Tax Competition with Heterogeneous Firms

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ABSTRACT

This paper studies tax competition in a setting that allows for agglomeration economies and heterogeneous firms. We find that the Nash equilibrium involves the large country charging a higher tax than the small nation, with this rate being too low from a social point of view. Tighter integration of markets leads to an intensification of competition, a drop in Nash tax rates, and a narrowing of the gap. Since large, productive firms are naturally more sensitive to tax difference in our model, large firms are the crux of tax competition in our model. This also means that tax competition has consequences for the average productivity of the big and small nations' industry; by lowering tax rates, the small nation can attract high-productivity firms.
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1. INTRODUCTION

The last few decades have seen OECD countries engaged in competition over corporate tax rates. These nations strive to balance tax revenue goals with their desire to avoid losing firms to low-tax nations. This has made corporate tax competition an important issue in both the theoretical and empirical public finance literature.

In some sense, one can view the theory development as capturing an ever wider range of real-world considerations that seem to be critical to the public debate. The early, classic tax-competition models such as Zodrow and Mieszkowski (1986) and Wilson (1986) – see Wilson (1999) for a survey – crystallised our thinking on the basic race-to-the-bottom logic. These models were not designed to capture the interaction between goods market integration and the intensity of tax competition. As this concern played a key role in the intra-European debate during the late 1980s and early 1990s, the theory literature responded with models that capture this interaction.

[1] We would like to thank Rikard Forslid, Gianmarco Ottaviano, and Masahisa Fujita for helpful comments and suggestions as well as participants in seminars at Tohoku University, and Kobe University for their comments.

While there are several ways of theoretically connecting international tax competition and goods market integration, one recent line relied on models with agglomeration economies where the degree of goods market integration has an important influence on the location of firms. One novel feature of these models was that they allowed for a rich set of outcomes between trade integration and tax competition.[2] For instance, in some cases, one observes first a 'race to the top' and then a 'race to the bottom' as trade costs fall. This is driven fundamentally by a well known feature of the agglomeration models, namely that agglomeration rents are greatest at intermediate trade costs. Since agglomeration rents are quasi-rents, nations can tax them up to a point without the firms relocating abroad to escape the tax.

More recently, focus has turned to taxation and difference among firms, in particular difference related to size and productivity. For instance, Ireland has the lowest rate in the EU at 10% and this has attracted many productive firms, whose average productivity is the highest in Europe (O'Mahony and Van Ark, 2003). Indeed, the public policy debate on international tax competition has long focused on large firms based on the premise that large firms are both the most likely to move in response to tax differentials and the sort of firms that a nation would be least happy about losing.

Our paper considers tax competition with a range of firm sizes. Our model is inspired by the so-called Heterogeneous Firms Trade (HFT) model introduced by Melitz (2003). Firms are heterogeneous in their size and productivity. In our tax model, which is automatically marked by agglomeration economies, the largest firms are the most sensitive to international tax gaps and so the cutting edge of the competition very naturally concerns large firms. Moreover, introducing firm heterogeneity allows us to consider the average productivity effect of firm relocation by size. The key is that size and profits differ across firms, so corporate tax gaps affect big and small firms in different ways.

The paper closest to our contribution is Burbidge, Cuff and Leach (2005, 2006). These authors model the impact of taxation in the framework of heterogeneous firms, i.e. different firm sizes. However, they do not consider monopolistic competition a la Melitz (2003) but rather work with perfect competition. Moreover they assume that firm productivity differences are location-specific as well as firm-specific, so the firm's productivity changes when it changes location. Our early paper, Baldwin and Okubo (2008), uses the basic HFT model with firm mobility, but that paper studied base-widening/rate-lowering tax reforms. Our current paper goes beyond this by introducing strategic interactions between tax setting authorities as in Devereux et al. (2008), although our model differs sharply from this paper in that ours involves heterogeneous firms.

To preview the paper's value added, we propose a setting where large firms are at the crux of the tax competition, as often discussed in the policy literature. Second, our framework redresses one of the problems of tax competition with homogenous goods model and agglomeration rents (e.g. Baldwin and Krugman 2004), namely they may have no Nash equilibrium. The basic problem with working with homogenous firms is that all the firms view the tax gap in the same way; if one wants to go, they all do. With heterogeneous firms, the firm's sensitivity to tax gaps varies with firm size, with large firms being the most sensitive to tax gaps. This differential responsiveness to tax allows us to derive Nash equilibrium that would not exist in previous studies.

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[2] For instance, Ludema and Wooton (1998), Kind, Midelfart-Knarvik and Schjelderup (1998), Trionfetti (2001), Forslid and Midelfart-Knarvik (2001), Ottaviano and Van Ypersele (2005), Andersson and Forslid (2003), and Baldwin and Krugman (2004).

The rest of the paper is organised in five sections. The next introduces the basic economic model without taxes in order to characterize the relocation tendency of firms according to size in the simplest possible setting. Section 3 studies the impact of exogenous tax differences on firm location choices. Section 4 explores tax competition, and Section 5 considers the relationship between tax competition and average productivity. Our concluding remarks are in the last section.

2. THE BASIC MODEL: NO TAXATION

This section introduces the basic economic framework; it can be thought of as combination of the agglomeration model of Martin and Rogers (1995) and the HFT model of Melitz (2003).

We work with two nations (North and South) each with two sectors (manufacturing and the numeraire sector). The numeraire-good sector is meant to be uninteresting; it is marked by constant returns, homogeneous firms, perfect competition and costless trade with labour as its only input. Its role is to alleviate general equilibrium considerations as it equalises unit labour costs internationally, balances trade and eliminates income effects (via quasi-linear preferences).

Manufacturing is the focus of the analysis, so we allow for a range of firm productivity levels, imperfect competition, trade costs and scale economies. Specifically, firms' marginal costs are flat, international trade is subject to iceberg trade costs, and firms compete according to Dixit-Stiglitz monopolistic competition. We assume the manufacturing sector is capital intensive since each manufacturing firm requires one unit of capital (its fixed cost); variable costs involve only labour. Since each firm requires a unit of capital and each firm has a different variable cost, it is useful to think of the capital as a blueprint that implies a firm-specific marginal cost.

Labour is immobile across nations but capital can move without costs between North and South. Since there is one unit of capital per firm, capital mobility is tantamount to firm relocation. To avoid issues of arising from where profits are spent, we assume all capital is owned by labour. That is, while capital is mobile, capital owners are not, so all capital rewards are spent in their native region.

Importantly, firms have heterogeneous marginal costs. In Melitz (2003) the distribution of these marginal costs is endogenised, but here we assume each nation's distribution is fixed – part of its endowment. Finally, we assume that the nations are asymmetric in size (North is bigger), but they are endowed with identical relative factor supplies. This makes the North larger in a pure sense (its endowment of labour and capital are proportionally larger than the South's) and rules out Heckscher-Ohlin motives for trade and/or capital movements.

The basic forces in this model are now well understood. The scale economies and trade costs make firms want to locate in the big market, all else equal. The imperfect competition, by contrast, makes firms want to locate away from their competitors, all else equal. The tension between the pro-agglomeration force and anti-agglomeration force is regulated by trade costs. On one hand, the cost disadvantage stemming from having to ship to customers in the other market rises with trade costs and this tends to favour agglomeration of all firms in one market. On the other hand, high trade costs provide protection from competitors located in the other market and so tend to favour a dispersed outcome.

More formally, tastes of the representative consumer in each region are:

$$U = \mu \ln C_M + C_A, \quad C_M \equiv \left(\int_{i \in \Theta} c_i^{\,1-1/\sigma} di \right)^{1/(1-1/\sigma)}, \qquad 1 > \mu > 0, \quad \sigma > 1$$

where C_M and C_A are, respectively, consumption of the composite of M-sector varieties and consumption of the A-sector good, and $\sigma > 1$ is the constant elasticity of substitution between any two M-sector varieties; Θ is the set of all varieties produced (pre-determined since each variety requires a unit of capital and the world capital stock is fixed).

Firm-level heterogeneity in our model stems from differences in firm's marginal costs. Thus, although all the Dixit-Stiglitz varieties enter consumers' preferences symmetrically, the cost of producing of each variety is different. The unit labour requirements are variety/firm specific and denoted as a_j for firm *j*. The distribution of firm-level efficiency, which part of each region's endowment, is assumed to be Pareto:

(1) $$G[a] = (\frac{a^{\rho}}{a_0^{\;\rho}}), \qquad 1 \equiv a_0 \geq a \geq 0, \quad \rho \geq 1$$

Here a_0 is the scale parameter (highest possible marginal cost) and ρ is shape parameter. We normalise a_0 to unity without loss of generality. For simplicity, G[*a*] is identical for the two nations.

While the distribution of *a*'s is the same in both nations, the big region (North) has more varieties with each level of marginal cost. The resulting distribution of *a*'s can be seen in Figure 1. Note that the distribution in the North is *n*G[*a*], in the South it is *n**G[*a*]. Of course this means that the total mass of firms in the North and South are '*n*' and '*n**', respectively.[3]

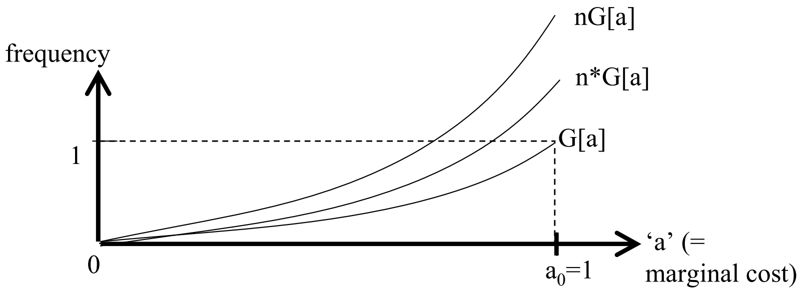

Figure 1: Endowed distribution of capital and marginal costs in North and South.

2.1. *The no-tax equilibrium and relocation tendencies*

Constant returns, perfect competition and zero trade costs in the numeraire sector equalise wages across countries (wages in terms of the numeraire). We choose the units of this good such that $w = w^* = 1$, so all differences in firms' production costs stem from differences in the *a*'s.

Utility maximisation generates the familiar CES demand functions in the manufactures sector. These, together with Dixit-Stiglitz monopolistic competition imply 'mill pricing' is optimal and a firm's operating profit is $1/\sigma$ times firm-level revenue. Thus, the operating profit of a North-based firm with marginal cost '*a*' is:

[3] Since we take the range of varieties to be continuous, we speak of the 'mass' of firms with a particular marginal cost. We assume that the mass is the same for every level of marginal cost (this is demonstrated in Melitz (2003) as the outcome of an endogenous entry/exit process).

$$\textbf{(2)}\qquad \pi[a]=\left(\frac{(\frac{a}{1-1/\sigma})^{1-\sigma}}{\int_i p_i^{1-\sigma}di}E \quad + \quad \frac{\phi(\frac{a}{1-1/\sigma})^{1-\sigma}}{\int_i p_i^{*1-\sigma}di}E^*\right)\frac{1}{\sigma}; \qquad \phi\equiv\tau^{1-\sigma}$$

The first term in this expression is the value of firm-specific sales in the Northern market; this rises as the firm's 'a' falls; E reflects the total Northern market expenditure. The second term shows the firm's export sales; the firm's price includes the iceberg trade cost raised to 1-σ, the denominator involves prices in the export market (the p^*'s), and the relevant expenditure is Southern expenditure, E^*. A parameter that plays a critical role in our paper is ϕ; we refer to it as the 'free-ness' (phi-ness) of trade, and note that ϕ ranges from zero when trade is perfectly un-free ($\tau=\infty$) to unity when trade is perfectly free ($\tau=0$). Southern demand functions are isomorphic.

There are several important features of **(2)**. First, all firms earn positive operating profit in equilibrium and this is the reward to capital, i.e. the Ricardian rent. Second, the most efficient firms, i.e. firms with low marginal costs, are the largest in the sense that they sell the most. Third, profitability and operating profits are proportional to sales.

In the initial situation where no capital mobility (i.e. delocation) is allowed, Northern and Southern operating profit, as a function of the firm's 'a', can be written as:[4]

$$\textbf{(3)}\qquad \pi[a]=a^{1-\sigma}\left(\frac{s_E}{\overline{\Delta}}+\frac{\phi(1-s_E)}{\overline{\Delta}^*}\right)\frac{E^w}{K^w\sigma}, \quad \pi^*[a]=a^{1-\sigma}\left(\frac{\phi s_E}{\overline{\Delta}}+\frac{(1-s_E)}{\overline{\Delta}^*}\right)\frac{E^w}{K^w\sigma}; \quad s_E=\frac{E}{E^w}$$

Here we have introduced s_E as shorthand for the North's share of world expenditure (we adopt the convention that North is bigger so s_E>½), K^w is world endowment of capital, and the Δ's are the denominators of the North and South CES demand functions (Δ is a mnemonic for denominator):

$$\overline{\Delta}\equiv s_K\int_0^1 a^{1-\sigma}dG[a] \quad + \quad (1-s_K)\phi\int_0^1 a^{1-\sigma}dG[a];$$

$$\overline{\Delta}^*\equiv s_K\phi\int_0^1 a^{1-\sigma}dG[a] \quad + \quad (1-s_K)\int_0^1 a^{1-\sigma}dG[a]; \qquad s_K\equiv\frac{K}{K^w},$$

where s_K is the North's share of K^w. Solving the integrals with **(1)** and assuming $1-\sigma+\rho>0$ so that the integrals converge, we have:[5]

$$\textbf{(4)}\qquad \overline{\Delta}=\lambda\left(s_K+\phi(1-s_K)\right); \quad \overline{\Delta}^*=\lambda\left(\phi s_K+1-s_K\right); \quad s_K\equiv\frac{K}{K^w}, \quad \lambda\equiv\frac{\rho}{1-\sigma+\rho}>0$$

Firms move to the region with the highest operating profit. Starting from the initial situation where no relocation has occurred, **(3)** and **(4)** imply that the operating profit gap is:

$$\pi[a]-\pi^*[a]=a^{1-\sigma}\left(\frac{(1-\phi)E^w}{\lambda\sigma K^w}\right)\left(\frac{2\phi\left(s-\frac{1}{2}\right)}{\left((1-\phi)s+\phi\right)\left(1-s+\phi s\right)}\right)\geq 0$$

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[4] This simplification uses mill pricing and cancels the (1-1/σ) terms.

[5] Since firms are atomistic, the first firm to move has no impact on the Δ's.

where 's' is the north's 'size', namely its share of world E and K. Notice that the first term in parentheses is positive since $\phi < 1$, and λ and σ are positive by our regularity conditions. The numerator of the second term is positive since s>½, and the denominator is positive since both s and ϕ are lie between zero and one. Thus π-π* is always positive in the initial situation, i.e. there is always a tendency for firms to move to the large region.[6] Importantly, the π-π* gap is highest for the largest firms, i.e. those with the smallest marginal cost, 'a'. In other words, the most efficient Southern firms are the ones who gain the most by moving to the big market. To summarise, the first firms to relocate from the small region (South) to the large region (North) are the most efficient small-region firms.

2.2. *The Location equilibrium*

As less and less efficient Southern firms move to the North, the degree of competition rises in the North and falls in the South. This tends to make the North less attractive and the South more attractive, i.e. the relocation extinguishes the forces that produced it. To characterise the location equilibrium, we find the level of 'a' for which the incentive to relocate drops to zero.

Once relocation occurs, the formula for the profit gap is more complicated than **(4)**. Defining the range as firms that have moved from South to North as $[0,a_R]$ (i.e. zero to a_R defines the range of firms that have moved by referring to their marginal cost), we have that the Δ and Δ* after relocation are:

$$\Delta = s\int_0^1 a^{1-\sigma}dG[a] + (1-s)\{\int_0^{a_R} a^{1-\sigma}dG[a] + \phi\int_{a_R}^1 a^{1-\sigma}dG[a]\},$$
$$\Delta^* = \phi s\int_0^1 a^{1-\sigma}dG[a] + (1-s)\{\phi\int_0^{a_R} a^{1-\sigma}dG[a] + \int_{a_R}^1 a^{1-\sigma}dG[a]\}; \qquad K^w \equiv 1$$

The first term in the top expression reflects the prices of North-made varieties sold in the North; the 's' in front of the integral reflects the north's share of firms, i.e. its share of world capital, namely s_K, but by symmetry of relative endowments s_K equals the relative size of the north's market, i.e. 's'. The second integration in the top expression reflects the prices of Southern varieties produced in the North (Southern firms with a's in the range $[0,a_R]$ have relocated). The third integral reflects the prices of Southern varieties that are made in the South and exported to the North. The second bottom expression is isomorphic, but reflects the situation in the South.

Solving the integrals using **(1)**:

(5)
$$\Delta = \lambda\left(s + (1-s)a_R{}^{\alpha} + \phi(1-s)(1-a_R{}^{\alpha})\right),$$
$$\Delta^* = \lambda\left(\phi s + \phi(1-s)a_R{}^{\alpha} + (1-s)(1-a_R{}^{\alpha})\right), \qquad \alpha \equiv 1-\sigma+\rho$$

Thus the operating profit gap is a function of a_R, so the 'location condition':

(6) $$\pi[a_R] - \pi^*[a_R] = 0$$

where

$$\pi[a_R] = a_R{}^{1-\sigma}\left(\frac{s_E}{\Delta[a_R]} + \phi\frac{1-s_E}{\Delta^*[a_R]}\right)\frac{E^w}{\sigma}, \quad \pi^*[a_R] = a_R{}^{1-\sigma}\left(\phi\frac{s_E}{\Delta[a_R]} + \frac{1-s_E}{\Delta^*[a_R]}\right)\frac{E^w}{\sigma}$$

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[6] This result is known as the 'home market effect' in international trade, e.g. Krugman (1980).

The location condition characterises the equilibrium range of firms that have moved from the South to the North. The solution (which defines the level of '***a***' where firms are indifferent between locations) is:

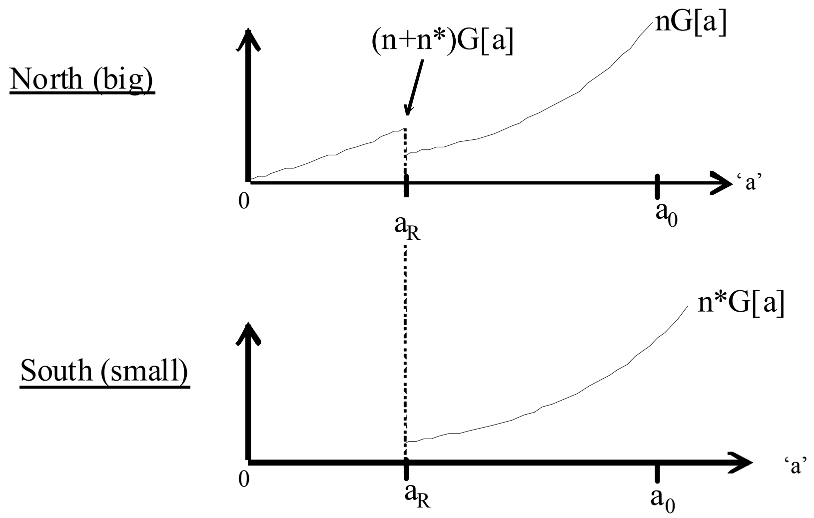
(7) $$a_R^{\alpha} = \frac{2\phi(s_E - \frac{1}{2})}{(1-\phi)(1-s_E)}, \qquad s_n = s_E + (1-s_E)a_R^{\rho}$$

where s_n is the share of all firms located in the North in equilibrium. Note that a_R rises with ϕ so more inefficient firms find it profitable to relocate as trade gets freer. Full agglomeration occurs when ϕ equals or exceeds the sustain point, ϕ^S:

(8) $$\phi^S = \frac{1-s_E}{s_E}$$

Figure 2 shows the impact of this agglomeration on the distribution of firm efficiencies by market. The diagram is drawn for an intermediate level of trade freeness, i.e. one where some but not all Southern firms have relocated to the North.[7]

Figure 2: Geographic distribution of firm efficiency with free delocation.



The location condition implies $s_E/(1-s_E)=\Delta^*/\Delta$. Since the CES price index for manufactures in the North and South are $P=\Delta^{1-\sigma}$ and $P^*= (\Delta^*)^{1-\sigma}$, the price index is lower in the North than it is in the South in equilibrium. Since the price of the numeraire good and labour are equalised, this implies that real incomes are higher in the North than they are in the South.

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[7] Note that the delocation process tends to raise the average efficiency of industry in the big region while it lowers the average efficiency in the South. This is what Baldwin and Okubo (2006) call 'spatial selection'.

Discussion

A number of features of this equilibrium are attractive when it comes to the analysis of tax competition. For example, compared to the small South, the large North is richer, has firms that are more efficient on average, and has a disproportionate share of the 'good' firms, i.e. large efficient firms with above-average profitability. The North is also a net exporter of manufactures and has a higher share of its work force in manufacturing. This is a second-best equilibrium since the imperfect competition means that too few resources are devoted to the manufacturing sector as opposed to the numeraire sector.

3. EXOGENOUS TAXATION

We now turn to considering the impact of taxes on the equilibrium location of firms and the distribution of firm types. For simplicity, we start from full agglomeration, i.e. when all manufacturing is in the North since trade is freer than the level necessary to sustain full agglomeration, i.e. $\phi > \phi^S$.

Consider a situation where firms in the bigger, richer North pay higher profit taxes than those in the South. To keep things simple, assume Southern taxes are zero. In this case, what matters for location is the post-tax profit gap for a North-based firm. This is given by:

$$a^{1-\sigma}\left(\frac{s_E}{\Delta}+\phi\frac{1-s_E}{\Delta^*}\right)\frac{E^w}{\sigma}T, \qquad T \equiv 1-t$$

where T is the tax factor, one minus the corporate tax rate. The net profit for a South-based firm is identical to the untaxed case, so the incentive to move to the North is:

$$\text{(9)}\qquad a^{1-\sigma}\left(\frac{s_E}{\Delta}+\phi\frac{1-s_E}{\Delta^*}\right)\frac{E^w}{\sigma}T - a^{1-\sigma}\left(\phi\frac{s_E}{\Delta}+\frac{1-s_E}{\Delta^*}\right)\frac{E^w}{\sigma} > 0,$$

all firms stay in the North and pay tax. Since the threshold we wish to solve for, a_R, enters the Δ's to the power of $1 - \sigma + \rho$ and directly to the power of $1 - \sigma$, we cannot solve the value of 'a' that sets the profit gap to zero. Numerical solutions, however, are readily available.

3.1. *The tax revenue curves*

As in many papers on tax competition in economic geography literature, we suppose Leviathan government and thus the government acts to maximise tax revenue by choosing an optimal corporate tax rate.

Simulations (not shown) confirm the intuitive result that the equilibrium range of Southern firms in the North falls as the Northern tax rate rises. Since it is the biggest Southern firms that have the most to gain from local access to the big Northern market, raising the Northern tax pushes out the biggest, most efficient Southern firms first. This creates a connection between average productivity and the corporate tax rate. Specifically, lowering the Northern rate tends to raise the average productivity of North's manufacturing sector and lower the South's.

As a background for tax competition, we characterise an important threshold tax rate namely the rate just low enough to attract all Southern firms to the North. We call this the full-relocation tax rate and denote it as T^{fr}.

As all firms are in the North, when T larger than or equal to T^{fr}, we can solve the location condition analytically to get:

(10)
$$T^{fr} = \frac{\phi^2 s + 1 - s}{\phi}$$

For taxes in the range $T > T^{fr}$, all manufacturing firms are in the North, so tax revenue is linear in T, specifically:

(11)
$$Tax\ revenue = \frac{E^w}{\sigma}(1-T)\,; \qquad T > T^{fr}$$

If the North raises taxes beyond the T^{fr} point, the Northern tax base falls with higher tax rates. This means that tax revenue is hump-shaped in terms of the tax factor T and there exists a Northern tax rate at which its tax revenue is maximised.

To explore the latter, note that the largest firms are naturally most sensitive to taxation – after all they pay the most taxes – so they are the ones to leave the North first when T falls below T^{fr}. Thus all firms with a's from 0 to a_R relocate to the South, while those with a's from a_R to 1 stay at the North. Solving the location condition, the cut off level is related to the North's tax rate by:

(12)
$$a_R^\alpha = \frac{(\phi^2 - 1)s - T\phi + 1}{(1-\phi)((1+\phi)(T-1)s - T\phi + 1)}$$

In this case, the tax revenue is related to a_R according and T to:

(13)
$$\begin{aligned} North\ Tax\ \mathrm{Re}venue &= \int_{a_R}^{1} \frac{E^w}{\sigma}\left(B + \phi B^*\right)a^{1-\sigma}(1-T)dG[a] \\ &= \frac{\lambda}{\sigma}\left(B + \phi B^*\right)(1-T)(1-a_R^\alpha) \end{aligned}$$

where

$B = \dfrac{s}{\lambda(\phi a_R^\alpha + 1 - a_R^\alpha)}$ and $B^* = \dfrac{1-s}{\lambda(a_R^\alpha + \phi(1-a_R^\alpha))}$. We can normalise as $E^w = 1$.

We plot Northern tax revenue against the Northern tax factor in **Figure 3**. This shows that the relationship is hump-shaped overall and linear for T's beyond above the rate of T^{fr}.[8] Higher tax rates increases tax revenue, while higher tax leads to firm relocation to South and decreases tax base and thus fall Northern tax revenue.

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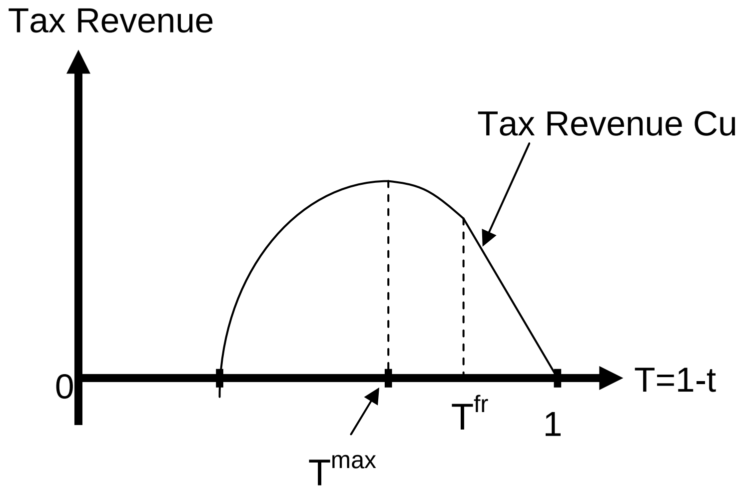
[8] The parameter values chose for the simulations that generated the figure are σ=2, ρ=2, ϕ=0.75 and s=0.6.

Figure 3: Tax Revenue Curves.



As it turns out, we can analytically identify the tax factor that maximise Northern tax revenue. This is:

(14)
$$T^{\max} = \frac{\phi s - (1-s)\phi^2 + \sqrt{\phi(1-\phi)^2(1+\phi)^2(1-s)s}}{s-(1-s)\phi^3}$$

To summarise, we write:

Result 1: Assuming the South imposes no tax, the Northern tax revenue curve is hump-shaped ("Laffer Curve"). The tax rates that maximise tax revenue vary with the relative size of the North and the freeness of trade. As the big country gets relatively bigger, the revenue maximizing tax rate rises, as long as trade is not too free.

4. CORPORATE TAX COMPETITION

In this section, we consider taxes set in the course of a strategic interaction among governments taking the relocation tendency of firms as given.

So far we have artificially prevented the South from taxing firms. If the North's tax rate is low enough to keep all firms in the North, the choice of Southern tax rate is irrelevant. However, if the North sets its tax factor such that $T < T^{fr}$, then the South may be tempted to impose a positive corporate tax and this will put the two governments into a situation of tax competition.

Consider first the Southern government's problem, taking the Northern tax rate as given. Tax revenue in the South is:

(15)
$$South\,tax\ \mathrm{Re}venue = \int_0^{a_R} \frac{E^w}{\sigma}\left(\phi B + B^*\right)a^{1-\sigma}(1-T^*)dG[a]$$
$$= \frac{\lambda E^w}{\sigma}\left(\phi B + B^*\right)(1-T^*)a_R^{\alpha}$$

The core of international tax competition is the classic rate-versus-base trade-off; charging a higher rate means losing some firms to the lower-tax location, but also means extracting more revenue from the firms that remain. To capture this trade-off most directly, we assume that governments choose taxes to maximise tax revenue. This simplifies the expressions since it eliminates considers that operate on welfare via relocation's impact on relative price indices. Note that in this model, tax has no distortionary impact beyond relocation. For example, in a closed economy a tax rate of even as high 99% would have no impact on prices, employment or investment since the capital stock is fixed and fully employed and each firm needs one and only one unit of capital. Taxation here merely transfers profits to the government.

4.1. *Nash Equilibrium of Tax Competition*

When both countries charge positive taxes, the equilibrium range of firms that relocate to the North (as measured by a_R) is now affected by T and T^*, the South's tax factor, i.e. $1-t^*$ where t^* is the Southern tax rate. Using the suitably modified location condition, a_R is now

$$a_R^\alpha = \frac{(s\phi^2 + (1-s))T^* - T}{(1-\phi)[\{(1-s) - s\phi\}T^* + \{s - (1-s)\phi\}T)]}.$$

Differentiating the cut off, we get $\frac{\partial a_R}{\partial T} < 0$ and $\frac{\partial a_R}{\partial T^*} > 0$.[9] These results are intuitively obvious; a lower Northern tax rate lowers a_R, i.e. it expands the range of Southern firms that prefer the North. A lower Southern tax rate has the opposite effect on a_R.

As far as market size is concerned, $\frac{\partial a_R}{\partial s} < 0$.[10] This means that as the North gets relatively larger, it attracts more firms for any given set of taxes.

Using location condition and tax revenue functions, the Nash first order conditions are:

(16) $$\left.\frac{\partial\ North\ tax\ revenue}{\partial T}\right|_{T^*} = 0 \quad \text{and}$$

(17) $$\left.\frac{\partial\ South\ tax\ revenue}{\partial T^*}\right|_{T} = 0.$$

Loosely speaking, we can think of these as the tax 'reaction functions'. With a good deal of simplification, they can be written as:

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[9] For convenience sake, we define $AR \equiv a_R^\alpha$.

$$\frac{\partial AR}{\partial T} = \frac{-A - \left[(s\phi^2 + (1-s))T^* - T\right](s - (1-s)\phi)}{A^2} < 0$$

$$\frac{\partial AR}{\partial T^*} = \frac{(s\phi^2 + 1 - s)A - \left[(s\phi^2 + (1-s))T^* - T\right](1 - s - s\phi)}{A^2} > 0, \text{ where}$$

$$A \equiv (1-\phi)[\{(1-s) - s\phi\}T^* + \{s - (1-s)\phi\}T\}]$$

[10] $\frac{\partial AR}{\partial s} = \frac{T^*(\phi - 1)A - \left[(s\phi^2 + (1-s))T^* - T\right](1-\phi)(1+\phi)(T - T^*)}{A^2} < 0$. Note that $T > T^*$ from Result 2.

(18) $$T[T^*] = \frac{(s\phi + (1-s)\phi^3)T^* - \phi^2 + \sqrt{\phi(1-\phi)^2(1+\phi)^2(1-T^*)(T^*-\phi)(1-s)s}}{(1-s)\phi^3(\phi T^* - 1) + s(T^* - \phi)} T^*$$

and

(19) $$T^*[T] = \frac{((1-s)\phi + s\phi^3)T - \phi^2 + \sqrt{\phi(1-\phi)^2(1+\phi)^2(1-T)(T-\phi)(1-s)s}}{s\phi^3(\phi T - 1) + (1-s)(T - \phi)} T \, .$$

Note that the above reaction functions are not fully symmetric, because firms are heterogeneous and the South (small country) in our model attracts high productivity firms by taxation, and vice versa.

To exclude the case of complex numbers, we assume $T > \phi$ and $T^* > \phi$. To ensure finite slopes, the denominators in **(18)** and **(19)** should not be zero, i.e. $T^* \neq \frac{(1-s)\phi^3 + s\phi}{(1-s)\phi^4 + s}$ and $T \neq \frac{(1-s)\phi + s\phi^3}{s\phi^4 + 1 - s}$.

Subject to these regularity conditions, we can show that the reaction functions are upward sloped, i.e. the tax rates are strategic complements (as expected) since

$$\frac{\partial^2 North\ Tax\ \mathrm{Re}venue}{\partial T\ \partial T^*} > 0 \text{ and } \frac{\partial^2 South\ Tax\ \mathrm{Re}venue}{\partial T^*\ \partial T} > 0$$ (See Appendix 1).

Figure 4 graphs the two reaction curves.[11] The intersection of the two curves, marked '*N*', is the Nash equilibrium tax rate. Tax revenue curves on the reaction curves are concave as seen in **Figure 4** since tax revenue is the objective of both governments.

It is easy to show numerically that the big North has higher taxes in the Nash equilibrium. To illustrate this intuitive results, we can consider an extreme case of when the North is much larger than the South, namely when the North's share of world expenditure and capital is in the neighbourhood of unity.

When s is approximately unity the Nash first order conditions simplify to:

(20) $$T[T^*] = \frac{\phi T^* - \phi^2}{T^* - \phi} T^* = \phi T^* \quad \text{and} \quad T^*[T] = \frac{\phi^3 T - \phi^2}{\phi^3(\phi T - 1)} T = \frac{T}{\phi}$$

Solving, we have $T = \phi T^*$, which says that the North always levies a higher tax, but the gap narrows as trade gets freer.

To summarise, we write:

Result 2: Tax rates are strategic complements. The big country always has higher tax rates than the small country in equilibrium.

Furthermore we have worked out the Stackelberg equilibrium in this model. The point 'S' in **Figure 4** is the equilibrium. Not surprisingly, both countries' tax rates as well as their tax revenues are higher in the Stackelberg versus Nash outcomes, as expected since the tax rates are strategic complements in the Nash game.

[11] The parameter values in Figure are σ=2, ρ=2, ϕ=0.75 and s=0.6.

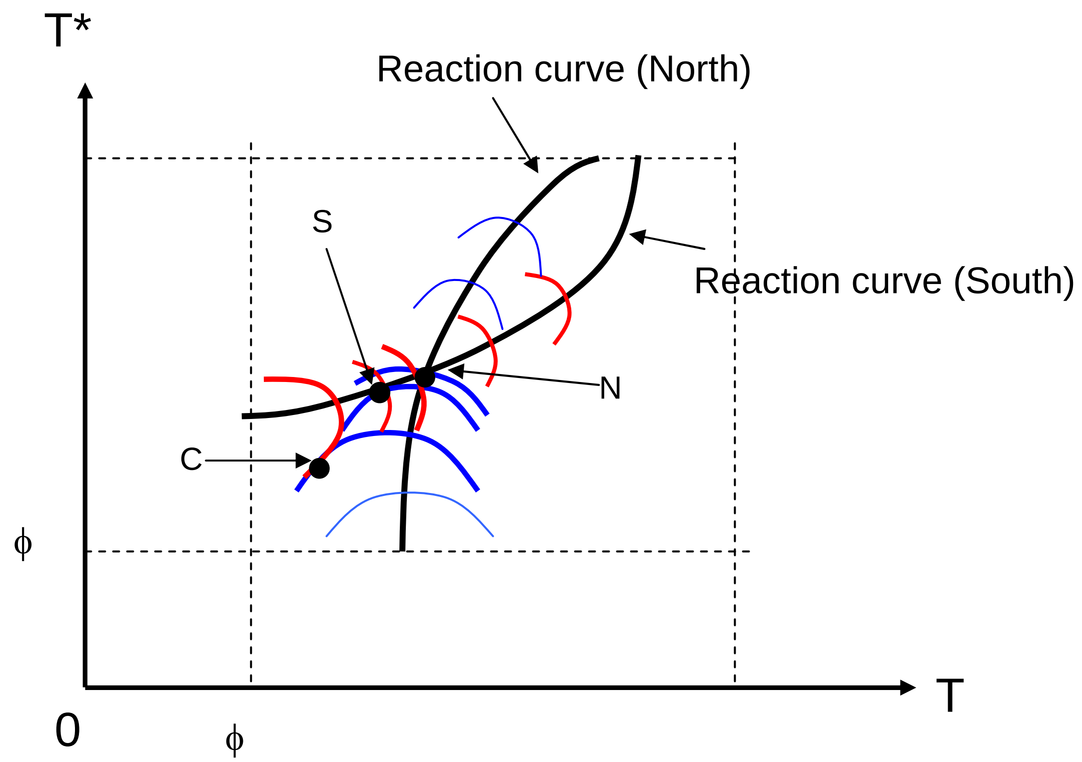

Figure 4: Tax Reaction Curves and Tax Revenue Curves.

4.2. *Trade costs and Tax competition*

The fundamental trade off for firms in this model weighs the net benefit firms receive from being in the large market against the higher tax rate. The fundamental trade off facing governments is the higher tax revenue they would receive by raising rates on firms that stay put versus the loss of some additional firms. Since the degree of trade freeness affects the net benefits of being in the large market, changes in ϕ will alter the trade-off faced by firms and thus indirectly the trade off faced by governments. As a result, tighter integration of goods market (i.e. $d\phi > 0$) will alter Nash equilibrium tariffs.

Given the non-linearity in the model, it is not possible to link the Nash tariffs to ϕ analytically, but numerical simulation for a wide range of parameter values show that both tax rates fall (i.e. the tax factors rise) as trade gets freer. The results of numerical simulations are shown in **Figure 5**. We see that rising freeness of trade leads to higher tax factors for both nations but that the gap narrows as they rise. Translating this into tax rates, it means that tax rates fall with tighter goods market integration. The small nation's Nash tax rate is everywhere lower than that in the large country, but the gap narrows as trade costs fall. In short, this model predicts a classic race to the bottom that intensifies as goods markets become better integrated.

Result 3: When trade costs are low, the forces of tax competition are relatively more important. This results in downward pressure on large country's tax rate and narrowing tax rate differential with small country.

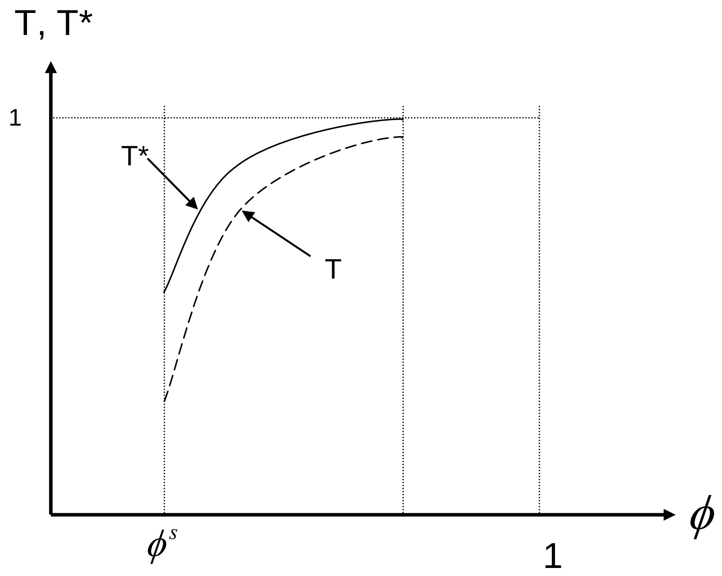

Figure 5: Tax factors and freeness of trade

4.3. *Tax Cooperation*

Above mentioned tax competition might not be the sole case. In order to maximise tax revenues, two Leviathan governments might collude. The possible combination of T and T* are determined by the maximisation of the sum of tax revenue, i.e. the differentiation of Northern tax revenue plus Southern tax revenue in terms of T and T*. In **Figure 4**, the equilibrium is '*C*'. Tax revenue curves are both below those of Nash and Stackelberg equilibrium. Both tax rates can be going up (T and T* go down). Since tax rates are collusively increasing, tax revenue rises in both countries.

4.4. *Welfare Analysis—Welfare-lowering Tax Competition*

Now we turn to whether tax competition by revenue maximising governments is socially optimal. A well known result for the Martin-Rogers model that we are using is that full agglomeration in the North is socially optimal when trade freeness is low enough to sustain full agglomeration in the absence of a tax gap (Baldwin et al. 2003 chapter 15). Since tax competition drives some firms to the South, we can intuitively understand that tax competition is not socially optimal.

More formally, social welfare in the North and the South are:

$$V = North\ tax\ revenue + capital\ income + labour\ total\ income - s + \frac{s}{\sigma - 1}\ln \Delta$$

$$V^* = South\ tax\ revenue + capital\ income + labour\ total\ income - (1-s) + \frac{1-s}{\sigma - 1}\ln \Delta^*$$

where we have normalised the world population to unity so North and South populations are *s* and 1-*s* just as the mass of varieties in firms are *s* and 1-*s*, respectively. Thus the global welfare is:

$$V + V^* = \textit{Total tax revenue} + \textit{Total capital income} + \textit{Total labour income} - 1 + \frac{1}{\sigma - 1}(s \ln \Delta + (1-s)\ln \Delta^*)$$

$$= \frac{1}{\sigma} + \frac{1}{\sigma - 1}(s \ln \Delta + (1-s)\ln \Delta^*)$$

Given the Dixit-Stiglitz competition and the fact that expenditure is fixed by quasi-linear preferences, total expenditure is invariant to the spatial allocation of firms. This in turn means that the world total operating profit is constant. Finally, since corporate taxation is merely a redistribution between the firms and the government, the world total of private profits post-tax and the tax revenue must also be constant. Consequently, global welfare depends only on Δ and Δ*, i.e. the impact of firm location affects the price indices (recall that $P^{1-\sigma} = \Delta$ and a similar expression holds for the Southern price index). Because Δ is proportional to the production of located firms and the North is larger (*s*>0.5), full agglomeration in the North can maximise $s \ln \Delta + (1-s)\ln \Delta^*$. It follows that full agglomeration in the North is socially optimal for global welfare. Anything the drives the spatial distribution of firms away from this lowers global welfare.

Because the competition for firms leads some firms to choose the South when they would have chosen the North had tax rates been equal, we can say that tax competition is welfare lowering.

4.5. *Implications for average productivity*

When firms are heterogeneous in terms of their productivity, firm relocation has an impact on a nation's average productivity. In our model, large firms are the most sensitive to tax rate gaps and so the productivity effects of tax competition are likely to be important.

In particular, the Nash tax equilibrium involves a Northern rate that is higher than the Southern rate. As a result, not all firms are in the North in the Nash equilibrium. Indeed, it is the highest productivity firms that move to the South. Comparing this to the benchmark where no firms have relocated, we see that tax competition enhances the average productivity in the South (although the South has few firms, all this firms have marginal costs below a_R). The productivity effects are zero sum, so it is clear that the North's average productivity falls, although it does have many more firms than it would with no firm-mobility at all. This result may have some resonance with the European situation where low but far from zero tax-rate peripheral countries such as Ireland and Nordic countries successfully attract high productivity firms.

5. CONCLUSION

This paper studies corporate tax competition in an economic framework that allows for agglomeration effects and heterogeneous firms. The addition of heterogeneous firms enriches the analysis in two ways. While previous studies on tax competition in agglomeration economy cannot discuss firm size or productivity, our contribution is that our model can take into account the location of firm when firms are different in size. Tax competition drives sorting of firm location. Since large, productive firms are naturally more sensitive to tax difference in our model, large firms are the crux of tax competition in our model. This also means that tax competition has consequences for the average productivity of the big and small nations' industry; by lowering tax rates, the small nation can attract high-productivity firms.

The other contribution is that we find the Nash equilibrium tax rates, albeit there is no Nash equilibrium in the homogenous firm tax competition model as shown in previous studies. Nash equilibrium tax rates involve the large country charging a higher tax than the small nation, with this rate being too low from a social point of view. Deeper economic integration leads to an intensification of competition, a drop in Nash tax rates, and a narrowing of the gap.

Our paper could be extended in many ways. For example, we could include infrastructure and the provision of public goods produced from the tax revenue. In these cases, welfare and transfer issues could be much more important than in our paper and would provide richer results. Also, we could involve full features of economic geography models such as backward and forward linkages and circular causality, although the basic outcome on tax competition not, we conjecture, be substantially modified.

APPENDIX 1 Strategic Complement

Here, we show the strategic complement in tax rates between two countries.

$$\frac{\partial^2 North\ Tax\ \mathrm{Re}venue}{\partial T\ \partial T^*} = \left(\frac{\partial B}{\partial a_R}\frac{\partial^2 a_R}{\partial T \partial T^*} + \phi \frac{\partial B^*}{\partial a_R}\frac{\partial^2 a_R}{\partial T \partial T^*}\right)(1-T)(1-a_R^\alpha) - (B + \phi B^*)(1-T)\frac{\partial^2 a_R}{\partial T\ \partial T^*} > 0$$

where

$\frac{\partial B}{\partial a_R} = \frac{s}{\left(\lambda(\phi a_R^\alpha + 1 - a_R^\alpha)\right)^2}(1-\phi)a_R^{\alpha-1} > 0$, $\frac{\partial B^*}{\partial a_R} = -\frac{1-s}{\left(\lambda\left(a_R^\alpha + \phi(1-a_R^\alpha)\right)\right)^2}(1-\phi)\alpha a_R^{\alpha-1} < 0$ and

$\frac{\partial^2 a_R}{\partial T \partial T^*} < 0$

This indicates that the North is subject to strategic complement in its tax with the South.

Note that Southern taxation is a mirror of the North's and we can get the same result of strategic complement.

APPENDIX 2 The impact of trade liberalisation on taxation

We can differentiate T in terms of freeness of trade:

$$\frac{dT}{d\phi} = \frac{\left((s + 3(1-s)\phi^2)T^* - 2\phi + E'\right)C - \left((s\phi + (1-s)\phi^3)T^* - \phi^2 + E\right)C'}{C^2}T^*$$

where $C \equiv (1-s)\phi^3(\phi T^* - 1) + s(T^* - \phi)$ $C' \equiv \frac{dC}{d\phi} = (1-s)(4\phi^3 T^* - 3\phi^2) - s < 0$

due to $\left(4\phi T^* - 3\right)\phi^2 < \frac{s}{1-s}$.

We define as

$$E \equiv \sqrt{\phi(1-\phi)^2(1+\phi)^2(1-T^*)(T^*-\phi)(1-s)s}$$

$$E' \equiv \frac{dE}{d\phi} = \frac{1}{2E}\begin{pmatrix} (1-\phi)^2(1+\phi)^2(1-T^*)(T^*-\phi)(1-s)s - 2\phi(1-\phi)(1+\phi)^2(1-T^*)(T^*-\phi)(1-s)s \\ + 2\phi(1-\phi)^2(1+\phi)(1-T^*)(T^*-\phi)(1-s)s - \phi(1-\phi)^2(1+\phi)^2(1-T^*)(1-s)s \end{pmatrix}$$

$$= \frac{1}{2E}\left((1-\phi)^2(1+\phi)^2(1-T^*)(T^*-2\phi)(1-s)s - 4\phi^2(1-\phi)(1+\phi)(1-T^*)(T^*-\phi)(1-s)s\right)$$

If a (necessary) condition,

$\left((s + 3(1-s)\phi^2)T^* - 2\phi + E'\right)C - \left((s\phi + (1-s)\phi^3)T^* - \phi^2 + E\right)C' > 0$ holds, $\frac{dT}{d\phi} > 0$ is always satisfied.

At extreme, when s=1, due to $(T^* - \phi)^2 > 0$, $\frac{dT}{d\phi} > 0$ always hold.

Similarly, we can differentiate *T** in terms of freeness of trade.

$$\frac{dT^*}{d\phi} = \frac{\left((1-s+3s\phi^2)T - 2\phi + F'\right)G - \left(((1-s)\phi + s\phi^3)T - \phi^2 + F\right)G'}{G^2}T$$

where $G \equiv s\phi^3(\phi T - 1) + (1-s)(T-\phi)$ $G' \equiv \dfrac{dG}{d\phi} = s(4\phi^3 T - 3\phi^2) - (1-s)$

$$F \equiv \sqrt{\phi(1-\phi)^2(1+\phi)^2(1-T)(T-\phi)(1-s)s}$$

$$F' \equiv \frac{dF}{d\phi} = \frac{1}{2F}\begin{pmatrix} (1-\phi)^2(1+\phi)^2(1-T)(T-\phi)(1-s)s - 2\phi(1-\phi)(1+\phi)^2(1-T)(T-\phi)(1-s)s \\ +2\phi(1-\phi)^2(1+\phi)(1-T)(T-\phi)(1-s)s - \phi(1-\phi)^2(1+\phi)^2(1-T)(1-s)s \end{pmatrix}$$

$$= \frac{1}{2F}\left((1-\phi)^2(1+\phi)^2(1-T)(T-2\phi)(1-s)s - 4\phi^2(1-\phi)(1+\phi)(1-T)(T-\phi)(1-s)s\right)$$

If a (necessary) condition, $\left((1-s+3s\phi^2)T - 2\phi + F'\right)G - \left(((1-s)\phi + s\phi^3)T - \phi^2 + F\right)G' > 0$ holds, $\dfrac{dT^*}{d\phi} > 0$ is always satisfied.

At extreme, when s=1, $\left(3\phi^2 T - 2\phi\right)(\phi T - 1) - \left((1+\phi^2)T - \phi\right)(4\phi T - 3) > 0$ and thus $\dfrac{dT}{d\phi} > 0$ always holds.

REFERENCES

Andersson, F and R. Forslid, (2003). "Tax Competition and Economic Geography", *Journal of Public Economics*; 5 (2), pp 279-303.

Baldwin, R E and P. Krugman (2004). "Agglomeration, Integration and Tax Harmonisation", *European Economic Review* 48 (1): pp1-23.

Baldwin, R E. and T. Okubo (2006) "Heterogeneous Firms, Agglomeration and Economic Geography: Spatial Selection and Sorting", *Journal of Economics Geography*, 6, pp.323-346.

Baldwin, R E. and T. Okubo (2008) "Tax reform, delocation and heterogeneous firms: Base widening and rate lowering rule", *CEPR Discussion Paper* 6843.

Burbidge, J and K. Cuff, (2005). "Capital tax competition and returns to scale," *Regional Science and Urban Economics*, Elsevier, vol. 35(4), pages 353-373, July

Burbidge, J and K. Cuff, and J. Leach, (2006). "Tax competition with heterogeneous firms," *Journal of Public Economics*, Elsevier, vol. 90(3), pages 533-549

Devereux, M.P., B. Lockwood, M. Redoano (2008) Do countries compete over corporate tax rates? *Journal of Public Economics* 92 pp.1210-1235

Dixit, A.K. and J.E. Stiglitz (1977) Monopolistic competition and optimum product diversity, *American Economic Review* 67, 297-308.

Forslid, R. and K. H. Midelfart Knarvik. 2002. Internationalisation, industrial policy and clusters. CEPR Discussion Paper No. 3129, Centre for Economic Policy Research.

Kind, H., K.H. Midelfart-Knarvik, and G. Schjelderup (2000) "Competing for Capital in a "lumpy" world", *Journal of Public Economics* 78, pp253-274.

Ludema, R and I. Wooton (2000). "Economic geography and the fiscal effects of regional integration", *Journal of International Economics* 52 (2): pp331-357.

Martin, P and C.A Rogers (1995) `Industrial location and public infrastructure' *Journal of International Economics* 39: pp335-351.

Melitz, M. (2003). The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity, *Econometrica*, Vol. 71, November 2003, pp. 1695-1725.

O'Mahony,M and van Ark,B (2003)"EU Productivity and Competitiveness : An industry perspective" European Commission

Ottaviano, G. I. P. and T. Van Ypersele. (2005). "Market access and tax competition" Journal of International Economics, 67 (1) pp25-46.

Trionfetti, F. (2001). "Public procurement, market integration, and income inequalities". *Review of International Economics* 9(1): 29–41.

Wilson, J.D., 1986. A theory of interregional tax competition. *Journal of Urban Economics* 19, 296–315.

Wilson J (1999), "Theories of tax competition". *National Tax Journal* LII 2, pp. 269–304.

Zodrow, G.R., Mieszkowski, P. (1986). Pigou, Tiebout, property taxation, and the underprovision of local public goods. *Journal of Urban Economics* 19, 356– 370.