Firm Exit, Technological Progress and Trade

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Abstract

The dynamics of export market exit and firm closure have found limited attention in the new heterogeneous-firms trade literature. In fact, several of the predictions on exit stemming from this new class of models are at odds with the stylized facts. Empirically, higher productivity firms survive longer, most firm closures are young firms, higher productivity exporters are more likely to continue to export compared to less productive exporters and market exits as well as firm closures are typically preceded by periods of contracting market shares. The present paper shows that the simple inclusion of exogenous economy wide technological progress into the standard Melitz (2003) model generates a tractable dynamic framework that generates endogenous exit decisions of firms in line with the stylized facts. Furthermore, we derive the effects of faster technological progress and trade liberalization on export market exit and firm closure.

JEL: F12, F15, O33, L11, L16

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1 Introduction

While heterogeneous-firms trade theory has a great deal to say about firm entry and their sorting into export markets and market serving modes, it has surprisingly little to add on the phenomenon of export market exit and firm closure. Yet, empirically, exit from markets is as important a phenomenon as entry, and since the introduction of Stigler’s (1958) survivor principle it has been a central parameter in the study of industrial organization. Most importantly, several of the predictions on exit stemming from the new models (e.g. Melitz, 2003) are at odds with the stylized facts.\footnote{In the Melitz (2003) model exit is solely driven by the random death parameter, $\delta$, such that high and low productivity firms (and thus large and small firms) have the same exit probability, low and high productivity exporters are equally likely to continue to export, and a firm exits – if it exits – on all markets simultaneously.} For example, empirically, high productivity firms survive longer, most firm closures are young firms, larger size entrants have lower exit probabilities, more productive exporters are more likely to continue to export compared to less productive exporters, market exit is typically preceded by a period of contracting market shares and firms are regularly observed to withdraw from some markets while staying active on others.\footnote{For these and other stylized facts on firm exit found in the Industrial Organization and International Trade literature, see for example Jovanovic (1982), Dunne et al. (1988), Caves (1998), Klepper (2002) and Eaton et al. (2008).}

The present paper shows that the single modification of including exogenous technological progress into the standard Melitz (2003) model with Pareto distributed productivities generates an analytically solvable dynamic framework that preserves all the established novelties of Melitz (2003) while adding endogenous exit decisions of firms in line with the stylized facts. In doing so, the present paper highlights how competition forces are an important transmission channel between technological progress and firm exit dynamics and shows how opening to trade amplifies these forces. With technological progress and vintage capital properties, the arrival of new competing producers squeezes the market share of existing producers such that incumbent firms will eventually exit the market. This perspective on creative destruction, and in fact the modeling choice of exogenous technological progress, is by no means new, but is based on the central contribution of Dasgupta and Stiglitz (1981) to the dynamics of oligopolistic industries, and relates closely to the problem of competency traps well-known from the literature on organizational learning, see Levitt and March (1988).

There are a number of papers that explicitly address issues of dynamics in heterogeneous firms trade settings. Rich exit dynamics are generated by...
the inclusion of some firm-specific random process affecting productivity, e.g. Irarrazabal and Opmoll (2008) and Arkolakis (2009). Thus, this branch of literature is in this respect similar to the seminal model presented by Hopenhayn (1992). Alternatively, firms may uncover firm-specific innovation advances that secure firm growth or the absence of which causes firm exit once a good innovation draw is lacking, see for example, Atkeson and Burstein (2007) and Constantini and Melitz (2007) who deal with the joint innovation and export decision of firms.\footnote{In a different line of modeling, Ederington and McCalman (2009) consider endogenous innovation choices by firms, but they focus on the interaction of international trade and the adoption of new technologies in causing industry shakeouts, i.e. large waves of exit. Finally, Baldwin and Robert-Nicoud (2007) model technological progress through falling fixed costs over time, yet they only study the entry of firms.}

What these previous works have in common is that exit of firms (be it from a foreign market alone or an actual firm closure) is driven by an intra-firm process, such as a random productivity development, that is paired with a constant firm death probability, as evoked in Melitz (2003). Thus, the driver of firm-level exit behavior is specific to the firm and occurs within the firm. In contrast, what the present paper highlights is a different – but no doubt central – channel of firm exit dynamics initially suggested by Dasgupta and Stiglitz (1981). It is exogenous to the firm, deterministic and specific to the economy. We introduce economy wide technological progress. New firms have, as in a vintage capital model, access to more efficient production methods, while incumbent firms are technologically locked.\footnote{Thus firms maintain – as in Melitz (2003) – their drawn productivity throughout their entire lifetime. Throughout the paper we employ the conventional interpretation that firm, plant and product are synonymous, and that firms are heterogeneous in marginal productivity. All our results are valid in a multi-product or multi-plant firm interpretation or in a heterogeneous product-quality interpretation. With multi-product (-plant) firms, the model would describe the life cycle of individual products (plants). In a product quality heterogeneity interpretation, the technological progress captured here would translate into continuous quality improvements, and accordingly low quality incumbents are squeezed from the market by higher quality newcomers.} This can be interpreted as firms facing competency traps, Levitt and March (1988). Stark examples of such situations are the late arrival of traditional camera and photographic film producers in the digital age, or the current focus in the motor vehicle industry on improvements of the internal combustion gasoline engine, despite the advent of next generation propulsion systems. An extensively studied episode comes from the telecommunications equipment industry: Olley and Pakes (1996) provide, apart from their widely cited methodological advances, important evidence that technological change affects industry-wide productivity, inter alia, by inducing the closure of older
and relatively unproductive plants.

There are several advantages of focusing on this competitive pressure channel of technological progress as a driver of firm-level exit dynamics compared to the other approaches discussed in the heterogeneous-firms trade literature. Firstly, the standard assumption of constant firm death probability has little economic foundation, and the specification presented here is able to abandon this assumption all together. Secondly, while dynamic model extensions based on random process often have to rely on numerical solutions, our dynamic extension of Melitz (2003) remains simple, tractable and based on analytical solutions throughout. Thirdly, technological progress and technological lock-in are well established phenomena and appear to be reasonable assumptions to impose on the model. In contrast, an assumption of random productivity developments is somewhat harder to grasp and random reductions in productivity may challenge the basic intuition of innovation, where new production methods are unlikely to replace existing ones, unless they are more efficient.

The paper perhaps closest to ours is that of Haruyama and Zhao (2008), albeit being set in a very different framework. In their model, product or market exit stems from a firm endogenous decision to improve quality of existing products, where successful quality jumps render former products obsolete. The innovation process is firm endogenous, labor absorbing, and the incentive to innovate interacts with trade liberalization via market size. Their model focuses on the interaction of the firm-level innovation process, trade and overall productivity growth. In contrast, our paper focuses on exit dynamics and the competition element in creative destruction, where entry of new and higher productivity varieties marginalizes existing products. Thus, in our paper firms decide endogenously to exit from export markets not because some specific innovation has rendered their product obsolete, but because their market share has been squeezed so much that further operation is unprofitable. Thus, we isolate the competition effect of technological progress in a Melitz (2003) intra-industry trade setting.

The model we develop is a straightforward extension of Melitz (2003), mapping dynamic firm-level behavior of exit under a situation of continuous technological progress. Progress is modeled as exogenous and continuous improvements in the productivity distribution that firms draw from before entering the market. With this simple assumption future firms will have expected higher productivity and larger market shares on all markets that they enter. Accordingly, potential new entrants realize prior to starting production of their drawn variety, that they will eventually be ousted from, first the foreign market (if they should choose to export) and eventually the domestic market. They take this expected time of exit into account when deciding
whether to enter the industry or not.

The resulting dynamic model, where exit and death of firms is endogenous and not depending on an exogenous death probability, has the following desirable properties concerning its cross-sectional and dynamic predictions for market exit and firm closure:

**Proposition 1.** Firms of all ages die. Conditional on firm-specific productivity, firm death is deterministic.

**Proposition 2.** More productive firms survive longer. Entry size of firms on a given market is positively linked to the duration of serving this market.

**Proposition 3.** Only small firms leave a market. Firms observe a declining market share before market exit and firm closure. Exporters that cease to export continue to exist as pure domestic firms for a while.

**Proposition 4.** For a given exit wave, younger (older) age cohorts account for a larger (smaller) share of the exiting firms.

**Proposition 5.** A higher growth rate in technological progress shortens the average time a firm survives in any market and softens selection (pushing the exit cut-off threshold down).

**Proposition 6.** Trade liberalization decreases the expected lifetime of exporters and non-exporters, yet the expected lifetime of all firms – conditional on starting production – is unaffected. Trade liberalization makes selection harder (pushing the exit cut-off threshold up).

The first three Propositions can be established for fairly general productivity distributions, however, for expositional simplicity we employ a Pareto distribution throughout. The remainder of the paper is structured as follows. The next section contains the central modeling elements including a detailed description of how we incorporate technological progress into the Melitz (2003) model. Section 3 presents proofs of the propositions laid out above. Section 4 concludes.

# 2 The Central Modeling Elements

In this section, we present the central modeling elements of the continuous time dynamic extension of the symmetric n-country Melitz (2003) model. Additional details on the full model are provided in the Appendix. We operate throughout with the well established Melitz (2003) notation and conventions such as the assumption of Pareto distributed productivity draws.
Technological Progress

We introduce technological progress as exogenous and continuous, thus following Dasgupta and Stiglitz’s (1981) work on the dynamics of oligopolistic industries and innovation. In particular, we introduce exogenous and continuous improvements in the distribution of productivities available to entering firms. At entry each firm pays sunk innovation costs of $f_e$ labor units and draws a firm-specific marginal productivity $\varphi$ that it maintains throughout its endogenous life cycle.\(^5\) Due to selection at the production cut-off the average productivity of the incumbents of today will be higher then the average productivity draw of tomorrow, i.e. at any point in time some of the drawn blue-prints are not worth bringing to the market. Still, the central implication of continuously improving productivity draws among new entrants paired with constant firm-specific productivities is that incumbent firms experience declining relative productivity and thus falling market shares over time, i.e. they face a competency trap. Eventually individual market shares decline to levels, such that firms cannot cover fixed costs and endogenously shut down.

To see this, consider the probability of drawing a lottery component below $\psi \geq 1$:

\[
\Pr (\Psi (\omega, t) < \psi) = \Pr \left( \frac{\varphi (\omega, t)}{\bar{\varphi}_t} < \psi \right) = \Pr (\varphi (\omega, t) < \psi \bar{\varphi}_t) = G_t (\psi \bar{\varphi}_t) = 1 - \psi^{-k}
\]

Hence $\Psi$ is Pareto with shape parameter $k$ and location parameter 1.\(^6\)

\(^5\)See footnote 4 for alternative interpretations of $\varphi$.

\(^6\)To see this, consider the probability of drawing a lottery component below $\psi \geq 1$:

\[
\Pr (\Psi (\omega, t) < \psi) = \Pr \left( \frac{\varphi (\omega, t)}{\bar{\varphi}_t} < \psi \right) = \Pr (\varphi (\omega, t) < \psi \bar{\varphi}_t) = G_t (\psi \bar{\varphi}_t) = 1 - \psi^{-k}
\]

Hence $\Psi$ is Pareto with shape parameter $k$ and location parameter 1.
where $\varphi_0(\omega)$ is a draw from the Pareto $G(\varphi_0(\omega)) = 1 - \left( \frac{\varphi_0(\omega)}{\bar{\varphi}_0} \right)^{-k}$ for $\varphi_0(\omega) \geq \bar{\varphi}_0$. To see this note that
\[
\Pr(\varphi_0(\omega) e^{\beta t} < x) = \Pr(\varphi_0(\omega) < x e^{-\beta t}) = 1 - \left( \frac{x e^{-\beta t}}{\bar{\varphi}_0} \right)^{-k} = 1 - \left( \frac{x}{\bar{\varphi}_0 e^{\beta t}} \right)^{-k} = \Pr(\varphi_t(\omega) < x).
\]

**Households**

The representative household supplies exogenously $L$ units of labor and chooses a consumption path $\{C_s\}_{s=t}^{\infty}$ to maximize utility\(^7\)
\[
U = \int_t^{\infty} \ln (C_s) \, ds
\]
subject to a budget constraint. The optimal expenditure path has $E_t = E = L$ for all $t$ which equals labor income as wages are normalized to unity. Expenditures in any period are spread over the set of available varieties, $\Omega_t$, to maximize
\[
C_t = \left[ \int_{\omega \in \Omega_t} [c_t(\omega)]^{\frac{1}{\sigma}} \, d\omega \right]^{\frac{\sigma}{\sigma - 1}}
\]
implying a demand of
\[
c_t(\omega) = \frac{E}{P_t} \left( \frac{p_t(\omega)}{P_t} \right)^{-\sigma} \quad \text{for all } \omega \in \Omega_t, \quad (3)
\]
where $p_t(\omega)$ is the price of variety $\omega$ and $P_t = \left[ \int_{\omega \in \Omega_t} [p_t(\omega)]^{1-\sigma} \, d\omega \right]^{\frac{1}{1-\sigma}}$ is the price index.

**Firms**

After the payment of the sunk innovation costs $f_e$ and the realization of the marginal cost drawn from the Pareto distribution (1), firm’s labor requirement conditional on production is $l_t(\omega) = f + \frac{q_t(\omega)}{\bar{\varphi}(\omega)}$, where $f$ is fixed production costs and $q_t(\omega)$ is output. A firm has to pay fixed costs of $f_e$ for each export market it serves, and moreover exports are subject to iceberg trade costs, $\tau \geq 1$. Upon entry and subsequently at each point in time, firms decide conditional on productivity and industry structure which markets to serve.

\(^7\)For simplicity we follow Melitz (2003) by imposing the assumption of no time discounting.
Given the constant elasticity of demand, prices are set as a constant mark-up, $\frac{\sigma}{\sigma-1}$, on marginal costs. Flow profits at time $t$ on the domestic market and on export markets for a firm with lottery component $\varphi_0$ and age $m$ are given by

$$\pi_{t,m}(\varphi_0) = B_t e^{\beta(\sigma-1)(t-m)} \varphi_0^{\sigma-1} - f \quad (4)$$

$$\pi_{t,m}^x(\varphi_0) = B_t \tau^{1-\sigma} e^{\beta(\sigma-1)(t-m)} \varphi_0^{\sigma-1} - f_x \quad (5)$$

where $B_t = \frac{1}{\sigma-1} \left( \frac{\sigma}{\sigma-1} \right)^{-\sigma} E P_t^{\sigma-1}$ is the market-specific demand component at time $t$. We assume at this point that a balanced growth path with a stable industry structure exists.\textsuperscript{8} A stable industry structure requires stable exit and export lottery thresholds (relative productivity), which in turn requires the above flow profits to be time-invariant. This can only be achieved if the market-specific demand component $B$ decreases to balance the technological improvement, i.e. $B$ must decrease at rate $\beta(\sigma-1)$.\textsuperscript{9} Writing $B_t = B_0 e^{-\beta(\sigma-1)t}$ the flow profit expressions read

$$\pi_{m}(\varphi_0) = B_0 \varphi_0^{\sigma-1} e^{-\beta(\sigma-1)m} - f \quad (6)$$

$$\pi_{m}^x(\varphi_0) = B_0 \tau^{1-\sigma} \varphi_0^{\sigma-1} e^{-\beta(\sigma-1)m} - f_x \quad (7)$$

A new firm serves the domestic market provided $\pi_0(\varphi_0) \geq 0 \Leftrightarrow \varphi_0 \geq \varphi_0^{exit} = \left( \frac{f}{B_0} \right)^{1/\sigma-1}$ and export markets provided $\pi_0^x(\varphi_0) \geq 0 \Leftrightarrow \varphi_0 \geq \varphi_0^x = \left( \frac{f}{B_0} \right)^{1/\sigma-1} \tau$.\textsuperscript{10} Due to the exogenously technological progress, a firm observes that its productivity falls over time relative to younger competitors. Eventually, the market share will fall to a level where the firm is unable to cover fixed costs in a given market, and the firm therefore endogenously exits the market at that point in time. The ages at which a firm shuts down ($m^{exit}$) and leaves a given export markets ($m^x$) are determined by (6) and (7) and

\textsuperscript{8}This assumption is proven correct in the Appendix.

\textsuperscript{9}The stable industry structure implies that the distribution of marginal productivities increases at rate $\beta$ which in turn implies that prices decrease at rate $\beta$. The time-invariant nominal expenditures and fixed/sunk costs imply, given the constant mark-ups, a time-invariant number of varieties, $M_t$. Thus it follows that $B_t = \frac{1}{\sigma-1} \left( \frac{\sigma}{\sigma-1} \right)^{-\sigma} E P_t^{\sigma-1}$ decreases at rate $\beta(\sigma-1)$. See the Appendix for further details.

\textsuperscript{10}We impose the conventional parameter restriction $f_x \tau^{\sigma-1} > f$ that ensures that firms, consistent with empirical evidence, are partitioned into exporters and non-exporters.
Industry Equilibrium

The industry equilibrium is pinned down by the free entry condition. Taking the endogenous market durations (8) and (9) into account, net present value of flow profits read\textsuperscript{11}

\[
\pi(\varphi_0) = \int_{m^\text{exit}(\varphi_0)}^\infty (B_0 \varphi_0^{\sigma-1} e^{-\beta(\sigma-1)m} - f) \, dm \\
= \frac{f}{\beta(\sigma-1)} \left[ B_0 \varphi_0^{\sigma-1} f^{-1} - 1 - \ln \left( \frac{B_0 \varphi_0^{\sigma-1}}{f} \right) \right] \\
\pi^x(\varphi_0) = \frac{f_x}{\beta(\sigma-1)} \left[ B_0 \tau^{1-\sigma} \varphi_0^{\sigma-1} f_x^{-1} - 1 - \ln \left( \frac{B_0 \tau^{1-\sigma} \varphi_0^{\sigma-1}}{f_x} \right) \right] 
\]

The free entry condition balances the expected net present value of flow profits with the sunk cost of entry costs, i.e.

\[
\int_{\varphi_0^\text{exit}}^\infty \pi(\varphi_0) \, dG(\varphi_0) + n \int_{\varphi_0^0}^\infty \pi^\text{export}(\varphi_0) \, dG(\varphi_0) = f_e, 
\]

which pins down the productivity thresholds as

\[
\varphi_0^{\text{exit}} = \left( \frac{(\sigma - 1)}{k - (\sigma - 1)} \frac{1}{\beta k f_x} \right)^{\frac{1}{\tau}} \left( 1 + n \left( \frac{f_x}{f} \right) \frac{\tau^{1-\delta}}{\tau^{-k}} \right)^{\frac{1}{\tau}} \varphi_0 \\
\varphi_0^x = \left( \frac{f_x}{f} \right)^{\frac{1}{\tau}} \tau \varphi_0^{\text{exit}}. 
\]

A comparison with the Melitz (2003) model conditional on productivities being Pareto-distributed reveals that the thresholds are only changed by a

\textsuperscript{11}On the balanced growth path, the interest rate equals the discount rate which by assumption equals zero.
scalar of \( \left( \frac{s}{\pi_t} \right)^k \), where \( \delta > 0 \) is the conventional exogenous death probability. Hence, all the novel predictions of the Melitz model also apply in this extended version. Indeed, the distribution of marginal productivities of active firms turns out to be Pareto with shape parameter \( k \) and location parameter \( \phi_{exit}^\varepsilon \).

3 Proof of Propositions

Proposition 1, that firm death is deterministic conditional on the firm-specific productivity draw, follow directly from (8) in Section 2. Furthermore, for each cohort the absolute number of exits declines with maturity. Accordingly, in the age distribution of a given exit-wave the most recent entries have the largest share (Proposition 4).\(^{12}\) Proposition 2, i.e. that more productive and larger firms survive longer, also follows directly from (8) in Section 2 when noting that entry size increases in marginal productivity.

The market shares of a firm with productivity \( \varphi \) on the domestic and export markets are

\[
s_t(\varphi) = \frac{E \left( \frac{p(\varphi)}{P_t} \right)^{-\sigma} p(\varphi)}{E} = \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \varphi^{\sigma-1} P_t^{\sigma-1}
\]

\[
s_x^\varepsilon(\varphi) = \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \tau^{1-\sigma} \varphi^{\sigma-1} P_t^{\sigma-1}
\]

Combining (4), (5), (11) and (12), we find that \( \pi_t(\varphi) = \frac{s_t(\varphi)E}{\sigma} - f \) and \( \pi_t^\varepsilon(\varphi) = \frac{s_x^\varepsilon(\varphi)E}{\sigma} - f_x \). Accordingly, flow profits increase in the market shares and exiting firms will thus be small (have a small market share). Market shares decline over time due to a decreasing price level, \( P_t \) (see (A.4) in the Appendix), and exiters thus observe periods of declining market shares prior to exit. Furthermore, exporters that cease their exporting activity continue for a while to exist as pure domestic firms, since \( \phi_x^\varepsilon > \phi^\varepsilon \), c.f. footnote 10. This proves Proposition 3.

The impact of the growth rate in technology on firm survival in any

\(^{12}\)It can be shown that the age distribution for the group of active firms exiting the market at any point in time is exponential with parameter \( \beta k \) (see the appendix for details on distributions).
market (Proposition 5) follows from the partial derivatives wrt. $\beta$ of

$$
E\left( m^{\text{exit}} \mid \varphi_0 \geq \varphi_0^{\text{exit}} \right) = \frac{\int_{\varphi_0^{\text{exit}}}^{\infty} \frac{1}{\beta(\sigma-1)} \ln\left( \frac{B_{\varphi_0^{\text{exit}}}}{f} \right) dG(\varphi_0)}{1 - G(\varphi_0^{\text{exit}})} = \frac{1}{\beta k} \quad (13)
$$

$$
E\left( m^{\text{exit}} \mid \varphi_0 \geq \varphi_0^{\text{exit}} \right) = \frac{\int_{\varphi_0^{\text{exit}}}^{\infty} \frac{1}{\beta(\sigma-1)} \ln\left( \frac{B_{\varphi_0^{\text{exit}}}}{f} \right) dG(\varphi_0)}{1 - G(\varphi_0^{\text{exit}})} = \frac{1}{\beta k}.
$$

The effect of trade liberalization on the expected life-time of exporters and non-exporters (Proposition 6) can be seen from the partial derivatives wrt. $\tau$ of

$$
E\left( m^{\text{exit}} \mid \varphi_0 \geq \varphi_0^{\text{exit}} \right) = \frac{1}{\beta k} \left( 1 + \ln \left[ \frac{f_x}{f} \right]^{\frac{k}{\sigma-1}} + k \ln \tau \right)
$$

$$
E\left( m^{\text{exit}} \mid \varphi_0^{\text{exit}} \leq \varphi_0 \leq \varphi_0^{\text{exit}} \right) = \frac{1}{\beta k} \left( 1 - \ln \left[ \frac{f_x^{\sigma-1}}{f} \right]^{\frac{k}{\sigma-1}} \right).
$$

The average lifetime of those firms that actually choose to start production remains, however, unaffected by trade liberalization. This happens, because the life-time reduction for the group of exporters and non-exporters is counterbalanced by shift from non-exporters to exporters. Trade liberalisation increases the share of exporters among the active firms; and exporters survive longer than non-exporters.

Finally, the effects on the exit cut-off thresholds, Propositions 5 and 6, namely that increasing technological progress softens selection and trade liberalization makes selection tougher, follow directly from the partial derivatives of $\varphi_0^{\text{exit}}$, equation (10). The intuition for the latter result is that faster technological progress reduces the value of the lottery, since firms will loose market shares faster. With fewer firms joining the lottery, the cut-off thresholds become less strict.

4 Conclusion

Firm exits from export markets and firm closures are prominent feature of modern international trade. The present paper develops an extension of the Melitz (2003) model to derive a simple analytically solvable dynamic framework able to address exit. The single modification we evoke is the inclusion
of exogenous technological progress, such that newer firms draw from an improved productivity distribution and accordingly older firms will eventually be ousted from the market place. Thus, the paper highlights how competition forces are an important transmission channel between technological progress and firm exit dynamics and how they interact with opening to trade. Our model extension is able to capture the stylized facts of export market exit and firm closure.

Firstly, high productivity firms (at any point in time) are likely also to produce in the future, i.e. they survive longer. That is, we have now a positive link between relative productivity and firm and market survival, respectively. Secondly, entry size of firms on a given market is positively linked to the duration of serving this market. Thirdly, large exporters do not exit export markets, but smaller exporters do, i.e. firms exit markets after they have lost market shares and exporters that cease to export will still serve their domestic market. Fourthly, opening to trade gives an economy wide productive redistribution by toughening entry thresholds, and by reducing the expected lifetime of exporters as well as non-exporters, even at constant rates of technological progress. Finally, dampened technological progress affects economy wide productivity directly, via smaller productivity improvements for new start-ups, and indirectly, via the longer survival of firms.
A Appendix

Households
Utility of the representative household is given by
\[ U = \int_0^\infty \ln \left( C_t \right) dt = \int_0^\infty \ln \left( \frac{E_t}{P_t} \right) dt \] (A.1)
where \( t \) denotes time, \( C_t \) is consumption, \( E_t \) is total nominal expenditures and \( P_t \) is the price level. The representative household maximizes utility subject to the flow budget constraint
\[ \dot{A}_t = r_t A_t + L - E_t \] (A.2)
and a no-ponzi-scheme condition
\[ \lim_{t \to \infty} A_t e^{-r_t t} \geq 0 \]
where \( A_t \) denote assets and \( r_t \) the interest rate. The optimality (Euler) condition reads
\[ \frac{\dot{E}_t}{E_t} = r_t \] (A.3)
The growth rate of assets read \( \frac{\dot{A}_t}{A_t} = r_t + \frac{L - E_t}{A_t} \). On a balanced growth path the growth rates of expenditures and assets must be constant. It is easy to see that this can only be the case for \( \frac{\dot{A}_t}{A_t} = \frac{\dot{E}_t}{E_t} = 0 \) implying that \( E_t = L \) for all \( t \) and \( r_t = 0 \).

Equilibrium
In the text, we assumed the existence of a balanced growth path with a stable industry structure with constant exit and export lottery thresholds and a fixed number of firms. Here, we show that this assumption is correct.
Provided the mass of firms, \( M \), is constant and average duration cf. (13) equals \( \frac{1}{\beta k} \), it must be the case that at any point in time a mass of \( M^e = \frac{\beta k M}{\Pr_{\text{ob}}(\varphi_0 \geq \varphi_{0, \text{exit}})} = \left( \frac{\varphi_{0, \text{exit}}}{\varphi_0} \right)^k \beta k M \) new firms pay sunk costs of \( f_e \) labor units to enter the industry in order for the mass of active firms to be constant. The equilibrium mass of firms is determined by the labor market clearing condition. Labor supply is exogenously given by \( L \). Labor demand at time \( t \) of the mass of firms with age \( m > 0 \) is given by
\[ L_{t,m} = M^e \left[ \int_{\varphi_{t, \text{exit}}}^{\infty} \left( f + E \left( \frac{p(\varphi)}{P_t} \right)^{-\sigma} \frac{1}{\varphi} \right) dG_{t-m} (\varphi) \right] \]
\[ + n \int_{\varphi_t}^{\infty} \left( f_x + E \left( \frac{p_x(\varphi)}{P_t} \right)^{-\sigma} \frac{1}{\varphi} \right) dG_{t-m} (\varphi) \],
where \( p(\varphi) = \frac{\sigma}{\sigma - 1} \frac{1}{\varphi}, p^x(\varphi) = \frac{\sigma}{\sigma - 1} \frac{x}{\varphi} \), \( B_t(\varphi^{\text{exit}})^{\sigma - 1} = f \) and \( B_t(\varphi_t^{\text{exit}})^{\sigma - 1} \tau^{1 - \sigma} = f_x \). Under the assumption of \( B_t = \frac{1}{\sigma - 1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} EP_t^{\sigma - 1} = B_0 e^{-\beta (\sigma - 1) t} \), we find that

\[
L_{t,m} = e^{-m \beta k} M \beta f \left( 1 + n \left( \frac{f_x}{f} \right)^{\frac{\sigma - 1 - k}{\sigma - 1}} \tau^{k} \right) \left( 1 + (\sigma - 1) \frac{k}{k - (\sigma - 1)} \right)
\]

and total demand for labor reads

\[
L_t = \int_0^\infty L_{t,m} dm + f_e M_e = \sigma f M \frac{k}{k - (\sigma - 1)} \left( 1 + n \left( \frac{f_x}{f} \right)^{\frac{\sigma - 1 - k}{\sigma - 1}} \tau^{k} \right)
\]

From the labor market clearing condition, \( L_t = L \), the mass of firms is determined as

\[
M = \frac{L}{\sigma f} \frac{k - (\sigma - 1)}{k} \left( 1 + n \left( \frac{f_x}{f} \right)^{\frac{\sigma - 1 - k}{\sigma - 1}} \tau^{k} \right)^{-1}
\]

Given the mass of firms, we can compute the price index as

\[
P_t = \left[ \int_{\omega \in \Omega_t} [p_t(\omega)]^{1 - \sigma} d\omega \right]^{-\frac{1}{\sigma}} = \left[ \int_0^\infty p_{m,t} dm + n \int_0^\infty p^x_{m,t} dm \right]^{-\frac{1}{\sigma}},
\]

where \( p_{m,t} = \frac{M}{\text{Pr}(\varphi_0 \geq \varphi_0^{\text{exit}})} \int_{\varphi_0^{\text{exit}}}^\infty [p(\varphi)]^{1 - \sigma} dG_{t-m}(\varphi) \) is the contribution to the price index of domestic firms of age \( m \) and \( p^x_{m,t} = \frac{M}{\text{Pr}(\varphi_0 \geq \varphi_0^{\text{exit}})} \int_{\varphi_0^{\text{exit}}}^\infty [p^x(\varphi)]^{1 - \sigma} dG_{t-m}(\varphi) \) is the contribution to the price index of foreign firms of age \( m \). It follows that

\[
P_t = \frac{\sigma}{\sigma - 1} \left[ \frac{L}{\sigma f \beta k} \right]^{-\frac{1}{\sigma}} \frac{e^{-\beta t}}{\varphi_0^{\text{exit}}}, \quad (A.4)
\]

which decreases at rate \( \beta \). This in turn implies that \( B_t = \frac{1}{\sigma - 1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} EP_t^{\sigma - 1} \) decreases at rate \( \beta (\sigma - 1) \).

We have thus shown that our assumption about the existence of a balanced growth path with a stable industry structure is correct.

**Distributions**

As shown above there is a fixed mass of firms entering the industry at any point in time \((M^e)\). The mass of firms at time \( t \) with productivity above \( \hat{\varphi} > \varphi_t^{\text{exit}} \) reads

\[
M_{t,\varphi > \hat{\varphi}} = M^e \int_0^\infty \left( \frac{\hat{\varphi}}{\varphi_{t-m}} \right)^{-k} dm = M^e \left( \frac{\hat{\varphi}}{\varphi_t} \right)^{-k} \frac{1}{\beta k}.
\]
Hence the fraction of firms with a productivity level below $\hat{\phi}$ at time $t$ is

$$G^t(\hat{\phi}) = \frac{M_{t,\hat{\phi} > \varphi^\text{exit}}}{M_{t,\hat{\phi} > \varphi^\text{exit}}} = 1 - \left(\frac{\hat{\phi}}{\varphi^\text{exit}_t}\right)^{-k} \quad \text{for } \hat{\phi} > \varphi^\text{exit}_t.$$ 

It follows, that at any point in time the distribution of marginal productivities among active firms is Pareto with shape parameter $k$ and location parameter $\varphi^\text{exit}_t$. In fact this applies to all cohorts. To see this consider a given cohort of age $m$ at time $t$ where

$$G^{t,m}(\hat{\phi}) = \frac{M^e\left(\frac{\varphi^\text{exit}_t}{\varphi_{t-m}}\right)^{-k} - M^e\left(\frac{\varphi}{\varphi_{t-m}}\right)^{-k}}{M^e\left(\frac{\varphi^\text{exit}_t}{\varphi_{t-m}}\right)^{-k}} = 1 - \left(\frac{\hat{\phi}}{\varphi^\text{exit}_t}\right)^{-k} \quad \text{for } \hat{\phi} > \varphi^\text{exit}_t$$

i.e. the productivity distribution for active firms is the same across cohorts.

Turning to the age distribution of firms we have that the mass of active firms of age $m$ at time $t$ is given by

$$M^m = M^e\left(\frac{\varphi^\text{exit}_t}{\varphi_{t-m}}\right)^{-k} = M^e\left(\frac{\varphi^\text{exit}_0 e^{\beta t}}{\varphi_{0} e^{\beta (t-m)}}\right)^{-k} = M^e\left(\frac{\varphi^\text{exit}_0}{\varphi_{0}}\right)^{-k} e^{-\beta km}.$$ 

The total mass of active firms can thus be written as $M = \int_0^\infty M^m dm = M^e\left(\frac{\varphi^\text{exit}_0}{\varphi_{0}}\right)^{-k} \frac{1}{\beta k}$ which in turn determines the age density as

$$g(m) = \frac{M^m}{M} = \beta k e^{-\beta km},$$

i.e. the age distribution of active firms is exponential with parameter $\beta k$. 

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References

