Trade liberalisation, heterogeneous firms and endogenous investment

Gonzague Vannoorenberghe *
University of Mannheim

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Abstract

This paper develops a Melitz (2003) type model where heterogeneously productive firms can decide on a level of investment in process innovation in order to increase their productivity. This intensive investment decision, which is in this form new in the literature, gives additional insights about the empirical relationship between firm size, export status and investment, while preserving the qualitative results of the original Melitz model. Trade liberalisation affects firms’ choices in different ways depending on their export status and on the investment technology. In addition, the model can address three recent empirical “puzzles” about standard heterogeneous firm models: (i) the negative relationship between firm size and Tobin’s Q (Nocke and Yeaple (2006)) (ii) the change in the skewness of the size distribution of firms when trade costs decrease (Nocke and Yeaple (2006)) (iii) the increase in the mass of consumed varieties following trade liberalisation (Baldwin and Forslid (2006)). I show that the properties of the investment technology can play an important role in their explanation.

Keywords: Process innovation, Firm heterogeneity, Trade liberalisation

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*Department of Economics, University of Mannheim, L7, 3-5 Mannheim, D-68131, Germany. Phone: +49 621 181 1797. e-mail: gonzague@uni-mannheim.de
1 Introduction

Models of heterogeneous firms have, in the last years been at the core of most theoretical developments in international trade. Their popularity is based on their ability to match a number of well-established stylised facts linking firm characteristics, such as size or productivity, to their export behaviour. These models moreover provide a new rationale for gains from trade. They suggest that trade liberalisation leads to a reallocation of productive factors from inefficient firms, which exit the market, to efficient firms, which export more. This source of gains from trade, which increases the productivity of the economy, has been confirmed by many studies\footnote{see among others Pavcnik (2002), Trefler (2004) and Bernard et al. (2006).}. However, empirical evidence points to another channel of productivity gains that occurs within firms, and not only through a reallocation of factors between firms as explained by Melitz (2003). This suggests that firms can affect their productivity by investment, which is the purpose of this paper.

I develop a model à la Melitz (2003) in which heterogeneously productive firms can decide on a level of investment in process innovation in order to increase their productivity. This decision is continuous in the sense that each firm decides how much to spend and is not restricted to a binary decision between investment and no investment. The first contribution of this paper is to examine the investment decision of all firms and how trade liberalisation affects it. Under minimal assumptions for a well-behaved problem, I obtain a number of predictions about investment and firm characteristics that are empirically well established. Larger exporting firms invest more than average and firms which enter the export market scale up their investment. Following a marginal trade liberalisation, small, non-exporting firms decrease their investment since the competition of foreign firms makes it more difficult for them to recoup the costs of investment. Large, exporting firms on the other hand find it profitable to invest more as they scale up their sales on the export market. The second contribution of this paper is methodological, in that I show that the main results of Melitz (2003) are unaffected by the introduction of a continuous investment decision. The changes in the cut-off levels, price index, or welfare are qualitatively similar to these of Melitz (2003).

Third, I examine a number of recent puzzles to the Melitz (2003) framework and show how the model can address them. Indeed, a number of empirical facts are still at odds with the Melitz (2003) model. (i) Nocke and Yeaple (2006) argue that trade liberalisation has reduced the skewness of the domestic size distribution of firms. They find that a decrease in trade costs does
not affect all firms proportionately, and that the size differential between two given firms tends to decrease. In the original Melitz (2003) model, all firms face the same proportional reduction in their domestic sales\(^2\), which makes it impossible to explain this fact. (ii) Nocke and Yeaple (2006) also show that the relationship between the Tobin’s Q of a firm and its size is empirically negative, and not positive as suggested by a straightforward extension of the standard model. (iii) Aguir (2007) and Baldwin and Forslid (2006) point out that in Melitz (2003), under the commonly used Pareto distribution, the mass of consumed varieties decreases in both countries following trade liberalisation. This is at odds with empirical evidence as presented by Feenstra (2006) and Broda and Weinstein (2006). I show in the present framework that the sensitivity of investment relative to firm size, which is a direct consequence of the properties of the investment function, can play a substantial role in explaining the puzzles. Indeed, if trade liberalisation induces firms of different sizes to adapt their optimal investment level in different proportions, the above puzzles can be - at least partially - explained.

This model relates to the rapidly growing literature on trade with heterogeneous firms, which follows the seminal contribution of Melitz (2003). A number of recent papers have allowed firms to take an additional investment decision in order to improve their productivity. Yeaple (2005), Ekholm and Midelfart (2005), Bustos (2005) or Navas and Sala (2007) allow firms to choose between two different production technologies: a low productivity, low cost technology, and a high productivity technology, which necessitates the payment of high fixed costs. This generates an endogenous correlation between exporter status and investment, in which large exporting firms find it profitable to invest, while small unproductive firms do not invest in technology. However, a discrete investment technology is too restrictive to explain a number of observed facts. It appears that following trade liberalisation, the largest exporters continue to scale up their spending in innovation\(^3\), which should not be the case if they had already adopted the high technology. The effects on the size distribution or on the investment intensity of firms as a function of size are also much poorer in that type of model, and insufficient to explain the aforementioned puzzles.

Costantini and Melitz (2007) develop a model of discrete investment decision in a dynamic framework, and obtain a similar intuition, but their focus is on the dynamic adjustment of firms to trade liberalisation, and on the

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\(^2\)The only channel that affects domestic sales is the decrease of the price index, which has the same proportional impact on all firms.

\(^3\)Table 7 in Bustos (2007) suggests that among the group of initial exporters, the largest scale up their spending in technology faster than others.
causality between export and investment decision. Ederington and McCalman (2006) use a dynamic model of trade liberalisation with ex-ante identical firms which can choose between two technologies. The use of the high technology gradually diffuses among firms, at a speed affected by trade liberalisation. The time dimension introduces an element of continuity, which makes their approach slightly closer to mine, but they restrict their attention to the speed of diffusion of technology. Their use of a discrete investment decision however bears similar problems to those mentioned earlier.

Van Long et al. (2007) and Atkeson and Burstein (2007) provide to my knowledge the only two models making the investment decision of firms continuous. The first is very different from the present setup, since it assumes strategic interaction where the investment decision is made before knowing the productivity draw. The second is however much closer to the present model, as it builds on Melitz (2003) with a continuous investment possibility in a dynamic framework. They however concentrate on different questions and make a strong assumption about the functional form of the technology, which makes the returns of process innovation proportional to firm profits. This is a special case of my - in this respect - more general formulation, which prevents them to obtain similar results to mine on the size distribution of firms or on the mass of consumed varieties.

In section 2, I present a closed economy version of the model to explain the basic mechanisms at stake. In section 3, I extend the model to two symmetric countries and examine the consequences of trade liberalisation. Section 4 concludes.

2 The closed economy

2.1 Demand

The representative consumer has a C.E.S. utility function over a continuum of varieties:

\[ U = \left[ \int_{\omega \in \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} \]

(1)

where the set \( \Omega \) represents all available varieties, \( q(\omega) \) stands for the consumption of variety \( \omega \) and \( \sigma \) for the elasticity of substitution between varieties, assumed to be strictly greater than one. The maximisation problem of the
consumer yields the following demand function for a variety \( \omega \):

\[
q(\omega) = \left( \frac{p(\omega)}{P} \right)^{-\sigma} Q
\]

(2)

where \( Q \) is an index of consumption \( Q \equiv U \) and \( P \) the ideal price index, i.e. the price of bundle \( Q \), defined as:

\[
P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}
\]

(3)

The consumer’s income consists exclusively of the proceeds of his labour, paid at a wage normalised to one. Labour supply \( L \) is inelastic, and is an index of the size of the economy. The aggregate budget constraint is therefore:

\[
PQ = L
\]

(4)

### 2.2 Production

There is a continuum of firms, each producing a different variety with a production technology using exclusively labour. Firms are heterogeneous with respect to a productivity parameter \( z > 0 \), drawn from a continuous distribution \( G(z) \) with support \( (0, Z] \) where \( Z \to \infty \). A firm drawing a large \( z \) has an advantage in the sense that it can, other things equal, produce with lower marginal costs. Upon learning its productivity parameter, the firm decides how much to invest in process innovation \( (i) \) in order to reduce its marginal costs. The production function of a firm having drawn \( z \) is as follows:

\[
y = zlt(i)^{\frac{1}{\sigma-1}}
\]

(5)

where \( l \) is the amount of labour used for production and the function \( t(i)^{\frac{1}{\sigma-1}} \) is the investment technology, linking the amount invested in process innovation \( i \) to its impact on marginal costs. \( t(i) \) is taken to the power \( \frac{1}{\sigma-1} \) for simplicity. I assume that the function \( t(i) \), defined on the positive reals, has the following properties:

\[
t(i) > 0 \quad t'(i) > 0, \quad t(0) = 0
\]

The last assumption is made to ensure that all producing firms invest a positive amount in process innovation, which simplifies the analysis\(^4\). This

\(^4\)Having firms producing without investing would require to deal with an additional cutoff level for the investment status.
does not seem implausible if one thinks of $i = 0$ as a case in which a manager would not even give a thought about the organisation of production in his plant.

This is in a sense the opposite assumption to that made in models where firms choose between discrete technology choices. Bustos (2005) among others assumes that the fixed costs of investing in the high technology are so high that only the most productive among exporting firms do invest. This is necessary due to the assumption of two discrete technologies in order to generate the result that trade liberalisation favours investment by exporting firms, but is not required in the present setup with continuous decision.

2.2.1 Timing

The timing of the model is as follows. In a first stage, there is an unbounded mass of entrepreneurs, who decide whether they want to enter the market or not. As in Melitz (2003), entering the market means paying a labour sunk cost $f_e$ in order to obtain a draw of the parameter $z$. I add here a second stage in which the firm decides how much to invest. In a third stage, the firm sets its price and decides how much to produce.

This timing is realistic, since the investment decision is usually made before the employment decision, and has been chosen for clarity of exposition. However, a simultaneous investment and production decision would yield the same results.

Firms live for a single period, which is another difference to Melitz (2003), who considers the effects of exogenous death and endogenous entry of firms. This dynamic dimension would however only complicate the matter in the present framework without adding much insight\(^5\).

2.2.2 The optimisation problem of the firm

In the third stage, the firm sets its optimal price given its investment decision. Since there is a continuum of firms, there are no strategic interactions, and each firm optimises given the market conditions summarised by the price index $P$. As is well-known in monopolistic competition models, the optimal pricing decision is a fixed markup over marginal costs:

$$p(z) = \frac{\sigma}{\sigma - 1} \frac{t(i)^{1-\sigma}}{z}$$

\(^5\)Helpman et al. (2004) also use a one-period model for simplicity.
Using this optimal price in the demand equation (2) yields:

\[ q(z) = \left( \frac{\sigma}{\sigma - 1} \frac{t(i)^{1/\sigma}}{zP} \right)^{-\sigma} Q \]  

(7)

By backward induction, in stage 2, the optimal choice of \( i \) should maximise profits, which, using (4), are given by:

\[ \pi_d(z) = A(zP)^{\sigma - 1} t(i)L - i - f \]  

(8)

where \( A \equiv \frac{1}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \). Choosing a high level of investment allows a firm to charge a lower price and therefore to sell more, but also requires the payment of higher fixed costs. \( f \) is the fixed cost component of production, which is common to all producing firms.

The first order condition for this problem is given by:

\[ A(zP)^{\sigma - 1} L t'(i_d) - 1 = 0 \]  

(9)

where \( i_d \) is the investment level solving the first order condition for firm \( z \).

To ensure that this condition yields a maximum, I assume that \( t''(i) < 0 \). Furthermore, to ensure the existence of a solution for any \( z \in [0, \infty) \):

**Assumption 1** \( \lim_{i \to 0} t'(i) = \infty \) and \( \lim_{i \to \infty} t'(i) = 0 \)

The first order condition (9) defines the optimal level of investment for a firm with productivity \( z \), which I will denote \( i_d(z) \). It appears that firms invest more (i) the higher the price index, (ii) the higher their own productivity \( z \). For a given firm, a higher price index or a high productivity means that it is relatively efficient in comparison to its competitors. The firm therefore sells higher quantities, which drives the returns of investment up. For a given \( P \), totally differentiating the F.O.C. with respect to \( i_d \) and \( z \) allows to compare the optimal investment of firms having drawn different \( z \):

\[ d i_d \frac{t''(i)}{t'(i)} \bigg|_{i=i_d} + (\sigma - 1) \frac{dz}{z} = 0 \]  

(10)

where \( \frac{t''(i)}{t'(i)} \) is the sensitivity of optimal investment to a change in the conditions faced by the firm. This establishes the first important result:
**Proposition 1** The optimal investment of a firm is increasing in its size.

The proof is as follows: from (10), it is immediate that a firm with a high $z$ will optimally invest more. It follows that $t(i_d(z))$ is increasing in $z$, and that, from (7), the size of a firm - whether defined as domestic sales or labour employed - is increasing in $z$. ■

**Lemma 1** The function $i_d: [0, \infty) \rightarrow [0, \infty)$ is bijective

This follows directly from Assumption 1 and (10).

Using the F.O.C., I rewrite the variable profits (VP) - given by the first term in (8) and defined as the sales minus the costs of labour used for production - as:

$$VP(z) = \frac{t(i_d(z))}{t'(i_d(z))} \equiv \frac{i_d(z)}{\epsilon(i_d(z))}$$

(11)

where $\epsilon(i) \equiv \frac{it'(i)}{t(i)}$ is the elasticity of the $t(i)$. $VP(z)$ is increasing in $z$ from the concavity of $t(i)$ and (10). I impose some additional regularity conditions on $\epsilon(i)$ to make sure that very productive firms make positive profits (i), and to simplify the interpretation (ii).

**Assumption 2**

(i) $\epsilon(i) < 1 - \mu$ for $\mu$ small and for all $i$

(ii) $\epsilon(i)$ is monotonic over the whole range of $i$ and bounded away from zero.

### 2.3 The cutoff productivity level

I define $z^*$ as the cutoff level of $z$ for which, given the optimal investment decision $i_d(z^*)$, the firm breaks even when it produces, i.e., it makes zero profits given its optimal investment decision. Using (11):

$$\frac{t(i_d(z^*))}{t'(i_d(z^*))} - i_d(z^*) - f = 0$$

(12)

**Lemma 2**

1. Under Assumption 1, the optimal level of investment of the cutoff firm $z^*$ is uniquely determined, and is independent of the level of $z^*$.  

2. There exists a strictly positive and unique cutoff level \( z^* \) such that all firms with \( z > z^* \) make strictly positive profits, and all firms with \( z < z^* \) decide to exit the market.

**Proof.** See Appendix ■

Part 1 of the lemma is a useful property, which allows to express \( i_d(z^*) \) independently of \( z^* \). The intuition for this result is that the optimal decision of a firm depends on its relative productivity compared to its competitors. Therefore, the least productive producing firm bases its decision on its rank, not on its absolute productivity level \( z^* \). Thus, its optimal investment level is independent of its exogenously efficiency draw \( z^* \). For convenience, I will therefore define: \( i^* \equiv i_d(z^*) \).

I now turn to general the equilibrium effect of the model. It is of particular interest to express the ideal price index \( P \) as a function of the cutoff firm \( z^* \). From (12):

\[
P^{\sigma - 1} = \frac{f + i^*}{\sigma - 1} A L t(i^*)^{z^* - \sigma}
\]

Since \( i^* \) is constant from Lemma 2, the price index is inversely proportional to the cutoff productivity level \( z^* \). This is qualitatively similar to the original Melitz (2003) model, and greatly simplifies the welfare analysis in the open economy case, which will only depend on the change in \( z^* \).

Two additional conditions are required to close the model. First, there is free entry of entrepreneurs in the first stage. An equilibrium therefore requires that the sunk cost of entry be equal to the expected profits:

\[
\int_{z^*} \pi_d(z) dG(z) = f_e
\]

where the subscript \( o \) denotes optimised profits.

Second, the labour market must be in equilibrium:

\[
L = M \left[ \left( \int_{z^*} (\sigma - 1) \frac{i_d(z)}{\epsilon(i_d(z))} \right) + i_d(z) + f dG(z) \right] + f_e
\]

where \( M \) is the mass of entrepreneurs paying the sunk entry cost \( f_e \). The square bracket summarises the labour used for production purposes, for investment and for the payment of fixed and sunk costs.
2.4 Capital intensity, Tobin’s Q and investment intensity

Empirical studies have examined the relationship between the investment intensity of a firm (usually measured as R&D spending per worker) and its size or its export status. Bustos (2007)\(^6\) or Bernard and Jensen (1995) argue that bigger, exporting firms have a higher investment intensity, while others such as Aw et al. (2007) find the opposite result. In the present setup, the investment intensity of a firm is defined as the ratio of investment \(i\) to the variable profits \(\epsilon(i)\), which is therefore equal to \(\epsilon(i)\). The relationship between investment intensity and firm size is thus as follows:

**Proposition 2** The investment intensity is constant for all firms if \(\epsilon'(i) = 0\) and is increasing (decreasing) in firm size if \(\epsilon'(i) > 0\) (\(\epsilon'(i) < 0\)).

The fact that empirical studies obtain mixed results for the correlation between firm size and investment intensity is not surprising considering that different sectors\(^7\) are likely to have different investment technologies (and therefore different \(\epsilon(i)\) functions).

Nocke and Yeaple (2006) find empirical evidence that the Tobin’s Q of a firm is negatively related to its size, and argue that the introduction of capital in the Melitz (2003) model would predict the opposite relationship. For this statement, they consider an extended version of the Melitz (2003) model where fixed costs of production are paid in terms of capital, and where the production function is Cobb-Douglas with capital and labour. Without fixed costs, all firms would use the same fraction of capital in production, and the value of capital (the book value) would be a constant fraction of variable profits (the market value in a static framework) for all firms. The fixed costs paid in terms of capital however account for a higher share of the market value for small firms than for large firms, and therefore yields a Tobin’s Q that is increasing with size. This, they argue, runs counter to empirical evidence.

It is straightforward to introduce capital in the present setup under the assumption that it is held by foreigners, and available to all firms in the economy at an exogenous price \(r\). I assume that the fixed costs \((f + i)\) are paid in terms of capital so as to be in line with the interpretation of

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\(^6\)Tables 9 and 10

\(^7\)Aw et al. (2007) for example look at the electronics industry in Taiwan, while Bustos (2007) takes different sectors of Argentinean manufacturing together
Nocke and Yeaple (2006), but abstract from the Cobb-Douglas production function. The main difference between the present model and traditional heterogeneous firm models is that larger firms pay larger fixed costs, since they invest more. This is precisely the mechanism that allows me to reverse the relationship between size and Tobin’s Q as I will show next.

A firm with productivity $z$ uses the following amount of capital for its fixed costs:

$$K(z) = i_d(z) + f$$

(16)

So that the capital labour ratio of a firm $z$ is given by:

$$\frac{K(z)}{l(z)} = \frac{(i_d(z) + f)t'(i_d(z))}{r(\sigma - 1)t(i_d(z))}$$

(17)

and its Tobin’s Q:

$$T(z) = \frac{t(i_d(z))}{t'(i_d(z))(i_d(z) + f)} = \frac{1}{\sigma - 1} \frac{l(z)}{K(z)}$$

(18)

**Proposition 3** The Tobin’s Q is:

- increasing in size if $\epsilon'(i) \leq 0$
- increasing in size for small firms, and decreasing in size for larger firms if $\epsilon'(i) > 0$

**Proof.** See Appendix

The intuition for this result is similar to that of the investment intensity. The difference with Proposition 2 comes from the $f$ component of the fixed costs, which is equal for all firms, and has a proportionately large impact on the capital labour ratio of small firms. If $\epsilon'(i) < 0$, the effect of $f$ goes in the same direction as that of $i_d(z)$, namely to increase the Tobin’s Q with firm size. If $\epsilon'(i) > 0$ on the other hand, the effect of $f$ dominates for small firms, for which the Tobin’s Q increases in size. However, for larger firms, the effect of the disproportional increase in investment relative to variable profits dominates, and the Tobin’s Q decreases with size. This therefore suggests an inverted U-shape relationship between the Tobin’s Q and the size of a firm. This is closer to the empirical results of Nocke and Yeaple (2006), especially considering that most dataset do not cover the smallest firms for which the Tobin’s Q would increase with size.

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8Introducing a Cobb-Douglas production function would not alter the qualitative results
3 The open economy

3.1 The setup

I now turn to the open economy version of the model, and assume that the world consists of two symmetric countries, Home and Foreign. As in Melitz (2003), exporting firms face two kinds of additional costs: iceberg trade costs and fixed costs of exporting. I define iceberg trade costs $0 < \tau < 1$ as the fraction of goods that arrive at destination per unit shipped. The fixed costs of exporting $f_x$, paid in units of labour, can be thought of as the cost of establishing a brand on a foreign market, knowing a business law, a culture. A Home firm which exports to the foreign market chooses the following optimal pricing rule:

$$p_F(z) = \frac{\sigma}{\sigma - 1} \frac{t(i)^{1-\sigma}}{\tau z} = \frac{p_H(z)}{\tau} \quad (19)$$

where the subscripts $F$ and $H$ denote Foreign and Home. The firm charges the same markup as on its home market, due to identical preferences, but the variable costs are higher, so that the c.i.f. price charged in a foreign country is higher. Given the optimal price, and using the symmetry assumption between the two countries, the sales of a firm on the foreign market are given by:

$$q_F(z) = q_H(z)\tau^\sigma \quad (20)$$

Due to the higher price charged for exports, a Home firm sells less in Foreign than at Home. The higher the price elasticity of demand, the higher this effect. If it decides to export, a firm $z$ faces the following maximisation problem:

$$\max_i \pi_x(z) = p_H(z)q_H(z) + p_F(z)q_F(z) - \frac{t(i)^{1-\sigma}}{z}(q_H(z) + q_F(z)) - i - f - f_x \quad (21)$$

Plugging the optimal prices and quantities in the above problem, the profits of an exporting firm can be rewritten as:

$$\pi_x(z) = (1 + \tau^{\sigma-1})A(zP)^{\sigma-1}t(i)L - (i + f_x + f) \quad (22)$$

And the the first order condition for optimal investment if a firm export $i_x$ is given by:

$$(1 + \tau^{\sigma-1})A(zP)^{\sigma-1}t'(i_x)L - 1 = 0 \quad (23)$$
which defines a function $i_x(z)$ as the optimal investment decision of a firm with productivity $z$ if it decides to export. The optimal investment level of a firm having drawn $z$ is higher if it exports than if it does not. This is due to the fact that an exporting firm sells more than an identical firm which would only produce for the domestic market. This is therefore more profitable to save on the variable costs by investing in fixed costs. This result is conform to that of Bustos (2005) or Navas and Sala (2007).

A firm will decide to export if it makes higher profits by exporting than by producing only for its domestic market. In contrast to Melitz (2003) and the subsequent literature, the decision to export is here not taken independently of domestic considerations, because it influences the optimal level of investment, which in turn affects domestic profits. A firm having drawn $z$ will therefore export if the difference between the profits it makes when exporting and when not is positive:

$$A(zP)^{\sigma-1}L \left[ (1 + \tau^{\sigma-1})t(i_x(z)) - t(i_d(z)) \right] - i_x(z) - f_x + i_d(z) \geq 0$$  \hspace{1cm} (24)$$

where $i_d(z)$ and $i_x(z)$ respectively denote the optimal investment decision of a firm $z$ when it decides not to export, and when it decides to export.

**Proposition 4** For $f_x$ sufficiently high, there exists a unique cutoff firm $z_x^* > z^*$ which is indifferent between exporting and producing only for its domestic market. Firms having drawn a $z$ above this cutoff export, while firms having drawn a lower $z$ do not. The firm $z_x^*$ invests discretely more than the most productive non-exporting firm.

**Proof.** See Appendix ■

There are two main differences with the literature. First, the condition for coexistence of non-exporting and of exporting firms is stronger than in the standard Melitz (2003) framework, in the sense that $f_x$ should be sufficiently higher than $f$. This is due to the fact that when deciding to export, a firm scales up its investment and therefore its revenues on the domestic market at the same time. This makes it more profitable to export than in the standard Melitz framework, and requires higher fixed costs of exporting in order to ensure that some producing firms do not export. Second, there is a discrete jump in optimal investment at the cutoff export level. This can not be replicated by static discrete technology models, but goes in the direction of Costantini and Melitz (2007), who argue that following an unanticipated trade liberalisation, firms will simultaneously invest and enter the export market. I therefore obtain a similar insight on this account in the present static model as in dynamic frameworks, which are less tractable.
3.2 Trade liberalisation

In the following, I examine the effect of a decrease in the variable costs of exports on the level of investment of exporting and non-exporting firms. A decrease in the transport costs has an effect on the price index $P$ and therefore on the domestic cutoff level $z^*$, which should be taken into account. For this, I use the condition that expected profits are equal to the fixed entry costs:

$$E(\pi) = \int_{z^*}^{z^*} \pi_d(z)dG(z) + \int_{Z}^{Z} \pi_x(z)dG(z) = f_e$$  \hspace{1cm} (25)

I rewrite this condition using (8) and (22):

$$f_e = \int_{z^*}^{z^*} AP^{\sigma-1} Lz^{\sigma-1} t(i_d(z)) - i_d(z) - f dG(z)$$

$$+ \int_{z^*}^{Z} (1 + \tau^{\sigma-1}) AP^{\sigma-1} Lz^{\sigma-1} t(i_x(z)) - i_x(z) - f - f_x dG(z)$$  \hspace{1cm} (26)

An increase in $\tau$ has a direct positive impact on the profit level of exporting firms, and tends to raise expected profits. The cutoff $z^*$ therefore has to increase in order for the expected profits to remain constant\(^9\). This decreases the price index from (13), and increases welfare, as in Melitz (2003). Following a marginal change in $\tau$, the envelope theorem states that the subsequent change in the level of investment of all firms has a second order effect on expected profits, and can be neglected. The entry of new firms on the export market also has negligible effects on the expected profits, since these firms are indifferent between exporting or not.

The percentage change in the domestic cutoff level is higher the larger the proportional size of the exporting sector. Indeed, if most firms export, a change in the variable costs of exporting has a large positive impact on the expected profits, and the corresponding adjustment of $z^*$ must be high. This, and the effect of trade liberalisation on the optimal investment of each firm is summarized in the following proposition:

**Proposition 5**

1. A marginal decrease in variable trade costs raises the domestic cutoff level $z^*$ and therefore induces a selection effect.

\(^9\)The same mechanism as in Melitz (2003) is at stake here. The drop in the price index increases the real wages and drives the least productive firms out of the market.
2. A marginal decrease in variable trade costs decreases the optimal investment level of all firms that remain non-exporters.

3. A marginal decrease in variable trade costs induces an increase in the investment of exporting firms.

Proof. See Appendix

Two factors play a role in these results. First, the drop in the price index makes it more difficult for all firms to sell on a given market. This mechanically reduces the incentives to invest, and accounts for the effect on domestic firms. For exporting firms however, the decrease in the costs of exporting allows to sell more on the export market. This second effect dominates the drop in the price index, and raises their incentives to invest. The above proposition constitutes a major difference with the models in which the investment decision is discrete. In these frameworks, domestic firms would continue production with the low technology, while very productive firms, which already produce with the high technology would not invest more$^{10}$.

I now turn to the evolution of the cutoff export level $z^*_x$ following a marginal change in $\tau$. This firm is by definition indifferent between exporting or not. From (24):

$$A(z^*_x P)^{\sigma-1} L \left[(1 + \tau^{\sigma-1}) t(i_x(z^*_x)) - t(i_d(z^*_x))\right] - i_x(z^*_x) - f_x + i_d(z^*_x) = 0 \quad (27)$$

Using the envelope theorem, changes in $i_d(z^*_x)$ and $i_x(z^*_x)$ only have second order effects on the above equation, which is the difference between the profit levels of exporting and of non-exporting firms. A marginal rise in $\tau$ has a direct positive effect on the left hand side of the above equation, because lower trade costs make it more profitable to export relative to not exporting. It also has an indirect negative effect through the implied decrease in the price index $P$, which has a larger absolute impact for an exporting than for a non-exporting firm. As shown in Appendix, the direct effect dominates and a marginal trade liberalisation has the same impact on the export cutoff level as in Melitz (2003).

Proposition 6 A small reduction in the variable costs of trade decreases the export cutoff level $z^*_x$, and raises the proportion of exporting firms.

Proof. See Appendix

$^{10}$This last fact especially is at odds with empirical evidence. Bustos (2007) shows that among the group of initial Argentinean exporters in 1992, spending in technology has been increased by trade liberalisation with Brazil, the more so for the largest exporters.
3.3 Distribution of firm size

Nocke and Yeaple (2006) empirically find that, for U.S. firms, trade liberalisation has reduced the skewness of the distribution of the logarithm of domestic sales. In other words, the relative size differential between two given firms appears to have decreased due to trade liberalisation.

In the traditional Melitz (2003) model, a decrease in trade costs affects domestic sales through a change in the price index, which has the same proportional effect on the domestic sales of all firms. In the present model, I show that the effect of a reduction in trade costs on the size distribution is non monotonic: depending on $\epsilon'(i)$, the size differential between two non-exporting firms rises (decreases) while that between two exporting firms decreases (rises). This does not necessarily run counter to the evidence in Nocke and Yeaple (2006), since they do not allow for non-monotonicity in the reaction to trade liberalisation. However, the difference in log size between the group of exporters and of non-exporters should increase.

For the interpretation of the results in this and the following sections, it is useful to define the relative sensitivity of investment to external conditions as $E(i)$:

$$E(i) \equiv -\frac{-t'(i)^2}{t''(i)t(i)}$$

Indeed, as seen in (10), $-\frac{\epsilon'(i)}{P'(i)}$ can be interpreted as the sensitivity of optimal investment by a firm investing $i$. The ratio of this sensitivity to the variable profits, which is a measure of the relative sensitivity of investment, is $-\frac{\epsilon'(i)^2}{P''(i)t(i)}$. The higher the $E$, the higher will be the proportional adjustment of investment by a firm to changes in external conditions. The relationship between $E(i)$ and the investment intensity of a firm is given by the following lemma:

**Lemma 3** Under Assumptions 1 and 2, $\epsilon'(i)$ and $E'(i)$ have the same sign.

**Proof.** See Appendix

This states that if large firms are relatively investment intensive (i.e. if the technology is such that the optimal investment intensitiy rises with the investment level: $\epsilon'(i) > 0$), their investment will be relatively sensitive to a change in market conditions.
Domestic sales ($s$) of a firm with productivity $z$ are given by:

$$s_{dd}(z) = \sigma A(zP)^{\sigma-1}t(i_d(z))L \quad \text{for } z < z^*_x$$  \hspace{1cm} (29)

$$s_{dx}(z) = \sigma A(zP)^{\sigma-1}t(i_x(z))L \quad \text{for } z \geq z^*_x$$  \hspace{1cm} (30)

where $s_{dd}$ and $s_{dx}$ respectively stand for the domestic sales of a non-exporting and of an exporting firm.

Using the equation for the price index (13), I rewrite these quantities as:

$$s_{dk}(z) = \sigma (f + i^*) \frac{t(i_k(z))}{t(i^*)} \left( \frac{z}{z^*} \right)^{\sigma-1} \quad \text{for } k \in \{d, x\}$$  \hspace{1cm} (31)

The percentage change in domestic sales for non-exporting firms following a marginal trade liberalisation is given by:

$$\frac{d\ln(s_{dd}(z))}{d\tau} = \frac{dz^*}{d\tau} \frac{1 - \sigma}{z^*} \left[ E(i_d(z)) + 1 \right]$$  \hspace{1cm} (32)

There are two effects that influence the domestic sales of non-exporting firms following trade liberalisation. First, there is a direct effect of the price index on sales, as shown by 1 in the bracket. Trade liberalisation increases the average productivity of competitors, thereby decreasing the price index and the domestic sales of all non-exporting firms. Second, the reduction in sales drives the incentives to invest down, as shown by the first part of the bracket. This further reduces domestic sales. If $\epsilon'(i) = 0$ this implies by Lemma 3 that $E(i)$ is constant for all $i$. The same result as in the Melitz (2003) model therefore holds, and the log of domestic sales of all non-exporting firms changes by exactly the same amount. In this case, the change in optimal investment by non-exporting firms of all sizes is such that the quantity they sell decreases by the same proportion. However, if $\epsilon'(i) < 0$, the drop in sales is more pronounced for the smallest of the non-exporting firms, while the larger non-exporting firms see their sales decrease by a lesser fraction. This is due to the investment technology, which is such that small firms are proportionately more reactive in their investment decision than larger firms. Under this assumption, the size differential between non-exporting firms increases. $\epsilon'(i) > 0$ would yield opposite consequences.

For exporting firms:

$$\frac{d\ln(s_{dx}(z))}{d\tau} = (\sigma - 1) \left[ -\frac{dz^*}{d\tau} \frac{1}{z^*} + \left( \frac{\tau^{\sigma-2}}{1 + \tau^{\sigma-1}} - \frac{dz^*}{d\tau} \frac{1}{z^*} \right) E(i_x(z)) \right]$$  \hspace{1cm} (33)
Again, the direct and the indirect effects mentioned above play a role here. The increase in the price index also affects exporting firms and makes it more difficult for them to sell on the domestic market. However, exporting firms benefit from a decrease in the transport costs on their export market, which raises their sales abroad, and therefore their investment and productivity on their home market (from Proposition 5). Which of the two effects dominates is unclear, so that the sign of the change in domestic sales remains undetermined for exporting firms. However, the group of exporters should see its domestic sales decrease proportionately less (or even increase) than that of non-exporting firms. This prediction cannot be obtained in the existing literature on heterogeneous firms.

The elasticity of the technology function plays again a central role in the determination of the skewness of the sales distribution. A constant \( E(i) \) yields the same proportional change in domestic sales by all exporting firms, although it is worth noting that this change is not equal to that of domestic firms. If \( E'(i) < 0 \), size differentials between exporting firms become smaller, since the smaller exporters scale up their investment proportionally more than larger exporters. A decreasing \( E(i) \) has the opposite effect.

These effects are summarised in the following Proposition:

**Proposition 7** The effect of trade liberalisation on the size distribution of firms depends on the investment technology \( t(i) \).

- If \( \epsilon(i) \) is a constant, the domestic sales of all non-exporting firms decrease by the same proportion. Those of all exporting firms also change by a constant proportion, albeit different from that of non-exporting firms.
- If \( \epsilon(i) \) is decreasing (increasing) in \( i \), the difference between the log of domestic sales of two given non-exporters decreases (increases), while that between two given exporters increases (decreases).

The link between the skewness of firm size distribution and trade liberalisation is therefore different from that of Nocke and Yeaple (2006). While they predict a decrease between the differences in the log size of two given firms, the present model suggests a non monotonic change.

### 3.4 Mass of consumed varieties

Traditional models of monopolistic competition built on Krugman (1979) emphasise the increase in the number of available varieties as one of the main
gains from trade, a result which has been supported by a number of empirical studies, such as Feenstra (2006) or Broda and Weinstein (2006). It is however puzzling that in Melitz (2003), the number of consumed varieties decreases with trade liberalisation under the usual assumption that productivity draws are Pareto distributed. I here show how the present framework can account for this puzzle.

The labour market equilibrium requires that in each country:

\[ L = M \left( \int_{z^*}^{\bar{z}} A(\sigma - 1)t(i_{d}(z))z^{\sigma - 1}P^{\sigma - 1}L + i_d(z) + fdG(z) \right) + M \left( \int_{z^*_x}^{\bar{z}} A(\sigma - 1)t(i_{x}(z))z^{\sigma - 1}P^{\sigma - 1}L + i_x(z) + f + f_xdG(z) \right) + Mf_e \] (34)

where \( M \) is the mass of entrepreneurs in a country who decide to pay the sunk cost in order to obtain a draw of productivity. The first part of the right hand side is the labour used by domestic firms for production and fixed costs purposes. The second bracket is the labour used by exporting firms, and \( Mf_e \) is the labour used for the payment of the sunk costs by all entrepreneurs who enter the market. This equation determines \( M \).

The mass of consumed varieties (\( N \)) is given by:

\[ N = M(1 - G(z^*) + 1 - G(z^*_x)) \] (35)

which is the mass of entrepreneurs entering the market in a country, times the fraction from which a consumer in a given country can buy products (i.e. home entrepreneurs who decide to produce and foreign entrepreneurs who decide to export). Trade liberalisation has different effects on the right hand side of (34), which I decompose into two groups: (i) the effect of the cutoff firms, and (ii) the effect of all other firms.

(i) The increase in the domestic cutoff level \( z^* \) and the decrease in the export cutoff \( z^*_x \) have a direct effect on the fraction of firms that serve a given market. They moreover have an effect on \( M \), the mass of entrepreneurs paying the sunk cost. Indeed, domestic firms exiting the market release labour, which tends to increase \( M \) from (34). On the other hand, new exporting firms will raise their demand for labour, and tend to decrease \( M \). These combined effects of the cutoff firms are qualitatively similar to those of Melitz (2003). Their quantitative effect depends, among others, on the distribution function \( G(z) \) of productivity draws. In Melitz (2003), under a Pareto distribution, these cutoff effects tend to decrease the number of consumed varieties.
(ii) I now turn to the effects of all non-cutoff firms on $M$. From the labour market condition (34), if their combined demand for labour increased (decreased), this would push $M$ down (up). In Melitz (2003), the aggregate labour used by all firms that are not at the cutoff level remains constant when trade liberalisation occurs. This is an important property of this model, which is such that the effect of the cutoff firms determines the change in the mass of consumed varieties. The intuition is as follows. First, the decrease in the price index following liberalisation reduces the demand for labour for domestic sales. The size of this effect is determined by the relative weight of exporters in the aggregate variable profits. The higher this proportion the higher the effect of a marginal trade liberalisation on the price effect since many exporters scale up their production on the domestic market, and the higher the reduction in the demand for labour for domestic sales. Second, exporting firms sell more on the export market, and scale up their demand for labour for this reason. This effect also depends on the exporters’ weight in aggregate profits, and the model is so constructed that both effects exactly cancel out.

In the present model, however, this will not be necessarily the case. As proved in Appendix, the change in aggregate demand for labour by all non-cutoff firms depends on $\epsilon'(i)$. If $\epsilon'(i) = 0$, the intuition of the Melitz (2003) model holds, because the proportionality between change in labour demand and change in variable profits is preserved. If $\epsilon'(i) < 0$ however, exporting firms scale up their demand for labour proportionately less because they are less sensitive. This releases resources and allows $M$ to increase, pushing up the mass of consumed varieties $N$. This provides an additional effect raising the mass of consumed varieties, which is potentially important since it concerns all firms and not only the cutoff firms.

4 Conclusion

This paper has developed a Melitz (2003) type model of trade, in which, after observing their efficiency, firms invest in productivity improvements. The investment decision is continuous, in the sense that each firm decides how much to invest. This framework preserves all main qualitative results of Melitz (2003), and provides additional insights in the investment decision of heterogeneous firms. Indeed, I am able to replicate a number of well established stylised facts, with weak assumptions for a well behaved problem. Exporters in the model invest more than non-exporters, and scale up investment following trade liberalisation. Among the group of exporters, the biggest
firms invest more and also invest more following trade liberalisation. This last empirical fact can not be replicated in models of discrete investment decision. I also generate a number of testable implications. (i) Non-exporting firms should scale down their investment following trade liberalisation. (ii) The domestic sales of non-exporting firms should decrease proportionately more than those of exporting firms\textsuperscript{11} following trade liberalisation.

The present model also addresses three puzzles that have arisen in the last years: (i) the negative relationship between firm size and Tobin's Q (Nocke and Yeaple (2006)) (ii) the change in the skewness of the size distribution of firms when trade costs decrease (Nocke and Yeaple (2006)) (iii) the increase in the mass of consumed varieties following trade liberalisation (Baldwin and Forslid (2006)). I show that the investment technology, and especially the sensitivity of investment relative to firm size, plays a central role in explaining them. These results cannot be obtained by the other existing models of process innovation in the Melitz (2003) framework, either due to the discrete investment decision\textsuperscript{12} or to an assumption on the specific form of the investment technology\textsuperscript{13}. The importance of the properties of the investment technology, which is arguably different across sectors, suggests more caution for the interpretation of empirical results on the above puzzles, which are usually investigated across sectors.

\textsuperscript{11}In some instances, the domestic sales of exporting firms should even increase.
\textsuperscript{12}Bustos (2005), Bustos (2007), Ederington and McCalman (2006) and others
\textsuperscript{13}Atkeson and Burstein (2007)
References


Proof of Lemma 2

Proof of part 1

In this section, I show that there is a unique $i$ solving:

$$\frac{t(i)}{t'(i)} - i - f = 0 \quad (36)$$

For this, note that $\frac{t'(0)}{t''(0)} = 0$ by Assumption 1. The left hand side of the above equation is therefore equal to $-f$ for $i = 0$.

Differentiating the left hand side with respect to $i$:

$$\frac{\partial}{\partial i} \left( \frac{t(i)}{t'(i)} - i - f \right) = -\frac{t''(i)t(i)}{t'(i)^2} > 0 \quad (37)$$

I now show that as $i$ goes to infinity, the left hand side will be positive, which is to say that at least some firms will make non-negative profits in the economy.

The left hand side of (36) can be rewritten as:

$$\frac{t(i)}{t'(i)} \left[ 1 - \frac{(i + f)t'(i)}{t(i)} \right] \quad (38)$$

As $i$ goes to infinity, $\frac{t(i)}{t'(i)} \to \infty$ by Assumption 1. Moreover, $\lim_{i \to \infty} \frac{(i + f)t'(i)}{t(i)} = \lim_{i \to \infty} \epsilon(i)$, which is bounded away from 1. The square bracket above is therefore bounded, and the whole expression therefore goes to infinity.

This completes the proof that there exists a unique $i$ so that there is a cutoff firm that makes zero profits. This is moreover independent of $z^*$

Proof of part 2

The proof of part 2 follows from part 1 and from Lemma 1.

Proof of Lemma 3

$\epsilon'(i)$ is equal to:

$$\epsilon'(i) = \frac{t'(i)}{t(i)} + i \frac{t''(i)t(i) - t'(i)^2}{t'(i)^2} = \left( \frac{t'(i)}{t(i)} \right)^2 \left[ \frac{t(i)}{t'(i)} + i \frac{t''(i)t(i)}{t'(i)^2} - i \right] \quad (39)$$

$\epsilon'(i)$ has therefore the sign of the square bracket on the right hand side. From Assumptions 1 and 2 (and using l'Hopital's rule to show that $\frac{\epsilon'(i)t(i)}{t'(i)^2}$ is bounded as $i \to 0$), the square bracket is equal to zero if $i \to 0$. Furthermore:

$$\frac{\partial}{\partial i} \left[ \frac{t(i)}{t'(i)} + i \frac{\epsilon'(i)t(i)}{t'(i)^2} \right] = i \frac{\partial \epsilon'(i)t(i)}{\partial i} \propto i E'(i) \quad (40)$$
\( \epsilon'(i) \) and \( E'(i) \) therefore have the same sign.

Q.E.D.

**Proof of Proposition 3**

The Tobin’s Q of a firm investing \( i \) is given by (18):

\[
T(i) = \frac{1}{\epsilon(i) + f \frac{\nu(i)}{t(i)}}
\]

(41)

Therefore, \( T'(i) \) is of the sign of:

\[
T'(i) \propto -\left( \epsilon'(i) - f \left( \frac{t'(i)}{t(i)} \right)^2 \left( 1 + \frac{1}{E(i)} \right) \right)
\]

(42)

The second term in bracket is always negative so that if \( \epsilon'(i) \leq 0 \), \( T'(i) \) is positive. Using that:

\[
\epsilon'(i) = \frac{t'(i)}{t(i)} + i \left( \frac{t'(i)}{t(i)} \right)^2 \left( 1 + \frac{1}{E(i)} \right)
\]

(43)

\( T'(i) \) can be rewritten as:

\[
T'(i) \propto J(i) \equiv -\left( \frac{t'(i)}{t(i)} + i + f \right) \left( \frac{t'(i)}{t(i)} \right)^2 \left( 1 + \frac{1}{E(i)} \right)
\]

(44)

so that \( \lim_{i \to \infty} \epsilon(i) = \lim_{i \to \infty} J(i) \). For the case that \( \epsilon'(i) > 0 \), this shows that for a sufficiently high \( i \), \( J(i) > 0 \) and therefore \( T'(i) < 0 \). The Tobin’s Q then decreases with investment (and therefore size). For \( i = i^* \), which is the smallest producing firms, it is immediate from (12) that:

\[
J(i^*) = -\frac{t'(i^*)}{t(i^*)} \left( 2 + \frac{1}{E(i^*)} \right) < 0
\]

(45)

The Tobin’s Q is therefore increasing in \( i \) for the smallest producing firms, which completes the proof.

**Proof of Proposition 4**

For clarity of exposition, I rewrite the condition for a firm \( z \) to export (24):

\[
\underbrace{A(zP)^{\sigma-1}L(1 + \tau^{\sigma-1})t(i_z(z)) - i_z(z) - f - f_z - A(zP)^{\sigma-1}L(i_d(z)) + i_d(z) + f}_{\pi_z(i_z(z))} \geq 0
\]

(46)

which requires that the difference in profits between the two strategies (domestic & export and domestic) be positive.

(i) A straightforward application of the envelope theorem shows that the derivative of the left-hand side with respect to \( z \) is positive, since the effects of \( z \) on optimal investment
are of second order importance. This means that the higher the \( z \), the larger the relative profit of the export strategy.

(ii) For \( z = z^* \), \( \pi_d = 0 \), and \( \pi_x(i_d(z^*)) \) should be negative for the cutoff firm \( z^* \) not to export. This obtains if \( f_x \) is sufficiently high for any \( \tau \).

(iii) As \( z \to \infty \), if \( \tau > 0 \), it is immediate from (46) that \( \pi_x(i_d(z)) - \pi_d(i_d(z)) \to \infty \). Since by definition, \( \pi_x(i_x(z)) > \pi_x(i_d(z)) \), this implies that the difference between the profits of the export and non-export strategies goes to infinity.

Combining (i), (ii) and (iii) shows Proposition 4

Proof of Proposition 5

- Part 1: the selection effect

Plugging (13) into (26) and totally differentiating yields:

\[
\frac{\partial z^*}{\partial \tau} = \frac{\int_{z^*}^{Z} \tau^{-1} z^{-1} t(i_d(z)) dG(z)}{\int_{z^*}^{Z} z^{-1} t(i_d(z)) dG(z) + \int_{z^*}^{Z} (1 + \tau^{-1}) z^{-1} t(i_d(z)) dG(z)}
\]

(47)

This is nothing else than the ratio of total exports to total sales. To derive the above expression, the envelope theorem has been used, as well as the indifference conditions of the \( z^* \) and \( z^*_x \) firms. It is clearly positive and shows that trade liberalisation yields a selection effect by increasing \( z^* \).

- Part 2: Change in optimal investment of non-exporting firms

For non-exporting firms, the first order condition for optimal investment rearranged with (13) is:

\[
(i^* + f) \left( \frac{z}{z^*} \right)^{\sigma - 1} \frac{t'(i_d)}{t(i^*)} = 1
\]

(48)

It is immediate that a change in \( \tau \) impacts the optimal decision of non-exporting firms only through the general equilibrium effect of a drop in the price index and therefore an increase in \( z^* \) (remember that from Lemma 2, \( i^* \) remains constant). The rise in \( z^* \) following trade liberalisation must incur a decrease in the optimal investment of a non-exporting firm since \( t(i) \) is concave. This can be immediately seen by totally differentiating (48):

\[
\frac{di_d}{d\tau} = \frac{\sigma - 1}{z^*} \frac{t'(i_d)}{t(i^*)} \frac{dz^*}{d\tau}
\]

(49)

- Part 3: Change in optimal investment of exporting firms

For exporting firms, the first order condition for optimal investment rearranged with (13) is:
\[(1 + \tau^{\sigma-1}) \left( \frac{z^*}{z} \right)^{\sigma-1} (i^* + f) \frac{t'(i_x)}{t(i_x)} = 1 \quad (50)\]

Totally differentiating the above equation and rearranging:
\[d\tau \left[ (\sigma - 1) \frac{\tau^{\sigma-2}}{1 + \tau^{\sigma-1}} t'(i_x) + \frac{dz^*}{d\tau} \frac{1 - \sigma}{z^*} t'(i_x) \right] + di_x t''(i_x) = 0 \quad (51)\]

The square bracket shows the two effects of trade liberalisation: the first term is the direct effect that exporting firms can sell more on their export market, the second term is the indirect general equilibrium effect of a smaller price index, which makes it more difficult for these firms to sell.

Plugging in (47):
\[d\tau \left( \frac{\tau^{\sigma-2}}{1 + \tau^{\sigma-1}} + \frac{dz^*}{d\tau} \frac{1 - \sigma}{z^*} \right) t'(i_x) + di_x t''(i_x) = 0 \quad (52)\]

The term in square bracket is strictly positive as long as there are non-exporting firms. To see this, assume that all firms export, so that \(\int z^* x z^* \sigma^{-1} t(i_d(z))dG(z) = 0\). In this case, the square bracket is equal to zero. If on the other hand, there are purely domestic firms, the second part of the square bracket gets smaller and the whole square bracket is positive. This shows that the direct effect of a change in \(\tau\) dominates the general equilibrium effect for the total sales of exporters.

Since \(t''(i_x) < 0\), trade liberalisation raises the investment of exporting firms.

**Proof of Proposition 6**

To highlight the general equilibrium effects of a change in trade costs on the export cutoff level, I rewrite (27) using (13):
\[(f + i^*) \left( \frac{z^*}{z} \right)^{\sigma-1} \left[ (1 + \tau^{\sigma-1})t(i_x(z^*_x)) - t(i_d(z^*_x)) \right] - i_x(z^*_x) - f_x + i_d(z^*_x) = 0 \quad (53)\]

Totally differentiating the above equation and rearranging gives:
\[
\frac{\tau^{\sigma-2} t(i_x(z^*_x))}{(1 + \tau^{\sigma-1})t(i_x(z^*_x)) - t(i_d(z^*_x))} - \frac{dz^*}{d\tau} \frac{1}{z^*} + \frac{dz_x^*}{d\tau} \frac{1}{z_x^*} = 0 \quad (54)
\]

Using (47) and following a similar argumentation as in the proof of Proposition 5 Part 3, the sum of the first two terms is positive, so that \(\frac{dz^*}{d\tau}\) must be negative, and trade liberalisation leads to a decrease in the cutoff export level.

**Mass of consumed varieties**

Plugging the free entry equation (26) in the labour market condition (34) yields:
conclusion holds for $E$.

Their investment decision, their positive effect on labour demand dominates. The opposite case exactly absorbs the labour released by the drop in investment of non-exporting firms. If firms in their investment decision, so that the increase in investment by exporting firm change when trade costs vary. This is due to the proportionality of the reaction of all standard case holds: the total labour used by existing firms not changing status does not change expression treating $M$ as given and neglecting the changes in the boundaries of the integrals in the square bracket. It is of the sign of:

Using the price index equation (13):

$$L = M \sigma \left[ \int_{z^*}^{Z} t(i_d(z)) A \sigma^{-1} P \sigma^{-1} L dG(z) + \int_{z^*}^{Z} (1 + \tau \sigma^{-1}) t(i_d(z)) A \sigma^{-1} P \sigma^{-1} L dG(z) \right]$$

(55)

Using the price index equation (13):

$$L = M \sigma \left[ \int_{z^*}^{Z} t(i_d(z)) A \sigma^{-1} P \sigma^{-1} L dG(z) + (1 + \tau \sigma^{-1}) \int_{z^*}^{Z} t(i_d(z)) A \sigma^{-1} P \sigma^{-1} L dG(z) \right]$$

(56)

I here examine the change in the aggregate labour demand by all non-cutoff firms, i.e. those firms that do not change status (between non-producing, domestic and exporting) following trade liberalisation for a constant $M$. This corresponds to the derivative of the above expression treating $M$ as given and neglecting the changes in the boundaries of the integrals in the square bracket. It is of the sign of:

$$\Delta \equiv \frac{1 - \sigma}{z^*} \frac{dz^*}{d\tau} \left[ \int_{z^*}^{Z} t(i_d(z)) A \sigma^{-1} P \sigma^{-1} L dG(z) + (1 + \tau \sigma^{-1}) \int_{z^*}^{Z} t(i_d(z)) A \sigma^{-1} P \sigma^{-1} L dG(z) \right]$$

$$+ (\sigma - 1) \tau \sigma^{-2} \int_{z^*}^{Z} t(i_d(z)) A \sigma^{-1} P \sigma^{-1} L dG(z) + \int_{z^*}^{Z} t(i_d(z)) \frac{di_d(z)}{d\tau} A \sigma^{-1} P \sigma^{-1} L dG(z)$$

$$+ (1 + \tau \sigma^{-1}) \int_{z^*}^{Z} t(i_d(z)) \frac{di_d(z)}{d\tau} A \sigma^{-1} P \sigma^{-1} L dG(z)$$

(57)

The first part is the general equilibrium effect, which tends to decrease the labour demand by all firms. The second part is the direct effect that increases the demand for labour by all exporting firms to satisfy the increased demand on their export market. Note that these first two terms cancel using (47), exactly as in the standard framework. The last two terms correspond to the change in labour demand for investment purposes.

Rearranging using (49) und (51), this expression is of the sign of:

$$\Delta^2 \equiv \frac{\tau \sigma^{-2} \int_{z^*}^{Z} t(i_d(z)) A \sigma^{-1} E(i_d(z)) dG(z)}{\int_{z^*}^{Z} t(i_d(z)) A \sigma^{-1} E(i_d(z)) dG(z) + (1 + \tau \sigma^{-1}) \int_{z^*}^{Z} t(i_d(z)) A \sigma^{-1} E(i_d(z)) dG(z)} - \frac{dz^*}{d\tau} \frac{1}{z^*}$$

(58)

Using (47), it is immediate that if $E'(i) = 0$, $\Delta^2 = 0$ and the same result as in the standard case holds: the total labour used by existing firms not changing status does not change when trade costs vary. This is due to the proportionality of the reaction of all firms in their investment decision, so that the increase in investment by exporting firm exactly absorbs the labour released by the drop in investment of non-exporting firms. If $E'(i) > 0$, the numerator of the first term above has relatively more weight than in the case $E'(i) = 0$, so that the whole expression is positive: if large firms are more sensitive in their investment decision, their positive effect on labour demand dominates. The opposite conclusion holds for $E'(i) < 0$.  

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