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Keywords: Unit Values, Importer Characteristics, Quality Expansion, Hierarchic Demand

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Import Prices, Importer Income and Importer Income Inequality: Comparing Competing Theories

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1 Introduction

Unit values of trade display variation as a function of various exporter characteristics, like the income and factor abundance of exporting countries (Schott (2004)). But unit values also vary with importer characteristics. Hummels and Lugovskyy (2009) study the effect of market size and income per capita on unit values in a Lancaster circle model. Market crowding as a consequence of a larger market size leads to lower prices. And as people become richer they are more eager (finicky) to consume their ideal variety and therefore the price elasticity goes down and the market price goes up.

In this paper we propose and explore two different channels through which unit values vary with income per capita of the importing country: through a larger demand for quality in a model with utility expanding both in quantity and quality and through a reduced
price elasticity as goods become more necessary in the consumption bundle in a hierarchic demand model. Also, we offer a way to discriminate between the two proposed theories and the theory of Hummels and Lugovskyy (2009) by examining the effect of income inequality on unit values. Income inequality theoretically increases unit values through an aggregate increase in demand for quality and through a decrease in the price elasticity of demand in the Hummels and Lugovskyy model and income inequality reduces unit values through an increase in the price elasticity of demand in the hierarchic demand model.

Empirically, we confirm the findings of Hummels and Lugovskyy (2009) that a higher GDP per capita raises importer unit values with a much broader dataset. The empirical analysis shows that unit values decline in income inequality, thereby rejecting the quality channel and the finickyness channel of Hummels and Lugovskyy (2009) and providing support for the price elasticity channel in the hierarchic demand model.

We model the quality channel and price elasticity channel separately. Catching the two channels in one model would become unduly complicated and has little to add. Still, we show that both models are a nested case of a more general model that is however not analytically solvable. The effect of quality is modeled using a utility function that expands both with quality and quantity of commodities consumed. Production is constant returns to scale, the market structure perfect competition and marginal costs rise in quality of a commodity. A higher income raises demand for quality and with marginal costs rising in quality this increases prices.

To address the effect of income on markups we use the hierarchical demand system proposed by Jackson (1982), which is a variant of Stone-Geary preferences but with negative instead of positive vertical intercepts. As such, agents expand their consumption set as they become richer. As the set of consumed goods becomes larger, the price sensitivity on goods lower in the hierarchy shrinks. The goods lower in the hierarchy become more

2
necessary and therefore their price elasticity declines. In a setting with market power this leads to higher prices. We model market power with small group monopolistic competition between firms within each sector. Hence, within each set of consumed goods (sector), there are various differentiated goods.

There is only 1 factor of production in the model, labor. We model the open economy as a 2-country world. Countries differ only in labor productivity in a uniform way across sectors. There are no trade costs. Hence, both countries produce all goods and charge different prices in the 2 different markets, either because of differences in demand for quality or because of differences in their price sensitivity due to differences in the set of goods consumed.

In the second model we can impose a free entry condition to address the long run. The implication is that an increasing income per capita might lead to a higher price elasticity if the amount of resources spent in a certain sector rises strongly. This leads to an increasing number of firms in the market possibly outweighing the direct effect reducing the price elasticity. However, in the open economy we assume the importing country to be small relative to the other country, interpreted as the rest of the world. This implies that a higher income in the importing country has hardly an effect on resources allocated to the sector and thus on competitive pressure in the sector. As such, also in the long run a higher income per capita will increase importer unit values.

A way to discriminate between the two models and the Hummels and Lugovskyy (2009)-model is by considering the effect of income inequality on import prices. When income inequality rises in the quality model, there is a shift in demand towards goods with higher quality and higher prices. Also, the price of the good consumed by the high incomes rises and the price of the good consumed by the low incomes declines, overall raising the average price. These effects are similar to the increased aggregate demand for high quality
goods as income inequality rises in a model with non-homothetic preferences like Francois and Kaplan (1996).

In the hierarchical demand model an increase in income inequality raises the price elasticity and thus reduces the market price, due to the fact that size of the change in the price elasticity is determined by the income elasticity. As the initial income of the higher incomes is smaller their price elasticity goes down by less than the price elasticity of the low incomes goes up. As a result the overal price elasticity goes up. This result of income inequality on the market price can be reversed if the low incomes would not consume a specific good. An increase in inequality would then simply raises relevant income and thereby reduces the price elasticity.

In the circle model of Hummels and Lugovskyy (2009) a higher income inequality decreases the price elasticity and thus raises the market price like in the quality model. Hence, two models predict that unit values rise in income inequality and one model predicts that they decline in income inequality.

In the empirical part we proxy prices with import unit values with data from the BACI database at the 6-digit level. This database is based on COMTRADE covering more than 200 countries and 5,000 products between 1995 and 2004. In the empirical analysis we include importer-exporter fixed effects and exporter-time fixed effects eliminating as much omitted variable bias as possible. We also perform a sample selection analysis, as a majority of the unit values in the trade matrix are zeros. The results of the sample selection model are in line with the estimation outcomes of the basic fixed effects estimates.

The empirical analysis shows that unit values rise with income, which is support for all three theories. However, in the empirical analysis we also find a clear and significant negative effect of income inequality on unit values. This supports the hierarchical demand model and contradicts the predictions of the effect of income inequality in the quality
expansion model and the finickyness circle model of Hummels and Lugovskyy (2009). The finding on income inequality does obviously not falsify the quality expansion model and the finickyness circle model. It is likely that all models provide part of the explanation for the positive effect of income per capita on unit values. But the empirical results on the effect of income inequality suggest that the price elasticity channel of the hierarchic demand model is an indispensable channel.

This paper fits in the literature explaining price differences across markets. The early literature on Harrod-Balassa-Samuelson effects focuses on the role of non-traded goods in these price differences. A more recent literature explores pricing to the market, with firms charging different prices for identical goods across different markets due to differences in market conditions, see Goldberg and Knetter (1997) for a review. The paper closest to our paper, by Hummels and Lugovskyy (2009), tries to relate these price differences in tradable goods with importer country characteristics, in particular income and income per capita. Hummels and Lugovskyy (2009) propose a model where the price elasticity also declines with income. In this paper we limit ourselves to exploring the importance of income per capita differences in explaining price differences and propose two alternative mechanisms for the mechanism proposed by Hummels and Lugovskyy (2009). Also we extend the analysis beyond income per capita by examining the effect of income inequality to discriminate between the different theories.

The current paper is also related to the literature that uses the large datasets on unit values of trade to analyse the relation of unit values with importer and exporter country characteristics. Schott (2004) relates unit values of trade with exporter characteristics, in particular income per capita and factor abundance. Baldwin and Harrigan (2007) analyse the relationship between unit values and distance between trading partners and importer country size in different firm heterogeneity models. Our approach differentiates itself from
this literature by focusing on demand side explanations for differences in unit values.

As we consider explicitly the importance of differential demand for quality, a third related strand of literature is the literature on the Linder hypothesis and the relevance of non-homothetic preferences in determining trade flows. As this literature is huge, we mention here only the recent literature that uses unit value data to pin down the importance of importer characteristics for the demand for quality (Hallak (2006)) and the literature that relates the demand for quality with income inequality. On the last topic, Francois and Kaplan (1996) and Dalgin, Mitra and Trindade (2008) examine the effect of income inequality on the type of goods imported finding that a higher income inequality leads to more demand for differentiated goods and for luxury goods, respectively. But these papers do not explore the effect of income inequality on unit values of trade. Choi, Hummels and Xiang (2009) explore explicitly the link between income distribution of the importer country and the price distribution of import prices, applying the model of Flam and Helpman (1987). Our approach is distinct from the mentioned paper by exploring the effect of average income inequality on average unit values to differentiate between different theories explaining differences in unit values as a function of importer characteristics.

The paper is organized as follows. The next section outlines the two theoretical models, with much derivations relegated to the appendix. Section 3 discusses data and empirical methods. Section 4 contains the empirical results and section 5 concludes with a discussion of the results.

2 Theory

Two channels through which income per capita of an importer country affects trade unit values are explored in this section. Empirically, we focus on the effect of variation of these variables over time on within sector variation in unit values. Hence, we concentrate on
within sector variation in unit values. We point out two different channels through which country characteristics affect unit values, differential demand for quality and differences in the price elasticity of identical goods.

2.1 Quality Channel

2.1.1 Basics

To focus on differential demand for quality within sectors as a function of income and market size, we work with Cobb-Douglas preferences across sectors $j$ and within each sector $j$, preferences depend both on quantity $q_j$ and quality $\alpha_j$:

$$U = \prod_{j=1}^{m} u_j^{\beta_j}; \quad 0 < \beta_j < 1 \text{ for all } j, \sum_{j=1}^{m} \beta_j = 1$$  \hspace{1cm} (1)

$$u_j = \left( \frac{q_j^{\rho \frac{1}{\rho}}}{\alpha_j^{\frac{1}{\rho}}} + q_j^{\frac{1}{\rho}} \right)^{\frac{\rho}{\rho - 1}} ; \quad 0 < \rho < 1; \quad \text{for all } j$$

$u_j$ is the sectoral utility, $\beta_j$ are the Cobb-Douglas parameters and $\rho$ is the substitution elasticity between quality and quantity. In the entire paper labor is the only factor of production and labor is homogeneous. Until we introduce inequality each agent has an amount $i$ of labor units and the total number of agents is equal to $L$. All agents have identical preferences. We normalize the wage $w$ at 1. Hence, each agent has income $i$.

In this subsection, there is perfect competition in all sectors. The cost function of a firm is defined as:

$$c_j(q_j, \alpha_j) = \alpha_j^\gamma q_j^\gamma; \quad 0 < \gamma < 1$$

Hence, marginal costs rise with quality, although less than proportional. $\alpha_j$ is a sector specific marginal cost shifter and $\phi$ is country specific cost shifter uniform across all sectors. Given the fact that perfect competition requires price to be equal to marginal cost, we can
find the equilibrium amounts of quality $\alpha_j$ and quantity $q_j$ by maximizing utility subject to the following budget constraint with the marginal costs substituted for prices:

$$\sum_{j=1}^{m} \alpha_j^\gamma a_j \phi q_j = i$$  \hspace{1cm} (2)

Maximizing utility in equation (1) s.t. (2) generates the following equilibrium outcomes for individual demand$^1$:

$$q_j = \gamma^{\frac{\mu}{\nu-1}} \left( \frac{\beta_j i}{a_j \phi} \right)^{\frac{1}{\nu}}$$  \hspace{1cm} (3)

$$\alpha_j = \gamma^{\frac{\mu}{\nu-1}} \left( \frac{\beta_j i}{a_j \phi} \right)^{\frac{1}{\nu}}$$  \hspace{1cm} (4)

$$p_j = \gamma^{\frac{\mu}{\nu-1}} \left( \frac{\beta_j i}{a_j \phi} \right)^{\frac{1}{\nu}} \left( a_j \phi \right)^{\frac{1}{\nu}}$$  \hspace{1cm} (5)

Hence, the price $p_j$ rises in income $i$.

2.1.2 Open Economy

There are 2 countries, indexed by subscripts $k$ and $l$. There is free trade between the two countries, i.e. there are no trade costs. Countries only differ in the productivity of labor, but this difference is uniform across all sectors. Hence, countries differ only in the marginal cost shifter $\phi$. Due to the absence of trade costs, the wage in country $l$ has to be equal to $w_l = \phi_k / \phi_l$, with wages in country $k$ normalized at 1. This allows us to focus on the impact of income differences through the demand side of the economy. Both countries produce in all sectors.

$^1$Derivations available upon request.
Individual demand in country \( l \) for good \( j \), \( q_{jl} \) is then given by:

\[
q_{jl} = \gamma \prod_{i=1}^{\gamma_j} \left( \frac{\beta_j i_l}{a_j \phi_l} \right)^{\frac{1}{\gamma_j}}
\]

And the level of quality and the (import) price are equal to:

\[
\alpha_{jl} = \gamma \prod_{i=1}^{\gamma_j} \left( \frac{\beta_j i_l}{a_j \phi_l} \right)^{\frac{1}{\gamma_j}} \tag{6}
\]

\[
p_{jl} = \gamma \prod_{i=1}^{\gamma_j} \left( \frac{\beta_j i_l}{a_j \phi_l} \right)^{\frac{1}{\gamma_j}} \left( \phi_l a_j \right)^{\frac{1}{\gamma_j}} w_l \tag{7}
\]

From equation (6) it is clear that the level of quality varies with income, this implies that firms produce different levels of quality for different markets. As there are no fixed costs in producing quality, this variation of quality across markets is costless for firms. As there are no trade costs, firms in both countries can serve both markets. Assuming that at least one firm from country \( k \) exports to country \( l \), the price \( p_{jl} \) is also the import price \( p_{jk,l} \) for goods going from country \( k \) to \( l \). Hence, we get the following result:

**Proposition 1**  
*In a model with utility expanding in both quantity and quality and constant returns to scale production, higher income per capita leads to higher import prices.*

### 2.1.3 Income Inequality

Now we introduce inequality. It is modeled such that agents differ in the amount of labor they have with labor still homogeneous. For our purposes it suffices to work with two types of agents indexed by \( H \) and \( L \). An agent of type \( G \), \( G = \{H, L\} \) has \( i_G \) labor at its disposal, hence his income is equal to \( w_i G \) with \( i_H > i_L \).\(^2\) We focus on country \( l \), but suppress country indexes to keep notation clean. The number of high and low income

\(^2\)We could interpret high and low income as skilled and unskilled labor with perfect substitutability between the two types of labor.
workers is indicated by $H$ and $L$. Therefore, total income in the economy is equal to:

$$I = w(i_H H + i_L L)$$

As there are no fixed costs, firms produce different quality for each income group. Concentrating on country $l$, we get the following expressions for quantity $q_{j,G}$, quality $\alpha_{j,G}$ and price $p_{j,G}$ in sector $j$ and in income group $G$:

$$q_{j,G} = \gamma^{\frac{\rho}{\rho+1}} \left( \frac{\beta_{j,G}}{a_j \phi} \right)^{\frac{1}{\rho+1}} \tag{8}$$

$$\alpha_{j,G} = \gamma^{\frac{\rho}{\rho-1}} \left( \frac{\beta_{j,G}}{a_j \phi} \right)^{\frac{1}{\rho-1}} \tag{9}$$

$$p_{j,G} = \gamma^{\frac{\rho}{\rho-1}} \left( \beta_{j,G} w \right)^{\frac{\gamma}{\rho-1}} (\phi a_j)^{\frac{1}{\rho-1}} w \tag{10}$$

To find the effect on unit values, we sum prices across different income groups, weighted by their share of spending on good $j$. Assume that the fraction of firms producing in country $k$ and selling in country $l$ for the different income groups, is proportional to the fraction of firms selling for the two income groups across the entire world economy. We get the following expression for unit values of trade from country $k$ to country $l$:

$$p_{j,k,l} = \omega_{j,l,H} p_{j,l,H} + \omega_{j,l,L} p_{j,l,L} \tag{11}$$

With $\omega_{j,G}$ the value share of good $j$ consumed by group $G$, defined as:

$$\omega_{j,l,G} = \frac{p_{j,l,G} q_{j,l,G}}{p_{j,l,H} q_{j,l,H} + p_{j,l,L} q_{j,l,L}} \tag{12}$$
As is clear from equation (12), to concentrate on income differences, we assume \( H = L = 1 \).

Straightforward log differentiation of equation (11) leads to:

\[
\dd p_{j;k;l} = \sum_{j;l;H} p_{j;l;H} \left( \sum_{j;l;L} p_{j;l;L} \right) + \sum_{j;l;L} p_{j;l;L} \left( \sum_{j;l;H} p_{j;l;H} \right)
\]

An increase in income inequality is modeled as an increase in the mean preserving spread of \( i_l = 1/2 (i_{l,H} + i_{l,L}) \). Hence, we can explore the effect of an increase in income inequality by log differentiating \( p_{j;k;l} \) with respect to \( i_{l,H} \) and \( i_{l,L} \) imposing \( \frac{i_{l,H}}{i_{l,L}} = \frac{i_{l,L}}{i_{l,H}} \) to keep mean income constant. Using equations (8) and (10), one can easily write equation (13) as:

\[
\dd p_{j;k;l} = \frac{i_{l,H}}{i_{l,H} + i_{l,L}} \left( \frac{i_{l,H}}{i_{l,L}} - \frac{i_{l,L}}{i_{l,H}} \right) + \frac{i_{l,H}}{1 + \frac{i_{l,H}}{i_{l,L}} + \frac{i_{l,L}}{i_{l,H}}} \left( \frac{i_{l,H}}{i_{l,L}} - \frac{i_{l,L}}{i_{l,H}} \right) \]

The two terms in equation (14) represent the two terms in equation (13), i.e. the shift in market share towards higher priced goods and the change in prices of the high and low quality good. As is clear from equation (14) both effects are positive. The shift in spending towards the higher quality goods consumed by the high incomes is positive. There is less consumption of the low priced good and more consumption of the high priced good leading to an increase in the average price. Also, the effect through the changes in prices themselves, because of changed demand for quality, is positive. The price of high quality goods goes up, as the rich get more income, but the price of low quality goods goes down as the poor get less income. Still, in its effect on average unit values the first effect dominates the second, because the initial price of the high quality good is higher. Hence, we find the following result:

**Proposition 2** *An increase in income inequality as modeled by an increasing income dif-
ference between two income groups keeping average income constant, leads to an increase in import prices.

2.2 Hierarchic Demand Model

2.2.1 Basics

Next we focus on the effect of income and market size on unit values through its effect on price elasticity and markup. Hummels and Lugovskyy (2009) use a Lancaster circle model to model (i) that a larger market size leads to market crowding and therefore to higher price elasticity and lower markups and (ii) that a higher income causes consumers to be willing to pay more to be closer to their ideal variety making them less price sensitive thus leading to a lower price elasticity and higher markups.

In this paper we focus on a different mechanism for the effect of income on unit values through the price elasticity. As people become richer, more goods become indispensable in their consumption bundle. This decreases the price elasticity of these goods and thus raises its markup in an imperfect competition setting. We use the following LHES utility function to model this notion first proposed by Jackson (1982):

\[
U = \sum_{j \in I} \beta_j \ln (u_j + \gamma_j) \, dj
\]

\[
u_j = \left( \frac{\gamma_j}{\sum_{i=1}^{n_j} q_{ij}^\sigma} \right)^{\frac{\sigma}{\sigma - 1}}
\]

Preferences characterized by this utility function are similar to the more well-known Stone-Geary preferences, where the intercept terms \(\gamma_j\) have a negative sign. Hence, in this model the intercept of the income expansion line with the vertical axis is negative. Therefore, as income rises, consumers extend their set of goods consumed. \(I\) is the set of potential varieties consumed. In the sector with subscript 1 there is perfect competition. Lower tier
utility is CES with substitution elasticity $\sigma$. There is monopolistic competition between a small group of identical firms. Firm $i$ in sector $j$ has the following cost function

$$C(q_{ij}) = (a_j q_{ij} + f_j) \phi$$

As the upper tier utility function is separable and the lower tier utility function is homothetic, we can first optimize within each nest, construct a price index and maximize the upper tier utility using this price index (see Blackorby, Primont and Russell (1998)). Demand within each sector is equal to:

$$q_{ij} = \frac{p_j^{\sigma-1}}{p_{ij}^\sigma} E_j; \quad p_j = \left( \frac{\sum_{i=1}^{n_j} p_i^{1-\sigma}}{n_j} \right)^{\frac{1}{1-\sigma}}$$

(16)

$p_j$ is the price index of composite $j$ and $E_j$ income spent on composite $j$, $E_j = p_j q_j$. Maximizing utility in (15) s.t. the budget constraint $\sum_j p_j q_j = i$ using Kuhn-Tucker, generates the following (rewritten) first order conditions:

$$q_j \left( \frac{\beta_j}{q_j + \gamma_j} - \lambda p_j \right) = 0; \quad j \in I$$

(17)

$$\begin{cases} \frac{\beta_j}{q_j + \gamma_j} = \lambda p_j \\
q_j \geq 0 \\
q_j = 0 \end{cases} \quad \begin{cases} j \in J \\
j \in J \end{cases}$$

(18)

$$\begin{cases} \frac{\beta_j}{\gamma_j} < \lambda p_j \\
q_j = 0 \end{cases} \quad \begin{cases} j \in K \\
j \in K \end{cases}$$

(19)

$J$ is the set of goods that are consumed, $K$ is the set of goods that are not consumed. The set of goods consumed $J$ is determined by the following condition:

$$j \in J \text{ if } \exists p_j \text{ s.t. } \frac{\beta_j}{\gamma_j} > \lambda p_j \text{ and } \pi_j(p_j, n_j = 1) > 0$$
With \( \pi_j(p_j, n_j = 1) \) the profit of a monopolist in sector \( j \) with a price of \( p_j \). Hence, the condition for a good to be in the consumption set is that there is a price \( p_j \) such that the marginal utility of the good at a consumption level of 0 is larger than this price and that with this price a monopolist can make positive profit.

Rearranging equation (18) and substituting back into the budget constraint generates an expression for \( \lambda \):

\[
\lambda = \frac{\sum_{j \in J} \beta_j}{i + \sum_{j \in J} \gamma_j p_j}
\]  

(20)

Substituting this back into equation (18), we get the following expression for demand:

\[
q_j = \frac{\sum_{j \in J} \beta_j \left( i + \sum_{j \in J} \gamma_j p_j \right)}{p_j} - \gamma_j; \ j \in J
\]  

(21)

The price elasticity of composite \( j \), \( \varepsilon_j \), can be derived easily from equation (21) as:

\[
\varepsilon_j = 1 + \frac{\gamma_j}{q_j} \left( 1 - \frac{\sum_{j \in J} \beta_j}{\sum_{j \in J} \beta_j} \right)
\]  

(22)

The number of firms within each sector \( j \) is small. This implies that the price elasticity facing an individual firm is non constant. In Appendix B it is shown that the price elasticity of firm \( i \) in sector \( j \), \( \varepsilon_{ij} \), is equal to:

\[
\varepsilon_{ij} = \sigma \frac{n_j - 1}{n_j} + \frac{1}{n_j} \varepsilon_j
\]  

(23)

Straightforward log differentiation shows that the price elasticity facing firm \( i \) in sector \( j \) declines in income \( i \) as follows (derivation in Appendix B; variables with a hat indicate
relative changes):\[3
\]

\[
\hat{\varepsilon}_{ij} = -\frac{\varepsilon_j - 1}{\varepsilon_j} \eta_{q_i,j}\]

(24)

With \(\eta_{q_i,j}\) the income elasticity of demand, given by:

\[
\eta_{q_i,j} = \frac{\beta_i \hat{j}}{\beta_i + \beta_j \sum_{j \in J} \gamma_j p_j - \sum_{j \in J} \hat{\beta}_j}
\]

(25)

Hence, given markup pricing, a smaller price elasticity leads to a larger markup and thus a higher price:

\[
\hat{p}_{ij} = -\frac{1}{\varepsilon_{ij} - \hat{\varepsilon}_{ij}}
\]

\[
= \frac{\varepsilon_j - 1}{\varepsilon_{ij} - \hat{\varepsilon}_{ij}} \left( \frac{\beta_j \hat{i}}{\beta_j \gamma_j p_j - \sum_{j \in J} \hat{\beta}_j} \right)
\]

Hence, we have derived the following result without imposing a free entry condition and thus valid in the short run:

**Proposition 3** A larger income per capita leads to higher prices through a decrease in the price elasticity of demand in the short run.

To address the effects in the long run, as a next step we can impose free entry. This will endogenise the number of firms \(n_j\). To solve for equilibrium sales \(q_{ij}\) and number of firms \(n_j\) in sector \(j\), we start by combining markup pricing and zero profit to get to the\[3\]

\[\text{We do not take into account the effect of a change in income on the price elasticity through the possible change in the budget set and therefore in } \sum_{j \in J} \beta_j, \text{ as this effect only occurs in the border case that the consumption set changes.}\]
following expression:

\[
\begin{align*}
    p_{ij} &= \frac{\varepsilon_{ij}}{\varepsilon_{ij} - 1} a_{ij} \phi \\
    p_{ij} q_{ij} &= (a_{ij} q_{ij} + f_{ij}) \phi \\
    q_{ij} &= (\varepsilon_{ij} - 1) \frac{f_{ij}}{a_{ij}}
\end{align*}
\]  

(26)

Next, in standard monopolistic competition models the model is closed by combining equation (26) with labor market equilibrium. As there is more than one sector and the upper-tier utility function is non-homothetic, we have to take into account that the budget share of sector \( j \) is not constant. Labor market equilibrium in sector \( j \) is given by:

\[
(a_{ij} q_{ij} + f_{ij}) n_j = s_j L
\]

(27)

With \( s_j \) the share of labor used in sector \( j \). Using \( (a_{ij} q_{ij} + f_{ij}) n_j = \frac{p_{ij} q_{ij}}{\phi} n_j = \frac{p_{ij} q_{ij}}{\phi a_{ij}} \), we know that the share of labor used in sector \( j \) is equal to the share of final demand in sector \( j \). Hence, equation (27) can be rewritten as:

\[
(a_{ij} q_{ij} + f_{ij}) n_j = \frac{\sum_{j \in J} \beta_j \left( i + \sum_{j \in J} \gamma_j p_j \right) - \gamma_j p_j}{\phi i} L
\]

(28)

Appendix B shows that one can log differentiate equations (26) and (28) taking into account the expression for \( \varepsilon_{ij} \) in equation (23), generating the following result:

\[
\hat{q}_{ij} = \frac{\sigma + 1 - 2\varepsilon_j}{n_j (\varepsilon_{ij} - 1) + \varepsilon_j - 1} \hat{n}_j
\]

(29)

\[
\hat{q}_{ij} = -\frac{n_j}{n_j - 1} \frac{\varepsilon_{ij} (\varepsilon_j - 1 + \sigma - 1)}{(\sigma - \varepsilon_j) (\sigma - 1)} \hat{n}_j
\]

(30)

Next, we assume that the substitution elasticity within a sector is large enough such that
\( \sigma > 2\varepsilon_j - 1 \). If at least one unit of each good has to be consumed, this assumption corresponds with \( \sigma > 2\gamma_j - 1 \). If we would not make this assumption, an increase in the number of firms would reduce the price elasticity facing an individual firm, \( \varepsilon_{ij} \). With this assumption, equation (26) is upward sloping and equation (28) downward sloping in \((q_{ij}, n_j)\) space, generating a unique equilibrium.

We can use equations (26) and (28) to explore the effect of changes in income \( i \). Appendix B shows that we get the following equation for the relative change in \( q_{ij} \) as a function of the relative change in \( i \):

\[
\frac{n_j (\varepsilon_{ij} - 1) + \varepsilon_j - 1}{\sigma + 1 - 2\varepsilon_j} + \frac{n_j - 1}{n_j} \frac{(\sigma - \varepsilon_j) (\sigma - 1)}{\varepsilon_{ij} (\varepsilon_j - 1 + \sigma - 1)} \frac{\eta_{q,i}}{q_{ij}} \]

\[
= -\frac{n_j (\varepsilon_{ij} - 1) + \varepsilon_j - 1}{\sigma + 1 - 2\varepsilon_j} \frac{\varepsilon_j - 1}{n_j (\varepsilon_{ij} - 1)} \frac{\eta_{q,i}}{q_{ij}} + \frac{\sigma - 1}{\varepsilon_{ij} (\varepsilon_j - 1 + \sigma - 1)} \frac{\eta_{q,i}}{q_{ij}} + \frac{1}{\varepsilon_{ij} (\varepsilon_j - 1 + \sigma - 1)} \frac{\eta_{q,i}}{q_{ij}} \hat{i}
\]

Using the zero profit condition, equation (26), we know that the price elasticity \( \varepsilon_{ij} \) rises in \( q_{ij} \). Hence, we can consider the effect on \( q_{ij} \) to determine how \( \varepsilon_{ij} \) and hence also \( p_{ij} \) change.

The first term on the RHS of equation (31) reflects the downward shift in the zero profit curve due to the initial (short term) decrease in the price elasticity. The second term represents the upward or downward shift in the resource curve. When the income elasticity is smaller than 1, the resource curve shifts downward and when the elasticity is larger than 1, the resource curve shifts up. This reflects the fact that an income elasticity larger than 1 implies that the amount of resources allocated to sector \( j \) rises. This raises the number of firms in sector \( j \), thereby raising the price elasticity. Hence, when the income elasticity for good \( j \) is smaller than 1, the effect of a larger \( i \) on market price \( p_{ij} \) is surely positive. It is easy to show with an example that the coefficient of \( (\eta_{q,i} - 1) \hat{i} \) can be larger than the coefficient of \( \eta_{q,i} \hat{i} \) implying that \( q_{ij} \) could actually rise in income \( i \) and \( \varepsilon_{ij} \) would then
rise as well and $p_{ij}$ fall. We summarize our findings in the following proposition:

**Proposition 4** In the long run the price elasticity of a good either rises or declines in income per capita $i$. When the income elasticity for good $j$ is smaller than 1, the price elasticity unambiguously declines in income per capita.

### 2.2.2 Open Economy

We assume like in the previous subsection that there are 2 countries that can trade freely. The countries are equal in all respects except in the productivity of labor. With wages in country $k$ normalized to 1, the wage in country $l$, $w_l$, is equal to $\phi_k/\phi_l$. As we do not focus on income differences within countries in this section but on wage differences between countries, we set income $i_k$ and $i_l$ equal to 1. As firms sell identical goods for different prices in different markets, we have to exclude parallel imports, for example because regulation forbids parallel imports for differentiated goods.

The demand facing firm $i$ in sector $j$ in country $k$ and country $l$, respectively $q_{ijk}$ and $q_{ijl}$ are then equal to:

$$q_{ijk} = \frac{p_{ijk}^2}{p_{ijk}^2} q_{jk}; \quad p_{jk} = \left( \frac{\sum_{i=1}^{n_j} p_{ijk}^{1-\sigma}}{\sum_{i=1}^{n_j} p_{ijk}^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}$$

$$q_{ijl} = \frac{p_{ijl}^2}{p_{ijl}^2} q_{jl}; \quad p_{jl} = \left( \frac{\sum_{i=1}^{n_j} p_{ijl}^{1-\sigma}}{\sum_{i=1}^{n_j} p_{ijl}^{1-\sigma}} \right)^{\frac{1}{1-\sigma}}$$

As the price elasticity varies with income, firms will charge a different price in each market. The set of consumed goods is also country dependent. Price elasticities in the two countries
are equal to:

\[
\varepsilon_{ijk} = \sigma \frac{n_j - 1}{n_j} + \frac{1}{n_j} \varepsilon_{jk} = \sigma \frac{n_j - 1}{n_j} + \frac{1}{n_j} \left(1 + \frac{\gamma_j}{n_j q_{ijk}}\right) \tag{32}
\]

\[
\varepsilon_{ijl} = \sigma \frac{n_j - 1}{n_j} + \frac{1}{n_j} \varepsilon_{jl} = \sigma \frac{n_j - 1}{n_j} + \frac{1}{n_j} \left(1 + \frac{\gamma_j}{n_j q_{ijl}}\right) \tag{33}
\]

Log differentiating \(\varepsilon_{ijl}\) wrt the wage in country \(l\), \(w_l\), we find like in the closed economy that the elasticity declines with the wage \(w_l\):

\[
\tilde{\varepsilon}_{ijl} = \frac{1}{n_j} \varepsilon_{jl} - \frac{1}{n_j} \varepsilon_{ijl} - \frac{w_l}{P_l q_{ijl}} \sum_{j \in J_l} \beta_j \tilde{w}_j \tag{34}
\]

As a next step we can introduce free entry and impose a zero profit condition. This will endogenise the number of firms \(n_j\). A solution of the model requires defining the following ratio of relative demands in country \(l\) and country \(k\) in sector \(j\), \(A_{jlk}\):

\[
A_{jlk} = \frac{q_{jil}}{q_{ijk}} = \frac{q_{jl}}{q_{jk}} = \frac{\sum_{j \in J_l} \beta_j \left(w_l + \sum_{j \in J_l} \gamma_j p_{jl}\right)}{\sum_{j \in J_k} \beta_j \left(w_k + \sum_{j \in J_k} \gamma_j p_{jk}\right)} - \gamma_j \frac{L_l}{L_k} \tag{35}
\]

Also, we define the ratio of relative revenues, \(B_{jlk}\) as:

\[
B_{jlk} = \frac{p_{jil} q_{jil}}{P_{jlk} q_{jlk}} = \frac{p_{jl} q_{jl}}{P_{jk} q_{jk}} = \frac{\sum_{j \in J_l} \beta_j \left(w_l + \sum_{j \in J_l} \gamma_j p_{jl}\right) - p_{jl} \gamma_j}{\sum_{j \in J_k} \beta_j \left(w_k + \sum_{j \in J_k} \gamma_j p_{jk}\right) - p_{jk} \gamma_j} \frac{L_l}{L_k} \tag{36}
\]
The prices $p_{jk}$ and $p_{jl}$ are equal to:

$$p_{jk} = n_j^{1-\sigma} \frac{\varepsilon_{ijk}^{-1}}{\varepsilon_{ijk} - 1} a_{ij} \phi_k w_k$$

(37)

$$p_{jl} = n_j^{1-\sigma} \frac{\varepsilon_{ijl}^{-1}}{\varepsilon_{ijl} - 1} a_{ij} \phi_l w_l$$

(38)

$\varepsilon_{jl}$ can be expressed as a function of $\varepsilon_{jk}$:

$$\varepsilon_{jk} = 1 + \frac{\gamma_j}{n_j q_{ijk}}$$

(39)

$$\varepsilon_{jl} = 1 + \frac{\gamma_j}{n_j q_{ijl}} = \varepsilon_{jk} + \frac{\gamma_j}{n_j q_{ijk}} \frac{1 - A_{jk}}{A_{jlk}}$$

(40)

The zero profit condition for a firm producing in sector $j$ in country $k$ can be expressed as follows:

$$\frac{q_{ijk}}{\varepsilon_{ijk} - 1} + \frac{q_{ijk} A_{jlk}}{\varepsilon_{ijl} - 1} = \frac{f_{ij}}{a_{ij}}$$

(41)

Finally, labor market equilibrium can be written as:

$$\frac{(a_{ij} (q_{ijk} + q_{ijl}) + f_{ij})}{\phi_k} n_j = S_{jk} L_k$$

(42)

With $S_j$ the share of labor in country $k$ allocated to sector $j$. We can calculate this share as:

$$S_{jk} = \frac{(a_{ij} (q_{ijk} + q_{ijl}) + f_{ij}) n_j}{\sum (a_{ij} (q_{ijk} + q_{ijl}) + f_{ij}) n_j}$$

$$= \frac{p_{jk} q_{jk} + p_{jl} q_{jl}}{\sum p_{jk} q_{jk} + p_{jl} q_{jl}}$$

$$= \frac{p_{jk} q_{jk} (1 + B_{jlk})}{L_k \alpha_k + w_l L_l (1 - \alpha_l)}$$

(43)

$\theta_k$ and $\theta_l$ are the shares of income spent domestically in country $k$ and $l$ respectively.
Balanced trade implies:

\[ L_k (1 - \theta_k) = w_l L_l (1 - \theta_l) \]  

(44)

Substituting equations (43) and (44) into equation (42), we get the following expression for labor market equilibrium:

\[ \frac{(a_{ij} q_{ijk} (1 + A_{jlk}) + f_{ij})}{\phi_k} n_j = \frac{p_{jk} q_{jk} (1 + B_{jlk})}{w_k L_k} L_k \]  

(45)

To show the existence of a unique equilibrium, we log differentiate equations (35)-(40) and (45) wrt \( q_{ijk}, n_j \), treating \( A_{jlk}, B_{jlk}, \varepsilon_{ijk}, \varepsilon_{ijl}, \varepsilon_{jk}, \varepsilon_{jl}, p_{jl} \) and \( p_{jk} \) as endogenous. Derivations in Appendix B show that this leads to the following two equations in \( \hat{q}_{ijk} \) and \( \hat{n}_j \) like in the closed economy:

\[ \hat{q}_{ijk} = \frac{\sigma + 1 - 2\varepsilon_j}{\varepsilon_j - 1 + n_j (\varepsilon_{ijk} - 1)} \hat{n}_j \]  

(46)

\[ \hat{q}_{ijk} = \frac{n_j}{n_j - 1} \frac{\sigma - 1 + \varepsilon_j - 1}{(\sigma - \varepsilon_j)(\sigma - 1)} \hat{n}_j \]  

(47)

Assuming as in the closed economy that the substitution elasticity within a sector is large enough such that \( \sigma > 2\varepsilon_j - 1 \), implying that equation (41) is upward sloping and equation (47) downward sloping in \((q_{ij}, n_j)\) space, generating a unique equilibrium.

We can do comparative statics on the effect of an increase in income per capita \( w_l \) in the importing country. Like in the closed economy, the effect of a larger income per capita in the importing country will generate two effects on the price in sector \( j \): a direct effect through the uppernest price elasticity driving down the price elasticity and an indirect effect through the change in the amount of resources allocated to sector \( j \) driving up the number of firms and thereby increasing the price elasticity. However, we can interpret the importing country as a small country if the exporting country is seen as the rest of the
world. This will imply that the resource allocation effect is negligible and only the direct effect remains. Technically, we assume that $A_{jlk}$ and $B_{jlk}$ are negligible. This implies that the ZP and LME do not change in response to a change in $w_l$. Consider first the ZP in equation (41). $w_l$ affects $A_{jlk}$ and $\varepsilon_{ijl}$. But when $A_{jlk}$ is negligible a change in these two variables will not affect the ZP equation. Similarly, the LME condition in equation (45) depends upon $w_l$ through $A_{jlk}$ and $B_{jlk}$, but these effects will again be zero when $A_{jlk}$ and $B_{jlk}$ are negligible. Hence, the change in price elasticity $\varepsilon_{ijl}$ is in the long run also given by equation (34). Therefore, we get the following result:

**Proposition 5** An increase in income per capita $w_l$ in country $l$ that is small relative to the rest of the world leads to a lower price elasticity and a higher markup on imported goods.

### 2.2.3 Income Inequality

Like in the quality model, we now address the effect of changes in income inequality. Again, there are 2 income groups with units of labor $i_H$ and $i_L$ and the number of these workers is respectively $H$ and $L$. We address the effect of an increase in the mean preserving spread, hence, we consider the effect of a change in $i_H$ with the corresponding change in $i_L$ equal to $\hat{i}_L = -\frac{i_H}{i_L} \hat{i}_H$. Like above we consider the effect on importer unit values of country $l$, assuming that country $l$ is small. This implies that the long run effect is equal to the short run effect for the same reason as above in response to a change in income.

Suppressing the country subscript $l$, total demand for sector $j$ goods in country $l$ is
equal to:

\[ q_j = q_{j,H}H + q_{j,L}L \]  

\[ q_j = \left( \frac{\sum_{j \in J_H} \beta_j \left( w_{i,H} + \sum_{j \in J_H} \gamma_j p_j \right)}{p_j} - \gamma_j \right) H + \left( \frac{\sum_{j \in J_L} \beta_j \left( w_{i,L} + \sum_{j \in J_L} \gamma_j p_j \right)}{p_j} - \gamma_j \right) L \]  

(48)  

(49)

Hence, \( q_{j,G} \) denotes individual demand in group \( G \). Notice from equation (48) that there is only one price for the two income groups, as the product is identical and the market cannot be segmented between income groups. Log differentiating equation (48) wrt the market price \( p_j \) generates the following expression for the price elasticity \( \varepsilon_j \):

\[ \varepsilon_j = 1 + \frac{H \left( 1 - \frac{\beta_j}{\sum_{j \in J_H} \beta_j} \right) + L \left( 1 - \frac{\beta_j}{\sum_{j \in J_L} \beta_j} \right)}{q_j} \]  

(50)

Log differentiating (50) wrt \( i_H \) imposing \( i_L = \frac{i_H}{i_H} \), we find the following effect of an increase in the mean preserving income spread on the price elasticity:

\[ \tilde{\varepsilon}_j = -\frac{\varepsilon_j - 1}{\varepsilon_j} \left( \frac{q_{j,H} H \eta_{q_{j,H},i_H} - q_{j,L} L \eta_{q_{j,L},i_L} i_H}{q_{j,H} H + q_{j,L} L} \right) \]  

(51)

Equation (51) can be rewritten as follows:

\[ \tilde{\varepsilon}_j = -\frac{\varepsilon_j - 1}{\varepsilon_j} \left( \frac{H \sum_{j \in J_H} \beta_j - L \sum_{j \in J_L} \beta_j}{p_j q_j} \right) \beta_j i_H \tilde{i}_H \]  

(52)

\footnote{We abstract from the effect through the possible change in the budget set \( J \), as before.}

\footnote{Derivation available upon request.}
Equation (52) shows that if the size of the two income groups is equal, i.e. $H = L$, the price elasticity rises with income inequality, as $\sum_{j \in J_H} \beta_j > \sum_{j \in J_L} \beta_j$ and hence the price declines in income inequality. The intuition for this result is the following: with an increase in inequality the price elasticity of high incomes goes down and of low incomes goes up. The effect of income on the price elasticity of demand is a function of the income elasticity of demand, as the price elasticity is a function of demand $q_j$. Hence, when income goes up, the price elasticity goes down. As the income elasticity for higher incomes is lower, their price elasticity declines by less than the price elasticity of low incomes goes up. As a result, the overall price elasticity rises with an increase in income inequality. Therefore, the market price declines in income inequality.

There is an important qualification to this finding. When the low income group does not consume a certain commodity, the only effect of an increase in income inequality is that the higher income of the high income group reduces the price elasticity and thus raises the market price. We summarize these results in the following proposition:

**Proposition 6** For goods consumed by all income groups, an increase in income inequality raises the price elasticity of demand and reduces the market price. For goods consumed only by the high income group, an increase in income inequality reduces the price elasticity and raises the market price

Hence, we find that for goods lower in the consumption hierarchy the effect of income inequality on market price through the elasticity channel is opposite to the effect of income inequality through the quality channel and for goods high in the consumption hierarchy the effect of income inequality on market price through the price elasticity channel has the same sign as the effect through the quality channel.
2.3 The Effect of Income Inequality in the Hummels-Lugovskyy Model

Hummels and Lugovskyy (2009) propose a different explanation for the fact that a higher income per capita in an importing country leads to a lower price elasticity and thus higher unit values. With higher incomes people are willing to pay more to get closer to their ideal variety. The implication is that the price elasticity is lower and firms can charge higher markups as people switch less easily between varieties. To differentiate their theory from ours, we slightly adjust their model to address the effect of income inequality. As will be shown the price elasticity rises and the market price declines in income inequality. Hence, the effect of income inequality on the market price (and unit values) has the same sign as in the quality model but the opposite sign from the hierarchic demand model.

Hummels and Lugovskyy (2009) introduce an additional term in the distance compensation function to catch the effect of a higher finickyness (eagerness) to buy the ideal variety as income rises. In concrete, finickyness rises with the amount consumed. In Hummels and Lugovskyy (2009) agents consume only one variety, i.e. there is no upper nest with a Lancaster circle in the lower nest. A model with preferences over more products is not feasible in the model of Hummels and Lugovskyy (2009) as the upper nest optimization depends upon the amount consumed and thus upon the lower nest compensation function. Therefore, we adjust the Hummels and Lugovskyy (2009)-model by creating a Cobb Douglas uppernest and including a finickyness effect in the compensation function that is a function of total consumption, i.e. not only of consumption of the specific variety. In the Hummels and Lugovskyy-case this would lead to the same result, as people consume only one variety.
We have the following preferences:

\[ U = \sum_{j=1}^{J} \beta_j \ln u_j \]  

\[ u_j = \int_{\omega_j \in \Omega_j} \frac{q_j}{h_j (\delta (\omega, \tilde{\omega}))} \]  

We specify the compensation function \( h_j \) for the cost of being further away from the ideal variety as rising in total income \( i \) and rising in distance \( \delta \) from the ideal variety:

\[ h_j (\delta, w) = 1 + iv^{\psi}; \quad v < 1; \quad \psi > 1 \]  

It is easy to show that an increase in income leads to a higher utility as long as \( v < 1 \).

There is an equal mass of consumers \( i_H \) with high income \( i_H \) and low income \( i_L \), which are both distributed uniformly across the circle. With the wage normalized at 1, the cost function for production of a variety \( j \) is equal to:

\[ C (q_j) = a_j q_j + f_j \]

It is easy to show\(^6\) that with this specification there exists a symmetric zero profit equilibrium like in Hummels and Lugovskyy (2009) with aggregate demand for any produced variety \( q_j \) equal to:

\[ q_j = d_j (i_H H + i_L L) \frac{\beta_j}{p_j} \]  

\( d_j \) is the equal distance between any two varieties. The price elasticity facing a firm consists of two components, one with a direct effect of price on demand and the other with an effect

\(^6\)Derivations are analogue to the derivation in Hummels and Lugovskyy (2009) and in Helpman and Krugman (1985) and are available upon request
through distance $d_j$, and is equal to:

$$
\varepsilon_j = 1 + \frac{i_H \left( 1 + \frac{1}{i_H \left( \frac{d_j}{2} \right)^v} \right) + i_L \left( 1 + \frac{1}{i_L \left( \frac{d_j}{2} \right)^v} \right)}{2\psi (i_H + i_L)}
$$

(56)

To address the effect of income inequality, we log differentiate equation (56) wrt $i_H$ and $i_L$ with the condition $i_H = i_L$, and hence distance $d_j$ fixed, generating the following result:

$$
\tilde{\varepsilon}_j = \frac{\varepsilon_j - 1}{\varepsilon_j} \left[ \frac{i_H H \left( 1 + \frac{1}{i_H \left( \frac{d_j}{2} \right)^v} \right) + i_L L \left( 1 + \frac{1}{i_L \left( \frac{d_j}{2} \right)^v} \right) - \tilde{i}_H}{i_H H \left( 1 + \frac{1}{i_H \left( \frac{d_j}{2} \right)^v} \right) + i_L L \left( 1 + \frac{1}{i_L \left( \frac{d_j}{2} \right)^v} \right)} \right]
$$

(57)

As $i_H > i_L$ and $v < 1$, an increase in the mean preserving spread reduces the price elasticity and hence increases the market price. In the long run, when $n_j$ is endogenous, we add the following zero profit condition:

$$
d_j = \frac{f\varepsilon_j}{(i_H + i_L) \beta_j}
$$

(58)

Taking into account equation (58), we find the following effect of income inequality on the price elasticity:

$$
\tilde{\varepsilon}_j = \frac{\frac{i_H H \left( 1 + \frac{1}{i_H \left( \frac{d_j}{2} \right)^v} \right) + i_L L \left( 1 + \frac{1}{i_L \left( \frac{d_j}{2} \right)^v} \right) - \tilde{i}_H}{i_H H \left( 1 + \frac{1}{i_H \left( \frac{d_j}{2} \right)^v} \right) + i_L L \left( 1 + \frac{1}{i_L \left( \frac{d_j}{2} \right)^v} \right)} \cdot \tilde{i}_H}{\frac{\varepsilon_j - 1}{\varepsilon_j}} \left[ \frac{i_H H \left( 1 + \frac{1}{i_H \left( \frac{d_j}{2} \right)^v} \right) + i_L L \left( 1 + \frac{1}{i_L \left( \frac{d_j}{2} \right)^v} \right) - \tilde{i}_H}{i_H H \left( 1 + \frac{1}{i_H \left( \frac{d_j}{2} \right)^v} \right) + i_L L \left( 1 + \frac{1}{i_L \left( \frac{d_j}{2} \right)^v} \right)} \right]
$$

(59)

Hence, also in the long run with the number of firms endogenous, an increase in income inequality reduces the price elasticity and raises the market price. We summarize our findings as follows:
Proposition 7 In the Lancaster circle model of Hummels and Lugovskyy (2009) an increase in income inequality as measured by an increase in the mean preserving spread causes a decrease in the price elasticity of demand and an increase in the market price.

The result in proposition 7 can be interpreted as follows: an increase in income raises the price elasticity of high incomes through one channel and reduces the price elasticity through another channel. The price elasticity rises with income, because the effect of a higher price through a loss in the number of customers is multiplied by the level of income and thus larger at a larger income level. The price elasticity declines with income, as agents become less price sensitive (more finicky) with a higher income. When \( v < 1 \), the first effect dominates the second and this effect is stronger for lower incomes, as their compensation function is smaller, so they change consumption more. As a result, the overall price elasticity declines.

3 Data and Estimation Method

We now turn to an empirical analysis of the impact of importer characteristics on unit values. More precisely, we examine how the price of disaggregated product categories change with the level of development of the country and with income inequality.

In our empirical analysis we proxy prices with import unit values. The data we use for unit values come from the BACI database \(^7\) which contains quantity and value of bilateral imports in 6-digit Harmonized System (HIS) classification. The database is based on COMTRADE (Commodities Trade Statistics database) and it covers more than 200 countries and 5,000 products between 1995 and 2004. BACI takes advantage of the double information on each trade flow to fill out the matrix of bilateral world trade providing a

\(^7\)http://www.cepii.fr/anglaisgraph/bdd/baci/baciwp.pdf
“reconciled’ value for each flow reported at least by one of the partners. Therefore the missing values in BACI are those concerning trade between non reporting countries.

Our income and income per capita data originate from the World Bank’s World Development Indicator database. We also use a measure of income inequality in our regressions. Data on income inequality come from the World Institute for Development Economics Research’s World Income Inequality Database.\footnote{http://62.237.131.23/wiid/wiid.htm}

We use 2 different estimation methods. We start with an analysis similar to Hummels and Lugovskyy (2009) including as many fixed effects as possible to purge as much unobserved heterogeneity as possible, but still being able to identify the effect of time varying importer variables. Next, we correct for possible sample selection bias applying the Heckman sample selection estimation procedure.

We start with a simple fixed effects analysis based upon the theoretical model in this paper, hence including income per capita and income inequality. As a control variable we add total income, based upon the theoretical and empirical analysis in Hummels and Lugovskyy (2009). Following the latter paper in its treatment of functional form and fixed effects, we write export prices as:

\[
\ln P_{i,j,t,k} = e_{j,t,k} b_{i,j,k} f (a_{i,k}, Y_{i,t}, Y/L_{i,t}, G_{i,t}) \tag{60}
\]

In equation (60) the subscript \(i\) stands for importer, \(j\) for exporter, \(k\) for product, and \(t\) for time. \(e_{j,t,k}\) captures any exporter-time-product specific effect on prices. \(b_{i,j,k}\) captures bilateral country-pair-product specific influences. The function \(f\) captures the effect of importer-commodity characteristics \((a_{i,k})\), the size of the market \((Y_{i,t})\), per capita income \((Y/L_{i,t})\), and income inequality measured by the Gini coefficient \((G_{i,t})\). These variables have their effect through the different channels mentioned in the theory section, so both
through exporter destination specific variations in quality and in markups.

Similarly to Hummels and Lugovskyy (2009) we proceed with two further steps before arriving at our estimating equation. First, we approximate our function $f$ as a separable log-linear function which allows us to write log export prices as follows:

$$\ln P_{i,j,t,k} = e_{j,t,k} + b_{i,j,k} + a_{i,k} + \beta_1 \ln Y_{i,t} + \beta_2 \ln Y_{i,t}/L_{i,t} + \beta_3 \ln G_{i,t}$$

(61)

Second, we also eliminate all time invariant variation, namely variations across bilateral country-pair-products and importer-product variations by taking log differences of the data. We take the difference between the end periods of our dataset which is 2000 and 2004. Adding a disturbance term $u_{ij1k} - u_{ij0k}$ we arrive at the following equation:

$$\ln P_{i,j,1,k} - \ln P_{i,j,0,k} = (e_{j,1,k} - e_{j,0,k})$$

$$+ \beta_1 (\ln Y_{i,1} - \ln Y_{i,0}) + \beta_2 (\ln Y_{i,1}/L_{i,1} - \ln Y_{i,0}/L_{i,0}) + \beta_3 (\ln G_{i,1} - \ln G_{i,0}) + u_{ij1k} - u_{ij0k}$$

(62)

In the next section we report the results of estimation of equation (62).

To correct for sample selection, we have to add a selection equation that determines whether there is trade or not. With a majority of the unit values in our data equal to zero, correcting for sample selection is necessary. Our use of selection modeling is a break from the general approach followed in the literature. Employing a sample selection model allows us to take account of the censoring process that leads to zero or missing bilateral trade flows. More precisely, in our estimating framework the outcome variable (the dependent variable in the second stage equation) is only observed if the defined selection criterion is met. (Helpman, Melitz and Rubinstein (2008) also employ a Heckman estimator to
examine bilateral trade flows including zeros, as do Francois and Manchin (2006)). This implies that unit values are not observed for those trade flows and simply dropping those observations from our empirical analysis could lead to biased results. Thus, apart from analyzing the effects of different factors on unit values, we also correct for possible bias arising from missing trade flows.

We work with Heckman’s selection model (Heckman (1979), Greene (2003)). We estimate a main equation for unit values based upon equation (61) and a selection equation determining whether there is trade or not. Hence, we get the following two equations:

$$
\ln P_{i,j,t,k} = e_{j,t,k} + b_{i,j,k} + a_{i,k} + \beta_1 \ln Y_{i,t} + \beta_2 \ln Y_{i,t}/L_{i,t} + \beta_3 \ln G_{i,t} + u_{1,i,j,t,k} \tag{63}
$$

$$
s_{i,j,t,k} = 1 \left[ h_{i,j,t,k}^* = \alpha_i + \gamma_j + \psi_k + \delta' z_{i,j,t,k} + u_{2,i,j,t,k} > 0 \right] \tag{64}
$$

$z_{i,j,t,k}$ is a vector of regressors in the selection equation and $u_1$ and $u_2$ are the error terms which are jointly normally distributed, independently of the regressors, with zero expectations. The unit values $\ln P_{i,j,t,k}$ are only observed if $h_{i,j,t,k}^* > 0$. The variable $s_{i,j,t,k}$ takes the value of one if $\ln P_{i,j,t,k}$ is observed, while it is 0 if the variable $\ln P_{i,j,t,k}$ is zero or missing. The first equation shows how the unit value of imports is affected by different factors, while the second gives some insight into why trade occurs at all between two partner countries.

The second equation in our Heckman selection model, which is the selection equation, with the dependent variable being a zero-one variable depending on whether trade occurs or not, is based on the gravity literature. Following the most recent advances of the gravity literature, we should include importer-time-product specific fixed effects, exporter-

---

9When examining the global pattern of bilateral trade flows, one striking feature of the landscape is that many country pairs do not trade. See Baldwin and Taglioni (2006) and Baldwin and Harrigan (2007)

10There are many paths that lead to the now standard functional relationship we use here. See Baldwin and Harrigan (2007) for an overview. Also see Evenett and Keller (2002); Anderson (1979); Anderson and Marcoullier (2002), Anderson and van Wincoop (2003); and Deardorff (1988).
time-product specific fixed effects and country-pair-product specific fixed effects. Including fixed effects for these three dimensions would minimize the omitted variable bias problem. Due to computational limitations, we only include an importer fixed effect, an exporter fixed effect and a product fixed effect and control variables.

Practically, we estimate equation (64) with Probit, include the Mills ratios in equation (63) and estimate the transformed version of this equation, i.e. equation (62) including the Mills ratio.

4 Empirical Results

Table 1 presents our empirical results. We report results of three different specifications. The first column in the table presents the results of OLS estimation of equation (62), hence with exporter-product fixed effects. The second column exposes the results for OLS regressions with separate exporter and product fixed effects. We present this specification along with the first specification given that in our main specification with the Heckman-selection model we also have separate exporter and product fixed effects. The coefficients are almost identical to those presented in the first column indicating that using separate product and country fixed effects instead of exporter-product fixed effects do not result in biased estimation results. The third and fourth columns in table 1 present results for our main specification using the Heckman-selection model in equations (63)-(64). The coefficients are close to those obtained with OLS regressions, however all somewhat smaller in magnitude.

The empirical results in table 1 provide support for the two models in this paper and the circle model in Hummels and Lugovskyy (2009): unit values rise in importer income per capita. The results also confirm the findings of Hummels and Lugovskyy (2009) that a larger market size of the importer as proxied by total GDP reduces unit values. The size of
<table>
<thead>
<tr>
<th></th>
<th>Column 1 OLS unit values</th>
<th>Column 2 OLS unit values</th>
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<th>Probit Pr(import)</th>
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<td><strong>ln GDP</strong></td>
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<td></td>
<td>(0.040)***</td>
<td>(0.040)***</td>
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<td>(0.040)***</td>
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<td><strong>ln GDP per capita</strong></td>
<td>0.942</td>
<td>0.933</td>
<td>0.796</td>
<td>(0.038)***</td>
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<tr>
<td></td>
<td>(0.038)***</td>
<td>(0.038)***</td>
<td></td>
<td>(0.038)***</td>
</tr>
<tr>
<td><strong>ln GINI</strong></td>
<td>-0.156</td>
<td>-0.158</td>
<td>-0.135</td>
<td>(0.006)***</td>
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<td></td>
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<td>no</td>
<td>yes</td>
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<tr>
<td>product fixed effects</td>
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<td>yes</td>
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<td>2815866</td>
<td>2815866</td>
<td>2815866</td>
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<tr>
<td><strong>Mills – ratio</strong></td>
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<td></td>
<td>-0.329</td>
<td>(0.007)***</td>
</tr>
</tbody>
</table>

Source: own calculations. Standard errors in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%

Table 1: Heckman and OLS estimates
the effects of income per capita and total income on unit values is considerably larger than in Hummels and Lugovskyy (2009). Our coefficients for GDP and GDP per capita are equal to 0.8 and -0.9 respectively, whereas the weighted average coefficients in Hummels and Lugovskyy (2009) of GDP and GDP per capita are equal to 0.5 and -0.5, respectively. Their data are taken from the Eurostats Trade Database for the period 1990-2003. This database contains data on values and quantities of trade from 11 EU exporters and 200 importers worldwide at the 8-digit level. The database we employ contains trade data at the 6-digit level for more than 200 exporters and importers.

This specification is the same as the one used by Hummels and Lugovskyy (2009), except that we also have a variable measuring income inequality. This specification allows us to compare our results with those of Hummels and Lugovskyy (2009). The sign of our GDP and income per capita coefficients are the same as those obtained by Hummels and Lugovskyy (2009), however the size of our coefficients are much higher (the coefficient of GDP was 0.85 and 0.2521 of the income per capita for Hummels and Lugovskyy (2009)).

To discriminate between the different theories, we also estimated the effect of income inequality in the importer country on unit values. We find a highly significant negative effect of income inequality on unit values. This finding provides support for the hierarchic demand model, that predicts a lower import price in response to higher inequality. The quality model with utility rising in quality and the finickyness circle model of Hummels and Lugovskyy (2009) both predict a positive effect of inequality on import prices. Therefore, the empirical findings do not directly falsify the quality and finickyness model. But they do indicate that these models have to be combined with a hierarchic demand model, where import price decline in income inequality.
5 Concluding Remarks

In this paper we modeled two new channels distinct from those existing in the current literature to account for the effect of importer income per capita on unit values (trade prices). On the one hand we modeled the increasing demand for quality at higher levels of income by utility expanding both in quality and quantity. At higher levels of income, consumers demand a higher level of quality and with marginal costs rising in quality this leads to higher prices. On the other hand we used a hierarchic demand system to model the notion that with a larger consumption set at higher levels of income, goods lower in the hierarchy become more necessary/indispensable in the consumption set and therefore people are willing to pay a higher price for these goods. Hummels and Lugovskyy (2009) provide a third theory to explain higher unit values for trade to richer countries: at higher income levels consumers are more eager to consume their ideal variety (more finicky) and are therefore willing to pay a larger markup.

Empirically, we found strong support for the theoretical predictions: an increase in importer income per capita by 1% raises importer unit values by 0.9%. By addressing the effect of income inequality on unit values, we find a way to discriminate between the different channels. Empirically we find that unit values of trade decline in income inequality of the importer country. This negative effect is predicted by the hierarchic demand model as derived in the theoretical section, whereas it is shown that the quality expansion model and the finickyness circle model of predict a positive effect of higher income inequality on unit values.

These findings do not invalidate the circle model and the quality expansion model; they rather show the importance of the hierarchic demand model: a larger demand for quality and an increased finickyness with higher levels of income can still be part of the story, but these channels have to be at least accompanied by the channel present in the hierarchic
demand model with an increased willingness to pay for necessary goods as consumers become richer.

The research findings in this paper can be extended in various interesting directions. First, we can include in the hierarchic demand model the condition that agents have to consume at least one unit of a good. The implication will be that it might be ideal for a firm to sell a good below its markup price, as otherwise the good will not be consumed at all. This would add further realism to the model, as in the real world for many goods one has to consume at least one unit (laptops, mobile phones). Second, we can use the hierarchic demand system to model the effect of higher world income per capita and a larger world economy on the availability of different varieties and the price of different varieties. In the current model we switched the market size effects off most of the time by assuming that the importing country is small relative to the rest of the world.

References


### Annex Table A.1: Sample countries

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Appendix A  Nesting the Quality and Hierarchic Demand Model

$L$ consumers have the following identical utility function:

\[ U = \sum_{j=0}^{\infty} \beta_j \ln (c_j + \gamma_j) \]

\[ c_j = \left( \sum c_{ij}^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{1}{\sigma - 1}} \]

\[ c_{ij} = \left( \delta_q q_{ij}^{\frac{\sigma - 1}{\sigma}} + \delta_{\alpha} \alpha_{ij}^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{1}{\sigma - 1}} \]

$U$ is maximized subject to the following budget constraint:

\[ \sum_{j=1}^{J} \sum_{i=1}^{n_j} P_{ij} q_{ij} = w \]

Production is increasing returns to scale with the following identical cost function for all producers within each sector $j$:

\[ c (q_{ij}, \alpha_{ij}) = \left( \alpha_{ij}^{\gamma_j} \right) a_j q_{ij} + f_j \]

There is monopolistic competition between producers of varieties $j$. To solve the model outlined, one has to determine a zero profit Nash equilibrium that solves for the set of goods in the consumption set, $J$, the number of commodities produced within each set of goods, $n_j$, and the level of quality and quantity produced, $\alpha_{ij}$ and $q_{ij}$. This model is not analytically solvable. Still, the two models discussed in the main text can be seen as nested cases of this more general model. For the quality model, we choose the following
parameters:

$$\delta_q = \delta_\alpha = 1$$

$$f_{ij} = 0$$

$$\sigma \to \infty$$

$$\gamma_j \to \infty; \ \forall j$$

For the hierarchic demand model, we fix the parameters as follows:

$$\gamma = 0$$

$$\delta_\alpha = 0$$

Appendix B  Hierarchic Demand Model

Some of the equations below are based upon rather lengthy derivations. A separate document, available upon request, contains the detailed derivations of these equations in the appendix.

Appendix B.1  Basics

To calculate the price elasticity facing firm $i$ in sector $j$, we rewrite demand facing firm $i$ in sector $j$ substituting $E_j = p_j q_j$:

$$q_{ij} = \frac{p_j^2}{p_{ij}^2} q_j$$
Log differentiating this equation we get:

$$\hat{q}_{ij} = \sigma (\hat{p}_j - \hat{p}_{ij}) + \varepsilon_j \hat{p}_j$$  \hspace{1cm} (B.1)

Using the expression for the price index $p_j$ in equation (16) we get for $\hat{p}_j$:

$$\hat{p}_j = \frac{p_j^{1-\sigma}}{n_j} \hat{p}_{ij} = \frac{1}{n_j} \hat{p}_{ij}$$

Substituting back in into equation (B.1) gives equation (23) in the main text.

To derive equation (24), we log differentiate the price index $\varepsilon_j$ with respect to income $i$ from equation (22):

$$\varepsilon_j = \frac{\gamma_j}{q_j} \left( 1 - \frac{\beta_j}{\sum_{j \in J} \beta_j} \right)$$

$$\varepsilon_j = - \frac{\varepsilon_j - 1}{\varepsilon_j} \eta_{qj, i\hat{e}}$$

With $\eta_{qj, i}$ the elasticity of $q_j$ wrt income $i$ given by:

$$\eta_{qj, i} = \frac{\beta_j i}{\beta_j + \beta_j \sum_{j \in J} \gamma_j p_j - \gamma_j p_j \sum_{j \in J} \beta_j}$$

There is also an indirect effect of $i$ on $q_j$, when $i$ is such that the budget set is extended.

It is straightforward to show that this effect is 0 (derivation available upon request). $\hat{\varepsilon}_{ij}$ can easily be derived from $\hat{\varepsilon}_j$ by log differentiating equation (23) wrt $\varepsilon_j$:

$$\hat{\varepsilon}_{ij} = \frac{\varepsilon_j}{\varepsilon_{ij} n_j} \hat{\varepsilon}_j$$  \hspace{1cm} (B.2)

Log differentiating the zero profit condition (ZP) and the labor market equilibrium
(LME), equations (26) and (28) wrt $q_{ij}$ and $n_j$, we proceed as follows. First, the expression for $\varepsilon_{ij}$ in equation (23) is log differentiated wrt $q_{ij}$ and $n_j$:

$$\hat{\varepsilon}_{ij} = \frac{\sigma + 1 - 2\varepsilon_j}{n_j\varepsilon_{ij}} \hat{n}_j - \frac{\varepsilon_j - 1}{n_j\varepsilon_{ij}} \hat{q}_{ij}$$

(B.3)

Log differentiating the ZP, equation (26), wrt $q_{ij}$ and $\varepsilon_{ij}$ yields:

$$\hat{q}_{ij} = \frac{\varepsilon_{ij}}{\varepsilon_{ij} - 1} \hat{\varepsilon}_{ij}$$

(B.4)

Substituting equation (B.3) into (B.4), one gets equation (29) in the main text:

$$\hat{q}_{ij} = \frac{\sigma + 1 - 2\varepsilon_j}{n_j(\varepsilon_{ij} - 1) + \varepsilon_j - 1} \hat{n}_j$$

(B.5)

Log differentiating the LME, equation (28), wrt $q_{ij}$ and $n_j$ leads to the following expression:

$$\frac{a_{ij}q_{ij}}{a_{ij}q_{ij} + f_{ij}} \hat{q}_{ij} + \hat{n}_j = -(\varepsilon_j - 1) \hat{p}_j$$

(B.6)

Hence, we need an expression for $\hat{p}_j$. $p_j$ can be written as:

$$p_j = n_j^{\frac{1}{1-\sigma}} p_{ij} = n_j^{\frac{1}{1-\sigma}} \frac{\varepsilon_{ij}}{\varepsilon_{ij} - 1} a_{ij}$$

(B.7)

Log differentiating equation (B.7) wrt $q_{ij}$ and $n_j$, using equation (B.4), we get:

$$\hat{p}_j = \frac{1}{\sigma - 1} \hat{n}_j - \frac{1}{\varepsilon_{ij} - 1} \hat{\varepsilon}_{ij} = \frac{1}{\sigma - 1} \hat{n}_j - \frac{1}{\varepsilon_{ij}} \hat{q}_{ij}$$
Substituting equation (B.7) into equation (B.6) gives:

$$a_{ij}q_{ij} + \frac{a_{ij}q_{ij}}{a_{ij}q_{ij} + f_{ij}} \hat{n}_j = -\frac{\varepsilon_j - 1 - \varepsilon_j}{\varepsilon_j - 1} \hat{n}_j + \frac{\varepsilon_j - 1}{\varepsilon_j - 1} \hat{q}_{ij}$$

(B.8)

The coefficient before $\hat{q}_{ij}$ in equation (B.8) can be rewritten using

$$\frac{\varepsilon_j - 1}{\varepsilon_j - 1} = \frac{q_{ij}a_{ij}}{q_{ij}a_{ij} + f_{ij}}$$

leading to the following expression:

$$\hat{q}_{ij} = -\frac{\varepsilon_j - 1 + \sigma - 1}{\varepsilon_j - 1} \hat{n}_j$$

Using $\varepsilon_{ij} - \varepsilon_j = (\sigma - \varepsilon_j) \frac{n_j - 1}{n_j}$, we get as final expression:

$$\hat{q}_{ij} = -\frac{n_j}{n_j - 1} \frac{\varepsilon_{ij} (\varepsilon_j - 1 + \sigma - 1)}{(\varepsilon_{ij} - \varepsilon_j) (\sigma - 1)} \hat{n}_j$$

(B.9)

Next, we log differentiate equations (26) and (28) wrt $q_{ij}$, $n_j$ and $i$. This gives the following two expressions:

$$\hat{q}_{ij} = \frac{\sigma + 1 - 2\varepsilon_j}{n_j (\varepsilon_{ij} - 1) + \varepsilon_j - 1} \hat{n}_j - \frac{\varepsilon_j - 1}{n_j (\varepsilon_{ij} - 1)} \eta_{q_{ij}, i}$$

(B.10)

$$\hat{q}_{ij} = -\frac{n_j}{n_j - 1} \frac{\varepsilon_{ij} (\varepsilon_j - 1 + \sigma - 1)}{(\varepsilon_{ij} - \varepsilon_j) (\sigma - 1)} \hat{n}_j + \frac{n_j}{n_j - 1} \frac{1}{\sigma - \varepsilon_j} (\eta_{q_{ij}, i} - 1) \hat{i}$$

(B.11)

Combining the two equations and writing $\hat{q}_{ij}$ as a function of $\hat{i}$ generates equation (31) in the main text.
Appendix B.2  Open Economy

To show that the ZP in the open economy is upward sloping in \((q_{ij}, n_j)\) space, we start by log differentiating the ZP condition in the open economy, equation (41), wrt \(q_{ijk}, A_{jlk}, \varepsilon_{ijk}\) and \(\varepsilon_{ijl}\) generating:

\[
\frac{q_{ijk}}{\varepsilon_{ijk} - 1} \left( \hat{\varepsilon}_{ijk} - \frac{\varepsilon_{ijk}}{\varepsilon_{ijk} - 1} \hat{\varepsilon}_{ijk} \right) + \frac{q_{ijl} A_{jlk}}{\varepsilon_{ijl} - 1} \left( \hat{\varepsilon}_{ijl} - \frac{\varepsilon_{ijl}}{\varepsilon_{ijl} - 1} \hat{\varepsilon}_{ijl} \right) = 0
\]

Rewriting gives:\(^{11}\)

\[
\hat{q}_{ijk} = -\frac{a_{ij} q_{ijk}}{f_{ij}} \left( \frac{\varepsilon_{ijk}}{(\varepsilon_{ijk} - 1)^2} \hat{\varepsilon}_{ijk} - \frac{A_{jlk}}{\varepsilon_{ijl} - 1} \hat{A}_{jlk} + \frac{A_{jlk} \varepsilon_{ijl}}{(\varepsilon_{ijl} - 1)^2} \hat{\varepsilon}_{ijl} \right)
\]

Next, log differentiating equations (39) and (40) wrt \(q_{ijk}, n_j\) and \(A_{jlk}\), one finds:

\[
\hat{\varepsilon}_{jk} = -\frac{\varepsilon_{jk}}{\varepsilon_{jk} - 1} \left( \hat{n_j} + \hat{q}_{ijk} \right)
\]

\[
\hat{\varepsilon}_{jl} = -\frac{\varepsilon_{jl}}{\varepsilon_{jl} - 1} \left( \hat{n_j} + \hat{q}_{ijk} \right) - \frac{\varepsilon_{jl} - \varepsilon_{jk}}{\varepsilon_{jl}} \frac{1}{1 - A_{jlk}} \hat{A}_{jlk}
\]

And log differentiating equations (32) and (33) using (B.13) and (B.14) wrt \(q_{ijk}, n_j, \varepsilon_{jk}\) and \(\varepsilon_{jl}\):

\[
\hat{\varepsilon}_{ijk} = \sigma + 1 - 2 \varepsilon_{jk} \hat{n_j} - \varepsilon_{jk} - 1 \frac{\varepsilon_{ijk} - 1}{\varepsilon_{ijk} \hat{q}_{ijk}}
\]

\[
\hat{\varepsilon}_{ijl} = \sigma + 1 - 2 \varepsilon_{jl} \hat{n_j} - \varepsilon_{jl} - 1 \frac{\varepsilon_{ijl} - 1}{\varepsilon_{ijl} \hat{q}_{ijk}} - \frac{1}{\varepsilon_{ijl}} \frac{\varepsilon_{ijl} - \varepsilon_{jk}}{1 - A_{jlk}} \hat{A}_{jlk}
\]

\(^{11}\)Use equation (41)
Finally log differentiating equation (35) wrt $\varepsilon_{ijl}$ and $\varepsilon_{ijk}$ generates:

$$\Delta_{jk} = \frac{\varepsilon_{ijl}}{\varepsilon_{ijl} - 1} \varepsilon_{ijl} - \frac{\varepsilon_{ijk}}{\varepsilon_{ijk} - 1} \varepsilon_{ijk}$$  \hspace{1cm} (B.17)

Next, we can substitute equation (B.17) in equation (B.16) to solve for $\varepsilon_{ijl}$ as a function of $\varepsilon_{ijk}$ and $q_{ijk}$ and $n_j$:

$$\varepsilon_{ijl} = \frac{1}{1 + \frac{1}{n_j (\varepsilon_{ijl} - 1)} \frac{\varepsilon_{ijl} - \varepsilon_{ijk}}{1 - A_{jk} \varepsilon_{ijl}}} \left( \frac{\sigma + 1 - 2 \varepsilon_{ijl}}{\varepsilon_{ijl} - 1} \varepsilon_{ijl} - \frac{\varepsilon_{ijl} - 1}{n_j \varepsilon_{ijl}} q_{ijk} + \frac{1}{n_j \varepsilon_{ijl}} \left( \frac{\varepsilon_{ijl} - \varepsilon_{ijk}}{\varepsilon_{ijl} - 1} \varepsilon_{ijl} \right) \right)$$  \hspace{1cm} (B.18)

Now we substitute equation (B.18) back into (B.17) to solve for $A_{jk}$:

$$\Delta_{jk} = \frac{\varepsilon_{ijl}}{\varepsilon_{ijl} - 1} \left( 1 + \frac{1}{n_j (\varepsilon_{ijl} - 1)} \frac{\varepsilon_{ijl} - \varepsilon_{ijk}}{1 - A_{jk} \varepsilon_{ijl}} \right) \varepsilon_{ijl} - \frac{\varepsilon_{ijk}}{\varepsilon_{ijk} - 1} \varepsilon_{ijk}$$  \hspace{1cm} (B.19)

Next, equations (B.18) and (B.19) are substituted back into equation (B.12), which gives after several steps the following equation:

$$\Delta_{ijl} = \frac{\varepsilon_{ijl}}{\varepsilon_{ijl} - 1} \left( 1 + \frac{1}{n_j (\varepsilon_{ijl} - 1)} \frac{\varepsilon_{ijl} - \varepsilon_{ijk}}{1 - A_{jk} \varepsilon_{ijl}} \right) \varepsilon_{ijl} - \frac{\varepsilon_{ijk}}{\varepsilon_{ijl} - 1} \varepsilon_{ijk}$$  \hspace{1cm} (B.20)

Substituting equation (B.15) into (B.21) finally leads to equation (46) in the main text:

$$\Delta_{ijl} = \frac{\varepsilon_{ijl}}{\varepsilon_{ijl} - 1} \varepsilon_{ijl}$$  \hspace{1cm} (B.21)

$$\Delta_{ijl} = \frac{\varepsilon_{ijl}}{\varepsilon_{ijl} - 1} \varepsilon_{ijl}$$  \hspace{1cm} (B.22)
Next, we log differentiate the LME, equation (45) wrt $q_{ijk}$, $n_j$, $A_{jlk}$, $p_{ijkl}$ and $B_{jlk}$:

$$\frac{a_{ij} (q_{ijk} (1 + A_{jlk}))}{(a_{ij} q_{ijk} (1 + A_{jlk}) + f_{ij})} \left( q_{ijk} + \frac{A_{jlk}}{1 + A_{jlk}} A_{jlk} \right) + \hat{n}_j = - (\hat{\varepsilon}_{ijk} - 1) \hat{p}_{ijk} + \frac{B_{jlk}}{1 + B_{jlk}} \hat{B}_{jlk}$$  \hspace{1cm} (B.23)

Log differentiating the expression for $B_{jlk}$, given in equation (36) gives:

$$\hat{B}_{jlk} = \hat{\varepsilon}_{ijl} - \hat{\varepsilon}_{ijkl}$$ \hspace{1cm} (B.24)

Substituting equation (B.24) and (B.17) and rewriting equation (B.23) leads after some steps to:

$$\frac{a_{ij} (q_{ijk} (1 + A_{jlk}))}{(a_{ij} q_{ijk} (1 + A_{jlk}) + f_{ij})} \left( q_{ijk} + (\varepsilon_{ijk} - 1) p_{ijk} \right) = \frac{p_{ijkl} q_{ijkl} - a_{ijl} q_{ijkl} \hat{\varepsilon}_{ijkl} w_k}{(a_{ij} (q_{ijk} + q_{ijkl}) + f_{ij}) w_k} \hat{\varepsilon}_{ijkl}$$ \hspace{1cm} (B.25)

$p_{ijk}$ can be calculated from equation (37) as:

$$\hat{p}_{ijk} = \frac{1}{\sigma - 1} \hat{n}_j - \frac{1}{\varepsilon_{ijkl} - 1} \hat{\varepsilon}_{ijkl}$$ \hspace{1cm} (B.26)

Substituting (B.26) into (B.25) generates:

$$\frac{a_{ij} (q_{ijk} (1 + A_{jlk}))}{(a_{ij} q_{ijk} (1 + A_{jlk}) + f_{ij})} \left( q_{ijk} + \frac{(\varepsilon_{ijk} - 1) \hat{n}_j}{\sigma - 1} \right) = \left( \frac{\varepsilon_{ijkl} - 1}{\varepsilon_{ijkl} - 1} \right) \frac{p_{ijkl} q_{ijkl} - a_{ijl} q_{ijkl} \hat{\varepsilon}_{ijkl} w_k}{(a_{ij} (q_{ijk} + q_{ijkl}) + f_{ij}) w_k} \hat{\varepsilon}_{ijkl}$$ \hspace{1cm} (B.27)
Solving for $\varepsilon_{ijk}$ from equation (B.21) and substituting into equation (B.27) leads after several steps to:

$$q_{ijk} = \frac{\varepsilon_{ijk} - 1}{\varepsilon_{ijl} - 1} a_{ij} q_{ijl}$$

We can rewrite the first term between brackets on the RHS of equation (B.28) as follows.

Rewrite equation (41) as:

$$q_{ijk} + \frac{\varepsilon_{ijk} - 1}{\varepsilon_{ijl} - 1} q_{ijl} = \frac{f_{ij}}{a_{ij}} (\varepsilon_{ijk} - 1) \quad (B.29)$$

Solving for $\varepsilon_{ijk} - 1$:

$$\varepsilon_{ijk} - 1 = \frac{1}{f_{ij}} \left( a_{ij} q_{ijk} + \frac{\varepsilon_{ijk} - 1}{\varepsilon_{ijl} - 1} a_{ij} q_{ijl} - \frac{\varepsilon_{ijk} - 1}{\varepsilon_{ijl} - 1} a_{ij} q_{ijkl} \right)$$

Hence, we get for the numerator of the first term of (B.28):

$$a_{ij} q_{ijk} + \frac{\varepsilon_{ijk} - 1}{\varepsilon_{ijl} - 1} a_{ij} q_{ijl} = (\varepsilon_{ijk} - 1) f_{ij} + \frac{\varepsilon_{ijk} - 1}{\varepsilon_{ijl} - 1} a_{ij} q_{ijkl} \quad (B.30)$$

Next, for the denominator of the first term between brackets on the RHS of equation (B.28), we use:

$$\varepsilon_{ijk} = \frac{1}{f_{ij}} \left( a_{ij} q_{ijk} + a_{ij} q_{ijl} - \frac{\varepsilon_{ijl} - \varepsilon_{ijk}}{\varepsilon_{ijl} - 1} a_{ij} q_{ijl} + f_{ij} \right)$$

Implying:

$$a_{ij} q_{ijk} + a_{ij} q_{ijl} + f_{ij} = f_{ij} \varepsilon_{ijk} + \frac{\varepsilon_{ijl} - \varepsilon_{ijk}}{\varepsilon_{ijl} - 1} a_{ij} q_{ijl} \quad (B.31)$$
After some tedious algebra it can be shown that dividing (B.30) by (B.31) reduces to \( \frac{\varepsilon_{ijk}^{-1}}{\varepsilon_{ijk}} \):

\[
\frac{a_{ij}q_{ijk} + \frac{\varepsilon_{ijk}^{-1}}{\varepsilon_{ijl}}a_{ijl}q_{ijkl}}{(a_{ij}q_{ijk} (1 + A_{jik}) + f_{ij})} = \frac{\varepsilon_{ijk} - 1}{\varepsilon_{ijk}}
\]

Hence, equation (B.28) reduces to:

\[
\hat{q}_{ijk} = -\frac{\varepsilon_{ijk}}{\varepsilon_{ijk} - \varepsilon_{jk}} \frac{\sigma - 1 + \varepsilon_{jk} - 1}{\sigma - 1} \hat{n}_j
\]

Using \( \varepsilon_{ijk} - \varepsilon_{jk} = (\sigma - \varepsilon_{jk}) \frac{\varepsilon_{ijk}^{n_j-1}}{n_j} \), we get equation (47) in the main text:

\[
\hat{q}_{ijk} = \frac{n_j}{n_j - 1} \frac{\sigma - 1 + \varepsilon_{jk} - 1}{(\sigma - \varepsilon_{jk}) (\sigma - 1)} \hat{n}_j
\]