Abstract
This paper explores the complementarity between exporting and product quality in a model of heterogeneous firms and endogenous technology. Product quality competition between firms makes domestic survival and exporting tougher, but the positive effects of quality upgrading on foreign demand encourage a greater proportion of surviving firms to export. The model also yields several testable hypotheses that can be tested with firm-level data. The model’s result that exporters increase their fixed cost spending and domestic firms reduce their fixed costs when trade liberalizes can offer an explanation as to why it has been difficult to measure "learning-by-exporting" effects.

1 Introduction
This paper explores a new channel by which trade affects the technology of heterogeneous firms. Previous efforts have focused on the idea of learning-by-exporting, whereby firms become more productive by the sheer act of exporting. The evidence for learning-by-exporting is mixed, however, leaving trade economists to prefer the selection-into-exporting story of Melitz (2003). However, a third mechanism could also be at play, namely that exporters can spread some of their fixed production costs over more markets, which allows them to produce higher-quality goods and gives them an advantage against domestic firms that

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only serve local consumers. It is this complementarity between exporting and quality that the following paper explores.

The idea that firms differ in their technology beyond simply exogenous differences in marginal cost has been largely ignored in the heterogeneous-firms-trade literature, despite the fact that many studies (Bernard and Jensen 1995, Bernard and Jensen 1999, Aw, Chung, and Roberts 2000, and Bernard and Wagner 1997) have empirically shown that exporters are more capital- and investment-intensive and employ a greater proportion of non-production workers than non-exporters. These characteristics suggest that exporters spend more on fixed costs compared to domestic firms. Moreover, the differences in capital- and investment-intensity and proportion of non-production workers between exporters and non-exporters are surely a result of manager decisions.

It is fairly reasonable to assume that a portion of these technological differences between exporters and non-exporters is driven by differences in product development or R&D, instead of simply by exporting costs. Indeed, Cohen and Klepper (1992) report that the correlation coefficient between business-unit sales and R&D spending is 0.78, while Griliches (1998) concludes that there is a high degree of correlation between firm productivity and the level of R&D expenditures. There is thus an abundance of evidence that higher-productivity firms and exporters spend more on R&D compared to other firms.

In addition to the well-known firm-level facts about exporters, Bernard and Jensen (1995) find that there is also a large variation in capital- and investment-intensity and the proportion of non-production workers across U.S. industries. Furthermore, only a small proportion of the the cross-industry variation can be explained by the firm-level facts about exporters. In particular, Bernard and Jensen report that exporters have 51% higher capital-intensity, 52% higher investment-intensity, and a 27% higher share of non-production workers. How-
ever, after controlling for industry and regional fixed effects, the exporter effect on these variables is only 9.3%, 3.6% and 12.4% respectively. The rest of the variation is due to the innate industry characteristics and regional characteristics.

There is also a strong correlation between R&D intensity and trade. Figure 1 shows a strong relationship between industry-level R&D intensity and the corresponding export share of production for U.S. manufacturing sectors in 2003. This figure suggests that industry characteristics, be they exogenously or endogenously driven, can have a large impact on the scope for complementarity between quality upgrading and exporting.

The theoretical model captures this empirical evidence by assuming that a firm’s capital, non-production workers and R&D spending are embodied in
the fixed cost component of firms’ increasing-returns-to-scale technology. This assumption is quite realistic, as firms’ spending on these costs often considered as fixed and sunk.

This paper describes a particular mechanism of "quality upgrading" whereby firms compete with each other in a monopolistically competitive environment not only on prices but also on product quality. Product quality enters the consumer demand function, and an increase in quality shifts out the demand for a firm’s product. The catch is that a firm must pay a larger fixed production cost in order to upgrade its product’s quality and shift out its demand curve, with decreasing returns to these fixed cost outlays. These costs represent product development or R&D, so firms pay this cost regardless of how many markets they serve. Firms with higher productivity are willing to spend more on the fixed cost since their marginal benefit from spending on the fixed cost is greater. Moreover, firms that export receive an additional marginal benefit from fixed cost outlays compared to domestic firms since higher fixed cost outlays increase foreign demand as well. This is the nature of the complementarity between exporting and quality upgrading that the theoretical model in this paper attempts to capture.

Fixed-cost quality upgrading is very plausible for a variety of industries, especially those where product upgrading is associated with fixed cost outlays instead of higher-quality materials. This paper thus examines a different set of circumstances than the Quality-Melitz literature by Baldwin and Harrigan (2007) and others, whereby firms invest in better quality materials in order to increase the price fetched for their product. My model closely follows the Melitz (2003) model of trade with heterogeneous firms and is directly comparable to the standard Melitz outcome.

The advantage with my approach is that firms choose their optimal quality
from a continuum of possibilities, instead of the binary all-or-nothing in much of the previous literature. Moreover, this is achieved in a CES framework. This allows for a much richer set of results than previous studies, and provides new testable hypotheses. It turns out that the more responsive demand is to fixed cost outlays, the "tougher" the domestic and export productivity cutoffs are. Quality upgrading has the same effect as trade liberalization in the sense that the domestic and export cutoffs converge, so a higher proportion of surviving firms export. The intuition is that the positive effect of quality upgrading on foreign demand make exporting more attractive to surviving firms.

The model predicts that exporting firms increase their fixed cost outlays due to trade liberalization, while domestic firms reduce their fixed costs. This pattern of changes in fixed costs can help to explain why it has been difficult to measure the "learning-by-exporting" effect, since the changes in fixed costs make the measured productivity boost from exporting smaller.

Another prediction of the model is that endogenous fixed costs dull the effect of trade liberalization. Intuitively, if most firms have already entered the export market due to complementarity between exporting and quality, then trade liberalization has very little effect on the extensive margin. This result can have implications for trade policy across industries that differ in their propensity for quality upgrading.

The paper is organized as follows. The related literature is briefly discussed in section 2. The model in autarky and with trade frictions is provided in sections 3 and 4 respectively. Testable implications follow in section 5, and conclusions are drawn in section 6.
2 Related Literature

Literature on the complementarity between exporting and quality has until recently been focused on technology spillovers, whereby firms gain access to new technologies by exporting to new countries. This hypothesis has recently been lent support by the work by Baldwin and Gu (2004) using Canadian data.


The idea that exporting allows firms to spread R&D benefits over larger markets has been studied by Aw, Roberts, and Winston (2007) and Aw, Roberts, and Xu (2008). These papers develop an estimable structural model and find evidence for the complementarity hypothesis.

Bustos (2008) takes a different approach by concentrating on changes in technology instead of productivity. In contrast to my approach that looks at "product innovation" Bustos examines "process innovation", whereby firms can pay a larger fixed cost in order to reduce their marginal cost of production. Bustos shows that Argentinian firms that enter the export market after trade liberalization become more skill and technology intensive than non-exporters. Antoniades (2008) develops a model of product innovation in a quadratic utility setting, with a tradeoff between fixed cost outlays and product quality.
There are several other variants of the Melitz model that assume heterogeneous fixed cost. Arkolakis (2006) assumes that country-specific fixed beachhead costs are a choice variable, with greater spending on the fixed cost associated with reaching more consumers. Akerman and Forslid (2007) assume that beachhead costs are increasing in country size.

3 The Autarky Model

3.1 Set-up

There are two industries: a differentiated good industry $M$ characterized by increasing returns to scale and a constant returns industry $A$. Wages are identical across sectors and are set equal to unity.

Consumer utility is assumed to be Cobb-Douglas between industries and CES within the differentiated good industry. In the same vein as the earlier work on endogenous sunk costs, starting with Sutton (1991), as well as the more recent quality-Melitz literature, an extra firm-specific quality parameter enters the utility function for differentiated goods. This parameter, $q_i$, can be influenced by firms’ fixed cost outlays in a manner that will be described in the next section. The utility function is specified as

$$U = C_M^{\mu} C_A^{1-\mu}, \quad C_M = \left( \int_0^N \frac{1}{q_i} c_i^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}}.$$

Utility is increasing and concave in $q_i$ since $\sigma > 1$. Demand for good $i$ is thus

$$x_i = \frac{p_i^{-\sigma} q_i}{p^{1-\sigma}} \mu L,$$
where the price index can be written as

\[ P = \left( \sum_{i=0}^{N} p_i^{1-\sigma} q_i d_i \right)^{\frac{1}{1-\sigma}}. \]

A larger \( q_i \) shifts firm demand outward. Firms set prices equal to the marginal cost, \( a_i \), times the markup:

\[ p_i = \frac{\sigma}{\sigma - 1} a_i. \]

This formulation is essentially a heterogeneous-firm version of the Schmalensee (1992) model of endogenous sunk costs.

### 3.2 Cutoff Conditions

Firm productivity is heterogeneous in this model, following Melitz (2003). Firms are endowed with a marginal cost parameter \( a \) that is randomly drawn from a continuous probability distribution \( G(a) \) when they are born. Dropping the \( i \) subscripts henceforth, the operating profit for any firm is:

\[ \pi = \frac{px}{\sigma} = qa^{1-\sigma}B \]

where \( B = \left( \frac{px}{\sigma a^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} \mu L \). The higher the quality parameter, \( q \), the greater the operating profit.

The "cutoff" firm is characterized by operating profits equal to total post-entry fixed costs, which can be written as:

\[ q(a_D) a_D^{1-\sigma} B = f(a_D) + F_D. \]

where \( f(a_D) \) is the cutoff firm’s firm-specific quality upgrading fixed cost and
3.3 Endogenous Quality Upgrading

This model departs from the standard Melitz formulation by assuming that firm-specific fixed costs, \( f \), influence firm-specific quality, \( q \). This assumption closely follows the seminal work of Sutton (1991) on endogenous sunk costs in the Industrial Organization literature. This assumption makes sense for fixed costs that enhance the attractiveness of a product to a consumer, such as product development or R&D. The model in this paper does not apply to advertising, however, since it tends to be a country-specific "beachhead" cost.

Firms choose the optimal fixed cost outlay by equating the marginal revenue of increasing \( q \) with its associated marginal cost. The firm’s optimal choice of \( q \) is the solution to the following profit maximization problem:

\[
\max_q [qa^{1-\sigma}B - f(q) - F_D]
\]

Firm \( i \)'s optimal quality-upgrading choice can be characterized by the following first order condition:

\[
\frac{\partial f(q)}{\partial q} = a^{1-\sigma}B. \tag{3}
\]

As long as \( \frac{\partial^2 f}{\partial q^2} > 0 \), a better productivity draw (lower \( a \)) leads to higher marginal revenue from an increase in \( q \), which means a higher equilibrium \( q \).

At this point it is useful to make an assumption about the cost of quality upgrading. I assume a very general functional form with increasing and convex costs:

\[
f(q) = q^\frac{1}{\theta}, \quad \frac{\partial f}{\partial q} = \frac{1}{\theta} q^{\frac{1-\sigma}{\theta}}, \quad \theta \in [0, 1]
\]

where \( \theta \) is a parameter common to all firms that determines the extent of de-
creasing returns to quality upgrading. The larger is \( \theta \), the easier it is for firms to affect consumer demand by raising fixed cost outlays. I henceforth refer to differences in \( \theta \) as differences in the "ease" of quality upgrading throughout the rest of the paper. It turns out that \( \theta \) equals the equilibrium industry "fixed cost intensity".

3.4 Equilibrium Quality for Marginal and Supra-Marginal Firms

One can combine (2), (3) and (4), in order to solve for each firm’s equilibrium quality and its associated fixed cost:

\[
q(a) = \left( \frac{\theta}{1 - \theta F_D} \right)^\theta \left( \frac{a}{a_D} \right)^{\frac{\theta}{1 - \theta}} (1 - \sigma), \forall a \in (0, a_D]
\]  

\[
f(a) = \frac{\theta}{1 - \theta} F_D \left( \frac{a}{a_D} \right)^{\frac{1 - \sigma}{1 - \theta}}, \forall a \in (0, a_D]
\]

One can see that a firm’s equilibrium quality is increasing in own productivity, \( \frac{1}{a} \), since a firm with higher productivity has a higher marginal revenue from quality upgrading. However, equilibrium quality is decreasing in the productivity of the cutoff firm, \( \frac{1}{a_D} \), since a tougher cutoff leads to a lower price index, which reduces all firms’ marginal revenue from quality upgrading. Equilibrium quality is also increasing in \( \theta \), so "easier" quality upgrading (higher \( \theta \)) results in all surviving firms spending more on endogenous fixed costs.

Inspecting (5) and (6), One can see that the endogenous fixed cost for the cutoff firm is thus independent of the productivity draw, and only depends on the exogenous beachhead cost, \( F_D \), and the ease of quality upgrading:

\[
q(a_D) = \left( \frac{\theta}{1 - \theta F_D} \right)^\theta
\]
\[ f(a_D) = \frac{\theta}{1-\theta} F_D \]  

(8)

The standard Melitz productivity (without quality upgrading) cutoff is given by equation (2) in the case where \( q(a_D) = 1 \) and \( f(a_D) = 0 \). This occurs in this model when \( \theta \) equals zero, meaning that quality upgrading is infinitely costly. Note that the equations above assume no particular probability distribution for the firm productivities.

### 3.5 Free Entry Condition

Firms must pay a fixed cost \( F_E \) to enter the market prior to finding out their respective productivities. Firms enter until the a priori expected profits from entry equal zero:

\[ E(\pi - F_E) = 0 \]

\[ \Leftrightarrow F_E = \begin{array}{c} \int_0^{a_D} [qa^{\sigma B - f(a) - F_D} g(a) da] \end{array} \]  

(9)

Substituting (1), (5) and (6) into (9), assuming a Pareto distribution, \( G(a) = a^k \), for firm productivities and integrating over all surviving firms provides an analytical solution for the cutoff firm productivity in autarky:

\[ a^k_D = \frac{F_E [\beta (1-\theta) - 1]}{F_D}. \]  

(10)

Note that \( \frac{\partial a^k_D}{\partial \theta} < 0 \), so "easier" quality upgrading (larger \( \theta \)) makes the cutoff "tougher". The standard Melitz solution is obtained in the special case where \( \theta \) approaches zero. One needs to assume that \( \beta > \frac{1}{1-\theta} \) in order to obtain a positive solution to this problem. If \( \theta \) is too high then endogenous fixed cost spending becomes so large that the number of firms becomes zero. This can be
seen in the expression for the equilibrium number of firms:

\[ n = \frac{\beta (1 - \theta) - 1 \mu L}{\beta F_D}. \]

One can see that \( \frac{\partial n}{\partial \theta} < 0 \), so "easier" quality upgrading (higher \( \theta \)) leads to fewer firms. This occurs because firms spend more on fixed costs and experience higher demand for their products, which pushes marginal, low productivity firms out of the market.

The solution for the price index is

\[ P^{1-\sigma} = \Psi \mu L (1 - \theta) \left( \frac{\theta}{1 - \theta} F_D \right)^{\theta} \left( \frac{F_D^{1-\beta}}{F_E [\beta (1 - \theta) - 1]} \right)^{\frac{1}{\beta}}, \]

where \( \Psi = \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \). Note that \( \frac{\partial P}{\partial \theta} < 0 \), so the price index decreases as quality upgrading becomes easier. Easier quality upgrading leads to a tougher firm cutoff, weeding out the low productivity, high-priced firms.

### 3.6 Frictionless Free Trade

The effect of free trade without trade frictions can be illustrated by simply increasing the market size, \( L \), in the model presented above. An increase in market size leads to a proportional increase in the number of firms, \( n \), but does not affect the endogenous fixed costs or the productivity cutoffs. The increase in variety lowers the price index and increases consumer welfare in the same way as previous models of trade with CES preferences.

### 4 Two Country Model with Trade Frictions

The autarky model can easily be extended to two countries, iceberg trade costs and fixed export beachhead costs. The two-country model follows many of the
same assumptions as the standard Melitz setup combined with the additional assumptions about endogenous fixed costs discussed in the autarky model. A new assumption is necessary, however, to incorporate quality upgrading with international trade. Namely, it is assumed that the fixed cost of quality upgrading, \( f \), is not country-specific, and each firm’s quality parameter, \( q \), affects consumers symmetrically in all countries. This means that a firm pays for quality upgrading only once and its benefits are spread over all of its markets. This contrasts with the exogenous fixed beachhead costs, which are country-specific. The assumption of demand spillovers resulting from quality upgrading makes sense because many fixed costs such as R&D can be spent once and then provide benefits in every market that the firm serves.

Extending the basic model to include two countries and trade frictions does not change any of the assumptions that were made in the Autarky model in the previous section. Consumers have identical utility functions in both countries, dubbed Home and Foreign. Wages are identical across sectors and countries are set equal to unity. All variables that refer to Foreign market are denoted with an asterix.

## 4.1 Cutoff Conditions

The domestic and export operating profits for any firm situated in Home are:

\[
\pi = \frac{px}{\sigma} = qa^{1-\sigma}B \tag{11}
\]

\[
\pi_x = \frac{px^*}{\sigma} = qa^{1-\sigma} \phi B^* \tag{12}
\]
where $B = \left(\frac{x}{\rho \beta}\right)^{1-\sigma} \mu L$ and $B^* = \left(\frac{x}{\rho \beta}^{1-\sigma}\right) \mu L^*$. The domestic and export operating profits for any firm situated in Foreign are:

$$\pi^* = qa^{1-\sigma}B^* \quad (13)$$

$$\pi^*_x = qa^{1-\sigma}\phi B \quad (14)$$

The cutoff firm is characterized by zero post-entry profits. There are thus four equations defining the conditions for marginal firms selling domestically and abroad, for both Home and Foreign.

Since an exporting firm at Home or Foreign spreads its quality upgrading costs, $f$ and $f^*$ respectively, over both markets, one cannot express the export cutoff as a function of export profits alone. The export cutoff firm is characterized by its net profits from serving both markets equaling the net profits from only serving the domestic market.

$$q(a_D)a_D^{1-\sigma}B = f(a_D) + F_D \quad (15)$$

$$[q_x(a_X) - q(a_X)]a_X^{1-\sigma}B^* + [q_x(a_X)]a_X^{1-\sigma}\phi B^* = f_x(a_X) - f(a_X) + F_X \quad (16)$$

$$q^*(a_D)a_D^{*(1-\sigma)}B^* = f^*(a_D) + F_D \quad (17)$$

$$[q_x^*(a_X) - q^*(a_X)]a_X^{*1-\sigma}B^* + [q_x^*(a_X)]a_X^{*1-\sigma}\phi B = f_x^*(a_X) - f(a_X) + F_X \quad (18)$$

where $f_x(a_X)$ is the endogenous cutoff firm-specific fixed cost and $F_X$ is the exogenous fixed cost to enter the domestic market. The domestic and export cutoff conditions for Home are illustrated graphically in Figure 2.

The exporter cutoff is tougher than the domestic firm cutoff by a parameter restriction which is given later in the paper. In addition, the case where the firm serves the export market only can be ruled out by parameter restrictions.
These are discussed in Appendix A.

As in Helpman, Melitz, and Yeaple (2004), the free entry condition means that both countries share the same cutoffs $a_D = a_D^*$, $a_X = a_X^*$, the same demand levels $B = B^*$. This also means that both countries share the same cutoff endogenous fixed costs $f(a_D) = f^*(a_D)$, $f(a_X) = f^*(a_X)$, and demand parameters $q(a_D) = q^*(a_D)$, $q(a_X) = q^*(a_X)$.
4.2 Endogenous Quality Upgrading and the Decision to Export

As in the autarky model described earlier, firms each choose their quality and its associated fixed cost outlay to maximize operating profits less post-entry fixed costs. This decision is made jointly with the decision to export or not. The firm thus compares the profits from exporting or not, given that they choose the optimal amount of quality upgrading in either case. The optimal degree of quality upgrading will differ between exporters and domestic firms, since exporters receive a demand response from both markets, which gives them a stronger incentive to undertake quality upgrading activities.

A domestic firm solves the following problem:

\[
\max_{q} \left[ qa^{1-\sigma} B - f(q) - F_D \right]
\]

A domestic firm’s optimal quality upgrading decision is determined by the following first order condition:

\[
\frac{\partial f}{\partial q} = a^{1-\sigma} B.
\]

The exporting firm problem differs from the domestic firm problem because it considers the additional operating profit in the export market when it chooses its optimal quality upgrading. An exporter solves the following problem:

\[
\max_{q_x} \left[ q_x a^{1-\sigma} B (1 + \phi) - f_x (q_x) - F_D - F_X \right]
\]

An exporting firm’s optimal decision is determined by the following first
order condition:
\[ \frac{\partial f_x(q_x)}{\partial q_x} = a^{1-\sigma} B (1 + \phi) \]  

I assume the same functional form as the autarky model for the cost of quality upgrading:
\[ f(q) = q^\xi, \quad f_x(q_x) = q_x^{\frac{\xi}{\sigma}}. \]  

### 4.3 Equilibrium Quality for Marginal and Supra-Marginal Firms

#### 4.3.1 Domestic Firm Quality

Each domestic firm’s equilibrium quality and its associated fixed cost are found by combining (15), (19) and (21):
\[ q(a) = \left( \frac{\theta}{1 - \theta} F_D \right)^\theta \left( \frac{a}{a_D} \right)^{\frac{\xi}{\sigma} (1-\sigma)}, \quad \forall a \in (a_X, a_D] \]  
\[ f(a) = \frac{\theta}{1 - \theta} F_D \left( \frac{a}{a_D} \right)^{\frac{1-\sigma}{1-\theta}}, \quad \forall a \in (a_X, a_D] \]  

Just as in the autarky model, domestic firm quality upgrading is increasing in own productivity, \( \frac{1}{a} \), while it is decreasing in the productivity of the domestic cutoff firm, \( \frac{1}{a_D} \).

#### 4.3.2 Exporter Quality

Each exporter’s equilibrium quality and its associated fixed cost are found by combining (15), (16), (20), (21), (22) and (23):
\[ q_x(a) = \left( \frac{\theta}{1 - \theta} F_X \left( \frac{1 + \phi}{1 + \phi} \right)^{\frac{1}{1-\sigma}} \right)^\theta \left( \frac{a}{a_X} \right)^{\frac{\xi}{\sigma} (1-\sigma)}, \quad \forall a \in (0, a_X] \]
\[ f_x(a) = \frac{\theta}{1-\theta} F_X \left( \frac{(1+\phi)^{\frac{1-\sigma}{\tau}}}{(1+\phi)^{\frac{1-\sigma}{\tau}} - 1} \left( \frac{a}{a_X} \right)^{\frac{1-\sigma}{\tau}} \right), \forall a \in (0,a_X] \]  

(25)

One can see that an exporter’s equilibrium quality upgrading is increasing in own productivity, \( \frac{1}{a} \), since a firm with higher productivity has a higher marginal revenue from quality upgrading. However, exporter quality upgrading is decreasing in the productivity of the cutoff exporter, \( \frac{1}{a_X} \), since a tougher cutoff leads to a lower price index, which reduces any domestic firm’s marginal revenue from quality upgrading.

### 4.4 Free Entry Condition

Firms must pay a fixed cost \( F_E \) to enter the market prior to finding out their productivity. Firms enter until the a priori expected profits from entry equal zero, i.e.,

\[ E(\pi_i - F_E) = 0 \]

\[ \Leftrightarrow F_E = \int_{a_X}^{a_D} [q a^{1-\sigma} B - f(a) - F_D] g(a) \, da + \int_0^{a_X} [q a^{1-\sigma} B (1+\phi) - f_x(a) - F_D - F_X] g(a) \, da \]  

(26)

Substituting (22), (23), (24) and (25) into (26), assuming a Pareto distribution for firm productivities and integrating provides analytical solutions for the domestic and export cutoff firm productivity:

\[ a_D^k = \frac{F_E \beta (1-\theta) - 1}{F_D \Omega \Theta} \]  

(27)

\[ a_X^k = \frac{F_E \Omega \beta (1-\theta) - 1}{F_X \Omega \Theta} \]  

(28)

where

\[ \beta = \frac{k}{\sigma - 1} > 1, \]
\[ \Omega = \phi^\beta \left( \frac{F_X}{F_D} \right)^{1-\beta} \in [0, 1], \]

and

\[ \Theta = \left( \left[ (1 + \phi) \frac{1}{\phi} - 1 \right]^{1-\theta} \frac{1}{\phi} \left[ \frac{F_X}{F_D} \right]^{\theta} \right)^\beta \in [1, \infty] \]

Just as in the autarky model, one needs to assume that \( \beta > \frac{1}{1-\theta} \) in order to obtain a positive solution to this problem.

The domestic cutoff and export cutoff equations closely resemble the productivity cutoff in the autarky model, (10). The only difference is the additional terms \( \Omega \) and \( \Theta \) that appear in (27) and (28). The term \( \Omega \) is a measure of trade freeness independent of the ease of quality upgrading. The term \( \Theta \) encompasses all of the additional trade friction effects that result from the additional impact of quality upgrading forces. Since \( \Theta \) is always greater than one (except when \( \theta = 0 \), when \( \Theta = 1 \)), this implies that trade frictions make the domestic cutoff "tougher" compared to the autarky model without trade frictions and compared to a two-country model with trade frictions but without quality upgrading.

Combining (15) and (16), provides the parameter restriction that guarantees that exporters have higher productivity is:

\[ \frac{F_X}{F_D} \frac{\phi^\beta}{(1 + \phi) \frac{1}{\phi} - 1} > 1 \]

Furthermore, the ratio of exporting firms to domestic-only firms in equilibrium is given by a very simple expression:

\[ \frac{a_X^k}{a_D^k} = \Omega \Theta \frac{F_D}{F_X} \]  

(29)
The solutions for the price index at Home and Foreign are:

\[
P^{1-\sigma} = \Psi \mu L (1 - \theta) \left( \frac{F_D^{1-\beta}}{F_E} \frac{1 + \Omega \Theta}{\beta (1 - \theta) - 1} \right)^{\frac{\alpha}{\beta}} \left( \frac{\theta}{1 - \theta} F_D \right)^{\theta} \left\{ \frac{(1 + \Omega \Xi)^2 - (\Omega \Phi)^2}{1 - (\Omega \Phi)^2} \right\}
\]

\[
P^{*(1-\sigma)} = \Psi \mu L^* (1 - \theta) \left( \frac{F_D^{1-\beta}}{F_E} \frac{1 + \Omega \Theta}{\beta (1 - \theta) - 1} \right)^{\frac{\alpha}{\beta}} \left( \frac{\theta}{1 - \theta} F_D \right)^{\theta} \left\{ \frac{(1 + \Omega \Xi)^2 - (\Omega \Phi)^2}{1 - (\Omega \Phi)^2} \right\}
\]

where \( \Psi = \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \), \( \Xi = \frac{(1 + \phi)^{\frac{1}{\sigma - 1}} - 1}{(1 + \phi)^{\frac{1}{\sigma - 1}} - 1} \) and \( \Phi = \frac{\phi (1 + \phi)^{\frac{1}{\sigma - 1}}}{(1 + \phi)^{\frac{1}{\sigma - 1}} - 1} \).

The solutions for the number of firms at Home and Foreign are

\[
n = \frac{\beta (1 - \theta) - 1}{\beta} \frac{\mu L (1 + \Omega \Xi) - L^* \Omega \Phi}{F_D} \frac{1}{1 - (\Omega \Phi)^2}
\]

\[
n^* = \frac{\beta (1 - \theta) - 1}{\beta} \frac{\mu L^* (1 + \Omega \Xi) - L \Omega \Phi}{F_D} \frac{1}{1 - (\Omega \Phi)^2}
\]

Just as in the autarky model, the total number of firms, \( n + n^* \), is decreasing as quality upgrading becomes easier (higher \( \theta \)). However, there is clearly a home market effect here so easier quality upgrading does not necessarily lead to fewer firms in both countries.

Firm revenues from the domestic and export markets can be expressed as

\[
y(a) = q_x(a) a^{1-\sigma} B = \frac{F_X}{1 - \theta} \frac{(1 + \phi)^{\frac{\sigma}{\sigma - 1}}}{(1 + \phi)^{\frac{\sigma}{\sigma - 1}} - 1} \left( \frac{a}{a_X} \right)^{\frac{\sigma}{\sigma - 1} (1 - \sigma)}
\]  

(30)

and

\[
y_x(a) = \phi q_x(a) a^{1-\sigma} B = \phi \frac{F_X}{1 - \theta} \frac{(1 + \phi)^{\frac{\sigma}{\sigma - 1}}}{(1 + \phi)^{\frac{\sigma}{\sigma - 1}} - 1} \left( \frac{a}{a_X} \right)^{\frac{\sigma}{\sigma - 1} (1 - \sigma)}
\]  

(31)

The ratio of export revenues to domestic revenues for any exporter is thus a
function of the variable trade cost:

\[ \frac{y_x(a)}{y(a)} = \phi \]  

(32)

This is the same result as the standard Melitz model without endogenous fixed costs. Adding quality upgrading to the model does not change this result because changes in quality benefit both the domestic and export markets in the same proportion for each exporter.

Fixed cost intensity, measured as the ratio of endogenous fixed costs to sales, is symmetric across firms and depends inversely on the ease-of-quality-upgrading parameter, \( \theta \). It can be found combining (23) and (30), or (25) and (31):

\[ \frac{f_x(a)}{y_x(a)} = \frac{f(a)}{y(a)} = \phi \]  

(33)

Easier quality upgrading thus leads to higher equilibrium "fixed cost intensity". This result also means that changes in trade costs do not affect firm-level fixed cost intensity in this model.

The volume of one-way trade is given by the following expression:

\[ V = \mu \Omega \Phi \frac{L (1 + \Omega \Xi) - L^* \Omega \Phi}{1 - (\Omega \Phi)^2} \]  

(34)

Two-way trade is:

\[ V + V^* = \mu \Omega \Phi (L + L^*) \left( \frac{1 + \Omega (\Xi - \Phi)}{1 - (\Omega \Phi)^2} \right) \]  

(35)

5 Explaining the Learning-by-Exporting Puzzle

One puzzle in the international trade literature is the weak evidence for learning-by-exporting. This is often tested by observing the change in a firm’s produc-
tivity resulting in the context of a trade liberalization event. Pavcnik (2002), among others, found that the productivity of exporters actually decreased due to trade liberalization. Wagner (2007), in his survey of the literature examining the relationship between exporting and productivity, concludes that the evidence for the learning-by-exporting is mixed.

The model in this paper predicts that changes in measured productivity are lower when endogenous fixed cost forces are stronger. This occurs because domestic firms reduce their fixed cost outlays when trade liberalizes, while exporters expand their fixed cost outlays. To illustrate this idea, the measured productivity of any firm in the economy can be expressed as the ratio of total revenue to total costs:

$$
\varphi_m = \frac{y}{\sigma^{-1} y + f + F_D + F_X},
$$

(36)

where $y$ is firm revenues. In a model without endogenous fixed costs the $f$ term is absent, so trade liberalization leads to an increase in revenue and measured productivity for exporters, but a decrease in revenue and measured productivity for domestic firms. However, if the $f$ term is included, then the increase in revenue is partially offset by a subsequent increase in exporter endogenous fixed cost outlays. This means that the increase in measured productivity will be smaller compared to leaving the $f$ out of the calculation. If exporters increase their fixed costs in the manner prescribed in this paper then this could partially explain why the literature has had difficulty measuring an increase in exporter productivity as a result of trade liberalization.

The same logic holds when discussing domestic firms. In this case, the reduction in revenues due to trade liberalization is partially offset by a reduction in fixed cost outlays, which means that the decrease in measured productivity will be smaller compared to the case where endogenous fixed costs are disregarded.
This mechanism can thus partially explain why the measured productivity of domestic firms is not always decreasing with trade liberalization.

6 Testable Implications

The testable implications of this model can be divided into firm-level implications and industry-level implications. Firm-level implications include the positive correlation between productivity and fixed cost spending and the effect of trade liberalization on fixed cost spending. Industry-level implications include systematic cross-sector differences in the extensive margin of trade, as well as the extensive margin’s response to trade liberalization, which depend on differences in the ease of quality upgrading across sectors.

6.1 Firm-Level Implications

The first obvious testable implication of this model is a positive correlation between a firm’s productivity and its spending on fixed costs, i.e. $\frac{\partial f}{\partial a} < 0$. R&D spending could be used as a measure of the cost of quality upgrading. Some care will need to be taken in order to control for firm age, since new firms with a low measured productivity ("high-tech startups", for instance) may invest heavily in research and development.

The effect of trade liberalization on fixed cost spending by domestic firms and exporters is also of interest. The model predicts that trade liberalization encourages exporters to increase quality upgrading, while domestic firms reduce their quality upgrading. This will correspond to an increase in fixed cost spending by continuing exporters and a decrease in fixed cost spending by continuing domestic firms.

An important characteristic of the model is that fixed cost intensity does not change due to trade liberalization, since revenues change in the same proportion
as fixed cost outlays when trade liberalizes. This can be seen by inspection of (33). This contrasts with Bustos (2008), where firms increase their technology spending intensity when trade liberalizes in the model. The difference stems from the different treatment of upgrading, since firms in Bustos’ model make discrete changes in fixed cost spending, whereas the model in this paper allows for incremental quality upgrading. Testing the model in this paper must look at absolute changes in firm-level fixed cost spending instead of changes in firm-level fixed cost intensity.

### 6.2 Industry-Level Implications

#### 6.2.1 Productivity Cutoffs

Comparative statics of the domestic and export cutoffs, equations (27) and (28) respectively, yield similar results to that of a two-country Melitz model with variable and fixed trade costs. As in the standard Melitz formulation, trade liberalization leads to tougher domestic cutoffs and easier export cutoffs:

\[
\frac{\partial a_k^D}{\partial \phi} < 0, \quad \frac{\partial a_k^X}{\partial \phi} > 0,
\]

\[
\frac{\partial a_k^D}{\partial F_X} > 0, \quad \frac{\partial a_k^X}{\partial F_X} < 0.
\]

A new result is that easier quality upgrading results in tougher productivity cutoffs for both domestic survival and exporting:

\[
\frac{\partial a_D^k}{\partial \theta} < 0, \quad \frac{\partial a_X^k}{\partial \theta} < 0.
\]
6.2.2 The Ease of Quality Upgrading and the Extensive Margin

The effect of changing the ease of quality upgrading, \(\theta\), on the proportion of exporters, equation (29), is positive:

\[
\frac{\partial \left( \frac{aX}{aD} \right)}{\partial \theta} = \frac{F_D}{F_X} \frac{\partial \theta}{\partial \theta} > 0.
\]

This occurs because easier quality upgrading encourages a greater proportion of surviving firms to export.

6.2.3 Variable Trade Cost Liberalization and the Extensive Margin

The model predicts that variable cost trade liberalization increases the export participation ratio, equation (29):

\[
\frac{\partial \left( \frac{aX}{aD} \right)}{\partial \phi} = \frac{F_D}{F_X} \beta \Omega \frac{(1 + \phi)^{\frac{\phi}{\Gamma + \phi}}}{(1 + \phi)^{\frac{\phi}{\Gamma + \phi}} - 1} > 0.
\]

This result means that variable cost trade liberalization brings the two cut-offs closer together, which increases the relative proportion of firms that are exporters.

Writing this in elasticity form and deriving with respect to \(\theta\) provides an expression for the interaction effect between variable cost trade liberalization and the ease of quality upgrading parameter, \(\theta\):

\[
\frac{\partial \varepsilon_{a,\phi}}{\partial \theta} = -\beta \frac{\phi}{(\phi - 1)^2} \left(1 + \phi\right)^{\frac{\phi}{\Gamma + \phi}} \ln \left(1 + \phi\right) \frac{1}{(1 + \phi)^{\frac{\phi}{\Gamma + \phi}} - 1} < 0
\]

where \(\varepsilon_{a,\phi} = \frac{\partial \left( \frac{aX}{aD} \right)}{\partial \phi} \frac{\phi}{\left( \frac{aX}{aD} \right)^{\phi}}\). The interpretation of this derivative is that easier quality upgrading dulls the positive effect of freer trade on the proportion of
exporters. The intuition is that easier quality upgrading encourages a greater proportion of firms to export prior to trade liberalization, so trade liberalization does not have such a large effect on the extensive margin.

6.2.4 Effect of Fixed Cost Trade Liberalization

The model predicts that fixed cost trade liberalization increases the export participation ratio, equation (29):

\[
\frac{\partial \left( \frac{a_X}{a_D} \right)^k}{\partial F_X} = -\Theta \Omega \frac{F_D}{F_X} \beta (1 - \theta) < 0.
\]

Writing this in elasticity form and deriving with respect to \( \theta \) provides an expression for the interaction effect:

\[
\frac{\partial \varepsilon a,F_X}{\partial \theta} = \beta > 0.
\]

The interpretation of this derivative is the same as the variable trade cost case. That is, easier quality upgrading dulls the effect of freer trade on the proportion of exporters.

6.2.5 Trade Volume

The effects of trade liberalization and the ease of quality upgrading on the volume of trade contains both intensive and extensive margin effects. The Home Country Effect makes it difficult to solve the comparative statics. However, we know that if countries are symmetric then the comparative statics for \( V \) will share the same properties as the extensive margin comparative statics above:

\[
\frac{\partial V}{\partial \phi} > 0.
\]
\[
\frac{\partial \varepsilon_{V,\phi}|_{L=L^*}}{\partial \theta} < 0,
\]
where \( \varepsilon_{V,\phi} = \frac{\partial V}{\partial \phi} \cdot \frac{\phi}{V} \).

\[
\frac{\partial V|_{L=L^*}}{\partial F_X} < 0,
\]
and

\[
\frac{\partial \varepsilon_{V,F_X}|_{L=L^*}}{\partial \theta} > 0.
\]

where \( \varepsilon_{V,F_X} = \frac{\partial V}{\partial F_X} \cdot \frac{F_X}{V} \). In other words, trade is increasing as trade liberalizes and the effect of trade liberalization is dulled by endogenous fixed cost forces.

7 Conclusions

This paper explores the complementarity between exporting and quality in a model of heterogeneous firms and endogenous technology. It is also one of the first papers to explore systematic differences in the pattern of trade across manufacturing industries.

I find that quality upgrading competition makes domestic survival and exporting tougher, but the positive spillovers of quality upgrading on foreign demand encourage a greater proportion of surviving firms to export. Higher quality upgrading intensity leads to more trade but a less elastic response to trade liberalization along the extensive margin. The tractability of the model allows for a rich set of testable hypotheses that can be tested using both aggregate and firm-level data.

Future research will entail taking the model to the data. There are several possible extensions for this modelling framework. Deriving a growth model in this context would be useful to understand the implications of R&D intensity for productivity growth. This framework may also be useful in understanding the effect of trade on technology in other contexts as well, such as services trade.
References


Appendix A: Ruling out the Export-Only Case

It is important to rule out the case where it is optimal for a firm to export without serving the domestic market since it is very rare occurrence in the real world. There are three potential cases to rule out. The first is the case where the export-only cutoff is the easiest cutoff. The second and third cases to rule out are that exporting-only is performed by firms with intermediate productivity or by firms with the highest productivity respectively.

The first case is ruled out by the following parameter assumption:

\[
\left( \frac{a_{X\text{only}}}{a_D} \right)^{1-\sigma} = \frac{1}{\phi} \left( \frac{F_X}{F_D} \right)^{1-\theta} > 1
\]

where \(a_{X\text{only}}\) is the marginal cost of the firm exporting only and earning zero profits. This restriction is similar to the restriction in the standard Melitz model.

The second case can be ruled out because the profits from serving the domestic market exceed the profits from serving the export market only, for any \(a \in [a_X, a_D]\). This is intuitive because export profits are always lower than domestic profits, for any given productivity level.

The third case can be ruled out because the profits from serving both markets exceed the profits from serving the export market only for any \(a \in [0, a_X]\). This is intuitive since any firm that can survive in the export market can make more profits by serving the domestic market as well.