Service Outsourcing and Specialization: A Theory based on Endogeneous Task Scope*

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VER Y PRELIMIN AR Y

First Draft: March, 2010
This version: July 29, 2010

Abstract

Firms outsource an increasing range of service activities to independent suppliers which tend to be specialized in providing a given service. This new form of outsourcing has the potential to raise aggregate productivity by allowing the “division of labour between firms”. We develop a model of outsourcing and trade in service inputs where the scope of tasks produced by both manufacturing firms and service providers is endogeneous. Manufacturing firms have to perform a fixed set of tasks in order to produce their final good but can decide to outsource some of these tasks to service providers, which, contrary to manufacturers, have the possibility to sell tasks to different manufacturers and thereby benefit from economies of scale in their task production. The key assumption is that the marginal cost of a firm (manufacturer or service provider) increases in the scope of tasks performed inside the firm: a firm which specializes in a narrow scope of tasks is more productive. Working against this incentive to produce as few tasks as possible “inhouse” is a fixed cost paid by each firm. The model yields several new predictions about trade liberalization and welfare as measured by aggregate productivity. An increase in the size of an economy raises the scale of all firms, facilitates greater specialization and therefore raises each firm’s productivity. The model therefore generates gains from trade or larger market size through a “specialization effect” as opposed to the classical “variety effect” usually generated by models building on Dixit Stiglitz utility structures. Detailed Swedish data on what tasks (or occupations) are performed by workers is used to test this prediction. Indeed, we find that manufacturing firms in larger cities (controlling for firm size) perform fewer tasks inhouse than firms in smaller cities.

Keywords: service outsourcing, division of labour, productivity, specialization.
JEL codes: F10, F43, L24, R10.

*We are grateful to Karolina Ekholm, Rikard Forslid, Joseph Francois, Wolfgang Keller, James Markusen, Philippe Martin, Thierry Mayer and to the participants of the second GIST conference for helpful comments and suggestions. All remaining errors are ours. Financial support from Jan Wallander’s and Tom Hedelius’ Research Foundation is gratefully acknowledged by Akerman. Financial support from the GIST (CEPR) network is gratefully acknowledged by Py.
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1 Introduction

There is by now large empirical evidence that firms outsource an increasing range of activities to independent suppliers, be they located in the domestic economy or abroad. However, the nature of outsourcing appears to be changing. First, while it has long involved the outsourcing of manufactured inputs, it increasingly concerns the outsourcing of services, see Amiti and Wei (2005). Second, this shift towards service outsourcing goes hand in hand with the emergence of firms which specialize in providing one or a few services. Manufacturing firms now have the possibility to outsource their tasks related to human resources to companies specialized in recruiting staff, the development of software to external IT engineers, the after sales services to call centres, the canteen to a catering firm and the cleaning of offices to a cleaning company. One important aspect of this new trend is that, contrary to the outsourcing of material inputs which involves indirect trade in tasks, service outsourcing involves direct trade in tasks and therefore leads to an emergence of more specialized firms. Indeed, the production of an intermediate good involves a whole production process in itself and implicitly a wide range of tasks, but providing a service only involves the performance of a few tasks strongly related to the specific service type. This, we believe, is strongly related to one of the fundamental ideas by Adam Smith (1776). He argued that production becomes more efficient when labour is divided such that workers focus on specific tasks instead of each worker performing all the tasks involved in the production of one unit of output:

“The greatest improvement in the productive powers of labour, and the greater part of the skill, dexterity, and judgment with which it is anywhere directed, or applied, seem to have been the effects of the division of labour.”

He proceeds by describing the classical case of the pin factory:

“To take an example, therefore, from a very trifling manufacture; but one in which the division of labour has been very often taken notice of, the trade of the pin-maker. [.....] Each person, therefore, making a tenth part of forty-eight thousand pins, might be considered as making four thousand eight hundred pins in a day. But if they had all wrought separately and independently, [.....] they certainly could not each of them have made twenty, perhaps not one pin in a day.”

We argue in this paper that service outsourcing facilitates the “division of labour” between firms and therefore has the potential to generate large productivity gains for economies. The aim of this paper is to provide a theoretical framework for this process
and to develop a model in which potential gains arise from service outsourcing and the induced specialisation of firms.

In order to do so, we develop a model of outsourcing and trade in service inputs in which the scope of tasks performed by both manufacturing firms and service providers is endogeneous. In order to produce one unit of a differentiated good, manufacturers have to perform a fixed range of tasks.\(^1\) However, manufacturers have the option to outsource some tasks in order to reduce the scope of tasks they perform in-house. The key assumption in the model is that the marginal cost of producing a given task increases with the average distance between tasks performed in-house. In other words, the narrower the range of tasks performed “inhouse” is, the more efficient the firm will be in the performance of these tasks or, equivalently, the lower marginal cost the firm will have. The incentive to outsource for manufacturers stems from this benefit of specialization. However, given the Leontief type production function of manufacturers, if the quality of service provided is insufficient, the manufacturer’s production will fall down to zero. In an imperfect contracting environment, manufacturers will have to share their surplus with service providers.\(^2\) Service providers do not produce any good and cannot outsource. They perform a range of tasks that they provide to all manufacturers. They face a cost structure similar to the one of manufacturers and also become more efficient the narrower is their task scope. However, contrary to manufacturers, service providers do not have to produce a fixed range of tasks and can instead specialize in a few tasks. They therefore benefit from economies of scale in specialization.

The model yields several new theoretical predictions. First, larger markets consist of more specialized firms. Second, aggregate productivity rises as both manufacturers and service providers perform fewer tasks in-house and become more efficient. Indeed, when the market grows, more service providers can be active given the higher demand and it becomes more profitable for manufacturers to outsource more tasks. Manufacturing firms decrease the scope of tasks performed in-house but instead expand output per good which spreads the fixed cost over more output units. Specialists act in the same way and also become more specialized. The model therefore generates gains from trade or larger market size through a “specialization effect” as opposed to the classical “variety effect” usually generated by models building on Dixit Stiglitz utility structures such as in Krugman (1980). Welfare increases due to adjustments in task scope allowed by service outsourcing and the emergence of specialized firms.

\(^1\)As in Grossman and Rossi-Hansberg (2008b), we assume that the task inputs for manufacturers are arranged along a line.

\(^2\)An important literature has already explored the determinants of the “make or buy” decision and highlights the importance of incomplete contracts theory, see in particular Grossman and Helpman (2002).
The main prediction of the model is tested using a Swedish dataset which contains detailed information on the task (occupation) of all workers employed in the private sector at the plant level. This dataset enables us to construct a proxy of how many tasks are performed within manufacturing plants in Sweden and to match the location of plants at the city level with the size of the cities (as proxied by population size). The results show that plants in smaller cities tend to perform more tasks inhouse than plants in larger cities, controlling for plant size. As predicted by the model, it therefore seems that in large cities, firms are more specialized in terms of task scope than in small cities.

Some of the mechanisms at work in the model are in line with the existing literature but we depart from previous work in several ways. First, an important body of the literature has already analyzed the determinants of the “make or buy decision”. This literature highlights the existence of a trade-off between outsourcing and vertical integration: outsourcing to specialists can be cheaper but there are costs associated with search and contractual frictions, see in particular Grossman and Helpman (2002) and Antràs and Helpman (2004). These models mainly focus on the extensive margin of outsourcing (being a vertically integrated firm or not) while this model focuses on the intensive margin of outsourcing (how many tasks should be performed in-house). Besides, our model focuses on service outsourcing instead on outsourcing of manufactured inputs. These two differences are key elements of the emergence of gains from specialization.

A second body of the literature has been concerned by the potential consequences of offshoring and several contributions have been made regarding the modelling of trade in tasks. Grossman and Rossi-Hansberg (2008b) develop in a Hecksher Ohlin setting a model of north-south trade in tasks where the range of tasks offshored varies continuously with the cost of offshoring. In this model, trade in tasks arises endogenously due to differences in factor endowments between countries. In a different theoretical framework, Grossman and Rossi-Hansberg (2008a) develop a model of trade in tasks between similar countries which differ in size. In this model, firms can decide to offshore tasks due to economies of scale external to the firm which provide an incentive to locate each task in countries where other forms are performing it. Finally, Baldwin and Robert-Nicoud (2010) present an analytic framework in which both trade-in-goods and trade-in-tasks arise endogenously in response to exogenous changes in the cost of moving goods and ideas and use this framework to integrate results from trade-in-tasks theory into mainstream trade theory. We build on these models by assuming that the production of a manufactured good requires the performance of a fixed range of tasks. However, in our model, trade in tasks between manufacturing firms and service providers arises between similar countries because of the efficiency gains stemming
from adjustments in tasks scope.

Third, our model is also in line with the recent literature on firm heterogeneity as it generates productivity gains at the industry level, see in particular Melitz (2003). However, while the firm heterogeneity literature focuses on reallocation of production across firms which are heterogeneous in marginal costs and generates productivity gains which are external to the firm, our model generates reallocation of tasks across firms which have homogeneous productivity and generates productivity gains that are internal to the firm.

Finally, the model is strongly related to the field of urban economics. It predicts that large cities are more specialized and therefore characterized by higher aggregate productivity. Chinitz (1961) noted that firms in New York are more specialized than in the smaller city of Pittsburgh. And the existence of a positive correlation between output per capita and city size has been largely documented in the literature, see in particular Puga (2010). Of course, this can be due to many factors. As emphasized by Behrens, Duranton, and Robert-Nicoud (2010), this can occur because more talented individuals sort into large cities, because large cities select more productive entrepreneurs and firms, or because of agglomeration economies. The mechanism at work in the model is in line with this last category and a large literature has already highlighted that output-input linkages are an important source of agglomeration economies (see for instance Holmes (1999) or Ellison, Glaeser, and Kerr (2010)). Keeping in mind that our model abstract from many of the forces that influence the formation of cities, it proposes a different mechanism. In the model, firms in larger cities are more specialized and more productive because of economies of scale in task scope which allow a deeper division of labour between firms. The empirical analysis, which is based on a very detailed dataset, tends to give support to the model. Indeed, the results confirm that in large cities, firms are more specialized (have a lower task scope) than firms in smaller cities, controlling for firm size.

The remainder of this paper is organized as follows. Section 2 describes the model we have developed. Section 3 uses Swedish data linking employee with employers to test the main implications of our model. Section 4 concludes.

2 Model

We begin by presenting our setting in autarky which generates our most important results as a trade liberalization can be seen as an increase in market size. In the Appendix, we also analyse a more complex open economy setting where we allow respectively for (i) trade in goods and (ii) trade in tasks.
2.1 Setup in autarky

The model depicts an economy with a primary production factor labour, \( L \), which is used in all sectors. The economy includes three sectors: the agricultural sector which is a Walrasian, homogenous-goods sector, the manufacturing sector, which is characterized by increasing returns and Dixit-Stiglitz monopolistic competition and the service sector, which consists of the service providers to which manufacturing firms can outsource some of the tasks needed in the manufacturing production. This service sector does not produce any consumer good, but instead performs tasks for manufacturing firms. All sectors have free entry and firms therefore make zero profit in equilibrium.

Consumers have two-tier utility functions with the upper tier (Cobb-Douglas) determining the consumer’s division of expenditure among the sectors and the second tier (CES) dictating the consumer’s preferences over the various differentiated varieties within the manufacturing sector. More specifically, individuals have the following utility function

\[
U = C_M^{\mu} C_A^{1-\mu},
\]

where \( \mu \in (0, 1) \), \( C_A \) is the consumption of the homogenous good and \( C_M \) is the consumption of manufactured goods. The utility function stemming from the consumption of manufactured goods is defined by

\[
C_M = \left[ \int_0^N c_i \frac{(\sigma-1)}{\sigma} \, di \right]^{\frac{\sigma}{\sigma-1}},
\]

\( N \) being the mass of varieties consumed, \( c_i \) the amount of variety \( i \) consumed and \( \sigma > 1 \) the elasticity of substitution between manufacturing goods.

Each consumer spends a share \( \mu \) of his income on manufactures, and demand for a variety \( i \) is therefore

\[
q_i = p_i^{\sigma} \frac{\mu}{P_0^{1-\sigma}} Y,
\]

where \( p_i \) is the consumer price of variety \( i \), \( Y \) is income and \( P = \left( \int_0^N p_i^{1-\sigma} \, di \right)^{\frac{1}{1-\sigma}} \) the price index of manufacturing goods. We will rewrite this as

\[
q_i = A p_i^{\sigma}
\]

where \( A \equiv \frac{\mu Y}{P_0^{1-\sigma}} \) and is taken as given by each manufacturing firm.

The unit factor requirement of the homogeneous good is one unit of labour and since it is chosen as the numeraire.
\[ p_A = w = 1; \]  
\( w \) being the nominal wage of workers. This also means that  
\[ Y = L. \]

Turning to the technologies, firms are assumed to be homogeneous in terms of productivity. Regarding production in the manufacturing sector, we assume that in order to produce one unit of a differentiated good, firms have to perform a fixed range of tasks, indicated by \( j \) and ranging from 0 to 1. We also assume that all tasks have to be performed in the same level to produce output \( q_i \) and more specifically that the production function is  
\[ q_i = \min_{j \in [0,1]} (q(j)). \]

Due to the Leontief nature of the production function, cost minimization means that the demand for each task, given the output level will be  
\[ q(j') = q(j) = q_M \]  
where \( j \neq j' \). What this means is that the level of each task input will equal the output of the final good in equilibrium.

Manufacturing firms face two costs, a fixed cost \( f_M \) and a marginal production cost per task \( j \), \( \tilde{\varphi}_M(j) \), which will depend on the task scope. The cost of performing \( q(j) \) units of a task is  
\[ C(j) = wq(j) \tilde{\varphi}_M(j). \]

The key assumption in our model is that the marginal cost, \( \tilde{\varphi}_M(j) \), of task \( j \) increases in the average distance of task \( j \) from all other tasks performed within the firm  
\[ \tilde{\varphi}_M(j) = \frac{\int_0^1 (j' - j)^\delta \gamma(j') \, dj'}{\int_0^1 \gamma(j') \, dj'}. \]

where \( \delta > 1 \) to attain a convex relationship between task scope and marginal cost, where \( \delta \) is restricted to the set of even numbers and where \( \gamma(j) \in \{0, 1\} \). It is 1 if the firm performs the task and 0 if it is outsourced.

This key assumption is founded upon our belief that it is more costly for firms to perform tasks of very different characteristics within the same firm than to specialize in a more narrow range of tasks. This is essentially how we capture Adam Smith’s argument of the division of labour. A firm that specializes in just a few tasks that are
close to each other in nature will be more productive than a firm that does a lot of tasks which differ within its boundaries.

Our intuition here is that if tasks are arranged along a line, tasks located close to each other are more similar in nature. Along the same argument, the further away two tasks are, the more different they are. To clarify this point, consider the following tasks: (i) hammering the pins so that they are completely straight, (ii) hammering the end of the pin so that it has a flat end, (iii) cleaning the factory floor and, (iv) disposing of waste created in the manufacturing process. This may be very stylized but tasks (i) and (ii) are likely to be more similar in nature than say (i) and (iii). So this assumption about the marginal cost ensures first that it will be less costly to perform tasks (i) and task (iii) than all the four tasks, but more importantly, that it will be less costly to perform tasks (i) and (ii) than tasks (i) and (iii).

The parameter $\delta$ is therefore very important in the model. It is a measure of how much more expensive it is to produce multiple tasks when they lie far from each other; or, equivalently, how expensive it is for a firm to operate a wide task scope.

The reduction in the marginal cost stemming from a reduction in task scope provides an incentive to outsource. A manufacturing firm can thus either perform a task and pay $w_\varphi (j)$ per unit or procure it from a specializing firm at the price $p_s (j)$ per unit. As it will be clarified shortly, in the model, the cost of outsourcing for the manufacturer will not involve to pay a price $p_s$ but rather to share some of its revenues with service providers.

The manufacturing firm’s total cost is therefore:

$$TC_i = f_M + w \int_0^1 \gamma (j) q (j) \varphi_M (j) \, dj + \int_0^1 (1 - \gamma (j)) q (j) p_s (j) \, dj$$

where $\gamma (j) \in \{0, 1\}$. It is 1 if the firm performs the task and 0 if it is outsourced.

Now, regarding the service sector, it consists of service providers. These service providers do not produce any goods but instead perform a set of tasks which they sell to manufacturers. These service firms cannot outsource. However they face a similar cost structure as the one of the manufacturer. First they pay a fixed cost $f_s$. Second, they face a marginal cost per task, $\varphi_s (j)$, that increases in its average distance from all other tasks performed within the firm. The total cost of the service provider $l$ is therefore:

$$TC_l = f_s + w \int_0^1 \gamma (j) q (j) \varphi_l (j) \, dj$$
\[
\tilde{\varphi}_l (j) = \frac{\int_0^1 (j' - j) \gamma (j') dj'}{\int_0^1 \gamma (j') dj'}.
\]

(11)

Especially important to note is that service providers (unlike manufacturing firms) do not have to produce all tasks along the line from 0 to 1. Instead, they choose a set of tasks, perform these tasks and then sell these tasks to manufacturers. We believe that this feature is a key difference between the supply of services versus intermediate inputs. An accounting firm, for example, focuses on a very narrow range of tasks in its production while a car tyre producer still needs a wide range of tasks in order to produce. Service outsourcing involves by nature direct trade in tasks while the outsourcing of a manufactured input involves indirect trade in tasks. This is why service outsourcing has the particularity to allow the division of labour between firms and to favour the specialization of firms in terms of task scope.

To sum up, the manufacturing production process involves two types of firms: manufacturers and service providers. Manufacturers produce final goods by performing a fixed range of tasks than can be performed inhouse or outsourced to service providers. Service providers, on the other hand, only perform some tasks and sell these to manufacturing firms.

Finally, we have to make assumptions about the competition structure between, first, manufacturers and service providers and, second, between service providers. Grossman and Helpman (2002) analyse in detail the case where the matching of these types of firms is characterized by search frictions and we therefore wish to abstract from such frictions in order not to duplicate their results and instead focus on our specific mechanism. In our model, the sequence of events will be as follows: (i) manufacturing firms enter and decide on their task scope. This process continues until their expected profits are driven to zero. Since it will not be profitable for manufacturers to produce different tasks in our model (manufacturers are homogenous), we simply assume that they always produce task 0 inhouse and then all tasks from 0 and onwards until some optimal level of \( t_M < 1 \). (ii) Service providers enter one by one and position themselves along the task scope (over the range of tasks that are outsourced) until their expected profits are zero. This means that they divide the scope of tasks not produced by manufacturers, \( j \in (t_M, 1] \), between themselves. Since service providers are identical it means that they will each produce a set of tasks \( t_S \) such that \( 1 - t_M = n_S t_S \) where \( n_S \) denotes the number of service providers. This setup is illustrated in figure 1. (iii) Manufacturers make contracts with task providers. This means, for example, that task providers install their operations in the manufacturing firm’s plants. The key assumption here is that once the manufacturer and task provider have set up their joint operations, the manufacturer cannot renege on the contract. Specifically,
we assume that a task provider can produce tasks of different quality and while the quantity can be verifiable by a court of law, the quality of the tasks cannot.\footnote{This assumption is similar to that in Grossman and Helpman (2002).} Due to the Leontief production function of the manufacturer, this gives the task provider full bargaining power over the manufacturer since it can stop the manufacturer’s production completely. (iv) Manufacturers and service providers bargain over the revenues generated by production. As mentioned, the service provider has full bargaining power and can therefore, as in Grossman and Helpman (2002), determine the level of output of the manufacturer (by deciding which quality of tasks to deliver, which indirectly determines the quantity). It subsequently receives all the revenues generated by the specific tasks it delivers.\footnote{Grossman and Helpman (2002) assumes some exogenous bargaining power parameter of the suppliers, $\omega$, but we assume here that service providers have full bargaining power over the revenues generated by their specific tasks. Our assumption is based on the fact that service providers have the power to completely stop production in the manufacturer’s plant.} (v) Production and consumption of manufacturing goods, agricultural goods and service inputs take place.

Another important point to clarify is that while manufacturers optimize their task scope, $t_M$, to maximize profits, service suppliers’ task scope is determined by free entry in their sector. We believe that this is reasonable given the fact that if service providers can optimize their task scope, they would make positive profits in equilibrium and this cannot be the case if there is free entry for service providers.

Figure 1 describes in detail how different tasks in the economy are produced. First, the tasks from 0 to $t_M$ are produced by each manufacturer inhouse. The range of tasks from $t_M$ to 1 are then produced by service providers to which manufacturers outsource. Since service providers are homogeneous, they divide the range $1 - t_M$ among themselves and each service provider therefore produces $t_S = \frac{1-t_M}{n_S}$ tasks.

To sum up, the conditions used to close the model are:
1. Consumers maximize utility through consumption given their income.

2. Manufacturers maximize their profits by determining their task scope (they take output as given since this is determined by the service providers as explained).

3. Service providers maximize their profits by determining what quantity of each task to produce (thereby also deciding on the output of the manufacturing firm).

4. Zero profits for manufacturers due to free entry.

5. Zero profits for task providers due to free entry.

Together, these equations will determine the output level of each final good and each task, \( q \), the price of each final good, \( p \), the task scope of manufacturers, \( t_M \), the task scope of service providers, \( t_S \), the number of manufacturers, \( n_M \), the number of service providers, \( n_S \), aggregate prices for manufacturing goods, \( P \), and aggregate utility, \( UL \).

Moreover, we restrict our analysis to equilibria with outsourcing taking place to some extent. The analysis of the binary cases of complete outsourcing versus full vertical integration is already thoroughly analysed by Grossman and Helpman (2002).

The following section will analyse the equilibrium of the model.

### 2.2 The autarky equilibrium

First, the total profit of a manufacturing firm is:

\[
\Pi_i = R_i - C_i = p_i q_i - f_M - \int_0^1 \gamma_i (j) q_i (j) \tilde{\varphi}_i (j) \, dj - \int_0^1 (1 - \gamma_i (j)) q_i (j) p_S (j) \, dj. \tag{13}
\]

The way the marginal cost increases in the difference between tasks means that all firms will produce tasks that are adjacent to each other. If the tasks are ordered on a line between 0 and 1, we can, for simplicity, assume that the tasks manufacturing firms produce inhouse are in the range between 0 and \( t_M \), where \( t_M \) is determined endogenously. Moreover, due to the homogeneity of firms, we now drop the index \( i \). With this setup, the marginal cost of a specific task \( j \) is

\[
\tilde{\varphi} (j) = \frac{\int_0^{t_M} (j' - j)^\delta \gamma (j) \, dj'}{\int_0^{t_M} \gamma (j) \, dj'} \tag{14}
\]

\[
= \frac{\int_0^{t_M} (j' - j)^\delta \, dj'}{\int_0^{t_M} \, dj'} \tag{15}
\]

\[
= \frac{1}{t_M} \int_0^{t_M} (j' - j)^\delta \, dj' \tag{16}
\]

\[
= \frac{1}{t_M (\delta + 1)} \left( (t_M - j)^{\delta + 1} - (-j)^{\delta + 1} \right). \tag{17}
\]
This means that the average marginal cost for a manufacturing firm, $\varphi_M(t_M)$ will be a function of $t_M$:

$$\varphi_M(t_M) = \frac{1}{t_M} \int_0^{t_M} \tilde{\varphi}_M(j) \, dj$$

(18)

$$= \frac{1}{t_M^2 (\delta + 1)} \int_0^{t_M} \left( (t_M - j)^{\delta+1} - (-j)^{\delta+1} \right) \, dj$$

(19)

$$= \lambda_1 t_M^\delta$$

(20)

where $\lambda_1 \equiv \frac{2}{(\delta+1)(\delta+2)}$. We note that the marginal cost of tasks produced inhouse strictly increases in the scope of tasks that are produced inhouse.

Service providers face a similar structure such that

$$\varphi_S(t_S) = \lambda_1 t_S^\delta.$$ (21)

Now, the demand of a manufacturing good (under a CES demand structure) can be denoted as:

$$q_M = A p_M^{\sigma}$$ (22)

where $A \equiv \frac{\mu_Y}{\sigma} = \frac{\mu_L}{\sigma - 1}$ and $P$ is the Dixit Stiglitz ideal price index.

The manufacturer produces a share $t_M$ of all tasks needed for production inhouse. Due to the competition structure described above, it is the case that manufacturers pay all the revenues generated by outsourced tasks to the service providers from which they buy the task. They therefore pay

$$\frac{(1 - t_M)}{n_S} p_M q_M$$

(23)

to each supplier where $n_S = \frac{1 - t_M}{t_S}$ is the mass of service providers in the economy. The total cost of the manufacturer for buying service inputs is therefore:

$$(1 - t_M) p_M q_M.$$ (24)

The profits of a manufacturer can then be rewritten as:

$$\pi_M = p_M q_M - t_M \varphi_M(t_M) q_M - (1 - t_M) p_M q_M - f_M$$

(24)

$$= t_M q_M (p_M - \varphi_M(t_M)) - f_M.$$ (25)

A supplier produces $t_S$ tasks taking demand $q = A p_M^{\sigma}$ and the scope of manufacturers $t_M$ as given. At this point, it also takes its own scope, $t_S$, as given. Since the supplier will have the same output of a task as the manufacturer will have of its final good, due to the Leontief production function in (5), we denote output of both items
simply as $q$ where $q = q_M = q_S$.

Per manufacturer customer, it gets paid $(1 - t_M) \frac{t_S q}{1 - t_M} p_M q = t_S p_M q$. As it performs tasks for all manufacturers, it gets paid $n_M t_S p_M q$ and its costs are $f_S + n_M t_S \varphi_S(t_S) q$. Therefore its profits are:

$$
\pi_S = n_M t_S q (p_M - \varphi_S(t_S)) - f_S.
$$

It faces the problem of how much task inputs to produce (knowing that it can completely control the output of the manufacturers):

$$
\max_q \pi_S = n_M t_S p_M q - n_M t_S \varphi_S(t_S) q - f_S
$$

which gives the following FOC (note that $\varphi_S(t_S) = \lambda_1 t_S^\delta$):

$$
q = \left( \frac{1}{\bar{\sigma} \lambda_1 t_S^\delta} \right)^{\sigma} A
$$

where $\bar{\sigma} \equiv \frac{\sigma}{\sigma - 1}$.

This gives the optimal output for a supplier given its own scope and the manufacturer’s scope. Note that our assumption is that the manufacturer (due to the holdup problem) has to accept the output volume chosen by the suppliers if it decides to outsource task production. We also note that the service supplier produces more if demand is high (high $A$) and less if its task scope is large (high $t_S$) since the price of its good in this case will be higher due to its lower productivity (higher marginal cost). As expected, a high elasticity of substitution makes output more sensitive to marginal costs.

The manufacturer faces the following profit function:

$$
\pi_M = p_M q - t_M \varphi_M(t_M) q - (1 - t_M) p_M q - f_M
$$

$$
= A^\frac{1}{\bar{\sigma}} q^{\frac{\sigma - 1}{\bar{\sigma}}} t_M - \lambda_1 t_M^{\delta + 1} q - f_M.
$$

It takes $q$ as given so it will use $t_M$ to maximize profits,

$$
\max_{t_M} \pi_M = A^\frac{1}{\bar{\sigma}} q^{\frac{\sigma - 1}{\bar{\sigma}}} t_M - \lambda_1 t_M^{\delta + 1} q - f_M,
$$

which gives the following FOC:

$$
t_M = \left( \frac{1}{\lambda_1 (\delta + 1)} A^\frac{1}{\bar{\sigma}} q^{\frac{\sigma - 1}{\bar{\sigma}}} S^\frac{\bar{\sigma}}{\sigma - 1} \right)^{\frac{1}{\bar{\sigma}}}.
$$
The solution for $t_M$ above gives the solution for a manufacturer’s optimal scope given the quantity produced by each intermediate supplier. We know the output of a supplier from (29) and $t_M$ can therefore be written as:

$$t_M = t_S \left( \frac{\tilde{\sigma}}{\delta + 1} \right)^{\frac{1}{\delta}}$$  \hspace{1cm} (34)

where $\tilde{\sigma} = \frac{\sigma}{\sigma - 1}$. This means that there is a monotonic and linear relationship between the task scope of a manufacturer and that of the service suppliers. The reason for this is simply that a higher task scope of service providers, $t_S$, make them less specialized and therefore less efficient. Less efficient service providers are not as attractive for the manufacturer to use for outsourcing and the manufacturer then prefers to perform relatively more tasks inhouse and raises $t_M$.

Now, we turn to the free entry condition in the manufacturing sector and the fact that they earn zero profits in equilibrium:

$$\pi_M = 0$$  \hspace{1cm} (35)

$$t_M q (p_M - \varphi_M (t_M)) = f_M$$  \hspace{1cm} (36)

$$A = t_S^{(\sigma - 1) - 1} \lambda_1^{\sigma - 1} \tilde{\sigma}^{\sigma - 1} \left( \frac{\delta + 1}{\sigma} \right)^{\frac{1}{\delta}} \left( \frac{\delta + 1}{\delta} \right) f_M$$  \hspace{1cm} (37)

which returns a relationship between $A$, the demand per firm in the economy, and $t_S$, the task scope of suppliers, and indirectly that of manufacturers too from (34), in the economy.

There is free entry for service providers too and this will drive down their task scope, $t_S$, such that they earn zero profits in equilibrium:

$$\pi_S = 0$$  \hspace{1cm} (38)

$$n_M t_S q (p_M - \varphi_S) = f_S$$  \hspace{1cm} (39)

$$n_M = \frac{f_S}{f_M} \frac{\tilde{\sigma}}{\sigma - 1} \frac{\delta}{\delta + 1} \left( \frac{\tilde{\sigma}}{\delta + 1} \right)^{\frac{1}{\delta}}.$$  \hspace{1cm} (40)

The free entry conditions yield a solution for the number of manufacturing firms, $n_M$, and this consists only of exogenous parameters. This is an important conclusion, especially that this variable is independent of population size which is otherwise the case with CES preferences, and we will see later that this means that there is no “variety effect” from trade. The reason for $n_M$ being fixed, however, stems from the fact that all surplus profits coming from an expansion in market size are passed on to the service suppliers and do not stay with the manufacturing firms. This result is
similar to what is found in Grossman and Helpman (2002).

Knowing this, we can use the expression for $A$ to find the other variables:

\[
\frac{\mu L}{P^{1-\sigma}} = t_s^{\delta(\sigma-1)-1}\lambda_1^{\sigma-1}\bar{\sigma}^{-\sigma-1}\left(\frac{\delta + 1}{\bar{\sigma}}\right)^{\frac{1}{\delta}}\left(\frac{\delta + 1}{\sigma}\right)^{\frac{1}{\delta}} f_M
\]

(41)

\[
t_s = \frac{f_s}{\mu L \bar{\sigma} - 1}
\]

(42)

where we note one of our key findings, that larger economies are more specialized. This is due to the fact that as population increases, the demand per each manufacturing variety increases. This leads to an expansion of output of each manufacturing variety since the number of varieties is fixed as was observed in (40). This also means an increase in the output of each task raising the profits of each service provider which leads to an inflow of new service firms in the economy. This entry process drives down the task scope of each service provider. Ultimately, this means that a larger economy consists of more specialized firms.

This intuition can be seen in the following equations using the result from (42):

\[
t_M = t_s \left(\frac{\bar{\sigma}}{(\delta + 1)}\right)^{\frac{1}{\delta}}
\]

(43)

\[
t_M = t_s \left(\frac{\bar{\sigma}}{(\delta + 1)}\right)^{\frac{1}{\delta}} = \frac{f_s}{\mu L \bar{\sigma} - 1} \left(\frac{\bar{\sigma}}{(\delta + 1)}\right)^{\frac{1}{\delta}}.
\]

(44)

\[
A = \left(\frac{1}{\mu L}\right)^{\delta(\sigma-1)} f_s^{\delta(\sigma-1)} \left(\frac{\bar{\sigma}}{\bar{\sigma} - 1}\right)^{\delta(\sigma-1)} \lambda_1^{\sigma-1}\bar{\sigma}^{-\sigma-1}\left(\frac{\delta + 1}{\bar{\sigma}}\right)^{\frac{1}{\delta}}\left(\frac{\delta + 1}{\sigma}\right)^{\frac{1}{\delta}} f_M f_S.
\]

(45)

\[
P = \left(\frac{1}{\mu L}\right)^{\delta} f_s^{\delta} \left(\frac{\bar{\sigma}}{\bar{\sigma} - 1}\right)^{\delta - \frac{1}{\sigma - 1}} \lambda_1 \bar{\sigma} \left(\frac{\delta + 1}{\bar{\sigma}}\right)^{\frac{1}{\delta}} \left(\frac{\delta + 1}{\sigma}\right)^{\frac{1}{\delta}} \left(f_M f_S\right)^{\frac{1}{\sigma - 1}}.
\]

(46)

The quantity per manufacturer is:

\[
q = \mu L^{(\delta + 1)} f_s^{\delta} \left(\frac{\bar{\sigma} - 1}{\bar{\sigma}}\right)^{\delta(\sigma)} \lambda_1^{1-\sigma} \bar{\sigma}^{-\sigma-1}\left(\frac{\delta + 1}{\bar{\sigma}}\right)^{\frac{1}{\delta}}\left(\frac{\delta + 1}{\sigma}\right)^{\frac{1}{\delta}} f_M \frac{1}{f_S \bar{\sigma}^\sigma}.
\]

(47)

which gives the following price per manufacturing good:

\[
p = \lambda_1 \bar{\sigma} \left(\frac{f_s}{\mu L \bar{\sigma} - 1}\right)^{\delta}.
\]

(48)
The number of suppliers can be found by using that:

\[ n_S = \frac{1 - t_M}{\ell_S} \]

\[ = \frac{L - \lambda_2 \lambda_3}{\lambda_3} \]  

which increases linearly in \( L \). We let \( \lambda_2 \equiv \left( \frac{\bar{\sigma}}{(\sigma+1)} \right)^{\frac{1}{\sigma}} \) and \( \lambda_3 \equiv \frac{\delta f_s \bar{\sigma}}{\mu \sigma - 1} \).

We focus especially on the result in (46) which gives the expression for the price index of the manufacturing goods. This is important since welfare can be expressed by:

\[ W = P^{-\mu} \]  

since the agricultural good is the numeraire and (51) therefore gives the real wage in the economy. The exponent for \( L \) in the expression for \( P \) is, in this model, no longer \( \frac{1}{1-\sigma} \) like in Krugman (1980) or other similar models building on CES preferences, but instead \( -\delta \) so it is still negative but this comes through a specialization effect rather than a variety effect; the elasticity is \( \delta \) rather than \( \frac{1}{\sigma-1} \). This outcome is a direct consequence of our new mechanism in this model. We therefore have a new margin of how welfare increases in market size. This margin is stronger the more difficult it is for firms to manage many tasks because this makes specialization relatively more important.

Dixit and Stiglitz (1977):

\[ -\frac{dP}{dL} \underbrace{L}_{\text{"Variety effect"}} = \frac{1}{\sigma - 1} \]  

Our model:

\[ -\frac{dP}{dL} \underbrace{L}_{\text{"Specialization effect"}} = \delta \]  

Moreover, the “market size per firm”, \( A \), changes with country size by the elasticity \( \delta (\sigma - 1) - 1 \) which can be either positive or negative. It increases with population size if specialization is important or if there is strong competition between manufacturing goods (higher elasticity of substitution).

**Theorem 1** Larger economies are associated with more specialization, higher aggregate productivity, lower prices and higher welfare. The elasticity of these relationships is greater the higher is the cost for firms to engage in many tasks simultaneously.

**Theorem 2** Trade liberalization leads to: a) increased specialization of firms and b) higher welfare due to greater efficiency in production.
Theorem 2 follows from Theorem 1 if it is assumed that trade liberalization can be proxied by an increase in population.

Smith (1776) argued that a division of labour could generate a great increase in production. One example he used was the making of pins. One worker could probably make only twenty pins per day. However, if ten people divided up the eighteen steps required to make a pin, they could make a combined amount of 48,000 pins in one day.

The exact equivalent in our model would be a decrease in $t_M$ and $t_S$ which makes firms more productive at what they do. Working against this mechanism is the presence of fixed costs (in Smith’s example this would be a fixed cost per step in the specialization process). The presence of fixed costs makes it possible for larger economies to engage in more specialization (because consumption is larger and we have a fixed set of varieties which causes larger economies to consume more of each variety). This also, interestingly, translates into higher aggregate productivity in larger economies since the marginal cost per output decreases. To summarize, the equivalent in our model of Smith’s division of labour in the pin factory is the division of production into more specialized service firms. In Smith’s economy, greater size made it possible for workers to specialize more, in our model it makes it possible for service providers to specialize more. The final outcome is equivalent for the two settings: larger economies specialize more and therefore have higher aggregate productivity and higher welfare.

3 Empirical analysis

3.1 Main prediction

The most important testable prediction of our model is Theorem 1: larger economies are characterized by more specialization and a more narrow task scope performed inside the firm’s boundaries. A convincing empirical test would therefore require information on firms and which tasks they decide to perform inhouse. We therefore use Swedish data which links employees to plants. Among the observables in the dataset on individuals is what occupation each individual has. This is the variable which we use as a proxy of a task. We link employees to plants and by calculating how many different occupations the employees of each plant have, we get a proxy of how many tasks are performed inside the organization.\(^5\) For each plant, we know its main business (sector by NACE codes) and its location (by city) and can therefore test whether plants in larger cities tend to perform fewer tasks inhouse.

\(^5\)We do not have specific information on traditional task measures but instead occupations of workers. We believe, however, that the type of outsourcing described in the model is more related to occupations rather than specific tasks.
3.2 Data

As stated, we use Swedish data (all data is from Statistics Sweden) from the year of 2005. For our purposes, we believe that a crosssectional approach is superior to using a panel estimation and therefore choose to use data only from a single year. 2,563,771 individuals are included in the dataset and this comprises all individuals employed in the private sector in Sweden in 2005. 413,387 plants are included and these are operated by 381,087 firms indicating that the vast majority of firms only operate one plant. For individuals, we use their occupation code and which plant they are employed at (by three and four digit respectively, e.g 114 occupations and 356 occupations). For plants, we use information on how many employees they have, their location (by city/commune of which there are 290 in Sweden) and their sector (by five digit NACE codes).

3.3 Method and results

As already described, we intend to test the hypothesis that plants in larger cities are more specialized (that they have a more narrow task scope). We will do this at the plant level so that we can exploit all the variation that we have and control for sector fixed effects. Moreover, larger plants tend to be more diversified so we control for plant size as well. Plant size is here proxied for by how many employees each plant has. The main specification that we run is therefore:

\[
\log t_{ij} = \beta_0 + \beta_1 \log Pop_j + \beta_2 \log Size_{ij} + f_l + \varepsilon_{ij} \tag{54}
\]

where \( t_{ij} \) denotes how many occupations employees have at plant \( i \) in city \( j \), \( Pop_j \) denotes the population of city \( j \) and \( Size_{ij} \) denotes how many employees plant \( i \) has. Due to the logarithmic functions used, we can interpret coefficients \( \beta_1 \) and \( \beta_2 \) as elasticities. We control for sectors at the two or three digit NACE level as shown here by \( f_l \). We cluster for standard errors at the city level in all regressions.

Table 1 shows the results. The first column estimates the regression in (54) using all manufacturing plants in Sweden in 2005. We find that the elasticity with respect to city size is indeed negative and significant: smaller city size tends to make plants hire a wider scope of occupations. This confirms the main prediction of our model, that larger cities are associated with a greater degree of specialization.

We then run a series of robustness tests for this specification. First, we note that many plants have very few employees and we therefore retain only the plants with more employees than the median plant (the median plant has 6 employees) in column (2). Then, in (3), we remove all plants that have less than the median number of occupations inhouse. This is equivalent to retaining only the plants with the most number of
Table 1: Task scope and city size in Sweden

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Emp &gt; Mean</th>
<th>Task scope &gt; Mean</th>
<th>City coverage</th>
<th>FE</th>
</tr>
</thead>
<tbody>
<tr>
<td>City Size (log population)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>City Size (log population)</td>
<td>-0.0026*</td>
<td>-0.010***</td>
<td>-0.008**</td>
<td>-0.009*</td>
<td>-0.0096*</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Plant Size (log employment)</td>
<td>0.589***</td>
<td>0.488***</td>
<td>0.475***</td>
<td>0.524***</td>
<td>0.523***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Industry fixed effect 2-digits</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Industry fixed effects 3-digits</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Cluster (city level)</td>
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<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Observations</td>
<td>38757</td>
<td>12886</td>
<td>12737</td>
<td>6558</td>
<td>6558</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.883</td>
<td>0.637</td>
<td>0.658</td>
<td>0.672</td>
<td>0.678</td>
</tr>
</tbody>
</table>
occupations inhouse. In column (4) we focus on the fact that some sectors could be substantially concentrated geographically. Our method here is then to retain in the sample only those sectors which are above the median sector as regards geographical coverage (as measured by the number of cities in which the sector is active). Finally, we use fixed effect at a more detailed sectoral level (three digit NACE instead of two digit). None of the robustness tests removes the conclusion from column (1): that firms in larger cities are associated with fewer occupations inhouse.

It should also be noted that our prior on task scope and plant size is highly significant and has a relatively high elasticity.

The empirical analysis therefore tends to confirm the main prediction of the model. However, more empirical work is needed. First, the results here are based on occupation code at the three digit levels while information is also available at the four-digit level. Besides, specialization in the model means that firms perform tasks which are more similar in nature. Future work would include a construction of the dependent variable not only based on the number of tasks performed within the plant but based also on the similarity of the tasks performed within a plant. Input-output tables at the industry level could also be used to check whether the results are stronger in industries which are more intensive in service outsourcing. This is ongoing work.

4 Conclusions

We develop a model of service outsourcing in which firms can choose how many tasks they wish to perform inhouse and how many to source from an external firm. Our key assumption is that it is more costly for firms to perform a wider range of tasks inhouse and that there therefore are benefits from specialization in a narrower range of tasks. Specifically, we allow service providers to focus on a narrow range of tasks and then sell these tasks to producers of final goods (manufacturers). The narrower is the scope of a service provider, the more productive it is. The same applies for manufacturers, the fewer tasks it produces inhouse, the more productive it is. We assume that there is a contracting friction due to a lack of legal verifiability of the quality of tasks provided by service firms and a holdup problem since a manufacturing firm cannot switch service provider once a contract has been made. This causes manufacturing firms to share their revenues with service providers.

The model generates analytical solutions for all variables and, most importantly, dynamics relating to market size: larger economies can sustain a greater degree of specialization since larger demand can make more firms afford the fixed costs involved in production while operating a narrower task scope. If trade liberalization is proxied by an increase in population size, the model generates benefits from trade through a
rise in specialization rather than an increase in the number of varieties (despite the fact that we use Dixit Stiglitz preferences).

We use detailed data from Sweden which links employees to plants. We find that, controlling for plant size, manufacturing plants in larger cities tend to employ fewer occupations. It therefore seems as if manufacturing firms are more specialized in larger economies, possibly due to larger benefits from outsourcing.

We also model two cases of incremental trade liberalization where we develop a two country model with iceberg trade costs for (i) goods and (ii) tasks. Goods trade liberalization does not affect specialization patterns while task trade liberalization actually lowers the degree of specialization. We view, however, using the autarky model and proxying trade liberalization by an increase in the population as the cleanest way of modelling trade liberalization.

Finally, we believe that our model lies close to the heart of Adam Smith’s theory of the benefits of the “division of labour within firms”. In Smith’s theory, a firm could divide tasks to different workers which raised their productivity substantially when specializing in these tasks. The equivalent in our model of service trade is that, in larger markets, manufacturing firms outsource more tasks to specialized service providers, all firms become more specialized in terms of task scope and therefore more productive. In this model, larger markets allow a greater “division of labour between firms” and a greater specialization of firms which translates into higher aggregate productivity and welfare.
5 Appendix: Open economy

This final section of the paper explores the open economy case in a slightly different way. In Section 2 we analysed the autarky setting and then proxied trade liberalization by an increase in population size. While this gives the analytically most robust, clearest and most intuitive results, we also wish to examine what happens in the case of incremental trade liberalization. We do this in this section.

We will explore two types of open economy settings. First, we will allow for trade in manufactured goods but not in tasks. This can be seen as a world with trade in final goods but where service trade is impossible due to too high trade costs for services. Trade costs are represented by an iceberg trade cost of \( \tau > 1 \). Second, we open, instead, trade in tasks which can be seen as an analysis of the recent rapid increase of service offshoring. The friction in task trade is represented by an iceberg trade cost of \( \beta > 1 \). As the notation suggests, the way we model is similar to Grossman and Rossi-Hansberg (2008b). We will, for now, maintain the assumption of equal wages across countries. Moreover, we will demonstrate our results in an economy consisting of two countries, Home and Foreign, where the latter is indicated by an asterix “∗”.

In order to maintain tractability and illustrate our main points, we assume symmetric country size. We are therefore abstracting from effects relating to economic geography and differences in relative country size.

5.1 Trade in goods

When there is trade in goods, the demand faced by each manufacturing firm in country \( i \) selling to country \( j \) is instead:

\[
q_{ij} = A_j \left( \tau_{ij} p_{ij} \right)^{-\sigma} \tag{55}
\]

where

\[
A_j \equiv \frac{\mu L_j}{P_j^{1-\sigma}} \tag{56}
\]

and where the price indices are

\[
P_j = \left( \sum_{i=\Lambda} n_i (\tau_{ij} p_{ij})^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{57}
\]
where Λ denotes the set of countries in the world. For now, we have only two countries and therefore

\[
P = (n_M p_M^{1-\sigma} + n^*_M (\tau p_M^{*})^{1-\sigma})^{\frac{1}{1-\sigma}} \quad (58)
\]

\[
P^* = (n_M (\tau p_M)^{1-\sigma} + n^*_M p_M^{1-\sigma})^{\frac{1}{1-\sigma}}. \quad (59)
\]

Now, the profits of a manufacturer in Home are:

\[
\pi_M = pq + p^*q^* - t_M \varphi_M (q + \tau q^*) - (1 - t_M) (pq + \tau p^*q^*) - f_M \quad (60)
\]

\[
= \left( A^{\frac{1}{\sigma}}q^{\frac{\sigma-1}{\sigma}} + A^{*\frac{1}{\sigma}}q^*^{\frac{\sigma-1}{\sigma}} \right) t_M - t_M \varphi_M (q + \tau q^*) - f_M \quad (61)
\]

where it should be noted that, for the foreign market, the manufacturing firm has to produce \( \tau q^* \) in order to sell \( q^* \) on the foreign market.

A supplier decides how much to produce for the manufacturing firm, \( q + \tau q^* \), by maximizing its operating profit \( \pi_S \) per manufacturing firm:

\[
\pi_S = t_S (pq + p^*q^*) - t_S \varphi_S (t_S) (q + \tau q^*) \quad (62)
\]

\[
= t_S \left( A^{\frac{1}{\sigma}}q^{\frac{\sigma-1}{\sigma}} + A^{*\frac{1}{\sigma}}q^*^{\frac{\sigma-1}{\sigma}} \right) - t_S \varphi_S (t_S) (q + \tau q^*). \quad (63)
\]

The FOCs from maximizing with respect to \( q \) and \( q^* \) yield:

\[
q = \left( \frac{1}{\sigma \lambda_1 t_S^6} \right)^{\sigma} A \quad (64)
\]

\[
q^* = \left( \frac{1}{\tau \sigma \lambda_1 t_S^6} \right)^{\sigma} A^* \quad (65)
\]

which means that

\[
q + \tau q^* = \left( \frac{1}{\sigma \lambda_1 t_S^6} \right)^{\sigma} (A + \phi A^*) \quad (66)
\]

where \( \phi \equiv \tau^{1-\sigma} \in (0, 1] \) is an index of “globalization” and takes the value 0 in autarky and 1 at free trade.

The manufacturer takes this as given and decides on its task scope \( t_M \):

\[
\max_{t_M} \pi_M = \left( A^{\frac{1}{\sigma}}q^{\frac{\sigma-1}{\sigma}} + A^{*\frac{1}{\sigma}}q^*^{\frac{\sigma-1}{\sigma}} \right) t_M - t_M \varphi_M (q + \tau q^*) - f_M \quad (67)
\]

23
and maximizing with respect to $t_M$ yields the following FOC:

$$A^{\frac{1}{\sigma}} q^{\frac{\sigma-1}{\sigma}} + A^{\frac{1}{\sigma}} q^{\frac{\sigma-1}{\sigma}} = (q + \tau q^*) \left( \delta + 1 \right) \lambda t_M^\delta$$

(68)

$$\tilde{\sigma} \lambda t_M^\delta = \left( \delta + 1 \right) \lambda t_S^\delta$$

(69)

$$t_M = \left( \frac{\tilde{\sigma}}{\delta + 1} \right)^{\frac{1}{\delta}} t_S$$

(70)

which is the same relationship between $t_M$ and $t_S$ as in the autarky model.

Now, we turn to the zero profit condition for manufacturers:

$$\pi_M = 0$$

(71)

$$A + \phi A^* = f_M (\tilde{\sigma} \lambda_1)^{\sigma-1} t_S^{-1+\delta(\sigma-1)} \left( \frac{\delta + 1}{\tilde{\sigma}} \right)^{\frac{1}{\delta}} \left( \frac{\delta + 1}{\delta} \right)$$

(72)

$$= \lambda_4 t_S^{-1+\delta(\sigma-1)}$$

(73)

where $\lambda_4 \equiv f_M (\tilde{\sigma} \lambda_1)^{\sigma-1} \left( \frac{\delta + 1}{\tilde{\sigma}} \right)^{\frac{1}{\delta}} \left( \frac{\delta + 1}{\delta} \right)$.

This expression for the market size can be used in the zero profit condition for service suppliers:

$$\pi_S = 0$$

(74)

$$n_M = f_S f_M \tilde{\sigma} \delta \left( \frac{\tilde{\sigma}}{\delta + 1} \right)^{\frac{1}{\delta}}$$

(75)

where we see that the mass of manufacturers is fixed in each country (as in autarky) and is independent of trade costs. Therefore we know that:

$$n_M = n_M^* = \frac{f_S}{f_M} \tilde{\sigma} \delta \left( \frac{\tilde{\sigma}}{\delta + 1} \right)^{\frac{1}{\delta}}$$

(76)

The price indices are:

$$P = \left( \int p_1^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

(77)

$$= n_M^{\frac{1}{1-\sigma}} \tilde{\sigma} \lambda_1 t_S^\delta \left( \left( \frac{t_S^*}{t_S} \right)^{\delta(\sigma-1)} + \phi \right)^{\frac{1}{1-\sigma}}$$

(78)

$$P^* = n_M^{\frac{1}{1-\sigma}} \tilde{\sigma} \lambda_1 t_S^\delta \left( \phi \left( \left( \frac{t_S^*}{t_S} \right)^{\delta(\sigma-1)} + 1 \right) \right)^{\frac{1}{1-\sigma}}.$$  

(79)
This gives:

\[
A = \frac{\lambda_4}{1 - \phi^2} t_S^{-1+\phi(\sigma-1)} \left( 1 - \phi \left( \frac{t_s}{t_S} \right)^{-1+\phi(\sigma-1)} \right). \tag{80}
\]

\[
A^* = \frac{\lambda_4}{1 - \phi^2} t_S^{-1+\phi(\sigma-1)} \left( \frac{t_s^*}{t_S} \right)^{-1+\phi(\sigma-1)} - \phi). \tag{81}
\]

Dividing the two gives:

\[
\frac{A}{A^*} = \frac{L P^{*1-\sigma}}{L^* P^{1-\sigma}} \tag{82}
\]

\[
\left( 1 - \phi \left( \frac{t_s^*}{t_S} \right)^{-1+\phi(\sigma-1)} \right) \left( \frac{t_s^*}{t_S} \delta(\sigma-1) + \phi \right) = \frac{L}{L^*}. \tag{83}
\]

Imposing size symmetry, \( \frac{L}{L^*} = 1 \) or \( L = L^* = \bar{T} \), yields:

\[
\left( 1 - \phi \left( \frac{t_s^*}{t_S} \right)^{-1+\phi(\sigma-1)} \right) \left( \frac{t_s^*}{t_S} \delta(\sigma-1) + \phi \right) = 1 \tag{84}
\]

where we note that \( t = t^* \) is a solution.

Moreover,

\[
A = A^* = \frac{\lambda_4}{1 - \phi^2} t_S^{-1+\phi(\sigma-1)} \left( 1 - \phi \right) \tag{85}
\]

\[
= \frac{\lambda_4}{1 + \phi} t_S^{-1+\phi(\sigma-1)}. \tag{86}
\]

when country size is the same. This also means that the prices are:

\[
P = P^* = n_M^{\frac{1}{\bar{\sigma}}} \tilde{\sigma} \lambda_1 t_S^\delta (1 + \phi)^{\frac{1}{\bar{\sigma}}}. \tag{87}
\]

Using this information yields:

\[
\frac{\mu \bar{L}}{P^{1-\sigma}} = \frac{\lambda_4}{1 + \phi} t_S^{-1+\phi(\sigma-1)}
\]

\[
\frac{\mu \bar{L}}{\left( n_M^{\frac{1}{\bar{\sigma}}} \tilde{\sigma} \lambda_1 t_S^\delta (1 + \phi)^{\frac{1}{\bar{\sigma}}} \right)^{1-\sigma}} = \frac{\lambda_4}{1 + \phi} t_S^{-1+\phi(\sigma-1)}
\]

\[
t_S = \frac{f S}{\mu \bar{L} \bar{\sigma} - 1}.
\]
Note that with no difference in country size, the level of specialization of both service providers and manufacturing firms is exactly the same as in autarky and independent of trade costs.

As in autarky, the mass of service providers increases in country size:

\[
\begin{align*}
n_S &= \frac{1 - \lambda_M}{t_S} \\ &= \frac{L - \lambda_2 \lambda_3}{\lambda_3}.
\end{align*}
\]

(88)

(89)

Finally, note that:

\[
A = A^* = \frac{\lambda_4}{1 + \phi} \left( \frac{f_S}{\mu L \tilde{\sigma}} \right)^{-1+\delta(\sigma-1)}
\]

(90)

\[
= \frac{\lambda_4}{1 + \phi} \left( \frac{f_S}{\mu L \tilde{\sigma}} \right)^{-1+\delta(\sigma-1)}
\]

(91)

and that

\[
P = P^* = n_M^{\frac{1}{1-\sigma}} \tilde{\sigma} \lambda_1 t_S^\delta (1 + \phi)^{\frac{1}{1-\sigma}}
\]

(92)

\[
= (1 + \phi)^{\frac{1}{1-\sigma}} \left( \frac{f_S}{\mu L \tilde{\sigma}} \right)^\delta \left( \frac{f_S}{f_M \tilde{\sigma}} \tilde{\sigma} \delta + 1 \left( \frac{\tilde{\sigma}}{\delta + 1} \right)^{\frac{1}{2}} \right)^{\frac{1}{1-\sigma}} \tilde{\sigma} \lambda_1.
\]

(93)

So all the welfare effects from trade liberalization comes from the fact that more varieties become available and that their price falls as trade becomes less costly, captured by the fall in the term \((1 + \phi)^{\frac{1}{1-\sigma}}\) when \(\phi\) increases.

**Theorem 3** Trade liberalization in goods trade does not affect the degree of specialization or the mass of firms. It does generate welfare effects, however, through the decrease in price of import goods.

Moreover, all the effects from country size \((L\) and \(L^*)\) are identical to the autarky case and do not change with trade in goods.

### 5.2 Trade in tasks

Now we turn instead to the case when tasks can be traded internationally. Here, we face some difficult decisions about what assumptions to use. This depends on the fact that, in autarky, all outsourced tasks are produced in both countries by local service providers. In perfectly free trade, however, no service provider will produce the same task, regardless of where it is located. Therefore, we have decided to analyse the case of some intermediate trade cost where service providers are specializing internationally.
meaning that only one service provider in the world will produce each task which is outsourced). Moreover, we will analyse the symmetric country case where \( L = L^* = L \) to ensure that \( t_M = t_M^* \) because of the difficulties in dealing with how the tasks that are between \( t_M \) and \( t_M^* \) are produced (only a subset of service providers would be affected in this case and behave differently).

To model trade frictions in tasks, we assume a standard iceberg cost of \( \beta > 1 \) where for one unit of a task to arrive in the foreign economy, \( \beta \) units have to be produced by the service provider.

The profits of a manufacturer are unchanged with respect to our baseline model:

\[
\pi_M = pq - t_M \varphi_M (q) - (1 - t_M) pq - f_M. \tag{94}
\]

The profit of a service provider has, however, changed. When it is selling to a domestic manufacturer, the problem is still the same but consider the service provider’s profits for exporting its task:

\[
\pi_S^X = t_S p^* q^* - \beta t_S \varphi_S (t_S) q^* \tag{95} = t_S q^* (p^* - \beta \varphi_S (t_S)) \tag{96} = t_S q^* \left( A^* q^* - \frac{\lambda_A^*}{\sigma} - \beta \varphi_S (t_S) \right). \tag{97}
\]

The FOC for the service provider’s profit maximization with respect to \( q^* \) yields:

\[
\frac{\sigma - 1}{\sigma} t_S A^* q^* - \frac{\lambda_A^*}{\sigma} q^* - \beta \varphi_S (t_S) = 0 \tag{98}
\]

\[
q^* = \left( \frac{1}{\sigma \beta A^* t_S} \right)^\sigma A^*. \tag{99}
\]

Since manufacturers now buy from both domestic and foreign suppliers, it means that they will have less production than before by a factor of \( \beta^{-\sigma} < 1 \) (also domestic service provider will produce this lower output for manufacturers due to the Leontief structure of the manufacturers’ production function).

The manufacturer’s problem is now:

\[
\pi_M = pq - t_M \varphi_M (q) - (1 - t_M) pq - f_M \tag{100} = t_M \left( A^* q^* - \lambda_A^* \right) - f_M. \tag{101}
\]
The FOC for profit maximization with respect to $t_M$ yields:

$$\left( A^{\frac{1}{2}} q^{\frac{\sigma - 1}{2}} - \lambda_1 t_M^{\frac{\lambda}{2}} \right) - \lambda_1 t_M q = 0$$  \hspace{1cm} (102)

$$t_M = \left( \frac{1}{\lambda_1 (1 + \delta)} A^{\frac{1}{2}} q^{\frac{1}{2}} \right)^{\frac{1}{\delta}}. \hspace{1cm} (103)$$

Using the knowledge of $q$ from before, we can rewrite this to:

$$t_M = \left( \beta^{\frac{1}{\tilde{\sigma}}} \right) \frac{1}{(1 + \delta)} t_S^*$$  \hspace{1cm} (104)

which is the same as in the autarky except for the presence of $\beta$. This is because service providers now become more expensive to use due to the cost of $\beta$ to ship some tasks. Here, we also assume that it is the specialization level of foreign service providers which will bound $q$ because the foreign service providers are the ones who face the iceberg trade cost $\beta$ of shipping tasks.

Now, we use the free entry condition for manufacturers:

$$pq - t_M \varphi_M (q) q - (1 - t_M) pq = f_M$$  \hspace{1cm} (105)

$$t_M q \left( A^{\frac{1}{2}} q^{\frac{1}{2}} - \frac{1}{\lambda_1} t_M^{\frac{1}{2}} \right) = f_M$$  \hspace{1cm} (106)

$$t_S^{\delta(\sigma - 1)-1} f_M (\beta^{\tilde{\sigma}})^{(\sigma - 1)-\frac{1}{2}} \lambda_1^{-1} (1 + \delta)^{\frac{1}{\delta} (1 + \delta)} = A.$$  \hspace{1cm} (107)

The free entry condition for service providers in Home yields:

$$\pi_S = 0$$  \hspace{1cm} (108)

$$t_S (n_M p q + n_M^* p^* q^*) - t_S \varphi_S (t_S) (n_M q + n_M^* \beta q^*) = f_S$$  \hspace{1cm} (109)

$$t_S^{1+\delta(1-\sigma)} \lambda_1^{-1} \left( \frac{1}{\sigma \beta} \right)^{\sigma-1} \left( n_M A \left( \frac{r^*}{t_S} \right)^{\delta(1-\sigma)} + n_M^* A^* \right) = f_S.$$  \hspace{1cm} (110)

Comparing this solution to the foreign equivalent shows that there exists a symmetric solution where:

$$t_S = t_S^*$$

$$A = A^*$$

$$n_M = n_M^*$$
and we proceed to analyse this solution. Equation (110) simplifies to:

\begin{align}
 f_S &= t_s^{1+\delta(1-\sigma)} \lambda_1^{1-\sigma} \left( \frac{1}{\bar{\sigma} \beta} \right)^\sigma n_M A (2\bar{\sigma} \beta - (1 + \beta)) \\
 A &= f_S t_s^{\delta(\sigma-1)-1} \lambda_1^{\sigma-1} (\bar{\sigma} \beta)^\sigma n_M^{-1} \frac{1}{2\bar{\sigma} \beta - (1 + \beta)}. 
\end{align}

(111) (112)

where we note that $2\bar{\sigma} \beta - (1 + \beta)$ is always positive since $\bar{\sigma} > 1$.

This solution can be equaled to the solution in (107):

\begin{align}
 t_s^{\delta(\sigma-1)-1} f_M (\bar{\sigma} \beta)^{(\sigma-1)-\frac{1}{2}} \lambda_1^{\sigma-1} (1 + \delta)^\frac{1}{\delta} &= f_s t_s^{\delta(\sigma-1)-1} \lambda_1^{\sigma-1} (\bar{\sigma} \beta)^\sigma n_M^{-1} \frac{1}{2\bar{\sigma} \beta - (1 + \beta)} \\
 n_M &= \frac{f_s}{f_M} \frac{1+\delta}{1+\delta} \frac{\beta}{2\bar{\sigma} \beta - (1 + \beta)} \frac{\delta}{1+\delta} (1 + \delta)^{-\frac{1}{2}}. 
\end{align}

(113) (114)

It can be noted that

\begin{align}
 \frac{\partial}{\partial \beta} \left( \frac{\beta}{2\bar{\sigma} \beta - (1 + \beta)} \right) &= \frac{2\bar{\sigma} \beta - (1 + \beta) - \beta (2\bar{\sigma} - 1)}{(2\bar{\sigma} \beta - (1 + \beta))^2} \\
 &= -\frac{1}{(2\bar{\sigma} \beta - (1 + \beta))^2} < 0
\end{align}

(115) (116)

but $\frac{\partial}{\partial \beta} \left( \beta^\frac{1}{ \delta} \right) > 0$ so the effect of $\beta$ on the mass of manufacturing firms is not clear.

\begin{align}
 \frac{\partial}{\partial \beta} \left( \frac{\beta^\frac{1+\delta}{\delta}}{2\bar{\sigma} \beta - (1 + \beta)} \right) &= \frac{1+\delta}{\delta} \frac{\beta^\frac{1+\delta}{\delta} (2\bar{\sigma} \beta - (1 + \beta)) - \beta^\frac{1+\delta}{\delta} (2\bar{\sigma} - 1)}{(2\bar{\sigma} \beta - (1 + \beta))^2} \\
 &= \beta^\frac{1}{\delta} \frac{1}{(2\bar{\sigma} \beta - (1 + \beta))^2}. 
\end{align}

(117) (118)

This is negative if

\begin{align}
 \frac{1}{\delta} (2\bar{\sigma} \beta - (1 + \beta)) - 1 < 0 \\
 \beta < \frac{\sigma - 1}{\sigma + 1} (1 + \delta). 
\end{align}

(119) (120)

To find the remaining variables, we use the expression for $n_M$ and the definition of $A \equiv \frac{\mu L}{\bar{L} \lambda_1^\sigma}$ in equation (112) and find:

\begin{align}
 t_s &= \frac{2^{1-\sigma}}{\mu L} f_s \frac{2}{\bar{\sigma} \beta - (1 + \beta)} \left( \frac{\beta}{2\bar{\sigma} \beta - (1 + \beta)} \right)^{1-\sigma} \left( \frac{f_s}{f_M} \frac{1+\delta}{1+\delta} (1 + \delta)^{-\frac{1}{2}} \right)^{-\sigma}.
\end{align}

(121)
Moreover, knowing $A$ and $t_S$ yield a solution also for the price index:

$$\frac{\mu L}{P^{1-\sigma}} = f_S t_S \delta^{(\sigma-1)^{-1}} \lambda_1^{-1-\sigma} (\sigma \beta)^{-1} n_M^{-1} \frac{1}{2\sigma \beta - (1 + \beta)}.$$  

$$t_S = \frac{1}{\mu L} f_S \sigma^2 \frac{\beta}{2\sigma \beta - (1 + \beta)}.$$  

This shows that specialization decreases when task trade is liberalized ($\frac{\partial t_S}{\partial \beta} < 0$) which can be seen in (115).

Note also that:

$$t_M = \frac{\lambda_4}{L} \beta^\frac{1}{\delta} \frac{\beta}{2\sigma \beta - (1 + \beta)},$$  

where $\lambda_4 \equiv \frac{1}{\mu} \left( \frac{\sigma}{\delta} \right)^{\frac{1}{\delta}} f_S \sigma^2$. We note that $t_M$ increases with trade liberalization ($d\beta < 0$) because $t_S$ increases but also decreases (due to the extra term $\beta^\frac{1}{\delta}$) because now the quantity supplied by foreign service providers increases.

The price index becomes:

$$P = \lambda_5 \left( \frac{1}{L} \right)^\delta \beta^{1-\frac{1}{\delta(\sigma-1)}} \left( \frac{\beta}{2\sigma \beta - (1 + \beta)} \right)^{\delta - \frac{1}{\delta - 1}}.$$  

where $\lambda_5 \equiv \mu^{-\delta} \lambda_1 (f_S)^{\delta - \frac{1}{\delta - 1}} \sigma^{(1+\delta)} \left( \frac{1}{f_M} \frac{\sigma^{\frac{1}{1+\delta}}}{1+\delta} (1 + \delta)^{\frac{1}{1+\delta}} \right)^{-\frac{1}{\delta - 1}}$.

The elasticity of the price index with respect to country size is the same as before, $\delta$. The net effect of $\beta$, the cost of task trade, is, however, uncertain. This is most likely due to the two main channels through which $\beta$ affects these variables: (i) a lower $\beta$ increases specialization which lowers the price index and increases welfare but (ii) a lower $\beta$ increases the output of each manufacturer which lowers the range of varieties available in the economy. The net effect ultimately depends on the relative size of $\delta$ and $\sigma$ or whether the preference for variety ($\sigma$) is stronger than the need for specialization ($\delta$).

**Theorem 4** Trade liberalization in task trade decreases the level of specialization among both service providers and manufacturing firms. The net effect on the range of manufacturing varieties and welfare is, however, uncertain.
References


W. Strahan and T. Cadell, London.