Heterogeneous multinationals and comparative advantage¹:

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Abstract

This paper presents a model of international trade that features heterogeneous and potentially multinational firms, relative endowment differences across countries, and consumer taste for variety. Firms are potentially multinational in that they can choose to supply the export market by means of either conventional exports or by establishing a production base abroad. Heterogeneity of firms is introduced via the Melitz model.

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1 INTRODUCTION

1.1 Diversity among firms

That firms vary greatly in size is well-known. However, the extent to which firms vary in size is astonishing. In most geographical areas or economic sectors, typically a relatively small number of large and very large firms are complemented by what is often an astoundingly large number of small and medium-sized firms. These differences between firms are compounded when we consider trade behaviour. Empirical evidence highlights a series of stylised facts about firms and trade. The first fact of note is that within a given country, the proportion of firms that export is very small. Eaton, et. al. using detailed firm-level data for France, show that only 17.6% of French firms export. The comparative figure for the US is 14.7%. Even within sectors that are relatively export intensive, the proportion of firms that export is relatively small in comparison with the total number of firms in that sector; for example, in the chemicals sector, 55% of French firms and 30% of US firms are involved in export activities, yet this is one of the most export intensive sectors in both countries.

Tybout (2003) demonstrates that exporting firms tend to be large relative to the size of non-exporting firms, and exporting firms typically enjoy higher levels of productivity than non-exporting firms. The corollary of high productivity is low marginal costs, and hence the stylised fact that informs the model developed in this paper: it is firms with low marginal costs that emerge as exporters.

1.2 The Melitz model

A relatively small proportion of firms export. Even fewer become multinationals. Yet sales from foreign affiliates are three times as large as trade flows. The diversity among firms detailed in Chapter 1 is not properly described by a representative firm; but the inadequacy of the representative firm as a description of the profile of industry had, until recently, not lead to a manageable way of dealing with this flaw. Motivated by the diversity in trade behaviour found among real world firms that appeared to be in stark contrast to the representative firm implicit in, for example, Krugman’s 1980 model, Mark Melitz sought to develop a model that embodied firm heterogeneity. The result is the innovative paper Melitz (2003), which is perhaps the most exciting development in the theory of international trade so far this decade.

The key contribution of Melitz is the integration of Hopenhayn’s (1992) mechanism to deal with heterogeneity in firm-level productivity to the theory of international trade. Combining firm heterogeneity with fixed market entry costs (or beachhead costs as in Baldwin (1988)) that occur in addition to the

The result is a model that allows us to describe and explain firm-level differences in trade behaviour. This new approach has been termed the HMCFMEC model, which stands for ‘heterogeneous marginal cost, fixed market entry cost’. [Baldwin, 2005] To enhance verbal elegance, throughout this paper we refer to the HMCFMEC model as the Melitz model.

Melitz shows how exposure to trade leads firms with relatively high productivity to export, while some less productive firms continue to produce only for the domestic market, and the least productive firms are forced to exit the industry. Further increases in the industry’s exposure to trade create inter-firm reallocations towards more productive firms. These phenomena had been empirically documented but were not explained by general equilibrium trade models that used a representative firm. The paper also shows how the aggregate industry productivity growth generated by the reallocations contributes to a welfare gain, thus highlighting a benefit from trade that has not been examined theoretically before.

1.3 Papers using the Melitz model

This subsection details two papers that have built on the Melitz model and that are important to the genealogy of this paper.

Helpman, Melitz and Yeaple (2004), (hereafter HMY.2004), one of the first papers to use the Melitz model and one of the most frequently cited papers of 2004, uses heterogeneous firms to re-examine the scale-versus-proximity trade-off. HMY.2004 incorporate the Meltiz model into a one-dimensional version of MV.2000. They build a multi-country, multi-sector, single factor model with a homogeneous good as numeraire. There are fixed costs of entry, a labour-per-unit-output coefficient drawn from a distribution (later specified as Pareto), and fixed overhead costs for entering the domestic market, the export market, or for engaging in FDI. After entry, producers engage in monopolistic competition, and traded goods are subject to iceberg trade costs in the usual way. Consumer preferences are standard CES. Endowments of the single factor, labour, is allowed to vary across countries, but differences in endowments are taken to be small enough that the homogeneous good is produced in all countries and hence the common wage rate equals one.

Helpman, et. al. (2004) find that only the most productive firms engage in foreign activity, and that of those firms that serve foreign markets, only the most
productive engage in FDI. The dispersion of productivity levels across firms is identified as a determinant of international trade. The theoretical model also suggests that firm-level heterogeneity is an important determinant of relative exports and FDI flows, and the data supports the prediction that more heterogeneity leads to significantly more FDI sales relative to export sales.

Bernard, Reading and Schott (2004) (hereafter BRS.2004) explore heterogeneous firms within the Helpman-Krugman framework. This paper is one of very few that combine the microeconomic modelling of firms with a general equilibrium analysis of trade. By combining factor endowment differences across countries, factor intensity differences across industries, and heterogeneous firms within industries, the model is able to explain simultaneously inter-industry trade, intra-industry trade, and selection into export markets. One important finding is that the existence of heterogeneous firms leads to a magnification of comparative advantage.

1.4 The current model

In this chapter I build a simple simulated general equilibrium model incorporating both heterogeneous firms and the scale-versus-proximity trade-off. The result is a model that allows us to describe and explain patterns of trade and the investment patterns of heterogeneous firms within a Helpman-Krugman framework. The model is built by integrating heterogeneous firms into the simulated general equilibrium model of international trade with multinationals developed in Markusen and Venables (2000), which itself added trade costs and introduced multinational firms to the general equilibrium models of international trade synthesised in Helpman and Krugman (1985). Heterogeneity in firms’ marginal costs is introduced using a Melitz-style mechanism with two simplifications: following HMY, the model omits the dynamic dimension of Melitz (retained by Baldwin (2005) and BRS) in favour of an instantaneous equilibrium; and following Baldwin and Forslid (2004) and Baldwin (2005) we take the distribution of firms’ marginal costs as given.

The remainder of this chapter is structured as follows. Section 4.2 develops the general equilibrium model. Section 4.3 explains the calibration of the model. Section 4.4 gives details of the numerical simulation and presents the results of the base case. Section 4.5 investigates comparative statics. Section 4.6 considers welfare. Section 4.7 concludes.
2 THE MODEL

2.1 Basic set-up

There are two countries, Home, \( H \) and Foreign, \( F \), each endowed with a mix of two factors, skilled labour, \( S \), and unskilled labour, \( L \). Country subscripts to the factors denote the endowment. The salaries and wage rates for skilled and unskilled labour are denoted \( z \) and \( w \) respectively and are determined endogenously in accordance with their marginal products. A country’s income, all of which is spent and hence is denoted \( E \) for expenditure, is given (for the home country) by

\[
E_h = z_h S_h + w_h L_h
\]

There are two sectors, \( Y \) and \( X \). The \( Y \) sector produces a homogeneous product, whilst the \( X \) sector produces differentiated products. The homogeneous good, produced using a CES combination of the two factors, is numeraire.

2.2 Production of the differentiated good

Differentiated goods, produced using only skilled labour, are subject to increasing returns via various fixed costs, and exports are subject to iceberg trade costs. To enter the industry, a firm bears a fixed cost of \( F_B \), expressed in units of skilled labour, which can be thought of as buying the blueprint for a unique, new variety. This is the standard Dixit-Stiglitz fixed cost of developing a new variety. Upon paying \( F_B \), the firm is assigned a unit labour requirement \( a_j \), which is a firm-specific marginal cost of producing one unit of that variety. The coefficient \( a_j \) is drawn randomly from a distribution \( G[a] \). Following Melitz, the distribution is assumed to be a Pareto distribution, namely,

\[
G[a] = \left( \frac{a}{a_0} \right)^k, \quad 0 \leq a \leq a_0
\]

where \( a_0 \) is the scale parameter and \( k \) the shape parameter.\(^1\) We choose units of \( X \) goods such that \( a_0 = 1 \).

After learning the marginal cost of production associated with the variety, the firm decides whether or not to actually produce anything. If the firm decides to enter production, it encounters a fixed cost \( F_D \), which can be understood as fixed cost associated with serving the domestic market. Think of this as building

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\(^1\)The Pareto distribution, that implies lots of little ones and a small number of big ones, is eminently suited to describing the distribution of firms in the market. Consequently, it is the Pareto distribution that has been adopted as standard within the trade literature on heterogeneous firms. See, for example, Baldwin and Forslid (2004), Helpman, Melitz and Yeaple (2004) and Baldwin (2005). Pareto developed this distribution to describe the allocation of wealth among individuals. The Pareto distribution describes a large number of real-world situations, such as the size of human settlements (few cities, many hamlets/villages) and the frequencies of words in English-language texts (lots of little words, fewer longer ones).
a factory and setting up a distribution network. The firm also decides whether to serve the foreign market. If the firm decides to serve the foreign market, it chooses one of two channels: either by export of domestically produced goods; or by local sales from affiliates (FDI). This choice is driven by the scale-versus-proximity trade-off: relative to exports, FDI saves transport costs, but requires higher fixed costs as certain facilities are replicated abroad. If the firm decides to serve the foreign market via exports, a fixed cost of $F_X$ is incurred; this is a beachhead cost that includes the cost of establishing a distribution network abroad. Alternatively, if the firm decides to engage in FDI abroad, it incurs a fixed cost of $F_M$, with $M$ standing for a Markusen-type multinational. After entry, firms engage in monopolistic competition.

All four of the fixed costs described are expressed in units of skilled labour, that must be engaged at the prevailing wage rate of $z$. Along the lines of HMY (2004), we assume that the relationship between the three types of beachhead costs are such that

$$F_D < \frac{F_X}{\phi} < \left( \frac{zh}{zf} \right)^{-\sigma} F_M. \quad (1)$$

These two inequalities are conditions on the ratios between the beachhead costs that are derived from manipulating equations 14, 15 and 17 from section 4.2.7 below. The assumption restricts the analysis to the empirically relevant focal case, in that the first inequality assumes that export costs are high enough to prevent some firms from exporting; the second inequality assumes that FDI costs are high enough to prevent some firms from becoming multinational.

The logic above gives rise to four types of firms. Extending the notation from Baldwin (2005) there are firms that serve only their own national market, ($D$ - types, short for domestic firms); firms that sell locally and export to the foreign market ($X$ - types, short for exporters; firms that sell locally and invest abroad ($M$ - types, short for multinationals); and firms that buy the blueprint but exit the industry without producing ($N$ - types, short for non-producers. A firm decides which type of firm to be after learning its marginal cost. If the $a$ is too high, a firm will be an $N$ - type. If the cost is lower than a certain level, which we define as $a_D$, a firm will produce for the domestic market. If the cost is low enough, a firm will also decide to enter the export market; we define this level of marginal cost as $a_X$. If a firm is very lucky, the marginal cost will be low enough to permit engaging in FDI abroad; we define this level of marginal cost as $a_M$.

### 2.3 Consumer behaviour

The utility of the representative consumer can be expressed:

$$U = X^\beta Y^{(1-\beta)} \quad (2)$$
where:
\[ X = \left[ \int_{i=1}^{n} \left( x_i^{1-\frac{1}{\sigma}} \right)^{\frac{1}{1-\frac{1}{\sigma}}} \right] di, \sigma > 1, \] (3)
the subscript \( i \) represents an individual variety and \( n \) is the mass of varieties.

Consumer optimisation yields demand for a particular variety \( j \) to be
\[ c_j = \frac{p_j^{-\sigma} \beta E}{\int p_i^{-\sigma} di} \] (4)
where \( \beta E \) is the fraction of income spent on the differentiated good. This well-know result is the building block for the six demand equations that form a core segment of the numerical model.

### 2.4 Producer behaviour
Consider prices charged by home firms. Assuming profit-maximising producers and a significantly large number of varieties, we obtain the usual pricing policy of \( p_j(1 - \frac{1}{\sigma}) = m_j \), where \( m_j \) equals marginal cost of the variety under consideration. The marginal cost for a particular home variety is \( a_j \) units of labour at the prevailing skilled wage rate \( z_h \), hence \( p_j = \left( \frac{\sigma}{\sigma-1} \right) a_j z_h \). The skilled wage rates, \( z_h \) and \( z_f \), are endogenously determined and so will vary with the endowments. The unit input coefficient \( a_j \) is different for each firm, hence each variety will have a unique marginal cost of \( z_h a_j \). In addition, varieties that are exported will be subject to an additional mark-up of \( t\% \) to account for trade costs. Hence the price of varieties produced at home for home consumption will be \( p_{hh} = \left( \frac{\sigma}{\sigma-1} \right) a_j z_h \); while the price of home varieties exported abroad for foreign consumption will be \( p_{hf} = (1 + t) \left( \frac{\sigma}{\sigma-1} \right) a_j z_h \); and the price of varieties produced abroad via foreign direct investment for foreign consumption will be \( p_{hf}^M = \left( \frac{\sigma}{\sigma-1} \right) a_j z_f \). In the last three expressions the first and second subscripts represent the country of origin of the firm and the country of sale respectively; and the superscripts \( X \) and \( M \) represent the mode of supply to the foreign market.

There are three analogous expressions for the prices charged by foreign firms.

### 2.5 The "Price index" equations
Consider the denominator from equation 4 above: note that the varieties sold in each market can be grouped into three types, according to the national origin of the firm and the place of production. First there are home varieties (produced at home) that retail at a price of \( \left( \frac{\sigma}{\sigma-1} \right) a_j z_h \); all home producers who have drawn
an \( a_j \) lower than \( a_D \) will be in this group. Then there are imported varieties that have a consumer price of \( (1 + t) \left( \frac{a}{\sigma - 1} \right) a_j z_f \); this category comprises foreign varieties with a unit input coefficient lower than \( a_X \) but greater than \( a_M \). Finally there are varieties that ensue from foreign plants established at home, which have a consumer price of \( (1 + t) \left( \frac{a}{\sigma - 1} \right) a_j z_h \); this expression for price is identical to that for home varieties, but only foreign varieties with a unit input coefficient lower than \( a_M \) will be in this category. Hence we can rewrite the denominator of (4) for consumption at home as:

\[
Z p_i \frac{1}{1 - \sigma} \frac{1}{d_i} = \int p_i^{1-\sigma} di = \left[ \int_0^{a_H} \left( \frac{a}{\sigma - 1} a z_h \right)^{1-\sigma} n_h dG(a) + \int_0^{a_M} \left( \frac{a}{\sigma - 1} a z_h \right)^{1-\sigma} n_f dG(a) \right] \nonumber
\]

\[ + \int_{a_M}^{a_f} \phi \left( \frac{a}{\sigma - 1} a z_f \right)^{1-\sigma} n_f dG(a) \]  

(5)

where \( n_h \) and \( n_f \) are the potential number of firms in Home and Foreign respectively, and \( \phi \), a symbol for the freeness of trade, is defined as \((1 + t)^{(1-\sigma)}\) as in Baldwin (2005).

As noted above, the "a"s come from a distribution \( G[a] = \left( \frac{a}{a_0} \right)^k \). Differentiating \( G[a] \) with respect to \( a \) and cross-multiplying we obtain \( dG[a] = \frac{ka}{a_0^k} da \). Thus we can rewrite 5 as:

\[
\int p_i^{1-\sigma} di = n_h \int_0^{a_H} \left( \frac{a}{\sigma - 1} z_h \right)^{1-\sigma} \frac{k}{a_0^k} a^{k-\sigma} da + n_f \int_{a_M}^{a_f} \phi \left( \frac{a}{\sigma - 1} z_f \right)^{1-\sigma} \frac{k}{a_0^k} a^{k-\sigma} da \nonumber
\]

\[ + n_f \int_{a_M}^{a_f} \left( \frac{a}{\sigma - 1} z_f \right)^{1-\sigma} \frac{k}{a_0^k} a^{k-\sigma} da \]  

(6)

Solving the integrals in 6 with respect to the truncated distribution and factorising the price-minus-marginal-cost markups, we obtain the following expression for the denominator of 4:

\[
\int p_i^{1-\sigma} di = \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \Delta_h \]

(7)

where

\[
\Delta_h = \frac{k}{(1 - \sigma + k)} \left[ z_h^{(1-\sigma)} \left( \frac{n_h(a_H)^{(1-\sigma+k)}}{(a_H)^k} + \frac{n_f(a_M)^{(1-\sigma+k)}}{(a_M)^k} \right) \right. \nonumber
\]

\[ + \Phi z_f^{(1-\sigma)} n_f \left( \frac{(a_f)^{(1-\sigma+k)}}{(a_f)^k} - \frac{(a_M)^{(1-\sigma+k)}}{(a_M)^k} \right) \]  

(8)

We refer to \( \Delta_h \) and \( \Delta_f \) as the "price index" equations because they are related to the price index in the following manner:

\[
\left( \frac{\sigma - 1}{\sigma} P_h \right)^{(1-\sigma)} = \Delta_h \]

We refer to \( \Delta_h \) and \( \Delta_f \) as the "price index" equations because they are related to the price index in the following manner:
where \( P_h \) is the price index for the home country.\(^2\) There is a corresponding expression for \( \Delta_f \). The equations \( \Delta_h \) and \( \Delta_f \) are used directly in the general equilibrium model.

### 2.6 The market-clearing inequalities

Using \( \Delta_h \) and \( \Delta_f \) the expression for consumption of a single variety is:

\[
c_j = \frac{p_j^{-\sigma} \beta E}{\Delta_h \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma}}
\]

where \( p_j \) takes on any of the three expressions for price identified above, depending on the national origin of the firm and the place of production. Thus we have three expressions for the consumption of individual varieties. Home consumption of a typical home variety will be:

\[
c_{jh} = \left( \frac{\sigma - 1}{\sigma} \right) \frac{(z_h a_j)^{-\sigma}}{\Delta_h} \beta E_h
\]

whilst consumption of imported foreign varieties is:

\[
c_{jf} = \left( \frac{\sigma - 1}{\sigma} \right) \frac{(1 + t)^{-\sigma} (z_f a_j)^{-\sigma}}{\Delta_h} \beta E_h
\]

and consumption of foreign varieties made in affiliates located in Home will be:

\[
c_{jf} = \left( \frac{\sigma - 1}{\sigma} \right) \frac{(z_h a_j)^{-\sigma}}{\Delta_h} \beta E_h
\]

There are three corresponding expressions for foreign consumption of an individual variety from each of the three groups of goods available on the foreign market. Hence we have identified consumption of all individual varieties of the myriad of \( X \) varieties as falling into one of six types: \( c_{jh} \), \( c_{jh}^{f(X)} \), \( c_{jh}^{f(M)} \), \( c_{jf}^{f(X)} \), \( c_{jf}^{f(M)} \) and \( c_{jf}^{f_f} \).

For the allocation of factors in the general equilibrium analysis, it is necessary to ascertain the total amount produced of \( X \)-varieties. This is most readily done by considering the total amount produced within each of the six types noted above. Consider, for example, home consumption of home varieties produced at home. Consumption of a single variety is as expressed by 9. Total production by all home firms for home consumption can be found by integrating

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\(^2\)See Fujita, et. al. p47.
with respect to \( dG [a] \). We can therefore express total output produced at home by home firms for home sales as:

\[
X_{hh} = \int_0^{a_h} \frac{\sigma - 1}{a} (z_h a)^{-\sigma} \frac{\beta E_h}{\Delta_h} n_h dG [a] \\
= n_h \int_0^{a_h} \frac{\sigma - 1}{a} (z_h)^{-\sigma} \frac{\beta E_h}{\Delta_h} k \frac{a^k}{a_0^k} (a^{k-\sigma}) da
\]

Solving this integral for the truncated distribution with respect to \( a \) we have:

\[
X_{hh} = n_h \frac{\sigma - 1}{\sigma} (z_h)^{-\sigma} \frac{\beta E_h}{\Delta_h} k \frac{a^k}{a_0^k} (a^{k-\sigma})
\] (10)

This is the total output produced at home by home firms for home sales.

In the same way we can work out the total output produced at home by foreign implants for local consumption:

\[
X_{fh} = \int_0^{a_f} \frac{\sigma - 1}{a} (z_h a)^{-\sigma} \frac{\beta E_h}{\Delta_h} n_f dG [a]
\]

which yields:

\[
X_{fh} = n_f \frac{\sigma - 1}{\sigma} (z_h)^{-\sigma} \frac{\beta E_h}{\Delta_h} k \frac{a^k}{a_0^k} (a^{k-\sigma})
\] (11)

Output produced at home for export abroad will be the amount consumed abroad, plus the amount that melts en route. A single firm that sells \( c_j \) abroad will need to produce a total amount of \((1 + t) c_j \). Total production for export will be the integral over all firms producing for the export market:

\[
X_{hf} = \int_0^{a_h} (1 + t)^{1-\sigma} \frac{\sigma - 1}{\sigma} (z_h a_j)^{-\sigma} \frac{\beta E_f}{\Delta_f} n_h dG [a] \\
= n_h \int_0^{a_h} (1 + t)^{1-\sigma} \frac{\sigma - 1}{\sigma} (z_h)^{-\sigma} \frac{\beta E_f}{\Delta_f} k \frac{a^k}{a_0^k} (a^{k-\sigma}) da
\]

The solution to this integral yields:

\[
X_{hf} = n_h (1 + t)^{1-\sigma} (z_h)^{-\sigma} \frac{\sigma - 1}{\sigma} \frac{\beta E_f}{\Delta_f} k \frac{(a^k M - (a^k D)^{k-\sigma})}{(a^k D)^{k}}
\] (12)
Combining 10, 11, 12 and the counterparts of each for the firms that are active abroad, we have six expressions that define the production of $X$ sector goods. The restriction on the relative sizes of fixed costs ensures that, for symmetric countries, if a firm produces at all, it will produce for the domestic market, and only more productive firms will become exporters, and only the most productive will engage in FDI. This completes the six market clearing equations required for the general equilibrium model.

Inspection of the market clearing conditions suggest a condition on the relationship between the parameters: it is immediately apparent that $k$, the dispersity of heterogeneity, must be greater than $\sigma$, the elasticity of substitution. In case $k < \sigma$, this gives rise to negative outputs. The condition $k > \sigma$ is a new condition, and occurs in addition to the "regularity condition" of $k > \sigma+1$, identified in HMY and required to ensure that both the distribution of productivity draws and the distribution of firm sales have finite variances.

### 2.7 The cut-off conditions

We now determine the cut-off coefficients, $a^h_D$, $a^h_X$, and $a^h_M$. Having drawn a random "$a$" from the distribution $G(a)$, a firm will decide whether to pay the fixed cost required to enter the domestic market. What is the maximum unit input coefficient a firm can draw and still find it worthwhile to enter the domestic market? The answer is, a firm will find it worthwhile employing $F_D$ units of skilled labour, the fixed cost to enter the domestic market, providing the operating profits cover this fixed cost. The firm will pay the beachhead cost $F_D$ provided operating profits ($\pi$) from sales in the domestic market are greater than $F_D$. Operating profits are defined as revenues minus variable costs, and this must be greater than or equal to the beachhead cost, hence for the home firm drawing $a_j$:

$$\pi_{hh} = c(p-m) = (a_j z_h)^{1-\sigma} \frac{\beta E_h}{\sigma \Delta_h} \geq F_D z_h$$

(13)

where $\pi_{hh}^D$ is the operating profits from sales in the home market for a single variety.

For the firm at the margin, with the highest unit input coefficient that makes it worth entering production, operating profits will exactly equate with fixed costs. In this case, equation 13 holds with equality and creates an implicit definition of $a_D$ for home firms as

$$(z_h)^{-\sigma} (a_D^h)^{1-\sigma} \frac{\beta E_h}{\sigma \Delta_h} = F_D$$

(14)

This expression is no less than fixed costs equals revenue over sigma, a familiar Dixit-Stiglitz result; but in this case, only for the firm with the marginal
cost of \( a_D \) will this expression hold with equality. All other firms with an \( a_j \) lower than \( a_D \) enjoy pure profits, such that \( \pi_{hh}^D > F_D z_h \).

Now consider a home firm’s decision to produce for the foreign market. Suppose a firm has drawn its marginal cost \( a_j \) from the distribution \( G(a) \), and finds itself with an "\( a \)" lower than the critical value of \( a_D \). What will be the condition under which the firm will want to produce for export? A firm will opt to produce for export providing pure profits from exporting are greater than the pure profits from being a domestic producer. This will occur if revenues from exports minus the cost of producing for export are greater than the beachhead costs associated with exporting. Formally,

\[
\pi_{hf}^X = \phi (a_j z_h)^{1-\sigma} \frac{\beta E_f}{\sigma \Delta_f} \geq z_h F_X.
\]

For the firm at the margin, this expression holds with equality and gives the following implicit definition of \( a_X^h \):

\[
\phi (z_h)^{-\sigma} (a_X^h)^{1-\sigma} \frac{\beta E_f}{\sigma \Delta_f} = F_X
\]

(15)

There is an analogous expression for foreign firms, which creates an implicit definition for \( a_X^f \), the critical value of \( a \) below which foreign firms will export to home.

Finally, consider a home-based firm’s decision to invest abroad. Investment abroad will occur if the pure profits from investment are greater than the pure profits from exporting. This can be expressed:

\[
\left[ (a_j z_f)^{1-\sigma} - \phi (a_j z_h)^{1-\sigma} \right] \frac{\beta E_f}{\sigma \Delta_f} \geq z_f F_M - z_h F_X
\]

(16)

Consider the firm whose "\( a \)" is just low enough to be indifferent between exporting and investing abroad. At the margin, equation 16 holds with equality, and provides a definition of the cut-off condition \( a_M^h \), above which a firm will not invest abroad:

\[
\left[ (z_f)^{1-\sigma} - \phi (z_h)^{1-\sigma} \right] a_M^h \frac{\beta E_f}{\sigma \Delta_f} = z_f F_M - z_h F_X
\]

(17)

Equations 14, 15, 17 and their counterparts for foreign firms define six cut-off conditions that collectively determine whether a firm will produce only for the domestic market or will serve a foreign market; and if the firm is to serve the foreign market, whether that market will be served by traditional exports or by sales from an affiliated plant via FDI. This completes the cut-off conditions module of the general equilibrium model.
2.8 Free entry

I now formulate the free entry condition. This allows us to determine the mass of firms that will purchase blueprints, namely $n_h$ in home and $n_f$ in foreign. Common to all Dixit-Stiglitz models, this model assumes zero-expected-profit-from-innovation. The idea is that, so long as profits can be earned, entry will continue, until the cost of entry exactly equals the expected benefit of entry. The cost of entry is clear - it will be $z_h F_B$, since $F_B$ units of skilled labour are needed to create a blueprint, employed at the prevailing wage rate of $z_h$. The expected benefit of entry is the total pure profits from each type of sale divided by $n_h$, which is the potential number of home firms earning pure profits. This is equivalent to the expected benefit of being a domestic producer conditional on the probability of being a domestic producer; plus the expected benefit of being an exporter conditional on the probability of being an exporter; plus the expected benefit of being a multinational conditional on the probability of being a multinational; plus the expected benefit of being a non-starter, which is zero.

For potential entrants in home, we can write:

$$z_h F_B = \frac{\Pi_{hh}^D + \Pi_{hf}^X + \Pi_{hf}^M}{n_h}$$

(18)

where $\Pi_{hh}^D$, $\Pi_{hf}^X$ and $\Pi_{hf}^M$ refer to pure profits earned by home firms on domestic, export and FDI sales respectively. Recall pure profits are defined as revenues minus all costs. Cross-multiplying equation 18 demonstrates that free entry works in such a way that the total sum of pure profits in the economy equates with the total spending of fixed entry costs, or blueprints.\(^3\) Equation 18 can be elaborated by assessing the pure profits from each type of sale:

$$n_h z_h F_B = \int_0^{a_h} (\pi_{hh}^D) n_h dG(a) - n_h^h z_h F_D + \int_0^{a_X} (\pi_{hf}^X) n_h dG(a) - n_h^X z_h F_X$$

$$+ \int_0^{a_M} (\pi_{hf}^M) n_h dG(a) - n_h^M z_f F_M$$

(19)

where $\pi_{hh}^D$, $\pi_{hf}^X$ and $\pi_{hf}^M$ refer to operating profits on domestic, export and FDI sales respectively. Recall operating profits are defined as revenues minus operating costs. Substituting the relevant expressions for operating profits into equation 19 and collecting similar terms yields:

\(^{3}\)It helps to envisage the zero-profit condition as an invisible tax mechanism, in which an invisible government costlessly taxes all firms to the full value of their pure profits. All tax revenue is then invested, again with zero transaction cost, into producing blueprints. In this model free entry works in such a way that the total sum of pure profits in the economy equates with the total spending on fixed entry costs.
\[ n_h z_h F_B = n_h \int_0^{a_h^b} \left( (a_j z_h)^{1-\sigma} \frac{\beta E_h}{\sigma \Delta_h} \right) dG(a) \]

\[ + n_h \int_{a_h^b}^{a_h^x} \left( \phi \left( (a_j z_h)^{1-\sigma} \right) \frac{\beta E_f}{\sigma \Delta_f} \right) dG(a) \]

\[ + n_h \int_0^{a_h^m} \left( (a_j z_f)^{1-\sigma} \frac{\beta E_f}{\sigma \Delta_f} \right) dG(a) \]

\[- (n_D^h z_h F_D + n_X^h z_h F_X + n_M^h z_f F_M) \]

Solving the integrals, factorising for \( n_h \) and rearranging yields:

\[ n_h = \frac{A}{B} \]

where

\[ A = n_D^h z_h F_D + n_X^h z_h F_X + n_M^h z_f F_M \]

and

\[ B = \frac{\beta k}{\sigma (1 - \sigma + k)} \left[ \frac{z_h^{1-\sigma} E_h (a_D^h)^{1-\sigma+k}}{\Delta_h} + \frac{z_f^{1-\sigma} E_f (a_M^h)^{1-\sigma+k}}{\Delta_f} \right] - z_h F_B \]

There is an analogous expression defining free entry in foreign.

### 2.9 Number of firms

Now we determine the number of firms of each types. These turn out to be expressed as ratios of \( n_h \), the mass of would-be firms. For firms that produce for the domestic market, this is:

\[ n_D^h = n_h G(a_D^h) = n_h \left( \frac{a_D^h}{a_0} \right)^k = n_h \left( a_D^h \right)^k \]

Note that this in not the number of firms that only produce for the domestic market, but also includes exporters and investors that also sell locally. Similarly, the number of home firms that export will be:

\[ n_X^h = n_h \left( a_X^h \right)^k \]

and firms that engage in FDI will be:

\[ n_M^h = n_h \left( a_M^h \right)^k \]

There are three analogous expressions for the numbers of foreign firms.
2.10 Factor market clearing conditions

To complete the general equilibrium model, it remains only to specify the factor market clearing conditions. Recall that as the $X$ sector uses only skilled labour, all the unskilled labour is used in production of $Y$. Factor demands for production of the $Y$ good are obtained in the usual manner using Shephard’s Lemma. Let $L_i$ be the endowment of unskilled labour in country $i$; the market clearing conditions for unskilled labour are:

$$L_i = Y_i w_i^{1-\gamma} \left( z_i^{(1-\gamma)} + w_i^{(1-\gamma)} \right)^{\frac{\gamma}{1-\gamma}} 2^{\frac{1}{1-\gamma}}; i = h, f. \quad (20)$$

This equation is familiar from Chapter 2. A similar expression describes the factor demand for skilled labour used in producing $Y$ in country $i$:

$$S_{yi} = Y_i z_i^{-\gamma} \left( z_i^{(1-\gamma)} + w_i^{(1-\gamma)} \right)^{\frac{\gamma}{1-\gamma}} 2^{\frac{1}{1-\gamma}}; i = h, f. \quad (21)$$

Before writing down the market clearing conditions for skilled labour, we need to specify where each of the four types of fixed costs are incurred, and how many of each type are needed. Recall each type of fixed cost details a requirement for units of skilled labour. The cost of the blueprint, $F_B$, is incurred in the country of origin of the firm; this is multiplied by the mass of (potential) firms, $n_h$, to obtain total allocation of skilled labour to the production of blueprints. The fixed cost of setting up in the domestic market, $F_D$, is incurred in the domestic market by the number of firms that produce for the domestic market, $n_{Dh}$. The beachhead cost of exporting, $F_X$ is also assumed to be incurred in the country of origin of the firm, by $n_{Xh}$ firms. The cost of setting up production in the foreign market, $F_M$, assumed to be incurred abroad; conversely, home labour is required for mass $n_{Mh}$ of foreign multinationals that establish production bases in home. Thus the total home skilled labour employed producing overheads is

$$n_h F_B + n_{Dh} F_D + n_{Xh} F_X + n_{Mh} F_M \quad (22)$$

Finally, we calculate the labour used in production of the $X$ good. For an individual producer, this is the unit input coefficient (the “$a$”) times the quantity produced, as defined for example in equation 9 above. Integrating over producers gives the production labour for that group of sales. Note that home resources are used to produce goods for home consumption made both by home firms and by foreign affiliates, and goods for export to foreign. The total home skilled labour used in the production of the $X$ good will be
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Notes: Endowments are 50 units of labour and 150 units of capital in each country.
Parameters are: \( \sigma = 2 \), \( \beta = 0.5 \), \( \gamma = 3 \), \( \theta = 0.5 \), \( B = 1 \), \( D = 1 \), \( F = 1 \) and \( M = 2 \).
At the centre, 25 firms buy blueprints; 18.2 enter production for the domestic market, 5.4 also export, and 0.7 also become multinationals.
Rows are market clearing conditions, columns are production and consumption activities.
\[
\int_0^{a_h} (z_h)^{-\sigma} \frac{\beta E_h}{\Delta_h} \left( \frac{\sigma - 1}{\sigma} \right) (a)^{1-\sigma} n_h dG(a) \\
\int_0^{a_M} (z_h)^{-\sigma} \frac{\beta E_h}{\Delta_h} \left( \frac{\sigma - 1}{\sigma} \right) (a)^{1-\sigma} n_f dG(a) \\
\int_0^{a_X} \phi (z_h)^{-\sigma} \frac{\beta E_f}{\Delta_f} \left( \frac{\sigma - 1}{\sigma} \right) (a)^{1-\sigma} n_k dG(a)
\]  

(23)

Combining 21, 22 and the solution to 23 determines the allocation of skilled labour in Home:

\[
S_h = Y_h z_h^{-\gamma} \left( z_h^{(1-\gamma)} + w_h^{(1-\gamma)} \right) \frac{z_h^{\gamma}}{2 \pi^{\gamma}} \\
+ n_A^h F_B + n_D^h F_D + n_X^h F_X + n_M^f F_M \\
+ \frac{(\sigma - 1) k (z_h)^{-\sigma} \beta}{(k - \sigma + 1) \sigma} \left[ \frac{E_h}{\Delta_h} \left( \frac{a_h^n (a_h^n)^{(k-\sigma+1)}}{(a_B^n)^k} + \frac{n_f (a_M^n)^{(k-\sigma+1)}}{(a_D^n)^k} \right) \\
+ \phi E_f \Delta_f n_k \frac{(a_X^n)^{(k-\sigma+1)} - (a_M^n)^{(k-\sigma+1)}}{(a_B^n)^k} \right]
\]  

(24)

There is a similar expression allocating skilled labour in Foreign.

To summarise the X sector of model, there are two "price index" equations associated with the intermediate variables \( \Delta_h \) and \( \Delta_f \); six output equations associated with the six output levels; six equations defining the cut-off levels of the marginal cost; six equations describing the mass of each type of producer; and two free-entry conditions associated with the potential mass of firms in each country. In addition to the X sector equations, there are two income equations; two equations defining the price of \( Y \), one for each of the two countries; a market clearing condition for \( Y \); and the market clearing condition for unskilled labour. The model is completed with, and the interaction between the two sectors defined by, the factor market clearing conditions for skilled labour, which is associated with the price of skilled labour.

The numerical model is thus solving thirty-one equations in thirty-one unknowns. The thirty-one equations are presented in full in Appendix A.

3 CALIBRATION OF THE MODEL

Table 4.1 summarises the calibration of the model at the centre of the Edgeworth Box.

The initial calibration of the model is to the centre of the world Edgeworth box, where countries are identical and a symmetric solution results. Country
subscripts are therefore superfluous for this subsection. In calibrating the model, I revert to Markusen’s dimensions of the Edgeworth Box: a world total of 100 units of unskilled labour and 300 units of skilled labour. It turns out that the smaller dimensions are easier for the solver to work with without changing the starting values every time we experiment with other parameter values.\(^4\) One of the beauties of these dimensions is that, when the consumer expenditure is divided equally between the two sectors (in which case \(\beta = 0.5\)), the arithmetic associated with the \(Y\) sector is very easy to work out. The ratio of endowments of skilled and unskilled labour ensures that, when \(\beta = 0.5\), the amount of skilled labour engaged producing the \(Y\) good will be equal to that of unskilled labour. This is a feature of the CES production function for the \(Y\) sector; the implication of equal amounts of the two factors being used to produce \(Y\) is that the price of the two factors will be equal to each other. Furthermore, providing the endowments are in the ratio 3:1 and \(\beta = 0.5\), the prices of the two factors is equal to the price of the \(Y\) good, which is set at 1.\(^5\)

Factor prices of unity make the income equation simple: it is the sum of the units of the two factors. When each country has 150 units of skilled labour and 50 units of unskilled labour, income will be 200 per country. As half of income is spent on \(Y\) and the price of \(Y\) is 1, production of \(Y\) will be 100 in each country. This leaves 100 units of income to be spent on \(X\) sector goods, to be produced with the remaining 100 units of skilled labour. These features of the model remain unchanged unless \(\beta\), the dimensions of the Edgeworth Box, or the set price of the \(Y\) good is changed.

This constancy of the \(Y\) sector turns out to be an extremely useful feature when modelling the \(X\) sector: providing the dimensions of the Edgeworth Box and \(\beta\) are unchanged, the \(Y\) sector and the prices of factors are unchanged. If the \(Y\) sector does appear to change, this acts as an alert, indicating there is a mistake somewhere in the modelling of the \(X\) sector.

Having solved the model for the \(Y\) sector, we choose arbitrary, easy-to-work-with values for the other parameters: let \(\sigma = 2\); \(k = 3\); \(t = 0.5\); \(F_B = 1\); \(F_D = 1\); \(F_X = 1\); and \(F_M = 2\). The cut-off conditions now reduce to:

\[
a_D = \frac{100}{2\Delta}; \quad a_X = \frac{100}{3\Delta}; \quad \text{and} \quad a_M = \frac{100}{6\Delta}.
\]

Inserting the above expressions for the cut-off conditions into the expression

\(^4\)The increase in the size of the Edgeworth box in the calibration case in chapter 3 had the pleasing effect of doubling the number of firms at the centre compared with Markusen’s dimensions; it was otherwise cosmetic, in that the regime remained unchanged. This is a feature of the monopolistic competition model.

\(^5\)The factor price equalisation line (Markusen and Venables 2000) passes through the centre of the Edgeworth box, so in any case we have \(z_f = z_h(= z)\) and \(w_h = w_f(= w)\); but only when endowments are in the ratio of 3:1 in favour of skilled labour and \(\beta = 0.5\) do we find \(z = w = 1\).
for $\Delta$ yields a value for $n$, the mass of potential firms, of 25. Using this value for $n$ with the free-entry condition, we obtain a value for $\Delta$ of 55.537.

The equations describing the mass of firms can now be solved to show that 18.2 firms will sell on their domestic markets; 5.4 firms will export; and 0.68 firms will become multinationals and establish plants abroad. The equations describing output are now easily solved, and yield $X_{hh} = 83.3$; $X_{hf} = 18.5$; and $X_{hf}^M = 27.8$.

Finally, I solve for the critical values of the "a"s: the set of parameters currently being worked with yield the very pleasing result of $a_D = 0.9$; $a_X = 0.6$; and $a_M = 0.3$. This means that all firms drawing an "a" of 0.9 or under will become producers and sell on the domestic market; those drawing an "a" above 0.3 and up to 0.6 will also become exporters; whilst those drawing an "a" of 0.3 or lower will become multinationals.

The implication of the values of $n_h$, $n_D$, $n_X$ and $n_M$ calculated above are that, in each country, 25 units of skilled labour are engaged in producing blueprints; a further 25 units in the various types of fixed costs; and the remaining 50 units of skilled labour are engaged in the production of $X$ sector goods.

Thus the full solution to the equations at the centre of the Edgeworth Box are:

$$
\begin{align*}
\Delta_h &= \Delta_f = 55.537 \\
E_h &= E_f = 200 \\
X_{hh}^D &= X_{ff}^D = 83.306 \\
X_h^X &= X_f^X = 18.512 \\
X_{hf}^M &= X_{fh}^M = 27.769 \\
a_D^h &= a_D^f = 0.900 \\
a_X^h &= a_X^f = 0.600 \\
a_M^h &= a_M^f = 0.300 \\
n_h &= n_f = 25 \\
n_D^h &= n_D^f = 18.243 \\
n_X^h &= n_X^f = 5.405 \\
n_M^h &= n_M^f = 0.676 \\
Y &= 100 \\
p_Y^h &= p_Y^f = 1 \\
Z_h &= Z_f = 1 \\
W_h &= W_f = 1
\end{align*}
$$
The GAMS model reproduces this result, hence the equations that form the core of the program can be considered free of important coding errors.

4 THE NUMERICAL GENERAL EQUILIBRIUM MODEL

The technique used to solve the model is similar to the method of solving the model in Chapter 3. The code that solves the general equilibrium model for this chapter is adapted directly from the previous chapter. Within the model, the Y sector is unchanged (aside from its notation, which is now Y as z is required to denote the skilled wage rate (z as a mnemonic for "salary"). There are new equations to represent the cut-off conditions, and the six output equations now define the total output per type of sale and per market, rather than the sales per firm as in the previous chapter.

One change between the current model and the previous chapter is that none of the equations are expressed in terms of complementary slackness. This implies that it may be possible to solve this model with another simulation package, whereas the model in the previous chapter required the use of one of the MCP solvers available in GAMS. There are additional constraints on some of the variables that need to be specified to prevent the solver from discovering solutions that contravene the logic of the model. Specifically, given that $a_0$ was set at unity, all of the cut-off conditions have a border condition of 1. Certain combinations of parameters could give rise to cut-off conditions that are larger than 1. This is impossible as it would imply that there were more firms producing than had bought blueprints. Luckily, in GAMS, an upper limit to a variable is easily specified.

As in Chapter 3, the GAMS program repeatedly solves the model for various combinations of factor endowments that represent different points within the Edgeworth Box. I limit the number of points within the Edgeworth Box for which the model is solved to nine. This reduction in dimensions to three rows by three columns is justified on the basis that nine points are sufficient to illustrate different instantaneous equilibria achieved for quite different endowments. The complexity of the model is such that it is already difficult to work out what is going on, without the complication of further points.
4.1 An integrated equilibrium?

One *modus operandi* commonly followed in solving trade models is to solve for the integrated world equilibrium, then move to a world comprising of two countries engaging in free trade. Often the free trade equilibrium reproduces the integrated world equilibrium. In this model the free trade equilibrium does not reproduce the integrated world equilibrium, because of the beachhead costs. In this simulation, where blueprint, domestic market setup and export beachhead costs are all set at 1, each active firm requires 3 units of skilled labour to supply fixed costs in the free trade equilibrium, compared with only 2 units of skilled labour per firm in the integrated world equilibrium. We can write

*Result 1: Beachhead costs prevent the free trade equilibrium from reproducing the integrated world equilibrium*

This implies that there will be a difference between the two cases, relating either to expenditure on fixed costs or to the number of varieties. This results appears to contradict Proposition 1 in BRS. One possible explanation for this is that in the current model we solve for an instantaneous equilibrium, and specifically allocate resources to production of blueprints. Other models do not. This is because the other models are dynamic. In a dynamic model there is a constant turnover of firms being created and dying, and there is no need to account for the production of the original blueprints.

One of the features of the simulation of an instantaneous equilibrium is that the total expenditure on fixed costs is fixed by the level of endowments, and is unrelated to the fixed cost parameters. This is shown in chapter 2. It is a feature of monopolistic competition that halving all fixed costs will double the number of firms, as will doubling the endowments. The implication of this for the current model is that total expenditure on fixed costs will be constant for any particular point in the Edgeworth Box, if the dimensions of the box are unchanged. In the case of comparing the integrated world equilibrium with the free trade equilibrium for symmetric countries, it is the potential mass of varieties that changes to account for the difference in fixed costs per firm. Table 4.2 shows changes to the mass of varieties produced and exported when moving from an integrated world equilibrium (i.e., a single country) to a two country model with free trade and then slightly increasing trade costs.
Here we see that the potential mass of varieties is much larger when the whole world is a single country, compared with when there are two countries with free trade. This loss in potential varieties is due to the beachhead costs. However, the actual number of varieties available in each country is the same in the two-country, free trade equilibrium, and total expenditure on fixed costs is constant at 100 units of skilled labour in the above example. This result suggest that moving from a free trade equilibrium to a single market (think of Europe) would have particular benefits. This peculiar feature of the simulation permeates the discussion below.

The base case simulation uses the following set of parameters: the allocation of expenditure between the two types of good is equal, thus $\beta = 0.5$; the elasticity of substitution between the two factors of production for $Y$ ($\gamma$) is 3; the elasticity of substitution between different varieties of $X$ is 2; $k$, the shape parameter describing the Pareto distribution, is set at 3; the fixed costs of blueprints ($F_B$), of setting up production for the domestic market ($F_D$) and the beachhead cost of exporting ($F_X$) are set equal to 1; whilst the fixed cost of setting up a production base abroad ($F_M$) is set at 2. All fixed costs are specified in terms of units of skilled labour.

### 4.2 A mixed equilibrium

When parameter values are such that an interior solution is achieved for the three pairs of cut-off conditions, we find that all six types of firm (domestic producers, exporters and multinationals in each of the two countries) are active at all nine of the points considered within the Edgeworth Box. Hence the first result established by integrating the Melitz model into the Markusen-Venables multinationals model is:

**Result 2:** In equilibrium, all types of firm are sustained throughout the Edgeworth Box.

This is an important result, in that it is unusual in a scale-versus proximity model; in such models, it is commonly found that there is either a knife-edge...
situation in that only one type of firm can be sustained except when the knife-edge condition is fulfilled, as in Chapter 2 and Brainard (1993); or in a general equilibrium model with more than one factor, a maximum of half of the potential types of firms can be sustained at any particular point within the Edgeworth Box, as in Chapter 3 and Markusen's models.

The conditions for the coexistence of the six types of firm is found by manipulating equations 14, 15 and 17. To attain an equilibrium in which some firms produce for the domestic market but not for the foreign market, we require $a^h_D > a^h_x$. This will be true when $F_X \frac{\Delta h}{\Delta x} > F_D \frac{\Delta h}{\Delta E}$. To arrive at a situation where some, but not all, suppliers to the foreign market opt to become horizontal multinationals, we require $a^h_x > a^h_M$. This will be true if $(\frac{z_h}{z_f})^{-\sigma} F_M > F_X \frac{\phi}{\phi}$. Putting these conditions together, it is clear that the condition for an equilibrium in which all six types of firm exist will be:

$$\left(\frac{z_h}{z_f}\right)^{\sigma} F_M > F_X \frac{\Delta h E_f}{\Delta f E_h} F_D$$

(25)

These conditions are akin to those in HMY, with a small difference. I obtain an identical condition to HMY when I change the assumption concerning the place where $F_M$ is incurred, from the country where the foreign subsidiary is located, to the firm's home country. It must be the case that HMY assumed that the costs associated with setting up the plant abroad would be incurred at home. This is despite the language that specifies that the firm “bears additional fixed costs $F_I$ in every foreign market”.

The other additions to equation 25 compared with HMY is that equation 25 specifies that the ratio of the price indices matters in establishing that some firms will produce only for the domestic market, as does the ratio of income levels in the two countries. These subtleties can be masked when using, as HMY do, an intermediate variable of $B_i$ to represent the "per firm level of demand" for an individual country. To facilitate comparison, following HMY, I define $B = \frac{\beta E_h}{\sigma \Delta h}$. Here it is helpful to note that $\Delta$, originally conceived as a mnemonic for denominator, is a direct measure of the diversity of the consumption profile. As $E_h$ and $\Delta_h$ are both increasing in country size, in it not immediately apparent how the $B$ will vary with country size. Table 4.3 shows the variation in the parameter $B_h$ throughout the Edgeworth Box, that occur even before any of the parameter values change.

---

6It is not wholly unreasonable to assume that the fixed cost of establishing a production base abroad are incurred at home. In practice it is likely that some expenses of establishing a base abroad will be incurred at home: it takes a senior executive time to draw up a shortlist of and gather information on possible locations, and commission consultants; but some expenses must surely be incurred in the destination country, notable those for local consultants and premises.

7The $B^i$ in HMY is identical to $\frac{\beta E_h}{\sigma \Delta h}$ in my notation.
Notice that the per firm level of demand is highest in the North East corner, where country $h$ is three times as large as country $f$. However, the implication of this is not immediately apparent, as inspection of Table 4.4 showing the number of varieties available in the home market reveals.

Here we see that the number of varieties available in the home market is high in the North East corner compared with the centre. This immediately suggests that welfare is likely to be relatively high at this point, due to the higher number of varieties. This also suggests that the production in the large country may be more competitive than in the smaller country.

### 4.3 Variations in the cut-off conditions

Table 4.5 presents the cut-off conditions throughout the Edgeworth Box for production for the domestic market, for export and for becoming a multinational.
As described in the calibration above, we see that the cut-off \( a^h_D \) is 0.9 at the centre of the Edgeworth Box. This means that all firms drawing an "a" of 0.9 or lower will produce for the home market. Firms drawing an "a" above 0.9 will exit the industry without producing. Similarly, at the centre of the Box, firms drawing an "a" of 0.6 or lower will also become exporters and firms drawing an "a" of 0.3 or lower will become multinationals.

Consider the point to the North East of the centre, where Home's endowments are 75% of both skilled and unskilled labour. At this point the cut-off level of \( a^h_D \) is 0.872, which is lower than at the centre. This is slightly counter-intuitive: it might have been reasonable to assume that when the country is larger, it could sustain less productive firms. This turns out not to be the case. At this point in the Box, salaries are 3.4% higher than at the centre, and
wages are 3.1% lower. This combination arises because of a relative scarcity of skilled labour, and in fact home producers have to be more productive at this point than in the case of equal-sized countries. This is the home market effect, whereby the large market is the most efficient producer of the differentiated good, and the smaller market produces, relative to its size, more of the homogeneous good. Here we see two aspects of the home market effect: the location with the larger home market has a more than proportionally larger differentiated goods sector; and the location with a higher demand for differentiated goods pays a higher nominal wage. (Krugman 1980).

Consider the changes to the cut-off conditions $a^h_X$ and $a^h_M$ when moving from the centre towards the North-East corner of the Box. The condition $a^h_X$ falls from 0.6 to 0.5, which means that a smaller proportion of home firms will export. As Foreign now represents a smaller market, Home’s exports are therefore doubly reduced: there are fewer firms exporting, and each one faces a lower per firm level of demand. Conversely, the condition $a^h_M$ increases from 0.3 to 0.42. The driving factor here seems to be the lower salaries that now prevail in Foreign, making it more attractive for Home multinationals to invest there, although the market is small. We can write:

Result 3: the cut-offs $a^h_D$ and $a^h_X$ fall as the home country increases in size

and

Result 4: the cutoff $a^h_M$ increases as the home country increases in size

Note that along the diagonal, the cut-offs $a^h_D$ and $a^h_X$ move in the same direction as each other, and in opposite directions to $a^h_M$.

Now consider the point to the South West of the centre, where Home is endowed with 25% of each of the world’s factors. The story here is the reverse of the one above. The smaller country now has lower salaries than in the case of equal countries, which attracts Foreign multinationals. Indeed, the bulk of Home’s supply of the differentiated good is via sales from affiliates. Thus the size of the country plays a major role in determining the mode of supply of $X$ sector goods: the large country enjoys a home-market effect, whilst the smaller country benefits from inward investment and sales from foreign affiliates. The logic here is that the blueprints are created in the larger country, and the smaller country benefits from shared technology. We can write

Result 5: the cut-offs $a^h_D$ and $a^h_X$ increase as the home country decreases in size

and

Result 6: the cutoff $a^h_M$ decreases as the home country decreases in size
Figure 4.2a Countries differ in size

Figure 4.2a Countries differ in endowments

Figure 4.2 Home's consumption by type of sale

Figure 4.3a Countries differ in size

Figure 4.3b Countries differ in endowments

Figure 4.3 Exports versus FDI: Home's consumption of Foreign goods by type of sale
Now consider the North-West, South-East diagonal along which countries differ greatly in endowment, but less so in size.\(^9\) When Home is skilled-labour rich (the North-West corner), the cut-offs \(a^h_D\) and \(a^h_X\) are both higher than at the centre. On the other hand, the cut-off \(a^h_M\) rises as Home becomes less skilled-labour rich. We can write:

**Result 7:** the cutoffs \(a^h_D\) and \(a^h_X\) increase as the home country becomes more skilled-labour abundant

and

**Result 8:** the cutoff \(a^h_M\) falls as the home country becomes more skilled-labour abundant.

Again, the cut-offs \(a^h_D\) and \(a^h_X\) move in the same direction as each other, and in opposite directions to \(a^h_M\).

### 4.4 Consumption

Now consider the same exercise focusing on consumption. Figure 4.2 shows Home’s consumption by type of sale as home varies in size (Figure 4.2a) and endowment (Figure 4.2b). The strong home-market effect discussed above is clearly visible in Figure 4.2a.

Figure 4.3 illustrates the export versus FDI trade-off: among equal-sized countries, affiliate sales are high relative to imports (trade costs are fairly high in this simulation.) However, imports are likely to be relatively important when home is large (less FDI into the large country - the foreign country prefers to export because of high skilled labour costs) and when home is skilled-labour poor (there is little availability of appropriate labour for foreign investors to employ.)

### 5 COMPARATIVE STATICS

Here I consider the impact of changes in key parameters, particularly trade costs and fixed costs, on the cut-off conditions.

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\(^8\)The impact on average exports per firm is not clear, as demand per firm is lower, but the average productivity per firm is higher.

\(^9\)There is also a difference in size, as the world endowment of skilled labour is three times that of unskilled labour.
Inspection of Table 4.6 reveals:

**Result 9:** A decrease in trade costs unambiguously increases $a_X^h$.

The logic here is rather obvious: when variable trade costs are higher, only the more productive firms will be able to export. This finding is in line with HMY (2004), BRS (2004) and Baldwin (2005). Conversely, when trade costs are higher, in this model it means firms are more likely to preference FDI.

Table 4.7 demonstrates:

**Result 10:** A decrease in trade costs unambiguously decreases $a_M^h$.

This finding is inline with HMY (2004), which is the only paper to have explicitly considered the scale versus proximity trade-off in the context of the Melitz model. But what about the impact of a rise in trade costs on $a_D^h$? Consider Table 4.8:
This leads to:

Result 11: A decrease in trade costs unambiguously increases $a_D$

This is an astonishing result. It is counter-intuitive, and is at odds with the findings in Melitz (2003), HMY (2004), and Baldwin (2005). An explanation is required. As Baldwin notes, the cut-off conditions are "potentially very complex since $\Delta$ depends upon $a_X$ and $a_D$, but these depend upon the $\Delta$ via (the demand function)". The numerical simulation is composed of thirty-one equations. Changing the parameter $t$ will change, even at the centre of the Box, no less than twenty-two of the variables. Away from the factor price equalisation set, all thirty-one variables will be jointly determined and any change in trade costs will lead to a change in all the variables.

So how can an increase in trade costs lead to an increase in $a_D$? When trade costs increase, this affects $\Delta$ in two ways. The first is via $\phi$: an increase in $t$ means that $\phi$ falls, which would also have a negative effect on $\Delta$; the intuition here is that there are fewer imported varieties due to higher trade costs, and as $\Delta$ is a measure of diversity, we might expect $\Delta$ to fall. However, in this model, a fall in imports will go hand in hand with an increase in sales from affiliates, as investment is now relatively more attractive than before. The real change is in the potential number of varieties, which we showed above will increase with trade costs, whilst the chance of success (the cut-off) falls. When $t$ increases, the denominator of $n$ becomes smaller, which results in a larger $n$ and hence and increase in $\Delta$. If the impact of $t$ via $n$ is greater than the impact of $t$ via $\phi$, $\Delta$ will rise. If $\Delta$ increases, then the cut-off gets smaller. Overall, in this simulation, an increase in trade costs leads to higher overall productivity, due partly to high trade costs favouring investment by very high productivity firms compared with exports, but also because the number of potential varieties increases. Correspondingly, the chance of success falls and so the average productivity increases.

6 WELFARE

The expression for welfare is

$$V = \beta^\beta (1 - \beta)^{(1-\beta)M_h} \frac{\sigma - 1}{\sigma} \Delta^{\frac{\sigma T}{(\sigma - 1)}}$$

showing that welfare is increasing in both income and in $\Delta$. Thus there are two components determining welfare. These are the number of varieties available in a country, which impacts upon $\Delta$; the returns to factors, as these drive income. Figure 4.4 illustrates the changes to welfare of the two types of labour from being in a large or small country compared with when two countries are of equal size.
The figure shows that both unskilled labour and skilled labour benefit from being in a large country, but that the factor used in the production of the differentiated good benefits more. This is another feature of the home market effect. Conversely, both factors are worse off in the small country, and again skilled labour is affected more intensely.

7 CONCLUSION

One of the contributions of this thesis is that it demonstrates some of the benefits of computer simulation: we have taken a problem that is well-defined and well-known in theory, and shown that when we enrich the problem to account for fairly large differences in factor endowments, such that an analytical result is no longer attainable, the computer simulation produces rich, illuminating results. Some of the results appear at first sight not to support the theory; but it is possible that some of the current results are due to the peculiarities of the general equilibrium between two countries, and possibly due to the way fixed costs are modeled. If the simulation were extended to cover three or more countries, the results may be different. Alternatively, in a dynamic model, where there is no need to account specifically for resources consumed by inactive firms, I would expect to obtain similar results to the established papers.
It would be possible to further interrogate the model developed for this paper, as there are further results that could be obtained. For example, it would be possible to run the simulation several times and obtain data to construct a figure similar to Figure 1 in HMY (2004); this would give profits from the various types of firms that are non-linear functions of the cut-off conditions.
Appendix 1

Equations for the general equilibrium model

The thirty-one equations used in the general equilibrium simulation are reproduced here, in eight groups.

Income equations (2)

\[ \begin{align*} 
E_h &= z_h S_h + w_h L_h \\
E_f &= z_f S_f + w_f L_f
\end{align*} \]

"Price Index" equations (2)

\[ \begin{align*} 
\Delta_h &= \frac{k}{(1 - \sigma + k)} \left[ z_h^{(1-\sigma)} \left( \frac{\nu_h (a^h_d)^{(1-\sigma+k)}}{(a^h_d)^{k}} + \frac{n_f (a^f_M)^{(1-\sigma+k)}}{(a^f_M)^{k}} \right) \right] \\
\Delta_f &= \frac{k}{(1 - \sigma + k)} \left[ z_f^{(1-\sigma)} \left( \frac{\nu_f (a^f_d)^{(1-\sigma+k)}}{(a^f_d)^{k}} + \frac{n_h (a^h_M)^{(1-\sigma+k)}}{(a^h_M)^{k}} \right) \right]
\end{align*} \]

Output equations (6)

\[ \begin{align*} 
X_{hh} &= \frac{\sigma - 1}{\sigma} n_h (z_h)^{-\sigma} \beta E_h \Delta_h \left( \frac{k}{(k-\sigma)} \right) \left( \frac{a^h_d)^{(k-\sigma)}}{(a^h_d)^{k}} \right) \\
X_{ff} &= \frac{\sigma - 1}{\sigma} n_f (z_f)^{-\sigma} \beta E_f \Delta_f \left( \frac{k}{(k-\sigma)} \right) \left( \frac{a^f_d)^{(k-\sigma)}}{(a^f_d)^{k}} \right) \\
X_{fh} &= \frac{\sigma - 1}{\sigma} n_f (z_f)^{-\sigma} \beta E_h \Delta_h \left( \frac{k}{(k-\sigma)} \right) \left( \frac{a^f_M)^{(k-\sigma)}}{(a^f_M)^{k}} \right) \\
X_{hf} &= \frac{\sigma - 1}{\sigma} n_h (z_h)^{-\sigma} \beta E_f \Delta_f \left( \frac{k}{(k-\sigma)} \right) \left( \frac{a^h_M)^{(k-\sigma)}}{(a^h_M)^{k}} \right) \\
X_{hf} &= \frac{\sigma - 1}{\sigma} (1 + t)^{1-\sigma} n_h (z_h)^{-\sigma} \beta E_f \Delta_f \left( \frac{k}{(k-\sigma)} \right) \left[ \frac{(a^h_d)^{(k-\sigma)}}{(a^h_d)^{k}} \right] \left[ \frac{(a^h_M)^{(k-\sigma)}}{(a^h_M)^{k}} \right]
\end{align*} \]
\[ X_f^X = \frac{\sigma - 1}{\sigma} (1 + t)^{1-\sigma} n_f (z_f)^{-\sigma} \frac{\beta E_h}{\Delta h} \frac{k}{(k-\sigma)} \left[ \frac{(a_X^f)^{(k-\sigma)} - (a_M^f)^{(k-\sigma)}}{a_D^f} \right] \]

Cut-off conditions (6 equations)

\[(z_h)^{-\sigma} (a_D^h)^{1-\sigma} \frac{\beta E_h}{\Delta h} \frac{1}{\sigma} = F_d\]

\[\phi (z_h)^{-\sigma} (a_X^h)^{1-\sigma} \frac{\beta E_f}{\Delta f} \frac{1}{\sigma} = F_x\]

\[\left[(z_f)^{1-\sigma} - \phi (z_h)^{1-\sigma}\right] (a_M^f)^{1-\sigma} \frac{\beta E_f}{\Delta f} \frac{1}{\sigma} = z_f F_M - z_h F_X\]

\[(z_f)^{-\sigma} (a_D^f)^{1-\sigma} \frac{\beta E_f}{\Delta f} \frac{1}{\sigma} = F_d\]

\[\phi (z_f)^{-\sigma} (a_X^f)^{1-\sigma} \frac{\beta E_h}{\Delta h} \frac{1}{\sigma} = F_x\]

\[\left[(z_h)^{1-\sigma} - \phi (z_f)^{1-\sigma}\right] (a_M^f)^{1-\sigma} \frac{\beta E_h}{\Delta h} \frac{1}{\sigma} = z_h F_M - z_f F\]

Number of firms (6 equations)

\[n_D^h = n_h (a_D^h)^k\]

\[n_X^h = n_h (a_X^h)^k\]

\[n_M^h = n_h (a_M^h)^k\]

\[n_D^f = n_f (a_D^f)^k\]

\[n_X^f = n_f (a_X^f)^k\]

\[n_M^f = n_f (a_M^f)^k\]
Free-entry conditions (2)

\[ n_h = \frac{n_D^h z_h F_D + n_X^h z_h F_X + n_M^h z_f F_M}{\beta k \sigma(1-\sigma+k)} \left[ \frac{z_h^{1-\sigma} E_h (a_h^k)^{1-\sigma+k}}{\Delta_h} + \frac{z_f^{1-\sigma} E_f (a_h^k)^{1-\sigma+k}}{\Delta_f} \right] - z_h F_B \]

\[ n_f = \frac{n_D^f z_f F_D + n_X^f z_f F_X + n_M^f z_h F_M}{\beta k \sigma(1-\sigma+k)} \left[ \frac{z_f^{1-\sigma} E_f (a_f^k)^{1-\sigma+k}}{\Delta_f} + \frac{z_h^{1-\sigma} E_h (a_f^k)^{1-\sigma+k}}{\Delta_h} \right] - z_f F_B \]

Y-sector equations (3)

\[ Y_h + Y_f = \frac{(1-\beta) E_h}{p_y} + \frac{(1-\beta) E_f}{p_y} \]

\[ p_h = \left( z_h^{1-\gamma} + w_h^{1-\gamma} \right)^{\frac{1}{m}} 2^{1\gamma} \left( z_h^{1-\gamma} + w_h^{1-\gamma} \right)^{\frac{1}{m}} 2^{1\gamma} \]

\[ p_f = \left( z_f^{1-\gamma} + w_f^{1-\gamma} \right)^{\frac{1}{m}} 2^{1\gamma} \left( z_f^{1-\gamma} + w_f^{1-\gamma} \right)^{\frac{1}{m}} 2^{1\gamma} \]
Factor market clearing conditions (4)

\[ L_h = Y_h w_h^{-\gamma} \left( z_h^{(1-\gamma)} + w_h^{(1-\gamma)} \right)^{\frac{\gamma}{\gamma+1}} 2^{\frac{1}{\gamma+1}} \]

\[ S_h = Y_h z_h^{-\gamma} \left( z_h^{(1-\gamma)} + w_h^{(1-\gamma)} \right)^{\frac{\gamma}{\gamma+1}} 2^{\frac{1}{\gamma+1}} + n_h^b F_B + n_h^b F_D + n_h^b F_X + n_h^M F_M \]

\[ + \frac{(\sigma-1) k (z_h)^{-\sigma}}{(k-\sigma+1) \sigma} \left( \frac{E_h}{\Delta h} \left[ \frac{n_h (a_{D}^{h})^{(k-\sigma+1)}}{(a_{b}^{h})^{\sigma}} + \frac{n_h (a_{M}^{h})^{(k-\sigma+1)}}{(a_{b}^{h})^{\sigma}} \right] \right) \]

\[ + \frac{\phi E_h n_h}{\Delta h} \left[ \frac{(a_{D}^{h})^{(k-\sigma+1)-(a_{b}^{h})^{(k-\sigma+1)}}}{(a_{b}^{h})^{\sigma}} \right] \]

\[ L_f = Y_f w_f^{-\gamma} \left( z_f^{(1-\gamma)} + w_f^{(1-\gamma)} \right)^{\frac{\gamma}{\gamma+1}} 2^{\frac{1}{\gamma+1}} \]

\[ S_f = Y_f z_f^{-\gamma} \left( z_f^{(1-\gamma)} + w_f^{(1-\gamma)} \right)^{\frac{\gamma}{\gamma+1}} 2^{\frac{1}{\gamma+1}} + n_f^b F_B + n_f^b F_D + n_f^b F_X + n_f^M F_M \]

\[ + \frac{(\sigma-1) k (z_f)^{-\sigma}}{(k-\sigma+1) \sigma} \left( \frac{E_f}{\Delta f} \left[ \frac{n_f (a_{D}^{f})^{(k-\sigma+1)}}{(a_{b}^{f})^{\sigma}} + \frac{n_f (a_{M}^{f})^{(k-\sigma+1)}}{(a_{b}^{f})^{\sigma}} \right] \right) \]

\[ + \frac{\phi E_f n_f}{\Delta f} \left[ \frac{(a_{D}^{f})^{(k-\sigma+1)-(a_{b}^{f})^{(k-\sigma+1)}}}{(a_{b}^{f})^{\sigma}} \right] \]
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