Globalization, Acquisitions and Endogenous Firm Structure

Larry D. Qiu and Wen Zhou*

The University of Hong Kong

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Abstract

A model of heterogeneous firms with multiple products and endogenous firm structure is developed to investigate how firms respond to trade liberalization. Firm-level productivity is endogenized as firms acquire capital through mergers and acquisitions to reach optimal product scope (number of products) and capital scale (the amount of capital in each plant). The analysis shows that while more efficient firms produce more products, plant and firm level capital have an inverse U-shaped relationship with firm efficiency. A firm responds to different types of trade liberalization differently, leading to different redistribution of capital and productivity. In an importing country that opens to trade unilaterally, capital moves from high efficiency firms to low efficiency firms, and firm and industry productivity increase. Under bilateral trade liberalization, capital moves from low efficiency firms to high efficiency firms, and productivity may increase or decrease. The analysis highlights the importance of resource reallocation after trade liberalization and the linkage between product and acquisition markets.

Keywords: firm heterogeneity, trade liberalization, mergers and acquisitions, scale, scope, multiproduct, firm structure

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1 Introduction

Drastic trade liberalization, unilateral, bilateral or multilateral, has been implemented in many places in the world as part of the globalization process. Traditional international economics has primarily been concerned

*Correspondence: larryqiu@hku.hk and wzhou@business.hku.hk. We thank Jiahua Che, Jee-Hyeong Park, Wing Suen, Shangjin Wei and seminar participants at the Chinese University of Hong Kong, Fudan University, the Hong Kong University of Science and Technology, Seoul National University, the University of Hong Kong and the 2008 International Industrial Organization Conference for their helpful comments.
about patterns of and gains from trade, but more recent studies have shifted the focus to how heterogeneous firms respond to trade liberalization differently and how industry productivity changes through firm entry and exit.\textsuperscript{1} The present study follows this trend and goes beyond by endogenizing firm-level productivity and by emphasizing the interaction between product and factor markets.

The majority of the new studies follows Melitz’s (2003) model, which assumes that every firm produces a single product.\textsuperscript{2} The reality is clearly very different. According to Bernard, Jensen and Schott (2005) and Bernard, Redding and Schott (2008), 41 percent of U.S. manufacturing firms produce more than a single product, and these firms account for 91 percent of U.S. manufacturing output and 95 percent of U.S. exports.\textsuperscript{3} When firms produce multiple products, they can respond to trade liberalization by changing the number of their products (i.e., product scope) as well as by the entry/exit and output adjustment that have been the focus of previous research. Investigating how heterogeneous firms change their product scope differently is in itself an interesting question. Furthermore, this extra channel of adjustment through product scope may affect responses in other channels and ultimately affect industry productivity. A few researchers, most notably Bernard, Redding and Schott (2009) and Nocke and Yeaple (2006), have already realized the need to study multiproduct firms. This study was designed with the distinctive feature that product scope and plant level productivity were both endogenized. Unlike the work of Bernard, Redding and Schott (2009) with exogenous plant productivity, the productivity was endogenized by allowing firms to acquire a productive resource called capital. Unlike Nocke and Yeaple (2006) who imposed a tradeoff between product scope and plant productivity, we allow the two variables to be chosen independently.

For firms to be able to adjust their productivity through changes in capital, there must be a channel through which capital moves across and within firms. The trading of capital was modelled as mergers and acquisitions (M&As) in an acquisition market. M&A is a major method of industrial restructuring (UNCTAD, 2000) and the quickest and least costly way to respond to external shocks such as trade liberalization.\textsuperscript{4} Waves of mergers have been documented as a consequence of trade liberalization and other industry shocks (Mitchell and Mulherin, 1996). Breinlich (2008) found that the Canada-United States Free Trade Agreement of 1989 increased domestic Canadian M&A activity by over 70%. Using data on Swedish firms for the period 1980-1996, Greenaway, Gullstrand and Kneller (2008) have shown that intensified international competition caused more M&As. M&A is particularly relevant to multiproduct firms, as acquisition is the most effective way to add new products quickly. In the context of firm heterogeneity, it would be interesting to investigate how firms behave differently in the acquisition market, i.e., who buys capital and who sells capital, in response to trade liberalization. Firms’ endogenous adjustment of product lines through M&A will also affect other decisions including product scope, and thus ultimately affect industry productivity. Breinlich (2008), for example, found that resources were transferred from low efficiency firms to high efficiency firms.


\textsuperscript{2}Another pioneer study is that of Bernard, Eaton, Jensen and Kortum (2003), who also focus on single-product firms.

\textsuperscript{3}The calculation is based on U.S. manufacturing and trade data from 1972 to 1997. Products are defined by five-digit Standard Industrial Classification categories.

\textsuperscript{4}According to the United Nations’ World Investment Report (UNCTAD, 2000), worldwide M&As grew at an annual rate of 42 percent over the period 1980-1999 to reach US$2.3 trillion in 1999. More than 24,000 M&As took place during that period, and the value of M&As relative to world GDP rose from 0.3 percent in 1980 to 2 percent in 1990 and to 8 percent in 1999.

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through M&As following the Canada-United States Free Trade Agreement.

M&A is not simply another channel through which firms respond to exogenous shocks. It highlights the important linkage between product and factor markets, which has not been thoroughly investigated in the literature. Trade liberalization is usually considered to affect firm behavior through either the product market or the factor (labor) market, but not both. Melitz and Ottaviano (2008) assume exogenous wage rate and focused only on the competition in the product market. Melitz (2003) and Bernard, Redding and Schott (2009) derive product demand from CES preferences, and in their model trade liberalization works its effect mainly through changes in the labor market. Although the trading of capital in the present study seems to work in a way similar to the labor market, there are two important differences between labor and capital. First, unlike labor, capital affects both firm-level and plant-level productivity. Second, trade liberalization affects both product and acquisition markets, which permits investigating the linkage between the two. The result are very different from those obtained when the two markets are considered separately.

Consider first a closed economy in which a continuum of firms produce differentiated products. Firms are distinguished by their intrinsic efficiency and may choose to produce multiple products. The marginal production cost of any product is assumed to be decreasing in both the firm’s intrinsic efficiency and the amount of capital allocated to the product. Capital is acquired in a perfectly competitive acquisition market before firms produce and compete in a monopolistically competitive product market. We will show that while more efficient firms maintain larger product scope (number of varieties), they do not necessarily possess larger size (capital at the firm level) or scale (capital at the plant level). In fact, size and scale are inversely U-shaped in intrinsic efficiency. An efficient firm sells its products at a relatively low price. Further reducing marginal production cost by increasing capital will not bring much extra benefit. As a result, an increase in efficiency may induce a firm to decrease its capital stock. Nevertheless, more efficient firms always produce more at the plant and firm level. Thus, this model helps reconciling seemingly conflicting predictions of previous studies. On the one hand, scope and plant-level output are positively correlated, confirming the theoretical and empirical findings by Bernard, Redding and Schott (2008, 2009). On the other hand, scope and firm-level size (measured by capital) may be negatively correlated, confirming the prediction of Nocke and Yeaple (2006).

Trade liberalization brings both opportunities (of exporting) and challenges (through increased competition from imports), and the two often have opposite effects. To isolate these effects, we consider two types of trade liberalization: unilateral and bilateral. Under unilateral trade liberalization in the industry concerned, firms in the importing country face more competition in the product market. They all try to downsize, pushing down the price of capital. Facing reduced profitability in the product market and reduced capital prices in the acquisition market, firms respond differently. Because they are efficient in using capital, high efficiency firms are mainly affected by changes in the product market, and they end up selling capital. Low efficiency firms, by contrast, are less responsive to the product market and are affected mainly by changes in the acquisition market. They will buy capital. Low efficiency firms expand their scope, while high efficiency firms reduce scope. The total number of varieties produced by home firms is reduced. As each plant expands scale and reduces output, firm- and industry-level productivity improve.

Under bilateral trade liberalization in the industry concerned, the effects of increased opportunity and
increased competition are both present. Interestingly, it turns out that when the two countries are symmetric, the effect of increased opportunity dominates and, as a result, capital moves from low efficiency firms to more efficient ones. Low efficiency firms reduce scope while high efficiency firms expand their scope. The total number of varieties consumed in each country increases. The impact on productivity is nonetheless ambiguous. When products are close substitutes, productivity tends to rise; when products are very differentiated, productivity tends to drop. Although we focused on two symmetric countries, the analysis can be easily extended to bilateral trade liberalization between two or more asymmetric countries.

Our model predicts that firm size (measured by capital or output) is distributed more evenly in the importing country under unilateral trade liberalization, and more skewed under bilateral trade liberalization. This helps explain another set of conflicting predictions. Bernard, Redding and Schott (2009) showed that size distribution is more skewed after trade liberalization, while Nocke and Yeaple (2006) showed that the distribution should be less skewed. It now appears that both are possible depending on the type of liberalization or the relative size of the two countries.5

Thus, by endogenizing firm structure through choices of scope and scale, and by introducing an acquisition market for the trading of capital, this study highlights M&A as an important channel through which firms respond to trade liberalization. It demonstrates a new source of productivity improvement through resource re-allocation, and highlights the linkage between product and acquisition markets. The implications are, first, that productivity adjusts not only at the industry and firm level, but also at the individual product level through the redistribution of capital across and within firms. Such a redistribution of productive resources within an industry is at the heart of industrial dynamics in the face of exogenous shocks. Its impact on industry and firm productivity are stronger and more direct than the re-allocation of output, which has been the focus of previous studies. A second implication is that the linkage between the product and acquisition markets cannot be ignored. Changes in the product market always induce an offsetting change in the acquisition market. Trade liberalization may reduce capital prices so much that capital moves from high efficiency firms to low efficiency firms, yet firm and industry productivity increases, a result impossible when the two markets are considered separately.

These findings relate to three strands of prior work. Most relevant is the international trade literature on heterogenous firms, where most studies (see the survey by Helpman, 2006) assume single-product firms and predict that industry productivity improves as a result of the exit of the least efficient firms (e.g., Melitz, 2003), i.e., through inter-firm output re-allocation. Bustos (2007) extended Melitz’s (2003) model by allowing firms to upgrade their technologies at some extra fixed cost. Recent models of multiproduct firms (e.g., Baldwin and Gu, 2005; Eckel and Neary, 2005; Bernard, Redding and Schott, 2009, 2008; Nocke and Yeaple, 2006; Feenstra and Ma, 2007) have emphasized intra-firm output re-allocation as a new source of productivity improvement. For example, Bernard, Redding and Schott (2009) start with the premise that productivity differs not only across firms but also among different products produced by the same firm.

5Bernard, Redding and Schott (2006) and Nocke and Yeaple (2006) reached their conclusions considering symmetric bilateral trade liberalization. Although in the model considered here, bilateral trade liberalization leads unambiguously to more skewed size distribution, this result obtains when the two countries are symmetric. If the two countries are asymmetric, it is reasonable to conjecture that depending on the parameters, either force may dominate and the size distribution may be more or less skewed.
They showed that each firm responds to trade liberalization by dropping its least efficient products. While they relied on output reallocation with fixed plant-level productivity, this study has emphasized resource reallocation with endogenous productivity. Nocke and Yeaple (2006) imposed an exogenous tradeoff between product scope and firm-level productivity and predicted that trade liberalization would induce high efficiency firms to shrink their product lines and low efficiency firms to expand product lines. In contrast, this study considered productivity adjustment at the plant level as independent of the choice of product scope. The modeling of M&As enabled us not only to generate within-firm rationalization through productivity and product scope adjustment, but also to reconcile the conflicting predictions of these prior studies. Furthermore, most previous formulations have relied on either the product market (Melitz, 2003; Nocke and Yeaple, 2006) or the factor market (Bernard, Redding and Schott, 2009; Melitz and Ottaviano, 2008) in explaining how trade liberalization affects firm choices. This study considered both markets.

This study is also relevant to M&A in an international context, particularly how trade liberalization affects firms’ merger incentives. Long and Vousden (1995) have shown that while unilateral tariff reduction discourages cost-reducing mergers, bilateral tariff reductions have the opposite effect. Long and Vousden focused solely on M&A incentives, but this study has examined how heterogeneous firms adjust their capital scale and product scope through M&A in response to trade liberalization. Moreover, like many others, Long and Vousden emphasized strategic effects in an oligopoly setting. This study assumed a perfectly competitive acquisition market. Spearot (2008) used the same cost function and two-stage game considered here, but assumed single-product firms and capital transactions in single units. His conclusion that only moderately efficient firms acquire capital was thus very different. Several empirical studies have found that trade liberalization can trigger merger waves (Mitchell and Mulherin, 1996; Breinlich, 2008; Greenaway, Gullstrand and Kneller, 2008). This work provides theoretical support for those findings.

Finally, it might be pointed out that this work is remotely related to industrial organization studies of endogenous product scope, which emphasize strategic effects (Brander and Eaton, 1984; Shaked and Sutton, 1990; and Johnson and Myatt, 2003). Since our purpose was to study how heterogeneous firms respond to trade liberalization by changing their firm structure, strategic effects were excluded by assuming that firms buy and sell capital in a perfectly competitive acquisition market.7

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6In this study, mergers were endogenous in the sense that all merger opportunities have been exhausted in the original equilibrium and only an exogenous shock to the economy such as trade liberalization can trigger new mergers. Elsewhere we have studied endogenous mergers between heterogeneous firms producing single and homogeneous products (Qiu and Zhou, 2007). The focus of that study was how mergers alter the incentives for subsequent mergers and why mergers occur in waves.

7Lommerud and Sorgard (1997) have shown that mergers may change a firm’s optimal choice of product scope. They did not consider the impacts of trade liberalization.
2 Closed Economy

2.1 The model

Consider a closed economy with a numeraire good and a differentiated goods industry. Production of the numeraire good displays constant returns to scale and requires two units of labor for one unit of output. Hence, wage rate, \( w \), is equal to \( \frac{1}{2} \) in the economy. In the differentiated goods industry, there are a continuum of firms. Each firm draws its efficiency level, \( a \), from a uniform distribution on \([0, 1]\). The parameter \( a \), which is the only exogenous characteristic that distinguishes firms, is called a firm’s intrinsic efficiency. A larger \( a \) denotes more efficient production.

A firm can maintain multiple plants, each producing a single variety. In doing so, it incurs a management cost, \( mv^2 \), where \( m > 0 \) and \( v \) is the number of varieties that the firm produces. The cost of producing each variety is derived from a constant-returns-to-scale technology \( q = (axl)^{\frac{1}{2}} \), in which \( x \) is capital input and \( l \) is labor input. In the short run when its capital is fixed, the variable cost is \( \min(lw) \) subject to \( q = (axl)^{\frac{1}{2}} \). The minimum cost is \( \frac{w}{ax}q^2 \). Since \( w = \frac{1}{2} \), this yields the cost function for the variety as

\[
c(q|a, x) = \frac{q^2}{2ax}.
\]

This cost function has been widely used in industrial organization studies of M&As (e.g., Perry and Porter, 1985).

The production technology in this model displays decreasing returns to (output) scale: marginal production cost, \( \frac{q}{ax} \), increases with \( q \). This cost structure ensures interior solutions for optimal quantity at arbitrary levels of capital, which is particularly convenient in a model where capital is endogenous. While it simplifies the calculation greatly, the assumption is not crucial. The major results hold even if marginal costs are constant, an assumption commonly used in models of international trade.

Given the above description, we note that the reciprocal of average production cost, i.e., \( \frac{q}{c} = \frac{2ax}{q} \), is the plant-level labor productivity. Industry-level labor productivity is defined similarly as the reciprocal of industry average cost, i.e., industry aggregate quantity divided by aggregate cost.\(^8\) Labor productivity thus increases with both the intrinsic efficiency, which is exogenous, and the amount of capital allocated to each plant, which is endogenous. While \( a \) is the intrinsic efficiency for a firm, \( ax \) represents the production efficiency of a particular variety.

In previous models of heterogeneous firms with single factor (labor), there is no difference between a firm’s intrinsic efficiency, \( a \), and its labor productivity. In models of single-product firms, firm level productivity has been fixed.\(^9\) In Bernard, Redding and Schott’s (2009) model of multiple-product firms,

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\(^8\)Thus, in discussing productivity, management cost \( mv^2 \) will be ignored to focus on the costs directly incurred in the production process.

\(^9\)The only exception is the work of Bustos (2007), who allowed single-product firms to upgrade their technology (i.e., a proportional reduction of marginal costs) at some additional fixed cost, giving rise to endogenized plant level productivity. She found
although the firm-level average productivity was endogenous, plant level productivity was still fixed. Nocke and Yeaple (2006) endogenized plant level productivity, but they imposed an exogenous tradeoff between productivity and product scope. In this model, by contrast, product scope and plant level productivity are both endogenized, and the two choices are independent. Although there is a tradeoff between scale and scope within a firm, the tradeoff disappears when firms can freely buy capital in the acquisition market. For example, a firm may buy capital to increase scope and scale at the same time. In this sense, firm structure is truly endogenized in this model.

Each firm is endowed with one unit of capital. With this endowment and the knowledge of their intrinsic efficiencies, firms play a two-stage game. In the first stage (the acquisition stage), firms buy and sell capital in a perfectly competitive acquisition market (as in Jovanovic and Rousseau, 2002). If a firm sells all of its capital, it becomes inactive in the product market, i.e., it exits the production. An inactive firm still keeps its assets and may choose to re-enter the product market later by buying capital from the acquisition market. Entry and exit incurs no extra cost. After the trading of capital, every firm which still possesses positive capital chooses its number of varieties and the allocation of capital into each plant. A firm’s choice of structure therefore consists of its product scope (the number of varieties), plant-level capital scale (the amount of capital used in each plant), and consequently firm-level capital size (total amount of capital). For expositional convenience, scale, scope and size will be treated as continuous variables. Moreover, capital is assumed perfectly divisible, and there is no minimum capital requirement for running a plant. In the second stage (the production stage), all existing firms (i.e., those who did not exit) produce and sell their products in a product market characterized by monopolistic competition.

In this model, capital represents industry-specific physical assets that are needed in production. Unlike financial assets, capital must be acquired through M&As rather than from financial institutions. Unlike variable inputs such as labor, capital are fixed inputs in production (in the short run), such as the distribution network, machinery and production capacity. Capital is thus a productive resource that directly determines production efficiency. We assume that a firm can acquire capital only from the acquisition market (i.e., only from other firms), which implies that the supply of capital in the industry is perfectly inelastic. Such an assumption is justified if the total amount of such physical assets cannot be increased within a short time. It is important to point out, however, that the results still hold even if new capital can be generated in response to a capital price increase. By assuming a perfectly competitive acquisition market, we imply that capital is homogeneous and acquisition is partial. That is, instead of acquiring a stand-alone target firm, the acquirer buys some productive assets from other firms and use them along with its own assets. As argued by Jovanovic and Rousseau (2002), transactions in such a used-capital market work just like those in an M&A market. Maksimovic and Phillips (2001) report that about half of U.S. M&A transactions are partial

that a reduction in variable trade cost induces more firms to upgrade and more firms to self-select into the export market. While technology upgrading is binary and irreversible (no possibility for downgrading), capital flow is continuous and can increase or decrease. More importantly, technology supply is perfectly elastic; there is no competition for the technology (the fixed cost of upgrading does not change when more firms try to upgrade). In contrast, there is competition for capital: When firms try to produce more in response to trade liberalization, they demand more capital, pushing up the capital price. Thus, capital is best regarded as a rare productive resource in limited supply, and trade liberalization changes both the benefit and cost of acquiring capital, both of which are at the heart of the model.
acquisitions or divestitures by multi-product conglomerates.

The model will now be analyzed backwards, first the product market, then the acquisition market, and finally the industry equilibrium.

2.2 The product market

Assume $L$ identical consumers in the economy. To analyze competition in the product market, we follow Melitz and Ottaviano (2008) to assume that the representative consumer has a quasi-linear preference over a numeraire good and all varieties in the industry concerned:

$$U = Q_0 + \alpha \int_{i \in \Omega} q_idi - \frac{1}{2} \beta \left( \int_{i \in \Omega} q_idi \right)^2 - \frac{1}{2} \gamma \int_{i \in \Omega} q_i^2 di,$$

where $Q_0$ is the consumption of the numeraire good, $\Omega$ is the set of all product varieties, $q_i$ is the consumption of variety $i$, $\alpha (> 0)$ and $\beta (> 0)$ capture the substitution between the industry’s products and the numeraire, and $\gamma (> 0)$ represents the degree of product differentiation between varieties.\(^{10}\)

Consumers maximize their utility subject to budget constraints. Utility maximization yields the inverse demand function for variety $i$:

$$p_i = A - b q_i,$$

where $A = \frac{\alpha \gamma}{\beta M + \gamma} + \frac{\beta}{\beta M + \gamma} P$ and $b = \frac{\gamma}{L}$.  \(1\)

In this demand function, $p_i$ is the price of variety $i$, $M$ is the measure of $\Omega$, and $P = \int_{i \in \Omega} p_idi$ is the aggregate price of all varieties. The vertical intercept $A$ of the demand function summarizes the competitiveness of the product market: If more products compete in the market, the demand for each product will drop, representing cannibalization. Compared with CES preferences which give rise to constant and exogenous markups, the quasi-linear preference used in this model leads to variable markups that are more consistent with empirical evidence (Helpman, 2006). More importantly, it allows capturing the competition effects in both the product and acquisition markets, and thus investigation of the linkage between them. Although linear demand is highly specific, it is no more specific than constant elasticity demands that had been used in most studies. More importantly, our results hold as long as elasticities are not constant.

As firms engage in monopolistic competition, each firm takes $A$ as given when it chooses its output. If firm $a$ (i.e., a firm whose intrinsic efficiency is $a$) uses $x_i$ amount of capital to produce variety $i$, its chooses

\(^{10}\)In the demand for any particular product, the cross coefficient is $\beta$ while the own coefficient is $\beta + \gamma$. As $\gamma$ decreases, varieties are less differentiated. In the limiting case of $\gamma = 0$, the industry’s varieties become perfect substitutes.
output $q_i$ to maximize its profit from this variety:\footnote{Thus, we assume away the cannibalization effect that a firm’s output in one variety has on the profitability of its other varieties. As shown by Schwartz and Thompson (1986) and Baye, Crocker and Ju (1996), in an effort to gain market share from competing companies or forestall entry, many companies instruct their divisions to act as independent firms despite the apparent cannibalization effect. If our assumption is relaxed to accommodate the cannibalization effect, our major conclusions still hold, as demonstrated by Feenstra and Ma (2007) in a setting similar to Melitz (2003) with the addition of the cannibalization effect.}

$$\max_{q_i \geq 0} \pi_i \equiv (A - bq_i)q_i - \frac{q_i^2}{2ax_i}. \tag{2}$$

The resulting quantity, price and profit for this variety are, respectively,

$$q_i(x_i) = \frac{ax_iA}{2abx_i + 1}, \quad p_i(x_i) = \frac{(abx_i + 1)A}{2abx_i + 1}, \quad \text{and} \quad \pi_i(x_i) = \frac{ax_iA^2}{2(2abx_i + 1)}. \tag{3}$$

Greater demand (i.e., a larger $A$) leads to more output, a higher price and more profit for each variety. As the variety’s production efficiency, $ax_i$, increases, the output and profit are larger but the price is lower.

Since a variety’s profit, $\pi_i(x_i)$, is increasing and concave in $x_i$, a firm will always allocate its total capital among its varieties equally. Consequently, the subscript $i$ can be dropped in (3), and firm level productivity always equals plant level productivity.

### 2.3 The acquisition market

Let $R$ be the market price of capital. If a firm chooses scope $v$ and scale $x$, its capital cost is $(vx - 1)R$, where $vx - 1$ is the firm’s net demand for capital, which can be negative (meaning that the firm is selling capital). Thus, the firm’s optimization problem in the acquisition market is

$$\max_{x \geq 0, v \geq 0} \Pi(v, x) \equiv v\pi(x) - (vx - 1)R - mv^2 = v\tilde{\pi}(x) + R - mv^2,$$

where $\tilde{\pi}(x) \equiv \pi(x) - xR$ is the firm’s profit from each single plant, taking into consideration the capital cost but not the management cost. It is as if the firm first sells its endowment of a unit of capital in the acquisition market and then chooses how much capital to buy for each of its plant. Since there is no transaction cost, selling and buying capital is fully reversible.

Given the above decomposition of the profit function, the firm’s optimization problem can be solved in two steps: The optimal scale is $x^* = \arg \max_x \tilde{\pi}(x)$, which is independent of the value of $v$, and the optimal scope is then $v^* = \arg \max_v \Pi(v, x^*)$.
2.3.1 Scale

Define

\[ y = \frac{A}{\sqrt{2R}} \] (4)

In the acquisition market, firms treat \( A, R \), and consequently \( y \) as given. Since \( \pi(x) \) is proportional to \( \frac{A^2}{x} \), \( y^2 = \frac{A^2}{2R} \) represents the value of expanding capital relative to the capital price.

>From the first-order condition (the second-order condition is always satisfied), \( \frac{\partial \pi}{\partial x} = 0 \), we obtain the optimal scale for each plant:

\[ x^* = \begin{cases} 
0 & \text{for } a \leq a_0, \\
\frac{y\sqrt{a} - 1}{2\sqrt{a}} & \text{for } a > a_0,
\end{cases} \] (5)

where \( a_0 \equiv \frac{1}{y^2} \). Later we will show that in equilibrium, \( y > 1 \) and therefore \( a_0 < 1 \). (5) shows that very inefficient firms will not operate in the product market (their holding of capital is zero). Intuitively, when \( a \) is small, the optimal firm scale is small. Each firm’s profit from the product market will also be small, and selling all its capital (i.e., exiting the industry) gives the firm a better payoff. The least efficient firms exit the product market because they can realize higher profits from the acquisition market.\(^{12}\)

For firms that hold positive amount of capital, (5) says that plant scale depends on \( y \), which is some ratio between \( A \) and \( R \). Thus, firms’ demand for capital depends on both the product market and the acquisition market. In previous models, trade liberalization affected firm choices through either the product market (Melitz and Ottaviano, 2008; Nocke and Yeaple, 2006) or the factor market (Melitz, 2003; Bernard, Redding and Schott, 2009). In this model, however, the impact of trade liberalization works through both markets, and the linkage between the two (as all changes in the acquisition market are induced by changes in the product market) becomes crucial. Note that scale depends only on \( y \), the ratio between \( A \) and \( R \), rather than on their individual values, and it increases with \( y \). This is easy to understand: The marginal benefit of expanding scale (i.e., acquiring capital) is proportional to \( \frac{A^2}{x} \), while the marginal cost of acquiring capital is \( R \). The optimal scale balances marginal benefit with marginal cost and therefore should increase when \( \frac{A^2}{x} \) increases. As a result, a higher demand in the product market or a lower price in the acquisition market allows firms to expand plant scale. It also allows firms with lower efficiency to remain active (i.e., \( a_0 \) is lower).

Only sufficiently efficient firms hold capital. However, firms with higher intrinsic efficiency do not

\(^{12}\)Most studies follow Melitz (2003) to assume fixed production cost for the least efficient firms to exit. Without assuming fixed cost, Melitz and Ottaviano (2008) obtain exit when the constant marginal costs that some firms draw are higher than the intercept of a linear demand function. In our model, demand is linear as in Melitz and Ottaviano (2008) and there is no fixed production cost. A firm’s marginal cost can be arbitrarily small (when its output is close to zero) even if it is very inefficient. It will always generate some profit, however small, in the product market. Thus, a firm exits only because it can sell its capital in the acquisition market.
necessarily have larger plant scale, as

\[
\frac{\partial x^*}{\partial a} = \frac{2 - y\sqrt{a}}{4a^2b} \begin{cases} > 0 & \text{for } a \in (a_0, 4a_0), \\ < 0 & \text{for } a > 4a_0. \end{cases}
\]

When \(4a_0 < 1\), capital scale increases with the firm’s intrinsic efficiency when the efficiency is low, and decreases with efficiency when the efficiency is high, an inverse U-shaped relationship. The intuition is the following. Although the marginal returns from capital \((\partial \pi / \partial x)\) are always positive, the magnitude of the marginal return depends on a firm’s intrinsic efficiency. Note that \(\frac{\partial^2 \pi}{\partial a \partial x} = \frac{A^2(1-2abx)}{4(2abx+1)^2}\). When \(a\) is small, the production efficiency \(ax\) is small; each variety is sold at a high price, where demand is highly elastic. Increasing \(x\) allows the firm to lower its price. Since demand is very elastic, lowering price brings a large benefit. By contrast, when \(a\) is large, the marginal production cost is low; each variety is already sold at low price, where the demand is barely elastic. Increasing \(x\) still allows the firm to lower its price, but the extra benefit is small because demand is not very elastic. Thus, the marginal benefit of expanding scale is inversely U-shaped in \(a\), which translates into an inverse U-behavior of optimal scale in \(a\), as the marginal cost of expanding scale is fixed.

Although firms with higher intrinsic efficiency \((a)\) may have smaller capital scale \((x^*)\), they still have higher production efficiency, as \(ax^* = \frac{y\sqrt{\pi - 1}}{2b}\) increases with \(a\). This monotonicity ensures that more-efficient firms produce more:

\[
q^* = \frac{A}{2b} \left(1 - \frac{1}{y\sqrt{a}}\right)
\]

and thus \(\frac{\partial q^*}{\partial a} > 0\). Furthermore, plant and firm level productivity, which is

\[
\frac{q}{c} = \sqrt{\frac{2a}{R}}
\]

also increases with \(a\).

### 2.3.2 Scope

Given the optimal scale, a firm’s profit from each variety (after paying for capital) is

\[
\tilde{\pi}(x^*) = \begin{cases} 0 & \text{for } a \leq a_0, \\ \frac{R(y\sqrt{\pi - 1})^2}{2ab} & \text{for } a > a_0, \end{cases}
\]
which is strictly increasing in $a$ for $a > a_0$. Given $\tilde{\pi}(x^*)$, the first-order condition for optimal scope (the second-order condition is always satisfied), $\frac{\partial \Pi}{\partial v} = \tilde{\pi}(x^*) - 2mv = 0$, leads to $v^* = \frac{\tilde{\pi}(x^*)}{2m}$ or

$$v^* = \begin{cases} 
0 & \text{for } a \leq a_0, \\
\frac{R(y\sqrt{a}-1)^2}{4abm} & \text{for } a > a_0.
\end{cases}$$

(7)

Notice that $\frac{\partial v^*}{\partial a} > 0$ for all $a > a_0$.

Thus, more-efficient firms have larger scope. The optimal scope balances the marginal benefit from adding a variety, which equals $\tilde{\pi}(x^*)$, with the additional cost, which is $2vm$. Since the marginal benefit increases with $a$, the optimal scope should be larger when $a$ is higher.

Since both $q^*$ and $v^*$ are increasing in $a$, $q^*$ and $v^*$ are positively correlated. This confirms that intensive margins (i.e., each variety’s output) and extensive margins (i.e., the number of varieties) are positively correlated (Bernard, Redding and Schott, 2009).13 The driving force behind the correlation is the same as that discussed by Bernard, Redding and Schott: The monotonic change of production efficiency with intrinsic efficiency. However, plant-level production efficiency is endogenous in this model but not in theirs.

### 2.3.3 Size

We now turn to a firm’s capital size, $x^*v^*$. Since $x^*v^* = 0$ for $a \leq a_0$, the least efficient firms will not operate in the industry. For those firms remaining,

$$\frac{\partial x^*v^*}{\partial a} = \frac{R(y\sqrt{a}-1)^2}{16a^3b^2m} (4 - y\sqrt{a}) \begin{cases} 
> 0 & \text{for } a < 16a_0, \\
< 0 & \text{for } a > 16a_0.
\end{cases}$$

The relationship between firm size and intrinsic efficiency is jointly shaped by the properties of scale and scope. Although scope increases with $a$, scale is inversely U-shaped. As a result, firm size is also inversely U-shaped.14 Note that the turning point for size (at $16a_0$) is larger than that for scale (at $4a_0$). Nevertheless, the important message is that more efficient firms do not necessarily require more capital. Also note that the envelope theorem predicts that a firm’s profit still increases with $a$ even though its capital size may not.

The relationship between a firm’s intrinsic efficiency and its scale, scope, size and profit is summarized

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13In contrast, Nocke and Yeaple (2006) predicted a negative correlation because they assumed a negative relationship between plant-level marginal cost and product scope.

14Spearot’s (2008) model also suggested an inverse U-relationship between firm capital and efficiency.
in the following proposition.

**Proposition 1.** *A firm’s optimal scale and size increase in its intrinsic efficiency a when a is small, but may decrease in a when a is large. A firm’s optimal scope and profit, however, always increase in a.*

Turn now to the acquisition market to determine the equilibrium $R$ for a given $A$. Market clearing requires total capital supply to equal capital demand at the industry level. Since each firm is initially endowed with one unit of capital, the total capital supply in the industry is $\int_0^1 da = 1$. The total capital demand is

$$k \equiv \int_{a_0}^1 x^* v^* da = \frac{R}{8b^2m} \rho,$$

where

$$\rho \equiv \int_{a_0}^1 \frac{(y\sqrt{a} - 1)^3}{a^2} da = 2y^3 + 3y^2(1 - 2\ln(y)) - 6y + 1.$$

**Lemma 1.** *Given A, there exists a unique equilibrium capital price $R^*$ and correspondingly a unique $y^*$. Moreover,

$$\frac{dR^*}{dA} > 0 \quad \text{and} \quad \frac{dy^*}{dA} < 0.$$  

**Proof.** See the Appendix.

At this point, $A$ has been regarded as an exogenous variable, although later $A$ will be solved from the product market equilibrium condition (1). Given $A$, $R$ and the corresponding $y$ are uniquely determined. Lemma 1 says that $R^*$ increases with $A$. That is, when demand is higher, capital will be more expensive; when demand is lower, capital will be cheaper. Thus, any change in the product market will induce a change in the acquisition market that always has an offsetting effect on firm profitability: When the product market becomes more profitable, capital price changes will reduce that profitability. When a CES utility function is used, as by Melitz (2003) and by Bernard, Redding and Schott (2009), there is no cannibalization. Trade liberalization intensifies competition in the factor market because the demand for production input (i.e., labor) increases as firms try to produce more for the export market. This model adopts the utility function used by Melitz and Ottaviano (2008), so the product market becomes more competitive after trade liberalization. At the same time, firms demand more capital as they try to produce for the export market, pushing up the capital price. So trade liberalization affects firm choices through both the product market and the acquisition market. As will become evident
when we analyze unilateral trade liberalization, incorporating both markets into a coherent model is not a simple combination of the two effects. Rather, it leads to important conclusions that cannot be found when the two effects are analyzed separately.

To proceed further, we need to express the equilibrium capital price as a function of \( y \). From the equilibrium condition, \( k(R^*) = 1 \),

\[
R^*(y) = \frac{8b^2 m}{\rho(y)},
\]

(9)

Note that this expression is not the reduced form solution for \( R \), as \( y \) is defined on \( R \).

2.4 Industry equilibrium

We now turn to the equilibrium in both acquisition and product markets (i.e., to determine the equilibrium \( A \) and \( R \)). The measure of varieties is

\[
M(y) \equiv \int_{a_0}^{1} v^* da = \frac{R}{4bm} \psi = \frac{2b \psi}{\rho},
\]

(10)

where the last equality has made use of expression (9), and

\[
\psi \equiv \int_{a_0}^{1} \frac{(y\sqrt{a} - 1)^2}{a} da = y^2 - 4y + 3 + 2 \ln(y) \quad \text{with} \quad \frac{d\psi}{dy} > 0 \quad \text{and} \quad \frac{d(\psi \rho)}{dy} < 0.
\]

Substituting the optimal scale \( x^* \) into (3) yields the equilibrium price for individual products (a firm charges the same price for all its varieties):

\[
p = \frac{A}{2y} \left( 1 + \frac{1}{y\sqrt{a}} \right)
\]

for all \( a > a_0 \). The aggregate price is therefore

\[
P(y) \equiv \int_{a_0}^{1} v^* p da = \frac{AR \phi}{8bm y} = \frac{4b^2 \sqrt{m\phi}}{\rho^2},
\]

(11)

where

\[
\phi \equiv y^3 - 2y^2 - 2 + 3y - 2y \ln(y) \quad \text{and} \quad \frac{d\phi}{dy} > 0.
\]
By the definition of $A$ in (1) and using (10) and (11), we have

$$A^* = \frac{\alpha \gamma + \beta P}{\beta M + \gamma} = \frac{\alpha}{1 + \frac{\beta}{\gamma} \frac{\eta}{y'}},$$

where $\eta \equiv 2y\psi - \phi = y^3 - 6y^2 + 2 + 3y + 6y \ln(y)$. (12)

The three equilibrium values $A^*$, $R^*$ and $y^*$ are jointly determined by equations (4), (9) and (12). Using (9) and (12) in (4) yields the following equation expressed in $y$ only:

$$Z(y) = 0, \quad \text{where} \quad Z(y) \equiv \frac{\rho^3}{(y\rho + \frac{\beta}{\gamma} \eta)^2} - \frac{16b^2m}{\alpha^2}. \quad (13)$$

It can be shown that $Z'(y) > 0$, $Z(1) < 0$, and $Z(y) > 0$ when $y$ is sufficiently large. Therefore, there exists a unique $y^* > 1$ satisfying (13). Once $y^*$ is determined, $R^*$ is determined from (9) and subsequently $A^*$ is determined from (4) or (12). As a result,

**Proposition 2.** There exist unique equilibrium $A^*$ and $R^*$ in the closed economy.

### 3 Unilateral Trade Liberalization

Assume two identical countries, home and foreign, which initially do not trade with each other. The closed economy equilibrium in each country is as analyzed in Section 2. Now consider trade liberalization, which presents firms with both opportunities (of exporting) and challenges (of increased competition from imports). To better understand the two forces, we first study them in isolation by analyzing unilateral trade liberalization. Specifically, we consider the following scenario: There is free trade between the two countries in the numeraire good. In the differentiated goods industry, the home country opens up to imports but it cannot export to the foreign country. We will investigate in the next section the combined effect when a country can both import and export the differentiated goods. In order to isolate the effects of trade liberalization, we exclude the possibility of cross-border M&A. Moreover, assume no variable or fixed cost of trade. It will be clear by the end of this section that the results do not rely on these simplifications of trade costs. Finally, trade is always balanced through the move of the numeraire good, and hence we skip the balanced trade condition in the following analysis.

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15Thus, any change in the acquisition market is induced by a change in the product market.
3.1 Scale and scope after liberalization

Let the subscripts \( h \), \( f \) and \( e \) stand for home, foreign and export, respectively, and define the following notation:

\[
y_h \equiv \frac{A_h}{\sqrt{2R_h}}, \quad y_f \equiv \frac{A_f}{\sqrt{2R_f}}, \quad y_e \equiv \frac{A_e}{\sqrt{2R_e}}.
\]

(14)

Unilateral trade liberalization affects the home country only through \( A_h \). Given \( A_h \) and \( R_h \), home firms face the same optimization problem as in the closed economy case. Hence, the equilibrium \( x_h \) and \( v_h \) can be obtained by replacing \( A \), \( R \) and \( y \) with \( A_h \), \( R_h \) and \( y_h \), respectively, for \( x^* \) in (5) and \( v^* \) in (7).

In the foreign country, each firm can sell its products to both the domestic (foreign country) and export (home country) markets. Suppose that a foreign firm sells \( q_f \) of a particular variety in its domestic market and \( q_e \) of the same variety in the export market. These quantities solve the following optimization problem:

\[
\max_{q_f \geq 0, q_e \geq 0} \pi_f \equiv (A_f - bq_f)q_f + (A_h - bq_e)q_e - \frac{(q_f + q_e)^2}{2ax_f}.
\]

The interior solution (i.e., when the non-negative constraints are not binding) is

\[
q_f = \frac{(2abx_f + 1)A_f - A_h}{4b(abx_f + 1)} \quad \text{and} \quad q_e = \frac{(2abx_f + 1)A_h - A_f}{4b(abx_f + 1)}.
\]

(15)

Note that \( A_f \) and \( A_h \) are endogenous, and the Appendix shows that \( A_f > A_h \) in equilibrium (imports intensify competition in the home country’s product market). Hence \( q_f \) is always positive, while \( q_e \) is positive if and only if \( ax_f > \frac{1}{2b} \left( \frac{A_f}{A_h} - 1 \right) \). That is, a foreign firm exports if and only if its production efficiency is sufficiently high. Thus, even without any (fixed or variable) trade costs, some foreign firms (the low efficiency ones) sell only to their domestic market without exporting. \(^{16}\) The intuition is the following. Since demand from the foreign country is higher than that from the home country \( (A_f > A_h) \), the marginal benefit of selling to the foreign country is higher than that of exporting to the home country if the sales in the two countries are the same. When \( a \) (and consequently \( ax_f \)) is low, output is small. The marginal benefit of selling the last unit in the foreign country is still higher than the marginal benefit of selling the first unit in the home country, so the product is not exported. When \( a \) is larger, output is larger. After selling some quantity in the foreign country, the marginal benefit of selling a further unit falls below the marginal benefit of selling the first unit in the home country, and the firm starts to export.

\(^{16}\) All existing studies in the heterogeneous firm literature require either fixed export costs (e.g., Helpman, Melitz and Yeaple, 2004) or variable trade costs such as tariffs (e.g., Melitz and Ottaviano, 2008) to separate pure domestic firms from exporters. That is because marginal costs are exogenous in those models. Once marginal costs are endogenized as in our model, neither fixed cost nor trade cost is needed.
So a foreign firm with efficiency $a$ sells in both markets if $x_f > \frac{1}{2ab} \left( \frac{A_f}{A_h} - 1 \right)$, and the sales in the two markets are given by (15). The corresponding prices and a variety’s total profit in the two markets are

\[
\begin{align*}
    p_e &= \frac{A_h(2abx_f + 3) + A_f}{4(abx_f + 1)}, \quad p_f = \frac{A_f(2abx_f + 3) + A_h}{4(abx_f + 1)}, \\
    \pi_f &= \frac{2abx_f(A_h^2 + A_f^2) + (A_h - A_f)^2}{8b(abx_f + 1)}.
\end{align*}
\]

If $x_f \leq \frac{1}{2ab} \left( \frac{A_f}{A_h} - 1 \right)$, then $q_e = 0$. The firm makes its choices as if there were no exporting opportunity, and the results are the same as in a closed economy:

\[
\begin{align*}
    q_f &= \frac{A_fax_f}{2abx_f + 1}, \quad p_f = \frac{A_f(ax_f + 1)}{2abx_f + 1}, \quad \pi_f = \frac{A^2fax_f}{2(2abx_f + 1)}.
\end{align*}
\]

Each plant’s profit net of capital cost is $\tilde{\pi}_f = \pi_f - x_f R_f$, where $\pi_f$ is given above depending on whether or not $x_f > \frac{1}{2ab} \left( \frac{A_f}{A_h} - 1 \right)$. The firm chooses $x_f$ to maximize $\tilde{\pi}_f$, which yields

\[
x_f = \begin{cases} 
0, & \text{if } a \leq a_f, \\
\frac{y_f \sqrt{\alpha - 1}}{2ab}, & \text{if } a_f < a \leq a_e, \\
\frac{y_f \sqrt{\alpha - 1}}{2ab} + \frac{y_e \sqrt{\alpha - 1}}{2ab}, & \text{if } a > a_e,
\end{cases}
\]

where $y_f$, $y_e$, $a_f$, and $a_e$ are given by (14). Note that $a_f < a_e$ because $A_f > A_h$. For $a \in (a_f, a_e]$, the firm focuses on the domestic market, so its optimal scale is determined in the same way as in the closed economy (i.e., replacing $y$ with $y_f$ in (5)). For $a > a_e$, the firm serves both markets and its optimal scale is the sum of the optima for serving the two markets separately.

Given this optimal scale, a firm chooses its scope, $v_f$, to maximize its profit $v_f \tilde{\pi}_f - mv_f^2 + R_f$. The optimal choice is

\[
v_f = \begin{cases} 
0, & \text{if } a \leq a_f, \\
\frac{R_f(y_f \sqrt{\alpha - 1})^2}{4\alpha m}, & \text{if } a_f < a \leq a_e, \\
\frac{R_f(y_f \sqrt{\alpha - 1})^2 + (y_e \sqrt{\alpha - 1})^2}{4\alpha m}, & \text{if } a > a_e.
\end{cases}
\]

Thus, the additive rule also applies to scope: When a firm serves both markets, its optimal scope is the sum of the scope optimal for serving the domestic market alone and the scope for serving the export market.
alone.

Given the optimal scale, a foreign firm’s domestic sales (in the foreign country) are \( q_f = \frac{A_f}{2b} \left( 1 - \frac{1}{y_f \sqrt{a}} \right) \) and the corresponding price is \( p_f = \frac{A_f}{2b} \left( 1 + \frac{1}{y_f \sqrt{a}} \right) \) for \( a > a_f \). Its export sales (in the home country) are \( q_e = \frac{A_e}{2b} \left( \frac{y_e}{y_f} - \frac{1}{y_f \sqrt{a}} \right) \) and the corresponding price is \( p_e = \frac{A_e}{2b} \left( \frac{y_e}{y_f} + \frac{1}{y_f \sqrt{a}} \right) \) for \( a > a_e \). The firm’s productivity is \( \frac{q}{a} R_f \) for \( a > a_f \). Thus, exporters are more efficient in production, produce more and earn more than firms that produce only for the domestic market, confirming well-established empirical findings.

3.2 Industry equilibrium

To solve for the industry equilibrium, use \( f_i \) to denote \( f_i(y_i) \) for \( i = h, f, e \). In the home country, given \( A_h \), the acquisition market equilibrium is derived in exactly the same way as in the closed economy case. Hence, as in (9), there exists a unique capital price that satisfies

\[
R_h = \frac{8b^2 m}{\rho_h}. \tag{17}
\]

Let \( M_h \) be the measure of all home varieties and \( M_e \) be the measure of all imported varieties. Then (making use of \( R_f \) in (18)),

\[
M_h(y_h) = \int_{a_h}^1 v_h da = \frac{2b \psi_h}{p_h}
\]

\[
M_e(y_f, y_e) = \int_{a_e}^1 v_f da = \frac{2b \left[ \psi_f + \psi_e - \tilde{\psi} \right]}{\rho_f + \rho_e + 2 \tilde{\rho}},
\]

where \( \tilde{\psi}(y_f, y_e) = \int_{a_f}^{a_e} (\frac{y_f \sqrt{a} - 1}{a})^2 da > 0 \) and \( \tilde{\rho} = 1 + 2y_e y_f - y_f^2 + 2y_e^2 + (y_e + y_f)(y_f y_e - 3) - 2 \ln(y_e)(y_e^2 + 4y_e y_f + y_f^2) \).

Substituting a firm’s optimal scale in its equilibrium price and making use of \( \frac{A_h}{\psi} = \frac{w}{y_h} \), we obtain \( p_h = \frac{A_h}{2b} \left( 1 + \frac{1}{y_h \sqrt{a}} \right) \) for \( a \geq a_h \) in the home country and \( p_e = \frac{A_e}{2b} \left( 1 + \frac{1}{y_e \sqrt{a}} \right) \) for \( a \geq a_e \) in the foreign country. The aggregate price in the home market is the sum of the prices of all products, both home-made and imported:

\[
P_h(y_f, y_e) = \int_{a_h}^1 v_h p_h da + \int_{a_e}^1 v_f p_e da = \frac{4b^2 \sqrt{m} \phi_h}{(\rho_h)^{\frac{3}{2}}} + \int_{a_e}^1 v_f p_e da.
\]

In the foreign country, given capital price \( R_f \) and using the optimal scale and scope derived earlier, we
obtain (see the Appendix) the total capital demand

\[ k_f = \int_0^1 x_f v_f da = \frac{R_f}{8b_2m} (\rho_f + \rho_e + 2\bar{\rho}). \]

>From \( k_f = 1 \) we obtain

\[ R_f = \frac{8b_2m}{\rho_f + \rho_e + 2\bar{\rho}}. \quad (18) \]

The total product varieties is

\[ M_f(y_f, y_e) \equiv \int_1^0 v_f da = \frac{2b(\psi_f + \psi_e)}{\rho_f + \rho_e + 2\bar{\rho}}. \]

Using \( x_f \) from (16) and the fact that \( A_h = \frac{y_e}{y_f} \), we obtain the individual product price sold in the foreign market:

\[ p_f = \frac{A_f}{2} \left(1 + \frac{1}{y_f \sqrt{a}}\right) \text{ for } a \in (a_f, 1]. \]

Thus, the aggregate price is

\[ P_f(y_f, y_e) = \int_{a_f}^1 v_f p_f da = \frac{4b^2 \sqrt{m}}{(\rho_f + \rho_e + 2\bar{\rho})^2} \left[ \phi_f + \int_{a_e}^1 \frac{(y_e \sqrt{a} - 1)^2(y_f \sqrt{a} + 1)}{a \sqrt{a}} da \right]. \]

Finally, by definition, \( A_h \) and \( A_f \) are jointly determined by

\[ A_h = \frac{\alpha \gamma + \beta P_h}{\beta (M_h + M_e) + \gamma}, \quad (19) \]

\[ A_f = \frac{\alpha \gamma + \beta P_f}{\beta M_f + \gamma}. \quad (20) \]

Having expressed \( R_h \) and \( R_f \) as functions of \( y_f, y_e \) and \( y_h \) (from (17) and (18)), we have five unknowns, \( A_f, A_h, y_f, y_e \) and \( y_h \), and five equations, (19), (20) and the definitions of \( y_f, y_e \) and \( y_h \) (from (14)), which jointly determine the equilibrium.

### 3.3 Equilibrium comparison

Although the calculation has involved quite a few steps and variables, the mechanism is very clear. When the home country opens to imports, foreign firms start to sell their products in the home country but home firms are unable to export because the liberalization is unilateral. Such an import shock affects home firms’
output and consequently their capital scale and product scope. These changes in firm structure lead to some firms selling capital and some others buying, which in turn affects the home country’s product market. Firm and industry productivity change. The following proposition summarizes the direction of the change in the importing country.

**Proposition 3.** After unilateral trade liberalization, the home (importing) country undergoes the following changes as compared to the closed economy equilibrium:

1. **markets:** the product market becomes more competitive \( A_h < A^* \) and capital prices drop \( R_h < R^* \);
2. **firm structure:**
   (i) capital moves from high efficiency firms to low efficiency firms: there exists \( a_h^* \in (a_h^0, 1) \) such that \( x_h v_h > x^* v^* \) for \( a < a_h^* \) and \( x_h v_h < x^* v^* \) for \( a > a_h^* \);
   (ii) all existing firms expand scale: \( x_h > x^* \) for all \( a \geq a_h^0 \);
   (iii) low efficiency firms expand their scope while high efficiency firms reduce their scope: there exists \( a_h^v \in (a_h^0, 1) \) such that \( v_h > v^* \) for \( a < a_h^v \) and \( v_h < v^* \) for \( a > a_h^v \);
3. **aggregate scope:** the total number of varieties produced by home firms decreases \( M_h < M^* \);
4. **productivity:** firm and industry labor productivity improves.

**Proof.** See the Appendix.

The change in firm structure is shown in Figure 1, where the solid lines represent the situation before liberalization and the dotted lines represent the situation after liberalization. When a country opens itself to trade, the initial change is an influx of foreign products, which immediately reduces \( A_h \) in the home country (a negative demand shock). The increased competition (or equivalently lower demand for any
particular product) lowers every home firm’s profit, which is proportional to $A_i^2$. If the capital price does not change, every firm will adjust its structure by reducing both scale and scope (as shown in (5) and (7)), which generates excess supply of capital in the industry. $R_{th}$ must then drop until the acquisition market clears. The question is, at the new capital price, which firms will increase their capital and which will decrease.

Both $A_{th}$ and $R_{th}$ drop, and $R_{th}$ must drop more so that $y_{th}$ rises (i.e., $y_{th} > y^*$). To see this, note that $x_h = \frac{y_h \sqrt{\sigma - 1}}{2ab}$ and $v_h^* = \frac{R_{th}(y_h \sqrt{\sigma - 1})^2}{abm}$; that is, scale is increasing in $y_{th}$, and scope is increasing in both $y_{th}$ and $R_{th}$. If $y_{th}$ also drops, every firm will reduce both scale and scope, and the acquisition market cannot clear. But because capital is cheaper relative to demand ($y_{th}$ increases), every plant expands scale. The question is which firms sell capital (and thus must reduce their scope). Recall that a firm’s profit in the product market increases with its intrinsic efficiency. A change in the product market thus affects high efficiency firms more heavily than it affects those that are less efficient. Facing reduced product demand (which reduces the demand for capital) and lower capital prices (which raises the demand for capital), more efficient firms will be affected mainly by the lower product demand. As a result, their demand for capital will decrease and they will sell capital and reduce their scope. By contrast, less efficient firms will be affected mainly by the lower capital price and they will buy capital and expand their scope. The total number of product varieties produced by home firms decreases. Note that the total number of varieties consumed by home consumers is likely to increase, as they consume both domestic products and imports.

All plants increase their scale, which implies that some plants originally not in the market now find it optimal to buy some capital and re-enter (the result $a_{th} < a_{th}^0$ is in the proof of the proposition). Such entries will be at the lower end of the efficiency spectrum. That trade liberalization may induce entry by low efficiency firms is a surprising but logical result. In the importing country, the influx of imports reduces the demand for home products ($A_{th}$ drops), which in turn reduces the capital price ($R_{th}$ drops). Because low efficiency firms are mainly affected by changes in the acquisition market, they buy capital and re-enter. Obviously, this result relies on the linkage between the product market and the acquisition market, a feature absent in all previous models. This analysis has shown that the response in the factor market tends to offset the change in the product market, and may even outweigh it so that unilateral trade liberalization induces the entry of low efficiency firms rather than their exit.\footnote{One must be cautious in comparing this result with empirical findings such as exit by less efficient firms after Chile opened to trade unilaterally in 1970s and 1980s (Pavcnik, 2002). Our result is obtained in a partial equilibrium framework, where the lower demand for capital in this industry reduces capital price, while the Chile case is obviously an effect of general equilibrium with the possible flow of extra capital to other industries.}

The big picture is that resources (capital) move from more-efficient to less-efficient firms, which leads to a more even distribution of firm size in terms of capitalization.\footnote{This direction of resource reallocation is opposite to empirical findings by Breinlich (2008) about how Canadian firms responded to the Canada-United States Free Trade Agreement (CUSFTA). The CUSFTA is a bilateral trade liberalization, while the results in Proposition 3 obtain under unilateral trade liberalization, which has not been tested empirically. The next section will show that resource reallocation under bilateral trade liberalization is consistent with Breinlich’s finding.} Measured by output, firm size is also distributed more evenly after trade liberalization. Despite the seemingly inefficient redistribution of resources between firms, labor productivity still increases. At the firm level, every firm expands the scale of each plant, and every product’s output is reduced due to the intensified competition. Both changes lower average
production cost and therefore raise labor productivity.\(^{19}\) At the industry level, two factors oppose the improvement of labor productivity: high efficiency firms reduce their product scope while low efficiency firms increase their product scope, and new entrants have lower TFP than all existing firms. Since industry productivity is the average of firm-level productivity weighted by scope, industry productivity performs worse than firm productivity. Nevertheless, firms’ productivity improves so much that industry productivity also increases.\(^{20}\)

So competition from imports forces home firms to rationalize resources so that labor productivity increases, which is hardly surprising. Nevertheless, it is surprising that productivity improves in the absence of export opportunities and as resources move from high efficiency to low efficiency firms. In models with fixed plant-level productivity, any change in industry productivity must be through entry/exit and output reallocation. There is no way that industry productivity can improve when resources move from high efficiency to low efficiency firms. In this model, trading capital and plant-level scale (which determines plant-level and firm-level productivity) are two separate variables, although they are connected. Despite the unfavorable movement of capital across firms, all plants increase scale, which is the driving force for labor productivity improvements.

In the foreign country, analytical comparisons are intractable,\(^{21}\) but numerical calculations confirm the intuition that almost every change is the opposite of what happens in the home country. Capital becomes more expensive. The least efficient firms exit. High efficiency firms buy capital and increase their scope while low efficiency firms sell capital and reduce scope, leading to a more skewed distribution of firm size measured by capital or output. Every firm reduces scale. Firm and industry labor productivity decrease. The only change that is similar to the home country is that the product market in the foreign country also becomes more competitive (\(A_f < A^*\)) even though the foreign country does not face any competition from imports. Note that what matters for the change in the acquisition market is profitability in the product market, where loosely speaking the total demand is \(A_f + A_h\), not \(A_f\).

## 4 Bilateral Trade Liberalization

Trade liberalization brings producers greater opportunities (for exporting) and greater competition (from imported products). The previous section investigated unilateral trade liberalization where only one effect is present in a country. This section will examine the combined effects of bilateral trade liberalization where

\(^{19}\)If marginal costs are constant with output, the effect of reduced output will disappear, but the effect of increased scale will still be present and the result will continue to hold.

\(^{20}\)An indication of the dominance can be obtained by comparing the productivity of the least efficient firm before and after trade liberalization. Although firm \(a_h\) has lower intrinsic efficiency than firm \(a_0^\ast\), the former’s productivity is higher due to increased capital scale: \(c(a_h^0) = \sqrt{2\lambda a_h} = \frac{A^*}{4}\). Similarly \(c(a_h^0) = \frac{A^*}{4}\). As a result, \(c(a_h^0) \frac{A^*}{4} < c(a_h^0) \frac{A^*}{4}\).

\(^{21}\)In the home country, although there are no closed-form solutions, analytical comparisons are possible because the expressions for scale and scope are exactly the same as in the closed economy, and we can prove that \(y_h > y^\ast\) because (19) and (12) differ only by an extra term. In the foreign country, by contrast, the expressions for scale and scope are different from those in the closed economy, and the expressions contain \(y_f\) and \(y_e\) that cannot be easily compared with \(y^\ast\).
everything is the same except that home firms in the differentiated goods industry can now export to the foreign country.

4.1 Equilibrium after bilateral trade liberalization

Since the two countries are symmetrical, there must be a symmetric equilibrium, which we focus on. Consider the home country. Define

\[ y_t = \frac{A_t}{\sqrt{2}R_t} \quad \text{and} \quad a_t = \frac{1}{y_t^2} \]

where the subscript \( t \) stands for bilateral trade. Given \( A_t \), a firm chooses the quantity of each variety it will produce for both markets, \( q_t \), to maximize its combined profit from both product markets for the variety:

\[
\max_{q_t \geq 0} \pi_t \equiv 2(A_t - bq_t)q_t - \frac{(2q_t)^2}{2ax_t}. \tag{21}
\]

The optimal output, price and profit are, respectively,

\[ q_t = \frac{ax_tA_t}{2(abx_t + 1)}, \quad p_t = \frac{(abx_t + 2)A_t}{2(abx_t + 1)}, \quad \text{and} \quad \pi_t = \frac{ax_tA_t^2}{2(abx_t + 1)}. \]

Given capital price \( R_t \), the firm chooses its optimal scale for each plant to maximize the plant-level profit

\[ \tilde{\pi}_t = \pi_t - x_tR_t. \]

The solution is

\[ x_t = \begin{cases} 
0, & \text{if } a \leq a_t, \\
\frac{y_t\sqrt{a_t-1}}{ab}, & \text{if } a > a_t. 
\end{cases} \tag{22} \]

The firm chooses its scope, \( v_t \), to maximize its total profit \( v_t\tilde{\pi}_t(x_t) - mv_t^2 + R_t \), yielding the following optimal scope

\[ v_t = \begin{cases} 
0, & \text{if } a \leq a_t, \\
\frac{R_t(y_t\sqrt{a_t-1})^2}{2abm}, & \text{if } a > a_t. \tag{23} 
\end{cases} \]

With this firm structure, the total capital demand \( k_t \equiv \int_0^1 x_tv_tda = \frac{R_t}{2b^2m}\rho_t \). A unique acquisition market equilibrium is then determined by \( k_t(R_t) = 1 \), or

\[ R_t(y_t) = \frac{2b^2m}{\rho_t}. \tag{24} \]
The aggregate variety supplied by firms in a country is \( M_t(y_t) = \int_{a_t}^{1} v_t \, da = \frac{b \psi}{\rho_t} \), and there are \( 2M_t \) varieties in each country’s product market. The aggregate price is \( P_t(y_t) = 2 \int_{a_t}^{1} v_t p_t \, da = \frac{2b^2 \sqrt{\psi \rho_t}}{(\rho_t)^{\frac{3}{2}}} \).

By definition, \( A_t \) is determined by

\[
A_t(y_t) = \frac{\alpha \gamma + \beta P_t}{2 \beta M_t + \gamma} = \frac{\alpha}{1 + \frac{\beta}{L} y_t \rho_t}.
\]

Equations (24) and (25) and the definitions of \( y_t \) jointly determine the three unknowns \( A_t, R_t \) and \( y_t \) as the equilibrium in bilateral trade liberalization.

In fact, instead of going through these steps, there is an alternative and much simpler way to obtain the equilibrium. Note that the plant’s maximization problem (21) can be rewritten as

\[
\max_{q_t} \pi_t = A_t - b \frac{1}{2} (2q_t) - \frac{(2q_t)^2}{2 \alpha x_t}.
\]

If we replace \( (2q_t) \) with \( q \), the problem is identical to that in the closed economy case, (2), except that the demand slope is \( \frac{b}{2} \) instead of \( b \). In other words, if in the closed economy we have equilibrium solutions \( p(A, a, x, b), q(A, a, x, b) \) and \( \pi(A, a, x, b) \), then under bilateral trade, the solutions will be \( p_t = p(A, a, x, \frac{b}{2}), q_t = \frac{1}{2} q(A, a, x, \frac{b}{2}) \) and \( \pi_t = \pi(A, a, x, \frac{b}{2}) \). It is straightforward to reach the following conclusion: if \( y^*(b), A^*(b) \) and \( R^*(b) \) are the equilibria in the closed economy, then the bilateral trade equilibria are \( y_t = y^*(\frac{b}{2}), A_t = A^*(\frac{b}{2}), \) and \( R_t = R^*(\frac{b}{2}) \).

These two approaches are not only mathematically equivalent, but also economically isomorphic. Without any trade cost, a firm views the two product markets (domestic and export) as identical. At the same time, a consumer is assumed not to treat domestic varieties differently from imported varieties. Compared to the closed economy, therefore, a firm’s demand under bilateral trade is doubled (population increases from \( L \) to \( 2L \)), which is captured by the change of the slope from \( b \) to \( \frac{b}{2} \) (because \( b = \frac{\gamma}{L} \)). From the firm’s point of view, for given \( A \) and \( R \), the change in the slope of the demand curve is the only difference between the closed economy and bilateral trade, so its optimal scale and scope are doubled after bilateral trade liberalization. Of course, \( A, R \) and consequently \( y \) will be different from those in the closed economy.

Since the acquisition market is not international, the change in the slope of the demand curve does not affect the acquisition market directly, but it does so indirectly through changes in each firm’s optimal scale and scope. This explains why \( R_t \) in (24) is obtained simply by replacing \( b \) with \( \frac{b}{2} \) from \( R^* \) in (9). Note that the demand intercept, \( A \), is not affected by the population size \( L \), or \( b \), directly. Also note that in addition to substituting \( b \) with \( \frac{b}{2} \) to get the bilateral trade equilibrium, the domestic market’s total varieties under bilateral trade are twice those produced by the domestic firms, and the aggregate price is also doubled.

These two observations allow us to map the equilibrium condition for \( A^* \) in (12) to that for \( A_t \) in (25).
4.2 Equilibrium comparison

We have seen how home firms respond to unilateral trade liberalization. Their responses to bilateral liberalization can be rather different, because they now have the opportunity to export their products, which in turn affects their optimal scale and scope.

Using (5) and (22), a firm expands its scale, i.e., \( x_t(a) > x^*(a) \), if and only if

\[
(2y_t - y^*)\sqrt{a} > 1.
\]  

(26)

Similarly, we can obtain (see Appendix) conditions for \( v_t(a) > v^*(a) \) and \( x_t(a)v_t(a) > x^*(a)v^*(a) \). In the Appendix we prove that \( y_t < y^* \) and so \( a_t > a_t^* \). These comparisons allow us to establish the following proposition about the impacts of bilateral trade liberalization.

**Proposition 4.** After bilateral trade liberalization,

1. **markets:** the demand in each country drops but the combined demand in the two countries increases \((A_t \in (\frac{A^*}{2}, A^*))\); capital prices may increase or decrease;
2. **exit:** the least efficient firms exit \( (a_t > a_t^*) \);
3. **firm structure:**
   1. capital moves from low efficiency to more efficient firms: there exists \( \hat{a}_s \in (a_t, 1) \) such that \( x_tv_t < x^*v^* \) for \( a \in (a_t, \hat{a}_s) \) and \( x_tv_t > x^*v^* \) for \( a \in (\hat{a}_s, 1] \);
   2. high efficiency firms expand while less efficient firms reduce their scale: there exists an \( \hat{a}_x \in (a_t, 1] \) such that \( x_t < x^* \) for \( a \in (a_t, \hat{a}_x) \) and \( x_t > x^* \) for \( a \in (\hat{a}_x, 1] \);
   3. high efficiency firms expand their scope while low efficiency firms reduce their scope: there exists an \( \hat{a}_v \in (a_t, 1] \) such that \( v_t < v^* \) for \( a \in (a_t, \hat{a}_v) \) and \( v_t > v^* \) for \( a \in (\hat{a}_v, 1] \).
4. **aggregate scope:** the total number of varieties consumed in each country increases \((2M_t > M^*)\);
5. **productivity:** if the capital price increases, firm-level labor productivity decreases; if the capital price declines, firm-level and industry-level labor productivity increase.

**Proof:** See the Appendix.

Note that \( \hat{a}_s < 1 \) (there must be some firms which buy capital) because some firms (at least those who exit) sell capital. It is possible that \( \hat{a}_x = 1 \) (i.e., all firms reduce scale) or \( \hat{a}_v = 1 \) (i.e., all firms reduce scope), but they will not happen at the same time, otherwise we would not have \( \hat{a}_s < 1 \). So after bilateral trade liberalization, high efficiency firms buy capital to increase scale or scope or both, low efficiency firms sell capital to reduce scale or scope or both, and the least efficient firms sell all their capital and exit the industry. Figure 2 illustrates the changes to firm structure. Every change is the opposite of what happens in the importing country under unilateral trade liberalization, except that all firms expand their scale after unilateral liberalization, while only high efficiency firms do so after bilateral liberalization.

As has been mentioned, the export opportunity for home firms under bilateral but not unilateral trade liberalization is responsible for these differences. Under unilateral liberalization, the home country’s capital
price drops because competition from imports reduces home firms’ profits in the product market, which induces every firm to cut down its scale and scope as the initial response. In the case of bilateral trade liberalization, home firms still face competition from imports, but they can now export, which increases their demand for capital. In the end, some firms’ net demand for capital will increase; the question is who’s. Although each single product market becomes more competitive after trade liberalization ($A_t < A^*$), the combined demand in the product market is higher ($2A_t > A^*$) because every firm now serves both domestic and export markets. The combined effect of greater opportunity and greater challenge is thus a higher product demand (a positive demand shock). As always, any change in the product market affects high efficiency firms more heavily than it affects low efficiency firms. Increased demand in the product market leads firms to demand more capital. Since all firms are affected equally by any change in the acquisition market, if some firms buy capital, they must be the more efficient ones. Low efficiency firms will sell capital, and firms with very low efficiency will sell all their capital and exit. Although the number of varieties produced in each country may decrease, the aggregate number of varieties consumed increases, as both home-made products and imports are available in each country.

So this analysis predicts that bilateral trade liberalization pushes capital from low efficiency firms to those more efficient, which leads to a more skewed distribution of firm size in terms of capital and output. The prediction is consistent with Breinlich’s (2008) finding about M&As among Canadian firms after the Canada-United States Free Trade Agreement. Maksimovic and Phillips (2001) contend that “industry shocks alter the value of the assets and create incentives for transfers to more productive uses”, and they showed that productive assets tend to move from less efficient firms to more efficient firms when an industry undergoes a positive demand shock.

The comparison between $R_t$ and $R^*$ is ambiguous. On the one hand, firms demand more capital when they have the opportunity to export. On the other hand, capital moves to more efficient firms who, due
to their higher efficiency in using capital, do not need much capital. The net effect depends, among other things, on the substitutability between products, i.e., $\frac{\gamma}{\beta}$. Capital prices are more likely to drop ($R_t < R^*$) when products are closer substitutes.\(^{22}\)

In terms of labor productivity, a firm’s average cost is $c = \sqrt{R \tau}$. Thus, if $R$ decreases, firm-level productivity improves; if $R$ increases, firm-level productivity drops. Firm productivity may decrease because each plant produces a much larger quantity of output. Industry-level productivity is the average of firm-level productivity weighted by each firm’s scope. Since more efficient firms increase their scope while less efficient reduce scope and least efficient exit, industry productivity performs better than firm productivity. When firm productivity improves (i.e., when capital becomes cheaper), industry productivity also improves. When firm productivity decreases (i.e., when capital becomes more expensive), industry productivity may increase or decrease.

While the movement of capital from low efficiency to more efficient firms is hardly surprising, the prediction that bilateral trade liberalization may reduce productivity has yet to find empirical support. What this analysis shows is a theoretical possibility. Real life situations may involve parameter values under which productivity is always improved by trade liberalization. Furthermore, the deteriorating productivity predicted is driven mainly by dramatically increased output, which matters when marginal cost is increasing with output. If marginal costs are constant or almost so, productivity is more likely to improve.

Firms’ responses to unilateral and bilateral trade liberalization are very different. These differences come from the different impacts trade liberalization has on the product markets. Under unilateral trade liberalization, firms in the importing country face a negative demand shock while firms in the exporting country face a positive shock. Under bilateral trade liberalization, firms face greater competition from imports and greater opportunities to export, and the combined effect is a positive shock in both countries.\(^{23}\) Once the situation in the product market is determined, every other response follows. Changes in the product market leads to changes in the acquisition market, and the two markets jointly determine each firm’s optimal scale and scope and consequently the redistribution of capital and productivity. Resources move to high efficiency firms under a positive demand shock, and to low efficiency firms under a negative demand shock.

It is instructive to compare these findings with those of Bernard, Redding and Schott (2009) and Nocke and Yeaple (2006), both of whom modelled endogenous product scope. Bernard, Redding and Schott emphasized intra-firm heterogeneity. Due to the CES preference they assumed, trade liberalization affected firm choices only through the labor market. Higher wages brought by trade liberalization forced marginal firms to exit and surviving firms to drop marginal products. In our model, capital seems to play a similar role in inducing less efficient firms to give up their productive resources and exit. However, there is an important difference between labor and capital. Labor is a variable input that does not affect productivity. Wage rate affects all products uniformly so that all firms reduce scope. The industry is re-organized through the exit

\(^{22}\)In our numerical example, products need to be extremely substitutable for the capital price to drop: $\alpha = 20, L = m = \gamma = 1$ and $\beta = 100$, i.e., $\frac{\gamma}{\beta} = \frac{1}{100}$. For more reasonable values of $\frac{\gamma}{\beta}$, capital prices increase after bilateral trade liberalization. In addition to very small $\frac{\gamma}{\beta}$, other factors that makes it more likely for the capital price to drop are small $m$ and large $L$ or $\alpha$.

\(^{23}\)It is reasonable to conjecture that when the two countries are asymmetric, either effect may dominate so that the product market may undergo a positive or negative demand shock depending on the situation.
of marginal firms and marginal products, which is still output reallocation. Product-level productivity does not change. By contrast, capital is a productive resource that directly determines productivity. Trade liberalization leads not only to entry or exit of marginal firms, but also to adjustments in plant-level productivity. And the adjustment is not uniform: while some firms increase scale, others reduce scale; while some firms expand their scope, others reduce it. The industry is re-organized by the redistribution of resources, which affects productivity more directly and more strongly than the redistribution of output (and the corresponding inputs).

Nocke and Yeaple (2006) allowed firms to trade product lines. The increase and decrease of product scope in their model can therefore be interpreted as M&A of product lines. However, they did not model the M&A market explicitly, a market which plays a central role in our model. Furthermore, because they imposed an exogenous tradeoff between product scope and productivity (only one was an independent choice), their predictions are the opposite of ours. For example, they found that high efficiency firms have low productivity (thus a negative correlation between internal and external margins), and that bilateral trade liberalization leads to a more even distribution of firm size. In our model, scope and scale are two independent choices. Firms may increase or decrease both, or increase one and reduce the other. We predict (as did Bernard, Redding and Schott (2009)) that high efficiency firms will have high productivity (thus a positive correlation between internal and external margins), and that bilateral trade liberalization leads to a more skewed distribution of firm size.

5 Concluding Remarks

This study treated heterogeneous firms with endogenous product scope and plant-level labor productivity. In order to truly endogenize productivity, the model incorporated capital and a market for trading capital through mergers and acquisitions. Capital differs from labor in that it directly determines productivity while labor is a variable input that does not affect productivity. The analysis shows how trade liberalization changes the demand in the product market, which induces a change in the acquisition market. Both markets affect firms’ choices of scale and scope and consequently the distribution of capital and productivity. While capital is homogenous and affects firms uniformly (i.e., the cost of acquiring capital is the same for all firms), the product market affects more efficient firms more heavily than it affects less efficient firms. So, if trade liberalization brings a positive demand shock to the product market, capital moves to high efficiency firms and the distribution of firm size becomes more skewed. When the product market undergoes a negative demand shock, the opposite happens. This analysis highlights the importance of resource allocation, which affects productivity more directly and more strongly than output reallocation. It also shows that product and acquisition should not be considered separately. By endogenizing plant level productivity, we show that firm-level and industry-level productivity may move in opposite directions, and that trade may raise labor productivity even in the face of decreasing returns to scale.

Given this central theme of resource reallocation as a response to changes in the product market, the supply of capital was assumed in this analysis to be perfectly inelastic. None of the findings, however,
depend on this assumption. It is clear that the results still hold if the supply of capital is upward sloping. The increasing marginal cost (in output) assumed here has been widely used in M&A studies in industrial organization (e.g., Perry and Porter, 1985). It can be justified as a short-run cost derived from a constant-returns-to-scale production function with a fixed level of capital, reflecting the idea that production and trade are short-run activities while M&A is a longer-run activity that affects short-run cost structure. Such a cost structure generates simpler expressions than the constant marginal costs commonly assumed in international trade studies because it ensures interior quantity choice for an arbitrary level of capital. Constant marginal cost relationships such as \( c(q|a, x) = \frac{q}{a} \) and \( c(q|a, x) = (c' - \sqrt{a}x)q \) have been tested, and Propositions 1, 3 and 4 still hold.

To focus on the linkage between the product and acquisition markets, we have assumed away the possibility of cross-border M&A so that trade liberalization affects the acquisition market indirectly through changes in the product market. When the two trading countries are symmetric, relaxing this assumption will not change any result under bilateral trade liberalization, as the capital price will be the same in any case. Under unilateral or bilateral trade liberalization with asymmetric countries, allowing cross-border M&A will add another important dimension to firms’ responses. This natural and interesting extension of the present analysis will be the topic of a separate study.

Appendices

**Proof of Lemma 1**

Since \( R = \frac{A^2}{2y^2} \), we have \( k = \frac{A^2}{16b^2m} \xi(y) \), where \( \xi(y) = \frac{\nu(y)}{y^2} \) and \( \xi'(y) > 0 \). Hence,

\[
\frac{\partial k}{\partial R} = \frac{A^2}{16b^2m} \xi'(y) \frac{\partial y}{\partial R} < 0.
\]

Moreover, when \( R = \frac{A^2}{2} \), we have \( y = 1, \xi = 0 \) and so \( k = 0 \). On the other hand, when \( R \) is sufficiently close to zero, \( y \) is sufficiently large, which means \( \xi \) can be large enough so that \( k > 1 \). Thus, given \( A \), there exists a unique equilibrium capital price, \( R^* \), which is determined by \( k(R^*) = 1 \).

From \( k(R^*) = \frac{A^2}{16b^2m} \xi(y^*) = 1 \) and \( \xi'(y) > 0 \), we have \( \frac{dy^*}{dx} < 0 \). Because \( R^* = \frac{A^2}{2y^2} \) and \( \frac{dy^*}{dx} < 0 \), clearly \( \frac{dR^*}{dx} > 0 \).

**Proof that \( A_f > A_h \) in unilateral trade liberalization**

Suppose \( A_f \leq A_h \). Then \( q_e \) is always positive, but \( q_f \) is positive if and only if \( ax_f > \frac{1}{2a} \left( \frac{A_f}{A_h} - 1 \right) \).

The following equilibrium derivation is similar to that in the text.

In the product market, if \( ax_f > \frac{1}{2a} \left( \frac{A_f}{A_h} - 1 \right) \), the equilibrium is the same as that given the text; if \( ax_f \leq \frac{1}{2a} \left( \frac{A_f}{A_h} - 1 \right) \), then \( q_f = 0 \) while the firm only gets profit from the export market with \( q_e = \frac{A_hax_f}{1+2ax_f} \), \( p_e = \frac{A_h(1+ax_f)}{1+2ax_f} \), and \( \pi_f = \frac{A_h^2ax_f}{2(1+2ax_f)} \).
From the profit function, \( \bar{\pi}_f = \pi_f - x_f R_f \), we get the optimal scale \( x_f = 0 \) if \( a < a_c \); \( x_f = \frac{y_c \sqrt{\pi - 1}}{2ab} \) if \( a_c \leq a \leq a_f \); and \( x_f = \frac{(y_f + y_c) \sqrt{\pi - 2}}{2ab} \) if \( a > a_f \). As a result, \( \bar{\pi}_f = \frac{R_f (y_c \sqrt{\pi - 1})}{2ab} \) if \( a_c \leq a \leq a_f \), and \( \bar{\pi}_f = \frac{(y_f + y_c) \sqrt{\pi - 2}}{2ab} \) if \( a > a_f \). The firm chooses its scope, \( v \), to maximize its profit \( v \bar{\pi}_f - v^2 m + R_f \). The optimal scope is \( v_f = 0 \) if \( a < a_c \), \( v_f = \frac{R_f (y_c \sqrt{\pi - 1})}{2ab} \) if \( a_c \leq a \leq a_f \), and \( v_f = \frac{(y_f + y_c) \sqrt{\pi - 2}}{2ab} \) if \( a > a_f \).

In the acquisition market, \( R_h \) and \( R_f \) are of the same in form as (17) and (18), respectively. In the home country’s product market, \( M_h \) is the same as given in the text. In the foreign country, the export variety is \( M_e = \int_{a_e}^{1} v f \, da \) while domestic variety is \( M_f = \int_{a_f}^{1} v f \, da = M_e - \int_{a_e}^{a_f} v f \, da \leq M_e \). The home country aggregate price \( P_h \) is the same as given in the text, but in the foreign country \( P_f = \int_{a_f}^{1} v f p f \, da \).

The equilibrium \( A_h \) and \( A_f \) are jointly determined by

\[
A_h[\beta(M_h + M_e) + \gamma] = \alpha \gamma + \beta P_h, \tag{A1}
\]
\[
A_f(\beta M_f + \gamma) = \alpha \gamma + \beta P_f, \tag{A2}
\]

which have the same form as (19) and (20), respectively.

(A1) can be rewritten as

\[
A_h(\beta M_f + \gamma) + \beta(A_h M_h - P_h) + A_h(\beta M_e - M_f) + \beta P_f = \alpha \gamma + \beta P_f. \tag{A3}
\]

Note that \( M_e - M_f \geq 0 \) and \( \beta(A_h M_h - P_h) > 0 \) because \( A_h - p_h > 0 \) for every home variety. Thus, comparing (A2) and (A3), we must have \( A_f > A_h \), contradicting the supposition \( A_f \leq A_h \). \( Q.E.D. \)

**Derivation of \( k_f \)**

Using the optimal scale and scope, \( k_f \equiv \int_{0}^{1} x_f v f \, da = \int_{a_e}^{a_f} \left[ \frac{y_c \sqrt{\pi - 1}}{2ab} \left( \frac{R_f (y_f \sqrt{\pi - 1})^2}{4abm} \right) \right] \, da + \int_{a_e}^{a_f} \left[ \frac{y_f \sqrt{\pi - 2}}{2ab} \left( \frac{R_f (y_f \sqrt{\pi - 2})^2}{4abm} \right) \right] \, da + \int_{a_e}^{1} \left[ \frac{y_c \sqrt{\pi - 1}}{2ab} \left( \frac{R_f (y_f \sqrt{\pi - 1})^2}{4abm} \right) \right] \, da + \frac{R_f}{8b^2 m} 2 \hat{\rho} = \frac{R_f}{8b^2 m} \rho_f + \frac{R_f}{8b^2 m} \rho_e + \frac{R_f}{8b^2 m} 2 \hat{\rho} \). In the last step, the first two terms are derived exactly as \( \rho \) was in Section 2.3, and the third term is obtained from the remaining part as

\[
\frac{R_f}{8b^2 m} 2 \hat{\rho} = \int_{a_e}^{1} \left[ \frac{y_c \sqrt{\alpha - 1}}{2ab} \left( \frac{R_f (y_c \sqrt{\alpha - 1})^2}{4abm} \right) \right] \, da + \int_{a_e}^{a_f} \left[ \frac{y_f \sqrt{\alpha - 1}}{2ab} \left( \frac{R_f (y_f \sqrt{\alpha - 1})^2}{4abm} \right) \right] \, da + \int_{a_e}^{1} \left[ \frac{y_c \sqrt{\alpha - 1}}{2ab} \left( \frac{R_f (y_f \sqrt{\alpha - 1})^2}{4abm} \right) \right] \, da
\]

\[
= \frac{R_f}{8b^2 m} 2[1 + 2y_c y_f - y_f^2 + 2y_c^2 + (y_c + y_f)(y_c y_f - 3) - 2 \ln(y_c)(y_c^2 + 4y_c y_f + y_f^2)].
\]
Because capital supply is fixed at 1, it is then straightforward to see that there exists a unique equilibrium $R_f$ that clears the capital market (regardless of the slope of $k_f$).

**Proof of Proposition 3**

We first compare $y_h$ and $y^*$. Given any $M_c$, equation (19) can be rewritten as $A_h(\beta M_h + \gamma) = \alpha \gamma + \beta \frac{A_h R_h \phi^h}{\text{sum} y_h} - g$, where $g \equiv A_h \beta M_c - \beta \int_{a_c}^{1} v_f p_e da = \beta \int_{a_c}^{1} v_f (A_h - p_e) da > 0$. Following the analysis of the closed economy (especially, (13)), we know that $y_h$ is determined by

$$Z_h(y_h) = \frac{\rho^3}{(y_h \rho_h + \frac{\rho}{\gamma})^2} - \frac{16b^2 m}{(\alpha - \frac{\beta}{\gamma})^2} = 0.$$ 

Since $Z'_h > 0$ and $Z_h(\cdot) < Z(\cdot)$, we must have $y_h > y^*$.

Recall that $x_h = \frac{y_h \sqrt{b}}{2a_h}$ for $a \geq a_h$. From $y_h > y^*$, we immediately obtain $x_h > x^*$ and $a_h < a_h^*$. We claim $R_h < R^*$. Suppose this were not true. Then, the inequality $R_h > R^*$, together with $y_h > y^*$, indicates that $v_h > v^*$ for all firms (recall that $v_h = \frac{R_h (y_h \sqrt{b})}{2 a_h \gamma}$) and so all firms expand their scope. As a result, all firms buy capital. This cannot occur at equilibrium.

Because $\frac{\partial R}{\partial y} > 0$ (Lemma 1) and $R_h < R^*$, we have $A_h < A^*$. In equilibrium, some firms must sell capital, which implies $v_h < v^*$ for some firms. This inequality is equivalent to $a > a_h^*$ where $a_h^* : (\frac{\sqrt{b}}{2} \frac{a_h}{\gamma})^2 \frac{y_h}{y} - \frac{\gamma}{\gamma} A_h - \frac{\gamma^2}{\gamma^2} a_h^2$. If $a_h^* \geq 1$, then $v^* > v^*$ for all firms and all firms buy capital, which cannot be true in equilibrium. Hence, $a_h^* < 1$. Because $v_h > 0$ for $a > a_h$, $v_h$ strictly increases with $a$, and $v^* = 0$ for $a = a_h^*$, we know that $v_h > v^*$ for $a = a_h^*$. This implies $a_h^* < a_h^*$.

A firm buys capital if and only if $x_h(a) v_h(a) > x^*(a) v^*(a)$, which is equivalent to (using (5) and (7)) $\frac{y_h \sqrt{b}}{2 a_h (\frac{\gamma}{\gamma})^2 \frac{y_h}{y}} - \frac{\gamma}{\gamma} A_h - \frac{\gamma^2}{\gamma^2} a_h^2$. The LHS is decreasing in $a$ because $y_h > y^*$. Because there are new entrants who buy capital, there must be some firms selling, and so $a_h^* < 1$. Since firm $a_h^*$ expands in both scale and scope, it buys capital, and so $a_h^* < a_h^*$.

$M(y) = \frac{2 y_0}{\rho}$ and $M'(y) < 0$. Because $y_h > y^*$, we have $M_h < M^*$.

Using the equilibrium scale (5) in the equilibrium output (3), we obtain an individual plant’s average cost

$$\frac{c}{q} = \sqrt{\frac{R}{2a}}.$$ 

Since $R_h < R^*$, the average cost of each plant (and therefore the average cost of each firm) decreases. The aggregate output of all home firms is $Q = \int_{a_0}^{1} qv da = \frac{A R_h}{8 \sigma m g}$, and their total production cost is $C = \int_{a_0}^{1} cuda = \frac{A^2 R_h}{16 \sigma m g}$. Industry’s average cost is then

$$\bar{c} = \frac{C}{Q} = \frac{A \rho(y)}{2 y \eta(y)}.$$
We can show that \( \frac{\rho(y)}{q(y)} \) is decreasing in \( y \). Because \( y_h > y^* \) and \( A_h < A^* \), we have \( c_h > \bar{c} \). \hspace{1cm} Q.E.D.

**Proof of Proposition 4**

We first prove \( y_t < y^* \). \( y^* \) is solved from equation (13): \( Z(y) = \frac{\rho^3}{(y^*_p + \frac{1}{2} y^*_q)^{\alpha}} - \frac{16k^2 \tau^2}{\alpha^2} = 0 \). \( y_t \) is solved from \( Z_t(y_t) = \frac{\rho^3}{(y^*_p + \frac{1}{2} y^*_q)^{\alpha}} - \frac{4R^2 \tau^2}{\alpha^2} = 0 \). Obviously, \( Z_t(y) > Z(y_t) \). Then \( Z_t(y^*) > Z(y^*) = 0 = Z_t(y) \). We already know \( Z_t(\cdot) > 0 \). Then \( y^* > y_t \).

(1). From (12), we have \( A(y) = \frac{a^2}{1 + \frac{a^2}{y^p}} \) and similarly for \( A_t = A(y_t) \). It is straightforward to verify that \( \frac{\eta(y)}{y_p(y)} \) decreases in \( y \). Therefore, \( A_t = A(y_t) < A(y^*) = A^* \) as \( y_t < y^* \). \( A^* = y^* \sqrt{2R^2} = y^* \sqrt{2 \frac{8k^2 \tau^2}{\alpha^2}} = 4b \sqrt{\bar{m}} \sqrt{\frac{q^2}{p^2}} \), while \( A_t = y_t \sqrt{2R_t} = y_t \sqrt{2 \frac{8k^2 \tau^2}{\alpha^2}} = 4b \sqrt{\bar{m}} \sqrt{\frac{q^2}{p^2}} \). We can show that \( \frac{y_t}{y_t} \) decreases in \( y_t \). Because \( y_t < y^* \), we have \( \sqrt{\frac{y_t}{y_t}} > \sqrt{\frac{y_t}{y_t}} \) and thus \( A_t > A^* \).

(2). Since \( y_t < y^* \), we have \( a_t > a_0 \). Therefore, for \( a \in [a_0, a_t] \), \( x_t v_t = 0 \), i.e., they sell capital.

(3-i). Using (5), (7), (22) and (23), a firm buys capital, i.e., \( x_t(a) v_t(a) > x^*(a) v^*(a) \), if and only if \( \frac{\sqrt{\bar{m}} \sqrt{\frac{q^2}{p^2}}}{y_t} > \left( \frac{R^*}{2R_t} \right)^{1/2} \). Note that the LHS of the condition is increasing in \( a \) because \( y_t < y^* \), while the RHS of is independent of \( a \). Therefore, if the inequality holds for \( a^* \), it also holds for all \( a > a^* \). Because some firms (at least those who exit) sell capital, there must be some firms who buy capital. Thus, \( a_t < 1 \).

(3-ii). If \( 2y_t - y^* > 0 \), we define \( \hat{a}_q \equiv \frac{1}{(2y_t - y^*)^2} \). Then, \( a_t < \hat{a}_q < 1 \) and condition (26) holds if and only if \( a > \hat{a}_q \). If \( 2y_t - y^* \leq 0 \), (26) never holds and all firms reduce their scale. In that case, define \( \hat{a}_q = 1 \).

(3-iii). Using (7) and (23), a firm expands its scope, i.e., \( v_t(a) > v^*(a) \), if and only if \( \frac{\sqrt{\bar{m}} \sqrt{\frac{q^2}{p^2}}}{y_t} > \sqrt{\frac{R^*}{2R_t}} \). This condition has the same property as that for the size comparison in (3-i), and thus a similar conclusion emerges. Note that the inequality never holds for \( a_t \) because \( y_t \sqrt{a_t} - 1 = 0 \). The existence of \( \hat{a}_q \) is obvious and \( a_t < \hat{a}_t \leq 1 \).

(4). \( M^*(y^*) = \frac{2b\bar{m}^*}{p^*} \) and \( M_t(y_t) = \frac{b\bar{m}_t}{p_t} \). Because \( \frac{d(y)}{dy} < 0 \) and \( y_t < y^* \), we have \( 2M_t > M^* \).

(5). Plant-level (and firm-level) average cost is \( \frac{\tau}{q} = \sqrt{\frac{R}{2\alpha}} \). Productivity thus increases (i.e., \( \frac{\tau}{q} \) decreases) if and only if \( R \) decreases. Recall that the industry’s average cost is \( \frac{C}{Q^*} = \frac{\dot{A}}{\dot{q}/q} = \frac{\dot{R}}{\dot{q}/q} \sqrt{\frac{\rho(y)}{q(y)}} \). We can show that \( \frac{\rho(y)}{q(y)} \) increases in \( y \). Since \( y_t < y^* \), if \( R_t < R^* \), we have \( \frac{\dot{C}_t}{Q_t} < \frac{\dot{C}^*}{Q^*} \). \hspace{1cm} Q.E.D.

**References**


