The Role of Information Costs in Direct and Intermediated Trade

Dimitra Petropoulou†

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Abstract

This paper is motivated by the observation that intermediaries play an important role in international trade such that direct trade between producers and consumers is observed alongside indirect trade through intermediaries. A growing body of evidence also documents the importance of information frictions for exporting. The paper examines the matching role of intermediaries through the building of contacts in a market characterised by information frictions. A pairwise matching model with two-sided information asymmetry is developed to analyse the channels through which information costs can affect the pattern of direct and intermediated trade. All traders can match directly through a stochastic matching process but some may match indirectly through an intermediary who invests in establishing a network of contacts. Intermediation is shown to unambiguously raise expected trade volume and welfare by expanding the set of matching technologies available to traders, while convexity in network-building costs with respect to network size gives rise to both direct and indirect trade in equilibrium. The trade pattern depends on the relative responsiveness of the direct and indirect matching technologies to information costs, which for some parameter values generates a non-monotonic relationship between information frictions and trade.

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†Hertford College, University of Oxford, Catte Street, Oxford, OX1 3BW, United Kingdom. Tel: 01865 279 457 Email: dimitra.petropoulou@economics.ox.ac.uk
1 Introduction

A growing body of empirical literature, using data from a range of countries, has highlighted the importance of intermediaries in international trade. The evidence suggests that the usual assumption made in the theoretical trade literature, namely that a producer trades directly with a final consumer, is a far cry from reality, where trade through wholesalers and retailers constitutes a significant proportion of international transactions. Ahn et al. (2010) report that intermediaries account for 20 per cent of China’s exports in 2005 whereas Blum et al. (2009b) find that intermediaries account for over 40% of Chilean importers of goods from Argentina\(^1\). Working with U.S. data, Bernard et al. (2009) find that ‘pure’\(^2\) wholesalers and retailers\(^3\) account for even large shares of trading firms\(^4\), reporting figures of over 40 per cent of exporting firms and over 50 per cent of importing firms.

Few cases are observed where small exporters match with small importers, suggesting that direct matching between small traders is difficult. Instead, Blum et al. (2009a, 2009b) find that small exporters typically match with one large importer, often an intermediary, whereas Ahn et al. (2010) find intermediaries are used by relatively small Chinese firms who find it difficult to penetrate export markets on their own. This evidence supports the notion that trading costs arise from the cost of buyers and sellers identifying each other; where both parties are small, this is particularly difficult. Moreover, Bernard et al. (2009) find that large, vertically integrated firms engage in both production and intermediation in-house, reinforcing the idea that scale is important for intermediation, whether this is achieved within the firm or by offering intermediation services to a broad range of relatively small firms. This paper presents a model in which the optimal scale of an intermediary’s network of trader contacts, and hence the proportion of intermediated trade, is endogenously determined as a function of information frictions and technological parameters.

Spulber (1996) defines an intermediary as ‘an economic agent that purchases from suppliers for resale to buyers or that helps buyers and sellers meet and transact’. There is a broad literature addressing the many functions of middlemen. They have been shown to reduce search costs (Rubinstein and Wolinsky, 1987; Yavas, 1992, 1994), to offer expertise in markets with adverse selection (Biglaiser, 1993), to operate as guarantors

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\(^1\)Wholesalers account for 35 per cent and retailers for 6 per cent of Chilean importers.

\(^2\)Where 100 per cent of employment is in these sectors.

\(^3\)For empirical evidence on retail chains see Basker and Van (2008a) and Basker and Van (2008b).

\(^4\)Note that while Bernard et al. (2009) find that intermediaries account for a large share of US trading firms, their share of export value is much lower than firms that purely or mostly produce and consume.
of quality under producer moral hazard (Biglaiser and Friedman, 1994), as well as to operate as investors in quality-testing technology (Li, 1998). More recently, Shevchenko (2004) endogenises the number of intermediaries who buy and sell goods and examines the optimality of the size and composition of their inventories.

Recent contributions to the international trade literature in heterogeneous firms\(^5\) include extensions of the heterogeneous firm framework to encompass intermediation. These theoretical contributions (Ahn et al 2010; Akerman 2009; Felbermayr and Jung, 2009; Blum et al., 2009a) focus on technological differences across firms and are mainly used to motivate empirical analysis. Common to all of these works is the exploration of the role of intermediaries as buyers and sellers of goods, i.e. the first aspect of intermediation identified by Spulber (1996). Effectively, intermediaries represent an alternative distribution technology, in which the trade-off between fixed costs and marginal costs interacts with firm productivity to determine the mode of distribution selected by firms. Blum et al. (2009a) argue that the fixed fee in their intermediation technology may reflect the cost of establishing an intermediation firm, investing in industry contacts through attendance at trade fairs or buying a database of producer etc, while variable costs depend on the number of firms the intermediary seeks to identify\(^6\). The literature motivates technological assumptions by appealing to information frictions and matching costs, yet the precise determinants of direct and indirect costs of exporting remain largely unexplained.

This paper focuses on developing theoretical microfoundations for the second aspect of intermediation as defined by Spulber (1996), namely the matching role of intermediaries through the development of contacts in a trading environment characterised by information frictions. Rauch and Watson (2004) present evidence from a pilot survey of US-based, international trade intermediaries that suggests half of trade intermediation in differentiated products does not involve taking title of goods and reselling (as compared to only 1 per cent for homogeneous-goods). This is consistent with the search based or network view of trade, pioneered by Rauch (2001), Rauch and Trindade (2002) and others, that posits that the information requirements for differentiated goods are much greater due to the need to match specific characteristics.

There is a general consensus in the literature that information matters for exporting and that information frictions generate costs that can impede trade, particularly for

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\(^5\)See Melitz (2003), Helpman, Melitz and Yeaple (2004), Chaney (2008) and Arkolakis (2009), among others.

\(^6\)Blum et al (2009a) consider costs of matching but do not follow a random matching methodology as used in this paper, and in Rauch and Watson (2004) and Antràs and Costinot (2009). Rather, they opt for a simpler approach based on Townsend (1983).
small firms. Although information costs cannot be directly observed in the data, the importance of information for trade has been highlighted repeatedly through surveys and other anecdotal evidence. For example, Roberts and Tybout (2007) report evidence based on interviews of Columbian firms in 1990 that identifies cost of information acquisition as a key component of entry costs into export markets. Firms emphasize the need to identify and make contact with buyers, to get information on foreign prices, market selection, as well as on standards and testing requirements. Many firms reported using both private (for a fee) and public external assistance (brokers, distributors, chambers of commerce etc) to overcome information obstacles. At the same time, many firms reported carrying out their own research in foreign market selection and buyer identification and contact, such that the direct and indirect channels of exporting were used equally.

The need to overcome information barriers is further evidenced by the growing number of international trade fairs and international trade shows (over 19000 worldwide in 2010 from 16000 in 2007), whose role is quite clearly to match buyers and sellers. With just a few exceptions (e.g. Maskell et al., 2004), the importance of trade shows has gone largely unstudied in the economics literature. There is, however, a growing international marketing literature that focuses on the role of international trade shows for the internationalisation of small firms (e.g. Evers and Knight, 2008) and on identifying motivations for international trade show participation (e.g. Hansen, 1996). Interestingly, international trade shows can facilitate both direct trade, by bringing buyers and sellers together, and indirect trade, by presenting opportunities for intermediaries to broaden their networks and for firms to appoint agents or distributors. This paper sheds light on the mechanisms through which trade facilitation policies may impact aggregate trade flows through their impact on both direct and indirect trade channels.

One strand of the intermediation literature investigates the role of information-sharing networks, such as ethnic minorities and business groups, in facilitating trade (e.g. Casella and Rauch, 2002; Combes et al., 2005; Krautheim, 2004). The emphasis of this literature is primarily on the effects of pre-existing social ties or contacts on trade. Another strand models exporters’ distribution problem of identifying and selling to customers as a random matching process (e.g. Antràs and Costinot, 2009 and Rauch and Watson, 2004). This paper draws on both the random matching and networks-based trade strands of the literature to examine theoretically how information frictions affect the realisation of trade matches and the pattern of trade through direct and indirect channels. The impact of information costs on both the technology of intermediation

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7 For details on international trade shows by country and industry visit www.fita.org/tshows.html.
and direct matching between importers and exporters is explicitly modeled, thereby inform- ing on the interplay between information costs and direct and indirect exporting. In doing so, the model fills a gap in the literature on intermediation by exploring the channels through which information costs can affect the pattern of direct and interme- diated trade. It does not rely on pre-existing ties between agents, but rather examines the incentives for contact-building by intermediaries and how intermediation can offer a more efficient means of matching.

The model considers a two-sided market formed by a continuum of uniformly dis- tributed, differentiated pairs of traders, whose chances of matching directly depend on the level of information frictions. For simplicity, the model abstracts from specifics of any particular model of international trade, allowing trade pairs in a partial equilibrium setting to reflect trading opportunities in reduced form; the focus is instead on the modes of international trade. Besides direct matching, traders may have the opportunity to match indirectly through a single intermediary who invests in establishing a network of contacts. Crucially, information frictions also impact on the intermediary’s cost of developing a network of contacts and the commission commanded for matching traders, such that the share of intermediated trade to total trade is shown to depend on the relative responsiveness of the direct and indirect matching technologies to information costs. This can be better understood through the example of trade fairs. Government trade facilitation schemes that encourage participation in trade fairs can be thought of as reducing information frictions, both in terms of improving the probability of di- rect matching (which lowers the fee the intermediary can command) and in terms of lowering the costs of contact-building by intermediaries. Intermediation is shown to un- ambiguously increase expected trade volume and welfare, however the nuanced impact of information costs on the different modes of transacting makes it possible for the relationship between information frictions and trade to be non-monotonic. This, in turn, sheds light on the possible mechanisms through which trade facilitation policies may impact on international trade.

The model is, of course, highly stylised. Nevertheless, it sheds light on the microfoundations of trade intermediation and informs on how the pattern of direct and intermediated trade are affected by changing information costs. While the model is motivated by the growing literature on intermediation in international trade, the model applies equally to matching of domestic sellers with domestic buyers. Of course, information frictions that render matching of domestic buyers and sellers costly are likely to be even more pertinent in the international context. Furthermore, the model can also be applied more broadly to intermediated markets where contact-building and matching
are key, for example headhunters, real estate agents or matchmakers in the marriage market.

The remainder of this paper is organised as follows. Section 2 introduces the intermediation model and section 3 discusses the key findings and concludes.

2 The model

This section introduces a pairwise matching model with a continuum of importers and exporters, and a single trade intermediary to capture the incentives for network-building and intermediation where there are barriers to the flow of information.

2.1 Model set-up

Consider a two-sided market\(^8\) where importers and exporters match in pairs to exchange a single unit of output. Let there be a continuum of exporters \((X)\) and a continuum of importers \((M)\), each distributed uniformly and with unit density over the interval \([0, 1]\).

Suppose that for each trader there is a unique partner on the other side of the market with whom they can trade. Each transaction generates a joint surplus \(S > 0\), but if agents fail to locate their match they receive a payoff of 0. All market participants are risk-neutral.

The framework best reflects trade in differentiated goods where specific characteristics have to be matched, whether these are features of the product, timing of delivery etc. In the absence of trade frictions, importers and exporters can identify each other costlessly and all trade opportunities are exploited generating a total surplus of \(S\).

Let there be two-sided information asymmetry such that traders on each side of the market do not know the location of their partner on the opposing side. Within the set of infinitely many traders, the probability of each exporter (importer) locating her partner by selecting a random trader from the measure of importers (exporters) is zero. Any pair \(j\) of trade partners \((X_j, M_j)\) can, however, match through a direct matching technology (‘direct trade’), which achieves successful matching with probability \(q(i)\), where parameter \(i \in [0, 1]\) reflects the level of information costs or barriers to information flow between the two sides of the market. Let \(q'(i) < 0\), so a higher prevailing level of information costs implies a lower probability of matching for each pair. Parameter \(i\) may be interpreted as reflecting the state of information and communication technology (ICT). An ICT improvement reflects a decline in \(i\), which in turn implies a higher

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\(^8\)For a discussion on two-sided markets see Spulber (2009), Chapter 10.
probability of a direct match. Further, let \( q(1) = 0 \) and \( q(0) = 1 \), so information cost level \( i = 1 \) prohibits any matching, while \( i = 0 \) corresponds to the full information case where all trade opportunities are exploited. \( q(i) \) is also the expected trade volume and \( q(i)S \) the expected joint surplus from direct trade. The two-sided market is represented in figure (1).

Suppose the market has a single intermediary \( (I) \) with access to a technology for developing contacts with importers and exporters and finding out their trade characteristics (location, product features etc.). The intermediary’s network is denoted by a measure of importers, \( P_M \in [0, 1] \), and a measure of exporters, \( P_X \in [0, 1] \), contacted by the intermediary. To form a network \( \{P_X, P_M\} \) the intermediary incurs a set up cost, \( F \), and a marginal cost of network expansion, \( c(i, P_X + P_M) \), which is increasing in both information costs and overall network size. Let \( C(P_X, P_M) \) denote the intermediary’s total investment cost for building a network of contacts of size \( \{P_X, P_M\} \):

\[
C(P_X, P_M) = F + c(i, P_X + P_M)(P_X + P_M)
\]  

Once network investment costs are sunk, it is costless for the intermediary to match trade pairs from within his network of contacts (‘indirect trade’). The intermediary’s marginal cost of trade intermediation is zero and he is able to match trade pairs from within his network\(^9\) with probability 1. The proportion \( P_X \) also reflects the \textit{ex ante} probability that any particular exporter \( X_j \) is a network member. Similarly, \( P_M \) is the \textit{ex ante} probability that any particular importer \( M_j \) is a network member. Thus, \( P_X P_M \) describes the \textit{ex ante} joint probability that pair \((X_j, M_j)\) is contacted by the intermediary.

The model focuses on the matching function of trade intermediaries, and hence abstractions from the idea that intermediaries take traded goods onto their books and sell them on. Instead, the intermediary raises revenue by charging a commission for matching trading partners through his network. Let \( \alpha_I \) denote the share of trade surplus, or commission rate, the intermediary demands for successful intermediation of trade.

### 2.1.1 Timing of the game

The timing of the game between traders and intermediary is as follows:

**Stage 1 - Network investment:** The intermediary invests in a network of size \( \{P_X, P_M\} \) by contacting a proportion of importers and exporters. Network investment costs,\(^9\) This assumption can easily be relaxed so that indirect matching takes place with a probability less than 1 but higher than the probability of direct matching.
C(P_X, P_M), are sunk. The intermediary offers contacts a take-it-or-leave-it contract specifying commission rate α_I for successful matching.

**Stage 2 - Contracting:** Traders in receipt of a contract accept or reject it.

**Stage 3 - Indirect trade:** Uncertainty over which trade matches are feasible through the network is resolved. The intermediary matches pairs of traders in his network, provided both parties accepted in stage 2.

**Stage 4 - Direct trade:** Any unmatched traders trade directly with probability q(i).

### 2.1.2 Equilibrium concept

The solution concept used is subgame perfect equilibrium (SPE) and the method employed is backward induction. A strategy for intermediary I is a set \{P_X(i), P_X(i), α_I(i)\} that describes network size and commission rate, given information costs i. A strategy for trader j is described by a rule \(R_a\) for accepting or rejecting a contract in stage 2, if such a contract is received. A set of strategies \{P_X^*(i), P_X^*(i), α_I^*(i), R_a^*\} can be said to form a subgame perfect equilibrium of the game if under these strategies the expected profit of the intermediary and the expected trade surplus of each trader are maximised, given the strategies of all other players.

### 2.2 Direct and indirect matching

The pool of unmatched traders in the final stage of the game includes three groups of traders: (a) those not contacted in stage 1, (b) those contacted but who rejected the contract in stage 2, and (c) those who were contacted and accepted, but could not be matched through the network in stage 3. Unmatched traders can expect to match directly with probability q(i) in the final stage of the game. Each direct match generates S, so the *ex ante* expected surplus from the direct trade route is q(i)S. Let α_X and α_M denote the surplus shares of exporters and importers, respectively, where α_X + α_M = 1.

For simplicity, assume both parties have equal bargaining power so gains from any transaction are split evenly\(^{10}\), such that α_X = α_M = \(\frac{1}{2}\). The expected payoff from direct trade for importers and exporters, denoted by \(E(\Pi^{DT}_M)\) and \(E(\Pi^{DT}_X)\), respectively, are thus:

\(^{10}\)The particular values of α_X and α_M have no bearing on the intermediary’s investment decision, or choice of commission rate. Symmetry is assumed for simplicity.
\[
E(\Pi_X^{DT}) = E(\Pi_M^{DT}) \equiv E(\Pi^{DT}) = \frac{1}{2} q(i) S
\]  

(2)

Intermediated trade transactions in stage 3 between network members who accept in stage 2 also generate \( S \) per match. The intermediary maximises stage 1 expected profit subject to participation constraints, thereby ensuring that all traders contacted find it optimal to accept in equilibrium\(^{11}\). Let \( \alpha_k \) denote the share of trade surplus captured by \( k \), given information costs \( i \), where \( k = \{X, M, I\} \). As with direct trade, exporters and importers are assumed to split (residual) surplus equally, so \( \alpha_X = \alpha_M \equiv \alpha_T \). It follows that:

\[
2\alpha_T + \alpha_I = 1
\]

(3)

Traders’ expected payoffs from indirect trade, denoted by \( E(\Pi_X^{IT}) \) and \( E(\Pi_M^{IT}) \), respectively, can thus be expressed as:

\[
E(\Pi_X^{IT}) = E(\Pi_M^{IT}) \equiv E(\Pi^{IT}) = \frac{1}{2} (1 - \alpha_I) S
\]

(4)

The measure of intermediated transactions in stage 3 varies depending on the degree of overlap between the two groups of contacts, \( P_X \) and \( P_M \). Let the measure of intermediated trade matches be denoted by the random variable \( T_I \). The largest measure of intermediated matches possible is \( \min \{P_X, P_M\} \), reflecting the maximal overlap between importer and exporter contacts, while the smallest measure of matches is \( \max \{P_X + P_M - 1, 0\} \), where mismatch between the two contact groups is greatest.

For any pair \((X_j, M_j)\), the \textit{ex ante} probability of matching through the intermediary is given by \( P_X P_M \), the joint probability of both partners being contacted in stage 1. Integrating over the range of possible pairs gives the expected measure of intermediated matches \( E(T_I) = P_X P_M \). The intermediary maximises profit from a given network investment by choosing \( P_X \) and \( P_M \) to maximise expected matches \( E(T_I) \). It follows directly from the first order conditions that symmetric investment in network-building is optimal, such that \( P_X = P_M \equiv P \) and hence \( E(T_I) = P^2 \). The subgame perfect equilibrium strategy set can thus be redefined as \( \{P^*(i), \alpha_I^*(i), R_n^*\} \) and marginal cost as \( c(i, P) \).

For any exporter (importer) evaluating whether to sign up with the intermediary in stage 2, the probability of her partner also being in the network is \( P \). Each trader can expect to receive \( E(\Pi^{IT}) \) with probability \( P \) and \( E(\Pi^{DT}) \) with probability \( 1 - P \). Trader

\(^{11}\)Since traders are identical in terms of their future trade prospects, they all either accept or reject the take-it-or-leave-it offer in stage 2.
expected payoff conditional on being contacted in stage 1, is thus:

\[ E(\Pi_X | X_j \in P) = E(\Pi_M | M_j \in P) = \frac{1}{2} [P (1 - \alpha_I) + (1 - P)q(i)] S \quad (5) \]

To ensure trader participation in stage 2, the intermediary must set \( \alpha_I \) such that expected payoff from signing up to the network, described by (5), is at least as large as the expected payoff from an exclusively direct trade route, given by (2). The highest commission rate consistent with trader participation is thus:

\[ \alpha_I = 1 - q(i) \quad (6) \]

Hence, traders’ optimal acceptance rule \( R^*_I \) in stage 2 is ‘accept the contract if \( \alpha_I \leq 1 - q(i) \); reject otherwise’. Anticipating traders’ incentives in stage 2, the intermediary sets\(^{12}\) \( \alpha^*_I(i) = 1 - q(i) \) in stage 1 and all contracts offered are accepted.

The intermediary is constrained by traders’ outside option to trade directly, which in turn depends on the level of information costs. The worse are traders’ prospects in the market, the higher the commission rate the intermediary can charge while ensuring trader participation. While larger network improves the chances of an indirect trade match, the option to trade directly remains available, so \( \alpha^*_I(i) \) is independent of \( P \).

At the outset of the game, any pair \((X_j, M_j)\) that anticipates a network of size \( P \) can expect to find themselves in one of four possible positions: (a) with probability \( (1 - P)^2 \), both trade partners are outside the network; (b) with probability \( P(1 - P) \), \( M_j \) is inside the network and \( X_j \) outside; (c) with probability \( P(1 - P) \), \( X_j \) is inside the network and \( M_j \) outside, and (d) both partners are members of the network, with probability \( P^2 \). The expected payoff for each partner is \( \frac{1}{2} q(i) S \) in (a)-(c) and \( \frac{1}{2} (1 - \alpha_I) S \) in (d). Weighing the expected payoffs with their respective probabilities yields the expected payoff to any trader \( j \) at the outset of the game:

\[ E(\Pi_X) = E(\Pi_M) = \frac{1}{2} [q(i)(1 - P^2) + (1 - \alpha_I) P^2] S \quad (7) \]

Since \( \alpha^*_I(i) = 1 - q(i) \), equation (7) simplifies to give \( E(\Pi_X) = E(\Pi_M) = \frac{1}{2} q(i) S = E(\Pi^{DT}) \). As all surplus over and above that generated through direct trade is appropriated by the intermediary in equilibrium, traders are indifferent between direct matching and the prospect of intermediated trade.

\(^{12}\)Assume that when indifferent between the two modes of trade, traders sign up with the intermediary. Alternatively, the intermediary could offer an infinitesimally small additional amount, \( \varepsilon \), to ensure traders sign up to the network.
An important simplifying assumption is that the probability of any pair \((X_j, M_j)\) matching directly, \(q(i)\), depends only on \(i\) and not on the mass of pairs already matched by the intermediary. In other words, there are no congestion externalities in the direct matching technology. Since the probability of matching through the intermediary is a function of network size \(P\), the mass of traders in the intermediary’s network is a crucial determinant of intermediated trade in this model\(^\text{13}\), while not so for the direct trade trade technology. This assumption makes starker the distinction between the network-based matching technology of the intermediary and the direct matching technology. In practice, trading firms may actively develop their own networks of business contacts through, for example, attendance at trade shows, the size of which is likely to be important for the chances of matching directly. This aspect is not considered in the model for the purposes of tractability.

2.3 Trade and welfare

Since any unmatched network members in stage 3 continue to have the opportunity to trade directly in stage 4, expected trade can never be lower with an active intermediary in the market than without. This is formalised in proposition 1.

**Proposition 1** An active intermediary raises expected trade volume unambiguously compared to when only direct trade is possible.

**Proof.** Let \(E(T)\) denote expected trade volume with an intermediary in the market and \(E(T^{DT})\) denote expected trade volume when only direct matching is possible. Network \(P \in [0, 1]\) generates \(P^2\) expected matches in stage 3 and a proportion \(q(i)\) of the remaining \((1 - P^2)\) pairs trade directly in stage 4. Hence:

\[
E(T) = q(i) + P^2 [1 - q(i)] \\
\geq q(i) = E(T^{DT})
\]

Expected trade volume with an intermediary is thus at least as large as when only direct trade is possible and unambiguously higher when the intermediary is active \((P > 0)\). □

Since traders are as well off (in expected terms) under the intermediation contract as through direct trade, the intermediary’s expected profit represents a pure welfare gain.

\(^\text{13}\)This is in line with the notion of intermediation in Antràs and Costinot (2009), where trading opportunities depend on the relative mass of unmatched intermediaries to producers. There is no direct trade route in their model, however, as producer access to the Walrasian market is assumed to occur exclusively through matching with an intermediary.
The gain arises from the fact that the intermediary expands the set of possible production technologies for matching, while exclusive appropriation of these welfare gains stems from being a monopolist provider of the indirect matching technology. Proposition 2 formalises this discussion\textsuperscript{14}.

**Proposition 2** An active intermediary raises expected welfare unambiguously compared to when only direct trade is possible.

**Proof.** Let $E(W)$ denote expected welfare with an intermediary in the market and $E(W^{DT})$ denote expected welfare when only direct matching is possible. The total surplus generated from direct and indirect trade is $P^2S$ and $q(i) (1 - P^2) S$, respectively, giving:

\[
E(W) = (1 - P^2) q(i) S + P^2 S - 2c(i, P)P - F = q(i) S + [1 - q(i)] SP^2 - 2c(i, P)P - F
\]

For values of $i$ where $E(\Pi_I) < 0$ the intermediary is inactive ($P^* = 0$); if $E(\Pi_I) \geq 0$ then $P^* \geq 0$, implying that $[1 - q(i)] S (P^*)^2 \geq 2c(i)P^* - F$. Hence:

\[
E(W^*) = q(i) S + [1 - q(i)] S (P^*)^2 - 2c(i, P)P^* - F \geq q(i) S = E(W^{DT})
\]

Equilibrium expected welfare with an intermediary is thus at least as large as expected welfare when only direct trade is possible and unambiguously greater where $P^* > 0$. \hfill \blacksquare

These results of propositions 1 and 2 emphasize that, through matching, intermediaries have a key role as trade facilitators, giving consumers access to a broader range of products than they would otherwise have access to.

\textsuperscript{14}Note the results of propositions 1 and 2 are general in that they do not depend on the functional forms of $c(i, P)$ and $q(i)$. 

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2.4 Equilibrium network size

To characterise the intermediary’s optimal choice of networks size some further restrictions are placed on the marginal cost of network expansion:

\[ c(0, \cdot) = 0 \ ; \ c(\cdot, 0) = 0 \]
\[ c_i(i, P) > 0 \ ; \ c_{ii}(i, P) \geq 0 \]
\[ c_p(i, P) > 0 \ ; \ c_{pp}(i, P) > 0 \]
\[ c_{ip}(i, P) = c_{pi}(i, P) > 0 \]

As described in (11), \( c(i, P) \) is monotonically increasing in \( i \), for any given network size \( P \), and monotonically increasing in \( P \), for given \( i \). Convexity of marginal cost in network size \( P \) (but not \( i \)) is necessary for an interior equilibrium \( P^* \in (0, 1) \) to exist for some range of \( i \); otherwise, only corner solutions exist\(^{15} \). This assumption implies the intermediary’s search process generates contacts that are increasingly inaccessible, so that it increasingly costly to widen the range of contacts. This is arguably a realistic assumption in that when building a network of contacts one can imagine some are easier to make than others and that when expanding a network those that can be made at lower cost are established first\(^{16} \).

The intermediary chooses \( P \in [0, 1] \) to maximise expected profits, \( E(\Pi_f) \), subject to \( \alpha^*_f(i) = 1 - q(i) \) and \( R^*_a \), where:

\[ E(\Pi_f) = [1 - q(i)]SP^2 - 2c(i, P)P - F \]  

For concreteness and to solve for \( P^* \) analytically let \( c(i, P) \) take the following form:

\[ c(i, P) = \gamma i^{\alpha} P^{\beta} \], where \( \alpha \geq 1, \beta > 1 \) and \( \gamma > 0 \)

Parameter \( \alpha \) is the elasticity of marginal cost \( c(i, P) \) with respect to information costs \( i \), \( \beta \) is the elasticity of marginal cost with respect to network size \( P \) and \( \gamma \) is a shift factor. Total network investment cost \( C(i, P) = F + 2\gamma i^{\alpha} P^{\beta+1} \) is thus convex in \( P \).

Further, let \( q(i) \) be described by:

\(^{15}\) Under alternative cost specifications, where marginal cost of network expansion is independent of network size e.g. \( c(i) \), where \( c'(i) > 0 \) and \( c(0) = 0 \), it can be shown that there exist only corner solutions to the intermediary’s profit maximisation problem, with contacts developed with all traders, or none.

\(^{16}\) There is evidence that both direct and indirect trade channels are observed amongst exporting firms e.g. see Roberts and Tybout (2007).
\[ q(i) = 1 - i^\delta, \text{ where } \delta \geq 1 \quad (14) \]

It follows that \( \alpha^*_f(i) = i^\delta \), where \( \delta \) denotes the elasticity of the equilibrium commission rate with respect to information cost \( i \). Substituting (13) and (14) into (12) yields the following expression for expected profits:

\[ E(\Pi_f) = Si^\delta P^2 - 2\gamma i^\alpha P^{\beta+1} - F \quad (15) \]

Maximising (15) with respect to \( P \) yields\(^{17}\) network size \( \tilde{P} \), expressed in terms of \( \alpha, \beta, \gamma, \delta \) and \( S \):

\[ \tilde{P} = \left[ \frac{Si^\delta}{\gamma(\beta+1)} \right]^{\frac{1}{\beta+1}} > 0 \quad (16) \]

Equilibrium network size is given by \( \min \left\{ \tilde{P}, 1 \right\} \) provided set-up costs \( F \) are sufficiently low relative to trade surplus \( S \) so that \( E(\Pi_f) \geq 0 \), and 0 otherwise. Lemma 3 describes the necessary condition for expected profit in the interior equilibrium to be increasing in \( i \).

**Lemma 3** The intermediary’s expected profit is monotonically increasing in information costs \( i \) if \((\beta + 1)\delta > 2\alpha\), where \( P \in (0,1) \).

**Proof.** For proof see Appendix B. \( \blacksquare \)

Condition \((\beta + 1)\delta > 2\alpha\) implies that as information costs increase, the direct matching route worsens relatively more than the cost of network provision. The intermediary can thus enjoy higher expected profits by relaxing the constraint on the commission fee the intermediary can demand.

There are four distinct equilibrium patterns of network investment. The parameter space is split into four ranges, denoted by (A)-(D), each corresponding to a different set of incentives for network investment:

(A) \( \delta > \alpha \geq 1 \): When the elasticity of the intermediary’s optimal commission rate with respect to information costs, \( \delta \), exceeds the elasticity of the marginal cost of network expansion with respect to information costs, \( \alpha \), then optimal network size is increasing with \( i \). As information costs rise, the increase in the commission rate the intermediary can command exceeds the increase in networking cost \( c(i, P) \) making a network expansion profitable.

\(^{17}\)A derivation of (16) is included in Appendix A.
(B) $\delta = \alpha \geq 1$: When the elasticities are exactly equal, then the effects of changing information cost $i$ on the intermediary’s cost and expected revenue exactly offset each other, so optimal network size is unchanging$^{18}$ with $i$.

(C) $\frac{2\alpha}{\beta+1} < \delta < \alpha$: When the elasticity of marginal networking cost exceeds the elasticity of the commission rate with respect to $i$, then it is optimal for the intermediary to reduce network size as information costs rise. Since $(\beta + 1)\delta > 2\alpha$, then from lemma 3 $c(i, P)$ is sufficiently elastic with respect to $P$ so as to offset the effects of information cost $i$, raising equilibrium profit overall.

(D) $\delta \leq \frac{2\alpha}{\beta+1} < \alpha$: When the commission rate is less responsive to $i$ than is $c(i, P)$, then it is optimal for the intermediary to reduce network size as information costs rise. Since $(\beta + 1)\delta \leq 2\alpha$, then from lemma 3 $c(i, P)$ is insufficiently elastic with respect to $P$ so as to offset the effects of information cost $i$, lowering equilibrium profit overall.

These patterns of intermediation shed light on how information frictions affect direct and indirect matching technologies. The model thus suggests that we can learn about the relative elasticities of the costs of network provision and the probability of direct matching from an empirical examination of the impact of changing information costs on intermediation. The rest of the section formally characterises the interior equilibrium path of network size, expected trade and expected welfare for parameter ranges (A) - (D), with further intuition provided through the discussion of illustrative examples.

### 2.4.1 Equilibrium pattern of intermediation and trade (A)

**Proposition 4** If $\delta > \alpha \geq 1$, then the interior equilibrium is characterised by the following:

(a) Network size is increasing in the level of information costs $i$ and trade surplus $S$ and decreasing in cost parameters $\beta$ and $\gamma$.

(b) The proportion of intermediated trade to total trade is increasing in the level of information costs $i$. The relationship between total expected trade and information costs is non-monotonic.

(c) The contribution of intermediation to social welfare is positive and increasing in the level of information costs $i$.

**Proof.** See Appendix C. ■

$^{18}$Note that while the intermediary’s investment decision is unaffected at the margin, it follows from Lemma (3) that unconstrained profits are increasing with $i$. 

15
Formally, equilibrium network size, \( P^* \), expected trade volume, \( E(T^*) \), and expected welfare, \( E(W^*) \), are:

\[
P^* = \begin{cases} 0 & \text{if } 0 \leq i < \min \{ \hat{i}, 1 \} \\ \frac{s_{i}^{\beta \alpha}}{(\beta + 1)} \frac{1}{i} & \text{if } \min \{ \hat{i}, 1 \} \leq i < \min \{ \hat{i}, 1 \} \\ 1 & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq 1 \end{cases}
\]

\[
E(T^*) = \begin{cases} 1 - \delta \cdot i \delta & \text{if } 0 \leq i < \min \{ \hat{i}, 1 \} \\ 1 - \delta + \left[ \frac{s}{\gamma(\beta + 1)} \right] \frac{2}{\beta - 1} \frac{\delta(\beta + 1)}{\beta - 1} & \text{if } \min \{ \hat{i}, 1 \} \leq i < \min \{ \hat{i}, 1 \} \\ 1 & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq 1 \end{cases}
\]

\[
E(W^*) = \begin{cases} (1 - \delta) S & \text{if } 0 \leq i < \min \{ \hat{i}, 1 \} \\ (1 - \delta) S + i \delta S A_{\beta - 1}^{2/\beta} - 2 \gamma A_{\beta - 1}^{2/\beta} i \alpha - F & \text{if } \min \{ \hat{i}, 1 \} \leq i < \min \{ \hat{i}, 1 \} \\ S - 2 \gamma i \alpha - F & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq 1 \end{cases}
\]

where \( A = \frac{s_{i}^{\beta \alpha}}{(\beta + 1)} \), \( \hat{i} = \left[ \frac{\gamma^{2/\beta} (F^{\beta + 1} S^{\beta - 1})}{\beta - 1} \right] \frac{1}{2^{(\beta + 1) - 2/\beta}} > 0 \) and \( \hat{i} = \left[ \frac{\gamma^{(\beta + 1) - 2/\beta}}{S} \right] > 0 \).

It follows from the interior equilibrium that \( \frac{\partial P^*}{\partial \delta} > 0 \), \( \frac{\partial P^*}{\partial S} > 0 \), \( \frac{\partial P^*}{\partial \gamma} < 0 \) and \( \frac{\partial P^*}{\partial \beta} < 0 \).

Let the equilibrium direct and indirect trade shares be denoted by \( s_D \) and \( s_I \), respectively, where \( E(T_D^*) \) denotes equilibrium direct trade and \( E(T_I^*) \) denotes equilibrium intermediated trade:

\[
s_D = \frac{E(T_D^*)}{E(T^*)} = \frac{q(i) \left[ 1 - (P^*)^2 \right]}{q(i) \left[ 1 - (P^*)^2 \right] + (P^*)^2} \tag{17}
\]

\[
s_I = \frac{E(T_I^*)}{E(T^*)} = \frac{(P^*)^2}{q(i) \left[ 1 - (P^*)^2 \right] + (P^*)^2} \tag{18}
\]

It is straightforward to show that \( \frac{\partial s_D}{\partial \delta} < 0 \) and \( \frac{\partial s_I}{\partial \delta} > 0 \) in the interior equilibrium. Higher information costs correspond to both a larger network size and a lower probability of direct matching. Both effects drive the result that the proportion of indirect trade to total trade is increasing in the level of information costs. For \( i \in [\hat{i}, 1] \), where \( P^* = 1 \), all trade is intermediated, so \( s_D = 0 \) and \( s_I = 1 \).

Recall that \( E(W^{DT}) \) is the expected welfare that would prevail if there were no intermediary in the market. It follows from (10) that \( E(W^*) - E(W^{DT}) = E(\Pi^*) \) is a measure of the intermediary’s contribution to social welfare. Moreover, since \( \delta > \alpha \geq 1 \), it follows from lemma 3 that \( E(\Pi_I^*) \) is increasing in \( i \) in the interior equilibrium, so the contribution of intermediation to social welfare is both positive and increasing in the
level of information costs where the intermediary is active.

Intuitively, if the probability of direct matching is more responsive to information costs than is the cost of network expansion, then indirect trade offers a relatively more attractive matching technology than direct trade when information costs are higher. Hence, the proportion of indirect trade to total trade is increasing in the level of information frictions, even though the relationship between total trade and information costs is non-monotonic.

**Numerical simulation** Figures (2) - (4) illustrate equilibrium network size, expected trade and expected welfare, respectively, for parameter values $\beta = 2$, $\gamma = 1$, $\delta = 4$, $\alpha = 2$, $F = 0.001$ and $S = \{2.5, 3, 4\}$, which satisfy $\delta > \alpha \geq 1$ and $\beta > 1$.

Figure (2) illustrates the positive relationship between optimal network size and prevailing information costs where the elasticity of the intermediary’s commission exceeds the elasticity of cost $c(i, P)$ with respect to $i$. The fixed set-up cost $F$ implies that information costs must be above a threshold level for intermediation to be profitable in the two-sided market. The optimal network path is illustrated for (a) $S = \gamma(\beta + 1)$, (b) $S > \gamma(\beta + 1)$ and (c) $S < \gamma(\beta + 1)$, verifying that network size and threshold level $\hat{i}$ are increasing in $S$ relative to cost parameters $\beta$ and $\gamma$.

Figure (3) illustrates the effect of intermediation on total expected trade between the two sides of the market. The intermediary’s network investment provides access to a more efficient matching technology than direct trade, thereby raising total trade relative to access to direct matching only. The relationship between expected trade volume and information cost $i$ is non-monotonic due to the conflicting effects of information cost $i$ on the constituent parts of expected trade. The intermediary finds it optimal to increase network size with $i$, thereby increasing the expected measure of intermediated trade matches. The impact on direct trade is twofold. First, higher information cost worsens the probability of a direct match, and second, the expansion in network size results in a smaller expected pool of unmatched traders in stage 4. The net effect is ambiguous, giving rise to a non-monotonic relationship between information costs and total expected trade in equilibrium. It follows that improvements in information technology may not necessarily give rise to higher trade volume. This implies that policy interventions that effectively lower information frictions, such as financial support to small firms to attend international trade shows\textsuperscript{19}, can, in some circumstances, actually lower aggregate trade flows.

\textsuperscript{19}For example, the Tradeshow Access Programme (TAP) of UK Trade and Investment (UKTI) provides grant support for eligible Small & Medium Sized Enterprises (SME’s) to attend trade shows overseas. For details visit https://www.uktradeinvest.gov.uk.
Figure (4) shows that intermediation is welfare improving and that it is more so when information cost is higher.

2.4.2 Equilibrium pattern of intermediation and trade (B)

**Proposition 5** If \( \delta = \alpha \geq 1 \) and \( S < \gamma (\beta + 1) \), then there exists an interior equilibrium characterised by the following:

(a) Network size is independent of the level of information costs \( i \), increasing in trade surplus \( S \) and decreasing in cost parameters \( \beta \) and \( \gamma \).

(b) The measure of intermediated transactions is independent of the level of information costs but represents an increasing proportion of total trade, which is unambiguously decreasing in information costs \( i \).

(c) The contribution of intermediation to social welfare is positive and increasing in the level of information costs \( i \).

**Proof.** See Appendix D. ■

If \( \delta = \alpha \geq 1 \) and \( S \leq \gamma (\beta + 1) \), then equilibrium network size is constant and less than 1. Otherwise, if \( S > \gamma (\beta + 1) \), the unit measure of market size poses a binding constraint and \( P^* = 1 \), provided \( E(\Pi_I) \geq 0 \).

Since equilibrium network size is constant it follows that the measure of intermediated trade is also constant. If \( P^* < 1 \) indirect trade is constant and direct trade decreases with \( i \) as the probability of successful matching declines. Hence, \( \frac{\partial s_D}{\partial i} < 0 \) and \( \frac{\partial s_I}{\partial i} > 0 \). At the limit where \( P^* = 1 \), all trade is intermediated, so \( E^*_D(T) = s_D = 0 \) and \( E^*_I(T) = s_I = 1 \). Furthermore, since \( \delta = \alpha \geq 1 \), it follows from lemma 3 that \( E^*(\Pi_I) \) is increasing in \( i \) in the interior equilibrium, so the contribution of intermediation to social welfare is again both positive and increasing in the level of information costs, where the intermediary is active.

It follows from patterns (A) and (B) that if the probability of direct matching is at least as responsive to information costs than networking cost, then intermediated trade is increasing relative to direct trade in the level of information frictions.

**Numerical simulation** Figures (5) and (6) illustrate\(^\text{20}\) the equilibrium network size where \( \delta = \alpha \geq 1 \). Figure (5) shows that optimal network size is constant with \( i \), but again increasing in \( S \) relative to cost parameters \( \beta \) and \( \gamma \). Figure (6) shows that expected trade volume decreases monotonically with \( i \), but lies above the expected trade path that would prevail without access to an intermediary.

\(^{20}\)Illustrated for \( \beta = 2, \gamma = 1, \alpha = \delta = 3, F = 0.001 \) and \( S = \{2, 3\} \).
2.4.3 Equilibrium pattern of intermediation and trade (C)

Proposition 6 If $\frac{2}{\beta+1}\alpha < \delta < \alpha$, then the interior equilibrium is characterised by the following:

(a) Network size is decreasing in the level of information costs $i$ and cost parameters $\beta$ and $\gamma$ and increasing in trade surplus $S$.

(b) Intermediated trade is decreasing and direct trade increasing in information costs $i$. Total expected trade is unambiguously decreasing in information costs $i$.

(c) The contribution of intermediation to social welfare is positive and increasing in the level of information costs $i$.

Proof. See Appendix E. ■

In this case, the optimal network size is decreasing in $i$ and cost parameters, but increasing in $S$. The decline in network size with higher $i$ implies there are fewer intermediated matches and hence a larger measure of traders seeking a direct match in stage 4 (though higher $i$ implies a lower probability of successful direct matching).

Finally, since $\delta > \frac{2}{(\beta+1)}\alpha$, it follows from lemma 3 that $E(\Pi^*_f)$ is increasing in information cost $i$ in the interior equilibrium. Hence, the contribution of intermediation to social welfare is once again both positive and increasing in the level of information costs, where the intermediary is active.

2.4.4 Equilibrium pattern of intermediation and trade (D)

Proposition 7 If $\delta \leq \frac{2}{\beta+1}\alpha < \alpha$, then the interior equilibrium is characterised by the following:

(a) Network size is decreasing in the level of information costs $i$ and cost parameters $\beta$ and $\gamma$ and increasing in trade surplus $S$.

(b) Intermediated trade is decreasing and direct trade increasing in information costs $i$. Total expected trade is unambiguously decreasing in information costs $i$.

(c) The contribution of intermediation to social welfare is positive but decreasing in information costs $i$.

Proof. See Appendix F. ■

The results in this case are similar to pattern (C), except for the fact that since $\delta \leq \frac{2}{\beta+1}\alpha$ it follows from proposition (3) that expected profit and thus the contribution of intermediation to social welfare is decreasing in the level of information costs $i$.

Intuitively, in both cases (C) and (D) networking costs are more responsive to information costs than the probability of a direct match, so direct trade offers a relatively
more attractive matching technology than indirect trade as information costs rise. Hence, indirect trade is decreasing relative to direct trade in the level of information frictions.

**Numerical simulation**  Figure (7) illustrates the pattern of network investment where \( \delta \leq \frac{\alpha}{\beta+\gamma} \). For this range of elasticities, the commission rate is less responsive to information cost \( i \) than is networking cost \( c(i, P) \), giving rise to a negative relationship between network size and information costs along the interior path. Moreover, as illustrated in figure (8), unconstrained expected profit, denoted by \( E(\Pi_I) \) rises without limit as \( i \to 0 \), which implies that in the absence of a binding market size constraint, the intermediary finds it profitable to invest in an an ever-increasing network size as information costs tend to zero. Thus below threshold \( \hat{i} \), equilibrium network size is constrained by the size of the market. For interval \( i \in [0, \hat{i}] \) the intermediary’s expected profits follow the constrained path, denoted by \( E(\Pi_I)|_{P=1} \). While unconstrained expected profit is increasing, constrained expected profit is declining as information costs tend to zero, rendering the network unviable below threshold level \( \hat{i} \).

### 3 Conclusion

This paper presents a stylised pairwise matching model with two-sided information asymmetry between trade partners, where a single intermediary has the opportunity to invest in a network of contacts and facilitate trade matching for a success fee. The framework innovates by explicitly examining the role of information costs on incentives for trade intermediation, thereby endogenising the pattern of direct and indirect trade.

The framework delivers four key results. First, intermediation unambiguously raises expected trade volume and social welfare by expanding the set of matching technologies available to traders. Second, convexity of network-building costs with respect to network size is necessary for an equilibrium characterised by both direct and indirect trade to exist. Third, assuming convexity, optimal network size and hence the equilibrium pattern of trade is shown to depend on the level of information costs as well as the relative responsiveness of direct and indirect matching technologies with information costs. In particular, if the probability of direct matching is at least as responsive to information costs...
costs than is the cost of network expansion, then indirect trade offers a relatively more attractive matching technology than direct trade when information costs are higher. Hence, the proportion of indirect trade to total trade is increasing in the level of information frictions. Conversely, if networking costs are more responsive to information costs than the probability of a direct match, then direct trade offers a relatively more attractive matching technology than indirect trade when information costs are higher.

Speculating on the plausibility of the relative size of the key parameter values is clearly very difficult. However, Ahn et al. (2010) provide evidence using Chinese data that intermediaries are relatively more important in markets that are more difficult to penetrate, offering support for the pattern of trade arising where probability of direct matching is at least as responsive to changing information costs than networking cost.

Finally, the model sheds light on the relationship between information frictions and aggregate trade volume, which is shown to be non-monotonic if the responsiveness of the direct matching probability to information costs exceeds that of networking cost. This arises due to the conflicting effects of information costs on the incentives for direct and indirect trade. Higher information costs worsen direct matching prospects but can, at the same time, provide an incentive for network-building and thus indirect trade through a trade network. Government trade facilitation policies, such as financial support to small firms to attend international trade fairs, while improving direct matching prospects, may actually lower aggregate trade flows by creating disincentives for intermediated trade.

References


3.1 Appendix A. Derivation of \( \tilde{P} \)

Maximising (12) with respect to \( P \) gives:

\[
\frac{\partial E(\Pi_l)}{\partial P} = 2P \left[ S \delta - \gamma (\beta + 1) P^{\beta - 1} i^\alpha \right] = 0
\] (19)

Solving yields the interior profit-maximising network size, \( \tilde{P} \), where:

\[
\tilde{P} = \left[ \frac{S \delta - \gamma (\beta + 1)}{\gamma (\beta + 1)} \right]^{\frac{1}{\beta - 1}} > 0
\]

The second order condition is:

\[
\frac{\partial^2 E(\Pi_l)}{\partial P^2} = 2 \left[ S \delta - \gamma \beta (\beta + 1) P^{\beta - 1} i^\alpha \right]
\] (20)

20 is negative provided \( \tilde{P} > \left[ \frac{S \delta - \gamma \beta (\beta + 1)}{\gamma (\beta + 1)} \right]^{\frac{1}{\beta - 1}} \). Since \( \beta > 1 \) this condition is satisfied, so \( \tilde{P} \) corresponds to a maximum.

3.2 Appendix B. Proof of lemma 3

Partially differentiating (16) with respect to \( i \) yields:

\[
\frac{\partial E(\Pi_l)}{\partial i} = -P \left[ 2c_i (i, P) + PSq' (i) \right]
\] (21)

It follows that expected profits are increasing with \( i \), if:

\[
c_i (i, P) < -\frac{PS}{2} q' (i)
\] (22)

Substituting \( c_i (i, P) \), \( q' (i) \) and \( \tilde{P} \) and rearranging gives the necessary and sufficient condition for equilibrium intermediary profits, \( E(\Pi_l^*) \), to be increasing in \( i \) where there is an interior solution for network size:

\[(\beta + 1)\delta > 2\alpha \] (23)
Appendix C. Proof of proposition 3

The intermediary sets $P = \min \{ \tilde{P}, 1 \}$ provided $E(\Pi_I) \geq 0$. Let $\hat{i}$ denote the threshold level of information costs at which $E(\Pi_I)|_{P=\hat{P}} = 0$. Since $\delta > \alpha \geq 1$, it follows that $(\beta + 1)\delta > 2\alpha$, so, from lemma 3, $E(\Pi_I)$ is increasing in $i$. Hence, $E(\Pi_I) > 0$ when $i > \hat{i}$. Solving $E(\Pi_I)|_{P=\hat{P}} = 0$ gives:

$$\hat{i} = \left[ \frac{\gamma (\beta + 1)}{S} \right]^\frac{1}{\beta - 1}$$

Hence $P^* = 0$ for $i \in \left[ 0, \min \{ \hat{i}, 1 \} \right]$. Furthermore, $\tilde{P}$ is increasing in $i$ since $\delta > \alpha \geq 1$, but network size is constrained by market size. Let $\hat{\gamma}$ denote the threshold level of information costs at which $\tilde{P} = 1$. Solving $\tilde{P} = 1$ for $i$ yields:

$$\hat{\gamma} = \left[ \frac{\gamma (\beta + 1)}{S} \right]^\frac{1}{\beta - 1}$$

Hence, equilibrium network size is $P^* = 1$ for $i \in \left[ \min \{ \hat{i}, 1 \}, 1 \right]$. For values $i \in \left[ \min \{ \hat{i}, 1 \}, \min \{ \hat{\gamma}, 1 \} \right]$, where $E(\Pi_I) \geq 0$ and $\tilde{P} \leq 1$, network size follows the interior path $P^* = \tilde{P} = \left[ \frac{S^{\delta - \alpha}}{\gamma(\beta + 1)} \right]^\frac{1}{\beta - 1}$. These results are summarised by:

$$P^* = \begin{cases} 
0 & \text{if } 0 \leq i < \min \{ \hat{i}, 1 \} \\
\left[ \frac{S^{\delta - \alpha}}{\gamma(\beta + 1)} \right]^\frac{1}{\beta - 1} & \text{if } \min \{ \hat{i}, 1 \} \leq i < \min \{ \hat{\gamma}, 1 \} \\
1 & \text{if } \min \{ \hat{\gamma}, 1 \} \leq i \leq 1
\end{cases}$$

Substituting $P^*$ into equation 8 yields the equilibrium expected (total) trade path over this range of information costs:

$$E(T^*) = \begin{cases} 
1 - i^\delta & \text{if } 0 \leq i < \min \{ \hat{i}, 1 \} \\
1 - i^\delta + \left[ \frac{S}{\gamma(\beta + 1)} \right]^\frac{2}{\beta - 1} i^{\frac{\delta(\beta + 1) - 2\alpha}{\beta - 1}} & \text{if } \min \{ \hat{i}, 1 \} \leq i < \min \{ \hat{\gamma}, 1 \} \\
1 & \text{if } \min \{ \hat{\gamma}, 1 \} \leq i \leq 1
\end{cases}$$

Finally, the piece-wise function $E(W^*)$ follows directly from substitution of $P^*$ into equation 10:

$$E(W^*) = \begin{cases} 
(1 - i^\delta) S & \text{if } 0 \leq i < \min \{ \hat{i}, 1 \} \\
(1 - i^\delta) S + i^\delta S A^{\frac{2}{\beta - 1}} - 2\gamma A^{\frac{\delta + 1}{\beta - 1}} i^\alpha - F & \text{if } \min \{ \hat{i}, 1 \} \leq i < \min \{ \hat{\gamma}, 1 \} \\
S - 2\gamma i^\alpha - F & \text{if } \min \{ \hat{\gamma}, 1 \} \leq i \leq 1
\end{cases}$$

where $A = \frac{S^{\delta - \alpha}}{\gamma(\beta + 1)}$.
It follows from the interior equilibrium that:

\[
\frac{\partial P^*}{\partial i} = \frac{(\delta - \alpha) S}{\gamma(\beta - 1)(\beta + 1)} \left( \frac{S}{\gamma(\beta + 1)} \right)^{\frac{2-\beta}{\beta+1+(\delta-\alpha)\beta-1}} > 0 \text{ when } \delta > \alpha
\]
\[
\frac{\partial P^*}{\partial S} > 0 ; \quad \frac{\partial P^*}{\partial \gamma} < 0 ; \quad \frac{\partial P^*}{\partial \beta} < 0
\]

(26)

Moreover, \(E(T^*)\) can be decomposed into direct and indirect equilibrium trade. Let direct\(^{23}\) and intermediated trade in equilibrium be denoted by, \(E(T_D^*)\) and \(E(T_I^*)\), respectively, where:

\[
E(T_D^*) = (1 - i^*) \left[ 1 - \left( \frac{Si^{\delta-\alpha}}{\gamma(\beta + 1)} \right)^{\frac{2}{\beta+1+(\delta-\alpha)\beta-1}} \right]
\]

(27)

\[
E(T_I^*) = \left( \frac{Si^{\delta-\alpha}}{\gamma(\beta + 1)} \right)^{\frac{2}{\beta+1+(\delta-\alpha)\beta-1}}
\]

(28)

Let the equilibrium direct and indirect trade shares be denoted by \(s_D\) and \(s_I\), respectively, where:

\[
s_D \equiv \frac{E(T_D^*)}{E(T^*)} = \frac{q(i) \left[ 1 - (P^*)^2 \right]}{q(i) \left[ 1 - (P^*)^2 \right] + (P^*)^2}
\]

(29)

\[
s_I \equiv \frac{E(T_I^*)}{E(T^*)} = \frac{(P^*)^2}{q(i) \left[ 1 - (P^*)^2 \right] + (P^*)^2}
\]

(30)

It is straightforward to show that \(\frac{\partial s_D}{\partial \delta} < 0\) and \(\frac{\partial s_I}{\partial \delta} > 0\). Higher information costs correspond to both a larger network size and a lower probability of direct matching. Both effects drive the result that the proportion of indirect trade to total trade is increasing in the level of information costs. Moreover, for \(i \in [\hat{i}, 1]\), where \(P^* = 1\), all trade is intermediated, so \(s_D = 0\) and \(s_I = 1\).

It follows from (10) that \(E(W^*) - E(W^{DT}) = E(\Pi^*)\) is a measure of the intermediary’s contribution to social welfare. Moreover, since \(\delta > \alpha \geq 1\), it follows from lemma 3 that \(E(\Pi_I^*)\) is increasing in \(i\) in the interior equilibrium. Hence the contribution of intermediation to social welfare is both positive and increasing in the level of information costs, where the intermediary is active.

\(^{23}\)\(E(T_D^*)\) is not to be confused with \(E(T^{DT})\). \(E(T_D^*)\) represents the equilibrium measure of direct trade matches, as a component of equilibrium total trade \(E(T^*)\). In contrast, \(E(T^{DT})\) represents the measure of equilibrium total trade if there were no intermediary in the market.
Appendix D. Proof of proposition 4

If \( \alpha \geq 1 \) and \( S \leq \gamma(\beta + 1) \), then equilibrium network size, \( P^* \), expected trade volume, \( E^*(T) \), and expected welfare, \( E^*(W) \), are described by:

\[
P^* = \begin{cases} 
0 & \text{if } 0 \leq i < \min \{ \hat{i}, 1 \} \\
\frac{S}{\gamma(\beta+1)} i & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq 1 
\end{cases}
\]

\[
E^*(T) = \begin{cases} 
1 - i^\delta & \text{if } 0 \leq i < \min \{ \hat{i}, 1 \} \\
1 - i^\delta (1 - B^2) & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq 1 
\end{cases}
\]

\[
E^*(W) = \begin{cases} 
(1 - i^\delta) S & \text{if } 0 \leq i < \min \{ \hat{i}, 1 \} \\
(1 - i^\delta) S + i^\delta S B^2 - 2 i^\delta \gamma B^2 + 1 - F & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq 1 
\end{cases}
\]

where \( B = \frac{S}{\gamma(\beta+1)} \) and \( \hat{i} = \left[ \frac{2}{\gamma(\beta+1)} \left( \frac{F}{\beta-1} \right) \left( \frac{\beta+1}{S} \right)^{2/\beta-1} \right]^{\frac{1}{\beta-1}} > 0 \).

If \( \alpha \geq 1 \) and \( S > \gamma(\beta + 1) \), then the unit measure of market size poses a binding constraint so \( P^* = 1 \), provided \( E(\Pi_I) \geq 0 \).

As the equilibrium network size is constant it follows that the measure of intermediated trade is also constant. In the interior equilibrium:

\[
E^*_D(T) = (1 - i^\delta) \left[ 1 - \left( \frac{S}{\gamma(\beta+1)} \right)^{2/\beta-1} \right] 
\]

\[
E^*_I(T) = \left( \frac{S}{\gamma(\beta+1)} \right)^{2/\beta-1} 
\]

It follows immediately that indirect trade is constant and direct trade decreases with \( i \) as the probability of successful matching declines. Hence, \( \frac{\partial E_I}{\partial i} < 0 \) and \( \frac{\partial E_I}{\partial \Pi} > 0 \). At the limit where \( P^* = 1 \), all trade is intermediated, so \( E^*_D(T) = s_D = 0 \) and \( E^*_I(T) = s_I = 1 \).

Furthermore, since \( \delta = \alpha \geq 1 \), it follows from lemma 3 that \( E^*(\Pi_I) \) is increasing in \( i \) in the interior equilibrium, so the contribution of intermediation to social welfare is both positive and increasing in the level of information costs, where the intermediary is active. The constrained profit path, where \( P^* = 1 \) is lower than if the intermediary could expand the trade network further, but increasing in \( i \) nonetheless, since \( E(\Pi_I)_{P=1} = i^\delta (S - 2\gamma) - F \).

Appendix E. Proof of proposition 5

The equilibrium path if \( \frac{2}{\beta+1} \alpha < \delta < \alpha \) follows from \( \tilde{P} \). Since \( \frac{\partial P}{\partial i} < 0 \), the interior equilibrium path of network size is declining with information cost \( i \). Moreover, the
second order condition in equation (20) is negative provided \( P > \left[ \frac{S}{\gamma \beta (\beta + 1)^{\alpha - \delta}} \right]^{1/\beta} \). Since \( \beta > 1 \), this condition is satisfied, so \( \hat{P} \) corresponds to an interior maximum.

Let \( \hat{i} \) denote the threshold level of information costs, at which \( \hat{P}_{|\delta < \alpha} = 1 \). Solving for \( i \) yields:

\[
\hat{i} = \left[ \frac{S}{\gamma (\beta + 1)} \right]^{1/\beta} \tag{33}
\]

Let \( \hat{i} \) denote the threshold level of information costs at which \( E(\Pi_I) = 0 \). Since the interior equilibrium path of network size is declining with information cost \( i \), then for sufficiently low \( F \), the threshold \( \hat{i} \) corresponds to a range where \( P = 1 \). If so, then \( \hat{i} \) solves \( E(\Pi_I)|_{P=1} = Si^\delta - 2\gamma i^\alpha - F = 0 \) and \( \hat{i} \leq \hat{i} \), where \( \hat{i} \) is described by equation (33).

If (for sufficiently high \( F \)) threshold \( \hat{i} \) corresponds to a range where \( P = \hat{P}_{|\delta < \alpha} \), however, then \( \hat{i} \) solves \( E(\Pi_I)|_{P=\hat{P}} = 0 \). This yields the threshold level in equation (24) and must exceed \( \hat{i} \), where \( \hat{i} \) is described by equation (33). If the value of \( \hat{P}_{|\delta < \alpha} \) at \( E(\Pi_I)|_{P=\hat{P}} = 0 \) exceeds 1, then this indicates that the constrained optimisation applies and the relevant threshold is \( \hat{i} \) solves \( E(\Pi_I)|_{P=1} = Si^\delta - 2\gamma i^\alpha - F = 0 \).

These results are summarised by:

\[
P^* = \begin{cases} 
0 & \text{if } 0 \leq i < \min \left\{ \hat{i}, 1 \right\} \\
1 & \text{if } \min \left\{ \hat{i}, 1 \right\} \leq i < \min \left\{ \hat{i}, \hat{i}_1 \right\} \\
\left[ \frac{S}{\gamma (\beta + 1)^{\alpha - \delta}} \right]^{1/\beta} & \text{if } \min \left\{ \hat{i}, 1 \right\} \leq i \leq 1
\end{cases}
\]

The piece-wise functions \( E(T^*) \) and \( E(W^*) \) follow directly from \( P^* \) and equations (8) and (10), respectively:

\[
E(T^*) = \begin{cases} 
1 - i^\delta & \text{if } 0 \leq i < \min \left\{ \hat{i}, 1 \right\} \\
1 & \text{if } \min \left\{ \hat{i}, 1 \right\} \leq i < \min \left\{ \hat{i}, \hat{i}_1 \right\} \\
1 - i^\delta + \left[ \frac{S}{\gamma (\beta + 1)^{\alpha - \delta}} \right]^{2/\beta} & \text{if } \min \left\{ \hat{i}, \hat{i}_1 \right\} \leq i \leq 1
\end{cases}
\]

\[
E(W^*) = \begin{cases} 
S - 2\gamma i^\alpha - F & \text{if } 0 \leq i < \min \left\{ \hat{i}, 1 \right\} \\
S + i^\delta SG^{2/\beta} - 2\gamma G^{\beta+1}i^\alpha - F & \text{if } \min \left\{ \hat{i}, \hat{i}_1 \right\} \leq i \leq 1
\end{cases}
\]

where \( G = \frac{S}{\gamma (\beta + 1)^{\alpha - \delta}} \), \( \hat{i} \) is the smaller positive root of \( E(\Pi_I)|_{P=1} = Si^\delta - 2\gamma i^\alpha - F = 0 \) and \( \hat{i} = \left[ \frac{S}{\gamma (\beta + 1)^{\alpha - \delta}} \right]^{1/\beta} > 0 \).
It follows from the interior equilibrium that:

\[
\frac{\partial P^*}{\partial \mu} = -\frac{(\alpha - \delta) S}{\gamma(\beta - 1)(\beta + 1)} \left( \frac{S}{\gamma(\beta + 1)} \right)^{\frac{2-\beta}{\beta-1}} \left( i^{\beta+1}(\delta - \alpha) \right)^{\frac{\beta+1+i(\delta - \alpha)}{\beta-1}} < 0 \text{ when } \alpha > \delta
\]

\[
\frac{\partial P^*}{\partial S} > 0 ; \quad \frac{\partial P^*}{\partial \gamma} < 0 ; \quad \frac{\partial P^*}{\partial \beta} < 0
\]

Hence, optimal network size is decreasing in \( i \) and cost parameters, but increasing in \( S \). Moreover, \( E(T_D^*) \), and \( E(T_I^*) \), are given by:

\[
E(T_D^*) = (1 - \delta^\mu) \left[ 1 - \left( \frac{S}{\gamma(\beta + 1)i^{\alpha - \delta}} \right)^{\frac{2-\beta}{\beta - 1}} \right]
\]

(35)

\[
E(T_I^*) = \left( \frac{S}{\gamma(\beta + 1)i^{\alpha - \delta}} \right)^{\frac{2-\beta}{\beta - 1}}
\]

(36)

The decline in network size with information cost \( i \) is mirrored by \( E(T_I^*) \) when \( \alpha > \delta \). The decline in intermediated matches with \( i \) increases the measure of traders seeking a direct match in stage 4. At the same time, a higher \( i \) implies a lower probability of successful direct matching.

Furthermore, since \( \delta > \frac{2}{(\beta + 1)\alpha} \), it follows from lemma 3 that \( E(\Pi_I^*) \) is increasing in information cost \( i \) in the interior equilibrium. Hence, the contribution of intermediation to social welfare is both positive and increasing in the level of information costs, where the intermediary is active.

**Appendix F. Proof of proposition 6**

If \( \delta \leq \frac{2}{\beta+1}\alpha \), then from lemma 3 it follows that \( E(\Pi_I) \) is decreasing in information cost \( i \) in the interior equilibrium. This implies that as \( i \to 0 \), \( \hat{P} \to \infty \), so the constraint that network size cannot exceed 1 is binding. Let \( \hat{i} \) denote the threshold level of information costs, at which \( \hat{P}_{|\delta<\alpha} = 1 \). This corresponds to the threshold given by equation (33). Further, let \( \hat{i} \) denote the threshold level of information costs at which \( E(\Pi_I) = 0 \). While unconstrained profit is decreasing with \( i \), constrained profit \( E(\Pi_I)_{|P=1} \) is increasing for low values of \( i \). Let \( \hat{i}_{|P=1} \) solve \( E(\Pi_I)_{|P=1} = 0 \) and \( \hat{i}_{|P=\hat{P}} \) solve \( E(\Pi_I)_{|P=\hat{P}} = 0 \). It follows from \( \delta \leq \frac{2}{\beta+1}\alpha \) and the definition of \( \hat{i} \) that \( \hat{i}_{|P=1} < \hat{i} < \hat{i}_{|P=\hat{P}} \). Thus, \( E(\Pi_I) \) is non-negative between these thresholds. Hence, the intermediary is inactive for low levels of information cost \( i \leq \hat{i}_{|P=1} \) and also for \( i \geq \hat{i}_{|P=\hat{P}} \).

If \( \delta \leq \frac{2}{\beta+1}\alpha \), then equilibrium network size, \( P^* \), expected trade volume, \( E(T^*) \), and expected welfare, \( E(W^*) \), are described by:
\[ P^* = \begin{cases} 
0 & \text{if } 0 \leq i < \min \{ \hat{i}_{|P=1}, 1 \} \\
1 & \text{if } \min \{ \hat{i}_{|P=1}, 1 \} \leq i < \min \{ \hat{i}, 1 \} \\
\left[ \frac{S}{\gamma(\beta+1)^{1-\alpha}} \right]^{\frac{1}{\beta-1}} & \text{if } \min \{ \hat{i}, 1 \} \leq i < \min \{ \hat{i}_{|P=P}, 1 \} \\
0 & \text{if } \min \{ \hat{i}_{|P=P}, 1 \} \leq i \leq 1 
\end{cases} \]

The piece-wise functions \( E(T^*) \) and \( E(W^*) \) follow directly from \( P^* \) and equations (8) and (10), respectively.

\[ E(T^*) = \begin{cases} 
1 - i^\delta & \text{if } 0 \leq i < \min \{ \hat{i}_{|P=1}, 1 \} \\
1 & \text{if } \min \{ \hat{i}_{|P=1}, 1 \} \leq i < \min \{ \hat{i}, 1 \} \\
1 - i^\delta + \left[ \frac{S}{\gamma(\beta+1)} \right]^{\frac{2}{\beta-1}} i^{\frac{\beta+1-2\alpha}{\beta-1}} & \text{if } \min \{ \hat{i}, 1 \} \leq i < \min \{ \hat{i}_{|P=P}, 1 \} \\
1 - i^\delta & \text{if } \min \{ \hat{i}_{|P=P}, 1 \} \leq i \leq 1 
\end{cases} \]

\[ E(W^*) = \begin{cases} 
(1 - i^\delta) S & \text{if } 0 \leq i < \min \{ \hat{i}_{|P=1}, 1 \} \\
S - 2\gamma i^\alpha - F & \text{if } \min \{ \hat{i}_{|P=1}, 1 \} \leq i < \min \{ \hat{i}, 1 \} \\
(1 - i^\delta) S + i^\delta S A^{\frac{2}{\beta-1}} - 2\gamma A^{\frac{\beta+1}{\beta-1}} i^\alpha - F & \text{if } \min \{ \hat{i}, 1 \} \leq i < \min \{ \hat{i}_{|P=P}, 1 \} \\
(1 - i^\delta) S & \text{if } \min \{ \hat{i}_{|P=P}, 1 \} \leq i \leq 1 
\end{cases} \]

where \[ A = \frac{S i^{\delta - \alpha}}{\gamma(\beta+1)} \]
\[ \hat{i}_{|P=1} \]

The trade effects follow. Expected profit is unconstrained in the interior equilibrium. Since \( \delta \leq \frac{2}{\beta+1} \alpha \) then it follows from proposition (3) that expected profit and thus the contribution of intermediation to social welfare is decreasing in the level of information costs \( i \).