Multistage production, cost linkages and the suboptimal location of firms

Tomasz Michalski*

June 19, 2010

Abstract

I present a model of trade with multiple production stages. With an intermediate intensive technology of production at each stage, there exists a unique equilibrium where cost linkages cause vertically linked firms to agglomerate in the country with a cost advantage in the initial stage of production. This location choice of firms while optimal from each firm's point of view can be jointly suboptimal and may constitute a barrier to industrialization of locations with lower costs from the perspective of the entire supply chain. Coordinating the location of vertically related firms can affect industrial location and hence the pattern of trade. The model adds to the understanding why industrialization policies in some countries could have been successful in the presence of a high degree of vertical integration of manufacturing industries.

JEL codes: F12, F15, L23, L52

Keywords: cost linkages, location, intermediates, vertical integration, agglomeration

*Address: Tomasz Michalski, HEC Paris, Economics and Decision Sciences Department, 1, rue de la Libération 78531 Jouy-en-Josas CEDEX, France. Phone: +33 (0)1 39 67 72 40. Fax: +33 (0)1 39 67 70 85. E-mail: michalski@hec.fr.
1 Introduction

The economic geography literature has provided many insights regarding the location choices of firms. In particular, in the presence of scale economies and transport costs, there is a tendency for firms to agglomerate in one location due to demand and/or cost linkages. Most of this literature assumes that production is carried out in either one or at most two stages. In reality however, most goods are produced in multiple sequential stages. Take an example of production of a simple good – clothing. First, there is production of cotton or synthetics, followed by production of fibres (spinning), cloth itself and then clothing. Moreover, each of these stages can have many substages and firms may not have the technology to produce all the intermediates by themselves. In this paper I argue that the fact that production takes place in more than two stages may matter not only for location choices of firms but also the optimality of the free market equilibrium location of firms. I show that there could be a difference in location choices of firms when the different stages are carried out by different firms than when they are controlled (or coordinated) by a single firm. Thus I add a new dimension of consideration – namely, the degree of integration of firms – for location choices of firms and hence the pattern of trade. The unique equilibrium location of firms when there are multiple stages of production can be suboptimal and there exists a Pareto improving policy intervention whereby firms are induced to relocate. This stands in contrast to the existing economic geography models with a unique equilibrium whereby the agglomeration of firms in one location is globally efficient and that any policy to induce firms to relocate necessarily reduces world welfare (see for examples, Krugman and Venables 1995, Amiti 2005).

I consider a simple two country general equilibrium trade model whereby the production of many manufacturing goods (sectors) involves several stages. Labor is the only primary factor of production. Each stage uses the intermediate good produced by the previous stage and labor as inputs. I assume that one country (call it Home) is the only producer of the initial (stage 0) stage (of all manufacturing goods) but subsequent stages can locate in either country.\(^1\) The initial stage is produced under constant returns and perfect competition while subsequent stages are produced under increasing returns to scale technology and monopolistic competition.\(^2\) Apart from the manufacturing good, there is also a homogenous (simple) good that is produced under constant returns to scale using

\(^1\) An example of an inefficient equilibrium with symmetric countries is presented in the Appendix, Section (A.3).

\(^2\) In line with existing economic geography models, I adopted the monopolistic competition framework. However, our results do not depend on the market structure of the intermediate good producers. An example with perfectly competitive industries is presented in the Appendix, Section (A.2). See also the section under discussion of results.
only labor as an input. All goods (intermediates, the final and the homogenous good) can be traded between countries with an iceberg trade cost applied.

There are in general two considerations affecting the choice of locations by firms – factor cost and demand and cost linkages due to the presence of transport costs. In this paper I abstract away from demand linkages and focus on factor cost and cost linkages. Firms want to locate where factor cost is lower but they also want to locate close to their suppliers to avoid transport costs on their intermediate inputs. Given that Home has a comparative advantage in stage 0, stage 1 firms may want to locate there to save on intermediate input costs if the share of intermediates in production is high enough. This cost linkage effect can be so strong that under certain conditions, even if the wage of Home is larger than the foreign country, the stage 1 firm will want to locate there. And since stage 1 firms are located in Home, the same cost considerations will lead the downstream stage 2 (and following) firms to locate in Home. This can lead to an equilibrium where Home obtains all the manufacturing firms and the foreign country will specialize in the homogenous good. Nevertheless, wages in Home will be higher than in Foreign in equilibrium.

This equilibrium outcome, however, may be inefficient. With a high enough (at least four, including the initial stage) number of production stages, placing at least one manufacturing industry (all except the initial stage) in the low wage country improves welfare globally. This is because with the number of intermediate stages large enough the final good embodies predominantly labor (which is cheaper in the low-wage country in equilibrium) and not the initial stage intermediate. In fact, as the number of production stages grows, the welfare loss from the (free market) equilibrium increases. Hence, the low-wage foreign country would have an advantage in producing some manufacturing goods if industries could use domestically produced intermediates. Yet, if firms at each stage of intermediate production pursue their own individual profit maximization, they are better off moving their production to the other country if their suppliers have the incentives to do so. Each firm in making its location choice does not take into account the fact that their location choice affects the profits of firms further down the production chain. The vertically linked firms could be jointly better off if they could somehow coordinate on their location choice and share through transfers the profits from doing so. I demonstrate therefore a new, interesting and realistic case (also in terms of parameters for which it holds) where agglomeration of sequentially linked firms leads to suboptimal outcomes. These effects can be a barrier to industrialization. Fragmentation of production into many consecutive stages hence may prevent comparative advantage from being revealed and lead to
suboptimal location equilibria. The result does not come from a coordination failure as there exist firms that do not want to locate in the low-wage country even if all others do but rather from the fact that the sequential decision making causes a globally efficient outcome to unravel. However, if the share of labor at each stage of production is high, the fragmentation of production may be efficient, with the sectors labor intensive at each stage of production locating in the low wage country.

I then proceed to show that a global welfare improving outcome can be reached by subsidizing stage 1 producers in one industry so that this industry will start producing in the low wage country. A Pareto improving outcome can be achieved by subsidizing the consumers in the high wage country at the same time with all subsidies being financed by lump sum taxes on consumers in the low wage country.

Subsidizing stage 1 producers is not the only way to induce firms to relocate to the low cost country. Another way would be for the low wage country to encourage the integration of vertically linked firms. Promoting industrialization through encouraging vertical integration is a typical example of policy intervention in Japan and South Korea (and in Taiwan in intermediate producing industries). Until today these countries have strongly vertically integrated intermediate industries that promoted forward linkages (see Feenstra et al. (1999) and Feenstra et al. (2003) for extended discussions). My result adds a new dimension to the literature on the benefits of vertical integration; from the viewpoint of a downstream firm exercising control over upstream firms may be valuable and important because it enables the control of the location decisions of upstream firms. Without such a control profits of the downstream firm can be negatively affected, especially in the case of countries which are resource poor, and have no advantage in the initial stages of production.

The literature on multistage production and trade was started by Dixit and Grossman (1982) who present a perfectly competitive model with chains of production where the division of production is determined by relative factor abundance. More recently, Yi (2008) examines how multistage production and transport costs could contribute to the home bias in trade. Amiti (2005) presents a two-sector and two-stage model and considers how trade liberalization affects the location of industries. She shows that industries may cluster together due to demand and cost linkages in a country seemingly against comparative advantage as the capital abundant country may receive labor-intensive industries when these follow their intermediate suppliers. Venables (1999) also shows that firms may locate in a country against comparative advantage. In both their models however, the equilibrium location of industries is globally welfare optimal. By contrast I show how multistage
production and cost linkages can lead to suboptimal concentration of firms. In this light, my model is related to Matsuyama and Takahashi (1998) who show that over concentration of firms in one region can occur due to the coordination failures between entry decisions of service firms and migration workers. My paper is also related to the literature on co-ordination failures due, for example, to the presence of increasing returns and forward and backward linkages and non-tradeability of some goods (e.g. Murphy et. al. 1989, Rodrik 1996). In this paper, however, all goods are tradeable, the demand linkages and expanding intermediate varieties do not play any role, and there is no co-ordination failure. My conclusion is not as optimistic as that of Trindade (2005) who analyzes trade in intermediates reinforcing the process of industrialization of small economies in the manner of the big-push theories. In the economies in this paper, a low-wage country may not industrialize with free trade in intermediates as it will remain in fact for individual firms the high-cost location because of the existence of transport costs.

In Section 2, I lay out a simple general equilibrium model with multistage production. In Section 3.1 I derive an equilibrium whereby agglomeration of manufacturing takes place in the country with a cost advantage in the production of the initial stage. In Section 3.2, I discuss some other equilibria that might prevail when the labor intensity of production at each stage is high enough. Section 4.1 discusses a subsidy cum tax cum transfer policy that induces one industry to relocate to the low wage country and shows that this policy intervention is Pareto improving. Section 5 concludes while the proofs are relegated to the Appendix.

2 The model

There are two countries (labeled Home and Foreign) with equal worker population $L$ and three types of goods: a homogenous good, $m$ manufactured final goods and intermediate goods of several stages. $^3$ Home is the only producer of the initial stage (it either has a patent to do this or some rare resource – coal, iron etc.) for all manufacturing industries. $^4, 5$ Worker productivity and technology

$^3$In discussing Pareto superior outcomes, I propose studying the change of location for one industry so that the factor prices remain constant.

$^4$We can have Foreign being able to produce by itself this intermediate with a technology that is costly enough in comparison to Home so that imports from Home dominate domestic production. Foreign may have bottlenecks at the initial stage of manufacturing production. Alternatively, although capital is not present in the model we can think that Home is capital abundant and the initial stage is capital intensive.

$^5$The assumption that Home is the only producer of the initial stage in all $m$ industries is a simplification. This helps us to avoid having to solve the trade balance equation. The main argument will carry through even if Foreign has sufficient cost advantage to be the only producer of the initial stage of some industries as long as the number of industries in which Home is the only producer is sufficiently larger than Foreign. See also an example with symmetric
is the same in both countries for all subsequent stages and firms producing the subsequent stages can locate in either country depending on labor and transport costs.

International trade is possible in all types of goods. There are iceberg trade costs in all goods of order \( \tau \). I set the wage in Home equal to 1 and let \( w \) be the relative wage of Foreign. The demand for the final goods is homothetic, with the shares of expenditures for the homogenous good \( \alpha_H > \frac{1}{2} \). The expenditure shares \( \alpha_m \) for each of the manufacturing goods are equal.

Each manufactured good has \( n + 1 \) stages of production with the initial stage (stage 0) being located in Home. This initial stage has a constant cost of production \( c \) normalized to 1 or equal to the cost of one unit of labor employed. For simplicity and without loss of generality I assume that there is only one variety of stage 0 intermediate and it is produced under perfect competition.

For each subsequent \( n \) stages there is a continuum of variety of intermediate goods indexed on \([0, 1]\) being produced. Each firm producing a variety uses all varieties of intermediate goods produced from the previous stage plus local labor to manufacture one variety of the next stage intermediate goods. A firm producing a stage \( s \) intermediate variety has the following production function\(^7\) \( ^8 \)

\[
f + Y_s = l_s^{1-\gamma} \left( \int_0^1 (z_{s-1}, j)^\sigma \, dj \right)^{\frac{\gamma}{\sigma}}
\]

where \( l_s \) is labor employed and \( z_{(s-1), j} \) are intermediate varieties of the previous stage, \( (1 - \gamma) \) is the labor share and \( 0 < \sigma < 1 \) is a parameter capturing the degree of substitution between inputs, \( f \) is the fixed cost and \( Y_s \) the output of a variety stage \( s \) intermediate. I assume that the inputs in a given industry cannot be used as inputs in other industries.

Given the above production function, I can obtain the marginal costs and input demands for each stage assuming that all intermediates of a given industry \( m \) are located in one country. Then the generic marginal cost \( \Psi_s \) of production of an intermediate variety at the \( s^{th} \) stage in country \( i \) is

\[
\Psi_s = (p_{s-1})^\gamma w_i^{(1-\gamma)} (1 - \gamma)^{(1-\gamma)} \gamma^{-\gamma}
\]
whereas the unit demand for the intermediate $i$ from the previous stage is

$$
\frac{\Psi_s \gamma}{\int_0^1 (p_{s-1,j})^{-\frac{s}{\gamma}} dj} (p_{s-1,i})^{-\frac{1}{1-\gamma}}
$$

where $p_{s-1}$ is the price of the representative intermediate of stage $s - 1$.

I assume that there is only one firm that is awarded a patent to produce each variety of intermediates at each stage. Each producer of the intermediates that is active in one of the $m$ sectors has a monopolistic power over its own variety. Observing the demand for its intermediate (as in eq. 3) each such firm active at stage $s$ sets an optimal price $p_s = \frac{\Psi_s}{\sigma}$.

Then the last stage final goods firms locating in country $i$ will have the following marginal cost (assuming that all firms locate in the same country):

$$
\Psi_n = (\Psi_1)^{\gamma - 1} \left[ \frac{1}{\gamma - 1} \right]^{\gamma - 1} \left[ \frac{1}{\gamma - 1} \right]^{\gamma - 1}
$$

where $\varrho = (1 - \gamma)^{(1 - \gamma)} \gamma^{-1} \sigma^{-1}$.

To obtain a patent, firms must invest in R&D. I assume that there is free entry in the R&D process and that each firm invests a fixed amount in the patent race. The probability that a given firm will get the patent for a variety depends on the number of firms that invest in R&D in developing the same variety. With free entry, the amount invested must be equal to the expected profit. This implies that the total amount of R&D expenditure by all the firms in the race plus the fixed cost $F \Psi_s$ must equal to the profit from production and sales of an intermediate variety.

The zero profit entry condition is then

$$
F \Psi_s = \Psi_s \left( \frac{1 - \sigma}{\sigma} \right) (D_H + D_F)
$$

where $D_H$ and $D_F$ represent the demand for those inputs in Home and Foreign (at the stage $s$) which can be rewritten as

$$
\bar{F}_c = (\Psi_{1,H})^{-\frac{1}{1-\sigma}} (A_H + \tau^{-\frac{s}{\gamma}} A_F)
$$

for a firm locating in home and

$$
\bar{F}_c = (\Psi_{1,F})^{-\frac{1}{1-\sigma}} (\tau^{-\frac{s}{\gamma}} A_H + A_F)
$$

for a firm locating in foreign.
for a firm locating in Foreign where $D_H = A_H (p)^{-\frac{1}{1+\sigma}}$ so that $A_H, A_F$ are demand parameters from the following stage producers and $\bar{F}$ is a constant.

A production structure of one of the $m$ industries with 2 intermediate stages is summarized in Figure 1.

3 Location equilibria

I study the incentives of firms locating simultaneously all along the production chain. Each firm when deciding upon its location makes its choice basing on where its expected profits would be larger. I concentrate on outcomes when firms would have, ex post, no incentives to deviate from their chosen domicile given the choice of others, also at different production stages.\footnote{I do not formalize the setup in a form of a game, but we study in fact outcomes which would constitute a Nash equilibrium of a subgame with a finite number of firms that have entered (for each stage of production).}

I discuss the equilibria in which industries locate only in one of the countries, restricting the parameter space of $\gamma$, the share of intermediates in production cost at each stage, and $\sigma$, the parameter measuring the elasticity of substitution between varieties. The characterization of other
equilibria is not tractable in general. The summary of the discussed equilibria in the \((\sigma, \gamma)\) parameter space is given in Figure (2).

3.1 The equilibrium when production is intermediate-intensive at each stage

Regardless of the distribution of demand, firms choose to locate in Home due to cost linkages if the share of labor in the production at each stage is low. This way, a low wage country may not be the most cost-effective location for an individual firm no matter what and where the located the downstream demand is. The decentralized equilibrium that results (see Proposition 4) may be inefficient and Pareto-inferior to some other outcomes.

**Proposition 1** Suppose that \(\gamma \geq \frac{1+\sigma}{2}\). Then all manufacturing industries locate in Home and this is a unique equilibrium.

**Proof.** See Appendix.

The condition \(\gamma \geq \frac{1+\sigma}{2}\) ensures that the intermediate and final manufactured industries move to Home irrespective of how large the markets for the intermediates (and the final good) are in a particular country. Hence, no demand linkages work here. If intermediate producers from an earlier stage have incentives to locate in Home, the ones further downstream will move as well because then they would be facing the same cost production differential between countries as there was for the earlier stage producers. The existing cost linkages deprive Foreign of the intermediate good industry in any of the manufacturing sectors. As a consequence, the final manufactured good production for “world” markets occurs only in Home. Home exports therefore final manufactured goods and obtains the homogenous good in return. Because the homogenous good is exported by Foreign, the wage in that country must be \(w = \tau^{-1}\) so as for that good to be competitive in Home.\(^{10}\) The result holds irrespective of the size of the transport costs \(\tau\).

The condition \(\gamma \geq \frac{1+\sigma}{2}\) holds for example for parameters \(\sigma = 0.8, \gamma = 0.9\) which would be plausible values characterizing the production in the U.S. economy on average under the assumption of only two production stages.\(^{11}\) With more stages of production, the labor share \((1 - \gamma)\) at each

\(^{10}\)All manufacturing industries would have even higher incentives to locate entirely in Home for example if the wage in Foreign would remain at \(w = 1\). Then with \(\gamma > \sigma\) would cause that all firms would want to locate in Home.

\(^{11}\)From the benchmark input-output tables for 1997 for the United States available at the Bureau of Economic Activity we find that more than 1/3 of six-digit manufacturing industries have a share of intermediate goods in costs (intermediate use + employee compensation) of over 0.8 which imply \(\gamma\) must be higher than that as typically there
stage would be even lower than 0.1. It guarantees the existence of a unique equilibrium. However, I can show that the location of all firms in Home may be an equilibrium (though not necessarily unique) for a milder set of restrictions.

**Proposition 2** If $\gamma > \frac{1}{2} > \sigma$ or $\gamma > \sigma > \frac{1}{2}$ then an outcome in which all industries at all stages locate in Home and $w = \tau^{-1}$ is an equilibrium.

**Proof.** See Appendix.

This is a sufficient condition for this equilibrium to exist. Finally, if the share of labor at each stage of the production process is high enough (or $\gamma$ low enough) one can assure that Home cannot have the entire manufacturing activity.

**Proposition 3** If $\gamma < \frac{1-\sigma}{2}$ then an outcome in which all industries at all stages locate in Home and $w = \tau^{-1}$ cannot be an equilibrium no matter what the demand linkages are.

**Proof.** See Appendix.

This result is due to the fact that all individual firms, no matter at which production stage, are going to recognize that costs are lower in Foreign if $w = \tau^{-1}$ and hence move there. In equilibrium, however, Foreign will not receive the entire manufacturing industry (which it would if the wage remained at $w = \tau^{-1}$) as the trade balance will tilt the wage above $w = \tau^{-1}$ as industries move so that Foreign does not export the homogenous good.

**Inefficiency of the decentralized equilibrium** The equilibrium with all manufacturing locating in Home and $w = \tau^{-1}$ may in fact be inefficient. Let us compare the relative costs of production of the final manufactured good for a particular industry $m$ assuming that all the intermediates (except stage 0) are located in the same country.

**Proposition 4** Suppose $w = \tau^{-1}$. In a given industry $m$, for a number of production stages large enough given $\gamma$ and $\sigma$, if the whole production chain (with the exception of stage 0) is located in Foreign, the marginal cost of production of the final good is lower than if the whole production chain is located in Home.

---

are more production stages than one. It is difficult to establish $\gamma$ at each stage of production for each industry. For entire manufacturing, a simple back of the envelope calculation (assuming that there are two stages of production) gives an implied usage of labor at each production stage of 0.1 (or $\gamma = 0.9$).
Figure 2: Equilibria with $w = \tau^{-1}$ and $\gamma$ and $\sigma$ values. Area I: Unique equilibrium with the manufacturing industry in Home. Area II: All firms locating in Home is an equilibrium (a sufficient condition). Area III: With $w = \tau^{-1}$ all firms have an incentive to move to Foreign. Point A: corresponds to average values for the United States assuming two stages of production and $\sigma = 0.8$ (data taken from the benchmark input-output tables for 1997 from BEA).
**Proof.** See Appendix.

Proposition 4 shows that if the intermediate goods in a given industry could be somehow moved to Foreign (without changing the factor prices) this could result in savings in the cost of production of the final good. This is because as the number of stages increases, more labor is embodied in the final product relative to the stage 0 input. Since labor is cheaper in Foreign in equilibrium this implies that Foreign is potentially the more efficient producer of the final good than Home (and potentially of some intermediates as well). The problem is, however, that firms in their individual quest for the maximum profit fail to achieve joint efficient production and profit maximization from the point of view of the entire supply chain. Higher cost of production and hence higher prices for the final good in turn imply lower consumer welfare.

The number of stages for which the suboptimality can be small. For example, for $\gamma = 0.7$ (and a corresponding $\sigma$ so that $\gamma > \frac{1+\sigma}{2}$) only three production stages including the initial stage 0 may be required so that Foreign is the cheaper producer of the final good. With $\gamma = 0.8$ this requires 4 and with $\gamma = 0.9$ only 7 stages. In perspective, even in industries with simple supply chains like textiles and clothing there are many stages of production that are performed typically by many firms that can lead to effectively more than 10 tasks even for relatively simple products.

The problem why market forces may lead to suboptimal allocations in comparison to a central planner is that at each stage the intermediate producers while making their location decisions do not take into account how it will affect the decisions and profits made by firms further downstream. The value added at each stage by a given producer is small; with many stages of production, a firm does not internalize the fact that the overall labor content (in the final good) may cost less in Foreign. The cost linkages created by the supply chain lead them to locate in the undesirable (from the global welfare point of view) location. Thus "free" trade (where only transport costs apply a surcharge on the producer price) and disintegration of production can lead to a welfare-inferior situation if the number of production stages is large enough. The low wage advantage of Foreign if the whole production chain is considered may not be exploited! One could say that the result presented here comes from a failure of a takeover market; if firms from downstream industries that recognize their cost production advantage at some stage if their supply chain remained in the low wage country could integrate backwards the problem would be solved. I therefore provide a new rationale for firms to consider vertical integration where the coordination of production location is key. The emergence of business groups (which may not necessarily involve one firm at each production stage) would be
a natural solution to the problem. Empirical studies (for example Holmes 1999) that attempted to validate Stigler’s (1951) theory that the division of labor is governed by the extent of the market find that there exists a correlation between the agglomeration of industries and a measure of vertical integration. Firms that are located far away from other establishments in the same industry tend to be more vertically integrated. It is unclear whether this correlation supports Stigler’s claim or, for example, this model, as the data does not permit determining causation.

The failure of the market is not related to the fact that firms apply monopolistic pricing. In fact, I can construct examples with a perfectly competitive market structure at each stage of production and the results will still go through (see Section A.2 in the Appendix). It is the technology that matters – the presence of fixed costs that have to be paid in a given locale. Any gain in the marginal cost while locating in Foreign and selling to the Foreign market can be wiped out by relatively higher fixed costs at that location and this may be decisive for the firm’s bottom line. Notice also that the demand here does not matter. Irrespective of how much the downstream producers demand (and where they are located), the earlier stages may want to locate in the high wage country. No matter what is the end demand, the "integrated" industry in the lower wage country can produce the final good at a lower cost than the same industry locating in the high wage country, when there is a sufficiently large number of stages. In Amiti (2005), there are only two stages and in an equilibrium whereby the downstream stage is located in a country against comparative advantage, the production cost of the final good will not be lowered even if the downstream stage were to move to the other country and the inefficiency does not manifest itself here. This is because with just two stages, the costs of inputs are important in the overall costs of production and the final good is produced more cheaply when the upstream firms are located in the same country as the downstream firms given the level of transport costs. In the model, however, the cost of the initial input becomes an insignificant part of overall costs of production of the final good as the number of stages increases and hence there exists Pareto improving intervention to induce firms to move to the low wage country (see section 4).

It has to be stressed that the result is not due to a coordination failure for example in the sense of Murphy et. al. (1989), Rodrik (1996) or Matsuyama and Takahashi (1998). Even if all firms would be initially located in Foreign, if $\gamma > \frac{1+\sigma}{2}$ the first stage producers would have always an incentive to move to Home.
3.2 Equilibria when production is labor-intensive at each stage

The low-wage country may not face a barrier to industrialization in sectors where $\gamma$ is low or where at each production stage labor is used intensively relative to intermediates (for example when $\gamma < \frac{1+\sigma}{2}$ as in Proposition 3). In such a case, in our model, the wage $w = \tau^{-1}$ will incite firms to locate in Foreign and this may be true irrespective of where the downstream firms would locate. The low-wage advantage of Foreign may be then exploited by individual firms. In fact, equilibria may exist with the entire manufacturing industry moving to Foreign. This may then require adjustments of the relative wage so as to equate the trade balance. In general, however, multiple equilibria may result that are not easy to characterize.

When the labor intensity at each production stage is high, demand linkages may be at play as well. Whenever $\gamma < \frac{1+\sigma}{2}$ and the final producers have the incentives to move to Foreign, the upstream suppliers will have the same incentives.

**Proposition 5** *(Demand linkages)* If the final producer in one manufacturing industry has the incentives to locate in Foreign (without changing the equilibrium wages) regardless of their suppliers’ potential location, then all the suppliers will have the incentives to locate in the same country as the final good producer.

**Proof.** See Appendix.

4 Policy interventions

Can the equilibrium exhibited in Section 3.1 be improved upon when it is inefficient? Simple trade protection of Foreign manufacturing will not attract the industry to Foreign. In what follows I consider the following policy interventions. First, I discuss a subsidy or tax that induces the production chain in one industry to move to Foreign and which is globally Pareto welfare improving. Secondly, I discuss coordinating the location of firms through vertical integration of industry.

4.1 Pareto-improving subsidies

To induce intermediate goods firms to move, one obvious policy intervention is to provide a subsidy high enough in Foreign to the stage 1 producers, making them want to move to that country. Once

\[\text{12}^\text{Of course, we there is a trivial welfare improvement if the monopoly power of the firms could be curbed.}\]
these would move, it is easy to check that stage 2 intermediate producers that use their inputs have the same incentive to move to Foreign. Hence, all intermediate producers further downstream in turn would have the same incentives. The question then is whether the cost of the subsidy will exceed the gains from instituting it. With the current production structure and monopolistic pricing, a cost subsidy to the stage 1 producers is always Pareto-improving (even in a closed economy) as it increases the output of the final good and lowers the distortion caused by monopolistic competition pricing. Hence, I want to design a scheme that does not affect the distortion caused by monopolistic pricing and show that there may be a Pareto gain from moving the production of some goods from Home to Foreign.\footnote{The Dixit-Stiglitz production function is particularly suitable here as it implies that the markups that firms charge at each stage are constant and depend only on the elasticity of substitution between inputs at each stage.} This in general requires the use of lump sum taxes and subsidies. I consider one such scheme that does not affect factor prices and hence incomes. Basically, I suggest a lump sum subsidy for stage 1 firms in one industry to induce them to move, a lump sum transfer to consumers in Home so as to ensure that their consumption remains the same and to finance the subsidy and transfers by lump sum taxes on consumers in Foreign.\footnote{In general, there exist many Pareto improving interventions each involving a different number of industries being moved to Foreign. We consider only the simplest case whereby we are sure that factor prices will not change so that we need not worry about solving the trade balance equation.} I then show that the consumers in Foreign are strictly better off (as they benefit from lower prices for the final good variety) even if they have to bear the taxes.

\textbf{Proposition 6} \textit{For a number of stages large enough, there exists a policy intervention that is Pareto improving and which induces stage 1 intermediate and consequently all producers further downstream in one industry $m$ to move to Foreign.}

\textbf{Proof.} See Appendix.

This policy intervention works and assures that the price of the good $m$ in Foreign falls, therefore causing a Pareto improvement and a rise in global welfare. The number of production stages for which such an intervention is Pareto welfare improving (intermediate + final good stages) for economies of equal population size as a function of a triple $(\gamma, \sigma, \tau)$ is shown in Table (1).

The increase in the number of required stages with an increasing trade cost comes from the fact that then the demand in Home increases relatively to that in Foreign because the wage gap between them is higher. As a result, assuring that consumers at Home enjoy the same consumption as before the intervention is more costly. A higher share of local labor in production (lower $\gamma$) requires less
Table 1: Number of production stages for which the intervention is Pareto welfare improving

<table>
<thead>
<tr>
<th>γ</th>
<th>0.7</th>
<th>0.7</th>
<th>0.7</th>
<th>0.7</th>
<th>0.8</th>
<th>0.8</th>
<th>0.8</th>
<th>0.8</th>
<th>0.9</th>
<th>0.9</th>
<th>0.9</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>τ</td>
<td>1.05</td>
<td>1.2</td>
<td>1.5</td>
<td>2</td>
<td>1.05</td>
<td>1.2</td>
<td>1.5</td>
<td>2</td>
<td>1.05</td>
<td>1.2</td>
<td>1.5</td>
<td>2</td>
</tr>
<tr>
<td>n</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>12</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

number of stages for there to exist Pareto improving intervention as the relative cost of production of the final good falls quicker with the increase in the number of production stages.

### 4.2 Coordinating location of a vertical production chain as a policy intervention

In the context of Propositions 1 and 4 vertical integration of industries can also improve global welfare. What matters here is that vertical integration allows for the control of location of any firm at any stage of the supply chain.\(^{15}\) This control eliminates the suboptimal (from the global welfare point of view) individual location decisions of firms that create the potential inefficiency and helps to create a supplier base that otherwise might not materialize.\(^{16}\) Welfare gains will not be obtained only from the avoidance of double marginalization but from making the counterpart units internalize the effect of their location decisions onto downstream producers.

Subsidizing initial stages of production and coordination of production and location decisions was a feature of the industrial policy for example in South Korea (Amsden, 1989). The development of shipbuilding and steel industries in the 1970s there provides examples where such measures were put in place, an asymmetry in the initial stages versus other countries existed and the industries proved to be competitive in the final product quickly after their creation.

POSCO, the steel producing company, had to import iron ore; Hyundai Heavy Industries (HHI), active in shipbuilding, at the beginning had to import steel and engines. Both POSCO and HHI benefited vis-a-vis its competitors through low wages (three times lower than in Japan per ton of hot-stock rolled product at the beginning), low construction costs and some government subsidies.\(^{17}\) HHI established a producer of engines HEMCO to overcome the problem of having to import pricy

\[^{15}\text{This applies even if, in general, a vertically linked firm would like to locate the production at different stages of production in different countries.}\]

\[^{16}\text{For a model, see Goh and Michalski (2010).}\]

\[^{17}\text{For example, transport and port services were subsidized. We could interpret these as directly affecting \(τ\) in the model, effectively lowering for example the cost of the initial input, iron ore.}\]
Japanese engines\textsuperscript{18} and other lower-tier component manufacturers to be able to compete; the establishment of the locally-based POSCO greatly helped. Note that there is a low usage of labor in the goods produced in those sectors at each production stage. The government encouraged also vertical integration and coordinating production in these sectors, either through helping to create chaebols or even directly through industry meetings held at relevant ministries (Amsden 1989, see also Wade 1990 for Taiwan).

5 Conclusions

I have shown that a unique equilibrium in a general equilibrium model with multistage production and trade can be suboptimal because of cost linkages and decentralized location decisions of firms. Policy interventions such as subsidies may be warranted if firms individually make their location choices and do not take into account while performing profit maximization the impact their decisions have on downstream producers. My model also suggests that fragmentation of production may lead to more clustering of vertically related firms in the ”wrong” location in the sense of leading to a higher cost of production of the final goods. I provide also a new rationale for why vertical integration may be beneficial through coordinating on the production location of its constituent vertical units.

More generally, the results point out that the vertical structure and industrial organization of industries in autarky prior to trade opening may matter for the equilibrium trade flows and the patterns of specialization, a subject worthy of further research.

\textsuperscript{18}HEMCO had to import initially key components from Japan (the fuel injection pump, turbo chargers, electronic control) which constituted up to 25-30\% of the cost. Hence it had to establish firms producing those components domestically as well.
A Appendix

A.1 Proofs

**Proof of Proposition 1.** Suppose first that the equilibrium relative wage is $w = \tau^{-1}$. Consider the decision of stage 1 intermediate firms in the $m^{th}$ sector. Since $\Psi_{1,H} = (c)^\gamma (1 - \gamma)^{\gamma - \gamma w_H^{1 - \gamma}}$ and $\Psi_{1,F} = (\tau c)^\gamma (1 - \gamma)^{\gamma - \gamma w_F^{1 - \gamma}}$, I have

$$\frac{\Psi_{1,F}}{\Psi_{1,H}} = \tau^{2\gamma - 1} > \tau^\sigma$$

which means, comparing the right hand sides of eqs. (6) and (7)

$$(A_H + \tau^{-\frac{1}{1-\sigma}} A_F) > \left((\tau)^{2\gamma - 1}\right)^{-\frac{1}{1-\sigma}} \left(\tau^{-\frac{1}{1-\sigma}} A_H + A_F\right)$$

Clearly the firms prefer to move to Home no matter what demand from buyers located in either country they are facing. An intermediate producer from Home has then a cost advantage in accessing both the home and foreign markets over the one that would enter in Foreign. Consequently, he would be earning higher profits.

Stage 2 firms, assessing the location preferences of stage 1 producers, notice that if all their suppliers move to Home the cost differential becomes

$$\frac{\Psi_{2,F}}{\Psi_{2,H}} = \tau^{2\gamma - 1} > \tau^\sigma$$

Stage 2 producers have then effectively the following zero profit conditions if they locate in Home

$$\bar{F} = \left(\Psi_{2,H}\right)^{-\frac{1}{1-\sigma}} \left(A_H + \tau^{-\frac{1}{1-\sigma}} A_F\right)$$

and in Foreign

$$\bar{F} = \left(\tau^{2\gamma - 1}\Psi_{2,H}\right)^{-\frac{1}{1-\sigma}} \left(\tau^{-\frac{1}{1-\sigma}} A_H + A_F\right)$$

Using the same comparison as in eq. (9) it is clear that the firms of stage 2 prefer to locate in Home if they expect that firms of stage 1 are going to locate in Home. The same reasoning can be made for producers at stage 3 and this is true of intermediate producers of all further stages. Therefore, Foreign specializes in the homogenous good and exports it to Home. Looking at the
expenditure shares and the trade balance, I confirm therefore that I described a viable equilibrium if \( w = \tau^{-1} \). Consider now that \( w > \tau^{-1} \). Then, the firms producing intermediate goods would have even stronger incentives to relocate to Home. Hence \( w > \tau^{-1} \) is not feasible in equilibrium as the trade balance would be positive in favor of Home, as the homogenous good could not be traded. ■

**Proof of Proposition 2.** Consider the decision of all intermediate firms at any stage in the \( m^{th} \) sector. I check for the incentives of firms to deviate to a different location. Inspect first the firm in the intermediate part of the supply chain. All suppliers and all demand is located in Home. Comparing the right hand sides of eqs. (6) and (7)

\[
A_H > \left( (\tau^{2\gamma-1})^{-\frac{1}{1-\sigma}} \tau^{-\frac{\sigma}{1-\sigma}} A_H \right)^{13}
\]

For \( \gamma > \frac{1}{2} \) there is no incentive for a firm to deviate from their location in Home. I also need to ascertain that the final good producers have no incentive to change their location. With \( w = \tau^{-1} \) and the assumptions about demand I may write \( A_F = \frac{\tau^{-1}}{N_H(\Psi_{H,s})^{-\frac{1}{1-\sigma}}} (\tau^{\frac{\sigma}{1-\sigma}}) \), \( A_H = \frac{1}{N_H(\Psi_{H,s})^{-\frac{1}{1-\sigma}}} \) the expected profits in Home are higher when \( 1 + \tau^{-1} \geq (\tau^{2\gamma-1})^{-\frac{1}{1-\sigma}} \left( \tau^{-\frac{\sigma}{1-\sigma}} + \tau^{-1} \right) \). A condition that \( \gamma > \frac{1}{2} \) for \( \sigma \leq \frac{1}{2} \) and \( \gamma > \sigma \) for \( \sigma > \frac{1}{2} \) is sufficient. ■

**Proof of Proposition 3.** Suppose \( w = \tau^{-1} \), and that all demand for any supplier (or the final goods producer) is located in Home. He will want to locate in Foreign if \( A_H < (\tau^{2\gamma-1})^{-\frac{1}{1-\sigma}} (\tau^{-\frac{\sigma}{1-\sigma}} A_H) \) or \( \gamma < \frac{1-\sigma}{2} \). ■

**Proof of Proposition 4.** Suppose first that the equilibrium wage remains at \( w = \tau^{-1} \). The cost differential of production of the final good between Home and Foreign when all intermediate goods (except stage 0) are located in the same country is

\[
\frac{\Psi_{n,F}}{\Psi_{n,H}} = \tau^{(2\gamma^n-1)} \geq \tau^{-1}
\]  

which makes the cost in Foreign smaller compared to the one in Home if the number of stages, \( n \) is large enough. Therefore, Foreign could be the least costly producer of good \( m \) for the world markets if the intermediate sector was located in that country.
I need to ascertain now that indeed \( w = \tau^{-1} \) can remain the wage. Home is providing the whole supply of industries \( k \in [1, m - 1] \). Suppose that all intermediates from industry \( m \) are placed in Foreign. If the wage \( w \) would increase from \( w = \tau^{-1} \) then there can be no more trade in the agricultural good. Since Home is producing more varieties of goods than Foreign which is producing just one variety and all varieties have equal expenditure share, this would create an imbalance in trade in favor of Home. Hence \( w = \tau^{-1} \). In Home, the labor freed from producing the intermediates and the final good in industry \( m \) covers the fall in imports of the agricultural good. ■

**Proof of Proposition 5** If the final producer has incentives to move holding the wage rate constant (assuming that all suppliers are located in Home) it must be true that

\[
(A_H + \tau^{-\frac{1}{n^*}} A_F) < \tau^2 \left( \tau^{-\frac{1}{n^*}} A_H + A_F \right)
\]

This implies, for a lower tier intermediate producer that is supplying directly the final good producer, that \( \tau^{-\frac{1}{n^*}} A_F < \tau^2 A_F \) holds (since all demand is now for that producer located in Foreign). Hence, in such a case lower tier suppliers will have the incentive to choose Foreign as a location. Note that if \( \gamma < \frac{1}{2} \) they have these incentives regardless of the behavior of the Final good producers if \( w = \tau^{-1} \). ■

**Proof of Proposition 6.** First, I want to guarantee the same consumption of the good \( m \) in Home as before the move of the industry to foreign. The consumers in Home demand

\[
\sigma \frac{\alpha_m L}{\Psi_{n,H}}
\]

units of the good \( m \) where \( \Psi_{n,H} \) is the cost of the final good if it were produced (and the whole supply chain located) in Home. Denoting by \( \Psi_{n,F} \) the cost of the final good \( m \) produced with stages \( s = 1, \ldots n \) in Foreign, I need a subsidy

\[
S_H = \tau \frac{\Psi_{n,F}}{\sigma} \frac{\alpha_m L}{\Psi_{n,H}} - \frac{\Psi_{n,H}}{\sigma} \frac{\alpha_m L}{\Psi_{n,H}}
\]

(15)

to guarantee the same consumption to Home after the move which, after substituting for \( \Psi_{n,F} \) from (14)

\[
S_H = \left( \tau^2 \gamma - 1 \right) \alpha_m L
\]

(16)
After some tedious algebra, one can find from (11) and (12) the lump sum subsidy to stage 1 producers that will encourage them to locate in Foreign.

\[ S_F = (1 - \sigma) \sigma^{\frac{2}{1 - \sigma}} \left( \tau^{2(\gamma - 1)} \right) \left[ \tau^{-\frac{2}{1 - \sigma}} - \left( \tau^{2(\gamma - 1)} \right)^{-\frac{1}{1 - \sigma}} \right] \left( \gamma \right)^{n} \left( \frac{\alpha_m L \tau^{-1}}{\tau^{(2(\gamma - 1))}} + \tau \alpha_m L \right) \]  

(17)

The sum of the subsidies \( S_H + S_F \) will be financed by a lump sum tax \( T \) on consumption of the \( m \) good in Foreign. Let \( C_{F,S} \) be the consumption in Foreign after subsidies. I have

\[ \alpha_m L \tau^{-1} - T = \tau^{(2(\gamma - 1))} \Psi_{n,H} C_{F,S} \]  

(18)

where the left hand side is the expenditures on good \( m \) in Foreign after tax. I want to check whether the consumption after tax in Foreign is greater than before the policy intervention. I look at

\[ \frac{C_{F,S}}{C_F} = \frac{\alpha_m L \tau^{-1} - T}{\tau^{(2(\gamma - 1))} \Psi_{n,H}} \frac{\tau \Psi_{n,H}}{\alpha_m L \tau^{-1}} \]  

(19)

or

\[ \frac{C_{F,S}}{C_F} = \frac{\alpha_m L \tau^{-1} - T}{\tau^{(2(\gamma - 1))} \Psi_{n,H}} \frac{\tau}{\alpha_m L} \]  

(20)

After substitutions and manipulations, I can evaluate eqs. (20), take the limit as \( n \to \infty \) and observe that

\[ \lim_{n \to \infty} \frac{C_{F,S}}{C_F} > 0. \]

no matter what the \( \tau \) is.

A.2 Example of the (unique) inefficient equilibrium with perfectly competitive industries

I show here that the result in Proposition 1 is not due to monopolistic pricing. I shall find the conditions on \( \gamma \), the usage of the lower stage intermediates in production at each level, for a unique equilibrium where all industry locates in Home. Suppose as in the equilibrium exhibited in Proposition 1, \( w = \tau^{-1} \). Consider the first stage intermediate producer. The marginal cost in Home is \( MC_H (q) = 1 \) while in Foreign it is \( MC_F (q) = (\tau)^{2(\gamma - 1)} \). Each firm is a price taker but has to sink a cost \( f \) prior to entry (produce \( f \) units of the product). Suppose that the intermediate product
is sold only in the Foreign market but there is the iceberg cost $\tau$ that is applied (this will simplify greatly the algebra). Then, for a firm located in Home the costs of delivering a unit of the good are $C_H(q) = \tau q + f$ whereas in Foreign $C_F(q) = (q + f)(\tau)^{2\gamma-1}$. Suppose that maximum capacity $\bar{q} = 1$. For firms to produce after entry $p > (\tau)^{2\gamma-1}$. Then, the profits of firms locating in Foreign are $\Pi_F = p - (1 + f)(\tau)^{2\gamma-1}$ and of a firm locating in Home $\Pi_H = p - \tau - f$. Suppose that all firms would locate in Foreign so that $p = (1 + f)(\tau)^{2\gamma-1}$ and $\Pi_F = 0$. The difference in profits $\Pi_H - \Pi_F = (\tau)^{2\gamma-1} - \tau + f ((\tau)^{2\gamma-1} - 1)$. As long as $\gamma > \frac{1}{2}$ for any $\tau > 1$ one can find a fixed cost $f$ such that $\Pi_H - \Pi_F > 0$. Each firm then has an incentive to swap location to Home. Since the downstream firms know what incentives the upstream firms face, they know that they will want to move to Home as well as their costs are going to make their incentives look exactly the same. The division of industries between Home and Foreign will be identical as under the conditions of Proposition 1.

Further examples with perfect competition at each stage of production can be generated, for example with decreasing returns to scale and no capacity constraints.

A.3 Example of the inefficient equilibrium with symmetric countries

Suppose that two countries, Home and Foreign, are symmetric in the sense that Foreign produces their own stage 0 intermediate as well. The technology for the first stage production of intermediates is Leontieff and $Y = \min\{z_H, z_F\}$ where $z_H$ is the Home produced and $z_F$ is the Foreign produced 0-stage intermediate while upstream intermediate producers have a technology as in Section 2. Therefore, inputs from both countries at the initial stage are perfect complements. I shall verify that an equilibrium with $w = \tau^{-1}$ and the entire industry moving to Home is an equilibrium.

The costs of the first stage intermediate firm locating in Home are $C_{H,1}(q) = \psi_{H,0} + \tau \psi_{F,0} = 2$ whereas in Foreign they are $C_{F,1}(q) = \tau \psi_{H,0} + \psi_{F,0} = (\tau + \tau^{-1})$. The marginal cost differential between Home and Foreign then is $\frac{C_{F,1}(q)}{C_{H,1}(q)} = \frac{(\tau + \tau^{-1})}{2} > 1$. For the firms of the first stage not to have an incentive to move to Foreign (no matter what the final demand would be) with the logic of the proof of Proposition 1 I need $\frac{(\tau + \tau^{-1})}{2} > \tau^\sigma$. For $\tau$ high enough, for any $\sigma$, this is true. Then, for further stages of production it suffices that the condition $\gamma > \frac{1 + \sigma}{2}$ holds. With a number of stages of production high enough and $w = \tau^{-1}$ will zero out the trade balance (one can show that the usage of the initial intermediate will constitute a small part of the final cost), all stages of production of the manufacturing goods have the incentive to locate in Home, and the entire cost of production in
manufacturing is higher in the country with the higher wage. If $w = \tau^{-1}$, there would be a Pareto improvement if at least a part of the industries moved to Foreign. This equilibrium is not unique, as an equilibrium with $w = 1$ and a symmetric division of industries between the two countries exists as well.

**References**


Amsden A., Asia’s Next Giant: South Korea and late industrialization. Oxford University Press. 1989

Dixit A. and Grossman G. Trade and protection with multistage production. Review of Economic Studies 1982; 159; 583-594

Feenstra R. C., Integration of trade and disintegration of production in the global economy. Journal of Economic Perspectives 1998; 31-50


Rodrik D. Coordination failures and government policy: A model with applications to East Asia and Eastern Europe. Journal of International Economics 1996; 40; 1-22


Stigler G. The division of labor is limited by the extent of the market. Journal of Political Economy 1951; 59; 185-193

