Capacity Constraining Labor Market Frictions in a Global Economy*

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Abstract

Convex vacancy creation costs shape firms’ responses to trade liberalization, since they induce capacity constraints by reducing firms’ capability to grow. A profit maximizing firm will not fully meet the increased foreign demand as in Melitz (2003), but will only serve a few export markets. As a consequence more productive firms will be able to profitably export to more countries and charge unlike in Melitz (2003) higher or similar prices than less productive firms. This is well in line with empirical findings. Trade liberalization also affects labor market outcomes. Increased profits by exporting firms reduces unemployment and increases the wage dispersion in the on-the-job search model.

Keywords: On-the-job search; capacity constraints; international trade; heterogeneous firms

JEL-Codes: F16, F12, J64, L11

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1 Introduction

Apple launched his new iPad on April 3, 2010 in the US. The pictures are well known: large queues in a lot of cities all over the US. The iPad was then consecutively launched in different countries.\footnote{On May 28, 2010 in Australia, Canada, France, Germany, Italy, Japan, Spain, Switzerland and the UK. On July 23, 2010 in Austria, Belgium, Hong Kong, Ireland, Luxembourg, Mexico, Netherlands, New Zealand and Singapore. On September 17, 2010 in China. Source: http://www.apple.com/pr/products/ipad/ipad.html.} In a press release from May 10, 2010, one can read: “Apple will announce availability, local pricing and pre-order plans for [...] additional countries at a later date.”\footnote{See http://www.apple.com/pr/library/2010/05/07ipad.html.} One may argue from different perspectives why this ordering of countries was chosen by Apple. But two things are striking: (i) It did not sell to all countries at the same time, and (ii) the countries that were served are very similar concerning their level of development and size. So why did Apple not sell the iPad to all countries at the same time? From empirical studies by Eaton, Kortum, and Kramarz (2004, 2010) we know that most exporting firms sell to only one foreign market, with the frequency of firms’ selling to multiple markets declining with the number of destinations. In order to explain these facts, they relate the export destination choice to country characteristics. But characteristics like market size can only partly explain the pattern of the iPad launch and the observed empirical regularities of exporting firms.\footnote{While Apple in the end served all countries with iPads, this is not true for most exporting firms even in the long-run.}

We propose a very different answer based on capacity constraining labor market frictions. Consider for example the IT branch, where Apple belongs to.\footnote{In an interview on June 1, 2010 Apple Inc. CEO Steve Jobs said that the idea for the iPad came before the iPhone. However, “...I put the tablet project on a shelf because the phone was more important.” This may hints at least indirectly to some constraints, because otherwise they could have developed both in parallel.} Lately The National Business Review wrote about the IT professional shortage in New Zealand\footnote{See http://www.nbr.co.nz/article/it-professional-shortage-continues-survey-118981.} and Webmaster Europe, the International-European labor union for Internet professionals stated that the IT professional shortage will continue in 2010 in Germany.\footnote{http://www.webmasters-europe.org/modules/news/article.php?storyid=95.}
firms face such capacity constraints due to labor market frictions, firms cannot serve all export markets. This can explain part of observed trade patterns that cannot be explained by export destination characteristics.

There are quite a few empirical studies emphasizing the importance of capacity constraints for exporters. Magnier and Toujas-Bernate (1994) argue that exporting firms may not always be able to meet the demands for its goods due to a capacity constraints. Based on this observations, they derive a theoretical model of export market share which explicitly accounts for capacity constraints through the inclusion of an aggregate investment variable. They confirm their theoretical predictions in a set of OECD countries. Madden, Savage, and Thong (1994) confirm the same results for Australia, Hong Kong, Japan, New Zealand and South Korea, for the period 1978 to 1993. Similarly, Eaton, Eslava, Kugler and Tybout (2008) see capacity constraints as one explanation for the differential growth across firms, where specifically large firms seem to face increasing resistance to foreign market penetration as their exports grow. Blum, Claro and Horstmann (2010) argue that the large fluctuations in the export behavior of Chilean firms can be explained by capacity constraints and fluctuations in domestic demand. Based on their empirical observations they develop a theoretical model where firms production capacities depend on fixed investments. Given these capacity constraints they explain the fluctuations in export behavior by fluctuations in domestic demand.\footnote{Babatunde (2009) addresses the problem of Sub-Saharan African countries’ inability to diversify exports from primary commodities and argues that supply capacity constraints are one important reason for this inability.}

Redding and Venables (2004) find that country specific components that influence the supply capacity have played a significant role in explaining the observed differentials in export performances. Similarly, Fugazza (2004) finds that while trade barriers continue to be a concern, poor supply-side conditions have often been the more important constraint on export performance in various regions, in particular in Africa and the Middle East. Manova (2008) provides evidence from 91 countries that credit constraints are an important determinant of international trade flows.

While there is by now a heavily growing literature that investigates the effects
of trade on unemployment and the wage distribution in the presence of labor market frictions, to the best of our knowledge with one exception no one has thought about the capacity constraining effects of labor market frictions in an open economy. Fajgelbaum (2011) studies the timing of the exporting decision in an on-the-job search equilibrium model based on Postel-Vinay and Robin (2002).

We merge a generalized version of the on-the-job search model by Burdett and Mortensen (1998) with the new trade model by Melitz (2003) and show how convex vacancy creation costs lead to capacity constraints. If the cost associate with an additional vacancy is constant like in Pissarides (2000) labor market frictions just act as additional labor cost and do not restrict the size of a firm. Labor market friction per se do not change the pattern of trade as shown by Felbermayr, Prat, and Schmerer (2011). Only if recruitment costs increase with the number of vacancies posted, firms’ capability to adjust their labor input to changes in demand is limited and changes the pattern of trade.

The empirical evidence on the shape of the vacancy cost function is small. Abowd and Kramarz (2003) and Kramarz and Michaud (2010) use French firm level data and Blatter et al. (2009) use Swiss firm level data to look at the shape of the hiring cost function. However, hiring cost functions do not necessarily have the same shape as vacancy cost functions, because the hiring rate per vacancy is generally not constant but increasing in the size of a firm and therefore increasing in the number of workers hired. This property holds in the Burdett-Mortensen model like in any monopsony wage model as shown by Manning (2006). Using firm level data from the Labour Turnover Survey in the UK Manning (2006) shows that there are increasing marginal costs of recruitment. In section 5, where we analyse the general model with vacancy creation, we show that a convex vacancy cost function is consistent with the mildly concave hiring cost function found by Abowd and Kramarz (2003) and Kramarz and Michaud (2010) for France as well as the convex hiring cost function found by Blatter et al. (2009) for Switzerland.

Analyzing labor market induced capacity constraints allows us to contrast the trade patterns arising in a perfect labor market or an imperfect labor market with constant
vacancy creation cost with the trade pattern arising in a frictional labor market with convex vacancy creation costs. Unlike in Melitz (2003) exporting firms do not grow in order to fully serve foreign demand, rather they react by selling only to a few markets at a higher price. Thus, even if only symmetric countries trade, exporting firms sell – depending on their productivity – to only part of the countries. As a consequence more productive firms export into more countries. This export reaction also implies a different price structure. In contrast to Melitz (2003) more productive exporting firms might charge higher prices in the domestic (and the export) market than less productive non-exporting firms. This result is supported by the empirical findings of Bughin (1996) based on panel-data of Belgian manufacturing firms. He shows that the extent of monopoly power is increased by capacity constraints and allows firms to boost prices.

We also show that additional demand from abroad increases firms’ expected profits and triggers entry of new firms. Unlike in Melitz (2003) the number of active firms increases, because the increase in foreign demand is also meet by additional firms not only by growing firms. This is also well in line with recent empirical findings that suggest that the large share of the adjustment comes from changes of the number of firms, i.e., the extensive margin, and not by adjustments of the amount sold by existing firms, i.e., the intensive margin. However, as in Melitz (2003) opening up to trade still forces less productive firms to leave the market.

While we are among the first to analyze the impact of labor market frictions on trade patterns in a new trade theory model, a large number of papers have analyzed the effect of labor market frictions and institutions on the patterns of trade based on comparative advantages. Brecher (1974) was the first to study minimum wages in the two-country, two-factor, two-good Heckscher-Ohlin model and Davis (1998) generalized

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8Fajgelbaum (2011) departs from the assumption of homogeneous firms in his numerical analysis, obtaining a similar result in a three country example. However, he does not study price and wage effects in the simulation, and assumes in the analytical part that prices stay constant when firms grow. Hence, in the analytical part with homogeneous firms prices are constant.

9Eaton, Kortum, and Kramarz (2004, 2010), for example, find that variation in market share translates nearly completely into firm entry, while about 60 percent of the variation in market size is reflected in firm entry.

By allowing firms with different productivities to pay different wages the on-the-job search model by Burdett and Mortensen (1998) also offers a natural environment to study the effects of trade liberalization on the wage distribution and on unemployment. Empirical findings show that intra-group wage inequality is an important and increasing part of overall inequality (Katz and Autor, 1999; Barth and Lucifora, 2006; Autor, Katz, and Kearney, 2008). In our model, trade liberalization leads to a higher wage dispersion, since search frictions pin down the lowest wage at the level of unemployment benefits, while all other wages depend on the profitability of firms. If trade is liberalized, exporting firms become more profitable and pay higher wages. Thus, wages become more dispersed in a global economy compared to autarky. Increased profits in response to trade liberalization also trigger job creation, which leads to a lower unemployment rate.

The effects of trade liberalization on unemployment and wage inequality have also been analyzed using the Krugman (1979, 1980) and Melitz (2003) model. Helpman, Itskhoki, and Redding (2009, 2010) allow firms to screen workers of different abilities. They find that lower variable trade costs shift the industry composition from low- to high-productivity firms, increase wage inequality and can increase or decrease unemployment. The wage dispersion in their framework arises from the assumed heterogeneity of workers. However, empirical results show that even very similar workers are paid different wages (Abowd, Kramarz and Margolis, 1999; Abowd and Kramarz, 1999). Egger and Kreickemeier (2008) explain intra-group wage inequality among ex ante identical workers due to a fair wage-effort mechanism and suggest that trade liberalization increases profits and can increase unemployment, if wage demands exceed increases in profits. The inequality of wage increases always like in our model. Similarly, Amiti and Davis (2008) assume a fair wage constraint and show that a fall in
output tariffs lowers wages at import-competing firms, but boost wages at exporting firms.

While the general trade pattern of the underlying Melitz (2003) model where all exporting firms serve all foreign markets does not change by introducing search-and-matching labor market frictions, it changes in our approach, since convex vacancy creation costs lead to capacity constraints inducing firms to respond to an increased foreign demand by serving only a subset of foreign markets and increasing prices.

The paper is structured as follows. In the next section we present the general framework that links the new trade model by Melitz (2003) with the on-the-job search model by Burdett and Mortensen (1998). In section 3 we analyze the equilibrium in a closed economy. In section 4 we investigate the effects of trade liberalization and compare the results with the literature focusing particularly on the comparison with Melitz (2003). Section 5 introduces vacancy creation with convex vacancy creation costs. If vacancy creation costs are linear, trade patterns resemble those in Felbermayr, Prat and Schmerer (2011). We then simulate the model with convex vacancy creation costs and show that our main effects prevail. Section 6 concludes and sets out future research objectives.

2 Framework

The model merges the new trade model of Melitz (2003), where firms face monopolistic competition with perfect labor markets, with the on-the-job search model by Burdett and Mortensen (1998), where firms have local monopsony power to set wages. This setup naturally incorporates wage dispersion into the trade model of Melitz (2003) and allows us to study the effects of trade liberalization on the wage distribution. Labor market frictions also change the pattern of trade, since they impose a capacity constraint on firms.
2.1 Labor market and workers’ search strategy

The model has an infinite horizon, is set in continuous time and concentrates on steady states. The measure of firms $M$ in the economy will be endogenously determined in the product market. In the basic framework, we assume that all firms face the same fixed contact rate $\eta \bar{v}$. This is identical to assuming that all firms open the same number vacancies $\bar{v}$ due to infinitely convex vacancy creation costs for $v > \bar{v}$. In section 5 we allow firms with different productivities to decide on the number of vacancies. In the basic model the total number of contacts made by active firms is given by $\eta M \bar{v}$.

Workers are risk neutral and infinitely lived. The measure of workers is normalized to one. Workers can either be unemployed receiving unemployment benefits $z$ or employed at a wage $w$ that might differ across firms. Both unemployed and employed workers are searching for a job with the same intensity. Following Burdett and Mortensen (1998) the probability of a worker to meet a firm follows a Poisson process with the rate $\lambda(M)$. The contact rate of a worker depends on the number of firms $M$ in the market. Since aggregation requires that the total number of firm contacts equals the total number of worker contacts, we get

$$\lambda(M) = \eta M \bar{v}. \tag{1}$$

The contact rate of a worker is therefore increasing in the number of active firms in the economy.

Production starts if a worker accepts the wage offer made by a firm. $F(w)$ denotes the wage offer distribution of employers. The employment relationship ends if either the worker quits to work for a better paying job, which happens at rate $\lambda(M) [1 - F(w)]$, or the worker quits for an exogenous reason, which happens at rate $\kappa$, or the entire firm has to close for an exogenous reason, which happens at rate $\delta$. If the worker quits for an exogenous reason or if the firm closes, workers become unemployed.

Given the wage offer distribution $F(w)$, a worker’s value $U$ of being unemployed, given in equation (2a), consists of unemployment benefits $z$, plus the expected gain from searching. The latter depends on the contact rate $\lambda(M)$ and the surplus of being employed rather than unemployed. The value of being employed $V(w)$ is given by the
current wage plus the expected surplus from finding a better paid job and the expected loss from becoming unemployed, i.e.,

\[ \begin{align*}
    rU &= z + \lambda(M) \int_{w}^{\bar{w}} \max[V(\bar{w}) - U, 0] \, dF(\bar{w}), \\
    rV(w) &= w + \lambda(M) \int_{w}^{\bar{w}} \max[V(\bar{w}) - V(w), 0] \, dF(\bar{w}) + (\kappa + \delta) [U - V(w)]
\end{align*} \tag{2a} \]

\[ \begin{align*}
    rV(w) &= w + \lambda(M) \int_{w}^{\bar{w}} \max[V(\bar{w}) - V(w), 0] \, dF(\bar{w}) + (\kappa + \delta) [U - V(w)], \tag{2b}
\end{align*} \]

where \( r \) is the interest rate with which workers discount future payments, \( w \) the lowest and \( \bar{w} \) the highest wage paid in the economy.

As shown by Mortensen and Neumann (1988) the optimal search strategy for a worker is characterized by a reservation wage \( w^r \), where an unemployed worker is indifferent between accepting or rejecting a wage offer, i.e., \( U = V(w^r) \). Using the above value functions it is straightforward to show that \( w^r \) is independent of \( F(w) \) and given by \( w^r = z \). Thus, only wages that are at least as high as unemployment benefits \( z \) are acceptable for unemployed workers. Equivalently, employed workers will only change employers if the wage \( \tilde{w} \) offered by the outside firm exceeds the current wage \( w \).

### 2.2 Product market and firms’ decisions

Firms are risk neutral and the life of a firm is exponentially distributed with parameter \( \delta \). Following Ethier (1982), Ludema (2002), Melitz (2003) and Helpman and Itskhoki (2010) the final output is a normalized CES-aggregate\(^{10}\), i.e.,

\[ Y = q_0 + \frac{1}{\rho} \left[ \int_{\omega \in \Omega} q(\omega)^\rho d\omega \right], \text{ where } 0 < \rho < 1. \tag{3} \]

The measure of the set \( \Omega \) equals the mass of available intermediate goods, and \( q_0 \) is the outside good serving as numéraire. Each intermediate good \( \omega \) is produced by a single firm in a monopolistic competitive market. Thus, the mass of intermediate goods producers is equal to the number of active firms \( M \) in the market.

We assume perfect competition in the final goods market. Profit maximization of competitive final goods producers leads to the following demand for intermediate good

\(^{10}\)The numéraire good \( q_0 \) in the production technology in (3) absorbs all changes in aggregate demand.
\[ q(\omega) = p(\omega)^{-1/\theta}. \]

(4)

\(l(\omega)\) is the unique factor of production. Firms differ in labor productivity such that the output of a firm that produces intermediate good \(\omega\) is given by \(q(\omega) = \varphi(\omega) l(\omega)\), where \(\varphi(\omega)\) denotes the labor productivity of intermediate input producer \(\omega\). As it is standard in the literature we use \(\varphi\) to index intermediate input producers. The productivity \(\varphi\) is drawn from a continuous distribution with c.d.f \(\Gamma(\varphi)\) and p.d.f \(\gamma(\varphi)\) and support \([\varphi, \bar{\varphi}]\).

The size of the labor force employed by a firm with productivity \(\varphi\) depends on its wage \(w(\varphi)\), since workers search on-the-job for higher wages, i.e.,

\[
\frac{\partial l(w(\varphi))}{\partial w(\varphi)} = \eta \tilde{v} [u + (1 - u) G(w(\varphi))] - [\varphi + \lambda(M) [1 - F(w(\varphi))] ] l(w(\varphi)).
\]

(5)

The number of workers recruited depends on the matching probability \(\eta \tilde{v}\) and on the probability that a contacted worker is willing to work for the wage \(w(\varphi)\). If the wage \(w(\varphi)\) exceeds the value of leisure \(z\), then all unemployed workers \(u\) and all employed workers \((1 - u)\) that currently earn a lower wage, which happens with probability \(G(w(\varphi))\), will accept it. \(G(w(\varphi))\) denotes the fraction of workers earning a wage \(w(\varphi)\) or below and is, therefore, equivalent to the cumulative wage earnings distribution.

Unlike in the competitive labor market assumed in Melitz (2003) firms are not able to adjust their labor input freely to produce the output they want. The size of a firm’s labor force \(l(w(\varphi))\) and consequently the output a firm produces is determined by the firm’s position in the wage offer and earnings distribution, \(F(w(\varphi))\) and \(G(w(\varphi))\), respectively.

Following Burdett and Mortensen (1998) we assume for simplicity that the discount rate is set to zero. Thus, the firm closure rate \(\delta\) acts as discount rate. The firms’ optimization problem is equivalent to maximizing steady state profit flows\(^{11}\), i.e.,

\[
\delta \Pi(\varphi) = \max_w [p(\varphi) q(\varphi) - w(\varphi) l(w(\varphi)) - f],
\]

(6)

\(^{11}\text{Coles (2001) and Moscarini and Postel-Vinay (2010) analyze the out of steady state dynamics of the Burdett-Mortensen model and show that the position of a firm within the wage distribution equals the position in the productivity distribution even if firm size is out of steady state.}\)
where \( p(\varphi) \) denotes the price that the firm will charge in the monopolistic competitive market and the fixed costs \( f \) reflects a per period cost component that is required to serve consumers. It is assumed to be identical for all firms.

Given its expected profit a firm decides whether it will enter the market depending on its productivity \( \varphi \). A firm will start to produce if profits are positive, i.e., \( \delta \Pi(\varphi) \geq 0 \). The cutoff productivity \( \varphi^* \) is therefore defined by,

\[
\delta \Pi(\varphi^*) = 0.
\]

(7)

Thus, only firms with productivity \( \varphi \in [\varphi^*, \bar{\varphi}] \) will be active in the market, assuming that \( \varphi^* \geq \varphi \).

Since all active firms – except the cutoff firm – make positive profits, the expected discounted profit of a firm before knowing its productivity type \( \varphi \) is also positive, i.e.,

\[ \Pi_e = [1 - \Gamma(\varphi^*)] \bar{\Pi} = [1 - \Gamma(\varphi^*)] \int_{\varphi^*}^{\bar{\varphi}} \Pi(\varphi) \frac{\gamma(\varphi)}{1 - \Gamma(\varphi^*)} d\varphi > 0, \]

where \( 1 - \Gamma(\varphi^*) \) equals the probability that a firm will draw a productivity level \( \varphi \) high enough to ensure that production is profitable. \( \bar{\Pi} \) equals the average discounted profit of all active firms.

Before a firm gets to know its productivity, it has to pay the fixed investment cost \( f_e \). Free entry of firms ensures that firms enter the market until the expected discounted profit before entering the market equals the fixed investment cost \( f_e \), i.e.,

\[ [1 - \Gamma(\varphi^*)] \bar{\Pi} = \int_{\varphi^*}^{\bar{\varphi}} \Pi(\varphi) \gamma(\varphi) d\varphi = f_e. \]

(8)

The zero-cutoff condition (7) and the free entry condition (8) determine the number of active firms \( M \) and the labor productivity \( \varphi^* \) of the active firm with the lowest productivity in an economy.

### 2.3 Aggregation and steady state conditions

Aggregation requires that total output produced per period equals the payments made by firms, i.e.,

\[
Y = q_0 + M (1 - u) \int_{\varphi^*}^{\bar{\varphi}} w(\varphi) dG(w(\varphi)) + M \delta \bar{\Pi}.
\]

(9)
Thus, wage payments plus aggregate profit per period have to equal total value of output produced each period. Aggregate profits are used to finance the fixed investment cost of new incumbent firms that attempt entry, i.e.,

\[ M \delta \bar{\Pi} = f_e M_e, \]  

where \( M_e \) is the total mass of firms that attempt entry and pay the fixed investment costs \( f_e \) each period. A large unbounded set of potential new firms ensures an unlimited supply of potential entrants that is able to replace firms that go out of business. Steady state requires that the flow into the pool of active firms is equal to the flow out of this pool, i.e.,

\[ [1 - \Gamma (\varphi^*)] M_e = \delta M. \]  

It is straightforward to show that the free entry condition (8) guarantees that steady state conditions (10) and (11) hold.

In steady state in- and outflows into and out of employment offset each other such that the unemployment rate and distribution of employment over firms are stationary. Equating the flows in and out of unemployment gives the steady state measure of unemployed, i.e.,

\[ u = \frac{\kappa + \delta}{\kappa + \delta + \lambda (M)}. \]  

Noting that the aggregate number of matches in the economy depends on the number of active firms \( M \) in the economy as stated in equation (1), the unemployment rate decreases if the number of active firms in the economy increases.

Equating the inflow and outflow into the group of workers employed at a wage \( w (\varphi) \) or less gives the steady-state measure of employed workers earning a wage less than \( w (\varphi) \), i.e.,

\[ \lambda (M) F (w (\varphi)) u = G (w (\varphi)) (1 - u) [\kappa + \delta + \lambda (M) [1 - F (w (\varphi))] \]  

\[ \Rightarrow G (w (\varphi)) = \frac{(\kappa + \delta) F (w (\varphi))}{\kappa + \delta + \lambda (M) [1 - F (w (\varphi))]. \]  

Labor market frictions constrain the number of workers a firm can recruit and restrict the number of workers a firm can employ. This is the reason why we call these labor market frictions capacity constraining. Using equation (5), the aggregate matching
condition (1), the steady state measure of unemployment (12) and employment (14) allows us to write the steady state labor force of a firm paying the wage \( w(\phi) \) as

\[
l(w(\phi)) = \frac{\eta \bar{v}}{[\kappa + \lambda(M)[1 - F(w(\phi))][\kappa + \delta + \lambda(M)[1 - F(w(\phi))]]^{\kappa + \delta}}.
\] (15)

Like in Burdett and Mortensen (1998) equation (15) implies that the size of a firm’s labor force \( l(w(\phi)) \) is increasing in the wage \( w(\phi) \), since on-the-job search leads to the fact that a high wage firm attracts more employed workers from firms paying lower wages and loses less workers to employers paying higher wages. Equation (15) also shows that a higher number of active firms \( M \) results in additional competition between firms and decreases the size of each firm’s labor force. As shown in section 5, this effect is still present but does not necessarily dominate if we endogenize the recruiting rate \( \eta \bar{v} \), i.e., if we allow firms to grow by posting vacancies.

3 Equilibrium in a closed economy

3.1 Equilibrium definition

We start by defining the product and labor market equilibrium in a closed economy. A labor market equilibrium is characterized by the contact rate of unemployed workers, the unemployment rate, the wage offer and wage earnings distribution, i.e., the set \( \{\lambda(M), u, F(w(\phi)), G(w(\phi))\} \). Firms offer wages \( w(\phi) \) that maximize profits given their productivity \( \phi \), the demand function they face in the monopolistic competitive market \( q(\phi) = p(\phi) \frac{1}{1 + \rho} \), the wage offer distribution \( F(w(\phi)) \) posted by other firms and the optimal search strategy of workers, i.e., \( w^r = \bar{v} \). The contact rate of unemployed workers \( \lambda(M) \), the unemployment rate \( u \) and the wage earnings distribution \( G(w(\phi)) \) in equations (1), (12) and (14) have to be consistent with steady state turnover given the productivity distribution \( \Gamma(\phi) \), the cutoff productivity \( \phi^* \) and the wage offer distribution \( F(w(\phi)) \) posted by firms.

A product market equilibrium is a set \( \{\phi^*, M\} \) such that intermediate good producers enter the product market if their productivity ensures a positive profit, i.e., \( \phi \geq \phi^* \), where the zero-cutoff productivity has to satisfy equation (7). The number of active
firms $M$ in the product market has to be consistent with profits of active firms being used to finance the fixed investment cost $f_e$ of potential new market entrants necessary to replace the firms exiting the product market. Thus, the number of active firms $M$ has to satisfy the free entry condition (8).

### 3.2 Firms’ wage offers

Since each firm has some monopoly power in the product market and some monopsony power in the labor market, a firm will choose its wage offer such that the marginal revenue of increasing labor input with a higher wage is offset by the additional wage cost. The optimality condition for a firm with productivity $\phi$, given the distribution of wages offered by all other firms $F(w(\phi))$, is given by

$$
\frac{\partial \delta \Pi(\phi)}{\partial w(\phi)} = \left[ \phi^\rho \rho l(w(\phi))^{(\rho-1)} - w(\phi) \right] \frac{\partial l(w(\phi))}{\partial w(\phi)} - l(w(\phi)) = 0.
$$

(16)

As in Mortensen (1990) more productive firms will pay higher wages\(^{12}\), if the marginal revenue is higher than the wage, i.e., $\phi^\rho \rho l(w(\phi))^{(\rho-1)} - w(\phi) > 0$ (See Appendix A).

If the marginal product is lower than the wage, firms will reduce their wage in order to reduce their labor force $l(w(\phi))$ and to increase their marginal revenue. Thus, the optimality condition (16) is only satisfied for all active firms with productivity $\phi \geq \phi^*$ if the marginal revenue of the least productive firm $\phi^*$ is higher than the level of unemployment benefits $z$. We therefore assume:

**Assumption 1:** The marginal revenue of the least productive firm $\phi^*$ is higher than the level of unemployment benefits $z$, i.e.,

$$
\rho [\phi^*]^{\rho} \left[ \frac{\eta \bar{v}}{\kappa + \lambda(M) \kappa + \delta + \lambda(M)} \right]^{\rho-1} > z.
$$

(17)

If Assumption 1 is violated, the wage distribution will be characterized by a mass point at the level of unemployment benefits, since firms with a marginal revenue below $z$ will not find it optimal to increase their labor input by increasing wages (see Appendix A).

\(^{12}\)This result only holds because more productive firms generate a higher marginal revenue. Fiedler (2010) shows in a setting with monopolistic competition that this need not be the case if firms with different productivities face different demand curves.
Given Assumption 1 wages $w(\phi)$ increase with productivity $\phi$ like in Mortensen (1990). Thus, the wage offer distribution $F(w(\phi))$ has to satisfy,

$$F(w(\phi)) = \frac{\Gamma(\phi) - \Gamma(\phi^*)}{1 - \Gamma(\phi^*)} \text{ for all } \phi \in [\phi^*, \bar{\phi}]. \quad (18)$$

The position of a firm in the wage offer distribution $F(w(\phi))$ is equivalent to its position in the productivity distribution of active firms. Thus, the position of a firm in the productivity distribution of active firms determines a firm’s labor input. The fact that a firm cannot adjust the size of its labor force freely to changes in output demand leads to a capacity constraint that implies that firms will adjust their output price to reflect demand changes.

Since a firm can only recruit workers, if it pays at least a wage that equals the level of unemployment benefits $z$, the least productive firm that is active in the market will offer exactly unemployment benefits $z$. The optimal wage $w(\phi)$ posted by a firm with productivity $\phi > \phi^*$ is given by,

$$w(\phi) = \frac{1}{l(w(\phi))} \left[ [\rho l(w(\phi))] \phi^\rho - \int_{\phi^*}^{\phi} \frac{\rho}{\phi} [\rho l(w(\bar{\phi}))]^\rho d\bar{\phi} - f \right]. \quad (19)$$

The derivation can be found in Appendix B. Multiplying equation (19) by $l(w(\phi))$ reveals that total wage payments are given by revenues (the first term on the rhs in brackets) minus total profits of a firm with productivity $\phi$ (the second term on the rhs in brackets), minus fixed costs. Note that in contrast to Melitz (2003), total profits are a function of wages, which reflects the fact that the firms have not only monopoly power on the product market but also monopsony power on the labor market.

### 3.3 Firm entry decision

Free entry of potential firms ensures that the expected discounted profit from entering the product market $[1 - \Gamma(\phi^*)] \bar{\Pi}$ equals the fixed investment cost $f_e$ as stated in equation (8). Substituting per period profit (6) and the optimal wage (19) implies the following free entry condition,

$$f_e = \frac{1}{\delta} \int_{\phi^*}^{\bar{\phi}} \left[ \int_{\phi^*}^{\phi} \frac{\rho}{\phi} [\rho l(w(\bar{\phi}))]^\rho d\bar{\phi} \right] \gamma(\phi) d\phi. \quad (20)$$
The expected discounted profit decreases with the number of active firms \( M \), because the size of a firm’s labor force \( l(w(\varphi)) \) is a decreasing function of the number of active firms in the labor market. At the same time the expected discounted profit increases if the cutoff productivity decreases, because the likelihood of having a productivity draw that is sufficiently high to make profits increases. Using the implicit function theorem, we show in Appendix C that the free entry condition defines a decreasing relation between the zero-cutoff productivity \( \varphi^* \) and the number of active firms \( M \) in the market.

\[
\begin{align*}
\phi^* & = z \eta \bar{v} + \frac{\phi^* \eta \bar{v}}{\phi^* + \eta M \bar{v}} \left( \frac{\phi^* + \delta}{\phi^* + \eta M \bar{v}} + \delta + \eta M \bar{v} \right) + f.
\end{align*}
\] (21)

Since the zero-cutoff productivity firm pays the wage \( z \), it will only attract unemployed workers and lose its workers to all other firms that pay higher wages. Consequently, a higher number of active firms \( M \) increases the number of quits at the zero-cutoff

![Figure 1: Number of firms and cutoff productivity](image-url)
productivity firm and, therefore, reduces its steady state labor input. This decreases the firm’s net revenue. The firm will subsequently no longer be able to cover the wage payments and the fixed cost $f$. Thus, only more profitable firms will be able to survive in the market, which increases the zero-cutoff productivity. Using the implicit function theorem and Assumption 1 we show in Appendix C that the zero profit condition defines an increasing relation between the zero-cutoff productivity $\varphi^*$ and the number of active firms $M$ in the market. Thus, the free entry condition and the zero-cutoff condition determine a unique equilibrium as shown in Figure 1, as long as unemployment benefits $z$ and fixed cost $f$ are low enough to ensure that an equilibrium exists.

4 Open economy

Assume that there are $n+1$ identical countries that differ only in the variety $\Omega$ of goods that they produce. Given that final output producers love variety, they are interested in trading with other countries. Due to the symmetry of countries, intermediate goods producers face the same demand curve in the export market as they face in the domestic market, i.e., $q(\varphi) = p(\varphi)^{1/(\rho-1)}$. Serving one export market involves some fixed cost $f_x \geq f$ per period and some proportional shipping costs per good shipped to the export market. Thus, the price of an export good at the factory gate is given by $p_x(\varphi) / \tau = p_d(\varphi)$, where $p_d(\varphi)$ denotes the price in the domestic market.

Given that an exporting firm with productivity $\varphi$ can only produce the fixed output $\varphi l(w(\varphi))$, it will chose the number of export markets $j$ such that the output sold in $j$ export markets and the domestic market maximizes revenues. Splitting the output for all export markets is not profit-maximizing given the capacity constraints and exporting fixed costs. Thus, a firm that decided to serve a subset $j \leq n$ of foreign markets maximizes its revenue $p_d(\varphi) q_d(\varphi) + j p_x(\varphi) q_x(\varphi)$ if it sells

$$q_d(\varphi) = \frac{1}{1 + j \tau^{\rho/(\rho-1)} q(\varphi)}, \quad \text{and}$$

$$q_x(\varphi) = \frac{\tau^{\rho/(\rho-1)}}{1 + j \tau^{\rho/(\rho-1)} q(\varphi)},$$

at the domestic and at each export market, respectively (see derivation in Appendix
D). The revenue of a firm serving $j$ export markets is therefore given by

$$\delta \Pi_{d+j}(\varphi) = \max_w \left[ 1 + j \tau \rho \left( 1 - \rho \right) \right]^{(1-\rho)} \left[ \varphi f (w(\varphi)) \right]^\rho - w(\varphi) l (w(\varphi)) - f - j f_x. \tag{24}$$

In addition to the closed economy a firm with productivity $\varphi$ decides not only on the wage $w(\varphi)$ but also on the number $j$ of countries it wants to export to. Hence, it will choose the number of export markets such that profits are maximized, i.e.,

$$\Pi^{\text{max}}(\varphi) = \max_j \Pi_{d+j}(\varphi).$$

## 4.1 Equilibrium characterization and trade pattern

Denote by $\varphi_x^j$ the export cutoff productivity for a firm that decides to export to $j \leq n$ countries. Firms with $\varphi \geq \varphi_x^j$ find it optimal to export to $j$ or more countries while firms with $\varphi < \varphi_x^j$ will only serve less than $j$ foreign markets and the domestic market (or only the domestic market). Wages chosen by firms have to satisfy the first order condition like in a closed economy. The non-exporting firm with the lowest productivity level $\varphi^*$ will pay the reservation wage $z$ such that unemployed workers are willing to start working. As shown in Appendix D the wage equation $w(\varphi)$ for exporting firms is given by,

$$w(\varphi) = \frac{1}{l(w(\varphi))} \left[ 1 + j \tau \rho \left( 1 - \rho \right) \right]^{(1-\rho)} \left[ \varphi f (w(\varphi)) \right]^\rho - f - j f_x \tag{25}$$

where $\varphi_x^{j+1} = \varphi$ and $\varphi_x^0 = \varphi^*$.

Note that profit maximization ensures that the wage function does not jump upward at $\varphi_x^j$, i.e., that the support of the wage distribution is connected. To see this suppose the opposite, i.e., that the exporting firm with the lowest productivity $\varphi_x^j$ were to pay a wage $w(\varphi_x^j) = w(\varphi) + \Delta$, where $\Delta > 0$ denotes the jump at $w(\varphi)$ where productivity is given by $\varphi = \varphi_x^j - \varepsilon$ for any small $\varepsilon > 0$. The wage jump does not increase the number of workers of the export-cutoff productivity firm since it has the same position in the wage distribution as before. It is, therefore, optimal for the firm to pay a wage that is only slightly above $w(\varphi)$ and save the wage costs $\Delta$ per worker. Thus, the wage function has to be continuous on $[\varphi^*, \varphi]$. 

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For low productive firms it is optimal to serve only the domestic market, while more productive firms will export. At the export-cutoff productivity $\varphi_x^j$ the firm is indifferent between serving $j$ export markets and the domestic market or serving $j - 1$ export markets and the domestic market, i.e., $\delta \Pi_{d+j} (\varphi_x^j) = \delta \Pi_{d+j-1} (\varphi_x^j)$. As proven in Appendix C more productive firms will export to more countries. Specifically, the export cutoff productivity is given by

$$\varphi_x^j = \frac{f_x}{\left[1 + j\tau\rho/(\rho - 1)\right]^{(1-\rho)} - \left[1 + (j - 1)\tau\rho/(\rho - 1)\right]^{(1-\rho)}}.$$  (26)

**Proposition 1** The number of export markets $j \leq n$ served by a firm is increasing in its productivity, i.e., the export cutoff productivity $\varphi_x^j$ is increasing in $j$.

**Proof.** See Appendix D. ■

This result is driven by the capacity constraint that firms face in a frictional labor market with convex vacancy creation costs. Given that expanding output is very costly, it is not optimal to serve all markets at the same time, but to concentrate on a few markets, charge in each market a higher price and thus maximize revenue given the capacity constraint.

Eaton, Kortum, and Kramarz (2004, 2010) find that most exporting firms sell to only one foreign market, with the frequency of firms’ selling to multiple markets declining with the number of destinations. They explicitly relate the number of countries served to export destinations characteristics. Firms selling to only a small number of markets sell only to the most popular destination countries. Only the most productive firms sell to less popular markets. However, Eaton, Kortum, and Kramarz (2004, 2010) are not able to explain why firms with similar products that serve only one foreign market export to different countries. Our model can explain such trade patterns with capacity-constraining labor market frictions and is therefore able to account for some variation in trade patterns that cannot be explained by export destination characteristics.\(^{13}\)

\(^{13}\text{As we assume perfectly symmetric countries, we only determine the number of countries, but not to which countries a firm exports. This indeterminacy could be easily solved by allowing for heterogeneous exporting fixed costs.}\)
As illustrated in Figure 2, the zero cutoff condition (21) is unchanged by opening up to trade, since the lowest productivity firm will pay the reservation wage \( z \) and only sells at the domestic market. However, the free entry decision changes if the economy opens up to trade, since average profits increase due to the additional foreign demand,

\[
f_e = \frac{1}{\beta} \int \frac{\varphi^{j+1}}{\varphi^*} \left[ 1 + (i - 1) \tau \right] \left( \varphi^{j+1} \right)^{(1-\rho)} \int_{\varphi^{j+1}}^{\varphi^*} \frac{\rho}{\varphi} \left[ \tilde{\varphi} \left( w (\tilde{\varphi}) \right) \right]^\rho \tilde{\varphi} \gamma (\varphi) d\varphi, \tag{27}
\]

where \( \varphi^{j+1} = \varphi \) and \( \varphi^0 = \varphi^* \). The derivation of equation (27) is given in Appendix D.

4.2 Firm structure and prices

The higher expected profit in an open economy given in equation (27) compared to the closed economy given in equation (20) triggers entry and increases the number of active firms \( M \) in the economy for a given cutoff productivity \( \varphi^* \), i.e., the free entry condition curve rotates upward as shown in Figure 2. Given the increased number of active firms in the economy, potential entrants realize that their labor force will be lower than in the closed economy and that they will not be able to produce enough to pay the per period fixed cost \( f \). Thus, the zero-cutoff productivity increases and low productivity firms do not enter the market.

As stated in Proposition 2 the move from the steady state equilibrium in autarky to the steady state in an open economy results in contrast to Melitz (2003) in a higher number of active firms \( M \) on the domestic labor market. While in a perfect labor market as in Melitz (2003) exporting firms grow in order to meet foreign demand, in the frictional labor market firms face a capacity constraint, which allows additional firms to enter the market and to meet the foreign demand.

**Proposition 2** Given Assumption 1, the zero-cutoff productivity \( \varphi^* \) and the number of active firms \( M \) in an open economy is higher than in autarky. The size of all firms \( l \left( w (\varphi) \right) \) decreases.

**Proof.** See Appendix D. ■

The monopsonistic labor market changes firms reactions to trade liberalization compared to the reaction of firms in a frictionless labor market like in Melitz (2003). In
a perfect labor market exporting firms increase labor input until their marginal product is reduced to equal the market wage. The higher demand for labor by exporting firms is meet at the cost of a lower labor input at non-exporting firms. In a frictional labor market without vacancy creation the size of a firm’s labor force is determined by the position of a firm in the wage offer distribution. Thus, exporting firms are not able to increase their output, since their labor input is given by their position in the wage distribution. Their position in the wage distribution decreases because the cutoff-productivity increases. Hence, opening up to trade will decrease a firm’s labor force. In addition it triggers entry of new firms that compete for the same number of workers reducing the number of workers per firm even further. In contrast to Melitz (2003), the increased foreign demand therefore leads to entry of additional firms and not to growth of existing exporting firms. Figure 3 shows the firm size reactions in a frictional labor market and compares it to the perfect labor market environment of Melitz (2003). This result is specific to the simple case of no vacancy creation. In section 5, where we allow for vacancy creation, highly productive exporting firms will grow while less productive firms will shrink.

Since exporting firms find it very costly to increase their output in response to the increase in foreign demand, they respond to the increased demand by increasing their
prices. The prices charged by exporting firms in the domestic market, i.e., without the shipping cost, are no longer lower for exporting firms compared to domestic firms like in Melitz (2003). As Figure 4 suggests, they are in the same range as the prices of domestic firms. The exact relation depends on the quantities of output sold as stated in the following Proposition.

**Proposition 3** Given Assumption 1, the highest domestic price of firms that export to \(0 \leq j \leq n\) countries is higher than the highest price of firms exporting to \(j - 1\) countries if and only if

\[
\frac{\varphi_j^{j-1}I(w(\varphi_j^{j-1}))}{1 + (j - 1)\frac{\tau}{\rho/(\rho-1)}} > \frac{\varphi_j^{j}I(w(\varphi_j^{j}))}{1 + j\frac{\tau}{\rho/(\rho-1)}},
\]

where \(\varphi_x^0 = \varphi^*\) and \(w(\varphi_x^0) = z\).

**Proof.** The firm that charges the highest price of all firms exporting to \(j\) countries is the firm with export-cutoff productivity \(\varphi_x^j\). It produces and sells the smallest quantity of the good and therefore charges the highest price of all firms exporting to \(j\), i.e.,

\[
\varphi_x^j = \arg \max_{\varphi \in [\varphi_x^j, \varphi_x^{j+1}]} p_d(\varphi).
\]

Due to the downward sloping demand functions we know that \(p_d(\varphi_x^j) < p_d(\varphi_x^{j-1})\) if and only if \(q_d(\varphi_x^j) > q_d(\varphi_x^{j-1})\).

While in the perfect labor market environment prices decrease with productivity, in a frictional environment exporting firms charge similar prices compared to domestic
firms, because capacity constraining labor market frictions induce exporting firms to maximize their revenue by selling only a limited quantity per market.

These findings are well supported by the empirical findings of Bughin (1996). First, Bughin (1996) finds some evidence of increasing returns-to-scale, which clearly rejects the assumption of price-taking behavior and supports the assumption of monopolistic competition in the product market. Second, and more importantly, the extent of monopoly power is increased by capacity constraints and allows firms to boost export prices, which corresponds to the mechanism presented here.

Another explanation for similar domestic prices of non-exporting and exporting firms that the empirical literature has suggested is the higher quality of the goods produced by exporting firms. Fajgelbaum (2011) shows that such a price pattern can arise, if workers can be perfectly substituted between the tasks of producing quantity or quality.

### 4.3 Unemployment and wages dispersion

Opening up to trade increases expected profits, triggers firm entry and reduces the unemployment. Like in Felbermayr, Prat, and Schmerer (2011) additional demand from abroad increases firms’ revenue and their demand for labor. While firms in Felbermayr, Prat, and Schmerer (2011) create additional vacancies in order to increase their
employment, in our frictional environment additional firms enter the market, since the simple framework does not allow them to increase their recruitment rate by opening new vacancies.

In the given context, opening up to trade still leads the lowest productivity firm to pay the level of unemployment benefits \( z \) like in autarky. Since the zero-cutoff productivity increases compared to autarky, i.e., \( \varphi_T^* > \varphi_A^* \), some firms with a productivity \( \varphi \geq \varphi_T^* > \varphi_A^* \) that paid a wage above \( z \) will decrease their wages, since they now occupy a lower position within the wage offer distribution. However, if we hold the position of a firm in the wage distribution constant, trade liberalization increases wages, because the marginal revenue of all firms increases due to the lower number of employees that they are able to recruit. Exporting firms experience even a higher increase in their marginal revenues since they can now charge higher prices by serving not only the domestic but also foreign markets. Thus, two counteracting effects drive wage changes: (i) the positive effect of an increase in the marginal revenue of a firm and, (ii) the negative effect of a lower position in the wage distribution. Of course, the negative effect is zero for the highest productivity firm \( \bar{\varphi} \), such that wages increase at the upper end of the wage distribution. Since wages at the bottom of the wage distribution are held constant by the level of unemployment benefits \( z \), it follows that the dispersion of wages is higher in an open economy than in autarky. The effect on the average wage is ambiguous and depends on the shape of the productivity distribution as well as the job finding and job destruction rate that translate the wage offer distribution into the wage earnings distribution as stated in equation (14). Proposition 4 summarizes the effect of trade liberalization on the unemployment rate and wage dispersion.

**Proposition 4** Given Assumption 1, opening up to trade reduces the unemployment rate \( u \) and increases wage dispersion, i.e., increases \( w(\bar{\varphi}) - z \), compared to autarky.

**Proof.** See Appendix E. ■

The results concerning the effects of trade liberalization on the wage distribution of ex-ante identical workers are similar to the papers by Egger and Kreickemeier (2008), Amiti and Davis (2008) and Helpman, Itskhoki, and Redding (2009, 2010). However,
in our context wage inequality is not the result of exogenously given fair-wage preferences\textsuperscript{14} or the result of monitoring or screening costs, but rather the result of continuous search for better jobs of workers, as introduced in Burdett and Mortensen (1998).

Building on the two-sector, two-country model by Helpman and Itskhoki (2010), Helpman, Itskhoki, and Redding (2009, 2010) allow in addition that workers differ according to an exogenously given ability. Firms engage into costly screening of their potential workers’ abilities. Lower variable trade costs and higher foreign labor market frictions shift the industry composition from low- to high-productivity firms. As more productive firms are more selective, wage inequality increases, since ability complementarities increase a firm’s productivity. While the wage inequality result in their paper is similar to our approach, the unemployment result can differ, since we abstract from the screening effect and its negative effect on unemployment.

The literature on the fair wage approach, such as Egger and Kreickemeier (2008, 2009), finds that trade liberalization increases firms’ profits of all firms but improves the relative position of less productive firms in relation to their more productive com-

\textsuperscript{14}Whereas in Egger and Kreickemeier (2009) fair-wage preferences are linked to productivity differences between firms, they are based on profits of firms in Egger and Kreickemeier (2008) and Amiti and Davis (2008).
petitors, since they have to pay higher wages due to the fair wage constraint. Thus, unemployment can increase, if wage demands due to fair-wage preferences dominate the increase in profits. Trade liberalization also increases wage inequality. The mechanism is in line with our approach and driven by increased profitability of exporting firms. Similar to Egger and Kreickemeier (2008) Amiti and Davis (2008) assume a fair wage constraint and show that a fall in output tariffs lowers wages at import-competing firms, but boost wages at exporting firms.

5 Vacancy creation in an open economy

5.1 The matching technology

In previous sections the analysis was based on the assumption that all firms have a constant recruitment rate $\eta \bar{v}$ and cannot expand their production by opening new vacancies in response to an increase in foreign demand. In this section we allow firms to influence their contact rate by posting vacancies like in Mortensen (2003). The contact rate of a firm with productivity $\varphi$ depends on the number of vacancies $v(\varphi)$ and is given by $\eta v(\varphi)$. The total number of contacts in an economy (and the contact rate of workers) is therefore given by

$$\lambda (M \bar{v}) = \eta M \int_{\varphi^*}^{\bar{\varphi}} \frac{v(\varphi)}{1 - \Gamma (\varphi^* )} d\Gamma (\varphi) = \eta M \bar{v}.$$  

The per period cost of vacancy creation is an increasing function of the vacancies opened, i.e., $c(v) = \xi v(\varphi)^\alpha$. This cost function allows us to compare our results with the case of constant vacancy creation cost, $\alpha = 1$, like in Felbermayr, Prat and Schmerer (2011), who link the Pissarides (2000) model with the Melitz (2003) model.

5.2 Labor market and trade pattern

In an open economy a firm with productivity $\varphi$ chooses its wage $w(\varphi)$ and its number of vacancies $v(\varphi)$ such that per period profits are maximized for a given number of
export markets $j$, i.e.,

$$
\delta \Pi_{d+j} (\varphi) = \max_{w,v} \left[ \left[ 1 + j \tau \varphi \right]^{(1-\rho)} \left[ \varphi l (\varphi, v) \right]^{\rho} - w (\varphi) l (\varphi, v) - \frac{\zeta}{\alpha} v^\alpha - f - j f_x \right].
$$

s.t. $l (\varphi, v) = \eta v \left( \zeta + \lambda (M \tilde{v}) \right) \left[ 1 - F (w (\varphi)) \right] \left( \zeta + \delta + \lambda (M \tilde{v}) [1 - F (w (\varphi))] \right).$ (29)

The number of employees $l (\varphi)$ working for a firm with productivity $\varphi$ increases proportionally with the number of vacancies like in Mortensen (2003) and with the wage like in Burdett and Mortensen (1998). Thus, firms can increase their labor input by increasing their wage and by opening more vacancies.

As long as the marginal revenue of a firm is higher than its wage, i.e., as long as Assumption 1 holds, more productive firms will pay higher wages. The reason is the same as in the original Burdett-Mortensen model. If a measure of firms pays the same wage, paying a slightly higher wage only marginally increases the cost per worker, while the additional revenue generated by the significantly higher labor force increases profits significantly. Thus, more productive firms will pay higher wages.

Because the contact rate between a worker and a specific firm is proportional to the number of vacancies posted by the firm and because wage offers are increasing in the productivity of a firm, the wage offer distribution is the vacancy weighted distribution of productivities, i.e.,

$$
F (w (\varphi)) = \frac{\int_{\varphi}^{\varphi^*} v (\tilde{\varphi}) d \Gamma (\tilde{\varphi})}{\int_{\varphi}^{\varphi^*} v (\tilde{\varphi}) d \Gamma (\tilde{\varphi})}. (30)
$$

Firms choose wages such that the resulting increase in labor balances marginal revenue with marginal labor cost. The number of vacancies are chosen such that the marginal net revenue generated by the last opened vacancy equals the marginal cost of creating the vacancy. The optimality conditions for wages and vacancies are therefore given by

$$
\rho \left[ 1 + j \tau \varphi \right]^{(1-\rho)} \varphi^\rho l (\varphi, v)^{(\rho-1)} - w (\varphi) \frac{\partial l (\varphi, v)}{\partial v} = cv^{\alpha-1}, (31)
$$

$$
\rho \left[ 1 + j \tau \varphi \right]^{(1-\rho)} \varphi^\rho l (\varphi, v)^{(\rho-1)} - w (\varphi) \frac{\partial l (\varphi, v)}{\partial \varphi} = l (\varphi, v) \frac{\partial w (\varphi)}{\partial \varphi}, (32)
$$

where the differential equation (32) follows from the fact that more productive firms pay higher wages. Substituting $F (w (\varphi))$ according to equation (30) and (28) in equation
(29) yields,
\[
l(\varphi, v) = \frac{\eta v}{\left[\kappa + \frac{\eta M}{1 - \Gamma(\varphi)}\right]} \left[\kappa + \delta + \frac{\eta M}{1 - \Gamma(\varphi)}\right].
\]
(33)

Inserting into the first order conditions implies the following first order differential wage equation, i.e.,
\[
\frac{\partial w(\varphi)}{\partial \varphi} = \left[p \left[1 + j\tau \varphi^{-1}\right]^{(1-\rho)} \varphi^\rho l(\varphi, v)^{(\rho-1)} - w(\varphi)\right] \frac{\partial l(\varphi, v)}{\partial \varphi} \frac{1}{l(\varphi, v)},
\]
with the terminal condition \(w(\varphi^*) = z\).

The number of vacancies created by the firm is implicitly defined by the vacancy creation condition (31), where the wage \(w(\varphi)\) is given by solution to the differential equation (34). The average number of vacancies \(\bar{v}\) per active firm is obtained by integrating the vacancies created by active firms, i.e.,
\[
\bar{v} = \int_{\varphi^*}^{\bar{\varphi}} \frac{v(\varphi)}{1 - \Gamma(\varphi^*)} d\Gamma(\varphi).
\]
(35)

The number of export countries a firm is willing to sell its products to depend like in the simple model on the comparison of profits from exporting to \(j\) or \(j - 1\) countries, i.e.,
\[
\Pi_{d+j}(\varphi) \gtrless \Pi_{d+j-1}(\varphi).
\]
(36)

5.3 Product market

The product market equilibrium is defined by two conditions, the free entry condition that determines the number of active firms in the economy \(M\) given the vacancy creation decision in the labor market that determines the average number of vacancies \(\bar{v}\) per active firm and the zero-cutoff productivity condition that determines the productivity level \(\varphi^*\) that guarantees non-negative profits.

Firms only enter the market, if the profits they are able to generate are positive. Using the vacancy creation condition (31) one can write total profits of a firm serving the home market and \(j\) export markets as,
\[
\delta \Pi_{d+j}(\varphi) = (1 - \rho) \left[1 + j\tau \varphi^{-1}\right]^{(1-\rho)} [\varphi l(\varphi)]^\rho + \left(1 - \frac{1}{\alpha}\right) cv(\varphi)^\alpha - f - jf_x.
\]
(37)
Since the firm with the lowest productivity pays a wage equal to unemployment benefits, the zero-cutoff productivity \( \phi^* \), defined as \( \delta \Pi_d (\phi^*) = 0 \), is given by the solution to the system of two equations determining the zero-cutoff productivity \( \phi^* \) and the number of vacancies \( v(\phi^*) \) created by the zero-cutoff productivity firm, i.e.,

\[
(1 - \rho) [\phi^* l(\phi^*)]^p + \left( 1 - \frac{1}{\alpha} \right) cv(\phi^*){\alpha} = f, \tag{38}
\]

\[
\rho [\phi^* l(\phi^*)]^p - zl(\phi^*) = cv(\phi^*){\alpha}. \tag{39}
\]

The labor force size of the zero-cutoff productivity firm is according to equation (29) given by

\[
l(\phi^*) = \frac{\eta v(\phi^*) (\kappa + \delta)}{[\kappa + \eta M \hat{v}] [\kappa + \delta + \eta M \hat{v}]}.
\]

The free entry condition ensures that the profits generated by all firms are used to pay the investment cost \( f_e \) of potential market entrants. The expected discounted profit of exporting and non-exporting firms can be written as follows,

\[
f_e = \int_{\phi^*}^{\hat{\phi}} \Pi_{d,\text{max}} (\phi) \gamma(\phi) d\phi, \tag{40}
\]

where \( \Pi_{d,\text{max}} (\phi) = \max_i \Pi_d (\phi_i) \) denotes the maximum profits attainable by a firm with productivity \( \phi \).

### 5.4 The case of linear vacancy creation costs (\( \alpha = 1 \))

If vacancy creation costs are linear, i.e., \( \alpha = 1 \), the vacancy creation condition reveals that firms increase their number of vacancies such that the marginal revenues are equalized across productivity levels (derivation in Appendix F), i.e.,

\[
\rho \left[ 1 + j \tau \frac{\sigma - 1}{p - 1} \right] (1 - \rho) \phi^l(\phi, j)^{p-1} = cv(\phi^*) \frac{v(\phi^*)}{l(\phi^*)} + z. \tag{41}
\]

Firms choose the number of export markets \( j \) such that profits are maximized, i.e., \( \max_j \Pi_{d+j} (\phi) \). Since marginal revenues are equalized across firms, all exporting firms increase their production in to order be able to serve all export markets.
Proposition 5 If vacancy creation costs are linear, then all exporting firms serve all $n$ foreign markets, i.e., the unique export cutoff is given by

$$\varphi_x = \frac{\tau}{\rho} \left[ \frac{f_x}{(1 - \rho)} \right]^{\frac{1 - \rho}{\rho}} \left[ \frac{v(\varphi^*)}{l(\varphi^*)} + z \right]. \quad (42)$$

Proof. Comparing the profits of exporting to $j$ or $j - k$ countries, i.e., $\delta \Pi_{d+j}(\varphi) = \delta \Pi_{d+j-k}(\varphi)$, gives

$$k f_x = (1 - \rho) \left[ 1 + j \tau \frac{\rho}{\rho-1} \right]^{(1-\rho)} \left[ \varphi l(\varphi, j) \right]^\rho - (1 - \rho) \left[ 1 + (j - k) \tau \frac{\rho}{\rho-1} \right]^{(1-\rho)} \left[ \varphi l(\varphi, j - k) \right]^\rho. \quad (43)$$

Using equation (41) to substitute the labor input $l(\varphi, j)$ into the profit comparison condition (43) gives the desired result.

Thus, with linear vacancy creation costs exporting firms create so many vacancies that their output is large enough to meet the additional demand of all $n$ export countries like in Felbermayr, Prat, Schmerer (2011) and and Helpman, Itskhoki, and Redding (2009, 2010).

Wages are still dispersed although marginal revenues are constant across productivities (derivation in Appendix F), i.e.,

$$w(\varphi) = c \left[ \frac{v(\varphi^*)}{l(\varphi^*)} - \frac{v(\varphi)}{l(\varphi)} \right] + z.$$

The reason is the same as in the simple Burdett-Mortensen model. If firms paid the same wage, each firm would have an incentive to deviate and offer a slightly higher wage, since it will then be able to recruit also workers employed at other firms and would therefore be able to recruit additional workers at no extra cost (i.e., could save on vacancy creation cost). Thus, in equilibrium high productivity firms pay high wages and have low turnover, while low productivity firms pay low wages and have high turnover.

5.5 The case of convex vacancy creation costs ($\alpha > 1$)

As shown by the following simulation, in the case of convex vacancy creation costs and on-the-job search labor market frictions all our propositions hold with one exception.
Sufficiently productive exporting firms will be larger in a global economy compared to autarky.

5.5.1 Simulation method

As the model with endogenous vacancy creation can no longer be solved analytically, we rely on numerical solutions. We assume productivity to be Pareto distributed

\[
\Gamma(\varphi) = \frac{\varphi^{-\gamma} - \bar{\varphi}^{-\gamma}}{\varphi^{-\gamma} - \bar{\varphi}^{-\gamma}}, \text{ and } \gamma(\varphi) = \frac{\gamma\varphi^{-\gamma-1}}{\varphi^{-\gamma} - \bar{\varphi}^{-\gamma}}
\]

In order to simulate the model, we proceed as follows. First we construct a grid of \(\varphi\), running from \(\varphi_0\) to \(\bar{\varphi}\) in equal steps. Afterwards we specify a starting value for \(\varphi^*\) somewhere above \(\varphi_0\) and below \(\bar{\varphi}\). For each element of the vector \(\varphi\) we check whether the value of \(\varphi\) is greater than \(\varphi^*\). If not, we assign the value zero to the vector of \(\varphi\).

Next, we multiply the values of this vector with the step size of \(\varphi\) and initialize the vector for the vacancies \(v(\varphi)\) assuming a constant value to begin with. Later in subsequent loops the vacancies \(v(\varphi)\) are determined according to equation (31) given wages \(w(\varphi)\) and labor inputs \(l(\varphi)\). We then construct a vector of size grid size \(\times 1\) that contains the integral

\[
\int_{\varphi}^{\bar{\varphi}} \frac{v(\tilde{\varphi})}{1 - \Gamma(\tilde{\varphi})} d\Gamma(\tilde{\varphi}) = \tilde{v}
\]

for each value of \(\varphi\).

To obtain the wages for each value of \(\varphi\), we start with the value \(z\) at \(\varphi^*\). Then, we add \(\partial w(\varphi)/\partial \varphi \times \text{step size of } \varphi\) to the previous wage, where \(\partial w(\varphi)/\partial \varphi\) is given by (34). Labor input per firm is calculated using equation (33).

Given wages \(w(\varphi)\) and labor inputs \(l(\varphi)\) the next steps within the same loop are to recalculate vacancies \(v(\varphi)\) according to (31) and labor inputs \(l(\varphi)\) according to (33). We then calculate the sum over the changes in \(v(\varphi)\) from the previous and current calculation in the loop. If this change is positive, we increase every element in \(v(\varphi)\) by multiplying the old values by 0.9999, and otherwise by 1.0001. We repeat this inner loop, until the sum of the changes of \(v(\varphi)\) is smaller than 0.01. We then recalculate the integral

\[
\int_{\varphi}^{\bar{\varphi}} \frac{v(\tilde{\varphi})}{1 - \Gamma(\tilde{\varphi})} d\Gamma(\tilde{\varphi})
\]

for each value of \(\varphi\).

Given the values of \(\tilde{v}\), \(v(\varphi)\), \(w(\varphi)\), and \(l(\varphi)\), we construct a matrix of size grid size \(\times\) (number of countries), where we calculate for each value of \(\varphi\) the (potential)

\[\text{We solve our model using Matlab Release R2009b. The m-file is available upon request from the authors.}\]
total profit if the firm would export to zero, one, two countries, and so on, up to the maximum number of trading partners. Profits are given by equation (37). Given the matrix we construct a vector of size grid size×1 that contains the number of countries that a firm with productivity \( \varphi \) should export to in order to maximum profits. The zero-cutoff productivity \( \varphi^* \) is given by the value of \( \varphi \) where total profits are equal to zero and where it is profit maximizing for a firm to serve only the home market.

After a first initialization of a chosen value of \( M \), we calculate the free entry condition as given in equation (40). If this value is negative, we reduce the number of firms \( M \) by 0.1%; otherwise we increase it by 0.1%. We then repeat the whole process with the new value of \( M \) until \( M \) converges.\(^{16}\)

For the simulations we have chosen the following parameter values, \( \chi = 0.05, \eta = 0.01, \delta = 0.02, \rho = 0.75, \tau = 1.5, \epsilon = 1000000, \alpha = 5, f = 0.003, f_x = 4f, f_e = 10, \gamma = 3.4, \varphi = 50, \bar{\varphi} = 100 \) and \( z = 5 \). For the case with trade we assume 49 trading partners.\(^{17}\)

5.5.2 Results

Throughout this section we focus on two scenarios: A world where the country is in autarky and a world where there are 49 symmetric trading partners.\(^{18}\)

In Figure 6 we plot the number of vacancies created (left panel) and the number of export markets served by a firm with productivity \( \varphi \) (right panel). In line with Proposition 1 the number of export markets served is an increasing function of productivity. We calibrated the model such that no firm is willing to export to all foreign markets. Firms with the highest productivities enter 42 out of the 50 markets. Like in the model with fixed vacancies the level of productivity where firms can successfully survive \( \varphi^* \) is higher in the open economy than in autarky.

With trade the number of vacancies per firm is lower than in autarky for low-productivity firms, but higher for high productivity firms. Additionally, the number

\(^{16}\)Our convergence criterion is \( \left| \int_{\varphi} \int_{\varphi^*} \Pi_{\text{max}}(\varphi) \gamma(\varphi) \, d\varphi \right| < 0.01. \)

\(^{17}\)The grid size is chosen to be 1000. However, results do not depend on the chosen grid size.

\(^{18}\)The number of (potential) trading partners is not crucial for the basic qualitative results.
Figure 6: Vacancies and number of countries served in autarky and in an open economy with endogenous vacancies

of vacancies are increasing with productivity in both scenarios. More importantly, the number of vacancies jumps up at each export-cutoff, because firms increase their labor input in response to additional demand from abroad. Convex vacancy creation costs, however, restrict firms in their ability to grow.

Figure 7 plots labor inputs (left panel) and outputs (right panel) per firm. The pattern of vacancies translates into labor input and output pattern. Labor input and output per firm is lower in the open economy as in autarky for low-productivity firms and higher for high-productivity firms. High productivity firms grow at the expense of low productivity firms, because the additional revenues from exports allow them to create more vacancies. Unlike in Melitz (2003) not all exporting firms grow, because the increased competition in the labor market due to the increased number of vacancies has a negative effect on employment per firm, similar to the negative impact that the increased number of active firms $M$ has on labor input in the basic framework without vacancy creation. Hence, the basic results of Proposition 2 for the case of fixed vacancies survive with the qualification that only less productive firms shrink when opening up to trade.

Let us now investigate domestic prices and quantities under autarky and in an open economy. Like in Melitz (2003) domestic variety prices are a monotonically falling
Figure 7: Firm size (labor input and output) in autarky and in an open economy with endogenous vacancies

function of $\varphi$ under autarky (Figure 8). However, with trade the domestic price profile of firms looks very different. First, firms only selling domestically charge a slightly higher price as firms under autarky, because the increased competition in the labor market reduce their output (see Figure 7). The firm that exports to one trading partner charges a higher price in the domestic market than the firm selling only locally. For more productive firms that export to more countries, the domestically charged price of the least productive firm in this group slightly falls as compared to the least productive firm exporting to only one country. However, it is still higher as the domestically charged price of the firm only serving the local market.\footnote{We set the number of (potential) trading partners large enough so that even the most productive firm does not serve all foreign markets. If we would allow a firm to hit the boundary for expanding the number of markets to be served, this firm can only expand by lowering the prices. This would be reflected by a fall of the price line at the right.} This results are similar to our results shown in Figure 4b.

Quantities are just the reverse image of prices charged in the domestic market. The right panel shows that the domestically sold quantities are much higher under autarky than in an open economy, specifically for very productive firms. The quantity of the least productive firm, i.e., the firm with productivity $\varphi^*$, is higher than the quantity of the least productive firm serving in addition to the domestic market one foreign market,
Figure 8: Domestic quantities and prices in autarky and in an open economy with endogenous vacancies

i.e., the firm with productivity $\phi^1_i$. The quantities of the least productive firms serving $j \geq 2$ markets are slightly lower. Hence, the results that we derived in Proposition 3 survive under endogenous vacancy creation.

Figure 9 shows total profits of firms as a function of productivity. In both scenarios, autarky and trade, profits are increasing in productivity. Even though there are jumps in prices and quantities there are no jumps in the profit function. The extra revenues from exporting are used to pay for the foreign market entry costs. This is equivalent to the export-cutoff condition, where the least productive firm entering $j$ markets has to be indifferent between entering $j$ markets or only serving $j - 1$ markets.

If we compare the profits of firms in autarky and in an open economy, we see that the profit function under trade is much steeper than under autarky. The reason is that by serving more than one market, a firm can demand higher prices in every market and therefore generate higher profits with the same output (which is constrained by convex vacancy creation costs). Furthermore, like in Melitz (2003) there are some low productivity firms that make lower profits in an open economy than under autarky, because the increased competition on the labor market reduces low productivity firms’ labor input and thus the output necessary to generate higher profits.

In Figure 10 we plot wages as a function of productivity (left panel) and the wage
distribution (right panel). Wages are an increasing function of productivity under both, autarky and trade. Interesting are the following three observations: (i) The wage distribution starts at lower productivity values in autarky than in an open economy. This reflects the fact that only more productive firms can survive in an open economy, i.e., the zero-cut-off productivity $\phi^*$ increases when opening up to trade.\(^{20}\) (ii) Wages are at least as high as unemployment benefits $z$. (iii) The wage function is much steeper in an open economy, because exporting generates higher profits and opens up the opportunity for firms to pay higher wages.

We can also compare the wage distribution in autarky and in an open economy. The right panel of Figure 10 shows that in both situations the lowest wage is given by $z$. Since wages increase at exporting firms, opening up to trade leads to a much larger wage dispersion as predicted in Proposition 4. Hence, allowing for vacancy creation does not lead to different conclusions regarding the effects of trade on the wage distribution. Note that with endogenous vacancy creation it still holds that in an open economy the number of firms is higher and the unemployment rate lower compared to autarky.

\(^{20}\)The effect is very small, though. Hence, it can not be seen in the figure.
5.5.3 Convex vacancy costs and concave hiring costs

Abowd and Kramarz (2003) and Kramarz and Michaud (2010) have shown that the shape of the hiring cost function for French firms is mildly concave, while Mhlemann, and Schenker (2009) have shown that the shape of the hiring cost function for Swiss firms is convex. In this section we show that a convex vacancy cost function is consistent with a concave and a convex hiring cost function. Hiring cost functions have the same shape as the vacancy cost functions, if the hiring rate \( h(v) \) per vacancy is the same for all firms. However, the hiring rate per vacancy is increasing in the wage, because job offers made by high wage firms are accepted by more employed workers. This property holds in the Burdett-Mortensen model like in any monopsony wage model as shown by Manning (2006).  

\[
h(v) = \eta [u + (1 - u) G(w)].
\]

In addition the number of vacancies are an increasing function of the wage paid by firms, i.e.

\[
\frac{\partial v(w)}{\partial w} > 0.
\]

\footnote{Using firm level data from the Labour Turnover Survey in the UK Manning (2006) shows that there are increasing marginal costs of recruitment, i.e. that the vacancy cost function is convex.}
Thus, the total number of workers hired $H = h(v)v$ increase with the wage for two reasons: (i) the number of vacancies created increase with the wage and (ii) the hiring rate per vacancy increases with the wage.

Now consider the shape of the hiring cost function $K(H)$ given any convex vacancy cost function $c(v)$ with $c'_v(v) > 0$ and $c''_{vv}(v) > 0$. Using the inverse function of $H = h(v)v$ and $v(w)$, the first derivative of the hiring cost function is given by,

$$\frac{\partial K(H)}{\partial H} = c'_v(v) \frac{\partial v}{\partial H} = c'_v(v) \frac{1}{h(v) + v \frac{\partial h(v)}{\partial w} \frac{\partial v}{\partial v}} > 0,$$

where the inequality follows from,

$$\frac{\partial h(v)}{\partial w} = (1 - u) g(w) > 0 \text{ and } \frac{\partial w(v)}{\partial v} > 0.$$

The second derivative that determines the shape of the hiring cost function is given by,

$$\frac{\partial^2 K(H)}{\partial H^2} = c''_{vv}(v) \left( \frac{\partial v}{\partial H} \right)^2 - c'_v(v) \frac{\partial^2 h(v)}{\partial w^2} \frac{\partial v}{\partial v} + v \frac{\partial^2 h(v)}{\partial w^2} \left( \frac{\partial w(v)}{\partial v} \right)^2 + v \frac{\partial h(v)}{\partial w} \frac{\partial^2 w(v)}{\partial v^2} \frac{\partial v}{\partial H},$$

where

$$\frac{\partial^2 h(v)}{\partial w^2} = (1 - u) g'_w(w) \geq 0 \text{ and } \frac{\partial^2 w(v)}{\partial v^2} \geq 0.$$

Thus, a convex vacancy cost function implies a concave hiring cost function, if and only if,

$$c''_{vv}(v) < c'_v(v) \left( 2 \frac{\partial h(v)}{\partial w} \frac{\partial w(v)}{\partial v} + v \frac{\partial^2 h(v)}{\partial w^2} \left( \frac{\partial w(v)}{\partial v} \right)^2 + v \frac{\partial h(v)}{\partial w} \frac{\partial^2 w(v)}{\partial v^2} \right) \frac{\partial v}{\partial H},$$

which is feasible, since $\partial h(v)/\partial w > 0$ and $\partial w(v)/\partial v > 0$. Thus, a convex vacancy cost function is consistent with a concave hiring cost function as found by Abowd and Kramarz (2003) and Kramarz and Michaud (2010) for French firms as well as a convex hiring cost function as found by Mhlemann, and Schenker (2009) for Swiss firms.

Our simulations also provide an example that a convex vacancy cost function leads to a convex hiring cost function as shown in the following Figure.
6 Conclusion

The implications of trade liberalization on wages and unemployment is one of the most heavily discussed consequences of increasing globalization. Recent evidence suggest that overall trade reduces unemployment, but has heavily asymmetric distributional consequences. Most recent models of trade and unemployment emphasize the role of trade on unemployment, while little is known about the consequences of labor market frictions on the structure of trade.

This paper uses the on-the-job search model from Burdett and Mortensen (1998) in combination with the Melitz (2003) trade model in order to investigate the effects of capacity constraining labor market frictions in a global economy.

We show that capacity constraints heavily alter the results compared to models with perfect labor markets or imperfect labor markets without capacity constraining effects, such as the recent works by Felbermayr, Prat, and Schmerer (2008) and Helpman, Itskhoki, and Redding (2009, 2010). With capacity constraining labor market frictions not all firms will serve all export markets. Rather the number of export markets served by a firm is increasing in its productivity. Even though exporting firms are more productive, they do not necessarily charge lower prices as in the Melitz (2003) model. Rather, they try to maximize profits by serving only part of the export markets and by charging the monopolistic price in each market. Given the capacity constraints

Figure 11: Hiring cost function for the convex vacancy cost function
that firms face if they want to recruit more workers in their domestic country, an obvious extension of our model is to allow for foreign direct investment, since it would allow firms to relax their capacity constraints by recruiting and producing in a foreign country.

Concerning trade liberalization we find that unemployment falls and wage dispersion increases with trade liberalization. Note that in our context wage inequality is the result of continuous search for better jobs and not of fair-wage preferences or the result of monitoring or screening.

References


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Appendix A: Wages offers

Wages increase with productivity

To show that wages increase with productivity we follow Mortensen (1990). On the support of the wage offer distribution it must be that

\[
\pi(\varphi) = \varphi^\rho l(w(\varphi)) - w(\varphi)l(w(\varphi)) - f \quad \text{for all } w(\varphi) \in \text{supp}(F),
\]

\[
\pi(\varphi) \geq \varphi^\rho l(w(\varphi)) - w(\varphi)l(w(\varphi)) - f \quad \text{for all } w(\varphi) \in \text{supp}(F).
\]

These equilibrium conditions imply for $\varphi > \varphi'$,

\[
\varphi^\rho l(w(\varphi)) - w(\varphi)l(w(\varphi)) \geq \varphi'^\rho l(w(\varphi')) - w(\varphi')l(w(\varphi'))
\]

\[
\geq (\varphi')^\rho l(w(\varphi')) - w(\varphi')l(w(\varphi')).
\]
The difference of the first and the last term of this inequality is greater than or equal to the difference of its middle terms, i.e.,
\[
[\varphi^\rho - (\varphi')^\rho] l(w(\varphi))^\rho \geq [\varphi^\rho - (\varphi')^\rho] l(w(\varphi'))^\rho.
\]
Since \( l(w(\varphi)) \) is an increasing function of the wage, it implies that wages are weakly increasing in productivity. Since firms always have an incentive to deviate if other firms offer the same wage, it follows that wages strictly increase with productivity.

**Wage offers, if Assumption 1 does not hold**

The optimality condition (16) implies
\[
l(w(\varphi)) = \left[ \varphi^\rho l(w(\varphi))^{(\rho-1)} - w(\varphi) \right] \frac{\partial l(w(\varphi))}{\partial w(\varphi)}.
\]
This equation can only hold for \( \varphi^\rho l(w(\varphi))^{(\rho-1)} - w(\varphi) > 0 \), since \( \partial l(w(\varphi)) / \partial w(\varphi) > 0 \). If this condition, i.e., Assumption 1 is not satisfied, firms with productivity \( \varphi \in [\varphi^*, \tilde{\varphi}] \) pay \( w(\varphi) = z \), where the productivity \( \tilde{\varphi} \) is defined such that \( \rho [\tilde{\varphi} l(\tilde{\varphi})]^\rho = z l(\tilde{\varphi}) \).

The size of the firm’s labor force \( l(\tilde{\varphi}) \) is derived using the generalization of equation (5). In steady state,
\[
l(\tilde{\varphi}) = \frac{\eta \tilde{v} u + (1 - u) G(w^{-}(\varphi))}{\kappa + \lambda(M)[1 - F(w(\varphi))]},
\]
where \( G(w(\varphi)) = G(w^{-}(\varphi)) + v(w(\varphi)) \) and \( v(w(\varphi)) \) denotes the mass of workers employed at firms offering the wage \( w(\varphi) \), and \( G(w^{-}(\varphi)) \) are all the workers getting a wage lower than \( w(\varphi) \). \( G(w(\varphi)) = G(w^{-}(\varphi)) + v(w(\varphi)) \) states that the whole wage distribution is given by all the workers earning wages lower than \( w(\varphi) \) plus the once earning exactly wages \( w(\varphi) \).

Since firms with a marginal revenue below \( z \) will not find it optimal to increase their labor input by increasing wage, the mass point is at the lower bound of the wage distribution. Thus, \( G(w^{-}(\varphi)) = 0 \). Using equations (12) and (18) we get,
\[
l(\tilde{\varphi}) = \frac{\eta \tilde{v} [1 - \Gamma(\varphi^*)]}{\kappa [1 - \Gamma(\varphi^*)] + \eta \kappa \tilde{v} [1 - \Gamma(\tilde{\varphi})]} \frac{\kappa + \delta}{\kappa + \delta + \eta M \tilde{v}}.
\]
We proceed by assuming that the marginal revenue of a firm exceeds the level of unemployment benefits, i.e., assume that Assumption 1 holds. If Assumption 1 holds, more productive firms offer higher wages and the first order condition (16) holds for all \( \varphi \in [\varphi^*, \tilde{\varphi}] \). Hence, the labor force of all firms operating is given by equation (15).
Appendix B: Derivation of the wage function \( w(\varphi) \)

Substituting \( l(w(\varphi)) \) form equation (15) into the first order condition (16) implies

\[
1 = \left[ \varphi^\rho \left( \frac{\eta \delta}{|x + \lambda(M)|} - w(\varphi) \right) \right]^{(\rho - 1)}
\]

Substituting \( F(w(\varphi)) \) using equation (18) gives

\[
\frac{\partial w(\varphi)}{\partial \varphi} = \left[ \varphi^\rho \left( \frac{\eta[1-\Gamma(\varphi^*)]}{|x + \lambda(M)|} - w(\varphi) \right) \right]^{(\rho - 1)}
\]

Define

\[
T(\varphi) = -\log \left( \frac{|x + \lambda(M)|}{|x + \lambda(M)| + \Gamma(\varphi^*)} \right)
\]

and

\[
T'(\varphi) = \frac{\lambda(M)v(\varphi)(2x + \delta + 2\lambda(M)|1 - \Gamma(\varphi)|)}{|x + \lambda(M)| + \Gamma(\varphi^*)}.
\]

Substitution simplifies the above differential equation to

\[
\frac{\partial w(\varphi)}{\partial \varphi} + w(\varphi)T'(\varphi) = [\eta\bar{u}(x + \delta)]^{(\rho - 1)} \rho\varphi^\rho \left[ e^{T(\varphi)} \right]^{\rho - 1} T'(\varphi).
\]

Any solution to this differential equation has to satisfy

\[
w(\varphi)e^{T(\varphi)} = [\eta\bar{u}(x + \delta)]^{(\rho - 1)} \int_{\varphi^*}^{\varphi} \rho \left[ \varphi e^{T(\varphi)} \right]^{\rho - 1} T'(\varphi) \, d\varphi + A,
\]

where \( A \) is the constant of integration. Note that

\[
\frac{d}{d\varphi} \left[ \varphi e^{T(\varphi)} \right]^{\rho} = \rho \varphi^{\rho - 1} \left[ e^{T(\varphi)} \right]^{\rho - 1} T'(\varphi) + \rho \varphi^{\rho - 1} \left[ e^{T(\varphi)} \right]^{\rho}.
\]

The integral can thus be written as

\[
\int_{\varphi^*}^{\varphi} \rho \left[ \varphi e^{T(\varphi)} \right]^{\rho} T'(\varphi) \, d\varphi = \left[ \varphi e^{T(\varphi)} \right]^{\rho} - \int_{\varphi^*}^{\varphi} \rho \varphi^{\rho - 1} \left[ e^{T(\varphi)} \right]^{\rho} \, d\varphi.
\]

Substituting into the wage equation (44) gives

\[
w(\varphi)e^{T(\varphi)} = [\eta\bar{u}(x + \delta)]^{(\rho - 1)} \left[ \varphi e^{T(\varphi)} \right]^{\rho} - \int_{\varphi^*}^{\varphi} \rho \varphi^{\rho - 1} \left[ e^{T(\varphi)} \right]^{\rho} \, d\varphi + A
\]

\[
w(\varphi) = [\eta\bar{u}(x + \delta)]^{(\rho - 1)} \left[ \varphi e^{T(\varphi)} \right]^{\rho} - \int_{\varphi^*}^{\varphi} \rho \varphi^{\rho - 1} \left[ e^{T(\varphi)} \right]^{\rho} \, d\varphi + \frac{\bar{u}}{e^{T(\varphi)}}\left[ \varphi e^{T(\varphi)} \right]^{\rho - 1} \, e^{T(\varphi)} d\varphi.
\]

(45)
since
\[ w(\varphi^*) e^{T(\varphi^*)} = ze^{T(\varphi^*)} = A. \]
Substituting \([\eta \bar{v} (\kappa + \delta)] e^{T(\varphi)} = l(w(\varphi))\) and \(z\) by using the zero-cutoff condition (21) gives the wage equation (19).

**Appendix C: Equilibrium in autarky**

Applying the implicit function theorem to the free entry condition (20) implies
\[
\frac{d\varphi^*}{dM} = -\frac{\int_{\varphi^*}^{\bar{\varphi}} \int_{\varphi^*}^{\bar{\varphi}} \rho^2 \left[ \varphi l(w(\varphi)) \right]^{(\rho-1)} \frac{\partial l(w(\varphi))}{\partial \varphi} d\varphi - \frac{\rho}{\varphi^*} \left[ \varphi^* l(w(\varphi^*)) \right]^{\rho} \gamma(\varphi) d\varphi}{\int_{\varphi^*}^{\bar{\varphi}} \int_{\varphi^*}^{\bar{\varphi}} \rho^2 \left[ \varphi l(w(\varphi)) \right]^{(\rho-1)} \frac{\partial l(w(\varphi))}{\partial M} d\varphi \gamma(\varphi) d\varphi} < 0,
\]
where the inequality follows because \(\partial l(w(\bar{\varphi})) / \partial \varphi^* < 0\) and \(\partial l(w(\bar{\varphi})) / \partial M < 0\).

Thus, the free entry condition defines a decreasing relation between the zero-cutoff productivity \(\varphi^*\) and the number of active firms \(M\) in the market.

Applying the implicit function theorem to the zero-profit condition (21) implies
\[
\frac{d\varphi^*}{dM} = \frac{\left[ \rho [\varphi^*]^{\rho} [l(z)]^{(\rho-1)} - z \right] \left[ \frac{\eta \bar{v}}{\kappa + \eta M \bar{v}} + \frac{\eta \bar{v}}{\kappa + \delta + \eta M \bar{v}} \right] l(z)}{\rho [\varphi^*]^{\rho-1} [l(z)]^\rho} > 0.
\]
Assumption 1 ensures an increasing relation between the zero-cutoff productivity \(\varphi^*\) and the number of active firms \(M\) in the market.

An equilibrium only exists if unemployment benefits \(z\) and the fixed cost \(f\) are low enough.

**Appendix D: The open economy**

*Quantities sold in the domestic and each export market*

An exporting firm that decided to serve \(j\) foreign countries maximizes its revenue
\[
R(\varphi) = p_d(\varphi) q_d(\varphi) + j \frac{p_x(\varphi)}{\tau} q_x(\varphi)
\]
\[
= p_d(\varphi) [q(\varphi) - j q_x(\varphi)] + j \frac{p_x(\varphi)}{\tau} q_x(\varphi)
\]
\[
= [q(\varphi) - j q_x(\varphi)]^\rho + j \left[ \frac{q_x(\varphi)}{\tau} \right]^\rho,
\]

by choosing its domestic and export sells according to,

\[
\frac{\partial R(\varphi)}{\partial q_x(\varphi)} = 0
\]

\[
\rho_j [q(\varphi) - jq_x(\varphi)]^{\rho-1} = \rho_j \frac{1}{\tau} \left[ \frac{q_x(\varphi)}{\tau} \right]^{\rho-1}
\]

\[
q(\varphi) - jq_x(\varphi) = \frac{1}{\tau^{\rho/(\rho-1)}} q_x(\varphi)
\]

\[
\tau^{\rho/(\rho-1)} q(\varphi) = [1 + j \tau^{\rho/(\rho-1)}] q_x(\varphi)
\]

Rearranging and using the fact that \(q_d(\varphi) = q(\varphi) - jq_x(\varphi)\) implies equations (22) and (23). The revenue of an exporting firm is, therefore, given by

\[
R(\varphi) = \left[ q(\varphi) - jq(\varphi) \frac{\tau^{\rho/(\rho-1)}}{1 + j \tau^{\rho/(\rho-1)}} \right]^\rho + j \left[ q(\varphi) \frac{\tau^{\rho/(\rho-1)}}{1 + j \tau^{\rho/(\rho-1)}} \right]^\rho
\]

\[
= \left[ 1 + j \tau^{\rho/(\rho-1)} \right] \left[ q(\varphi) \frac{\tau^{\rho/(\rho-1)}}{1 + j \tau^{\rho/(\rho-1)}} \right]^\rho
\]

\[
= \left[ 1 + j \tau^{\rho/(\rho-1)} \right]^{(1-\rho)} [q(\varphi)]^\rho.
\]

**Export-cutoffs**

The export-cutoff productivity \(\varphi_x^*\) is defined by \(\delta \Pi_{d+j} (\varphi_x^*) = \delta \Pi_{d+j-1} (\varphi_x^*)\), where

\[
\delta \Pi_{d+j} (\varphi) = \left[ 1 + j \tau^{\rho/(\rho-1)} \right]^{(1-\rho)} \left[ \varphi l(w(\varphi_x^*)) \right]^\rho - w(\varphi) l(w(\varphi)) - f - j f_x.
\]

Since profit maximization implies that the wage is continuous at \(\varphi_x^*\), i.e., that both wages are the same, and since the same wage implies that the number of workers employed by both type of firms are identical and given by \(l(w(\varphi_x^*))\) we may write:

\[
\left[ 1 + j \tau^{\rho/(\rho-1)} \right]^{(1-\rho)} [\varphi_x^* l(w(\varphi_x^*))]^\rho - w(\varphi_x^*) l(w(\varphi_x^*)) - f - j f_x
\]

\[
= \left[ 1 + (j - 1) \tau^{\rho/(\rho-1)} \right]^{(1-\rho)} [\varphi_x^* l(w(\varphi_x^*))]^\rho - w(\varphi_x^*) l(w(\varphi_x^*)) - f - (j - 1) f_x.
\]

Thus, the export-cutoff condition (26) can be derived:

\[
[\varphi_x^* l(w(\varphi_x^*))]^\rho = \frac{f_x}{\left[ 1 + j \tau^{\rho/(\rho-1)} \right]^{(1-\rho)} - \left[ 1 + (j - 1) \tau^{\rho/(\rho-1)} \right]^{(1-\rho)}}.
\]
The rhs of the last equation (and therefore the export-cutoff productivity \( \varphi_x^j \)) is increasing in \( j \), i.e.,

\[
\left[ \varphi_x^j \left( w \left( \varphi_x^j \right) \right) \right]^\rho - \left[ \varphi_x^{j-1} \left( w \left( \varphi_x^{j-1} \right) \right) \right]^\rho = \int_{\varphi_x^j}^{\varphi_x^{j-1}} \left[ \varphi_x^j \left( w \left( \varphi_x^j \right) \right) \right]^\rho \, d\varphi_x^j
\]

where the last inequality follows from Jensen’s inequality, i.e.,

\[
\frac{1}{2} \left[ 1 + (j - 2) \rho \right]^{(1-\rho)} + \frac{1}{2} \left[ 1 + j \rho \right]^{(1-\rho)} < \left[ \frac{1}{2} \left[ 1 + (j - 2) \rho \right]^{(1-\rho)} + \frac{1}{2} \left[ 1 + j \rho \right]^{(1-\rho)} \right]^{(1-\rho)}
\]

\[
= \left[ 1 + \frac{1}{2} (j - 2) + \frac{1}{2} j \right]^{(1-\rho)}
\]

\[
= \left[ 1 + (j - 1) \rho \right]^{(1-\rho)}.
\]

**Wages in an open economy**

The wage equation for exporting firms follows from the first order condition and the equilibrium condition (18), leading to the following differential equation:

\[
\frac{\partial w (\varphi)}{\partial \varphi} + w (\varphi) T' (\varphi) = \left[ \frac{1 + j \rho}{\eta \bar{v} (\varphi + \delta)} \right]^{(1-\rho)} \rho \varphi^{\rho} \left[ e^{T(\varphi)} \right]^{\rho - 1} T' (\varphi),
\]

where \( T (\varphi) \) and \( T' (\varphi) \) are defined in Appendix A. The solution to this differential equation is obtained by following the same steps as in Appendix A,

\[
w (\varphi) e^{T(\varphi)} = \left[ \frac{1 + j \rho}{\eta \bar{v} (\varphi + \delta)} \right]^{(1-\rho)} \left[ \varphi e^{T(\varphi)} \right]^{\rho} - \left[ \varphi_x^j e^{T(\varphi_x^j)} \right]^{\rho} - \int_{\varphi_x^j}^{\varphi} \frac{\rho}{\varphi} \left[ \varphi e^{T(\varphi)} \right]^{\rho} \, d\varphi + A,
\]

where

\[
A = w \left( \varphi_x^j \right) e^{T(\varphi_x^j)}
\]

\[
= \left[ \frac{1 + (j - 1) \rho}{\eta \bar{v} (\varphi + \delta)} \right]^{(1-\rho)} \left[ \varphi_x^{j-1} e^{T(\varphi_x^{j-1})} \right]^{\rho} - \left[ \varphi_x^{j-1} e^{T(\varphi_x^{j-1})} \right]^{\rho} - \int_{\varphi_x^{j-1}}^{\varphi_x^j} \frac{\rho}{\varphi} \left[ \varphi e^{T(\varphi)} \right]^{\rho} \, d\varphi
\]

\[
+ w \left( \varphi_x^{j-1} \right) e^{T(\varphi_x^{j-1})},
\]

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and
\[ w \left( \varphi_x^j \right) e^{T(\varphi_x^j)} = \frac{1}{\eta(x+\delta)^{(1-\rho)}} \left[ \left[ \varphi_x^j e^{T(\varphi_x^j)} \right]^\rho - \left[ \varphi_x^* e^{T(\varphi^*)} \right]^\rho - \int_{\varphi^*}^{\varphi_x^j} \frac{\rho}{\varphi_x} \left[ \varphi e^{T(\varphi)} \right]^\rho d\varphi \right] + z e^{T(\varphi^*)}. \]

Since \( A \) depends on the wage payments of those firms with export-cutoff productivities \( \varphi_x^j < \varphi \) we need to rewrite the wage equation as follows, i.e.,
\[ w \left( \varphi \right) e^{T(\varphi)} = \frac{1}{\eta(x+\delta)^{(1-\rho)}} \left[ \left[ \varphi e^{T(\varphi)} \right]^\rho - \left[ \varphi_x^* e^{T(\varphi^*)} \right]^\rho - \int_{\varphi^*}^{\varphi_x^j} \frac{\rho}{\varphi} \left[ \varphi e^{T(\varphi)} \right]^\rho d\varphi \right] + z e^{T(\varphi^*)}, \]

or
\[ w \left( \varphi \right) e^{T(\varphi)} = \frac{1}{\eta(x+\delta)^{(1-\rho)}} \left[ \left[ \varphi e^{T(\varphi)} \right]^\rho - \int_{\varphi^*}^{\varphi_x^j} \frac{\rho}{\varphi} \left[ \varphi e^{T(\varphi)} \right]^\rho d\varphi \right] - \sum_{i=1}^{j} \left[ \frac{1}{\eta(x+\delta)^{(1-\rho)}} \left[ \varphi_x^j e^{T(\varphi_x^j)} \right]^\rho - \frac{1}{\eta(x+\delta)^{(1-\rho)}} \left[ \varphi_x^{i-1} e^{T(\varphi_x^{i-1})} \right]^\rho \right] \]
\[ - \sum_{i=2}^{j} \int_{\varphi_x^{i-1}}^{\varphi_x^j} \frac{\rho}{\varphi} \left[ \varphi e^{T(\varphi)} \right]^\rho d\varphi \]
\[ - \frac{1}{\eta(x+\delta)^{(1-\rho)}} \left[ \varphi_x^* e^{T(\varphi^*)} \right]^\rho + \int_{\varphi^*}^{\varphi_x^j} \frac{\rho}{\varphi} \left[ \varphi e^{T(\varphi)} \right]^\rho d\varphi \] + z e^{T(\varphi^*)}. \]

Substituting \( \eta(x+\delta)^{(1-\rho)} e^{T(\varphi)} = l(w(\varphi)), z \) by using the zero-cutoff condition (21) and \( [\varphi_x^j l(w(\varphi))]^\rho \) using the export cutoff condition (26) gives
\[ w(\varphi) l(w(\varphi)) = \left[ 1 + j \tau \frac{\rho}{\eta(x+\delta)^{(1-\rho)}} \right]^{(1-\rho)} [\varphi l(w(\varphi))]^\rho - j f_x \]
\[ - \left[ 1 + j \tau \frac{\rho}{\eta(x+\delta)^{(1-\rho)}} \right]^{(1-\rho)} \int_{\varphi^*}^{\varphi_x^j} \frac{\rho}{\varphi} [\varphi l(w(\varphi))]^\rho d\varphi \]
\[ - \sum_{i=2}^{j} \left[ 1 + (i-1) \tau \frac{\rho}{\eta(x+\delta)^{(1-\rho)}} \right]^{(1-\rho)} \int_{\varphi_x^{i-1}}^{\varphi_x^j} \frac{\rho}{\varphi} [\varphi l(w(\varphi))]^\rho d\varphi \]
\[ - \int_{\varphi^*}^{\varphi_x^j} \frac{\rho}{\varphi} [\varphi l(w(\varphi))]^\rho d\varphi - f. \]
The wage equation (25) follows immediately by defining $\varphi = \varphi^j_{x+1}$ and $\varphi^* = \varphi^0_{x}$.

**Average profits in an open economy**

Rearranging the wage equation (25) implies that the profit of an exporting firm is given by

$$
\delta \Pi_d (\varphi) = \left[1 + j \tau \rho^{-1}\right] \int_{\varphi^j_{x}}^{\varphi^*} \frac{1}{\varphi} \left[\varphi l (w (\varphi))\right]^\rho \gamma (\varphi) d\varphi,
$$

where $\varphi^j_{x+1} = \varphi$ and $\varphi^0_{x} = \varphi^*$. Since free entry implies $f_e = \Pi_e (\varphi^*) = \int_{\varphi^*}^{\varphi} \Pi (\varphi) \gamma (\varphi) d\varphi$, integrating over all firms with productivity $\varphi \in [\varphi^*, \varphi]$ implies the free entry condition for an open economy as stated in equation (27).

**Upward rotation of the free entry condition**

We need to show that for each $\varphi^* \in [0, \varphi]$ the number of active firms increases, i.e., $M_T > M_A$. Suppose the opposite, i.e., $M_T \leq M_A$. Thus, labor input for a firm with productivity is given by $l_T (w (\varphi)) \geq l_A (w (\varphi))$ according to equation (15). Since $\left[1 + (i - 1) \tau \rho^{-1}\right] > 1$, it follows that $\Pi_e (\varphi^*)|_{Trade} > \Pi_e (\varphi^*)|_{Autarky} = f_e$. This contradicts, however, the free entry condition in an open economy. Thus, $M_T > M_A$.

The increase in the number of active firms $M$ increases the zero-cutoff productivity $\varphi^*$. It is easy to verify from equations (15) and (18) that the size of all firms $l (w (\varphi))$ decreases.

**Appendix E: Unemployment and wage dispersion in an open economy**

It follows from equation (12) that the unemployment rate decreases as the number of active firms $M$ increases in response to opening the economy for trade.

The increase in the number of active firms $M$ and the increase in the zero-cutoff productivity $\varphi^*$ in response to opening up to trade implies that wages increase if the
position of a firm in the wage offer distribution, i.e., $F(w)$, is kept constant. This can be proven by using equation (25), i.e.,

$$w(\varphi) = \left[1 + j \tau^{(\rho - 1)}\right]^{(1-\rho)} \varphi^\rho l(w(\varphi))^{(\rho - 1)} - f - j f x - j \sum_{i=1}^{j+1} \left[1 + (i - 1) \tau^{(\rho - 1)}\right]^{(1-\rho)} \int_{\varphi_{x}^{i-1}}^{\varphi_{x}^{i}} \rho \left[\tilde{\varphi} l(w(\tilde{\varphi}))\right]^{(\rho - 1)} \frac{l(w(\tilde{\varphi}))}{l(w(\varphi))} d\tilde{\varphi},$$

where $\varphi_{x}^{j+1} = \varphi$ and $\varphi_{x}^{0} = \varphi^*$. First, keep $l(w(\varphi))$ constant and notice that for a given labor input $l(w(\varphi))$ wages increase when opening up to trade, because $\left[1 + j \tau^{(\rho - 1)}\right] > \left[1 + i \tau^{(\rho - 1)}\right]$ for all $i < j$. Second, an increase in $M$ leads to a lower labor size $l(w(\varphi))$ and an increase in $\varphi^*$ while keeping $F(w)$ constant reduces the integral in the second line of the above equation and thereby increases the wage. Since the position of the wage distribution of the highest productivity firm, i.e., $F(w(\bar{\varphi})) = 1$, remains unchanged, it follows that the highest wage increases. Since the lowest wage equals the level of unemployment benefit $z$ in the closed and open economy, it follows that wage dispersion $w(\bar{\varphi}) - z$ is higher in an open economy than in a closed economy.

**Appendix F: Vacancy creation condition for linear vacancy costs**

The optimality condition for vacancies (31) for $\alpha = 1$ and wages (32) imply the equality of marginal cost, i.e.,

$$cv(\varphi) \frac{\partial l(\varphi)}{\partial w(\varphi)} \frac{\partial w(\varphi)}{\partial \varphi} = \frac{\partial w(\varphi)}{\partial \varphi} l(\varphi), \quad (47)$$

where we used $\partial l(\varphi, v)/\partial v = l(\varphi, v)/v$. Similar to Appendix A we define,

$$l(\varphi) = \eta v(\varphi) (\kappa + \delta) e^{T(\varphi)},$$

where

$$T(\varphi) = -\log \left[\kappa + \eta M \int_{\varphi}^{\bar{\varphi}} \frac{v(\tilde{\varphi})}{1 - \Gamma(\varphi^*)} d\Gamma(\tilde{\varphi}) \right] \left[\kappa + \delta + \eta M \int_{\varphi}^{\bar{\varphi}} \frac{v(\tilde{\varphi})}{1 - \Gamma(\varphi^*)} d\Gamma(\tilde{\varphi}) \right].$$
\[ T'(\varphi) \equiv \frac{\eta M e(\varphi)\gamma(\varphi)\left[ \frac{2\kappa + 2\eta M}{\eta M} \int \bar{\varphi} \, d\Gamma(\bar{\varphi}) \right]^{T'(\bar{\varphi})}}{\left[ 2\kappa + 2\eta M \int \bar{\varphi} \, d\Gamma(\bar{\varphi}) \right]^{\kappa + 2\eta M} \left[ 2\kappa + 2\eta M \int \bar{\varphi} \, d\Gamma(\bar{\varphi}) \right]} = \frac{\partial l(\varphi)}{\partial \varphi} \frac{\partial w(\varphi)}{\partial \varphi}. \]

Equation (47) can therefore be written as
\[
\frac{\partial w(\varphi)}{\partial \varphi} = \frac{c}{\eta (\kappa + \delta)} \frac{T'(\varphi)}{e^{T(\varphi)}},
\]

Integration gives
\[
w(\varphi) = \frac{c}{\eta (\kappa + \delta)} \int_0^\varphi T'(\bar{\varphi}) \frac{1}{e^{T(\bar{\varphi})}} d\bar{\varphi} + A = \frac{c}{\eta (\kappa + \delta)} \left[ e^{-T'(\varphi^*)} - e^{-T(\varphi)} \right] + A = c \left[ \frac{v(\varphi^*)}{l(\varphi^*)} - \frac{v(\varphi)}{l(\varphi)} \right] + z,
\]

where \( A = z \) follows from \( w(\varphi^*) = z \). Substituting the wage \( w(\varphi) \) into the optimality condition for vacancies (31) for \( \alpha = 1 \) gives the stated result.