Risk, Returns, and Multinational Production

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(PRELIMINARY AND INCOMPLETE)

Abstract

This paper starts by unveiling a new empirical regularity: multinational corporations tend to exhibit systematically higher returns and earnings yields than non-multinational firms. Within non-multinationals, exporters tend to have higher earnings yields than firms selling only in their domestic market. To explain this pattern, we develop a real option value model where firms are heterogeneous in productivity, and have to decide whether and how to sell in a foreign market where demand is risky. Firms can serve the foreign market through trade or foreign direct investment, thus becoming multinationals. Multinational firms are more exposed to risk: following a negative shock, they are reluctant to exit the foreign market because they would forgo the option premium (sunk cost) that they paid to become multinationals. The theory provides a complementary explanation for the cross section of returns by exploiting the production side from an international point of view. We calibrate the model to match U.S. export and FDI dynamics, and use it to explain cross-sectional differences in earnings yields.

Keywords: Multinational firms, option value, cross-sectional returns

JEL Classification: F12, F23, G12

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1 Introduction

Multinational corporations tend to have higher earnings yields than non-multinational firms. Within non-multinationals, exporters tend to have higher earnings yields than firms selling only in their domestic market. Variation in earnings yields across firms is directly related to the variation in the stock returns across firms. Many studies in the new trade literature have focused on firm level features determining selection into foreign markets: exporters and multinational firms tend to be larger, more productive, to employ more workers, and sell more products than firms selling only domestically.\(^1\) To our knowledge, none of this literature has studied whether the international status of the firm matters for its investors. Similarly, there has been little attention in the financial literature in attempting to explain the features that set multinationals apart from other types of firms.

In this paper we present a real option value model, where firms choose optimally whether to produce only domestically, export, or set up an offshore affiliate to serve a foreign market where aggregate demand is uncertain. Firms in the domestic market effectively purchase an option that allows them to enter into the foreign market through exports or direct investment (henceforth, FDI). Productivity and prospects of growth of the foreign demand for goods determine the equilibrium choice. In equilibrium, firms differ in the covariance of their earnings yields with the aggregate uncertainty. The model provides a complementary explanation for the cross section of returns exploiting the production side from an international point of view.

The selection mechanism is modeled following Helpman, Melitz, and Yeaple (2004). Exports are characterized by low fixed costs and high variable costs, due to the necessity of shipping goods every period, while FDI entails high fixed costs of setting a plant abroad but low variable costs. We adapt this cost structure to a dynamic stochastic environment, using Dixit (1989) as a benchmark to model entry decision under uncertainty. Firms have to pay a sunk cost to enter the foreign market, which they weight against expected profit flows to decide whether and how to enter. The dynamic model generates imperfect sorting of firms’ productivities into international statuses and is able to explain the observed yields’ variation across firms. The main idea is the following: since foreign investment is associated to larger sunk costs compared to trade, it generates more hysteresis in the firm’s strategies over time. This implies larger losses if the foreign economy is hit by negative shocks. Multinationals are more exposed to aggregate risk since – in case of a negative shock – by

\(^1\)See Bernard, Jensen, and Schott (2009).
exiting they would forgo the option premium (sunk cost) that they paid to become multinationals. Pulling back is a more expensive strategy for multinationals than for exporters, who paid a lower option premium to serve the foreign market. Due to the imperfect sorting, more productive firms have more incentives to become multinationals but the hysteresis induced by fixed costs associated with uncertain demand may generate different dynamics for very similar firms. Each choice of status is associated with a different option value, which creates a wedge between per-period profits and the firm’s valuation over time. As a result, earnings yields depend on the choice of whether and how to serve foreign markets, therefore vary across firms with different international status.

The solution of the model delivers a series of predictions related to a firm’s productivity and to the realization of aggregate demand. First, as expected given the cost structure of the economy, foreign investment is associated to a wider hysteresis (or inaction zone) than trade. Second, more productive firms need smaller positive shocks to enter the foreign market, and larger negative shocks to exit. Moreover, for the same choice of whether and how to serve the foreign market, more productive firms are characterized by less hysteresis than less productive firms. Nonetheless, like in Helpman, Melitz, and Yeaple (2004), they tend to choose the strategy (FDI) that is associated to more hysteresis. This interaction generates an ambiguous result, which gives to the model the degree of freedom to explain the cross-sectional variation we observe in the data.

Why are we interested in the cross section of returns and earnings yields? Historically, average returns vary across stocks. Fama and French (1996) is a comprehensive description of the cross-sectional picture of returns. Stocks with high book-to-market ratios have yielded higher average returns than stocks with low book-to-market ratios. Since this variation is not captured by the traditional CAPM (or consumption-based alternatives), it has been called an anomaly, specifically, the “value premium puzzle”. In this paper we address the risk-return trade-off concerning multinational and non-multinational firms. We introduce a production based model in which firms select optimally whether to serve the foreign market at all, and how to do it. The model endogenously determines a cross-sectional difference in earnings-to-price ratios and returns. We focus on cash flow dynamics of the firm and how these are determined by endogenous decisions and exogenous risk. Multinational firms are exposed to foreign demand risk for longer due to the hysteresis result of the model. This risk must be rewarded by a higher asset returns in equilibrium. Investors will be willing to hold these companies if the returns are high enough to compensate for the risk. We find that this risk is not fully captured by the multifactor model in ?).

The existing financial literature that focuses on cross-sectional differences in earnings-to-price
ratios and returns abstracts from the international organization of the firm. For instance, the
answers of the value premium puzzle in the finance literature are based on a decomposition of
aggregate risks. Campbell and Vuolteenaho (2004), Santos and Veronesi (2005), and Hansen,
Heaton, and Li (2008) identify two types of risk: long-run and transient. Shocks to expected growth
rates are persistent, therefore they affect real variables in the long run as well. Alternatively, i.i.d.
risks that impact current period growth vanish in the short run. The different exposure of firms’
cash flows to these two types of shocks determines cross sectional differences in returns observed
in the data. Turns out that value firms are more exposed to the former. Thus, agents require a
higher reward for those risks that they fear the most. We contribute to the financial literature by
endogenizing the exposure of cash-flows to these types of shocks. Exposure is directly linked to the
decision of when and how to serve the foreign market, which is ultimately driven by productivity.
The conclusion of the model is that more productive firms select themselves into the multinational
status and offer higher returns in the long run than less productive firms.

To our knowledge, Rob and Vettas (2003) is the only other paper that developed a model of
trade and FDI with uncertain demand growth. In their framework FDI is irreversible, so it can
generate excess capacity, but has lower marginal cost compared to export. The authors show
that uncertainty implies existence of an interior equilibrium where export and FDI coexist. Our
work generalizes their model to one with many heterogeneous firms and a more general process for
demand growth, and focuses on the implications for the cross section of earnings. Alessandria and
Choi (2007) and Irrarazabal and Opromolla (2009) model entry and exit into the export market
with idiosyncratic productivity shocks and sunk costs. Our model is closer to the framework in
Irrarazabal and Opromolla (2009) for the use of the real option value analogy in solving the firm’s
optimization problem. While Irrarazabal and Opromolla (2009) concentrate their attention on the
impact of idiosyncratic productivity shocks for firm dynamics, we model aggregate demand shocks
that affect firms differently only through their endogenous choice of international status. Moreover,
we add the choice of the mode of entry. Alessandria and Choi (2007) study the impact of firms’
shocks and sunk costs on the business cycle. While the focus of our exercise is different from their
paper, we follow their calibration methodology and extend it to consider the mode of entry into a
foreign market.

Roberts and Tybout (1997) and Das, Roberts, and Tybout (2007) address empirically the issue
of market participation for export. Our model has similar predictions for both exports and FDI
sales, and can be estimated by using information from both trade/FDI flows and stock market
prices. Ramondo and Rappoport (2008) introduce idiosyncratic productivity shocks and aggregate shocks in a model where firms can locate plants both domestically and abroad. Multinational production allows firms to match domestic productivity and foreign shocks, and works as a mechanism for risk sharing. We abstract from the risk sharing/diversification motive: the risks in our model are aggregate, hence not insurable. Moreover, we model both trade and multinational production as different modes of dealing with uncertainty in foreign markets.

The remainder of the paper is organized as follows. Section 2 presents preliminary evidence on the ranking in earnings-to-price ratios according to the firms’ internationality status, and on the excess returns exhibited by multinational firms. Sections 3 and 4 develop the model and provide analytical properties of the solution. Section 5 contains the calibration and the computation of the predicted earnings yields. Section 7 concludes.

2 Motivating Evidence

In this section, we document the sorting of earnings-to-price ratios according to firms’ international status, and show evidence on excess returns displayed by multinational corporations. Our sample consists of all US-based manufacturing firms in the Compustat database, and tracks about 2000 firms from 1979 to 2007. We define a firm to be a multinational (MN) if it reports the existence of a foreign geographical segment associated with positive sales. Similarly, we define a firm to be an exporter if it reports a positive level of exports. According to this definition, on average, about 34.73% of firms sell only in the U.S. market, 20.66% also export to foreign countries, and the remaining 44.61% have positive levels of FDI sales.

Figure 1 shows earnings-to-price ratios over time for three portfolios of firms. Each portfolio is composed by firms with the same international status (only domestic sales, exporters, multinational-

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2 Multinational and exporter dummies are constructed based on Compustat geographic and operating segments data. Appendix A contains a summary of the Financial Accounting Standards Board which sets the standards for data reporting of publicly traded firms present in Compustat. ******% of firms in the data set report both positive exports and FDI sales. We classify these firms as multinationals, based on the criterium that the existence of FDI sales is sufficient to make them subject to the risk of owning a plan in a foreign market. One of FASB’s roles is to “require significant disclosures about the separate operating segments of an entity’s business so that investors can evaluate the differing risks in the diverse operations.” More details on how standards set the disclosure requirements are in the appendix.

3 Notice that the shares of firms belonging to each group are very different from what reported in other papers. Particularly, the shares of exporters and multinational firms are disproportionately large. This is due to the fact that Compustat collects data for publicly listed firms only, which tend to be the largest firms in the population.
The solid line represents earnings-to-price ratios of multinational firms, the dashed line the ones of exporters, and the dash-dotted line the ones of firms selling only domestically. Multinational firms exhibit higher earnings-to-price ratios than non-multinational firms, consistently over the entire time period. The ranking between domestic firms and exporters is less clear from the raw data, but emerges when controlling for the effect of other variables. The average $e/p$ ratios for the three groups are 5.25\% for MN firms, 3.01\% for exporters, and 1.89\% for firms selling only domestically. To separate the effects of the international status from other firm characteristics, Table 1 displays the results of the following firm-level regression:

$$(e/p)_{it} = \alpha + \gamma_1 D_{it}^{MN} + \gamma_2 D_{it}^{EXP} + \gamma_3 \beta_i^{MKT} + \gamma_4 X_{it} + \delta_{NAICS} + \delta_t + \epsilon_{it}$$

where $D_{it}^{MN}$ and $D_{it}^{EXP}$ are multinational and exporter dummies, respectively, $\beta_i^{MKT}$ is the market

\footnote{The portfolios are constructed as follows. For each firm $i$, determine its status $S$ ($S = D, X, I$) at the end of year $t - 1$, and collect data on earnings ($e_i$) and market capitalization ($p_i$) in year $t$ to compute the earnings-to-price ratio $e_i/p_i$. Aggregate earnings-to-price ratios for each portfolio are computed by weighting each firm’s observation by the firm’s market share:}

$$\left(\frac{E}{P}\right)_t^S = \sum_{i \in S} \omega_i \frac{e_i}{p_i}$$

where: $\omega_i = \frac{p_i}{\sum_{i \in S} p_i}$, and $S = D, X, I$. 

Figure 1: Earnings-to-price ratios, portfolios of firms in each group.
Table 1: Earning-price regressions. Firm-level regressions of earnings-to-price ratios on multinational and exporter dummies, market betas and other controls, with year and industry fixed effects. (Top and bottom one percent of sample excluded, all dollar values expressed in billions).

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MN dummy</td>
<td>.068</td>
<td>.069</td>
</tr>
<tr>
<td></td>
<td>(.004)***</td>
<td>(.004)***</td>
</tr>
<tr>
<td>EXP dummy</td>
<td>.039</td>
<td>.039</td>
</tr>
<tr>
<td></td>
<td>(.004)***</td>
<td>(.004)***</td>
</tr>
<tr>
<td>$\beta^{MKT}$</td>
<td>-.051</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.005)***</td>
<td></td>
</tr>
<tr>
<td>K/L</td>
<td>2.568</td>
<td>3.369</td>
</tr>
<tr>
<td></td>
<td>(8.283)</td>
<td>(9.285)</td>
</tr>
<tr>
<td>sales per emp.</td>
<td>32.043</td>
<td>32.846</td>
</tr>
<tr>
<td></td>
<td>(18.812)*</td>
<td>(19.538)*</td>
</tr>
<tr>
<td>total revenue</td>
<td>.0005</td>
<td>.0005</td>
</tr>
<tr>
<td></td>
<td>(.0003)**</td>
<td>(.0003)*</td>
</tr>
<tr>
<td>market cap.</td>
<td>.001</td>
<td>.001</td>
</tr>
<tr>
<td></td>
<td>(.0002)***</td>
<td>(.0002)***</td>
</tr>
</tbody>
</table>

No. of obs. 52096 52096
$R^2$ .077 .082

beta of the primary security of firm $i$, $X_{it}$ is a set of controls, including capital/labor ratio, sales per employee (our measure of productivity), total revenues and market capitalization (measures of size). $\delta_{NAICS}$ and $\delta_t$ are 4-digit industry and year fixed effects, respectively, and $\epsilon_{it}$ is an orthogonal error term.

The coefficients associated to exporters and MN dummies are positive and significant. Moreover, the coefficient associated to multinationality appears to be larger than the one associated to export status, identifying a further difference between these two groups. The purpose of adding the market betas is to control for aggregate market risk and to highlight the contribution of the international status to the magnitude of earning yields once market risk is accounted for.

As we argued in the Introduction, earnings-to-price ratios carry information about returns on the firms’ stocks. Table 2 is taken from Lettau and Wachter (2007), and shows how portfolios delivering higher earning yields are also associated to higher excess returns. Our data confirm this correlation for multinational firms versus non-multinational firms. Table 3 reports the results of

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5The market betas have been computed by running a regression of individual security returns on the market aggregate returns (NYSE, AMEX, and Nasdaq) for the entire sample period. Adding the market betas in the regression captures the exposure of the security to aggregate market risk.
Table 2: **Mean Excess Return (% per year)**. Portfolios are formed by sorting firms into deciles on the earnings yield (E/P), ratio of cash flow to prices (C/P), dividend yield (D/P), and book-to-market ratio (B/M). Monthly (annualized) data, from 1952 to 2002. From Lettau and Wachter (2007).

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Growth to Value</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>G</td>
<td>2</td>
</tr>
<tr>
<td>E/P</td>
<td>4.71</td>
<td>5.02</td>
</tr>
<tr>
<td>C/P</td>
<td>5.05</td>
<td>6.07</td>
</tr>
<tr>
<td>D/P</td>
<td>7.35</td>
<td>6.41</td>
</tr>
<tr>
<td>B/M</td>
<td>5.67</td>
<td>6.55</td>
</tr>
</tbody>
</table>

The coefficient on the MN dummy is positive and significant, confirming that multinational firms tend to have higher returns than non-multinational firms, controlling for market risk, capital intensity, and size. Any cross-sectional differences in returns generated by exposure to aggregate risk is captured by cross sectional differences in their market betas. Hence, the significant coefficient on the multinational dummy identifies a separate source of risk. The coefficient on the exporter dummy does not appear to be significant. In the regression, we exclude firms that switch status at some point in the sample period. The reason for this exclusion is that the switches provide noisy information about returns per status. Data on returns are monthly, while the segments data on which our status dummies are based are annual: for this reason, it is hard to correctly attribute returns to status, as we do not observe when exactly the change in status took place.

After an exploration of earnings-to-price and returns across the three groups of firms, it seems natural to explore the source of higher returns. Higher returns do not constitute by themselves a puzzle. From a CAPM point of view, higher returns must be explained by higher betas, or co-movements with the aggregate risks. Beyond the one-factor CAPM model, ?) introduced a multifactor extension to the original CAPM. ?) argue that a unique source of risk is not able to explain the cross section of returns. Instead, a three factor model is developed and explain a higher fraction of the variation in expected returns. Higher returns must be explained by higher exposures to either of these three factors: market excess returns, high-minus-low book-to-market, or small-minus-big portfolio. Each of the three factors is assumed to mimic a macroeconomic

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6We identify firm-level returns with the returns of the firm’s common equity. Data on returns are taken from CRSP.

7In the data, about 43% of firms switch status at least once during the sample period.
Table 3: **Returns Regressions.** Firm-level regressions of returns on multinational and exporter dummies, market betas and other controls, with year and industry fixed effects. (Firms switching status excluded, top and bottom one percent of sample excluded, all dollar values expressed in billions).

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MN dummy</td>
<td>.057</td>
<td>.057</td>
</tr>
<tr>
<td></td>
<td>(.008)**</td>
<td>(.008)**</td>
</tr>
<tr>
<td>EXP dummy</td>
<td>-.006</td>
<td>-.008</td>
</tr>
<tr>
<td></td>
<td>(.009)</td>
<td>(.009)</td>
</tr>
<tr>
<td>$\beta^{MKT}$</td>
<td></td>
<td>.041</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.007)**</td>
</tr>
<tr>
<td>K/L</td>
<td>3.612</td>
<td>3.449</td>
</tr>
<tr>
<td></td>
<td>(2.158)*</td>
<td>(1.771)*</td>
</tr>
<tr>
<td>sales per emp.</td>
<td>46.118</td>
<td>44.980</td>
</tr>
<tr>
<td></td>
<td>(19.045)**</td>
<td>(18.671)**</td>
</tr>
<tr>
<td>total revenue</td>
<td>.0005</td>
<td>.0005</td>
</tr>
<tr>
<td></td>
<td>(.0003)</td>
<td>(.0003)</td>
</tr>
<tr>
<td>market cap.</td>
<td>-.001</td>
<td>-.0008</td>
</tr>
<tr>
<td></td>
<td>(.0003)**</td>
<td>(.0003)**</td>
</tr>
<tr>
<td>Obs.</td>
<td>24340</td>
<td>24340</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.194</td>
<td>.196</td>
</tr>
</tbody>
</table>
Table 4: Firm level regressions. Summary statistics of the firm-level time series coefficient estimates of Fama-French 3 factor regressions: Firm level annual excess returns are regressed on the three Fama-French factors: market excess return, SMB (Small Minus Big) which is the average return on the three small-size portfolios minus the average return on the three big-size portfolios, and HML (High Minus Low) which is the average return on the two value portfolios minus the average return on the two growth portfolios. The \( \alpha \)'s capture the pricing errors of the three-factor model. There are 10005 regressions for multinational firms (MN), 4388 for exporting firms (X), and 10971 for domestic firms (D).

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{MN} )</td>
<td>0.0262</td>
<td>0.5879</td>
</tr>
<tr>
<td>( \alpha_X )</td>
<td>-0.0258</td>
<td>0.7662</td>
</tr>
<tr>
<td>( \alpha_D )</td>
<td>-0.0393</td>
<td>0.8301</td>
</tr>
<tr>
<td>( \beta_{mkt}^{MN} )</td>
<td>1.139</td>
<td>5.879</td>
</tr>
<tr>
<td>( \beta_{mkt}^X )</td>
<td>0.2793</td>
<td>8.4256</td>
</tr>
<tr>
<td>( \beta_{mkt}^D )</td>
<td>1.135</td>
<td>5.2011</td>
</tr>
<tr>
<td>( \beta_{SMB}^{MN} )</td>
<td>0.9357</td>
<td>8.4495</td>
</tr>
<tr>
<td>( \beta_{SMB}^X )</td>
<td>1.106</td>
<td>9.0135</td>
</tr>
<tr>
<td>( \beta_{SMB}^D )</td>
<td>1.6518</td>
<td>7.4945</td>
</tr>
<tr>
<td>( \beta_{HML}^{MN} )</td>
<td>-0.3216</td>
<td>6.6390</td>
</tr>
<tr>
<td>( \beta_{HML}^X )</td>
<td>-0.6967</td>
<td>7.8190</td>
</tr>
<tr>
<td>( \beta_{HML}^D )</td>
<td>-0.0219</td>
<td>6.0135</td>
</tr>
</tbody>
</table>

aggregate risk, and the three together span the entire space of assets. Therefore, any asset is a linear combination of the three Fama-French factors. Table 4 shows the results of running one time-series regression per manufacturing firm in Compustat. The results displayed are sample averages and standard deviations of the sample of estimated coefficients. Strikingly, multinationals feature a higher pricing errors on average than exporters and domestics. An explanation for this initial anomaly is that the risk incurred by a firm which decides to serve foreign market is not fully captured by the three Fama-French factors. This initial results shift the explanation of the higher returns. The production-based model that we used to explain firms behavior is indeed a “new” source of risk, being new defined as a risk factor beyond value or growth.

Firm level regressions are subject to substantial idiosyncratic noise. Next, we compute three portfolios composed by firms according to their international status: domestic, exporters, and multinationals. Every year portfolios are formed by value-weighted firms belonging to each of the three categories. The same multifactor regressions are computed and we present the results
Table 5: **Portfolio regressions.** Time series coefficient estimates of Fama-French 3 factor regressions for the three value-weighted portfolios based on multinational status, *Domestic* (DOM), *Exporter* (EXP), and *Multinational* (MN). Portfolio annual excess returns are regressed on the three Fama-French factors: market return in excess of the 3 month T-bill rate, SMB (Small Minus Big) which is the average return on the three small-size portfolios minus the average return on the three big-size portfolios, and HML (High Minus Low) which is the average return on the two value portfolios minus the average return on the two growth portfolios. The $\alpha$s capture the pricing errors of the three-factor model. Firms with less than 10 million market capitalization have been dropped from the sample.

<table>
<thead>
<tr>
<th></th>
<th>DOM</th>
<th>EXP</th>
<th>MN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^{mkt}$</td>
<td>1.029***</td>
<td>0.752***</td>
<td>0.915***</td>
</tr>
<tr>
<td></td>
<td>(0.113)</td>
<td>(0.134)</td>
<td>(0.0606)</td>
</tr>
<tr>
<td>$R^{SMB}$</td>
<td>0.140</td>
<td>0.464***</td>
<td>-0.131*</td>
</tr>
<tr>
<td></td>
<td>(0.132)</td>
<td>(0.156)</td>
<td>(0.0709)</td>
</tr>
<tr>
<td>$R^{HML}$</td>
<td>0.127</td>
<td>-0.197</td>
<td>-0.139**</td>
</tr>
<tr>
<td></td>
<td>(0.109)</td>
<td>(0.129)</td>
<td>(0.0584)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.0132</td>
<td>0.00537</td>
<td>0.0194*</td>
</tr>
<tr>
<td></td>
<td>(0.0197)</td>
<td>(0.0233)</td>
<td>(0.016)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
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<th>EXP</th>
<th>MN</th>
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<tbody>
<tr>
<td>Obs.</td>
<td>29</td>
<td>29</td>
<td>29</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.803</td>
<td>0.733</td>
<td>0.931</td>
</tr>
</tbody>
</table>

In Table 5. It is worth to mention that the risk to which the multinationals are exposed and the corresponding higher returns that they provide to investors is not captured at all by the three existing Fama-French factors. On the contrary, we find that the market betas are significantly lower than those of only domestic firms. Exposure to the HML factor, related to the value premium, is also significantly lower than the exposure of domestic and exporter firms to the same factor. Evidently, if the exposure to the 3 factors does not explain the higher reward that multinational stocks provide, it must be reflected in the pricing errors of the model. Effectively, the *alpha* of the portfolio of multinational firms is significantly different from zero.

[*TBA: GRS tests on $\alpha$*]
3 Model

3.1 Preferences and Technology

The economy is composed of two countries, a home country and a foreign one. Variables related to consumers and firms from the foreign country are going to be denoted with an asterisk (*). In both countries, agents are infinitely lived, and have time-separable intertemporal preferences. Agents derive utility from two goods: a homogeneous good, that we denote with \( H \), and a differentiated good, that we denote with \( Q \). Preferences are defined by:

\[
U = \int_0^\infty e^{-\rho t} \left[ H(t)^{1-\zeta} Q(t)^\zeta \right] dt
\]

where \( \rho > 0 \) is the discount rate, \( \zeta \in [0, 1] \), \( Q \) is an aggregate of differentiated varieties, \( Q(t) = \left( \int q_i(t)^{1-1/\eta} di \right)^{\eta/(\eta-1)} \), and \( \eta > 1 \) is the elasticity of substitution across varieties.

Agents maximize \( U \) subject to their budget constraint. While income in the home country is deterministic, we assume that income in the foreign country is hit by aggregate random shocks affecting aggregate demand for the differentiated product \( Q^* \). More precisely, \( Q^* \) follows a geometric Brownian motion:

\[
\frac{dQ^*}{Q^*} = \mu dt + \sigma dz
\]

where \( 0 \leq \mu < \rho \) and \( dz \) is the increment of a standard Wiener process, satisfying: \( E(dz) = 0 \) and \( E(dz^2) = dt \).

Labor is the only factor of production. The homogeneous good is produced by firms in both countries with a one-to-one linear technology under perfect competition, and is perfectly tradeable.\(^8\) We assume that firms from the foreign country only produce the homogeneous good, while firms from the home country produce goods in both sectors and sell both homogeneous and differentiated products in both countries.\(^9\) One could think for example about the foreign country as an economy whose income is subject to natural resource shocks, or whose purchasing power is subject to exchange rate shocks. The homogeneous good could be interpreted as a standard good that both

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\(^8\)The existence of a perfectly tradeable good that is produced and consumed in both countries pins down relative wages as the relative productivity in producing the good. Here we assume that the two countries produce the homogeneous good with identical technologies, so that \( w = w^* = 1 \).

\(^9\)We introduce this artificial asymmetry between the two countries in order to present in the simplest possible way the problem of entry in a risky foreign market. Solving the model for a symmetric world where both countries produce differentiated varieties and are hit by aggregate shocks is straightforward. We develop the results of the paper in the simpler asymmetric case, and we show that the results easily extend to the symmetric case in section 6.
countries produce and consume, and the differentiated good can be seen as a set of more advanced goods that the foreign country does not have the technology to produce.

The differentiated sector is characterized by a monopolistically competitive market structure. The differentiated sector is populated by a continuum of firms, whose mass we denote by $n$. Each firm produces a differentiated variety with a linear technology defined by a unit labor requirement $a$, which is a random draw from a distribution $G(a)$. $a$ indicates the number of units of labor that a firm needs in order to produce one unit of a differentiated variety.

Let’s now turn to the description of the production costs in the differentiated sector. All firms in the differentiated sector produce in the home market. We assume that there are no fixed costs associated to production for the domestic market, so every firm makes positive profits from domestic sales. Besides producing for the domestic market, firms from the home country must decide whether to produce also for the foreign country one. Production in the foreign market involves fixed operating costs, to be paid every period, and sunk costs of entry. If a firm decides to sell in the foreign market, it can do so either via exports or via foreign direct investment. We call multinationals those firms that decide to serve the foreign market through FDI sales.

We model the choice between trade and FDI along the lines of Helpman, Melitz, and Yeaple (2004): exports entail a relatively small sunk cost of entry $F_X$, but a per-unit iceberg transportation cost $\tau$ to be paid every period, while FDI is associated to a larger sunk cost $F_I$ ($F_I > F_X$), but there are no transportation costs to be covered every period, as both production and sales happen in the foreign market. Both options also entail fixed operating costs to be paid every period, that we denote with $f_I$ and $f_X$ for FDI and exports, respectively. Once entered the foreign market, a firm can exit at no cost. However, if it decides to re-enter, it will have to pay the sunk cost again. Sunk costs and stochastic demand imply that firms decide to enter when their expected profits are well above zero, and – once entered – are reluctant to exit even in case of losses due to negative shocks. We will show that this dynamic behavior, labeled “hysteresis” by the literature (see Dixit and Pindyck (1994)), is more sever for multinational firms than for exporters, due to the larger sunk costs of FDI. Notice also that the cost structure and the nature of uncertainty in the model imply that if a firm decides to enter the foreign market, it will do so either as an exporter or as a

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10 We could have introduced positive fixed costs of domestic production, and modeled the initial decision of entry in the domestic market, like in Helpman, Melitz, and Yeaple (2004). This would have introduced additional complications in solving for firms’ optimal dynamics, without any gains for our empirical analysis. Compustat includes only publicly listed firms, so when a firm enters or exits Compustat we do not have any information about whether the firm is in fact entering or exiting the market, or whether it just started or stopped being listed.

11 $\tau > 1$ units of good need to be shipped for one unit of good to arrive to the destination country.
multinational firm, but it will never adopt the two strategies at the same time.\footnote{This feature of the model is the same as in Helpman, Melitz, and Yeaple (2004). Rob and Vettas (2003) obtain the existence of an equilibrium where firms can optimally choose to adopt simultaneously the two strategies because in their model firms choose the amount of the foreign investment, and given the structure of demand there may be the possibility of overinvestment. In their framework, FDI can be adopted to cover certain demand, and exports may be used to serve the additional random excess demand without incurring the cost of a larger investment that could be underutilized. In the data we do observe firms that both export and have FDI sales. This fact could be rationalized within our framework by having multiproduct firms that choose different strategies for different product lines, or in a multi-country model where firms choose different strategies to enter different countries. Unfortunately, there is not enough information in the segments data to check whether any of these is the case. Explaining the choice of firms to adopt both entry strategies would probably need a differently tailored framework, and is beyond the scope of this paper.}

Hence for a given realization of $Q^*$, a firm with productivity $1/a$ must choose its optimal status $S \in \{D, X, I\}$, i.e. domestic, exporter, or multinational, the current selling price $p_S(a)$ and an updating rule (how to change the optimal price and status following changes in aggregate foreign demand). The state of the economy is described by the realization of foreign aggregate demand $Q^*$.

The CES aggregation over individual varieties implies that individual pricing rules are independent on $Q^*$. However, marginal costs of production and optimal pricing rules vary with the status of the firm. Let $w, w^*$ denote the wages in the home and foreign countries, respectively. The marginal cost of domestic production is given by the labor requirement times the domestic wages, $MC_D = aw$. The marginal cost of exporting is augmented by the iceberg transportation cost: $MC_X = \tau aw$. When the firm serves the foreign market through FDI, firm-specific productivity is transferred to the foreign country and the firm employs foreign labor: $MC_I = aw^*$. CES preferences across varieties of the differentiated good imply that the optimal prices are $p_S(a) = \frac{\eta}{\eta-1}MC_S(a)$.

Let $\pi_D(a; Q)$, $\pi_X(a; Q^*)$ and $\pi_I(a; Q^*)$ denote the per-period profits from domestic sales, from exports and from FDI sales abroad, respectively, for a firm with productivity $1/a$, given a realization of the aggregate quantity demanded in the domestic (foreign) market equal to $Q$ ($Q^*$):

\begin{align*}
\pi_D(a; Q) &= B(aw)^{1-\eta}P^{\eta}Q \\
\pi_X(a; Q^*) &= B(\tau aw)^{1-\eta}(P^*)^\eta Q^* - f_X \\
\pi_I(a; Q^*) &= B(aw^*)^{1-\eta}(P^*)^\eta Q^* - f_I
\end{align*}

where $B \equiv \frac{1}{\eta-1} \left( \frac{\eta}{\eta-1} \right)^{-\eta}$, and $P$ ($P^*$) is the aggregate price of the differentiated good in the domestic (foreign) market, that firms take as given while solving their maximization problem. In order to assure the existence of exporters in equilibrium, we assume that the cost parameters satisfy
the following inequality:\footnote{Condition (6) is the “present discounted value equivalent” of the analogous assumption in Helpman, Melitz, and Yeaple (2004). It is derived imposing that the profit functions of an exporter and of a multinational firm – expressed as functions of the productivity level $a$ – cross at a point associated to positive profits.}

**Assumption 1.**

\[
\left( \frac{w^*}{w} \right)^{\eta-1} (f_I + \rho F_I) > \tau^{\eta-1} (f_X + \rho F_X). \tag{6}
\]

### 3.2 Value Functions

We solve the model along the lines of Dixit (1989). Let $V_S(a; Q^*)$ denote the expected net present value for a firm with productivity $1/a$ in status $S$ ($S = D, X, I$) starting with aggregate demand $Q^*$ in the foreign market and following optimal policy. As we assume no uncertainty in the domestic market, firms in all statuses $S$ have positive profits $\pi_D(a; Q)$ from domestic sales. We abstract from those profits and interpret the value functions as net of domestic profits.

Over a generic time interval $\Delta t$, the Bellman equation for a firm that is currently selling only in the domestic market is:

\[
V_D(a, Q^*) = \max \left\{ \frac{1}{1 + \rho \Delta t} E[V_D(a, (Q^*)')|Q^*] ; V_X(a, Q^*) - F_X ; V_I(a, Q^*) - F_I \right\}. \tag{7}
\]

The right-hand side of the Bellman equation expresses the firm’s possibilities. If it sells only domestically, it gets the continuation value from not changing status, equal to the expected value of the firm conditional on the current realization of foreign demand $Q^*$. If it decides to switch to export (FDI) it gets the value of being an exporter, $V_X$ (multinational, $V_I$) minus the sunk cost of entry $F_X$ ($F_I$). Similarly, the Bellman equation for an exporter is:

\[
V_X(a, Q^*) = \max \left\{ \pi_X(a, Q^*) \Delta t + \frac{1}{1 + \rho \Delta t} E[V_X(a, (Q^*)')|Q^*] ; V_D(a, Q^*) ; V_I(a, Q^*) - F_I \right\}. \tag{8}
\]

and for a multinational:

\[
V_I(a, Q^*) = \max \left\{ \pi_I(a, Q^*) \Delta t + \frac{1}{1 + \rho \Delta t} E[V_I(a, (Q^*)')|Q^*] ; V_D(a, Q^*) ; V_X(a, Q^*) - F_X \right\}. \tag{9}
\]

Notice that the continuation value of an exporter (a multinational) also includes the flow profits from sales in the foreign market $\pi_X(a, Q^*) \Delta t$ ($\pi_I(a, Q^*) \Delta t$). There are no costs associated with
exiting the foreign market: if a firm decides to exit, its value is simply the one of a domestic firm: 
\[ V_D(a, Q^*) \].

In the continuation region, the Bellman equation of a domestic firm reduces to: 
\[ \rho V_D(a, Q^*) = \frac{E(dV_D(a, Q^*))}{dt} \] (the instantaneous return must be equal to the expected capital gain). By applying Ito’s lemma, we obtain:

\[ \rho V_D(a, Q^*) = V_D'(a, Q^*) \mu Q^* + \frac{1}{2} V_D''(a, Q^*) \sigma^2(Q^*)^2. \] (10)

The solution takes the form 
\[ V_D(Q^*) = (Q^*)^{\xi}, \] where:

\[ \xi = \frac{(1 - m) \pm \sqrt{(1 - m)^2 + 4r}}{2} \]

and \( m = 2\mu/\sigma^2, \ r = 2\rho/\sigma^2 \). Hence the value function of a domestic firm takes the form:

\[ V_D(a, Q^*) = A_D(a)(Q^*)^\alpha + B_D(a)(Q^*)^\beta \] (11)

where \( \alpha \) and \( \beta \) are the negative and positive values of \( \xi \), respectively, and \( A_D(a) \) and \( B_D(a) \) are parameters to be determined.

To compute the value functions of exporters and multinationals, we need to take into account also the current profits from sales in the foreign market. In the continuation region, the value of a firm starting as an exporter with foreign aggregate demand \( Q^* \) satisfies:

\[ \rho V_X(a, Q^*) = B(aw)^{1-\eta}(P^*)^{-\eta}Q^* - f_X + V_X'(a, Q^*) \mu Q^* + \frac{1}{2} V_X''(a, Q^*) \sigma^2(Q^*)^2 \] (12)

and the value of a firm starting as a multinational with aggregate demand \( Q \) satisfies:

\[ \rho V_I(a, Q^*) = B(aw^*)^{1-\eta}(P^*)^{-\eta}Q^* - f_I + V_I'(a, Q^*) \mu Q^* + \frac{1}{2} V_I''(a, Q^*) \sigma^2(Q^*)^2. \] (13)

For \( S = X, I \), the value function takes the affine form \( V_S(Q^*) = (Q^*)^{\xi} + c_{s0} + c_{s1}Q^* \). By substituting it in the expressions above, the value functions of an exporter and a multinational firm can be
written as:

\[ V_X(a, Q^*) = A_X(a)(Q^*)^\alpha + B_X(a)(Q^*)^\beta + \frac{B(\tau aw)^{1-\eta}(P^*)^\eta Q^*}{\rho - \mu} - \frac{f_X}{\rho} \]  

(14)

\[ V_I(a, Q^*) = A_I(a)(Q^*)^\alpha + B_I(a)(Q^*)^\beta + \frac{B(aw^*)^{1-\eta}(P^*)^\eta Q^*}{\rho - \mu} - \frac{f_I}{\rho} \]  

(15)

where \( A_X(a), B_X(a), A_I(a) \) and \( B_I(a) \) are parameters to be determined.

Equations (11), (14), and (15) describe the value functions in their continuation regions. We still need to solve for the updating rule, which in this case consists of thresholds in the realizations of the state \( Q^* \) that induce firms to switch status. Let \( Q^*_SR(a) \) denote the quantity threshold at which a firm with productivity \( 1/a \) switches from status \( S \) to status \( R \), for \( S, R \in \{D, X, I\} \). In order to find the 6 quantity thresholds \( Q^*_SR(a) \) and the 6 value function parameters \( A_S(a), B_S(a), \) for \( S \in \{D, X, I\} \), we impose the following value-matching and smooth-pasting conditions:

\[ V_D(a, Q^*_{DX}(a)) = V_X(a, Q^*_{DX}(a)) - F_X \]  

(16)

\[ V_D(a, Q^*_{DI}(a)) = V_I(a, Q^*_{DI}(a)) - F_I \]  

(17)

\[ V_X(a, Q^*_{XD}(a)) = V_D(a, Q^*_{XD}(a)) \]  

(18)

\[ V_X(a, Q^*_{XI}(a)) = V_I(a, Q^*_{XI}(a)) - F_I \]  

(19)

\[ V_I(a, Q^*_{ID}(a)) = V_D(a, Q^*_{ID}(a)) \]  

(20)

\[ V_I(a, Q^*_{IX}(a)) = V_X(a, Q^*_{IX}(a)) - F_X \]  

(21)

\[ V_R'(a, Q^*_{RS}(a)) = V_S'(a, Q^*_{RS}(a)), \text{ for } S, R \in \{D, X, I\}. \]  

(22)

For each \( a \), equations (16)-(22) are a system of 12 equations in the 12 unknowns given by the 6 quantity thresholds \( Q^*_SR(a) \) and by the value functions parameters \( A_S(a), B_S(a), \) for \( S, R \in \{D, X, I\} \). The system is highly nonlinear, and as such is associated to multiple solutions. To get an economically sensible solution, we follow Dixit (1989) and impose \( A_D(a) = 0, \forall a \): the option of entering a foreign market is nearly worthless for a domestic firm experiencing a very low \( Q \). Consistently, it must be that \( B_D(a) \geq 0 \) to insure non-negativity of \( V_D(a, Q^*) \). The option of quitting FDI for another strategy is nearly worthless for a multinational firm experiencing an extremely high \( Q^* \), hence \( B_I(a) = 0 \). Similarly, a multinational firm has expected value \( \frac{B(aw^*)^{1-\eta}(P^*)^\eta Q^*}{\rho - \mu} - \frac{f_I}{\rho} \) from the strategy of never changing status, hence the optimal strategy must yield a no lesser value.
\[ A_I(a) \geq 0. \] Finally, an exporter has expected value \( \frac{B(\tau \omega)^{1-\eta}(P^*)^{\eta}Q^* - \mu}{\rho} \) from the strategy of never changing status, hence its optimal strategy must yield a no lesser value for any realization of \( Q^* \): \( A_X(a), B_X(a) \geq 0 \). Consequently, the value function of a domestic firm \( V_D \) is increasing on the entire domain, indicating the fact that, as the realized aggregate demand in the foreign market \( Q \) increases, the value of the option of entering the foreign market (either through trade or FDI) increases. The value functions of an exporter and of a multinational (\( V_X \) and \( V_I \) respectively) are U-shaped: for low levels of \( Q \), the term with the negative exponent \( \alpha \) dominates, and the value is high thanks to the option of leaving the market. Conversely, for high levels of \( Q \) the value is high due to the profit stream that the firm derives from staying in the market and – for exporters – due to the additional option value of becoming a multinational firm (the term with the positive exponent \( \beta \)).

### 3.3 Equilibrium

Under our assumption, whereby firms from the foreign country only produce the homogeneous good, the aggregate price of the differentiated good in the two countries is given by:

\[
P^{1-\eta} = n \int \left( \frac{\eta \omega}{\eta - 1} \right)^{1-\eta} dG(a) \quad (23)
\]

\[
(P^*)^{1-\eta} = n \left[ \int_{\Omega_X(Q^*)} \left( \frac{\eta \tau \omega}{\eta - 1} \right)^{1-\eta} dG(a) + \int_{\Omega_I(Q^*)} \left( \frac{\eta \omega^*}{\eta - 1} \right)^{1-\eta} dG(a) \right] \quad (24)
\]

where \( n \) is the mass of firms from the home country in the differentiated goods sector, and \( \Omega_X \) (\( \Omega_I \)) is the subset of these firms that export (have multinational sales) when the realization of the state is \( Q^* \).

Since we abstract from the problem of entry in the domestic market, we take the mass of firms \( n \) as given\(^{14} \). The aggregate quantity \( Q \) and the initial value of \( Q^* \), \( Q^*_0 \), are also taken as given. The existence of a perfectly tradeable homogeneous good produced in both countries implies: \( w = w^* = 1 \).

\(^{14}\)Since we do not impose free-entry, we can normalize the mass of firms to \( n = 1 \), and present the results in terms of shares of total firms.
3.4 Earnings-to-Price Ratios

In Section 2 we showed data on earnings-to-price ratios, displaying a ranking across firms belonging to different groups according to their international status. Our earnings yields measure in the model is given by the ratio $\pi_t/V_t$, where $\pi_t$ represents per-period profits and $V_t$ is the market value of the firm. In a static model, or in a dynamic but deterministic model, $\pi_t/V_t$ is constant and independent on the firm’s status, since per-period profits and value of the firm coincide. Dynamics and uncertainty introduce a wedge between these two magnitudes, which reflects the option value.

Let $ep_S(a, Q^*)$ denote the earning yield of a firm with productivity $1/a$ in status $S$ when the realization of aggregate foreign demand is $Q^*$. Earning yields in the model are given by:

$$ep_D(a, Q^*) = \frac{\pi_D}{(r-\mu)} + B_D(Q^*)^\beta$$  \hspace{1cm} (25)

$$ep_X(a, Q^*) = \frac{\pi_D + \pi_X(Q^*)}{(r-\mu)} + A_X(Q^*)^\alpha + B_X(Q^*)^\beta$$  \hspace{1cm} (26)

$$ep_I(a, Q^*) = \frac{\pi_D + \pi_I(Q^*)}{(r-\mu)} + A_I(Q^*)^\alpha.$$  \hspace{1cm} (27)

The empirical evidence presented in Section 2 suggests the following ordering in aggregate earnings yields across groups:

$$\int_{\Omega_D(Q^*)} ep_D(a, Q^*)dG(a) < \int_{\Omega_X(Q^*)} ep_X(a, Q^*)dG(a) < \int_{\Omega_I(Q^*)} ep_I(a, Q^*)dG(a)$$  \hspace{1cm} (28)

where $\Omega_D(Q^*)$, $\Omega_X(Q^*)$, and $\Omega_I(Q^*)$ are the sets of firms that have domestic sales only, exporters, and multinationals, respectively.

The objective of Section 5.2 will be to find a suitable parameterization, able to reproduce features of trade and multinational production dynamics that we observe in the data, and also able to generate the observed ranking in average earnings-to-price ratios. Before moving to the numerical exercise, in the next section we show a series of qualitative properties of the model that illustrate its amenability to reproduce features of the trade dynamics data.
4 Qualitative Properties of the Solution

4.1 Ordering of the Quantity Thresholds

The relationship between the sunk costs of exporting and FDI, $F_I > F_X$, implies a precise ordering of the quantity thresholds that are solution of (16)-(22). Theorem 1 contains this result.

**Theorem 1.** If $F_I > F_X$, the quantity thresholds $Q_{RS}^*(a)$, for $R, S \in \{D, X, I\}$, solution of system (16)-(22), satisfy the following ordering:

$$Q_{IX}^*(a) < Q_{ID}^*(a) < Q_{XD}^*(a) < Q_{DX}^*(a) < Q_{DI}^*(a) < Q_{XI}^*(a) (29)$$

for each given productivity level $1/a$.

**Proof:** See Appendix B.

Like in Dixit (1989), the pure presence of sunk costs implies that “entry” thresholds are higher than “exit” thresholds: $Q_{DX} > Q_{XD}$, $Q_{DI} > Q_{ID}$, and $Q_{XI} > Q_{IX}$.\(^{15}\) A higher quantity demanded $Q^*$ is needed to induce a firm to enter a foreign market through FDI with respect to the quantity necessary to induce the firm to export: $Q_{DI} > Q_{DX}$. An even larger positive shock is needed to induce and exporter to become a multinational, since he is already serving the foreign market with exports: $Q_{XI} > Q_{DI}$. Similarly, a larger negative shock is needed to induce a multinational to exit the foreign market with respect to the shock needed to induce an exporter to exit: $Q_{ID} < Q_{Xd}$. Finally, an even larger negative shock is needed to induce a multinational to divest but serve the foreign market as an exporter: $Q_{IX} > Q_{ID}$.

4.2 Comparative Statics: Value and Productivity

System (16)-(22) makes clear that both the quantity thresholds and the parameters of the value functions depend on the productivity level $1/a$. Figure 2 shows the value function of a domestic firm as a function of aggregate quantity demanded in the foreign market $Q^*$ and of productivity $1/a$. As observed in the previous section, $V_D$ is increasing in $Q^*$, as the option value of entering the foreign market is increasing in the quantity demanded. $V_D$ is also increasing in firm’s productivity, as more productive firms can get higher profits from entering the foreign market.

\(^{15}\)Results about hysteresis, together with the analytical proof of the ordering of the thresholds $Q_{RS}$, for $R, S \in \{D, X, I\}$, are contained in Appendix B.
Figure 2: Value function of a domestic firm.

Figure 3 shows the value functions of an exporter and of a multinational firm as functions of $Q^*$ and $1/a$. As previously observed, $V_X$ and $V_I$ are U-shaped functions of $Q^*$, indicating the high option value of exiting for low realizations of $Q^*$ and the high option value of not changing status for high realizations of $Q^*$. The behavior of the value functions for $Q^* \to 0$ does not vary much across the productivity dimension: when $Q^*$ is low, the value is high as firms of all productivity levels associate a high value to the option of exiting. The behavior of the value functions when $Q^*$ is “large”, conversely, varies with individual productivity: the value function is steeper for higher productivity firms, indicating that more productive firms obtain higher returns from staying in the foreign market when the realized aggregate demand is high.

From Figure 3, the qualitative behavior of $V_X$ and $V_I$ appears very similar. Figure 4 plots the difference between the value functions of firms serving the foreign market and firms selling only domestically, $V_X - V_D$ and $V_I - V_D$. For each productivity level $1/a$, each plot has two stationary points, a local maximum and a local minimum. The value matching and smooth pasting conditions imply that the local maxima correspond to the “entry” thresholds ($Q_{DX}^*$ and $Q_{DI}^*$ in the left and right plot respectively), while the local minima correspond to the “exit” thresholds ($Q_{XD}^*$ and $Q_{XD}^*$).

Notice that for $Q^* \to \infty$, the value function of an exporter is steeper than the one of a multinational, because the exporter gets high value both from staying in the market as an exporter and from the option value of becoming a multinational.
Figure 3: Value functions of an exporter and of a multinational firm. \(Q^*_{ID}\). The picture shows that both entry and exit thresholds are decreasing in \(1/a\), indicating that more productive firms enter the foreign market for lower realizations of aggregate demand \(Q^*\) with respect to less productive firms. Similarly, more productive firms need larger negative shocks to demand to be induced to exit the foreign market with respect to less productive firms.

Notice that for \(Q^* \to 0\), \(V_X - V_D\) and \(V_I - V_D\) tend to infinity, because the option value of exiting the foreign market is extremely high for very low realizations of \(Q^*\) (and irrespective of firm’s productivity). Conversely, for \(Q^* \to \infty\), \(V_X - V_D\) and \(V_I - V_D\) tend to negative infinity, because the domestic firms’ option value of entering the foreign market is extremely high, compared to the flow profits of staying for firms that are already serving that market.

Figure 5 plots the difference between the value functions of a multinational firm and of an exporter, \(V_I - V_X\), as a function of \(Q^*\) and \(1/a\). In this picture, for each value of \(1/a\), the peak of the surface represents the quantity threshold where the firm switches from being an exporter to being a multinational, \(Q_{XI}\). The figure shows that also the threshold \(Q_{XI}\) is decreasing in \(1/a\), consistent with the prediction of Helpman, Melitz, and Yeaple (2004), according to which the most productive firms are more likely to become multinationals than exporters. Notice also that, for constant \(Q^*\), the excess value \(V_I - V_X\) decreases as productivity \(1/a\) increases: for the same level of \(Q\), a more productive exporter has a higher option value of switching to FDI compared to a less productive one.

Figures 4 and 5 suggest a systematic relationship between the quantity thresholds \(Q^*_{RS}\) and the firm productivity level \(1/a\). Theorem 2 establishes this result.
Figure 4: Difference between the value functions of exporters and multinationals and the value function of domestic firms.

Figure 5: Difference between the value functions of an exporter and of a multinational firm.
Theorem 2.

\[ \frac{\partial Q^*_{RS}(a)}{\partial a} \geq 0, \text{ for } R, S \in \{D, X, I\}, \forall a. \]  \hfill (30)

**Proof:** See Appendix B.

Theorem 2 establishes that the six thresholds \( Q^*_{RS}(a) \) are decreasing in productivity \( 1/a \), indicating that more productive firms need smaller positive shocks to demand to enter the foreign market, and larger negative shocks to exit. The one-to-one correspondence between productivities and quantity thresholds established by Theorem 2 implies that for each realization of aggregate foreign demand \( Q^* \) we can compute 6 productivity thresholds \( a_{RS}(Q^*) \), for \( R, S \in \{D, X, I\} \), that determine the selection of heterogeneous firms into the three statuses, and their likelihood of switching across different statuses. This redefinition of the thresholds in terms of productivity is going to be extremely helpful to compute the model numerically. The sets of firms belonging to each status can be written as functions of the productivity thresholds \( a_{RS} \). At time \( t = 0 \):

\[
\Omega_{D0} = [a_{DX}, \infty) \hfill (31)
\]

\[
\Omega_{X0} = [a_{XI}, a_{XD}] \hfill (32)
\]

\[
\Omega_{I0} = (0, a_{ID}] \hfill (33)
\]

and for the following periods:

\[
\Omega_{Dt+1} = \left\{ \Omega_{Dl} \cap [a_{DX}, \infty) \right\} \cup \{ \Omega_{Xl} \cap [a_{XD}, \infty) \} \cup \{ \Omega_{Il} \cap [a_{ID}, a_{IX}] \} \hfill (34)
\]

\[
\Omega_{Xt+1} = \left\{ \Omega_{Dt} \cap [a_{DI}, a_{DX}] \right\} \cup \{ \Omega_{Xl} \cap [a_{XI}, a_{XD}] \} \cup \{ \Omega_{It} \cap [a_{IX}, \infty) \} \hfill (35)
\]

\[
\Omega_{It+1} = \left\{ \Omega_{Dt} \cap (0, a_{DI}] \right\} \cup \{ \Omega_{Xl} \cap (0, a_{XI}] \} \cup \{ \Omega_{It} \cap (0, a_{ID}] \} \hfill (36)
\]

where we omitted the dependence of \( \Omega_{St} \) and \( a_{RS} \) on \( Q^* \) to ease the notation. Notice that the sets \( \Omega_X \) and \( \Omega_I \) vary with the realization of \( Q^* \), as firms may switch status, but only depend on the firms’ status in the previous period, due to the Markov property of Brownian motions.

Figure 6 illustrates Theorem 2. In addition, from the convexity of the schedule \( Q^*_{RS}(1/a) \), we can investigate how hysteresis combines with productivity. On the one hand, for the same choice of status, the convexity of the thresholds displayed in Figure 6 implies that more productive firms exhibit less hysteresis. On the other hand, more productive firms self-select into the status (I)
that is associated with more hysteresis.\textsuperscript{17} This generates an imperfect sorting of productivities into status, which contributes to generate the variation in earnings yields that we observe in the data.

5 Empirical Results

Objective of this section is an evaluation of the performance of the model in matching qualitatively and quantitatively features of trade and FDI dynamics, and of the pattern of earning yields across firms. In Section 5.1 we show that the ordering of thresholds established by Theorem 1 is consistent with the pattern of firms switching status in the data. In Section 5.2 we calibrate the parameters of the model to match quantitatively moments related to the switching pattern, and to relative size and presence of the three types of firms in the data. In Section 5.3 we use the calibrated version of the model to compute earning yields (not targeted in the calibration) and show that the theory is able to replicate the observed ranking in earning yields that we documented in Section 2.

\textsuperscript{17}Dixit and Pindyck (1994) show that hysteresis is increasing in the sunk cost of investment.
Table 6: **Switching behavior.** Average percentages of firms switching status each year (Source: Compustat).

<table>
<thead>
<tr>
<th>$t$</th>
<th>$t+1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>1.48</td>
</tr>
<tr>
<td>$X$</td>
<td>1.07</td>
</tr>
<tr>
<td>$I$</td>
<td>0.71</td>
</tr>
</tbody>
</table>

5.1 **Qualitative Results: Switching Pattern in the Data**

The ordering in quantity thresholds that is the result of Theorem 1, associated with the Markov properties of Brownian motions, has implications for the expected switching pattern of firms across statuses. Since $Q^*_{DX} < Q^*_{DI} < Q^*_{XI}$, on average, it should be more likely for a domestic firm to start exporting than to do FDI, and that it should be more likely for an exporter to start FDI than for a domestic firm. Similarly, $Q^*_{IX} < Q^*_{ID} < Q^*_{XD}$ implies that it should be more likely that multinationals go back to domestic sales only instead than switching to exports following a negative shock, and it should be more likely for an exporter to go back to domestic sales only than for a multinational.

We find that the average shares of firms switching across statuses in the data are consistent with these predictions. In Compustat, each year on average 6.69% of firms switch status. Of those, 1.48% switch from domestic to exporters, more than the 1.36% that switch from domestic to multinational. 1.59% of firms switch from exporters to multinational, more than the 1.36% that switch from domestic to multinational. 0.71% of firms switch back from being multinational to domestic only, more than the 0.47% that switch back from being multinational to being an exporter. Finally, 1.07% of firms switch back from exports to domestic sales only, more than the 0.71% that switch back from being multinational to being a domestic firm. Table 6 summarizes the average percentages of firms switching status in the data.

This qualitative evidence raises our confidence on the fact that the model can be successfully calibrated to match quantitatively the switching pattern and other firm characteristics. This will be the object of the next section.
5.2 Calibration

The calibration exercise presented in this section is designed to match a series of facts on exports and FDI sales dynamics. We do not make any use of financial variables in this exercise. In the next section we show that the model calibrated by targeting trade facts only performs well also in matching untargeted moments like earning yields and returns.

To calibrate the model, we need to choose a functional form for the cost distribution \( G(a) \), and assign values to its parameters. We need to parameterize the Brownian motion, and to choose values for preference parameters and parameters describing trade and FDI costs. We refer to the literature to assign parameters to preferences and to the firms’ productivity distribution. We choose the remaining parameters such that the model matches a series of moments from the data. We start describing the calibration with the parameters we adopt from the literature.

Several studies document that the tail of the empirical firm size distribution is well approximated by a Pareto distribution (see for example Luttmer (2007)). As in the model firm’s size (sales) is linked to the productivity distribution, we assume that firms’ productivities \( 1/a \) are distributed according to a Pareto law with location parameter \( b \) and shape parameter \( k \).\(^{18}\) We normalize \( b = 1 \), and choose \( k \) to match the coefficient of the empirical sales distribution: if productivity is Pareto-distributed with parameter \( k \), sales in the model are also Pareto-distributed with shape parameter \( k/(\eta - 1) \). By regressing firm rank on firm size, Luttmer (2007) finds that \( k/(\eta - 1) = 1.06 \). We then choose \( k \) accordingly, given a value for \( \eta \). There is little agreement in the literature on the value to attribute to the elasticity of substitution across differentiated varieties, \( \eta \). Many papers focusing on long-run macroeconomic predictions use a standard value of 2. Other papers, focusing on matching data at business cycle frequencies, choose much higher values. Alessandria and Choi (2007), for example, set \( \eta \approx 10 \) (to match markups of about 11%). We set \( \eta = 2.54 \), equal to the median value in Broda and Weinstein (2006) 3-digit estimates.\(^{19}\) This choice implies \( k = 1.63 \). We set the discount rate \( \rho \) to match a long-run risk-free rate of 2%. The existence of a perfectly tradeable homogeneous good that is produced in both countries ensures that \( w = w^* = 1 \).\(^{20}\) Moreover, we impose that the drift of the Brownian motion has value \( \mu = 0 \). The need to impose zero expected

\(^{18}\)The Pareto distribution is also a convenient choice for computational reasons, since it allows to solve explicitly for the aggregate prices \( P, P^* \) as functions of the productivity thresholds \( a_{RS} \) and of the other parameters of the model.

\(^{19}\)Broda and Weinstein (2006) estimate sectoral elasticities of substitution from price and volume data on U.S. consumption of imported goods. By using data at the 10-digit Harmonized System, they estimate how much demand shifts between 10-digit varieties when relative prices vary, within each 3-digit SITC sector.

\(^{20}\)In general equilibrium, in the absence of a homogeneous sector, wages would depend on \( Q, Q^* \).
demand growth arises from the fact that we abstract from firms’ productivity growth.\textsuperscript{21}

It remains to calibrate the variance parameter $\sigma$, the variable trade cost $\tau$, fixed operating costs $f_X$, $f_I$, sunk costs $F_X$, $F_I$, domestic aggregate demand $Q_D$ and the initial value for the foreign aggregate demand, $Q^*(0)$. We follow the methodology of the calibration in Alessandria and Choi (2007), and select values for these parameters to match a set of moments related to trade and FDI dynamics. We target data on firms’ persistence in the same status, on size differentials across firms in different statuses, and on the relative presence of the three types of firms in the data.

We compute all the moments from Compustat data. In terms of status persistence, on average, 89.98\% of domestic firms remain domestic the following year, while 5.04\% of them become exporters, and the remaining become multinationals. 84.31\% of exporters continue exporting the following year, while 7.83\% of them become multinationals, and the remaining exit the foreign market to sell domestically only. Multinational firms exhibit even higher persistence, with 97.25\% of them continuing being multinationals the following year, and only 1.06\% of them becoming exporters the following year. Domestic firms’ and exporters’ persistence moments are close to the ones reported in Alessandria and Choi (2007), but we are unaware of other papers computing moments related to persistence in multinational activity. Next, we look at employment differences as a measure of size differences. In Compustat, multinational firms appear to employ about 6 times more workers than exporters, but we do not observe any significant difference in employment between exporters and domestic firms. This fact is due to the selection problem associated with Compustat: since the data set includes only publicly traded firms, it is biased towards larger firms, and excludes most small firms, that tend to be the majority of domestic ones. For this reason, domestic firms in the sample do not look as dramatically different from exporters as other literature shows.\textsuperscript{22} We conclude the list of moments with the average share of firms in each status. In Compustat, the average share of firms selling only domestically is 34.73\% of the sample, while exporters are 20.66\% of the sample, and multinational firms account for the remaining 44.61\% of the sample. As previously noted, also these numbers reflect selection of the largest firms in the data set.\textsuperscript{23} Despite the divergence of our moments with the ones reported from other papers, to be internally consistent we decided to match Compustat data in this exercise. While not representative of the entire population of U.S. firms,

\textsuperscript{21}If $\mu > 0$, $E(Q^*)$ would be increasing over time and for $t \to \infty$ all firms would become multinationals. By setting $\mu = 0$, we are implicitly assuming that $Q^*$ and $b$ grow at a rate such that the distribution of firms in the three groups does not degenerate over time.

\textsuperscript{22}For example, Alessandria and Choi (2007) report that exporters tend to hire between 77 and 95\% more workers than domestic firms.

\textsuperscript{23}Bernard et al. (2007), among others, report that the average share of manufacturing firms that export is about 18\%, while Bernard and Jensen (2007) report that multinational firms represent only 1\% of manufacturing firms.
Table 7: **Summary of calibrated parameters.** Values of the Pareto distribution parameters and elasticity of substitution are from Luttmer (2003) and Broda and Weinstein (2006). Discount rate matches the long run average of 3-month T-bills, and the rest of parameters are calibrated to match aggregate data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brownian motion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>drift</td>
<td>0</td>
<td>no productivity growth</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>variance</td>
<td>0.1</td>
<td>to match data</td>
</tr>
<tr>
<td>Pareto distribution</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>lower bound</td>
<td>1</td>
<td>normalization</td>
</tr>
<tr>
<td>$k$</td>
<td>shape parameter</td>
<td>1.63</td>
<td>Luttmer (2007)</td>
</tr>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>discount rate</td>
<td>0.02</td>
<td>risk-free rate</td>
</tr>
<tr>
<td>$\eta$</td>
<td>el. of substitution</td>
<td>2.54</td>
<td>Broda and Weinstein (2006)</td>
</tr>
<tr>
<td>Trade and FDI costs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w,w^*$</td>
<td>wages</td>
<td>1</td>
<td>homogeneous sector</td>
</tr>
<tr>
<td>$\tau$</td>
<td>iceberg export cost</td>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td>$f_X$</td>
<td>fixed export cost</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>$F_X$</td>
<td>sunk export cost</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>$f_I$</td>
<td>fixed FDI cost</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>$F_I$</td>
<td>sunk FDI cost</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Aggregate demand</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q_D$</td>
<td>domestic demand</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>$Q(0)$</td>
<td>initial foreign demand</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Compustat offers a detailed portrait of the largest firms in the economy, which are the major actors when talking about volumes of trade and FDI.  

Table 7 summarizes the calibrated parameters. Table 8 displays jointly the moments computed from the data and the moments generated by the model with the calibrated parameters. The model slightly overpredicts the persistence of domestic and multinational status, and underpredicts the upward switches (from domestic only to export, and from export to FDI). Overall though, the status persistence moments are close to the ones in the data. Both shares of exporters and multinational

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24The choice of a Pareto distribution for firms’ productivity is robust to the selection problem associated with dealing with Compustat data. The Pareto distribution is invariant to lower truncations, hence if we assume that the entire productivity distribution is Pareto with parameters $(b,k)$, the distribution of firms in Compustat will also be Pareto, with parameters $(b',k)$, $b' > b$. As the lower bound of the distribution does not enter the computation of the moments, we normalize it to one.
firms are lower than in the data, and while the relative size of multinationals compared to exporters is close to the data, the model predicts a size advantage of exporters compared to domestic firms.

Table 8: Moments. Comparison of the moments, model versus data. (Source: Compustat)

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D \rightarrow D$ (%)</td>
<td>89.98</td>
<td>96.25</td>
<td>$X$ (%)</td>
<td>20.66</td>
<td>16.06</td>
</tr>
<tr>
<td>$D \rightarrow X$ (%)</td>
<td>5.04</td>
<td>2.98</td>
<td>$I$ (%)</td>
<td>44.61</td>
<td>36.52</td>
</tr>
<tr>
<td>$X \rightarrow X$ (%)</td>
<td>84.31</td>
<td>84.8</td>
<td>empl. $X/D$</td>
<td>1</td>
<td>3.33</td>
</tr>
<tr>
<td>$X \rightarrow I$ (%)</td>
<td>7.83</td>
<td>3.4</td>
<td>empl. $I/X$</td>
<td>6</td>
<td>6.99</td>
</tr>
<tr>
<td>$I \rightarrow I$ (%)</td>
<td>97.25</td>
<td>99.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I \rightarrow X$ (%)</td>
<td>1.06</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.3 Quantitative Results: Earning Yields

With the calibrated version of the model, we compute average earnings yields across the three groups. The model generate average earnings yields of 1.91% for multinational firms, 1.33% for exporters, and 0.48% for firms selling only domestically, which are consistent with the ordering we found in the data.

TO BE COMPLETED

6 Robustness: Symmetric Case

TBA

7 Conclusions

We presented a novel fact distinguishing multinational firms from exporters and from firms selling only domestically. Multinational corporations tend to have higher earnings yields and returns than non-multinational firms. Within non-multinationals, exporters tend to have higher earnings yields than firms selling only in their domestic market. To explain this fact, we presented a real option value model where firms choose optimally whether to produce only domestically, export, or serve
the foreign market through FDI. Productivity and prospects of growth of the foreign demand for goods determine the equilibrium choice. Firms in the domestic market effectively purchase an option that allows them to enter into the foreign market through exports or direct investment. The value of this option varies across firms with different productivities, and determines their choice of international status. In equilibrium, firms differ in the covariance of their earnings yields with the aggregate economy.

The model provides a complementary explanation for the cross section of returns exploiting the production side from an international point of view, and is able to generate the observed yields’ variation across firms. Since foreign investment is associated to larger sunk costs compared to trade, it generates more hysteresis in the firm’s strategies over time, and this implies larger losses if the foreign economy is hit by negative shocks. Multinationals are more exposed to aggregate risk since – in case of a negative shock – by exiting they would forgo the option premium (sunk cost) that they paid to become multinationals. Pulling back is a more expensive strategy for multinationals than for exporters, who paid a lower option premium to serve the foreign market.

The solution of the model delivers a series of predictions related to a firm’s productivity and to the realization of aggregate demand. First, foreign investment is associated to a wider hysteresis (or inaction zone) than trade. Second, more productive firms need smaller positive shocks to enter the foreign market, and larger negative shocks to exit. Moreover, for the same choice of whether and how to serve the foreign market, more productive firms are characterized by less hysteresis than less productive firms. Nonetheless, they tend to choose the strategy (FDI) that is associated to more hysteresis. This interaction generates an ambiguous result, which gives to the model the degree of freedom to explain the cross-sectional variation we observe in the data.

We calibrated the model to match a set of moments about trade and FDI dynamics, and with the calibrated model we computed the predicted earning yields for the three groups of firms. While matching fairly well the overall aggregate dynamics of trade and FDI, the model is able to reproduce the ordering in earning yields we observe in the data.

We see this paper as the first step in a novel research agenda linking trade and FDI dynamics to asset pricing. In the current framework, we do not explore the demand-side implications of risk implied by international exposure. A natural extension of this model would feature risk-averse agents discounting future cash flows in such a way to keep into account firms’ different exposure to international risk. We think this is a promising avenue for research in finance and international
trade, that we plan to pursue in future work.

Appendix

A Accounting Standards

We obtain our data from the Standard & Poors Compustat Database, in particular data from the Compustat Segments. The Financial Accounting Standards Board, in their Statement No. 131, set the standards for the way that public business report information about operating segments in their annual financial statements. Operating segments are defined by the FASB as components of an enterprise about which separate financial information is available that is evaluated regularly by the chief operating decision maker in deciding how to allocate resources and in assessing performance.

FAS 131 establishes standards for the way firms should disclose data about products and services, geographic areas, and major customers. The FAS 131 determines that firms should report data about revenues derived from the firm’s products or services, countries in which they earn revenues and hold assets, and about major customers regardless of whether that information is used in making operating decisions. However, the statement does not require firms to disclose the information on all the different segment types if it is not prepared for internal use and reporting would be impracticable. Therefore, the firms decide how to report the data, disaggregated in several different ways: either by product, geography, legal entity, or by customer, but they do not necessarily have to report all of them. This method is referred to as the management approach. The statement establishes a minimum threshold to report separately information about an operating segment: either revenues of the segment are 10% or more of the combined revenue of all operating segments, or profits or losses are 10% or more of the combined reported profit or losses, or its assets are 10% or more of the combined assets of all operating segments.

Hence, if a given firm considers best practice to aggregate the information upstream to the management level is by customer, they may elect not to disclose geographical segments information. That contrasts with the previous FAS No. 14, superseded by FAS No. 131 in 1998, in which firms were required the financial statements of a business include information about the enterprise’s operations in different industries, its foreign operations and export sales, and its major customers.
B Proofs

B.1 Hysteresis

In this section we extend the analytical results about the nature of hysteresis in Dixit (1989) to our model, where firms also choose the mode of entry in a (foreign) market.

Let $G_{SR}(Q) \equiv V_S(Q) - V_R(Q)$, for $S,R \in \{D,X,I\}$. We omit the dependence of the value functions on the productivity parameter $a$, as the proof carries over $\forall a$.

Let us consider the value matching and smooth pasting conditions for a firm switching from being domestic only to also export and vice versa. Value matching implies: $G_{DX}(Q_{DX}) = F_X$ and $G_{DX}(Q_{XD}) = 0$. Similarly, smooth pasting implies: $G_{DX}^\prime(Q_{DX}) = G_{DX}^\prime(Q_{XD}) = 0$.

By using the expressions for the value functions in equations (11), (14), we can compute the limits of $G_{DX}^\prime(Q)$:

$$G_{DX}(Q) = A_XQ^\alpha + B_XQ^\beta + \frac{B(\tau aw)^{1-\eta}P^nQ}{\rho - \mu} - \frac{f_X}{\rho} - B_DQ^\beta$$

which implies:

$$\lim_{Q \to 0} G_{DX}(Q) = +\infty$$

since $\alpha < 0$ and $\beta > 1$. Similarly:

$$\lim_{Q \to +\infty} G_{DX}(Q) = \pm\infty$$

(according to whether $B_X \geq B_D$ or $B_X < B_D$). Hence $G_{DX}^\prime(Q_{DX}) \leq 0$ and $G_{DX}^\prime(Q_{XD}) > 0$.

From Ito’s Lemma (combining equations (10) and (12)):

$$\rho G_{DX}(Q) = G_{DX}^\prime(Q)\mu Q + \frac{1}{2}G_{DX}^{\prime\prime}(Q)\sigma^2Q^2 + B(\tau aw)^{1-\eta}P^nQ - f_X$$

which, evaluated at $Q_{DX}$ reduces to:

$$\rho F_X - B(\tau aw)^{1-\eta}P^nQ_{DX} + f_X \leq 0$$
since $G'_{DX}(Q_{DX}) = 0$ and $G''_{DX}(Q_{DX}) \leq 0$. Hence:

$$Q_{DX} \geq \frac{\rho F_X + f_X}{B(\tau aw)^{1-\eta} P_\eta} \equiv W_{DX}$$

where $W_{DX}$ is the corresponding threshold in absence of uncertainty. By evaluating the Ito’s lemma condition at $Q_{XD}$ one can also show that:

$$Q_{XD} \geq f_X B(\tau aw)^{1-\eta} \eta \equiv W_{XD}.$$ 

With the same procedure, the result follows for the other four thresholds as well:

$$Q_{DI} > \frac{\rho F_I + f_I}{B(aw^*)^{1-\eta} P_\eta} \equiv W_{DI}$$

$$Q_{ID} < \frac{f_I}{B(aw^*)^{1-\eta} P_\eta} \equiv W_{ID}$$

$$Q_{XI} > \frac{\rho F_I + (f_I - f_X)}{B(aw^*)^{1-\eta} P_\eta - B(\tau aw)^{1-\eta} P_\eta} \equiv W_{XI}$$

$$Q_{IX} \leq \frac{-\rho F_X + (f_I - f_X)}{B(aw^*)^{1-\eta} P_\eta - B(\tau aw)^{1-\eta} P_\eta} \equiv W_{IX}.$$ 

Notice also that for $f_X = f_I$, like in the numerical example in Section 3, $W_{IX} < 0$ and hence $Q_{IX} < 0$.

### B.2 Ordering of the quantity thresholds

It is easy to prove that, if $F_I > F_X$, $f_I \geq f_x$, and $w^* < w$, the following order of the deterministic thresholds holds:

$$W_{IX} < W_{ID} < W_{XD} < W_{DX} < W_{DI} < W_{XI}.$$ 

Dixit and Pindyck (1994) show that hysteresis, here defined as the difference $Q_{RS} - W_{RS}$, for $R,S, \in \{D,X,I\}$ is increasing in sunk costs. Hence: $Q_{DI} - W_{DI} > Q_{DX} - W_{DX}$. Since $W_{DI} > W_{DX}$, we have $Q_{DI} > Q_{DX}$. By applying the same reasoning, one can also show that $Q_{XI} > Q_{DI}$. Symmetrically, one can also show the ordering of the exit thresholds, so that:

$$Q_{IX} < Q_{ID} < Q_{XD} < Q_{DX} < Q_{DI} < Q_{XI}.$$
B.3 Monotonicity of the quantity thresholds

Everything else constant, a higher productivity $a^{1-\eta}$ is equivalent to a lower variable operating cost. Dixit and Pindyck (1994), pp. 221-223, show that entry and exit thresholds are decreasing in operating costs, hence $\frac{\partial Q}{\partial a} < 0$.

References


