. The equations defining the equilibrium number of effective producers for each quality within each country are then the following:

\[
\begin{align*}
f_H \sigma n_H^r &= \mu(1-\beta d_r)\theta L_r - (1-\beta)N(1-\mu)\frac{\sigma}{\sigma - 1} a_H(n_H^r) \gamma a_H(n_H^r)^{1-\gamma} \\
f_L \sigma \bar{n}_L^r &= (1+r^{1-\sigma})\theta L_r - f_H \sigma \bar{n}_H^r \\
f_L \sigma \bar{n}_L^s &= (1+r^{1-\sigma})\theta L_s
\end{align*}
\]

3. Case O3. \( l_r > l_{r}^*, l_{R} > l_{D}^*, l_{PS} < l_{F}^*, l_{RS} < l_{F}^* \): the high-quality bundle in consumed by both the poor and the rich consumers in country \( r \) (\( r = D, F \)) and by none in country \( s \). Only low quality varieties are traded. The equations defining the
equilibrium number of effective producers for each quality within each country are then the following:

\[ f_H \sigma n_H^r = \mu \theta L_r - N(1 - \mu)\gamma \frac{\sigma}{\sigma - 1} a_H(\tilde{n}_H^r) \]  
\[ f_L \sigma n_L^r = (1 + \tau^{1 - \sigma})(\theta L_r - f_H \sigma n_H^r) \]  
\[ f_L \sigma n_L^s = (1 + \tau^{1 - \sigma})\theta L_s \]

4. Case O4. \( l_{PD} < l_{DR}^*, l_{RD} > l_{DR}^*, l_{PF} < l_{PF}^*, l_{RF} > l_{RF}^* \): the high-quality bundle is consumed only by the rich consumers in both countries. Both high and low quality varieties are traded. The equations defining the equilibrium number of effective producers for each quality within each country are then the following:

\[ f_H \sigma n_H^D = (1 + \tau^{1 - \sigma})(\mu(1 - \beta d_D)\theta L_D - (1 - \beta)N(1 - \mu)\gamma \frac{\sigma}{\sigma - 1} a_H(\tilde{n}_H^D) \]  
\[ (1 + \tau^{1 - \sigma})\theta L_D = f_H \sigma n_H^D + f_L \sigma n_L^D \]  
\[ f_H \sigma n_H^F = (1 + \tau^{1 - \sigma})(\mu(1 - \beta d_F)\theta L_F - (1 - \beta)N(1 - \mu)\gamma \frac{\sigma}{\sigma - 1} a_H(\tilde{n}_H^F) \]  
\[ (1 + \tau^{1 - \sigma})\theta L_F = f_H \sigma n_H^F + f_L \sigma n_L^F \]

5. Case O5. \( l_{Pr} < l_{Pr}^*, l_{Rs} > l_{Pr}^*, l_{Ps} > l_{Rs}^*, l_{Rs} > l_{Rs}^* \): the high-quality bundle is consumed only by the rich consumers in country \( r \) \( (r = D, F) \) and by both rich and poor consumers in country \( s \). Both high and low quality varieties are traded. The equations defining the equilibrium number of effective producers for each quality within each country are then the following:

\[ f_H \sigma n_H^r = (1 + \tau^{1 - \sigma})(\mu(1 - \beta d_r)\theta L_r - (1 - \beta)N(1 - \mu)\gamma \frac{\sigma}{\sigma - 1} a_H(\tilde{n}_H^r) \]  
\[ (1 + \tau^{1 - \sigma})\theta L_r = f_H \sigma n_H^r + f_L \sigma n_L^r \]  
\[ f_H \sigma n_H^s = (1 + \tau^{1 - \sigma})(\mu \theta L_s - N(1 - \mu)\gamma \frac{\sigma}{\sigma - 1} a_H(\tilde{n}_H^s) \]  
\[ (1 + \tau^{1 - \sigma})\theta L_s = f_H \sigma n_H^s + f_L \sigma n_L^s \]

6. Case O6. \( l_{PD} > l_{DR}^*, l_{RD} > l_{DR}^*, l_{PF} > l_{PF}^*, l_{RF} > l_{RF}^* \): the high-quality bundle is consumed by both the rich and poor consumers in both countries. Both high and low quality varieties are traded. The equations defining the equilibrium number of effective producers for each quality within each country are then the following:

\[ f_H \sigma n_H^D = (1 + \tau^{1 - \sigma})(\mu \theta L_D - N(1 - \mu)\gamma \frac{\sigma}{\sigma - 1} a_H(\tilde{n}_H^D) \]  
\[ (1 + \tau^{1 - \sigma})\theta L_D = f_H \sigma n_H^D + f_L \sigma n_L^D \]  
\[ f_H \sigma n_H^F = (1 + \tau^{1 - \sigma})(\mu \theta L_F - N(1 - \mu)\gamma \frac{\sigma}{\sigma - 1} a_H(\tilde{n}_H^F) \]  
\[ (1 + \tau^{1 - \sigma})\theta L_F = f_H \sigma n_H^F + f_L \sigma n_L^F \]