Free Trade Area and Welfare: Is A Bigger Trade Diversion More Detrimental?

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Abstract

This paper analyzes how the welfare of a member country may be affected by the formation of a new free trade area (FTA). In particular, it examines whether a change in its welfare may be related to the volume of trade diverted from a non-member country to another member country. It has been long and widely believed that a bigger trade volume diverted from a non-member country to another member country is an indication that the welfare of country has been more deteriorated. This practice can be traced back to Viner’s work (Viner, 1950), and has been used in many empirical investigation and policy recommendations. This paper seriously and carefully examines the validity of this assumption. We show that this assumption is not necessarily true, especially if imperfect competition exists in trade, which partly rejects Krishna’s (1998) finding that a bigger trade volume diverted implies a higher likelihood of an FTA to be approved.

Keywords: Imperfect Competition, Preferential Trade Agreements, Trade Diversion

JEL: F10, F11, F12, F13, F15
1 Introduction

The welfare impacts of a new preferential trade agreement (PTA) such as a customs union (CU) or a free trade agreement/area (FTA) on the member countries and non-member countries has long been an interesting issue for economists and policy makers. In particular, they want to know how a country may be affected if it chooses to form a new FTA with other countries.\(^1\)

For some time, economists had held the position that anything can happen as a PTA represents a movement of a second-best equilibrium to another second-best equilibrium, while it is argued that the first-best position of the world is free trade by all countries.\(^2\) Viner, in his pioneering work (Viner, 1950), suggested an approach to identifying welfare-improving PTAs and welfare-deteriorating PTAs. He argued that a trade-creating PTA (one in which a member country imports more from a country where the cost of production is lower) is beneficial but that a trade-diverting PTA (one in which a member country switches its import from a country with a lower cost of production to a country with a higher cost of production) is detrimental.

Viner’s approach has been criticized. First, it had been argued that a trade diverting PTA may still be beneficial to a member country (Gehrels (1956-57), Lipsey (1957), Wonnacott (1996), and Panagariya (1999)).\(^3\) Second, it had been realized that Viner’s criteria for welfare improvement are difficult to test, as it could be costly to estimate the costs of production of different goods in different countries. Third, since Viner’s analysis is based on a partial equilibrium framework, it is not clear how the analysis can be extended to a multi-good economy. In particular, it is not clear how the welfare may change if one finds trade creation for some goods but trade diversion for some goods.

Because of these limitations, economists who tried to apply Viner’s approach to examine the impacts of a PTA chose to focus on the change in the

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\(^1\)Panagariya(1999) investigates the effects on member countries under four headings: (i) the welfare effects of a PTA based on Vinerian analysis, (ii) the implication of differences in transport costs, (iii) the implications of rules of origin, and (iv) nontraditional gains. In this paper, we only focus on the welfare effects on member countries.

\(^2\)By the Second-best Theory, two second-best positions *in general* cannot be ranked in terms of the welfare of the world or the welfare of a country.

\(^3\)Panagariya (1999) says that unions which are primarily trade diverting are harmful to member countries taken together, however, an individual member of the union can be still be beneficial by shifting in intra-union terms of trade in its favor.
trade volumes to get hints on how welfare may change. In the literature, it has been long and widely believed that a bigger trade volume diverted from a non-member country to another member country is an indication that the welfare of a member country has been deteriorated to a greater extent, and a number of empirical studies tend to interpret their results with little caution in making welfare conclusions, and policy recommendations. One of the theoretical arguments is the “Natural Trading Partner Hypothesis” which are strongly advocated by Wonnacott and Lutz (1989), Krugman (1991), and Summers (1991). It says that it would be more beneficial if an FTA is formed between countries with significant bilateral trade flows because it is less likely trade-diverging.

For example, Wonnacott and Lutz (1989) say that “trade creation is likely to be great, and trade diversion small, if the prospective members of an FTA are natural trading partners. Several points are relevant: Are the prospective members already major trading partners? If so the FTA will be reinforcing.”

Trefler (2004), Krishna (2003), and Panagariya (2000) show that trade creation should dominate trade diversion for sufficient welfare gains of a member country. Clausing (2001), which tests the Canada-United states Free Trade Agreement (CUSFTA), concludes that little evidence of trade diversion from non-member countries suggest an encouraging assessment of the CUSFTA.

Surprisingly, however, theoretical studies do not provide rigorous analysis

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4 Most common way to test the ex post welfare effect of PTAs is to use a gravity model by including dummies which capture a change in trade flows between members, and/or a member country and a non-member country. Eicher, et al. (2007) provide a nice summary of the literature. As different approaches, Balassa (1967) introduces income elasticities of demand for imports to estimate trade effects, and Winters (1984, 1985) uses systems of import demand equations.

5 For instance, in a gravity model, it seems that more significant (negative) coefficient for the dummy brings more concern to researchers. As a different view, Kowalczyk (2000) argues that if a good from a non-member country is complementary to a good exported from a member country, a non-member country may end up exporting more to a member country even after the FTA, and a member country may increase the volume of trade with a non-member country due to increased income after eliminating tariffs. Freund (2000) also shows that PTAs yield trade creation effects between a member country and a non-member country (Open Bloc Trade Creation effects) so that we may observe a positive coefficient in the dummy. In the empirical literature, Eicher, et al. (2007) find strong evidence for significant Open Bloc Trade Creation among PTAs in Europe and Asia.
to test the validity of the assumption of a negative relation between the changes in trade volumes and the changes in welfare levels. As an exception of theoretical analysis, Krishna (1998) provides an examination of the volume of trade diverted and the profits of local firms in a member country.\footnote{Krishna (1998) follows the approach in Grossman and Helpman (1994, 1995) and assumes that the government of a member country put a big emphasis on the profits of local firms.} Using a model of intra-industry trade with oligopoly, he argues that a larger trade volume diverted from non-member country to a member country represents a bigger increase in the profits of local firms, and thus makes a FTA more attractive and likely. Krishna’s result, however, seems to be incompatible with the general belief that a larger trade volume diverted implies a higher likelihood of a drop in welfare.

This paper examines how the volume of trade diverted from a non-member country affects change in welfare (or profits of local firms) of a member country, and thus, try to sort out the seemingly conflicting results about what a bigger trade diversion may mean to welfare. We provide a systematic analysis of the relation between changes in trade volume diverted and changes in welfare in order to determine whether an FTA may be supported by the government and local firms. We find out that such a relation depends crucially on the type of trade one is considering. If inter-industry trade with perfect competition is assumed, then a rise in the trade volume diverted will generally imply a drop in the change in the welfare that a member country will get. This seems to be compatible with Viner’s instinct about trade diversion and to confirm the belief in some recent studies about the welfare implications of volume of trade diverted. If, however, one examines intra-industry trade in the presence of oligopoly, the relation between trade volume diverted and profit change is more complicated, and the relation cannot positive or negative. In particular, we find that Krishna’s conclusion about a positive relation may not hold even though we follow his model. Most important difference of his finding and ours comes from consideration of the positive relationship between the level of competition and trade diversion in the oligopolistic market, which is ignored by Krishna (1998). This paper shows that more competitive market generating bigger trade diversion may reduce a local firm’s profit. We also find that the relation between trade diversion and change in welfare is not straightforward in this model.

The rest of this paper is organized as follows. In Section 2, we introduce
a framework that allows us to examine the relation between trade volume diverted and welfare change. To allow a comparison between our analysis and previous analysis, first we follow Krishna’s approach and use the profits of local firms as a criterion for the formation of an FTA and then investigate the relation between trade diverted. In Section 3, the focus of the analysis is on inter-industry trade. Section 4 concludes.

2 Intra-industry Trade: Trade Diversion and Welfare Change

We first consider on intra-industry trade. We adopt the model of Krishna (1998) and use it to examine the relation between trade diversion and welfare change.

2.1 The Model

Consider three countries labeled X, Y, and Z, and a homogeneous product. In country \( i \), \( i = X, Y, Z \), there are \( n_i \) firms producing the homogeneous product and competing in a Cournot fashion. Assume for simplicity that all firms face the same marginal cost of \( c \), which is independent of output level. The demand for the product by the consumers in country \( i \) is \( P_i = A_i - Q_i \), where \( P_i \) is the market price and \( Q_i \) is the demand.

Before the formation of any free trade area, each country imposes the same specific tariff rate, \( t \), on the product imported, independent of the country of origin. We assume that the demand is sufficiently large and the tariff sufficiently small so that there is intra-industry trade in the good among the countries. (Brander and Krugman, 1983). Denote the supply of the product by a firm in country \( i \) to the market in \( j \) by \( q^j_i \), \( i, j = X, Y, Z \). In equilibrium, \( Q_j = \sum_i n_i q^j_i \). The profit of a representative firm in country \( i \), \( \pi_i \), consists of the profit from market \( j \), \( \pi^j_i \), i.e., \( \pi_i = \sum_j \pi^j_i \), where

\[
\pi^j_i = q^j_i [A_j - Q_j - (c + t)] .
\] (1)

The firm chooses the outputs, \( q^j_i \), to maximize its profit, taking the tariff rate and the outputs of all other firms are given. The first-order conditions
(assuming intra-industry trade) are:

\[ A_x - q^i_x - \sum_j n_j q^j_x - c - t = 0 \]  
\[ A_y - q^i_y - \sum_j n_j q^j_y - c - t = 0 \]  
\[ A_z - q^i_z - \sum_j n_j q^j_z - c - t = 0. \]

Denote the total number of firms by \( n = n_x + n_y + n_z \). Solving the first-order conditions (2), we get the Nash equilibrium supply by a firm in country \( i \) to country \( j \):

\[ q^j_i = \frac{A_j - c + \sum_k n_k t^k_{ij}}{n + 1} - t^j_i, \]  

(3)

where \( t^j_i = t \) if \( i \neq j \) or \( t^j_i = 0 \) if \( i = j \). The summation in (3) is over X, Y, and Z. For example, condition (3) gives country X’s import from a firm in country Z:

\[ q^x_z = \frac{A_x - c - t(1 + n_x)}{n + 1}. \]  

(4)

From (1) and (3), we can get the profit received by a firm in country \( i \) from the market in country \( j \):

\[ \pi^j_i = [q^j_i]^2. \]  

(5)

Condition (5) shows a monotonic positive relation between the profit of a firm in country \( i \) from a market and the output to that market. Condition (5) also gives the total profit received by a firm in country \( i \):

\[ \pi_i = \sum_j \pi^j_i = \sum_j [q^j_i]^2. \]  

(6)

### 2.2 Formation of An FTA

Suppose now that countries X and Y form a free trade area (FTA), removing the tariff on the good imported from each other while maintaining the tariff on the good from Z. Note that this case can be analyzed by applying the above analysis by noting that the FTA is an integrated economy with the number of identical firms given by \( n_x + n_y \). Let us use a subscript “xy” before a variable to represent it in the presence of the FTA; for example, \( x_yq^z_x \) is the export of a firm in country Z to country X after the formation of the FTA.
Applying (4), the Nash equilibrium FTA-volume of country X’s import of
the good from a firm in country Z is

\[ x_y q_x^z = \frac{A_x - c - t(1 + n_x + n_y)}{n + 1}. \] (7)

Condition (6) can be applied to find the resulting profit of a firm in country
X after the formation of the FTA:

\[ x_y \pi_x = \Sigma_j [x_y q_j^x]^2. \] (8)

We now compare the pre-FTA equilibrium with the post-FTA equilib-
rium. In particular, we want to see whether the FTA will likely to be ac-
cepted by country X. We assume a political-economic approach similar to the
one in Grossman and Helpman (1994, 1995) and Krishna (1998), so that the
decision of whether an FTA is chosen is based solely on whether the profits
of local firms increase.\(^7\) We will examine the relations between the volume
of trade diverted and the local firms’ profits.

We say that for country X trade is diverted from country Z to country Y
if there is a drop in the volume of import from country Z, or if

\[ q_x^z > x_y q_x^z. \] \(^8\)

In this case, we define for country X the volume of trade diverted (VTD) from
country Z, \(D\), by

\[ D = D(n_x, n_y, n_z, t) = n_z [q_x^z - x_y q_x^z] = t \left( \frac{n_y n_z}{n + 1} \right). \] (9)

\(^7\)The objective function of each government in the model of Grossman and Helpman
(1994, 1995) is a weighted sum of campaign contributions from the lobbies and overall
welfare of voters while the decision of an FTA solely depends on the local firm’s profits in
the model of Krishna (1998). Kowalczyk and Davis (1996) test the political-economy model
with the United States data and find that import-competing lobbies were the strongest in
the sectors which were allowed the longest tariff phase out periods.

\(^8\)For the purpose of this paper, we do not examine whether the volume of import from
country Y will increase by the amount of volume of trade diverted.
The derivatives of the VTD from Z can be obtained from (9):

\[
\frac{\partial D}{\partial t} = \frac{n_y n_z}{n + 1} > 0 \quad (10a)
\]

\[
\frac{\partial D}{\partial n_x} = -\frac{t n_y n_z}{(n + 1)^2} < 0 \quad (10b)
\]

\[
\frac{\partial D}{\partial n_y} = \frac{tn_z(1 + n_x + n_z)}{(n + 1)^2} > 0 \quad (10c)
\]

\[
\frac{\partial D}{\partial n_z} = \frac{tn_y(1 + n_x + n_y)}{(n + 1)^2} > 0. \quad (10d)
\]

The intuition for the signs of the derivatives in (10) is simple. If the initial tariff rate is higher, it means a greater drop in the tariff on the good from country Y. Thus a higher initial tariff rate, or a larger number of firms in country Y or Z will result in a bigger impact on trade and thus a bigger volume of trade diverted. A larger number of firms in country X will have a smaller impact, however, because it will tend to diminish the impact of the FTA.

As explained, we use the change of a firm in country X to determine whether the FTA will be accepted by the country. Subtract condition (6) from (8), we get the change in the profit of a firm in X:

\[
\Pi_x = \Pi_x(A_x, A_y, n_x, n_y, n_z, t, c) \equiv xy\pi_x - \pi_x
\]

\[
= \left( (xyq_x^r)^2 + (xyq_y^r)^2 + (xyq_z^r)^2 \right) - \left( (q_x^r)^2 + (q_y^r)^2 + (q_z^r)^2 \right)
\]

\[
= \frac{t \Phi}{(n + 1)^2}. \quad (11)
\]

where \( \Phi = 2(A_y-c)(1+n_y+n_z)+t(n_z)^2-t(1+n_y)^2-2(A_x-c)n_y-tn_y^2-2tn_y n_z \). Condition (11) can be used to derive how these exogenous variables may affect the change in profit. First, we get the effects of a change in the size of the markets:

\[
\frac{\partial \Pi_x}{\partial A_x} = -\frac{2tn_y}{(n + 1)^2} < 0 \quad (12a)
\]

\[
\frac{\partial \Pi_x}{\partial A_y} = \frac{2tn_y(1 + n_y + n_z)}{(n + 1)^2} > 0. \quad (12b)
\]
Conditions (12) imply that a smaller local demand or a bigger demand in country Y will allow country X to gain more from the FTA. This result is not surprising as a bigger market in a member country will allow the local firms to export more while a bigger local market will attract more competition from the firms in a member country.

We then turn to the effects of the number of firms in each of the countries.

\[
\begin{align*}
\frac{\partial \Pi_x}{\partial n_x} & = -\frac{2t\Phi}{(n+1)^3} \quad (13a) \\
\frac{\partial \Pi_x}{\partial n_y} & = -\frac{2t[(A_x - A_y) + t(1 + 2n_y + n_z)]}{(n+1)^2} - \frac{2t\Phi}{(n+1)^3} \quad (13b) \\
\frac{\partial \Pi_x}{\partial n_z} & = \frac{2t[(A_y - c) + t(n_z - n_y)]}{(n+1)^2} - \frac{2t\Phi}{(n+1)^3}. \quad (13c)
\end{align*}
\]

Note that for the purpose of our analysis, we assume that \( \Pi_x > 0 \), i.e., country X is willing to form an FTA with country Y. This implies that \( \Phi > 0 \), and that by (13a) \( \frac{\partial \Pi_x}{\partial n_x} < 0 \). For (13b), if it is further assumed that

\[
A_x \geq A_y, \quad (14)
\]

then \( \frac{\partial \Pi_x}{\partial n_y} < 0 \). For (13c), \( \frac{\partial \Pi_x}{\partial n_z} < 0 \) if and only if

\[
n_z > n_y \frac{2(A_x - c) - (A_y - c) - t(1 + n_x)}{(A_y - c) - t(1 + n_x + 2n_y)} + (A_y - c)(n_x - 1) + t(1 + n_y)^2. \quad (15)
\]

We now examine the impacts of a change in \( t \) or \( c \).

\[
\begin{align*}
\frac{\partial \Pi_x}{\partial t} & = \frac{\Phi + t(n_z^2 - (1 + n_y)^2 - n_y^2 - n_y n_z)}{(n+1)^2} \quad (16a) \\
\frac{\partial \Pi_x}{\partial c} & = -\frac{2(t + tn_z)}{(n+1)^2} < 0. \quad (16b)
\end{align*}
\]

The effect of a higher initial tariff rate \( t \) on the change in the profit of a firm in country X is complicated: If the initial tariff rate is larger, both countries X and Y will experience a substantial drop in the tariff rate. For firms in country X, it is good because it will be easily to invade into the market in country Y, but it is also bad because it will be easier for firms in country
Y to invade into the local market. In general, the net effect is ambiguous. Condition (16a) can be rearranged to show that if

$$t > \frac{2(A_x - c)n_y - 2(A_y - c)(1 + n_y + n_z)}{2n_x^2 - 2(1 + n_y)^2 - 2n_y^2 - 3n_y n_z},$$

(17)

then \(\partial \Pi_x / \partial t > 0\). On the other hand, condition (16b) means that if the marginal cost of all firms is lower, the gain in the profit of each firm in country X will be higher.

The above results are summarized by the following proposition:

**Proposition 1** If the firms in country X would support an FTA with country Y, each of them will get a bigger profit improvement if

1. the size of country X’s market \(A_x\) is smaller; or
2. the size of country Y’s market \(A_y\) is larger; or
3. the number of firms in country X \(n_x\) is smaller; or
4. the number of firms in country Y \(n_y\) is smaller, if condition (14) is satisfied;
5. the number of firms in country Z \(n_z\) is smaller, if condition (15) is satisfied; or
6. the initial tariff rate \(t\) is higher, if condition (17) is satisfied; or
7. the common marginal cost \(c\) is lower.

### 2.3 Volume of Trade Diverted and Profit Change

Conditions (9) and (11) show that for country X the volume of trade diverted from country Z to country Y, \(D\), and the change in the profit of each firm in country X, \(\Pi_x\), are dependent on some exogenous variables. A change in some of exogenous variables could change \(D\) and \(\Pi_x\) simultaneously. We now examine how \(D\) and \(\Pi_x\) may change. The analysis in the previous section shows that the \(D-\Pi_x\) relationship depends on which exogenous variable is changing. A general theory can be provided as follows. Suppose that an
exogenous variable \( v \) changes and that it may affect both \( \Pi_x \) and \( D \). Thus the \( D-\Pi_x \) relations can be given by

\[
d\frac{\Pi_x}{dD} \bigg|_v = \frac{\partial \Pi_x}{\partial D} / \frac{\partial v}{\partial v}.
\]  

(18)

Condition (18) immediately gives the following lemma:

**Lemma 2** \( d\Pi_x/dD|_v < 0 \) if and only if \( \text{sign}(\partial \Pi_x/\partial v) \neq \text{sign}(\partial D/\partial v) \).

We now make use of the lemma to see how trade volume diverted and the change in firm profit may be related to each other. We can consider the following cases:

(a) **The Smaller-Trade-Diversion-the-Better Case**

We note that an increase in the number of firms in Y or Z will enlarge the trade volume diverted, \( D \), but will lower the profit improvement each firm in X will experience, under the conditions stated in Proposition 2. This means that

\[
d\frac{\Pi_x}{dD} \bigg|_v = \frac{\partial \Pi_x}{\partial D} / \frac{\partial v}{\partial v} < 0,
\]  

(19)

where \( v = n_y \) or \( n_z \). In these cases, a bigger volume of trade diverted from country Z to country Y is not good in terms of the profit of the firms in country X.

(b) **The Larger-Trade-Diversion-the-Better Case**

If there is a decrease in the number of firms in country X or a larger initial tariff rate, both the trade volume diverted and the profit improvement experienced by each firm in country X will go up. Thus we have

\[
d\frac{\Pi_x}{dD} \bigg|_u = \frac{\partial \Pi_x}{\partial u} / \frac{\partial D}{\partial u} > 0,
\]  

(20)

where \( u = n_x \) or \( t \). In these cases, a bigger volume of trade diverted from country Z to country Y represents a bigger profit improvement experienced by each firm in country X.

(c) **The Trade-Diversion-Does-Not-Matter Case**
If, however, there is a decrease in $A_x$ or $c$, or there is an increase in $A_y$; or there is an increase in $A_y$; then each firm in $X$ will experience a bigger profit improvement but the volume of trade diverted will not be affected. This means that there is no direct relation between trade diversion volume and the profit improvement of the firms in country $X$.

In the present case with intra-industry trade, we can identify three types of relations between trade volume diverted and profit improvement. In the case of inter-industry trade, we find only the smaller-trade-diversion-the-better case and the trade-diversion-does-not-matter case, but not the larger-trade-diversion-the-better case.

The direct relation between the trade diversion volume and profit improvement was probably first pointed out by Krishna (1998). He argued that a country facing a non-member country with more firms producing the product is more likely to form an FTA because of a bigger profit improvement for local firms. Our results are quite different from his even if we adopt his model in our analysis. First, with intra-industry trade, an increase in the trade diversion volume may indicate an increase, a decrease, or no change in the trade diversion increase, depending on the factor that causes a change in the trade diversion volume in the first place. Second, even if we consider only the case in which there is a change in the number of firms in the non-member country Z (Krishna (1998) considers only this case.), we note that the relations between the trade diversion volume and the profit improvement is in general ambiguous, and is negative if condition (15) is satisfied.

The followings explain why our findings are different from Krishana (1998).

The key factor of the trade diversion in Krishna (1998) is the number of firms of the non-member country. He points out that the larger trade volume diverted comes along with the bigger number of firms of the non-member country.

Krishna (1998) provides the condition where the FTA is accepted by the country $X$.

\[(A_x - c) < \frac{1}{2n_y}\{ (A_y - c)(2 + 2n_y + 2n_z) - 2t n_y n_z + t(n_z)^2 - t(n_y)^2 - t(1 + n_y)^2 \}\]

(21)

The condition above can be derived from equation (11). From (11) we see that the signs of both $\Phi$ and $\Pi_x$ are the same. Note also that the satisfactory condition for (21) is the same as that for $\Phi > 0$. We can
safely say that $\Phi > 0$ is the condition for the FTA to be concluded in this model. Krishna (1998) points out that $\Phi$ is increasing in $n_z$. He also figures that the volume of trade diverted increases in $n_z$. This is shown in equation (10d). Based on the positive relationship between $\Phi$ and $n_z$, he concludes that the bigger trade diversion volume brings a higher profit to local firms and it results in a higher chance for a member country to attain the FTA due to more supports from the interest groups through the lobbying process.

One can, however, easily notice that $n_z$ is in the denominator of equation (11). It implies that $\Pi_x$ may not increase in $n_z$ while $\Phi$ is always increasing in $n_z$.10 Thus, the bigger trade diversion with larger $n_z$ may not guarantee a positive improvement of a firm’s profit of X. This is because of the fact that gains from an FTA by each individual firm would be less as the number of firms in the economy including that of the rest of world increases. More competition in the market generating bigger trade diversion may reduce a local firm’s profit. It indicates that bigger number of firms in the rest of world does not always imply larger gains to a local firm with an FTA. We verify this in (13c).

Equation (13c) also gives us the critical point of $n_z$ where $\frac{\partial \Pi_x}{\partial n_z} = 0$:

\[
\left. n_z \mid \frac{\partial \Pi_x}{\partial n_z} = 0 \right\} n_z = \frac{n_y \{2(A_x - c) - (A_y - c) - t(1 + n_x)\} + (A_y - c)(n_x - 1) + t(1 + n_y)^2}{(A_y - c) - t(1 + n_x + 2n_y)}.
\]

(22)

Equation (22) implies that if $n_z$ is greater than the value of the right-hand side, the change in profits declines as the volume of trade diverted gets bigger.

The other fact which makes our main results different from Krishna (1998) is that the volume of trade diverted is determined by not only a factor from the rest of world, the number of firms in the third country, but also other factors, such as the number of firms in a partner country or even the number of firms in the member country. We see these in equations (10a), (10b), and (10c).

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9 This is easily verified by examining $\Phi$.

10 The number of firms from each of countries are in the denominator of (11). It is because each firm supplies less as the number of firms in the economy is bigger. Since each firm’s profit moves in the same direction in the quantities it sells, the individual firm’s profit may decrease in the number of firms of each country and total number of firms in the economy.
Variable changed \( (v) \) & \( \frac{\partial D}{\partial v} \) & \( \frac{\partial \Pi_x}{\partial v} \) & \( \frac{\partial W_x}{\partial D} \) \\
\hline
\( n_z \) & > 0 & > 0, or < 0 & > 0, or < 0 \\
\( n_x \) & < 0 & < 0 & > 0 \\
\( n_y \) & > 0 & < 0 & < 0 \\
\( t \) & > 0 & > 0, or < 0 & > 0, or < 0 \\
\( A_x \) & \cdot & < 0 & NA \\
\( A_y \) & \cdot & > 0 & NA \\
\( c \) & \cdot & < 0 & NA \\
\hline

Table 1: Intra-industry trade with perfect competition: Trade diversion and profit change

The summary of our findings of this section is shown in Table 1. We also examine how the volume of trade diverted is related to country X’s welfare, not profits alone. This would be of help to show the difference more clearly between results from the model with perfect competition and those from the model with imperfect competition. We find that the relationship is not straightforward and hard to determine without pre-conditions in most of cases. Appendix 1 shows calculations and results.

2.4 Numerical Experiment

To provide better insights on ambiguous relations between the volume of trade diverted and the change in welfare levels in the presence of imperfect competition, we simulate the economy with an FTA under the intra-industry trade model. We assign values to each of the fixed exogenous variables as follows: \( n_x = 2, n_y = 4, n_z = 2, A_x = 10,000, A_y = 15,000, t = 300, \) and \( c = 1,500. \)

Figures 1-4 illustrate the relations between the volume of trade diverted and the change in firm’s profits \( (D-\Pi_x) \) and the change in welfare levels \( (D-W_x) \) due to the change in one of exogenous variables, \( n_z, n_x, n_y, \) and \( t \) respectively. In each figure, section (a) depicts the relation between the volume of trade diverted and one of exogenous variables. Sections (b) and (c) show
how welfare change (or profits of a local firm) is related to one of exogenous variables and how it is related to the volume of trade diverted respectively. In sections (b) and (c), dashed lines represent change in the firm’s profit ($\Pi_x$) and solid lines represent change in welfare ($W_x$).

Figure 1, section (a) shows the positive relation between the volume of trade diverted and $n_z$. In Figure 1, section (b) we see that the firm’s profit change start to fall around the point when $n_z = 3$. Accordingly, in figure 1, section (c) the $D-\Pi_x$ curve is down-ward sloping after $n_z = 3$. It indicates that the bigger trade diversion with larger $n_z$ does not guarantee a positive improvement of the firm’s profits of a member country and thus, a large trade diversion does not always imply a higher likelihood of an FTA to be approved as argued by Krishna (1998) in his political-economy model. In this example, however, the welfare improvement is positively related to the trade diversion as shown in Figure 1 section (c).

We also observe that the signs of $D-\Pi_x$ and $D-W_x$ are the same when $n_z$ is small (positive) but different as $n_z$ and the volume of trade diverted increase. Thus, the difference between these two become larger as the volume of trade diverted rises. Note that the change in profits per se could be positive while the change in profits falls. After an FTA, the change in profits become negative after the volume of trade diverted reaches 1,300. More interestingly, if the volume of trade diverted is fairly small, the FTA is welfare-deteriorating ($W_x$ is negative) while it is welfare-enhancing if the trade diverted is greater than 165. This result is opposite to that from the perfect competition model in section 2.

Figure 2, section (a) shows that the volume of trade diverted and $n_x$ is negatively related. Because a larger number of firms from X brings less gain to a local firm, we see the upward-sloping $D-\Pi_x$ curve in section (b), which says that the bigger trade diversion is better. Figure 2, section (c) shows that the change in welfare is positive and rises when the trade diversion is small, but it becomes negative and decreases as the volume of trade diverted reaches at some point.

In figure 3, sections (a) and (b) we see that the volume of trade diverted increases in $n_y$, and as the profit change and welfare improvement become smaller as $n_y$ rises respectively. Accordingly, the profit change and welfare improvement decline as the trade diversion increases as shown in section (c).

Figure 4, section (a) illustrates the positive linear relation between the volume of trade diverted and the tariff rate. In sections (b) and (c) we observe that the profit changes increase initially and then decrease as $t$ and
trade diversion rise respectively while the welfare change is always increasing. Also notice that the welfare change is always positive in all ranges in this case.

These figures clearly show that the $D-\Pi_x$ and the $D-W_x$ relations are not straightforward. The sign of the profit and welfare improvement depends on the exogenous variable which changes the level of the trade diversion and the conditions of other variables. We also find that the direction of their movement is not necessarily opposite.

3 Inter-industry Trade: The Vinerian Analysis

We now examine the implication of trade diversion on welfare using Viner’s approach. Consider a model of three countries labelled X, Y, and Z. Countries X and Y are forming a free trade area (FTA) so that goods can flow freely between them while both of them keep their initial tariffs on the goods imported from Z. Country Z is the non-member country, and its tariffs on the goods from countries X and Y are not affected by the new FTA. For simplicity, goods can flow between any two countries without transport costs.

Consider a competitive industry of a homogeneous product in country X. Denote its inverse import demand function by $P_x = A_x - M_x$, where $P_x$ is the import price (including any possible tariff) and $M_x$ is the import level. The

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11The question of the choice of the external tariff on imports from non-member countries has been raised and studied separately both in theoretical and empirical literature. In the political economic literature, Panagariya and Findlay (1996) show that the external tariff rises along with the magnitude of lobbying in the importing sectors after an FTA while Cadot, et al. (1999) and Ornelas (2006) argue that the external tariff declines or will be removed completely. With data of over fifty developing countries Foroutan (1998) shows that PTAs affect tariffs positively against third country while using US and European Union data respectively Limao (2005) and Karacaoglu and Limao (2005) find the negative effects of PTAs on external tariff. For simplicity, we assume that external tariff of member countries remains constant throughout this paper.

12The literature shows the controversial views on the role of transport costs in the formation of PTAs. Krugman (1991) and Frankel, et al. (1995) suggest that transport costs are critical factor to choose a PTA partner. However, Bhagwati(1993), Bhagwati and Panagariya (1996) and Panagariya (1997a) argue that transport costs are no different than any other costs.

13In this paper, we allow the possibility of positive production of the good in country
constant \( A > 0 \) is a measure of the size of the market in X for importers. The import demand function is illustrated by curve AB in Figure 5. The country can import the good from countries Y and Z at constant marginal costs equal to \( C_Y \) and \( C_Z \), respectively, \( C_Z < C_Y \).

Initially, country X imposes a specific tariff of \( t_x \) on the good imported from Y and Z. Thus the total cost of importing one unit of the good from countries Y and Z are \( C_Y + t_x \) and \( C_Z + t_x \), respectively, as shown in Figure 5, assuming that \( C_Y < C_Z + t \). Thus country X chooses to import the good from Z, and the domestic price in X is \( P_x = C_Z + t_x \), and the corresponding import level is \( M_x^e \).

Now, countries X and Y form an FTA. This means that the tariff on the good from Y is no longer subject to the tariff, but the good from Z is. Because \( C_Y < C_Z + t \), country X will choose to import the good from Y, and the new import level is \( M_y^e > M_x^e \). This is trade diversion. Viner seemed to regard this type of trade as detrimental to country X because it represents a switch of the import from a place where the marginal cost is lower to a place where the marginal cost is higher. However, as pointed out by Gehrels (1956, 1957) and Lipsey (1957), trade diversion may or may not be harmful to country X. What has not been analyzed is how the change in the welfare of X is related to the volume of trade diverted. This is what we now turn to.

The volume of trade diverted, \( D \), from Z to Y for country X is \( M_x^e \), which is the initial import level from Z, i.e., \( D = M_x^e \). From country X’s import demand function,

\[
D = D(A_x, C_z, t) = A_x - C_z - t. \tag{23}
\]

The dependence of the diverted trade on the exogenous variables can be obtained easily by differentiating both sides of (23) to give:

\[
dD = dA_x - dC_z - dt. \tag{24}
\]

The welfare of this industry of country X can be represented by the sum of consumers’ surplus, producers’ surplus, and the tariff revenue. As the market price drops from \( C_Z + t \) to \( C_Y \), the corresponding change in net surplus is equal to area \((a + c)\) in Figure 5.\(^{14}\) At the same time, the government of X loses the tariff revenue that it initially received. The tariff revenue is equal to area X.

\(^{14}\)Net surplus is the consumers’ surplus less producers’ surplus. The change in net surplus is equal to the change in consumers’ surplus less the change in producers’ surplus.
As a result, the change in welfare of country X, $W_x$, is equal to area $(c - b)$ in Figure 5. The area $c$ represents the regular gains from trade: the welfare improvement from an increase in trade, and the area $b$ is the net loss in tariff revenue. Using Figure 5, the welfare improvement is equal to

$$W_x = W_x(A_x, C_y, C_z, t)$$

$$= (C_z + t - C_y)(A_x - C_z - t) + \frac{1}{2}(C_z + t - C_y)^2 - t(A_x - C_z - t)$$

$$= \frac{1}{2}(C_y^2 - C_z^2) + \frac{1}{2}t^2 - A_x(C_y - C_z).$$

(25)

The sign of the change in X’s welfare, $W_x$, in equation (25) is ambiguous. It has been argued that there are cases in which it is positive, meaning that country X may still gain from a trade-diversion FTA.

The dependence of the change in welfare can be given by differentiating both sides of (25) to give:

$$dW_x = (A_x - C_z)dC_z - (A_x - C_z)dC_y - (C_y - C_z)dA_x + tdt_x.$$  

(26)

What we want to examine is how the change in welfare, $W$, is related the volume of trade diverted, $D$. Equations (23) and (25) show that both of them depend on several exogenous variables. We can derive the relation between them by allowing one of the exogenous variables to change.

(a) A Change in $C_z$

When only $C_z$ can change while all other exogenous variables are kept constant, both $D$ and $W_x$ can be inverted to find how a change in $W_x$ can be dependent on a change in $D$. More explicitly, assuming that $dA_x = dC_y = dt = 0$, equations (24) and (26) can be combined to give

$$dC_z = -dD = dW_x/(A_x - C_z),$$

which, after rearranging terms, gives

$$\left.\frac{dW_x}{dD}\right|_{C_z} = -(A_x - C_z) < 0.$$  

(27)

Equation (27) implies that a rise in the volume of trade diverted as caused by a change in $C_z$ hurts the welfare improvement that country X can experience
from the new FTA. In other words, a bigger volume of trade diverted will diminish country X’s willingness to form the FTA.

The relation between \( W_x \) and \( D \) is shown by curve DW in Figure 6. The slope of the curve is equal to \(-(A_x - C_z)\). The vertical and horizontal intercepts can be obtained by making use of equation (23) and (25): When \( D = 0 \), the corresponding welfare change is \( W_{x0} = (A_x - C_y)^2/2 > 0 \) or when \( W_x = 0 \), the corresponding trade volume diverted is \( D_0 = -t_x + \sqrt{(A_x - C_y)^2 + t_x^2} > 0 \). In the present case, an FTA with a small diverted trade volume benefits country X.

**Lemma 3** A bigger volume of trade diverted from a more efficient non-member country to a member country lowers the welfare improvement that a country is able to get from a new FTA.

The above lemma can be explained easily by using Figure 5. When \( C_z \) decreases, the initial volume of import from country Z is larger, meaning there is a bigger trade volume diverted. At the same time, the decrease in \( C_z \) diminishes the size of area \( c \) (because of a small increase in trade) but enlarges the net loss in tariff revenue (because of a large pre-FTA trade tariff). We call this case the smaller-trade-diversion-the-better case.

(b) A Change in \( t \)

When \( A_x \), \( C_z \), and \( C_y \) are held constant, equations (23) and (25) can be combined together, after eliminating \( t \), so that \( W_x \) can be expressed as a function of \( D \). The derivative of this function can be obtained by combining equations (24) and (26) together after setting \( dA_x = dC_z = dC_y = 0 \). We have

\[
\frac{dW_x}{dD} \bigg|_{t_x} = -t < 0,
\]

which means that a rise in the diverted trade volume because of a smaller initial tariff diminishes the welfare improvement of country X. The relation between the diverted trade volume and the welfare improvement can be illustrated by a curve similar to curve WD in Figure 6. Thus we have

**Lemma 4** A bigger volume of trade diverted from a non-member country to a member country because of a small initial tariff lowers the welfare improvement that a country is able to get from a new FTA.

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15See Appendix 2.
This lemma can be explained in terms of Figure 5. If the initial tariff, $t$, is smaller, the initial import level from country Z will be bigger, meaning a bigger trade volume diverted. At the same time, area $c$ is smaller but area $b$ is bigger. Thus the welfare improvement country X gets from the new FTA is smaller.\textsuperscript{16} Note that this is another the-smaller-trade-diversion-the-better case.

(c) A Change in $A_x$

Suppose now that we treat $A_x$ as a parameter while keeping $C_z$, $C_y$, and $t$ constant. Like we did earlier, equations (23) and (25) can be combined together to eliminate $A_x$, thus giving the relation between trade diversion and welfare improvement. More specifically, equation (24) gives $dD = dA_x$, which can be substituted into (26) to give

$$\frac{dW_x}{dD} \bigg|_{A_x} = -(C_y - C_z) < 0,$$

which implies that a rise in trade diversion because of a bigger size of the domestic market diminishes the welfare improvement of the FTA.\textsuperscript{17} This relation can also be illustrated graphically by a curve similar to curve WD in Figure 6. Thus we have:

**Lemma 5** A bigger volume of trade diverted from a non-member country to a member country because of a bigger size of the local market lowers the welfare improvement that a country is able to get from a new FTA.

This result can also be explained in terms of Figure 5. An increase in $A_x$ is represented by an upward shift of curve AB. Since the marginal costs and the tariff rate do not change, area $c$ remains the same as before, but area $b$ becomes bigger. Thus the welfare improvement drops. This is another example of the-smaller-trade-diversion-the-better case.

(d) A Change in $C_y$

\textsuperscript{16}Pangariya (1997b,1999), however, argues that the loss from the PTA will be larger for a higher tariff member country.

\textsuperscript{17}This finding supports Panagariya (1995, 1996, 1997b) whose argument is based on the presumption is that a member country loses by giving the preferences to another member country while it receives benefits from the partner.
Variable changed $(v)$  $D$  $W_x$  $\frac{\partial W_x}{\partial D}|_v$

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Table 2: Inter-industry trade with perfect competition: Trade diversion and welfare change

Note that the trade volume diverted is independent of $C_y$. From (26),

$$\frac{\partial W_x}{\partial C_y} = -(A_x - C_z) < 0.$$  

Thus we have,

**Lemma 6** A decrease in a member country’s marginal cost will not affect the volume of trade diversion but will improve the welfare of country X as a new FTA is formed.

This lemma can also be explained in terms of Figure 5. A rise in $C_y$ will not affect the volume of trade diversion, $M_x^x$, but will lower the size of area $c$ but increase that of area $b$. In this case, which be called the trade-diversion-does-not-matter case, there is no direct relation between the trade diversion volume and the welfare improvement of country X.

The above results are summarized below:

**Proposition 7** An increase in the trade diversion volume due to a more efficient non-member country, a lower pre-FTA tariff, or a bigger local market represents a lower welfare improvement of a country from a new FTA. Trading with a more efficient member country will not affect the volume of trade diversion but will increase the welfare improvement of the country.

The summary of the results of this section is shown in Table 1.
4 Concluding Remarks

In this paper, we examine the relation between the trade volume diverted and the change in welfare (or profits of local firms) using two different types of trade models: inter-industry trade in the presence of perfect competition and intra-industry trade with oligopoly. We argue that the relation depends on the type of trade considered. We show that if trade is of the inter-industry trade type, a rise in the trade volume diverted in general is related to a smaller change in welfare. This is because a bigger trade diversion generally results in a larger drop in consumer’s gains and/or a higher degree of tariff loss in the presence of perfect competition. However, if intra-industry trade with oligopoly is considered, then the relation between trade volume diverted and profit (welfare) change is not so straightforward. It can be either positive, negative or ambiguous depending on economic variables which affect the volume of trade diverted and the level of other variables. If imperfect competition exists, it is difficult to define how the change in profits, consumer’s gains and tariff loss are affected by the trade diversion. These results show that Krishna’s (1998) finding that larger trade diversion yield higher profits may not hold.

An interesting result is that if the trade volume diverted is small, the welfare change could be positive in the inter-industry trade. This means that a trade diversion can be welfare improving if the volume of trade diverted is small. If the intra-industry trade model with imperfect competition is taken into account, a member country’s welfare may drop in the presence of small trade diversion, but improve as the trade diversion increases. It presents a contrast to common belief that small trade diversion can be ignored if the trade-creating effect is significant. We also find that in the existence of imperfect competition profit and welfare improvement move to the same direction in some cases. This implies that a welfare-detioriating FTA commonly identified by a bigger trade volume diverted is not necessarily more likely to be approved as previously found in the political economic literature.\(^\text{18}\)

Figure 1. Volume of Trade Diverted and Change in Welfare due to $n_z$

Notes 1. $n_x=2$, $n_y=4$, $t=300$, $A_x=10,000$, $A_y=15,000$, $c=1,500$

2. $\Pi_x$  $\Pi_y$  $\Pi_z$
Figure 2. Volume of Trade Diverted and Change in Welfare due to $n_x$
Figure 3. Volume of Trade Diverted and Change in Welfare due to $n_y$

Notes 1. $n_1=1$, $n_2=2$, $f=300$, $A_1=10,000$, $A_2=15,000$, $c=1,500$

2. $\Pi_1$ (red dashed)  $\Pi_2$ (blue solid)  $W_1$ (red dashed)  $W_2$ (blue solid)
Figure 4. Volume of Trade Diverted and Change in Welfare due to $t$
Figure 5. Equilibrium Prices
Figure 6. The Smaller-Trade-Diversion—the Better Case
Appendix

Appendix 1.

As the government welfare function is composed of the firms profit, consumers’ surplus, and the tariff revenue from the imports, the welfare improvement with an FTA is equal to

\[
W_x = W_x(A_x, A_y, n_x, n_y, c, t) \equiv_{xy} w_x - w_x
\]

\[
\begin{align*}
W_x &= n_x[\Pi_x(A_x, A_y, n_x, n_y, c, t)] \\
&+ \left[ \left( \int_{Q}^{Q_y} p_{xy}Q_x(v)dv - p_{xy}Q_xQ_x \right) - \left( \int_{0}^{Q_x} p_{xy}Q_x(v)dv - p_{Q_x}Q_x \right) \right] \\
&+ t[n_z(xyq_x^z - q_y_q_x^z) - n_y(q_y^y)] \\
&= n_x \left[ (xyq_x^x)^2 + (xyq_y^y)^2 + (xyq_z^z)^2 - (q_x^x)^2 + (q_y^y)^2 + (q_z^z)^2 \right] \\
&+ \frac{1}{2} \left[ n_x(xyq_x^x) + n_y(xyq_y^y) + n_z(xyq_z^z) \right]^2 - (n_x(q_x^x) + n_y(q_y^y) + n_z(q_z^z))^2 \\
&+ t[n_z(xyq_x^z - q_y_q_x^z) - n_y(q_y^y)] \\
&= \frac{tn_x \Phi}{(n + 1)^2} + \frac{tn_z \{2n(A_x - c) - t(n_y + 2n_z)\}}{2(n + 1)^2} \\
&- \frac{tn_y \{(A_x - c) - t(n_x + 1 - n_z)\}}{n + 1}.
\end{align*}
\]

where \( w_x \) is the welfare of country X. In equation (31), the term with the first square bracket represents the change in all firms profits in country X, the second square bracket represents the improvement of consumers’ surplus, and the last term is the change in the tariff revenue on the imports from countries Y and Z.

Equation (31) can be used to derive how the exogenous variables may affect the change in welfare. First, we get the effects of a change in the size of markets:

\[
\begin{align*}
\frac{\partial W_x}{\partial A_x} &= \frac{t(n_x^2 - n_y^2 - 3n_xn_y + n_xn_z - n_y)}{(n + 1)^2} \quad (32a) \\
\frac{\partial W_x}{\partial A_y} &= \frac{2t(n_xn_y(1 + n_y + n_z))}{(n + 1)^2} > 0. \quad (32b)
\end{align*}
\]

28
The sign of (32a) is not determined and depends on the number of firms in each of countries. From condition (12a) we know that as the local demand increases the change in the profit of a firm in X declines. We also know that the bigger initial imports from a member country due to higher demand in local market implies greater loss of the tariff revenue with an FTA. However, the consumers’ gain increases in their demand. Thus, overall welfare impact due to an increase in the size of market is ambiguous.

Since $A_y$ does not affect consumers’ surplus and the government tariff revenue of a country X, the sign of (32b) is the same as that of (12b). For (32a), $\partial W_x/\partial A_x > 0$ if

$$n_z > \frac{n_y^2 + 3n_x n_y + n_y}{n_x + n_z}.$$  \hspace{1cm} (33)

We then turn to the effects of the number of firms in each of the countries.

---

19The country X’s initial volume of trade with Y is $(\frac{A_x - c}{n+1})t(n_y + 1)$. In this model, if a member country forms an FTA, the tariff revenue of a member country always falls: First, it does not collect the tariffs from a partner country. Second, the tariff revenue from the non-member countries decreases along with the drop of the volume of trade with them.
The relations of welfare change and the number of firms in each country are not very clear even though change in welfare levels is most likely to decline with an increase in the number of firms in partner country from (34b). As the number of firms from each country increases, the quantity sold in the X’s market goes up. It brings larger gains to consumers. However, the firms in country X lose more if there exist more foreign firms competing in the local markets while gains less in the other countries. After the formation of an FTA with Y, the impact (net loss) of profits due to change in the number of foreign firms would be larger in case of a change in number of firms in Y. In addition, the tariff revenue declines the most with an increase in the number of firms in Y after an FTA.
We now examine the impacts of a change in $t$ or $c$.

$$\frac{\partial W_x}{\partial t} = \frac{n_x\{\Phi + t(n_z^2 - (1 + n_y)^2 - n_y^2 - n_y n_z)\}}{(n + 1)^2}$$

$$+ \frac{n_z\{n(A_x - c) - t(n_y + 2n_z)\}}{2(n + 1)^2}$$

$$- \frac{\{n_y(A_x - c) - 2t(n_x + 1 - n_z)\}}{n + 1}.$$  \hfill (35a)

$$\frac{\partial W_x}{\partial c} = \frac{t(n_y(n_x + n_y + 1) - 2n_x - 3n_x n_z - n_z)}{(n + 1)^2}. \quad \hfill (35b)$$

Tariff elimination on the imports from $Y$ makes consumers better off, and if the initial tariff rate is higher, this effect increases while the tariff revenue drops more. The impacts on profits are ambiguous as shown in (16a). It results in an undetermined sign of (35a). The marginal cost effect on welfare is also not clear. The firm’s profits are negatively related to change in marginal costs as shown in (16a), and consumers’ surplus is expected to increase less if the marginal cost is higher, but the government loses less tariff revenue due to the smaller initial imports from outside.

In summary, the signs of $\partial W_x/\partial v$ where $v$ is an exogeneous variable, are not straightforward and hard to determine without pre-conditions in most of cases. Accordingly, the $D-W_x$ relations which can be given by $\frac{dW_x}{dD}|_v = \frac{\partial W_x/\partial v}{\partial D/\partial v}$ are not straightforward.

**Appendix 2.**

From figure 5 the welfare improvement is

$$W = \frac{1}{2}\{M^g(A_x - C_y) - M^e(A_x - C_z - t_x)\} - M^e(t_x). \quad (36)$$

In figure 5, $M^g = A_x - C_y$, and $M^e = A_x - C_z - t_x$. Thus, $M^g = M^e + C_z + t_x - C_y$.

It gives us

$$W = \frac{1}{2}\{(M^e + C_z + t_x - C_y)(A_x - C_y) - M^e(A_x - C_z - t_x)\} - M^e(t_x). \quad (37)$$

Now, we need to express $W$ as a function of $D$ and other exogenous variables, but not $C_z$ in this case. In section 1, we know that $M^e = D$. From
equation (23) $C_x = A_x - t_x - D$. These give

$$W = \frac{1}{2} \{(D + A_x - C_y)(A_x - C_y) - D^2\} - D(t_x).$$  \hspace{1cm} (38)

Plugging $D = 0$ into equation (38) gives us

$$W_{x0} = (A_x - C_y)^2/2 > 0.$$  \hspace{1cm} (39)

We also let $W$ be equal to 0 in order to get $D_0$ in equation (38). It gives

$$D_0 = \sqrt{(A_x - C_y)^2 + t_x^2} > 0.$$  \hspace{1cm} (40)

Equations (39) and (40) imply positive vertical and horizontal intercepts respectively.
References


