ABSTRACT

Less productive firms are more likely to exit markets than more productive ones. Moreover, the risk of insolvency that business firms face differs widely across countries. These two outstanding facts are not accounted for by the standard general equilibrium model of international trade due to Melitz (2003). We reconcile these facts with this seminal model and explore the links between the default risk at the firm level, the default risk at the country level and the conditions to do business under international trade. Our welfare and policy analysis for the open economy delivers a number of novel results.

JEL-Classification: F12, L25, L26
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1 Introduction

Less productive firms are more likely to exit markets than more productive ones. This finding has consistently been obtained for a large number of countries. Drawing on a panel of manufacturing plants in the United States, Bailey et al. (1992) and Doms et al. (1995) document that productivity has a sizable negative effect on the probability of firm exit. Similar findings have been obtained for the UK (Disney et al. 2003), for France (Bellone et al. 2006), for Spain (Esteve-Pérez and Mañez-Castillejo 2008), and for Portugal (Carreira and Teixeira 2009). Apparently, the organizational capital in form of the general and financial management skills of entrepreneurs and managers helps firms to adapt to their environment and to become more productive, thus contributing to their survival. Furthermore, more productive firms are more likely to make higher profits and, hence, have a greater buffer against adverse shocks.

A second robust finding is that the risk of insolvency that business firms face differs widely across countries. For example, CreditReform (2007; 2009), a private research institute and consultancy, documents that the risk of insolvency varies markedly even within Europe: using Germany as the benchmark indexed with a value of 100, the risk of insolvency gets as low as 47 for Scandinavian countries and as high as 161 for countries in Eastern Europe. Clearly, countries offer different business environments. These business conditions comprehend legal and institutional factors (e.g. the enforcement of contracts, the protection of investors, the proneness to corruption), the supply of infrastructure and of public goods, microeconomic factors and policies (e.g. the workings of credit markets, product and labor market regulations), as well as the macroeconomic conditions and stability (aggregated demand policies, inflations and unemployment). Importantly, the evidence suggests that these business conditions and the insolvency rates are highly correlated. For example, the ranking of European countries in terms of their insolvency risks just alluded to is highly in line with the ranking of these countries in the World Bank's Doing Business Report (World Bank 2010).

Surprisingly, these two outstanding facts are not accounted for by the now standard general equilibrium model of international trade due to Melitz (2003) which allows for productivity differences across firms, firm entry and firm death. That model assumes that all firms, irrespective of their productivity, face an identical default risk which is also identical across countries.

The aim of this paper is to reconcile these facts with this seminal model and to explore the links between the default risk at the firm level, the default risk at the country level and the conditions to do business under international trade. We modify the standard model in two respects: first, in
line with the first fact, we assume that the default risk at the level of the firm is negatively related to the firm's productivity. Second, we introduce a comprehensive set of factors that determine business conditions and that may differ across the two countries that we consider.

We obtain a number of remarkable results. Fundamentally, our model is able to explain the fact that the default risk at the country level is determined by the conditions to do business, which itself are heavily influenced by policy. For example, firms have lower default risks in countries where it is cheaper to enter the market or where the technology potential is superior. Moreover, in the open economy, the default risk of a country's firms falls if the market access to consumers in the other country improves. Among the novel results that our welfare and policy analysis for the open economy delivers are the following. First, there are gains to trade even if one country is driven into full specialization on traditional goods by a trading partner which offers its firms far advantageous business conditions. Second, improvements in the business conditions in one country (e.g. due to trade policies, infrastructure policies or industrial policies) have negative welfare effects on the other country. Third, trade integration is immiserizing for a country which trades with another country which offers far superior business conditions. Fourth, the default risk of firms falls in both countries under symmetric trade integration.

Our paper is related to the new literature that explores policy issues in the standard model with heterogeneous firms. Importantly, none of these works accounts for heterogeneous default risk at the firm level, however, and so they are also at odds with the heterogeneity of default risks at the country level. Demidova (2008) studies differences in the technology potential across countries. Her work is the one most closely related to our analysis. We shall therefore explain in detail how our results deviate from her contribution, as we go along. Baldwin (2005) and Baldwin and Forslid (2006) are also related in that they study the welfare effects of trade integration, albeit in a model which lacks the comprehensive set of business conditions that we account for. Feenstra and Kee (2008) and Demidova and Rodriguez-Clare (2009) conduct welfare analyses. Jorgenson and Schröder (2006) study the effects of tariffs. Cole and Davis (2009) analyze optimal tariffs. Pflüger and Südekum (2009) focus on entry costs and entry subsidies and study the non-cooperative and cooperative choice entry subsidies.

Our paper is also related to the literature on trade policies, infrastructure policies and industrial policies with homogeneous firms (Venables 1987, Helpman and Krugman 1987, Flam and Helpman 1987, Martin and Rogers 1995 and Baldwin et al. 2003). Of course, this literature

\footnote{See also Falvey et al. (2005).}
ignores the heterogeneity of firms altogether and this literature also neglects the process of business exit that we highlight. Nonetheless, some parallels emerge that we discuss as we proceed.

The paper's structure is as follows. The basic model is laid out in section 2. Section 3 derives the open economy equilibrium. Section 4 covers the welfare and policy analysis in the open economy. Section 5 offers some concluding remarks.

2 The Model

2.1 General set-up

Our model is based on a version of the standard monopolistic competition model with heterogeneous firms (Melitz 2003) due to Demidova (2008). There are two industries, a traditional numéraire industry, $n$, which produces a homogeneous good under constant returns to scale and perfect competition and a monopolistic competitive industry, $c$, which produces a continuum of differentiated manufacturing varieties under increasing returns. Each variety is produced by a single firm and firms are heterogeneous in their productivity. Labor is the only factor of production in both industries. There are $L$ workers who supply one unit of labor each. Demidova (2008) highlights country asymmetries with respect to the access to manufacturing technologies and also with respect to size (the number of workers). Following up on our discussion in the introduction, we consider an extensive list of factors which affect the conditions of doing business: we allow for country asymmetries concerning the effective entry costs, the fixed costs to serve domestic and foreign consumers, respectively, trade and transport infrastructure, and Ricardian productivity differences in the competitive sector.\(^2\) We first consider a single country in autarky.

2.2 Preferences

Preferences of household $h$ are defined over the homogenous numéraire commodity and the set of differentiated varieties, $z \in \Omega$, according to a logarithmic quasi-linear utility function with CES sub-utility\(^3\)

$$u^h = \beta \ln c^h + n^h$$

$$c^h = \left[ \int_{z \in \Omega} q^h(z)^\rho dz \right]^{1/\rho}$$

(1)

\(^2\) Differences in entry costs due to governmental entry subsidies are analyzed in Pfüger and Südekum (2009).

\(^3\) Demidova (2008) assumes a Cobb-Douglas upper tier utility function rather than a quasi-linear one.
where \(0 < \rho < 1\) and \(\beta > 0\) are constant parameters and where \(q^h(\mathbf{z})\) expresses household \(h\)’s consumption of variety \(z\). The elasticity of substitution between any two varieties is given by \(\sigma = 1/(1-\rho) > 1\). It is well-known from Dixit and Stiglitz (1977) that \(e^h\) can be understood as the consumption of the manufacturing aggregate with aggregate price

\[
P = \left[ \int_{z \in \Omega} p(z)^{1-\sigma} \, dz \right]^{1/(1-\sigma)}
\]

(2)

The budget constraint of an individual is \(Pe^h + \nu^h = y^h\), where \(y^h\) denotes income. Standard utility maximization implies that per-capita expenditure on the manufacturing aggregate and the numéraire are given by \(Pe^h = \beta\) and \(\nu^h = y^h - \beta\), respectively. Indirect utility is of the form \(\nu^h = y^h - \beta \ln P + \beta(\ln \beta - 1)\). The index \(h\) will be dropped from now on since households are identical. We impose the assumption \(\beta < \gamma\) in order to ensure that the demand for the homogeneous good is non-negative. Aggregate demand for a single variety \(z\) is given by \(q(z) = p(z)^{-\sigma} P^{\sigma-1} \beta L\), and total revenue for that variety is \(r(z) = p(z)q(z) = [P/p(z)]^{\sigma-1} \beta L\).

Overall manufacturing expenditure, \(PeL\), equals \(\beta L\).

### 2.3 Production and pricing

In the numéraire-sector \(a\) units of labor are transformed into one unit of output. This pins down the wage at \(w = 1/a\). Technologies in the modern sector are such that \(l = f + q / \phi\) units of labor are needed to produce \(q\) units of output. The fixed overhead labor \(f\) is the same for all firms, but the variable labour requirement \((1/\phi)\) differs across firms. Firms have zero mass. Each firm thus faces a residual demand curve with constant price elasticity of demand \(-\sigma\).

Profit maximization implies that a firm with marginal cost \((w/\phi)\) charges the price:

\[
p(\phi) = \frac{\sigma}{\sigma - 1} \frac{w}{\phi} = \frac{w}{\rho \phi}
\]

(3)

Revenue and profits of this firm are then given by \(r(\phi) = \beta L(\rho \phi P / w)^{\sigma-1}\) and \(\pi = r(\phi) / \sigma - w f\), respectively. Hence, the firm with higher productivity level \(\phi\) charges a lower price, sells a larger quantity and has higher revenue and profits. Since all firm-specific variables differ only with respect to \(\phi\), the CES price index (2) can be rewritten as

\[
P = M^{1/(1-\sigma)} \frac{w}{\rho \bar{\phi}} \quad \text{with} \quad \bar{\phi} \equiv \left[ \int_{\phi}^{-\infty} \phi^{\sigma-1} \mu(\phi) \, d\phi \right]^{1/(\sigma-1)}
\]

(4)
where $M$ denotes the mass of manufacturing firms (and varieties) in the market, $\mu(\phi)$ is the productivity distribution across these active firms (with positive support over a subset of $(0, \infty)$) and $\bar{\phi}$ is an average productivity level as introduced by Melitz (2003).

### 2.4 Entry and exit

There exists a mass of potential entrepreneurs who can enter the manufacturing sector subject to a sunk entry investment in terms of labor $f_c$. At each point in time a mass of $M^E$ entrepreneurs decides to enter. Upon entry these entrepreneurs learn about their productivity $\phi$, which is drawn from a common and known density function $g(\phi)$ with support $(0, \infty)$ and cumulative density function $G(\phi)$. Call this the 'productivity lottery'. After the productivity level is revealed, an entrant can decide to exit immediately or to remain active in the market, in which case the firm earns constant per-period profits $\pi(\phi)$. It will exit immediately if $\pi(\phi) < \sigma w f$. Only those firms remain active whose productivity draw exceeds the cutoff $\phi^* > 0$ at which profits are zero, $\pi(\phi^*) = 0$. A tractable way to express this notion is to assume that the firm-specific death rate is given by $\delta(\phi) = 1/\phi$.

Once in the market, every firm may be hit by a lethal shock which forces it to shut down and exit the industry. The empirical evidence that we have discussed in section 1 strongly suggests that less productive firms face a higher risk of market exit than more productive ones. We focus on a stationary equilibrium without time discounting such that in each period the mass of entrants which successfully enter the market equals the mass of firms that are forced to shut down. Analytically, $\text{prob}_t M^E = E[\delta(\phi) | \phi > \phi^*] M$, where $E[\delta(\phi) | \phi > \phi^*]$ is the expected rate of firm death and $\text{prob}_t = 1 - G(\phi^*)$ is the probability to draw a productivity no smaller than the cutoff $\phi^*$. The endogenous productivity distribution among surviving firms, $\mu(\phi)$, is thus the conditional (left-truncated) ex-ante distribution $g(\phi^*)$ on the domain $[\phi^*, \infty)$.

### 2.5 Equilibrium in the closed economy and parameterization

The equilibrium within the manufacturing sector can be characterized as in Melitz (2003) by two conditions, a free entry condition (FEC) and a zero cutoff profit condition (ZCPC).

To derive the FEC note that, assuming risk neutrality, potential entrepreneurs enter the market (i.e. incur the entry cost $w f_c$ to participate in the productivity lottery) until the value of entry,
\( v^E = E \left[ \sum_{t=0}^{\infty} (1 - \delta(\varphi))^t \pi(\varphi) \right] - w f_c = E[\pi(\varphi) / \delta(\varphi)] - w f_e = [1 - G(\varphi')]E[\pi(\varphi) / \delta(\varphi) | \varphi > \varphi'] \), is driven to zero. Using \( \pi(\varphi) = r(\varphi) / \sigma - w f \), \( r(\varphi) = (\varphi / \varphi')^{\sigma-1} r(\varphi') \) and \( \delta(\varphi) = \delta(\varphi') \varphi' / \varphi \) where \( \varphi' = E[\varphi^\alpha | \varphi > \varphi']^{1/\alpha} \) is a suitably defined auxiliary average productivity, and imposing \( v^E = 0 \), the FEC can be derived as (see appendix A):

\[
(FEC) \quad \pi(\varphi') = \frac{\delta(\varphi') w f_c}{1 - G(\varphi')} + w f \delta(\varphi') \left\{ E \left[ \frac{1}{\delta(\varphi)} | \varphi > \varphi' \right] - \frac{1}{\delta(\varphi')} \right\} \quad (5)
\]

The ZCPC states that the cutoff firm makes zero profits, \( \pi(\varphi^*) = 0 \Leftrightarrow r(\varphi^*) = \sigma w f \). Using \( \pi(\varphi) = \left[ r(\varphi') / \sigma \right] - w f \), and \( r(\varphi^*) = (\varphi^* / \varphi')^{\sigma-1} r(\varphi') \), this condition can also be expressed as a function of the auxiliary average productivity level \( \varphi' \):

\[
(ZCPC) \quad \pi(\varphi') = \left[ \frac{\varphi'}{\varphi^*} \right]^{\sigma-1} - 1 \left. w f \right. \quad (6)
\]

Although this ZCPC corresponds qualitatively to the one stated in Melitz (2003) we have formulated it in terms of \( \varphi' \) to facilitate the derivations that follow. The equilibrium is determined by the cutoff productivity \( \varphi^* \) which simultaneously satisfies the FEC and the ZCPC.

In order to conform with the empirical evidence and to obtain closed-form solutions we assume Pareto-distributed productivities, \( G(\varphi) = 1 - (\varphi_{\text{min}} / \varphi)^k \) and \( g(\varphi) = G'(\varphi) = k \varphi_{\text{min}} \varphi^{-k-1} \) where \( \varphi_{\text{min}} > 0 \) is the lower bound for productivity draws and \( k > 1 \) is the shape parameter.\(^4\) The ex post probability of productivities is then conditional on successful market entry, \( \mu(\varphi) = g(\varphi) / [1 - G(\varphi^*)] = k \varphi^k \varphi^{-k-1} \) if \( \varphi > \varphi^* \) and \( \mu(\varphi) = 0 \) otherwise. Moreover, under the Pareto specification, \( \bar{\varphi} = [k / (k - (\sigma - 1))]^{1/\sigma} \varphi^*, \varphi^* = [k / (k - \sigma)]^{1/\sigma} \varphi^* \) where we assume \( k > \sigma \).

Using these expressions and \( \delta(\varphi) = 1 / \varphi \) in FEC and ZCPC and then solving these two conditions yields the autarky equilibrium cutoff (see appendix B):

\(^4\) For empirical support see e.g. Del Gatto et al. (2006) and Ikeda and Suoma (2009). The Melitz-model with the Pareto-parameterization has been popularized by Bernard et al. (2003), Helpman et al. (2004), Baldwin (2005), Helpman et al. (2008) and Melitz and Ottaviano (2008).
\[
\varphi_{\text{aut}}^* = \left[ \frac{(\sigma-1)k f_k}{(k-\sigma)(k-1) f_e} \varphi_{\text{min}} \right]^{1/(k-1)}
\]  

(7)

The equilibrium cutoff is independent of the number of workers \(L\), positively related to the elasticity of substitution \(\sigma\), the fixed labor \(f\) to serve the market and the lower bound \(\varphi_{\text{min}}\) and negatively related to the fixed investment of labor at the entry stage \(f_e\) as well as the Pareto-shape parameter \(k\), as in Melitz (2003) and Demidova (2008).\(^5\) Moreover, \(\varphi_{\text{aut}}^*\) is unaffected by the labour coefficient in the competitive sector \(a\) since this coefficient affects the wage and hence the fixed costs both to enter and serve the market equi-proportionately. We show below that countries' labour coefficients play a role in the open economy equilibrium, however.

With Pareto-distributed productivities and the specification \(\delta(\varphi) = 1/\varphi\) it follows that the expected (average) exit rate of the economy is given by \(E[\delta(\varphi)|\varphi > \varphi^*] = [k/(k+1)]\varphi^{*-1}\). Making use of the equilibrium cutoff \(\varphi_{\text{aut}}^*\) given by (7) immediately entails:

**PROPOSITION 1. (Country-specific default risk under autarky).** The expected (average) risk of business exit in the closed economy, is independent of country size \(L\) and the labour coefficient in the traditional sector \(a\), negatively related to the degree of competition \(\sigma\), the fixed labour input to the serve market \(f\) and the minimum productivity \(\varphi_{\text{min}}\) and positively related to the fixed investment of entry labour \(f_e\) and the shape parameter \(k\).

Taking into account that more productive firms have a larger buffer to withstand negative shocks and taking an extensive list of business conditions into account, allows us to provide an endogenous explanation for the stylized empirical fact that the risk of business exit varies with the business condition across countries as expressed through the parameters highlighted in proposition 1.

Once the equilibrium cutoff is determined, all other endogenous variables can be derived in the usual way. In equilibrium, the aggregate expenditure on manufacturing has to be equal to the aggregate revenue of manufacturing firms, \(\beta L = M r(\varphi)\). Using \(r(\varphi) = (\varphi/\varphi^*)^{\sigma-1} \sigma w f\), \(\varphi = [k/(k-(\sigma-1))]^{1/(\sigma-1)}\varphi^*\), and the equilibrium cutoff (7), the number of active firms can be

\(^5\) The statements concerning \(\varphi_{\text{aut}}^*\) and \(k\) refer to versions of the Melitz-model with Pareto-distributed productivities (cf. the references in footnote 4).
derived, $M_{aut} = \frac{\beta L[k-(\sigma-1)]}{\sigma k w f}$. The condition for the stationarity of the equilibrium then implies the number of entrants, $M_{aut} = \frac{[k-(\sigma-1)]k(\sigma-1)}{(k^2-1)(k-\sigma)\sigma} \frac{\beta L}{w f_c}$. Using $M_{aut}$ and $\tilde{\varphi} = \left[\frac{k}{(k-(\sigma-1))}\right]^{1/(\sigma-1)} \varphi^*$ in (4), yields the price level, $P_{aut} = \left(\frac{\beta L}{\sigma f}\right)^{1/(\sigma-1)} w^{\sigma/(\sigma-1)} \left(1/\rho \varphi_{aut}^*\right)$ and the indirect utility of a household is then:

$$v_{aut} = w - \beta \ln \left[ \frac{\beta L}{\sigma f} \right]^{1-\sigma} w^{\sigma/(\sigma-1)} \left(1/\rho \varphi_{aut}^*\right) + \beta (\ln \beta - 1)$$

(8)

Countries with a greater endowment of labor $L$ and a higher cutoff are better off. Moreover, it is readily derived that wage increases resulting from productivity increases in the numéraire sector raise (lower) the indirect utility iff $(1/a) > (\sigma/\beta)$.  

3 The Open Economy

3.1 Assumptions

We now turn to an open economy setting with two countries $i, j \in [H, F]$, say home $H$ and foreign $F$. These two countries potentially differ in a number of characteristics which determine the conditions of doing business. There may be differences in country size $L_i$ and in the labour coefficient in the competitive sector $a_i$. Technologies in the manufacturing sector may be different: we assume that entrants in country $i$ draw their productivity from a country-specific Pareto-distribution with common shape parameter $k$ but with potentially different lower bounds, $\varphi_{min}$. We also allow the fixed labour input for entry in the manufacturing sector $f_{ei}$ and the fixed labour input $f_i$ to serve domestic markets to differ across countries. If (after learning its productivity $\varphi_i$) a firm from country $i$ decides to export to region $j$ it faces an additional country-specific fixed cost $f_{st}$, on top of the domestic per-period fixed costs $f_i$ that accrue irrespectively of export status. Moreover, firms have to incur variable iceberg costs to serve foreign consumers: for one unit to arrive in $j$, a firm from country $i$ has to ship $\tau_{ij} > 1$

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6 Demidova's (2008) treatment of productivity differences in the manufacturing sector is more general than ours. She allows for general country-specific productivity distributions $G_i(\varphi)$ which may dominate the productivity distribution $G_j(\varphi)$ of the other country in terms of the hazard rate order. Since we consider further asymmetries across countries and since we want to keep the model tractable, we have chosen to sacrifice some generality here.
units. We shall allow for the possibility that \( \tau_{ij} \neq \tau_{ji} \), e.g. due to different trade policies or trade infrastructures. Trade in the competitive sector is costless. As long as both countries produce this good, an assumption that we shall maintain throughout the paper, the law of one price dictates that the foreign wage is tied to the domestic wage, \( W = w_f / w_d = a_d / a_f \) where \( W \) denotes the relative foreign wage. Note that \( w_i = 1 / a_i \) by our choice of the numéraire.

3.2 Domestic cutoffs and export cutoffs

Following the approach pioneered in Demidova (2008) we now derive the domestic cutoff productivities \( \varphi^*_i \) and \( \varphi^*_F \) drawing on the conditions of free entry and zero cutoff profits which become interdependent across countries in the open economy. If a manufacturing firm from country \( i \) exports to country \( j \), its profits from exporting are given by \( \pi_{xi}(\varphi) = r_x(\varphi) / \sigma - w_i \cdot f_{xi} \) where \( r_x(\varphi) = (\tau_{ij} w_i / \rho \varphi)^{1-\sigma} P_j^{\sigma-1} \beta L_j \) is the export revenue. There is a critical productivity threshold \( \varphi^*_i \) where such a firm just breaks even on the export market, i.e. \( \pi_{xi}(\varphi^*_i) = 0 \Leftrightarrow r_x(\varphi^*_i) = \sigma w_i f_{xi} \). We call this the export ZCPC. Furthermore, a manufacturing firm from country \( i \) that serves her home market \( i \) derives profits \( \pi_i(\varphi) = r_i(\varphi) / \sigma - w_i f_i \) where \( r_i(\varphi) = (w_i / \rho \varphi)^{1-\sigma} P_i^{\sigma-1} \beta L_i \) is the associated revenue. The cutoff \( \varphi^*_i \) where this firm breaks even is defined by \( \pi_i(\varphi^*_i) = 0 \Leftrightarrow r_i(\varphi^*_i) = \sigma w_i f_i \). We call this the domestic ZCPC. The revenue equations imply a link between export cutoffs and domestic cutoffs, \( \varphi^*_xi = W^{-\sigma^{(\sigma-1)}} t_{xi} \varphi^*_F \) and \( \varphi^*_Fi = W^{\sigma^{(\sigma-1)}} t_{Fi} \varphi^*_H \) where \( t_i = \tau_{ij}^{(f_{xi} / f_{ji})^{1/(\sigma-1)}} \) (see appendix C).

Throughout the paper we impose the assumption \( f_{xi} / f_{ji} > \tau_{ij}^{1-\sigma} (w_j / w_i)^{(\varphi^*_i / \varphi^*_j)^{\sigma-1}} \) to ensure that only firms that produce in the domestic market can export (i.e. \( \varphi^*_xi > \varphi^*_j \)).

The free entry condition (FEC) for country \( i \) commands that firms enter the market until the value of entry is zero, \( \text{prob}_i \left[ \frac{\pi_i(\varphi)}{\delta(\varphi)} > \varphi^*_i \right] + \text{prob}_j \left[ \frac{\pi_j(\varphi)}{\delta(\varphi)} > \varphi^*_j \right] = w_i f_{xi} \). The first term on the LHS formalizes the expected profits on the domestic market and the second term expresses expected profits on the export market where \( \text{prob}_i = 1 - G(\varphi^*_i) \) denotes the probability for a productivity draw high enough to enter the export market. The RHS expresses the entry costs. Using the FECs and the domestic and export ZCPC for each of the two countries, employing the links between export cutoffs and domestic cutoffs, and imposing the Pareto
parameterization the domestic equilibrium cutoff productivities for $H$ and $F$, are easily derived as (see appendix D):

$$\phi^*_{H} = \left[ \frac{(\sigma-1)k}{(k-\sigma)(k-1)} f_{EH} \phi^k_{min H} \right]^{1/(k-1)} \left[ \frac{1 - \Phi^H_{F} \Phi^H_{H}}{1 - \Delta^{L_{\sigma_{min, w}}^{w}} \Phi^H_{H}} \right]^{1/(k-1)} = \phi^*_{H, aut} \cdot \left[ \frac{1 - \Phi^H_{F} \Phi^H_{H}}{1 - \Delta^{L_{\sigma_{min, w}}^{w}} \Phi^H_{H}} \right]^{1/(k-1)}$$

$$\phi^*_{F} = \left[ \frac{(\sigma-1)k}{(k-\sigma)(k-1)} f_{EF} \phi^k_{min F} \right]^{1/(k-1)} \left[ \frac{1 - \Phi^F_{F} \Phi^H_{H}}{1 - \Phi^F_{H} / \Delta^{L_{\sigma_{min, w}}^{w}} \Phi^F_{H}} \right]^{1/(k-1)} = \phi^*_{F, aut} \cdot \left[ \frac{1 - \Phi^F_{F} \Phi^H_{H}}{1 - \Phi^F_{H} / \Delta^{L_{\sigma_{min, w}}^{w}} \Phi^F_{H}} \right]^{1/(k-1)}$$

where the $\Phi_i = \tau^{i-1}_j \left( f_j / f_{sl} \right)^{(k-\sigma)\sigma/(\sigma-1)}$ are measures of trade openness which rise as variable trade costs $\tau_j$ and the fixed cost ratio $f_{sl} / f_j$ (i.e. the fixed cost that a firm from $i$ faces to serve the foreign market in relation to the fixed costs of a foreign competitor) fall. We shall assume that $f_{sl} / f_j > 1$ which implies that $0 \leq \Phi < 1$. The parameter $\Delta^{L_{\sigma_{min, w}}^{w}} \equiv F \sigma \left( \frac{w_{fe}}{w_{fe}} \right)$ captures international differences (ratios) concerning entry investments $F \equiv f_{EF} / f_{el}$, technologies in the manufacturing sector as proxied by the respective lower productivity bounds of the Pareto-distribution $T = (\phi_{min H} / \phi_{min F})$ and wage differentials $W = w_{F} / w_{H} = a_{H} / a_{F}$ caused by productivity differences in the competitive sector. $\Delta^{L_{\sigma_{min, w}}^{w}}$ rises when home business conditions turn in favour of domestic firms (i.e. when market entry becomes less expensive in home, when technological conditions are such that the domestic productivity lottery 'dominates' the foreign one, or when domestic wages fall relative to foreign wages).

Absent international differences in business conditions (with $\Delta^{L_{\sigma_{min, w}}^{w}} = 1$ and $\Phi_H = \Phi_F$) the cutoffs are given by $\phi^*_{i} = \phi^*_{i, aut} \cdot (1 + \Phi)^{1/(k-1)}$ both for $H$ and for $F$ as in Melitz (2003). When we allow for country asymmetries, we have to impose the condition $1 / \Phi_H > \Delta^{L_{\sigma_{min, w}}^{w}} > \Phi_F$ to ensure meaningful solutions (such that $\phi^*_i > 0$ for $i = H, F$). Intuitively, this condition commands that the overall business conditions for the manufacturing sectors in the two countries must not be too different. Notice that it clearly is conceivable that business conditions are so disparate that a country, call it the 'laggard', is driven into full specialization in the traditional industry and that all manufactures are produced in the 'leading' country. We take this case up in section 4.6. An immediate implication of (9) is that the foreign and the domestic equilibrium cutoffs are tied, $\phi^*_{F} = W^{\sigma/(\sigma-1)} B \phi^*_{H}$, where
\[ B = \left[ \frac{f_H}{f_F} \left( 1 - \Delta^L_{\Phi_{\text{aut}}} - \Phi_H \right) \right]^{1/(k-1)} \] is an inverse measure of (relative) conditions favouring business in \( H \) (as opposed to \( F \)).

Once the domestic equilibrium cutoffs are determined, the export cutoffs are immediately implied by the links \( \varphi^{*}_{\text{xF}} = W^{\sigma/(\sigma-1)} t_H \varphi^{*}_H \) and \( \varphi^{*}_{\text{xF}} = W^{\sigma/(\sigma-1)} t_F \varphi^{*}_F \). The probability to become an exporter in country \( i \) can then be derived from \( \text{prob}_{xH} = \left( \varphi^*_i / \varphi^{*}_{\text{xH}} \right) \). The average productivity of domestic firms \( \bar{\varphi}_i \) and the average productivity of exporting firms \( \bar{\varphi}_{xH} \) follow from \( \bar{\varphi}_i = [k/(k-(\sigma-1))]^{\varphi^*_H} \) and \( \bar{\varphi}_{xH} = [k/(k-(\sigma-1))]^{\varphi^{*}_{xH}} \). Finally, the average profits can be calculated as \( \pi(\bar{\varphi}) = \pi(\bar{\varphi}_i) + \text{prob}_{xH} \pi_{xH}(\bar{\varphi}_{xH}) = \frac{(\sigma-1)w_i}{k-(\sigma-1)} (f_i + \text{prob}_{xH} f_{xH}) \).

### 3.3 Trade balance and open economy equilibrium

To complete the characterization of the open economy equilibrium we have to derive the share of labor employed in a country's manufacturing sector \( \gamma_i \). This can be achieved by exploiting the trade balance condition which, from the perspective of the domestic economy, is given by:

\[ \text{prob}_{xH} M_H r_{xH} (\bar{\varphi}_{xH}) = \text{prob}_{xF} M_F r_{xF} (\bar{\varphi}_{xF}) + (w_H - \beta)L_H - (1 - \gamma_H)L_H / a_H \] (10)

The LHS of eq. (10) gives the value of country \( H \)'s manufacturing exports and the first term on the RHS gives the value of manufacturing imports. The second and third term on the RHS are the values of domestic consumption and production of the traditional good, respectively. Any imbalance in trade in manufacturing must be matched by a trade surplus or deficit in this numéraire. Making use of the fact that for each country the total revenue of manufacturing firms must equal the sum of wages in that sector, i.e. \( M_i \bar{r}_i = \gamma_i L_i w_i \) where \( \bar{r}_i = r_i (\bar{\varphi}_i) + \text{prob}_{xH} r_{xH} (\bar{\varphi}_{xH}) \) is the average revenue of a manufacturing firm in country \( i \), and taking into account that worldwide expenditures on manufacturing goods must match the sum of wages earned in this sector in the two countries together, \( \beta(L_H + L_F) = \gamma_H L_H w_H + \gamma_F L_F w_F \), we can derive the \( \gamma_i \) and thus also the masses of firms (see appendix E):

\[ M_H = \frac{k - (\sigma - 1)}{\sigma k f_H w_H} \left( 1 - \frac{\phi_H / \Delta_{f-H}^{\psi \psi \Phi}}{1 - \phi_H / \phi_F} \right) = M_{H,\text{aut}} \left( 1 - \phi_H / \Delta_{f-H}^{\psi \psi \Phi} \right) \] (11)

\[ M_F = \frac{k - (\sigma - 1)}{\sigma k f_H w_F} \left( 1 - \frac{\phi_F / \Delta_{f-F}^{\psi \psi \Phi}}{1 - \phi_H / \phi_F} \right) = M_{F,\text{aut}} \left( 1 - \phi_H / \Delta_{f-F}^{\psi \psi \Phi} \right) \]
where \( 0 \leq \phi_i = \Phi_i / \mathcal{I}_i = \tau_{ij} \left( f_i / f_{ii} \right)^{(1-\sigma)\sigma-1} < 1 \) is a further measure of trade openness from the point of view of exporters from country \( i \) to country \( j \). \( \phi \) is proportional to the measure \( \Phi_i \) and thus also rises as \( \tau_{ij} \) or \( f_{ii} / f_i \) fall. The parameter \( \Delta^{1-\sigma} \) is an increasing measure of relative conditions favouring business in \( H \) (against \( F \)). The number of exporting firms is implied by \( M_{ij} = \text{prob}_i M_i \) and the masses of entrants follows according to

\[
M_{ij}^E = \frac{k}{k+1} \phi^{*-k} \phi^{*-1} M_i.
\]

The consumption variety available in country \( i \) is \( M_u = M_i + M_{ij} \).

Country \( i \)'s CES price index is given by

\[
P_i = \left[ \int_0^{\infty} p_j^{-1-\sigma} M_j \mu_j(\phi) d\phi + \int_0^{\infty} \left( \tau_{ij} p_j \right)^{-\sigma} M_{ij} \mu_{ij}(\phi) d\phi \right]^{1/(1-\sigma)}
\]

in this open economy setting. With the price setting rule defined by eq. (3) it can be rewritten as

\[
P_i = \frac{1}{1-\sigma} \cdot p_i(\tilde{\phi}_i) \quad \text{where} \quad \tilde{\phi}_i = \left( M_i \phi_i^{-1-\sigma} + M_{ij} \left( w_j / w_i \right)^{-\sigma} \phi_{ij}^{-1-\sigma} \right)^{1/(1-\sigma)} \]

can be interpreted as an average productivity of all (domestic and foreign) firms that serve consumers in country \( i \). These consumers spend \( M_{ui} r_i(\tilde{\phi}_i) = \beta L_i \) on manufacturing varieties and the average firm revenue is related to the revenue of the cutoff firm according to

\[
r_i(\tilde{\phi}_i) = \frac{\sigma w_i f_i}{\phi_i^*} \quad \text{With} \quad r_i(\phi_i^*) = \sigma w_i f_i \quad \text{it follows that} \quad M_u = \beta L_i \left( \tilde{\phi}_i / \phi_i^* \right)^{-\sigma} / \sigma w_i f_i.
\]

On substitution, this yields for the price level,

\[
P_i = \left( \beta L_i / \sigma f_i \right)^{(1-\sigma)/\sigma} w_i^{\sigma/(\sigma-1)} \left( \rho \phi_i^* \right)^{-1}.
\]

Notice that this derivation of the price level is independent from the derivation of the productivity thresholds and observe that it is also completely general (it does not depend on the Pareto parameterization). However, making use of eq. (11) that obtains under the Pareto-distribution, a closed-form solution can be derived for the price level as well and the same holds true for the indirect utility, which in the open economy setting is given by:

\[
v_i = w_i - \beta \ln \left( \frac{\beta L_i}{\sigma f_i} \right)^{(1-\sigma)/\sigma} w_i^{\sigma/(\sigma-1)} \left( \frac{1}{\rho \phi_i^*} \right) + \beta (\ln \beta - 1)
\]

(12)
4 Welfare and Policy Analysis

4.1 The gains from trade

The welfare effect of opening up an economy from the state of autarky to trade is unambiguously positive as given in our first proposition.

**PROPOSITION 2. (Gains from trade).** Both countries have higher welfare under free trade than under autarky.

*Proof.* Proposition 2 is immediately implied by eqs. (9) and (12). Both without or with asymmetries across countries, equilibrium cutoffs are higher in the two countries under trade than under autarky by eq. (9). The price level is then lower in both countries under trade than under autarky and this entails by eq. (12) that welfare (indirect utility) is higher under trade than under autarky, $v_i - v_{aut} > 0$.

Proposition 2 generalizes previous results. Melitz (2003) has proved the gains from trade for the case of identical countries and Demidova (2008) has extended this result to the case of countries which are asymmetric with respect to technologies in the manufacturing sector. We generalize this result to economies which are asymmetric with respect to a comprehensive set of factors determining the conditions to do business and whose exit/default risk is endogenously explained. In our generalized model, the welfare gain associated with the move from autarky to trade derives fully from the selection effect which drives up the productivity cutoffs as described in Melitz (2003).

4.2 Unilateral trade integration and infrastructure policies

We now turn to analyse the welfare effects associated with a reduction of trade costs between the two countries. We start with the case of unilateral trade integration where one country (say $j$) allows firms located in $i$ better access to its consumers. This is captured by an increase in $\Phi_i$ which may stem from reductions in variable trade costs $\tau_y$ and/or reductions in the fixed export costs $f_{xj}$. Our results are summarized in:

**PROPOSITION 3. (Welfare gains and losses from unilateral trade integration).** A unilateral reduction in trade costs to serve market $j$ (captured by $d\Phi_j > 0$) leads to welfare gains in country $i$ and welfare losses in country $j$. 
Proof. To prove proposition 3 first note that, by eq. (9), \( \partial \varphi_i^* / \partial \Phi_i > 0 \) and \( \partial \varphi_j^* / \partial \Phi_i < 0 \). Taking this into account in the indirect utility, eq. (12), immediately implies our claim.

The intuition behind proposition 3 is the following. Granting firms located in country \( i \) better access to consumers located in country \( j \) raises the profitability to produce manufacturing varieties in country \( i \). This stimulates entry and tightens competition. The least productive firms are thus driven out of the market in \( i \) and the cutoff is raised. This benefits domestic consumers. Firms in \( i \) also gain a competitive advantage over firms located in \( j \). The foreign market becomes less profitable for local firms which reduces the incentive for foreign firms to enter. Competition is thus weakened resulting in a reduction in the foreign productivity cutoff which negatively affects the welfare of foreign consumers.

The policies comprehended by proposition 3 involve both trade and infrastructure policies. Reductions in variable trade costs \( (d \tau_{ij} < 0) \) can both be thought of as being due to lower import tariffs or similar trade costs or due to infrastructure policies (such as greater and more efficient harbours or airports) in country \( j \).

4.3 Symmetric trade integration

We now turn to the case of a symmetric reduction in trade costs \( d \Phi_H = d \Phi_F > 0 \). Note that this comprehends a reduction in variable (iceberg) trade costs and/or a reduction in fixed costs to serve the foreign market (since \( \partial \Phi_i / \partial \tau_{ij} < 0 \) and \( \partial \Phi_i / \partial f_{ij} < 0 \), respectively). We obtain:

PROPOSITION 4. (Welfare gains and losses from symmetric trade integration). A symmetric reduction in trade costs \( (d \Phi_H = d \Phi_F > 0) \) leads to an immiserization of one country (the 'laggard') and welfare gains in the other country (the 'leader') if 

\[ \Delta^{L,F_{an}} < (\Phi_H + \Phi_F)/(1 + \Phi_H^2) \]  

(then \( H \) looses) or if 

\[ \Delta^{L,F_{an}} > (1 + \Phi_F^2)/(\Phi_H + \Phi_F) \]  

(then \( F \) looses). Otherwise both countries reap welfare gains.

Proof. A country' welfare rises (falls) when the productivity cutoff rises (falls). Totally differentiate \( \varphi_i^* = \varphi_i^*(\Phi_i, \Phi_j) \), take the derivatives of the equilibrium cutoffs \( \partial \varphi_i^* / \partial \Phi_i \) and \( \partial \varphi_j^* / \partial \Phi_j \) for \( i, j \), impose \( d \Phi_H = d \Phi_F > 0 \), and then explore the condition when the reaction is positive or negative.
This proposition delivers the important result that the possibility of immiserization through trade integration that was first noted by Demidova (2008, proposition 1) is far more general than was conceived in her paper. Demidova allows technology potentials in the manufacturing sector to differ across countries and she shows that it is possible that the 'laggard' (the country with the inferior technology potential) may lose from falling trade costs.

We generalize this result in two important dimensions. First, we show that asymmetric business conditions in a much more comprehensive sense are accountable for the possibility of immiserization. In fact, there is the possibility of immiserization even without differences in technology potentials. To see this consider the case where country $F$ is the laggard and $H$ is the leading country and remember that $\Delta_{L,\Phi_{w}}^{w} = F^{\frac{\sigma(k-1)}{T^{w}W^{\sigma-1}}}$. Then note that the condition $\Delta_{L,\Phi_{w}}^{w} > \left(1 + \Phi_{F}^{k}\right) \left(\Phi_{H} + \Phi_{F}\right)$ can be fulfilled even if $T = \left(\Phi_{\min H} / \Phi_{\min F}\right) = 1$, indicating identical technology potentials, if entry investments are relatively more favourable in country $H$ (i.e. if $F_{e} = f_{eF} / f_{eH}$ exceeds 1 large enough) and/or wages are relatively low in country $H$ (i.e. $W = w_{F} / w_{H} = a_{H} / a_{F}$ exceeds 1 large enough) such that the left-hand side (LHS) is large. Moreover note that this condition is more easily fulfilled if the right-hand side (RHS) is small, which is the case if firms from $H$ can easily accede consumers in $F$ (i.e. $\Phi_{H}$ is large) or, when trade costs are identical and low (i.e. high $\Phi_{H} = \Phi_{H}$). Moreover, the RHS is low, if it is difficult for firms from $F$ to accede consumers in $H$. Also note that size differences as proxied by the number of workers, $L_{H}$ and $L_{F}$, are inconsequential.

Second, our analysis is general that we allow that the symmetric trade integration proceeds from an initial situation where firms face different conditions to accede consumers in the other country, i.e. $\Phi_{H}$ and $\Phi_{F}$ may differ in the initial equilibrium.

### 4.4 Business conditions and policy options in the open economy

We now turn to the effects of key determinants of business conditions on the productivity of firms and on country welfare under international trade. We first have:

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7 Remember that Demidova (2008) allows technology potentials to differ in a general sense. Our specification which only involves the support of the technology distribution suffices to make the point, however.

8 Caveat: this holds when $2\Phi_{H} \Phi_{F} - \left(1 - \Phi_{H}^{k}\right) < 0$.

9 This result was already found in Baldwin and Forslid (2006), see also Baldwin (2005). However, since these authors did neither account for differences in technology potentials nor the comprehensive set of business conditions that we highlight, they found that symmetric trade integration raises welfare in both countries.
PROPOSITION 5. (The effect of business conditions under trade). Lower domestic entry investments \( f_{ai} \), lower labour productivity in the traditional sector \( 1/a_i \), and/or a lower bound of technologies \( \varphi_{\text{min}_i} \) in country \( i \) raises the cutoff productivity and welfare in this country and decreases productivity and welfare in country \( j \).

Proof. This proposition follows immediately by considering the effects of changes in \( f_{ai} \), \( 1/a_i \), and \( \varphi_{\text{min}_i} \) on eq. (9) and (12).

Intuitively, any improvement in business conditions in country \( i \), such as a better technology potential, lower entry investments and lower wages, raises the profitability of the domestic market and gives local firms a competitive edge over their foreign competitors. This stimulated entry in country \( i \) and reduces the incentive to enter the manufacturing industry in country \( j \), which sets in a selection effect that leads to higher cutoffs and welfare in \( i \) and lower cutoffs and welfare in \( j \) (similarly to the case of unilateral improvements in market access that we discussed before). The importance of proposition 5 derives from the fact that it implies a considerable generalization of the finding that productivity improvements in one country hurt the other country (Demidova 2008, proposition 2). In fact our proposition shows that the very same result holds with respect to competitive advantages due to lower wages and easier market entry.

In contrast to the factors considered in proposition 5 the effect of changes in the domestic fixed labour input necessary to serve the domestic market has an ambiguous effect on the domestic productivity cutoff, but an unambiguous effect on welfare as stated in:

PROPOSITION 6. (The effect of domestic fixed labour input under trade). An increase in domestic fixed labour inputs \( f_n \) leads to (i) an increase in the domestic productivity cutoff iff the domestic market is sufficiently protected from foreign competition, (ii) unambiguous welfare losses in country \( i \), and (iii) an unambiguous increase in the cutoff productivity and welfare in country \( j \).

Proof. The method of proof follows the one employed to prove the previous proposition.

Proposition 6 shows a remarkable difference to our finding for the closed economy. In the closed economy, an increase in \( f \) necessarily drives up the productivity cutoff (see eq. (7)) due to a stronger selection effect which drives the least efficient firm out of the market. In the open
economy, an increase in $f_i$ has a further effect, it facilitates the access of foreign firms to the domestic market, as $d\Phi_i / df_i > 0$. This implies a competitive disadvantage for domestic firms vis-à-vis their foreign competitors whose effect it is to reduce the productivity cutoff. This leads to the ambiguity. However, the effect on domestic welfare is unambiguously negative, as the unproductive use of fixed labour input reduces the domestic number of firms and hence the product variety available. Furthermore, the impact on foreign productivity and welfare is positive, as firms from $j$ now enjoy a comparative advantage.

Propositions 5 and 6 are of crucial importance from a policy perspective. Fixed investments that are needed to enter and serve the domestic market and the technology potential can be influenced by policy. For example the necessary fixed investments to start and do business are associated with a country's level of corruption, the costs to enforce contracts, the costs to provide protection against crime, product piracy and product imitation. Technology policies have an influence on a country's technological potential. Crucially, any improvement from the point of view of one economy has a negative welfare effect on the other economy.

One may be interested in how sensitive the effects noted in propositions 5 and 6 are with respect to the level of trade integration. This we cover in:

**PROPOSITION 7. (Trade cost sensitivity of policies).** Consider possible changes in country $i$'s technology potential, fixed market entry investment, wage or fixed labour input. (i) Suppose $\Phi_F = \Phi_H = \Phi$. The effect of any such change on the domestic productivity cutoff (as captured by $d\Phi_i / \varphi_i^*$) is the greater, the greater is the level of trade freeness ($\Phi$). (ii) Suppose $\Phi_F \neq \Phi_H$. The effect of any such change on the domestic productivity cutoff is the greater, the higher is $\Phi_i$, i.e. the better is the market access of firms from country $i$ to market $j$. The effect of any such change is insensitive to $\Phi_j$.

Proof. The proposition follows from differentiation of eq. (9).

Part (i) of proposition 7 carries the important message that the impact of policies that affect the conditions to do business gets magnified as the general level of trade freeness is raised. This finding has previously been obtained in models of the new trade theory and the new economic geography with homogeneous firms (cf. Helpman and Krugman 1985; Baldwin et al. 2003), the underlying mechanism being the same one as here. Our analysis extends this result to a comprehensive set of factors affecting business conditions. Part (ii) of proposition 7 is entirely
novel. It reveals that domestic policies are more powerful when domestic firms have easy access to foreign markets.

4.5 Business conditions and default risks in the open economy

One of the main findings of this model is that country-specific default risks arise endogenously. The determinants of this risk under international trade are summarized by:

**PROPOSITION 8.** *(Country-specific default risk under international trade)*. The expected (average) risk of business exit in country $i$, $E[\delta(\varphi) | \varphi > \varphi^*_i] = [k/(k + 1)] \varphi_i^{-1}$, (i) is independent of country size $L_i$ and $L_j$, (ii) increases when entry investment $e_{i}$ is higher, when domestic labour productivity in the traditional sector $1/a_i$ is higher, and when the technology bound $\varphi_{min}^i$ is lower, and (iii) decreases when domestic fixed labour input $f_i$ is higher if market $i$ is sufficiently protected from foreign competition (i.e. if $\sigma_i \Phi_i < (\sigma - 1)/(k - \sigma)$), (iv) decreases when foreign entry investment $e_{j}$ is higher, when foreign labour productivity in the traditional sector $1/a_j$ is higher, when the foreign technology bound $\varphi_{min}^j$ is lower and foreign fixed labour input $f_j$ is higher.

**Proof.** Proposition 8 is proved by using $E[\delta(\varphi) | \varphi > \varphi^*_i] = [k/(k + 1)] \varphi_i^{-1}$ and the effects on $\varphi_i^*$ that were stated in the previous propositions.

Intuitively, we find that all measure which stimulate market entry in country $i$ raise the cutoff productivity in this country. As the empirical evidence strongly suggests that more productive firms have a lower bankruptcy risk, the average risk of business default falls. However, if the incentives for market entry increase in country $j$, firms in country $i$ face fiercer competition. Hence, in country $i$ less potential entrepreneurs invest into the productivity lottery so that less productive firm can survive in the market.

As national productivity cutoffs do not only dependent on fundamental differences in the business environment but also on the level of trade integration, we have:

**PROPOSITION 9.** *(Default risk under trade integration)*. The expected (average) risk of business exit in country $i$, $E[\delta(\varphi) | \varphi > \varphi^*_i] = [k/(k + 1)] \varphi_i^{-1}$ (i) decreases when firms from $i$ have better access to market $j$, (ii) increases when firms from $j$ have better access to market $i$, and
(iii) decreases by symmetric trade integration (i.e., \( d\Phi_H = d\Phi_F = d\Phi \)) iff
\[ A^{\ldots, \Delta, \Phi_{\text{min}}, w} > (\Phi_H + \Phi_F)/(1 + \Phi_H^2) \] (if you consider the default risk in country \( H \)) or iff
\[ A^{\ldots, \Delta, \Phi_{\text{min}}, w} < (1 + \Phi_F^2)/(\Phi_H + \Phi_F) \] (if you consider the default risk in country \( F \)).

Proof: Consider 
\[ E[\delta(\Phi | \Phi > \phi^*_i)] = [k/(k+1)]\phi_i^{-1} \], proposition 4 and the proof of proposition 3.

The interesting insight of Proposition 9 is that country-specific default risks vary with the level of trade integration which, to the best of our knowledge, is novel to the literature. As before, better access for firms from country \( i \) to market \( j \) induces a selection process of firms which leads to a higher cutoff productivity and, hence, a lower average risk of business exit in \( i \). This holds true vice versa, if firms from country \( j \) can more easily serve costumers in country \( i \).

Part (iii) picks up the possibility of immiserizing trade integration as discussed in section 4.3.

4.6 Full specialization in the traditional industry

Our analysis has so far rested on the assumption that the two countries are diversified in production both under autarky and under trade, i.e. that each country has an active manufacturing sector in addition to a traditional industry. However, we have already noted that it is conceivable that one country (the 'laggard') may be forced into specialization in the traditional industry if asymmetries are very strongly in favour of doing business in the other country (the 'leading economy'). This section considers this possibility. We shall assume that the 'leading country' is still diversified in production. We highlight the key results here and refer the reader to appendix F for an extended technical exposition of this case.

Condition for specialization. We start out with an exploration of the condition under which one country is driven into full specialization in the traditional industry. Using eq. (11) and imposing \( M_i \geq 0 \) we can immediately derive the condition where both countries have manufacturing producers,

\[ \phi_F < A^{\ldots, \Delta, \phi^*} < 1/\phi_H \] where
\[ A^{\ldots, \Delta, \phi^*} = \frac{f_F}{f_H} \frac{L_H}{L_F} \left[ \frac{f_F}{f_H} \left( \frac{1 - A^{\ldots, \Delta, \Phi_{\text{min}}, w} \cdot \Phi_H}{A^{\ldots, \Delta, \Phi_{\text{min}}, w} - \Phi_F} \right)^{\frac{k}{k-1}} \right] \]

Outside this range, one country will be fully specialized in the production of the traditional good: country \( H \) is fully specialized if \( \phi_F \geq A^{\ldots, \phi^*} \) and country \( F \) is fully specialized if \( A^{\ldots, \phi^*} \geq 1/\phi_H \). On inspection of this condition we see that countries are fully specialized if business conditions are strongly against doing business in that economy (i.e. if wages are high,
the economy is small, fixed investments needed for domestic and foreign market supply are high, entry investment is high, the technology potential is weak and trade access is difficult).

Gains from trade. The switch from autarky to trade may already force one country into full specialization in the traditional industry. Even in this case there are gains from trade to both countries. We can state:

PROPOSITION 10. (Gains from trade under specialization). Both countries have higher welfare under international trade than under autarky even if trade opening forces one country into full specialization in the traditional industry whereas the other country is diversified in production.

Proof. Since we assume that both countries produce the traditional good both under autarky and under trade (such that a consumer has the same wage under autarky and trade), the welfare comparison boils down to a comparison of the price levels. We show in appendix F that even if a country is forced into full specialization by opening up to trade, its price level is lower than under autarky (where it produces both types of goods). The country which produces both types of goods has a lower price level for the same reason as in proposition 2. Hence, it holds true for that $v_i - v_{aut} > 0$ for both countries.

To the best of our knowledge, proposition 10 is entirely novel. The country which is driven into full specialization in the traditional industry benefits from the productivity increase of the trading partner. Our proposition shows that this beneficial effect is so strong that it compensates for the fact that the ‘laggard’ country has to incur trade costs for all manufacturing goods.

No immiserizing trade integration with full specialization. Proposition 4 which was derived under the assumption that both countries are diversified in production showed that one country may experience immiserization under trade integration. This result no longer holds true under specialization. In fact, it is immediate to see that a country that is and remains specialized during trade integration always experiences welfare gains through trade cost savings. We thus have:

PROPOSITION 11. (No immiserization under trade integration). If trade opening forces one country into full specialization in the traditional industry whereas the other country is diversified in production, no country is worse off by trade integration.
Proof: The welfare of a country increases when the price level falls. Use the price indices under specialization (as stated in appendix F) to see that they do not rise by trade integration.

**Default risk in the leading country.** For completeness, let us explore the risk of default in the present scenario. It is immediately clear that this question is irrelevant for the 'laggard' which has no manufacturing industry. For the leading country we summarize the results by

**Proposition 12. (Default risk under specialization).** Under specialization, the expected (average) risk of business exit in country \(i\), \(E[\delta(\varphi)|\varphi > \varphi^*_i] = \left[\frac{k}{(k + 1)}\right] \varphi^{*-1}_i\), (i) is smaller than under autarky, (ii) is independent of domestic labour productivity in the traditional sector \(1/\alpha_i\) and the level of trade integration \(\Phi_i\) and \(\Phi_J\), (iii) increases when entry investment \(f_{ai}\) is higher, when the technology bound \(\varphi_{\min_i}\) is lower, when the domestic market is size \(L_i\) is larger and the foreign market size \(L_J\) is lower, and when fixed investment for doing business \(f_i\) and for export activities \(f_{si}\) are lower.

Proof: Consider \(E[\delta(\varphi)|\varphi > \varphi^*_i] = \left[\frac{k}{(k + 1)}\right] \varphi^{*-1}_i\). Part (i) follows immediately from the domestic cutoff productivity given by Eq. (F1). For part (ii) derive this cutoff productivity with respect to the parameter of interest.

The intuition behind the independency from domestic labour productivity in the traditional sector carries over from the autarky case (see section 2.5.). The same holds true for fixed labour inputs: it is intuitive that only more productive firms can bear higher fixed input requirements as they have a greater profit – fixed cost – margin. Furthermore, the rationale of the role of entry investments and the technological potential is the same as under non-specialization (as discussed in section 4.5.). The novelty of Proposition 12 is that under specialisation now market sizes influence the national risk of bankruptcy. The larger is the export market, the more exporting firms can withstand competition for foreign costumers, so that the incentive to participate in the production lottery increases, which drives unproductive producers out of the market. Instead, in a larger domestic market even unproductive firms can survive.

### 5 Conclusion

The paper develops a general equilibrium model of international trade which on the one hand takes into account that the default risk at the level of the firm is negatively related to the firm's
productivity, and on the other hand introduces a comprehensive set of factors that determine business conditions and that may differ across the two countries that we consider.

We obtain a number of remarkable results. Fundamentally, our model is able to explain the fact that the default risk at the country level is determined by the conditions to do business, which itself are heavily influenced by policy. For example, firms have lower default risks in countries where it is cheaper to enter the market or where the technology potential is superior. Moreover, in the open economy, the default risk of a country's firms falls if the market access to consumers in the other country improves. Among the novel results that our welfare and policy analysis for the open economy delivers are the following. First, there are gains to trade even if one country is driven into full specialization on traditional goods by a trading partner which offers its firms far advantageous business conditions. Second, improvements in the business conditions in one country (e.g. due to trade policies, infrastructure policies or industrial policies) have negative welfare effects on the other country. Third, trade integration is immiserizing for a country which trades with another country which offers far superior business conditions. Fourth, the default risk of firms falls in both countries under symmetric trade integration.

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Appendices

Appendix A  The free entry condition (FEC) in the closed economy

From $\pi(\phi) = \frac{r(\phi)}{\sigma} - w^r$ it follows that

$$E \left[ \frac{\pi(\phi)}{\delta(\phi)} \phi > \phi^* \right] = \frac{1}{\sigma} E \left[ \frac{r(\phi)}{\delta(\phi)} \phi > \phi^* \right] - w^E E \left[ \frac{1}{\delta(\phi)} \phi > \phi^* \right]$$

Using $r(\phi) = (\phi / \phi')^{\sigma-1} r(\phi')$ and $\delta(\phi) = \delta(\phi') \phi / \phi'$, it holds true that

$$E \left[ \frac{\pi(\phi)}{\delta(\phi)} \phi > \phi^* \right] = \frac{1}{\sigma} r(\phi') (\phi')^{-\sigma} E \left[ \phi^* \phi > \phi^* \right] - w^E E \left[ \frac{1}{\delta(\phi)} \phi > \phi^* \right]$$

where $E[\phi^* | \phi > \phi^*] = (\phi')^\sigma$. Adding and subtracting $f / \delta(\phi')$ on the RHS leads to

$$E \left[ \frac{\pi(\phi)}{\delta(\phi)} \phi > \phi^* \right] = \frac{\pi(\phi')}{\delta(\phi')} - w^f \left( E \left[ \frac{1}{\delta(\phi)} \phi > \phi^* \right] - \frac{1}{\delta(\phi')} \right)$$

Using this expression in the value of entry, $v^E = [1 - G(\phi^*)] \cdot E[\pi(\phi)/\delta(\phi) | \phi > \phi^*] - w \cdot f_e$, equating this to zero and then solving for $\pi(\phi')$ gives the FEC stated in eq. (5).

Appendix B – Equilibrium cutoff under Pareto specification under autarky

With Pareto-distributed productivities $G(\phi) = 1 - (\phi_{\min} / \phi)^k$ and $g(\phi) = G'(\phi) = k \phi_{\min}^{k-1}$ where $\phi_{\min} > 0$ and $k > 1$. Then $\mu(\phi) = g(\phi) [1 - G(\phi^*)] = k \cdot \phi_{\min}^{k-1} \cdot \phi^{-(1+k)}$ if $\phi > \phi^*$ and $\mu(\phi) = 0$ otherwise. With this parameterization and assuming $k > \sigma$, it follows that

$$\bar{\phi} = \left[ k / (k - (\sigma - 1)) \right]^{1/(\sigma-1)} \phi^*$$

$\phi' = \left[ k / (k - \sigma) \right]^{1/\sigma} \phi^*$ and $E[\delta^{-1}(\phi) | \phi > \phi^*] = \left[ k / (k - 1) \right] \phi^*$. The value of entry reads $v^E = \phi^{\sigma-1} \phi_{\min}^{k-1} \pi(\phi') - w^f \left( \left( \frac{k}{k - 1} \right)^{1/\sigma} \pi(\phi') - w^f \phi_{\min}^{k-1} \right) - w f_e$. The ZCPC and the FEC are given by

$$\pi(\phi') = \left[ \left( \frac{k}{k - \sigma} \right)^{\frac{\sigma-1}{\sigma}} - 1 \right] \cdot w^f$$

and

$$\pi(\phi') = \left( \frac{k}{k - \sigma} \right)^{-\frac{1}{\sigma}} w^f \frac{\phi_{\min}^{k-1}}{\phi_{\min}^{k-1}} + w^f \left( \left( \frac{k}{k - 1} \right)^{\frac{1}{\sigma}} - 1 \right)$$

respectively. Solving these two conditions yields the autarky equilibrium cutoff given by eq. (7).

Appendix C – The link between the productivity cutoffs in the open economy
(i) From the ZCP conditions it follows that

\[ r_i(\phi^*_i) = \left(\frac{\rho \phi^*_i}{P_i}\right)^{-\sigma} P_j^{-\sigma} \beta L_j = \sigma w_j f_{j,i} \]

Consequently, we have

\[ \frac{r_{ih}(\phi^*_h)}{r_f(\phi^*_f)} = \frac{w_h f_{ih}}{w_f f_f} \Rightarrow \phi^*_h = W^{-\sigma/(\sigma-1)} \frac{P_E}{P_H} \left( \frac{f_{ih} L_H}{f_f L_f} \right)^{1/(\sigma-1)} \]  

(B1)

\[ \frac{r_{ai}(\phi^*_a)}{r_i(\phi^*_i)} = \frac{f_{ai}}{f_i} \Rightarrow \phi^*_a = \tau_i \left( \frac{f_{ai}}{f_i} \right) \left( \frac{P_i}{P_j} \right) \left( \frac{L_i}{L_j} \right)^{1/(\sigma-1)} \]  

(B2)

Combining (B1) and (B3) leads to \( \phi^*_a = W^{-\sigma/(\sigma-1)} t_h \phi^*_f \) and \( \phi^*_f = W^{\sigma/(\sigma-1)} t_f \phi^*_h \) where

\[ t_f = \tau_f \left( f_{ai} / f_i \right)^{1/(\sigma-1)} \]

(ii) We assume that only firms that serve the domestic market can export, i.e. \( \phi^*_a > \phi^*_f \). From (B3) it follows that this holds true whenever

\[ \tau_i \left( f_{ai} / f_i \right)^{1/(\sigma-1)} \left( \frac{P_i}{P_j} \right) \left( \frac{L_i}{L_j} \right)^{1/(\sigma-1)} > 1. \]

Substituting

\[ P_i = \left( \beta L_i / \sigma \right)^{1/(1-\sigma)} w_i^{1/(1-\sigma)} \left( \rho \phi^*_i \right)^{-1} \]

and rearranging yields

\[ f_{ai} / f_i > \tau_i \left( w_i / w_j \right)^{1/(\sigma-1)} \left( \phi^*_a / \phi^*_f \right)^{1-\sigma} \]

Note that in Demidova (2008) the condition \( \phi^*_a > \phi^*_f \) implies \( \phi^*_a > \phi^*_f \) (i.e. that a domestic firm finds it easier to break even in its domestic market than a foreign exporter does) since her model assumes \( W = 1 \). However, in the presence of a possibly large wage differential it is quite conceivable that an exporting firm might find it easier to break even than a local firm does. Hence, the implication will not carry over to our model, in general.

Appendix D: Determination of equilibrium cutoffs in the open economy

The free entry condition (FEC) for country \( i \) is given by

\[ \left( 1 - G(\phi^*_i) \right) \cdot \left[ \frac{\pi_i(\phi)}{\delta(\phi)} \phi > \phi^*_i \right] + \left( 1 - G(\phi^*_a) \right) \cdot \left[ \frac{\pi_a(\phi)}{\delta(\phi)} \phi > \phi^*_a \right] = w_i \cdot f_{ei} \]  

(D1)

As \( \pi_i(\phi) = r_i(\phi) / \sigma - w_i f_i \), we can write the expected domestic profits as

\[ E \left[ \frac{\pi_i(\phi)}{\delta(\phi)} \phi > \phi^*_i \right] = \frac{1}{\sigma} E \left[ r_i(\phi) \delta(\phi) \phi > \phi^*_i \right] - w_i f_i E \left[ \frac{1}{\delta(\phi)} \phi > \phi^*_i \right] \]

Using \( r_i(\phi) = \left( \rho \phi / w_i \right)^{\sigma-1} P_j^{-\sigma} \beta L_j \), \( \delta(\phi) = 1 / \phi \) and the Pareto parameterization we get

\[ E \left[ \frac{\pi_i(\phi)}{\delta(\phi)} \phi > \phi^*_i \right] = \left( \frac{r_i(\phi^*_i)}{\sigma} \frac{1}{k-\sigma} w_i f_i k \right) \phi^*_i \]
On substituting \( r_i(\phi_i^*) = \sigma w_i f_i \) which is implied by the domestic ZCPC \( \pi_i(\phi_i^*) = 0 \), we have:

\[
E \left[ \frac{\pi_i(\phi)}{\sigma(\phi)} \right]_{\phi > \phi_i^*} = \frac{\sigma - 1}{k - \sigma} \frac{k}{k - 1} w_i f_i \phi_i^* \tag{D2}
\]

The expected export profits are determined in the same manner. Now we use export profits, export revenue, the previous parameterizations as well as the export ZCPC to obtain:

\[
E \left[ \frac{\pi_{xi}(\phi)}{\sigma(\phi)} \right]_{\phi > \phi_{xi}^*} = \frac{\sigma - 1}{k - \sigma} \frac{k}{k - 1} w_{ixi} \phi_{xi}^* \tag{D3}
\]

Substituting (C2) and (C3) into (C1) and using \( G_i(\phi) = 1 - \left( \frac{\phi_{min}}{\phi_i} \right)^k \) yields

\[
\frac{\sigma - 1}{k - \sigma} \frac{k}{k - 1} \phi_{min}^k f_i (\phi_i)^{-k} + \frac{\sigma - 1}{k - \sigma} \frac{k}{k - 1} \phi_{min}^k f_{xi} (\phi_{xi})^{-k} = f_{ei} .
\]

Writing this equation out for \( i = H, F \) and using the relationships between export cutoffs and domestic cutoffs,

\( \phi_{st}^* = W^{-\sigma(\sigma-1)} t_H \phi_F^* \) and \( \phi_{st}^* = W^{-\sigma(\sigma-1)} t_F \phi_H^* \) as derived in appendix C yields two equations which can be solved for the cutoffs \( \phi_H^* \) and \( \phi_F^* \) as stated in eq. (9).

**Appendix E: The share of labor in manufacturing and the masses of firms in the open economy equilibrium**

Start with the condition of balanced trade (eq. (10))

\[
prob_{st} M_i^{r_{st}} (\phi_{st}) = prob_{ixi} M_{ixi}^{r_{ixi}} (\phi_{xi}) + (w_H - \beta) L_H - (1 - \gamma_H) L_H / a_H
\]

and then substitute \( M_i = \gamma_i L_i w_i / \pi_i \) where \( \pi_i = r_i(\phi_i) + prob_{xi} r_{xi} (\phi_{xi}) \), \( w_H = 1 / a_H \) and \( \beta(L_H + L_F) = \gamma_H L_H w_H + \gamma_F L_F w_F \). Solving for the \( \gamma_i \) then gives:

\[
\gamma_H = \frac{\gamma_i w_H}{r_H(\phi_H)} \frac{1 - \phi_F / \Delta^{1, \phi^*}}{1 - \phi_F \phi_H^*} \quad \text{and} \quad \gamma_F = \frac{\gamma_i w_F}{r_F(\phi_F)} \frac{1 - \phi_H / \Delta^{1, \phi^*}}{1 - \phi_H \phi_F^*}
\]

where \( 0 \leq \phi_i = \Phi_i / t_i = \tau_{ij}^k \left( f_j / f_{xi} \right)^{k \sigma - 1} < 1 \) and \( \Delta^{1, \phi^*} \equiv \frac{f_{xi} f_H L_H}{f_{H} L_F} B^{-k} \). Using \( \gamma_i \), the masses of firms are immediately implied by \( M_f H = \gamma_i L_i w_i \) where \( r_i(\phi_i) \) follows from the domestic ZCPC and is given by \( r_i(\phi_i) = \sigma k f_i w_i [k - (\sigma - 1)] \). The firm numbers are stated in eq. (11).

**Appendix F: The model with specialization on the traditional industry in \( H \)**

Consider that only \( F \) has manufacturing firms, whereas country \( H \) only produces the homogeneous good. The price indices are then given by \( P_{xi} = \left[ \int_{z \in \Omega} p^{1, \sigma} M_{ixi} dz \right]^{1/(1-\sigma)} \) and
\[ P_{H} = \left[ \left( \tau_{F} p_{F} \right)^{-\sigma} M_{xF} \mu_{x} dz \right]^{-1/(1-\sigma)} \] where \( M_{xF} \) is the number of exporting firms from \( F \).

Like under non-specialization, potential entrepreneurs in \( F \) invest into the productivity lottery until the value of firm entry is driven to zero, so that the FEC is the same and is given by
\[
\frac{k - (\sigma - 1)}{k - \sigma} \frac{\phi_{xF}^{k}}{k_{F}} + \frac{1}{k} \frac{\phi_{xF}^{k}}{k_{F}} = f_{xF}.
\]
The trade balance condition from the perspective of \( F \) is given by
\[
M_{xF} r_{xF}(\phi_{xF}) = L_{HF} - (w_{HF} - \beta)L_{HF}.
\]
The LHS gives the value of country \( F \)'s manufacturing exports, whereas the LHS represents the difference between the value of domestic production and consumption of the homogeneous good. Using \( 1/a_{HF} = w_{HF} \) and
\[
r_{xF}(\phi_{xF}) = \sigma k f_{xF} w_{HF} \left[ k - (\sigma - 1) \right] which follows from the export ZCPC \( \pi_{F}(\phi_{xF}) = 0 \), the mass of exporting firms is given by \( M_{xF} = \left[ k - (\sigma - 1) \right] \beta L_{HF} / k \sigma w_{HF} f_{xF} \). Furthermore, the total revenue of manufacturing firms must equal the sum of wages in that sector, i.e. \( M_{F} r_{F} = \gamma_{F} L_{F} w_{F} \) where \( r_{F} = r_{F}(\phi_{F}) + \text{prob}_{xF} r_{xF}(\phi_{xF}) \) is the average revenue of a manufacturing firm in country \( F \).

Taking into account that worldwide expenditures on manufacturing goods must match the sum of wages earned in this sector, \( \beta(L_{HF} + L_{F}) = \gamma_{F} L_{F} w_{F} \), we derive the mass of domestic firms as
\[
M_{xF} = \text{prob}_{xF} M_{F} \text{, where } \text{prob}_{xF} = \left( \frac{\phi_{xF}}{\phi_{xF}} \right)^{k} \text{, shows that the domestic and the export cutoff are tied, } \phi_{xF}^{*} = \left( \frac{L_{F}}{L_{HF}} \right)^{k} \phi_{xF}^{*}. \]

To ensure that only domestic firms can export, i.e. \( \phi_{xF}^{*} > \phi_{xF}^{*} \), we assume \( (L_{F} / L_{HF}) (f_{xF} / f_{F}) > 1 \). Using this link in the FEC yields the equilibrium cutoffs:
\[
\phi_{xF}^{*} = \phi_{xF}^{*} \left[ 1 + \left( \frac{L_{HF}}{L_{F}} \right)^{k} \left( \frac{f_{xF}}{f_{F}} \right) \right]^{1/k} \text{ and } \phi_{xF}^{*} = \phi_{xF}^{*} \left[ 1 + \left( \frac{L_{HF}}{L_{F}} \right)^{k} \left( \frac{f_{xF}}{f_{F}} \right) \right]^{1/k} \]

The equilibrium masses of firms immediately follow by
\[
M_{xF} = \left[ k - (\sigma - 1) \right] \beta L_{F} / k \sigma w_{HF} f_{xF} \text{ and } M_{xF} = \left[ k - (\sigma - 1) \right] \beta L_{HF} / k \sigma w_{HF} f_{xF} \]
To show that country $H$, which specializes in the homogeneous good, has gains from trade, we depart from the utility differential between the case with trade but no manufacturing firms in $H$ and autarky, $\Delta v \equiv v_{TS,H} - v_{aut,H} = \beta \ln(\frac{P_{aut,H}}{P_{FTS,H}})$. The utility differential is determined by the difference in price indices under autarky and specialization. Using

$$ P_{Hs} = \left[ \int_{z \in \Omega} (\tau_F p_f)^{1-\sigma} M_{s_F} \mu_s dz \right]^{\frac{1}{1-\sigma}} = \tau_F M_{s_F}^{1/(1-\sigma)}(\rho_{s_F} w_F)^{-1}, $$

the number of exporting firms from Eq. (F2), $\tilde{\phi} = \left[ k/(k-(\sigma-1)) \right]^{1/(\sigma-1)} \phi^*$ and the equilibrium cutoffs as given by Eq. (F1), the ratio of price indices is given by

$$ \left( \frac{P_{aut,H}}{P_{S,H}} \right)^{k-1} = \Phi_F \Delta_{f,F,\phi_{min,w}} \left[ 1 + \left( \frac{L_F}{L_H} \right)^{k-1} \left( \frac{f_H}{f_F} \right)^{\frac{k-1}{k}} \left( \frac{f_F}{f_{s_F}} \right)^{\frac{k-1}{k}} \right] $$

(F3)

From Eq. (11) we know that there are no manufacturing firms in $H$ whenever $\Delta_{f,L,\phi^*} \leq \phi_F$, which can be rewritten as

$$ \left( \frac{L_F}{L_H} \frac{f_H}{f_F} \right)^{k-1} \geq \left( \frac{1}{\phi_F} \right)^{1/k} B^{(k-1)}. $$

Substituting this condition as equality (note that this keeps Eq. (F3) artificially small), making use of $\Phi_F = t_F \cdot \phi_F$, $\phi_F = \tau_F k (f_H / f_{s_F})^{(k-1)/(\sigma-1)}$ and $B = \left[ (f_F / f_H) (1 - A^\phi_{min,w} - \Phi_F) / (A^\phi_{max,w} - \Phi_F) \right]^{1/(k-1)}$ we get

$$ \left( \frac{P_{aut,H}}{P_{Hs}} \right)^{k-1} = \frac{1 - \Phi_F \cdot \Phi_H}{1 - \Delta^\phi_{min,w} \cdot \Phi_H} $$

(F4)

Whenever $\Phi_F < A^\phi_{min,w} < 1/\Phi_H$, the price index under autarky is greater than under trade, so that country $H$ reaps welfare gains from trade. If business conditions favouring country $F$ are even stronger, so that $\Phi_F > A^\phi_{max,w}$ or $\Phi_F \Delta_{f,F,\phi_{min,w}}^{1} > 1$, there are also gains from trade which immediately follows from (F3).