Price Competition in International Mixed Oligopolies

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Abstract In this paper we analyze the effects of international competition in a mixed oligopoly framework, with price competition and differentiated products. The properties of equilibria, and the impact of policy measures such as privatizations and cross-border acquisitions, are studied both in a single-country and in a two-country framework, under the hypothesis that all firms share the same linear technology. Besides showing that a mixed-market internationalization allows for efficiency gains which are consistent with binding budget constraints for the public firm, we identify the market structures and the competitive environment which support welfare enhancing privatization policies, independently of any endogenous or exogenous cost differential between public and private producers. In particular, we suggest that the cross-country distribution of firms, the degree of product substitutability and the overall density of the market are the key elements in the assessment of the desirability of public ownership.

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1 Introduction

This paper studies the outcomes of competition in those international oligopolistic markets, where strategic interaction involves welfare-maximizing public and profit-maximizing private firms. The reference framework is therefore that of mixed-oligopoly theory, which we address under the hypothesis that firms compete over prices, sharing the same linear technology in a market where products are imperfectly substitutable. These assumptions are consistent with recent competitive patterns observed on the international scenarios. In these, the presence of state-owned firms, still massive on the domestic markets, is increasing despite the worldwide waves of privatization which have taken place over the past twenty years. Public ownership, which is commonly observed in network industries like telecommunications, transports, energy and utilities, characterizes also a range of services like insurances and banking, postal services, health care and education. Private and public firms frequently coexist in these sectors, which are moreover becoming increasingly exposed to international competition, in response to the international liberalizations and demand growth.

When dealing with international mixed oligopolies, the key issue arises of distinguishing between the within-country market interactions, and the interactions among countries. While the former are related to the existence of firms (public and private) characterized by different motives and to the contribution of foreign firms to domestic welfare, the latter concern international competition in the realization of the governments' objectives. In order to capture this distinction, the paper analyzes two different open market frameworks: a single-country model, where a public firm interacts with domestic and foreign private firms on the home market, and a so-called two-country model, in which two domestic welfare-maximizing public firms (one for each country) compete strategically on a single international market with a number of private firms from both countries. A full characterization of equilibria and their welfare properties in these two scenarios is the basis for an assessment of the effects of changes in the market structure, such as cross-border acquisitions and privatizations. When relevant, these effects are studied both in a country-specific perspective, and in a global perspective.

As far as the single country model is concerned, our main result is that under price competition and differentiated product the presence of international competitors in the domestic market induces an overall convergence to efficient pricing, and is consistent with binding budget constraints of the public firm even under constant average and marginal costs. This also holds under Stackelberg price competition, provided that the presence of foreign competitors is not too large. In the more articulated two-country set-up, we show that the degree of product substitutability, the degree of asymmetry in the cross-country distribution of firms, and the overall number of firms, determine the conditions for a price reversal between public and private firms to occur, and crucially affect the country-specific and the aggregate welfare evaluation of privatizations.

The paper is organized as follows. Section 2 put our results in context, by sketching the main relevant related literature. The single-country model with
simultaneous and sequential moves, and the two-country model are developed in Sections 3 and 4, respectively. Some conclusions are gathered in Section 5.

2 The related literature

Mixed oligopoly theory has remarkably developed in the last two decades by pointing out those situations in which interactions between private and public firms lead to a higher social desirability as compared to a fully privatized context, thus contributing to the debate on privatization. Efficiency, strategic and political arguments have been invoked to provide a theoretical support to the idea that mixed public-private markets can dominate the alternative of pure private markets. The most commonly adopted framework is one in which public ownership is associated with pure welfare maximization objectives, and firms compete over quantities under increasing marginal costs.

In a framework of strategic interaction, a number of works attributes the welfare-enhancing character of a mixed market to the incentives of the public firm to expand total output, thus indirectly regulating markets. Indeed, under quantity competition a publicly owned firm produces the quantity at which the clearing price equals the marginal cost; this implies a greater aggregate market production than that observed in a market with only profit-concerned firms, and therefore a higher efficiency. In this context, however, De Fraja and Delbono (1989) make an important warning about the role of increasing marginal costs: the presence of this high producing firm may impact negatively on welfare due to its higher average and marginal costs. While welfare is positively affected by the boost to production, it is negatively affected, under quadratic costs, by an unequal sharing of production between the public and the private firms. The spread in the produced quantities and the associated welfare losses are higher, the larger is the number of private firms in the market. In these circumstances privatization may lead to welfare improvements.

In the context of mixed oligopolies, privatization is to be positively considered also when it induces a firm’s restructuring which improves the overall efficiency and productivity. Many works focus on these cost-saving reasons which justify a change of ownership from public to private, on the assumption, often derived from a conventional wisdom, that public firms are less efficient. As pointed out by De Fraia (1991), in these circumstances the beneficial effects

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1 Some authors assume that public firms maximize a weighted average of welfare and their own profits, thus tackling partial privatization. The search for the optimal degree of government ownership in the privatized firms is a core issue of this literature. See Matsumura (1998) as a major reference.

2 In this case, though public and private firms share the same technology, at equilibrium the public firm is endogenously less efficient, the efficiency gap basically reflecting the differences in firms’ objectives.

3 Managerial slackness and higher agency costs are often invoked as reasons for the lower efficiency attributed to public companies. Willner (1999) points to the higher wages paid under public ownership as an explanation for lower cost efficiency. This belief, however, has been often challenged and does not receive unanimous consensus, neither in the theoretical nor in the empirical literature (see Björkroth et al (2006), p.180, and the papers referred therein).
of privatization crucially depend on the size of the technological efficiency gains, which must be high enough to offset the welfare loss due to the firm’s output contraction once the privatization is realized. A rationale for public ownership has also been found in its being an instrument to achieve political, social, industrial and environmental goals. White (2002) shows that governments can strategically manipulate the public firms’ objective functions in order to disguise their real political orientation and to actually pursue aims which differ from those publicly stated. Moreover, the presence of public firms has been considered socially beneficial when in the bargaining process they allow for higher wages (Willner, 1999), or when their positive impact on social welfare is through their contribution to investment in R&D (Poyago-Theotoky, 1998). Finally, public more than private ownership is to be invoked when governments pursue an environmental policy: public firms may internalize their environmental damages and ensure a higher revenue from environmental taxes (Bárcena-Ruiz and Garzón, 2006).

The recent extension of the analysis of mixed oligopoly to an international framework has raised several issues, which are of interest for both industrial organization and international trade theory: from the impact of privatizations in international markets to the effectiveness of open-door policies and cross-borders acquisitions. Several papers analyze international competition in a single-country mixed market. Among these, Fjell and Pal (1996) extend the model by De Fraia and Delbono to allow for the competition of foreign private firms in addition to the domestic ones, while Pal and White (1998) analyze privatizations in the presence of subsidies or tariffs. More recent works tackle the public-private interactions in international markets within a two-country model, in which competition among private and public firms is addressed as part of strategic competition between governments. In this line, Dadpay and Heywood (2006) offer an exhaustive analysis of the equilibria under quantity competition and decreasing returns to scale, showing that welfare gains are typically associated to coordinated privatization, though the strategic motives of governments do not create the appropriate unilateral incentives to privatize. Using a similar framework, Han and Ogawa (2008) examine the optimal extent of privatization, while Bárcena-Ruiz and Garzón (2005) develop a setting with asymmetric constant marginal costs, where the decision of one government to privatize depend on the relative cost advantage of the private firms over the publicly-owned firm.

With the exception of the duopoly model by Ohnishi (2010), the analysis of international mixed oligopolies has been developed under quantity competition in a homogeneous product market. This calls for the additional assumption of

4 Most of the economic literature, however, looks at the political interference within state-owned enterprises as a reason for their privatization. By assigning external objectives to public firms, Estrin and Perotin (1991) show how politician may contribute to amplify the agency problem at the firm or state level, and to weaken the capital market pressure which would motivate public managers towards efficiency. Public enterprises are also viewed by Boycko et al (1996) as a means to pursue political objectives, such as excess employment; also in this case privatization is invoked for the efficiency gains it may generate in underperforming firms.
decreasing returns to scale, constant returns being inconsistent in that framework with a non-negative profit condition for the public firm. This paper contributes to the existing literature by developing a general international mixed-oligopoly model under price-competition and imperfect product substitutability. The main theoretical advantage of this set-up is that price competition, by enlarging significantly the set of market configurations in which the public firms’ budget constraint is consistent with constant average and marginal costs, allows us to rule out any exogenous or endogenous technological asymmetry between public and private firms, so that the properties of equilibria and their policy implications rely exclusively on the characteristics of strategic interaction.

3 The single country framework

We consider a country (for simplicity, country $H$), in which a public domestic firm $i$ interacts in the market for a differentiated product with a number of private firms, $m$ of which are domestic, and $n$ come from the rest of the world (for simplicity, $F$). The total number of firms operating in the market is therefore $m + n + 1$. All firms are characterized by the same linear technology, and produce at a constant average and marginal cost $c$.

The representative consumer shows quasi-linear preferences à la Bowley, so that the direct demand function faced by the generic firm $s$ is given by:

$$q_s = \frac{1 - \gamma (1 + \gamma (m + n - 1)) p_s + \gamma \bar{P}}{(1 - \gamma (1 + \gamma (m + n)))}$$

where $\bar{P} = \sum_{v \neq s} p_v$ is the sum of the prices of all the other firms and $\gamma$ is the degree of product substitutability, ranging from 0 (absence of substitutability) to 1 (homogeneous products).\(^5\)

As standard in mixed oligopoly models, the public firm maximizes social welfare, while all private firms are profit-maximizing. In a single country framework, the social welfare is defined as the sum of consumers’ surplus ($SC$) over all the $m + n + 1$ varieties, and the aggregate profits of domestic firms ($\Pi_h$): obviously the profits of foreign firms do not enter the public firm’s objective function. Firms compete simultaneously over prices. In the sequel, domestic private firms are indexed with $h$, while foreign private firms are indexed with $f$.

Let us consider the optimal behavior of the public firm. It solves the following problem

$$\max_{p_i} (SC + \Pi_h)$$

where the consumer surplus

$$SC = \frac{(1 - \gamma)(q_i^2 + \sum_{h \in H} q_h^2 + \sum_{f \in F} q_f^2) + \gamma (q_i + \sum_{h \in H} q_h + \sum_{f \in F} q_f)^2}{(1 - \gamma (1 + \gamma (m + n)))}$$

\(^5\)Our analysis is also robust to a model specification with the Shubik and Levitan demand function, which captures product substitutability under the hypothesis that the market size is constant as the number of varieties increases.
can be expressed in terms of prices by using (1) for all domestic and foreign varieties. The solution yields the best reply of the public firm as a function of the rivals’ prices:

\[ p_i = \frac{c(1+\gamma(n-1)) + \gamma \sum_{h \in \{H\}} p_h}{1+\gamma(m+n-1)} \]  

(2)

Notice that this reaction function exhibits the standard strategic complementarity of price decisions. However, this occurs with respect to the private domestic firms only: the public firm’s price is strategically independent of the price of the foreign private firms, notwithstanding the fact that the latter enters the public firm’s objective function through both the consumers’ surplus and the aggregate domestic profits. Under constant average and marginal costs, the foreign rivals’ behavior affects the impact of a marginal change of the public firm’s price on the consumers’ surplus with the same magnitude but opposite sign as it affects the marginal impact of the public firm on aggregate domestic profits. In the extreme case in which the public firm interacts with foreign firms only \((m = 0)\), its optimal reaction is to set a price equal to marginal cost, independently of \(n\) and independently of the prices set by the rivals.\(^6\)

As far as private domestic firms are concerned, for the generic firm \(h\) profit maximization yields the following reaction function:

\[ p_h = \frac{1+c+\gamma(c(m+n)-(1+c)) + \gamma(p_i + \sum_{k \in \{H-h\}} p_k + \sum_{f \in \{F\}} p_f)}{2(1+\gamma(m+n-1))} \]  

(3)

where \(\sum_{k \in \{H-h\}} p_k\) denotes the sum of the prices of the private domestic firms other than \(h\). Similarly, the reaction function of the generic private foreign firm \(f\) is

\[ p_f = \frac{1+c+\gamma(c(m+n)-(1+c)) + \gamma(p_i + \sum_{h \in \{H\}} p_h + \sum_{g \in \{F-f\}} p_g)}{2(1+\gamma(m+n-1))} \]  

(4)

where \(\sum_{g \in \{F-f\}} p_g\) denotes the sum of the prices of the foreign domestic firms other than \(f\). Aggregating (3) over \(h\) and (4) over \(f\), we get

\[ \sum_{h \in \{H\}} p_h = \frac{m(1+c+\gamma(c(m+n)-(1+c)) + \gamma(p_i + \sum_{f \in \{F\}} p_f))}{2+\gamma(m+2n-1)} \]  

(5)

\[ \sum_{f \in \{F\}} p_f = \frac{n(1+c+\gamma(c(m+n)-(1+c)) + \gamma(p_i + \sum_{h \in \{H\}} p_h))}{2+\gamma(2m+n-1)} \]  

(6)

Equations (5)–(6) and (2) can be solved simultaneously for \(p_i, \sum_{h \in \{H\}} p_h\) and \(\sum_{f \in \{F\}} p_f\). By substituting the solutions into (3) and (4) and recalling the definitions of \(\sum_{k \in \{H-h\}} p_k\) and \(\sum_{g \in \{F-f\}} p_g\), we obtain the following equilibrium prices:

\[ p_i^N = \frac{c\gamma m(\gamma m+2\gamma(n-1)+2) + n\gamma(\gamma(n-2)\gamma+3)+c(1-\gamma)(2-\gamma)+\gamma m(1-\gamma)}{\gamma^2 m(m+2n)+n\gamma((n-2)\gamma+3)+(1-\gamma)(\gamma(3m-1)+2)} \]

\[ p_f^N = \frac{(1-\gamma)^2+c(1-\gamma)+c\gamma m(\gamma m+2\gamma(n-1)+2)+\gamma n(1-\gamma)+n\gamma(\gamma(n-1)+2)}{\gamma^2 m(m+2n)+n\gamma((n-2)\gamma+3)+(1-\gamma)(\gamma(3m-1)+2)} \]

\(^6\)This result extends to a general oligopoly setting the mixed-duopoly model by Ohnishi (2010).
with \( z = h, f \). This solution confirms the result by Anderson et al (1997) and Ghosh and Mitra (2008), that in a closed-economy framework (which can be recovered by setting \( n = 0 \) in our solution) a public firm competing over prices sets a price higher than the marginal cost, though lower than that of its private rivals. This is in sharp contrast with the behavior we would observe under quantity competition, where for all quantities produced by the private firms, the public firm reacts by producing the amount of its own product for which the market clearing price equals the marginal cost. Indeed, under quantity competition a marginal increase in the public firm’s production would not affect, for given quantities of the rivals, the contribution of the private firms to welfare, while it would increase the specific contribution of the public firm – i.e. \( \frac{1}{2} q_i^2 + \pi_i \) – so long as its clearing price exceeds the marginal cost. Under price competition, for given choices of the rivals, a price reduction by the public firm reduces the marginal contribution of the private firms to welfare, and this mitigates its incentive to price at marginal cost. Aggressiveness in prices is more detrimental for the private firms contribution to welfare than aggressiveness in quantities, and this explains why we do not observe efficient pricing of the public firm under price competition. Strategic complementarity plays also a role in the equilibrium configuration, in that the differences in public and private prices are lower under price competition.

The existence of foreign firms obviously makes for a stronger aggressiveness of the public firm both under quantity and under price competition. In the latter case, the properties of equilibrium are summarized in the following proposition.

**Proposition 1** In a single-country mixed oligopoly with price competition, the public firm sets a price \( p_i^N \) such that the mark-up over the marginal cost is decreasing in the share of foreign firms, and converges to zero when all private firms are foreign. The price of the public firm is lower than the price of the private firms – the difference \( p_i^N - p_i^N \) being decreasing in \( \gamma \).

Foreign firms contribute to the public firm’s objective function only through the consumer surplus, so that the negative effect of a reduction of \( p_i \) on their profits is neglected when domestic welfare is maximized. However, \( p_i \) converges to the marginal cost only when no domestic private firms are active in the market. The relative weight of foreign firms in satisfying consumers’ demand exerts a marked downward pressure on prices, but this effect of market internationalization does not rely on the competition among private firms; rather, it derives from the objectives pursued by the public firm. Notice that under quantity competition and constant marginal costs, market openness would be inconsistent with a non-negative profit constraint of the public firm.

These properties of equilibrium have clear consequences in terms of the welfare evaluation of cross-border acquisitions and privatization policies. If the domestic country acquires a foreign firm, the domestic CS decreases, due to a

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7 Calculations and simulations are available from the authors upon request.

8 However our result is consistent with the findings by Pal and Fjell (1996) in a Cournot setting with homogenous product and convex costs.
generalized increase in prices. As far as welfare is concerned, it certainly in-
creases if the acquisition price is not taken into account, since the profits of
the acquired firm now enter the welfare computation. If, on the contrary, the
acquisition price is computed when evaluating welfare, then the latter increases
if the foreign firm is paid according to its pre-acquisition profitability, while it
decreases if it is paid according to its post-acquisition profitability. The oppo-
site considerations apply for the acquisition of a domestic firm by foreign agents.
Again, a comparison ceteris paribus with the quantity setting case shows a rel-
levant difference: under quantity competition the acquisition of a domestic firm
would never be welfare improving, even if the revenues from the acquisition were
added to welfare.

Finally, the privatization of the public firm is clearly both CS and welfare
detrimental.

3.1 The sequential game

The above discussion should have clarified that the properties of equilibria under
price competition, and the differences with the correspondent outcome under
quantity competition, rely upon the very basic fact that the price decisions
of the public firm along its reaction function are taken for given prices of its
private rivals. While under quantity competition the expansion of a public firm
implies a reduction of the allocative inefficiency on the given quantity sold by the
private firms, under price competition the public firm can reduce the demand
faced by its private rivals at their given prices, but cannot affect the price
over cost margin at which these quantities are sold. Strategic complementarity
and substitutability do not play a relevant role in defining the key features of
equilibria.

On the contrary, the nature of strategic interaction becomes extremely rel-
levant when we extend the above set-up to allow for price competition in a
sequential game, with the public firm in the role of the leader and with the m
domestic and the n foreign private firms in the role of followers. All private
firms take the leader’s price as given and compete simultaneously in the second
stage of the game, while the leader moves in the first stage, anticipating the
followers’ reactions. Without loss of generality, in the analysis of this game we
assume \( c = 0 \).

Following the usual solution procedure of this Stackelberg game, we ob-
tain the following equilibrium prices of the sequential game for the welfare-
maximizing public leader and the private profit-maximizing firms:

\[
p_i^S = \frac{(1-\gamma)(\gamma m(1+\gamma(m-1))=\gamma n(1+\gamma(n-1)))}{\gamma^3(m+n)^3+\gamma^2 n^2(5-3\gamma)+2\gamma^2 mn(5-4\gamma)+(1-\gamma)(\gamma m(5\gamma(m-1)+8)+\gamma n(8-3\gamma)+2\gamma^2 mn)}
\]

\[
p_z^S = \frac{(1-\gamma)(1-\gamma)(2-\gamma)+\gamma(\gamma m(1+\gamma(n-1)))}{\gamma^3(m+n)^3+\gamma^2 n^2(5-3\gamma)+2\gamma^2 mn(5-4\gamma)+(1-\gamma)(\gamma m(5\gamma(m-1)+8)+\gamma n(8-3\gamma)+2\gamma^2 mn)}
\]

with \( z = h, f \). The main implications of this solution are gathered in the
following proposition.
Proposition 2. In a single country mixed-oligopoly with price competition and sequential moves, the public firm leader sets a price $p^S_i \leq p^N_i$, while the private firms set $p^S_z \leq p^N_z$; moreover, $p^S_i < p^S_z$, for any degree of product differentiation and for any given number of private and foreign firms. The price $p^S_i$ is again decreasing in the share of foreign firms and is equal to the marginal cost when $m = n$.

Sequentiality in price decisions with the public firm in the role of the leader would create a more competitive and welfare enhancing environment even in a closed-economy framework (Bárcena-Ruiz, 2007). The public firm anticipates that a reduction of its own price will be followed by a reduction of the unit profit margin of the private firms, due to strategic complementarity; this lowers the marginal negative impact on private profits of the public firm’s price decrease, allowing for $p^S_i$ to be closer than $p^N_i$ to marginal cost. The same positive effect on welfare would be observed under quantity competition, though in that case it would be associated, due to strategic substitutability, to a decrease in the production of the public firm (the price of which would exceed the marginal cost) and an increase in that of the private firms.\(^9\)

The presence of foreign firms causes a further decrease of the price set by the public firm at equilibrium, along the same lines described for the simultaneous game; again, market openness creates an additional impulse towards a generalized price reduction. The more aggressive attitude shown by the public firm when it takes the role of the market price leader is such that the latter behaves like a public monopolist when faced with an equal number of foreign and domestic private competitors: only a market with $n < m$ is consistent with a positive price-over-cost margin, while a market where foreign firms play a dominant role is one in which the public firm suffers from profit losses and is therefore inconsistent with a budget-balancing policy.

4 The two-country framework

We consider now a two-country model, in which two public firms, firm $i$ and firm $j$, located respectively in country $H$ and $F$, compete in a common market with $m$ private firms from country $H$ and $n$ private firms from country $F$, operating on this market regardless of their origin country. Therefore, we have $m + n + 2$ firms, each producing a variety of a differentiated product, under the same cost conditions of the previous section. The consumers of the two countries are identical in tastes and size, so that the demand faced by each firm on this international market now includes an identical demand from each of the two countries.

\(^9\)If the public firm anticipates the rivals’ contraction associated to its expansion, it perceives a trade-off between its own positive marginal contribution to welfare and the negative marginal contribution from the rivals. This result relies on increasing marginal costs if product homogeneity is assumed (Fjell and Heywood, 2002), but it is also observed under constant marginal costs if we allow for product differentiation - case in which $\gamma$ affects the extent to which the simultaneous and sequential solution differ.
The solution of problem (B) gives the reaction function for firm where the profits of a generic foreign firm as a function of \( p_i \) with 
\[
π_h = (p_h - c)(q_h + q_f)
\]
with 
\[
q_h = \frac{1-γ-(1+γ(m+n))p_h+γ(p_i+p_j+∑_{k∈(F,H)}p_k+∑_{j∈(F)}p_j)}{1-γ(1+γ(m+n))}
\]
and 
\[
q_f = \frac{1-γ-(1+γ(m+n))p_f+γ(p_i+p_j+∑_{k∈(H)}p_k+∑_{j∈(F)}p_j)}{1-γ(1+γ(m+n))}
\]
Notice that the consumer surplus is in both countries given by 
\[
SC_h = SC_f = \frac{(1-γ)(q_h^2+q_f^2+∑_{k∈(H)}q_k^2+∑_{j∈(F)}q_j^2)+γ(q_h+q_f+∑_{k∈(H)}q_k+∑_{j∈(F)}q_j)^2}{2}
\]
The solution of problem (A) yields the best reply of the public firm of the home country as a function of \( p_j \) and the private firms’ prices: 
\[
p_h = \frac{1+2c+γ(2c-1)+γp_j+3γ∑_{k∈(H)}p_k+γ∑_{j∈(F)}p_j}{3(1+γ(m+n))}
\]
Similarly, the public firm in the foreign country faces the following problem: 
\[
max_{p_f} (SC_f + Π_f)
\]
where the profits of a generic foreign firm \( f \) are 
\[
π_f = (p_f - c)(q_h + q_f)
\]
The solution of problem (B) gives the reaction function for firm j: 
\[
p_j = \frac{1+2c+γ(2c-1)+γp_h+3γ∑_{k∈(F)}p_k}{3(1+γ(m+n))}
\]
Let us now consider the optimal behavior of each private domestic firm. Maximizing \( π_h \) with respect to \( p_h \) we obtain the optimal reply function of firm h: 
\[
p_h = \frac{1+γ+γ(c(m+n)-1)+γ(p_i+p_j+∑_{k∈(H)}p_k+∑_{j∈(F)}p_j)}{2(1+γ(m+n))}
\]
In the same way, maximization of \( π_f \) with respect to \( p_f \) gives the optimal reply function of firm \( f \) :
\[
p_f = \frac{1+γ+γ(c(m+n)-1)+γ(p_i+p_j+∑_{k∈(H)}p_k+∑_{j∈(F)}p_j)}{2(1+γ(m+n))}
\]
Summing (9) over the \( m \) domestic firms and (10) over the \( n \) foreign firms and using (7) and (8), we obtain the following solution for the equilibrium prices: 
\[
p_i = \frac{(1-γ)(2c(2m+n)+γ+2)+cγm(3m+2γ+2)+3cγ^2n(2m+n)+2c(γ+2)+7cγn}{(γ+2)(3-γ)+(n+m)(γ(3(n+m)-2)+9)}
\]
\[ p_j = \frac{(1-\gamma)(2\gamma n + 2(\gamma + 2) + 3\gamma n + 2\gamma + 5) + 3\gamma^2 m(2\gamma + 2) + 2(\gamma + 2) + 7\gamma m}{(\gamma + 2)(3-n) + (n+m)(\gamma(3(n+m) - 2)+9)} \]

By substituting the aggregate equilibrium prices of private firms in (9) and (10), we obtain the individual prices of the private domestic and foreign firms:

\[ p_h = p_f = \frac{3c(\gamma + 1)(\gamma + 3 + 3\gamma(m+n)) + \gamma c(m+n)((6+\gamma) + 3\gamma(m+n))}{(\gamma + 2)(3-n) + (n+m)(\gamma(3(n+m) - 2)+9)} \]

Inspection of the equilibrium prices allows to establish the following proposition.

**Proposition 3** The prices \( p_h \) and \( p_f \) set respectively by the private domestic and the private foreign firms always coincide, as well as their profits. The prices set by the public firms differ provided that the number of private domestic and private foreign firms differs, with \( p_i \geq p_j \) when \( m \geq n \), and vice versa. Moreover, for any given \( m \), we find that \( p_i \geq p_h \) when \( n \leq \frac{2m-1}{\gamma} \), with \( \lim_{\gamma \to 1} \left( \frac{2m-1}{\gamma} \right) = m - 1 \). Similarly, for any given \( n \), \( p_j \geq p_f \) when \( m \leq \frac{2n-1}{\gamma} \), with \( \lim_{\gamma \to 1} \left( \frac{2n-1}{\gamma} \right) = n - 1 \).

The first two statements of Proposition 3 have an easy explanation. All private firms set the same price, since they have identical objective function and face the same market conditions. But if they are unevenly distributed across countries, the objective functions of the two public firms differ, with the profit component of welfare having a higher relative weight for the public firm operating in the country with the largest share of private firms. Therefore, the optimal reaction of this public firm to any given profile of the prices of the rivals is to set a higher price than the one which maximizes welfare for the other public firm. Indeed, while the marginal benefit in terms of higher consumer surplus of a price reduction is the same for both public firms – produced quantities affecting the consumer surplus of both countries symmetrically and independently of the origin country – the marginal cost in terms of lower domestic profits is higher for the public firm of the country where most private firms are located. The balance is therefore obtained at a higher price.

These considerations also help to understand why in the presence of an asymmetry in the cross-country distribution of firms the price of the public firm can be higher than that of the private firms. Suppose that most private firms are located in the domestic country. If the asymmetry is sufficiently large, the public firm of the foreign country perceives a strong incentive to set its price very close to marginal cost, for any given profile of the prices set by the rivals; this implies that all the other firms (foreign and domestic) face a downward shift of their demand functions. Under these tougher demand conditions, for the public domestic firm the marginal benefit on the consumer surplus of a price reduction is very low, and the balance with its marginal cost in terms of domestic profits may well occur at a price higher than the individual profit-maximizing price. When its marginal impact on the consumer surplus through price changes becomes very low, a welfare-maximizing behavior at the margin resembles a collusive behavior, which in our framework results into a protectionist-like attitude.
This result extends to price competition the idea already put forth by Dad- pay and Heywood (2006) in a quantity-setting framework with homogeneous product. In their two-country model the degree of asymmetry required for the domestic public firm to produce less than the private firms depends on the shares of the two countries in market demand. In our model, the reversal in the level of prices occurs for a cross-country asymmetry in the distribution of firms which depends on the degree of product differentiation. Indeed, this reversal occurs as a consequence of the reduction of demand faced by the private firms due to the aggressiveness of the foreign public firm; as $\gamma$ increases, the markets of the various firms become more interconnected and the spillover of the price decisions of each firm on the demand faced by the others becomes stronger. Given that the foreign public firm sets its price close to marginal cost, the demand contraction faced by the other firms is higher the higher is $\gamma$, so that for a low degree of product differentiation the incentive for the public domestic firm to take its protectionist-like role emerges even in the presence of a moderate asymmetry.

Through the asymmetry in the country-distribution of firms, and the related asymmetries in the public firms’ objective functions, the two-country model allows for more complicated interactions among public and private firms. This suggests that the answers to the related policy issues might be more complex than in the simple single-country approach.

4.1 Effects of cross-border acquisitions

In the two-country model the competition between public and private firms is framed within a competition between public firms – i.e. between governments interested in their own domestic welfare. If firms are symmetrically distributed, the objectives of the two public firms are perfectly aligned, while any asymmetry in the distribution of firms creates an asymmetry in their objective functions and in their behavior. This simple fact is at the basis of the welfare evaluation of cross-border acquisitions summarized in the following proposition.

Proposition 4 In a two-country model the aggregate welfare is maximum and the aggregate consumer surplus is minimum when firms are evenly distributed across countries. The cross-border acquisition of a private firm increases welfare in the acquiring country and decreases welfare in the other. The overall effect on aggregate welfare depends on whether the acquisition widens (the effect is negative) or narrows (the effect is positive) the asymmetry of the distribution of firms.

When firms are unevenly distributed, the actual share of aggregated profits and consumers’ surplus in domestic welfare is different in the two countries. This implies that in an aggregate perspective, one public firm is ‘too’ aggressive and the other ‘too’ cautious – an imbalance which favours consumers but reduces aggregate welfare. The country-specific effects are unambiguous and consistent with the standard findings of the literature.
4.2 Effects of privatization

When investigating the welfare effects of privatizations in a two-country model, we may take two different perspectives. The first is to evaluate the welfare effects (in aggregate and on the individual countries) of unilateral privatizations; the second is to assess the effects (aggregate and country-specific) of coordinated privatizations. In the first case we take the non-cooperative perspective of the strategic competition among governments; in the second we take the cooperative view of a supra-national authority.

The analysis of unilateral privatization amounts to comparing the outcome of the model with \( m + n + 2 \) firms described above with that of a single-country model where the demand coming from the two countries is satisfied only by private firms (increased by one) in the country which privatizes, and by \( m + 1 \) or \( n + 1 \) (the private and the remaining public) firms in the other.

Calculations are tedious but straightforward and yield the results summarized in Proposition 5.\(^{10}\)

**Proposition 5** The qualitative and quantitative effects of unilateral privatizations depend on \( \gamma \) and the degree of asymmetry in the distribution of firms. If firms are evenly distributed unilateral privatization is welfare detrimental for the country which privatizes, while the other country is positively affected only for sufficiently high values of \( \gamma \) - the threshold degree of product differentiation negatively depending on the number of private firms; aggregate welfare slightly increases for \( \gamma \) close to 1. If firms are unevenly distributed and the privatization is realized in the most populated country, then welfare slightly increases in the privatizing country if \( \gamma \) approaches from below the value at which the price reversal in the initial situation occurs; welfare in the other country and aggregate welfare both increase if \( \gamma \) is higher than this value, and decrease if this condition is not met. If firms are unevenly distributed and the privatization is realized in the less populated country, then it is always welfare detrimental for the privatizing country, while it is welfare improving for the other country for a large set of values of \( \gamma \) - aggregate welfare increasing provided that the value of \( \gamma \) is sufficiently high.

There are several interesting results in the above proposition. The first is that there are market structures in which unilateral privatization is welfare enhancing in the privatizing country. This occurs when the distribution of firms is uneven, and \( \gamma \) approaches the value at which the price reversal occurs. In this situation the private and public price are almost aligned, and the increase in profits associated to privatization turn out to overcompensate the decrease in the CS. The second is the positive impact on the non-privatizing country under even distribution of firms, for high values of \( \gamma \). When \( \gamma \) is sufficiently high, there is a relevant demand shift from the privatizing to the non-privatizing country, the market structure of which allows for a full exploitation of this

\(^{10}\)Details on these calculations are available from the authors on request. Simulations are presented in the Appendix.
demand benefit in terms of welfare. Finally, we stress the positive aggregate welfare impact of unilateral privatization, when it is realized in the country in which the price reversal between private and public firms is initially observed. Since in that case the welfare-maximizing role of the public firm in the most populated country collapses into a sort of protectionist attitude, its privatization implies a generalized price decrease, which generates a large welfare gain in the other country, where the CS has a relatively higher weight in national welfare. The welfare (profits) loss in the privatizing country does not compensate in aggregate, due to the low level of demand faced by its firms which compete with a very aggressive public firm in the non-privatizing country. Were instead the less populated country to privatize, then the overall market competitiveness would be relaxed, and price would increase, with obvious benefits for the firms of the other country. CS decreases, but aggregate welfare may increase, if the demand spillovers from the privatizing to the non-privatizing country are large enough – which occurs for high values of $\gamma$.

Notwithstanding this rich set of implications in terms of country-specific and aggregate effects of unilateral privatizations, the result that it generates a welfare loss for the privatizing country has the obvious consequence that in this two-country model non-privatization is a dominant strategy for welfare-maximizing, self-interested governments. This brings us to consider the alternative perspective of coordinated privatizations, which could be possibly considered by supranational authorities. In order to assess the impact of this cooperative approach to privatization, we compare the model with $m + n + 2$ firms with a standard Bertrand oligopoly in the common international market. Welfare comparisons between these different set-ups lead to the following proposition.

**Proposition 6** If firms are evenly distributed across countries, coordinated privatization reduces the individual country and global welfare, except in the case in which $\gamma$ is close to 1. If firms are unevenly distributed, welfare decreases unambiguously in the less populated country, while it increases in the other for a large range of sufficiently high values of $\gamma$. There is also a (high) threshold value of $\gamma$ above which global welfare increases.

The fact that under a symmetric distribution of firms, an aggregate welfare increase can occur only for very high values of $\gamma$ should not come as a surprise. As we approach the conditions of the standard Bertrand competition with homogeneous product, the interaction between private firms is sufficient to attain maximum welfare. In this environment, the presence of competing public firms concerned with their domestic welfare may paradoxically create a friction in the aggregate welfare enhancing competition among private firms. When the distribution of firms is asymmetric, the disadvantages of the coordinated privatization fall on the country in which the CS has a relatively higher weight in the welfare function. For a large set of values of $\gamma$, the other country – where profits have a larger relative role – benefits from the redistribution of demand and the increase in the prices of the private firms associated to the joint privatization. Again, when $\gamma$ becomes sufficiently high, this demand effect in the
most populated country and the competition effect among private firms create the scope for an aggregate welfare increase.

This analysis suggests a more careful evaluation of the advantages of joint privatization than that presented by Dadpay and Heywood (2006): under price competition with constant marginal costs and differentiated products a coordinated privatization is often detrimental for aggregate welfare, while for the latter to increase very well defined market conditions are required.

4.3 Concluding remarks

This paper is a first attempt to provide a systematic analysis of price competition with imperfect product substitutability in international oligopolistic mixed markets. By exploring the properties of market equilibria in a single country and in a two-country model, and by investigating the welfare consequences of privatizations and acquisitions, we have confirmed that in most cases public firms actually play on these markets a role of market regulators. Within a single-country approach, we have shown that, for any degree of product substitutability and any market structure, public firms are always successful in enforcing internal market discipline, by inducing all private firms to keep lower prices and by reacting to international competition with further beneficial price reductions.

Public firms can be looked at as an instrument of indirect regulation, also in the presence of interactions between governments in international markets. In a two-country model, this indirect regulatory role is always preserved at the country level, but it may vanish in a global perspective if the degree of product substitutability is sufficiently high. Indeed, the policy prescriptions which emerge in this context suggest that the welfare improving character of public enterprises is preserved in aggregate, provided that the market is not too close to homogeneous product conditions and provided that the domestic welfare objectives do not result in a sort of protection of domestic profits. If a protectionist-like behavior arises, with a public firm setting the highest price observed in the market, there are arguments in favour of coordinated privatizations. This occurs when there are relevant cross-country asymmetries in the distribution of firms, which disalign the objectives pursued by the public firms of different countries. Any increase in the degree of product substitutability enlarges the set of market configurations supporting this outcome. When products become very close substitutes, coordinated privatizations increase aggregate welfare even in the presence of a symmetric distribution of firms: the strategic interaction of firms pursuing domestic-wide – and not market-wide – objectives generates less efficient outcomes than the simple interaction of profit-maximizing firms.

The overall implication of this analysis is that international markets where governments compete through their firms, and compete with private firms, may require a supra-national coordination to achieve global welfare gains. While single governments never perceive incentives to privatize, supra-national bodies should suggest coordinated privatization policies when the tension between the objectives of the public firms becomes welfare detrimental. However, these
prescriptions are appropriate only in very well defined market configurations and cannot be considered as a general rule.

References


Appendix

In this Appendix we provide graphical simulations for the results stated in Propositions 5 and 6.

Unilateral privatization under even distribution of firms.

Let us start with an even distribution of firms, i.e. $m = 4$ and $n = 4$ and consider a unilateral privatization in one of the two countries. Figures A1-A3 show its effects on the welfare of the privatizing country, the welfare of the non-privatizing country and aggregate welfare, for the different relevant values of $\gamma$.

Figure A1. Effect on the privatizing country

Figure A2. Effect on the non-privatizing country
Unilateral privatization under uneven distribution of firms

Assume now an uneven distribution of firms, e.g. $m = 2$ and $n = 4$. In this case the price reversal occurs in the foreign country for $\gamma = 0.5$. We have now to consider two case: privatization in the most populated and in the less populated country. Figure A4-A6 show the effects of the privatization in the first case, while Figures A7-A9 refer to the second.

Privatization in the most populated country

Figure A4. Effect on the privatizing country
Figure A5. Effect on the non-privatizing country

A6. Effect on aggregate welfare

Privatization in the less populated country

A7. Effect on the privatizing country
Coordinated privatization

Again, we distinguish between the two cases of even and uneven distribution of firms.

Coordinated privatization under even distribution of firms
In this case the effects on welfare are obviously the same in the two countries. Aggregate welfare is simply the double of country-specific welfare. Figure A10 shows the country-specific effect for $m = n = 4$. 

Figure A8. Effect on the non-privatizing country

Figure A9. Effect on aggregate welfare
Coordinated privatization under uneven distribution of firms

Assume, as before, $m = 2$ and $n = 4$. Figure A11-A13 show the effects of a coordinated privatization on the most populated country, the less populated country and aggregate welfare.
Figure A13. Effect on aggregate welfare