Vertical Production Chain
and Global Sourcing

(Preliminary Draft)

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Abstract
I extend the “global sourcing” framework by Antràs and Helpman (2004, JPE) by introducing a continuum of intermediate inputs in the production function. This allows to study input specific sourcing strategies. I find new effects that can exclusively emerge in a setting with strictly more than two inputs. First, I find that the optimal revenue share contributed to a supplier systematically varies in the input intensities of the other suppliers. It increases if the production process becomes more unequal in the inputs of the other suppliers. Second, the introduction of a vertical production chain leads to fact that a higher position in the vertical production chain favors integration of that the intermediate input. This finding is perfectly in line with recent empirical evidence on the role of a vertical production chain in the organization of production.

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1 Introduction

Increasing fragmentation of production due to technological progress has been a crucial trend in the last decades. Multinational enterprises can nowadays choose from a richer pattern of sourcing strategies than ever before. For a multitude of intermediate inputs firms face the classical “make or buy” decision (i.e. outsource vs. integrate) and may also opt to offshore the intermediate input production to a foreign country. Various authors provide empirical evidence and examples from the business press. A recent study by Linden, Kraemer and Dedrick (2007) reports that Apple has completely outsourced the production of the 451 mostly generic inputs that are needed to fabricate the newest iPod. Only final assembly is done by Apple itself. Alfaro and Charlton (2007) find for General Motors Corporations that out of 2,248 entities 455 subsidiaries are located outside the United States.

The theory of the firm predicts that the decision to outsource or offshore intermediate input production relates to the characteristics of the countries and the characteristics of the products being produced. In their seminal contribution “global sourcing” Antràs and Helpman (2004) introduce a North-South model of international trade where firms choose from a variety of organizational forms, depending on their individual productivity and sector characteristics. Their framework, which combines firm heterogeneity in spirit of Melitz (2003) with organizational structures as in Antràs (2003), is especially helpful for coming to grips with newly emerged empirical facts about arm’s length outsourcing and intra-firm trade. The main result is that firms in headquarter-intensive sectors are more likely to choose integration strategies, whereas component-intensive sectors solely focus on outsourcing strategies.

As it is common in the outsourcing and offshoring literature Antràs and Helpman (2004) study a production process with two intermediate inputs. The firm’s decision whether to integrate or outsource intermediate input production is modeled with respect to one intermediate input. In the light of increasing fragmentation it is however straightforward to search for new effects that exclusively emerge in a setting with strictly more than two inputs.
My model builds on “global sourcing” by Antràs and Helpman and extends their framework by introducing a continuum of intermediate inputs in the Cobb-Douglas type production function. This approach allows to differentiate between intermediate inputs on the one hand and on the other hand to explore input specific sourcing strategies. The production process in Antràs and Helpman can be regarded as a two-stage game. In the first stage a final-good producer distributes revenue shares from the potential sale of the final-goods. In the second stage intermediate input suppliers take their revenue shares, which were assigned in the first stage, as given and maximize their individual profits by providing optimal input contributions. Various new effects appear exclusively in a setting with strictly more than two intermediate inputs.

First, I derive the suppliers’ profit maximizing intermediate input contribution in the second stage. I find that the optimal input relation of two representative suppliers is independent of other suppliers.

Second, I derive the profit maximizing distribution of revenue, i.e. the optimal revenue shares that should ideally be assigned to each intermediate input supplier in the first stage. I find that the optimal revenue share contributed to a supplier systematically varies in the input intensities of the other suppliers, even if the own input intensity remains constant. It increases if the production process becomes more unequal in the inputs of the other suppliers, i.e. if the input intensity dispersion increases. Similar joint profits and revenues increase in the input intensity dispersion.

Third, I differentiate between intermediate inputs by considering the proximity to the final good, i.e. I introduce a vertical production chain. If, for example, technology-services are higher in the vertical production chain than labor services, I find that it is more likely that firms will integrate the production of technology-services. This theoretical prediction is perfectly in line with new empirical evidence by Alfaro and Charlton (2007). They find that multinationals tend to own the stages of production proximate to their final-good. Therefore, the novelty of my approach is that it introduces a new determinant of sourcing strategies, namely a vertical production chain.

My extension of the Antràs and Helpman (2004) framework is comple-
mentary to the theory of the firm and in particular the strand of the literature that investigates the ownership structure of firms. The ownership decision of firms is a lively field of both theoretical and empirical research as the survey by Helpman (2006) illustrates. Various aspects are under investigation. Grossman and Helpman (2002) address the choice between outsourcing and integration within a one-input production function. McLaren (2000) and Grossmann and Helpman (2003) focus on the matching between a final-good producer and unaffiliated intermediate input suppliers. Antràs (2003) uses a setting of incomplete contracts to study the ownership decision of firms with respect to the input intensities of the production process. The impact of legal contractibility with respect to specific inputs is addressed by Antràs and Helpman (2006).

To study the organizational choice of individual firms, new theories have to account for the empirical fact of within-in sectoral heterogeneity. The seminal contribution by Melitz (2003) allows productivity to differ across firms and has become the cornerstone in the “new” new trade theory. In this framework, the interaction between productivity differences across firms and fixed costs of exporting leads to the prediction that only the most productive firms export. However, final-goods production can also be offshored to a foreign country. Helpman, Melitz and Yeaple (2004) extend the Melitz (2003) model by incorporating the horizontal motive of FDI, i.e. transport cost savings. They show that among firms that serve foreign markets, the less productive firms export while the more productive firms engage in horizontal FDI. Unlike horizontal FDI the classical motive for vertical FDI is the firms’ attempt to take advantage of cross-border factor cost differences like in Helpman (1984) and Helpman and Krugman (1985).

The first ones who combine firm heterogeneity in spirit of Melitz (2003) with the organization of firms as in Antràs (2003) are Antràs and Helpman (2004). They introduce a North-South model of international trade where firms choose from a variety of organizational forms, depending on their individual productivity and sector characteristics. Their main result is that in a sector that is component intensive, i.e. the input provided by the component supplier is relatively more important for the production process than the one
of the final-good producer, firms solely focus on domestic and international outsourcing. In case the sector is intensive in the input provided by the final-good producer, i.e. headquarter-services, integration and outsourcing can coexist.

However, all these contributions focus on one single intermediate input that can either be integrated or outsourced. This strand of the literature often neglects that final good production is accomplished by combining various inputs. The major drawback of this approach is that it does not allow to explore interdependencies between intermediate input specific sourcing strategies. Shy and Stenbacka (2005) study for example a Cournot model with a continuum of intermediate inputs. They derive the equilibrium fraction of outsourced intermediate inputs, which is a result that can exclusively be derived in a setting with a continuum of intermediate inputs. My extension of the Antràs and Helpman (2004) framework can therefore be seen as a new impulse to study the interdependencies between input specific sourcing strategies.

2 Model

Consider an economy with a set of sectors $S$. Consumers have identical preferences which are represented by

$$U = x_0 + \frac{1}{\mu} \int_S X(k)^{\mu} \, dk$$

(1)

with $0 < \mu < 1$ and where $x_0$ is consumption of a homogeneous good. Aggregate consumption $X_s$ in sector $s$ is a constant elasticity of substitution function

$$X_s = \left[ \int_{V_s} x_s(k)^{\alpha} \right]^{\frac{1}{\alpha}}$$

(2)

with $0 < \alpha < 1$ and where $V_s$ is the set of final-good varieties in sector $s$. The elasticity of substitution between any two final-good varieties in a given sector is $1/(1 - \alpha)$. For simplicity, I assume that the final-good varieties $x_s$ are constant. Aggregate consumption simplifies to $X_s = x_s d_s^{1/\alpha}$
and rises in the number of final-good varieties $d_s$. Consumers find it more easy to substitute varieties within a sector than across sectors if $\alpha > \mu$ holds. Consumer preferences lead to an inverse demand function for each variety $x_s$ of

$$p_s = X_s^{\mu-\alpha} x_s^{\alpha-1}.$$  

Each final-good variety $x_s$ is accomplished by combining a continuum of intermediate inputs $m_s$. Each intermediate input $m_s(i)$ is provided by a unique intermediate input supplier $A(i)$. Output of a final-good variety $x_s$ is given by a sector specific Cobb-Douglas type production function of the form

$$x_s(\theta, m_s(i)) = \theta \exp \left[ \int_{I_s} e_s(k) \ln \left( \frac{m_s(k)}{e_s(k)} \right) dk \right].$$

with $0 < e_s(i) < 1$. The productivity parameter $\theta$ is firm specific, whereas the parameters $e_s$ are sector specific. The larger $e_s(i)$ the more intensive is the production of the final-good variety $x_s$ in the intermediate input $m_s(i)$. As in Antràs (2003) I call $e_s(i)$ the input intensity of the intermediate input $m_s(i)$ provided by the intermediate input supplier $A(i)$. Total revenue $R_s = x_s p_s$ of a final-good variety is given by

$$R_s = d_s^{(\mu-\alpha)/\alpha} \theta^\mu \exp \left[ \mu \int_{I_s} e_s(k) \ln \left( \frac{m_s(k)}{e_s(k)} \right) dk \right].$$

Production of intermediate inputs requires only one factor of production, named labor. An intermediate input supplier $A(i)$ faces a perfectly inelastic supply of workers at the wage rate $w(i)$. Intermediate input production involves variable costs of $w(i)$ and fixed costs $f(i)$. Final-good production can be seen as a two stage production process. In the following I drop the sector index $s$ for convenience. Now consider the production process in detail:

**1st Stage: Distribution of Revenue**

In the first stage intermediate input suppliers distribute the potential future revenue from the sale of final goods varieties. Each agent's revenue share is given by $b(i)$ with $\int_{I_s} b(k) dk = 1$. One of the intermediate input suppliers is the final-producer $A(h)$ that provides headquarter services. The
final-good producer offers a contract to each other intermediate-input supplier $A(i)$, $h \neq i$, that involves a share of revenue $b(i)$ and a participation fee. The participation fee has to be paid by the other intermediate-input suppliers to the final-good producer. The final-good producer has an incentive to raise the participation fee as much as possible, as long as the participation constraint for the intermediate input supplier is satisfied. Once a relationship between a final-good producer and an intermediate-input supplier is formed the participation fee has no further effects on the outcomes. As a result, the final-good producer is interested in the distribution of revenue $b^* \equiv \left(b^*_1, \ldots, b^*_n \right)$ that maximizes the potential revenue from the sale of the final goods.

**2nd Stage: Production Stage**

In the second stage intermediate input suppliers take their revenue share $b(i)$ assigned in the first stage as given and maximize their individual profit $\pi(i)$. The intermediate input suppliers’ individual profit at the second stage is given by the share of revenue less the cost of producing the intermediate input and fixed costs. The maximization problem of a supplier $A(i)$ is

$$\max_{m(i)} \pi(i) = b(i) R - w(i) m(i) - f(i). \quad (6)$$

I solve this two-step production process via backward induction.

**Solution 2nd Stage**

The maximization problem of an agent $A(i)$ in the 2nd stage leads to the first order condition

$$\frac{\partial \pi(i)}{\partial m(i)} = b(i) R \mu e(i) - w(i) = 0. \quad (7)$$
Solving the system of first order conditions leads to the optimal input contribution \( m^* (i) \) of an agent \( A(i) \) and is given by

\[
m^* (i) = \frac{e(i)}{w(i)} b(i) \left( \mu \theta^\mu d^{w-a} \right)^{1-\mu} \exp \left[ \frac{\mu}{1-\mu} \int e(k) \ln \left( \frac{b(k)}{w(k)} \right) dk \right].
\] (8)

Proof see Appendix.

**Proposition 1**

Two intermediate input suppliers \( A(i) \) and \( A(j) \) contribute optimal inputs \( m^* (i) \) and \( m^* (j) \) if and only if

\[
\frac{m^* (i)}{m^* (j)} = \frac{b(i)}{b(j)} \frac{e(i)}{e(j)} \frac{w(j)}{w(i)}
\] (9)

holds.

The optimal input relation solely depends on the relative distribution of revenue \( b(i)/b(j) \), the relative input intensities \( e(i)/e(j) \) and the inverse wage differential \( w(j)/w(i) \) of the intermediate input suppliers. Note that it does neither depend on the aggregate consumption in the economy \( X \) nor on factors that are associated with other suppliers \( A(k), k \neq j \neq i \), involved in the final-goods production. Whether two representative input suppliers provide an optimal contribution relation is therefore independent from the supplement of other suppliers.

The economic intuition is the following. Assume that there are no intermediate input wage differences and the inputs are both equally important for the production process. In this case intermediate-input suppliers are only willing to contribute relative quantities equal to the relative revenue shares they capture from the relation. If one allows for differences in the input intensities the intermediate-input supplier with the more important input also invests relatively more in order to avoid underinvestment. If the wage gap between the two intermediate-input suppliers increases, the supplier of the relatively more expensive input decreases the own contribution. This is intuitive since the input suppliers have to bear an increase in the variable
costs of intermediate-input production for their own.

After having solved the second stage I now proceed with the first stage of the production process.

Solution 1st Stage

The total value of the final-good production is given by \( \pi = \int_I \pi(i)dk \).

If all agents contribute optimal inputs \( m^* \) the total value of the final-good production is given by

\[
\pi^* = d^{\frac{\mu-\alpha}{\beta-\alpha}} \theta^{\frac{\mu}{\beta-\alpha}} \psi - \int_I f(k)dk \quad (10)
\]

with

\[
\psi = \left[ 1 - \mu \int_I e(k) b(k)dk \right] \exp \left[ \frac{\mu}{1-\mu} \int_I e(k) \ln \left( \frac{b(k)}{w} \right) dk \right] \quad (11)
\]

Proof see Appendix.

One of the intermediate input suppliers is the final-producer \( A(h) \) that provides headquarter services. This agent offers a contract to each other intermediate-input supplier \( A(i), h \neq i \), that involves a share of revenue \( b(i) \) and a participation fee \( t(i) \). The participation fee has to be paid by an intermediate input supplier and is captured by the final-good producer. The profit of an intermediate input supplier \( A(i) \) at the 1st stage is then

\[
\pi(i) = b(i) R - w(i) m(i) - f(i) - t(i).
\]

The final-good producer \( A(h) \) has an incentive to raise the participation fee \( t(i) \) as much as possible, as long as the participation constraint \( \pi(i) \geq 0 \) for each other intermediate input supplier \( A(i) \) is satisfied. It is important to note that if the participation constraint is satisfied the participation fee has no further effects on the outcomes and the intermediate input supplier will contribute optimal inputs \( m^*(i) \) as derived in (8). As a result the equilibrium value of \( t(i) \) satisfies \( \pi(i) = 0 \) for all intermediate-input suppliers \( A(i) \) except the final-good producer \( A(h) \). The final-good producer captures
the total value of the final-good production, i.e. $\pi(h) = \pi$, due to the sum of participation fees. The final-good producer is therefore interested in the ex-post distribution of revenue that maximizes potential revenue from the sale of the final-goods.

In the following, I derive the distribution of revenue $b^* = (b^*_1, \ldots, b^*_n)$ that maximizes the potential revenue from the sale of the final goods. Potential revenue is maximized if $\psi$ is maximal. A necessary condition for a maximum of $\psi$ with respect to the optimal distribution of revenue $b^*$ is

$$\frac{d\psi}{db(i)}|_{b^*} = 0.$$  \hfill (12)

**Proposition 2**

The distribution of revenue $b^*$ that maximizes the potential revenue from the sale of the final-goods is implicitly given by

$$e(i) - b(i) - (1 - \mu) e(i) b(i) + \left[ b(i) - \mu e(i) \right] \int_I e(k) b(k) \, dk = 0 \hfill (13)$$

due to the fact that it provides a solution to (12).

Proof see Appendix.

An explicit form of $b^*$ is possible to derive for the two intermediate input cases (headquarter-services and manufactured components) Antràs and Helpman (2004) examine\(^1\). For other cases with strictly more than two intermediate inputs numerical methods help to provide $b^*$.

However, a basic economic intuition can be established analytically from proposition 2. If an intermediate input supplier provides an input that is not required for the final-good production the optimal assigned revenue share is zero, i.e. $b(i) = 0 \iff e(i) = 0$. Furthermore if the production process solely relies on only one single input, the intermediate-input supplier completely captures the revenue, i.e. $e(i) = 1 \Rightarrow b(i) = 1$.

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\(^1\)I provide the derivation of $b^*$ for the Antràs and Helpman (2004) case in the Appendix.
Three Input Production

In the following, I study a production process with three intermediate inputs. Even with only one additional input new effects appear that cannot be derived in the original Antràs and Helpman (2004) framework with two intermediate inputs. The production function is given by

\[ x(\theta, m(i)) = \theta \exp \left[ \sum_{k=1}^{3} e(k) \ln \left( \frac{m(k)}{e(k)} \right) \right] \tag{14} \]

with \( e(3) \equiv 1 - e(1) - e(2) \) and \( b(3) \equiv 1 - b(1) - b(2) \). I consider the agent \( A(1) \) as the final-good producer that supplies headquarter services \( m(1) \).

Optimal Distribution of Revenue

In the following, I discuss the characteristics of the distribution of revenue \( b^* \equiv (b^*_1, b^*_2, b^*_3) \) that maximizes the potential revenue from the sale of the final goods. The final-good producer opts for this distribution of revenue that maximizes total revenue since it also maximizes total profits.

In particular, I am interested in the revenue share \( b^*_1 \) that should ideally be contributed to the final-good producer \( A(1) \). Proposition 2 states that the revenue share \( b^*_1 \) is a function of all three input-intensities \( e(1) \), \( e(2) \) and \( e(3) \). From the perspective of the final-good producer \( A(1) \) I label the input intensity \( e(1) \) the final-good producer’s associated input intensity. The input intensities \( e(2) \) and \( e(3) \) are non-associated input intensities with respect to the final-good producer since they are contributed by the other intermediate input suppliers \( A(2) \) and \( A(3) \). With this terminology the optimal revenue share for the final-good producer is a function of associated and non-associated input intensities.

A good starting point for variations in both types of input intensities is the most moderate sector in which all input intensities are equal, i.e. \( e(1) = e(2) = e(3) = \frac{1}{3} \). First, I consider an increase in the final-good producer’s associated input intensity \( e(1) \). Figure 1 provides \( b^*_1 \) with respect to \( e(1) \) if \( e(2) = \frac{1}{3} \) is fixed. It is clear that \( b^*_1 \) increases monotonically in the associated input intensity \( e(1) \). The economic intuition has already
Figure 1: Optimal Incentives Effect

Figure 2: Dispersion Effect
been delivered in Antràs (2003): A supplier’s severity of underinvestment is reversely related to the fraction of surplus that it appropriates. Ex-ante efficiency then requires giving a larger share of revenue, i.e. a higher \( b(i) \), to the agent undertaking the relatively more important investment, i.e. to the agent with the higher \( e(i) \). I refer to this insight as the optimal incentives effect.

Second, I consider changes in the final-good producer’s non-associated input intensities \( e(2) \) and \( e(3) \). I find a new effect that can only arise in a setting with strictly more than two intermediate inputs. In Figure 2 I provide \( b^*(1) \) for variations in the non-associated input intensities \( e(2) \) and \( e(3) \) while the final-good producer’s associated input intensity is fixed to \( e(1) = \frac{1}{3} \). It is unclear whether and how a change in the other suppliers’ intensities should have an effect on the final-good producer’s assigned optimal revenue share. Nevertheless, I actually find that the final-good producer’s share \( b^*(1) \) varies systematically in the non-associated input intensities \( e(2) \) and \( e(3) \). In particular, the final-good producer’s share \( b^*(1) \) reaches a minimum if both non-associated input intensities \( e(2) \) and \( e(3) \) are equal. If one of the non-associated inputs becomes relatively more important, i.e. the input intensity dispersion \( ||e(2) - e(3)|| \) increases, also the optimal final-good producer’s share \( b^*(1) \) increases. Moreover, total revenue and profits from the sale of the final goods rises in the dispersion of the non-associated inputs, see Figure 7 in the Appendix. I refer to this insight as the dispersion effect.

What is the economic intuition that the optimal share \( b^*(1) \) contributed to the final-good producer rises in the input intensity dispersion of the non-associated inputs? Two aspects have to be considered due to the two stage-production process: First, how do the assigned optimal revenue shares \( b^* \) change for variations in the non-associated input intensities. Second, how do the change in the assigned optimal revenue shares then alter the optimal contributions \( m^* \) of the intermediate input suppliers.

I start with the first aspect. I consider a decrease in the input intensity dispersion, e.g. a decrease in \( e(2) \) from Point \( Y \) to \( Z \) in Figure 3. With a decrease in \( e(2) \) the optimal incentives effect predicts that a final-good pro-
Figure 3: Optimal Revenue Shares

Figure 4: Optimal Contributions
ducer will decrease the optimal revenue share \( b^*(2) \). Vice versa, an increase in \( e(3) \), due to a fall in \( e(2) \), leads to an increase in the assigned optimal revenue share \( b^*(3) \). As Figure 3 illustrates, the decrease in \( b^*(2) \) is weak compared to the strong increase in \( b^*(3) \). This reallocation of revenue shares induces a decrease in the final-good producer’s own revenue share \( b(1) \).

Now, consider the second aspect. How does the weak decrease in the revenue share \( b^*(2) \) change the optimal contribution \( m^*(2) \)? As Figure 4 illustrates even the weak decrease in \( b^*(2) \) leads to a a sharp decrease in the optimal contribution \( m^*(2) \). In case of a decrease in \( e(2) \) from \( Y \) to \( Z \), it is therefore optimal for a final-good producer to decrease the assigned revenue share \( b^*(2) \) only weakly to counteract the sharp decrease in the contribution \( m^*(2) \) of that supplier. Now to the other intermediate input supplier. As Figure 4 illustrates a strong increase in \( b^*(3) \) leads to a weak decrease in the optimal contribution \( m^*(3) \). In case of an increase in \( e(3) \) from \( Y \) to \( Z \), it is therefore optimal for a final-good producer to increase the assigned revenue share \( b^*(2) \) sharply to counteract the weak increase in the contribution \( m^*(3) \) of that supplier. The mechanism can be summarized as follows: From the perspective of a final-good producer a decrease in the input intensity dispersion leads to the assignment of over proportionally high revenue shares to the other suppliers in order to counteract an over proportional decrease in the contribution of the now relatively more unimportant input supplier and an under proportional increase in the contribution of the now relatively more important input supplier. Proposition 3 summarizes the new effect.

**Proposition 3: Dispersion Effect**

1. The optimal revenue share \( b^*(i) \) assigned to an agent \( A(i) \), \( i = 1, 2, 3 \), increases in the dispersion of the non-associated input intensities, i.e. \( b^*(i) \) increases in \( ||e(j) - e(k)|| \) with \( i \neq j \neq k \), \( i, j, k = 1, 2, 3 \).

2. Total revenue and profits from the sale of the final goods rises in the dispersion of the non-associated input intensities.
4 Ownership Structure of Firms

In this section I introduce the ownership structure of firms. Therefore, I modify the two stage production process. In a setting with incomplete contracts the final-good producer and the intermediate-input suppliers cannot sign ex ante enforceable contracts specifying the purchase of specialized intermediate inputs. The original first stage is supplemented by the following decision of the final-good producer. The final-good producer cannot freely assign optimal revenue shares anymore. The input suppliers now bargain about the potential revenue from selling the final-good. I model the ex post bargaining as a multilateral generalized Nash bargaining game in which each agent $A(i)$ has a bargaining power $b(i) \in (0, 1)$ with $\int_I b(k) dk = 1$, and consequently obtains a fraction $b(i)$ of potential revenue from selling the final-goods. Following the property rights approach of the theory of the firm, ex post bargaining takes place both under outsourcing and under integration\(^2\). If the intermediate input production is accomplished within the boundaries of the firm and final-good production fails, the final good producer can enforce better residual rights as if the production is accomplished by an unaffiliated intermediate input supplier. The final-good producer’s outside option in the Nash bargaining is therefore better under integration than under outsourcing. Vice versa, intermediate input suppliers’ outside options are better under outsourcing than under integration. Let $\delta$ be the outside option of a representative intermediate input supplier $A(i)$, $i \neq h$, in case of outsourcing and assume for simplicity that the outside option in case of integration is zero. The ordering of the bargaining powers for a representative intermediate-input supplier $A(i)$, $i \neq h$, is

$$b^O (i) = \delta^\alpha + b^V (i) [1 - \delta^\alpha] > b^V (i).$$  \hfill (15)

The exogenous revenue share for an supplier $A(i)$, $i \neq h$, is $b^O (i)$ if the supplier is unaffiliated while it is $b^V (i)$ in case it is integrated within the boundaries of the firm.

\(^2\)See Grossman and Hart (1986)
Besides the difference in the revenue shares the ownership types also differ in the fixed costs of intermediate input production. As motivated in Antràs (2003) the fixed costs of intermediate-input production are higher if production is accomplished within the boundaries of the firm than under outsourcing, i.e.

\[ f^V (i) > f^O (i). \]  

(16)

With this supplement of the first stage the final-good producer cannot freely distribute the potential revenue from selling the final-goods anymore. For each intermediate input supplier \( A (i), i \neq h \), the final-good producer is now restricted to choose a revenue share \( b (i) \) from the the exogenous set \( \{b^O (i), b^V (i)\} \) with \( b^O (i), b^V (i) \in (0, 1) \). With this restriction the final-good producer now opts for the profit maximizing ownership type, by assigning either \( b^V \) or \( b^O \), and not for the optimal distribution of revenue \( b^* \) as derived in Proposition 2. As one will see, they are however, closely related. The formal maximization problem of the final-good producer in the extended first stage is therefore

\[
\max_{\{b^O (i), b^V (i)\}} \pi = \int I \pi (i).
\]  

(17)

\textbf{Solution 1st Stage}

I continue with the production process of the three intermediate inputs. The characteristics of the optimal distribution \( b^* \) of revenue is already discussed in the solution to the original first stage. However, the final-good producer now chooses \( b (2) \) and \( b (3) \) from the exogenous binary set \( \{b^O (i), b^V (i)\} \), \( i = 2, 3 \), in order to maximize total profits \( \pi \).

The decision whether it is optimal for the final-good producer to integrate or outsource intermediate input production now depends on the firm specific productivity level \( \theta \) and sector characteristics. As already discussed, the sector characteristics determine whether the assigned optimal revenue share \( b^* (i) \) to a supplier \( A (i) \) should be high or low. As the optimal incentives effects predicts the more important an intermediate input is for the final-

\[^3\text{I follow Antràs (2003) and assume that managerial overload dominates managerial economies of scope.}\]
goods production, the higher should be the assigned revenue share. Figure 4 assumes that the exogenous bargaining shares for each ownership type, a final-good producer can choose from, are identical for the two intermediate inputs, i.e. $b^O(2) = b^O(3)$ and $b^V(2) = b^V(3)$. Moreover, the final-good producers’ input intensity is fixed to $e(1) = \frac{1}{3}$.

![Figure 5: Bargaining Shares and Optimal Distribution of Revenue](image)

The trade-off between outsourcing and integration is two-sided: On the one hand, the final-good producer opts to provide optimal incentives by assigning revenue shares from the given exogenous set $\{b^O(i), b^V(i)\}$. On the other hand fixed costs are lower with outsourcing than with integration. With the help of Figure 5, the following sector-specific sorting pattern with respect to the firm specific productivity $\theta$ results. In a sector that is $m(2)$ intensive, e.g. $e_H(2)$ in Figure 5, joint profits are increasing with a higher revenue share $b(2)$ for the $m(2)$ intermediate input supplier $A(2)$. Vice versa, this sector is relatively low intensive in $m(3)$ intermediate inputs and joint profits are increasing with a lower revenue share $b(3)$ for the $m(3)$ intermediate input supplier $A(3)$. Now, the final-good producer examines the two aspects of the ownership type decision. First, outsourcing provides better incentives for the $m(2)$ input supplier $A(2)$. Second, fixed costs are also lower with outsourcing. Therefore, a final-good producer solely relies on outsourcing with respect to $m(2)$ inputs if and only if the sector is $e(2)$-intensive since out-
sourcing has both lower fixed costs and provides better incentives. However, with respect to $m(3)$ inputs, a coexistence of integration and outsourcing strategies prevails. The low productive firms benefit relatively more from the lower fixed costs of outsourcing while the high productive final-good producers favor the $m(3)$ suppliers’ lower revenue share in case of integration. Profits are increasing with lower levels of revenue for the $m(3)$ intermediate input supplier.

These sorting patterns are perfectly in line with what one would expect from an extension of the original Antràs and Helpman (2004) framework. They derived that in component-intensive sectors, read $m(2)$ intensive sectors, only outsourcing is prevalent. With a continuum of inputs the mechanism can be summarized as follows: The more important the input is for the final-goods production the more likely it is that the final-good producer will outsource intermediate input production.

5 Vertical Production Chain

The main assumption in the last section was that the exogenous bargaining shares for each ownership type were identical across agents. Now I relax this assumption. In particular, I am interested in introducing a vertical production into the models’ framework. The following definition allows to distinguish between different intermediate inputs and introduces a vertical production chain.

**Vertical Production Chain**

The intermediate input $m(i)$ is higher in the vertical production chain relative to the intermediate input $m(j)$, if and only if $b^V(i) > b^V(j)$ and $b^O(i) > b^O(j)$ holds.

I motivate this definition by the following economic intuition. Intermediate inputs on a higher stage of the vertical production are typically more specialized relative to generic intermediate inputs on lower levels. After the intermediate input production is accomplished a two sided hold-up problem
occurs with incomplete contracts. A tailor-made intermediate input makes it more difficult for the final-good producer to substitute it, while it is also more difficult for the intermediate input supplier to sell to an alternative final-good producer. My definition assumes that the first effect dominates the latter one. Intermediate input suppliers that provide an intermediate input that is on a higher stage of the vertical value chain capture relatively higher revenue shares. Note that this definition is independent of the input intensities.

Figure 6: Bargaining Shares with a Vertical Production Chain

Again, I study the three intermediate input production process. I consider the case that the $m(2)$ input is on a higher stage of the vertical production chain relative to the $m(3)$ input. The exogenous revenue shares differ substantially in Figure 6. The $m(2)$ input is so high in the vertical production chain such that $b^V(2) > b^*(2)$ holds for all $e(2) \in (0, \frac{3}{2})$. The $m(3)$ input is on a low stage of the vertical production chain and $b^O(2)$ is close to zero. The following sorting pattern results. In contrast to the case where both intermediate inputs were on the same vertical production chain the sorting pattern with respect to the $m(2)$ is independent of the input intensity $e(2)$. Now a coexistence of integration and outsourcing strategies prevails with respect to the $m(2)$ intermediate input supplier. The low productive firms benefit relatively more from the lower fixed costs of outsourcing while the
high productive final-good producers favor the \( m(2) \) suppliers’ lower revenue share in case of integration. With respect to the \( m(3) \) input supplier \( A(3) \) for almost all \( e(3) \in (0, \frac{2}{3}) \) the condition \( b^O(3) < b^*(3) \) is satisfied. A final good producer favors both the lower fixed costs and better incentives of outsourcing with respect to the \( m(3) \) input supplier. From the reasoning above and the comparison to the case without a vertical production chain it is clear that a higher position in the vertical production chain favors integration. I refer to this insight as the \textit{vertical production chain effect}.

**Proposition 4: Vertical Production Chain Effect**

\textit{A higher position in the vertical production chain favors integration.}

### 6 Conclusion

I extended the “global sourcing” framework by Antràs and Helpman (2004) by introducing a continuum of intermediate inputs in the production function. This approach allowed to study input specific sourcing strategies and various new effects emerged.

First, I derived the suppliers’ profit maximizing intermediate input contribution in the second stage. I find that the optimal input relation of two representative suppliers is independent of other suppliers.

Second, I derive the profit maximizing distribution of revenue, i.e. the optimal revenue shares that should ideally be assigned to each intermediate input supplier in the first stage. I find that the optimal revenue share contributed to a supplier systematically varies in the input intensities of the other suppliers, even if the own input intensity remains constant. It increases if the production process becomes more unequal in the inputs of the other suppliers, i.e. if the input intensity dispersion increases. Similar joint profits and revenues increase in the input intensity dispersion.

Third, dealing with a continuum of inputs enables to explicitly explore intermediate specific sourcing strategies. As already introduced by Antràs (2003) the sourcing strategies crucially depend on the input intensities. Recent empirical evidence by Alfaro and Charlton (2007) suggests that the
outsourcing versus integration decision does not solely relate to the characteristics of the countries and the characteristics of the products being produced. The authors suggest that intra firm trade and foreign investment activity may be better explained by more complex production processes involving several vertical production stages. Alfaro and Charlton (2007) find strong empirical evidence that final-good producers integrate stages of the production process which are close to the own final-good.

In the light of these new empirical findings the novelty of my approach is that it introduces a new determinant of sourcing strategies, namely a vertical production chain. I investigate how the introduction of a vertical value chain alters the predictions of the original Antràs and Helpman (2004) framework. With my definition of a vertical production chain I find that a higher position in the value chain makes it more likely that the corresponding intermediate input is integrated within the boundaries of the firm. Such a sourcing pattern seems to be consistent with the outsourcing practice reported in the business press, i.e. for European, Japanese and US producers of electronic appliances. Moreover, this prediction is also empirically testable and consistent with the recent study by Alfaro and Charlton (2007). They provide strong empirical evidence that multinationals tend to own the stages of production proximate to their final production.
7 Appendix

Proposition 1

I claim in (8) that

\[ m^*(i) = \frac{e(i) b(i)}{w(i)} \left( \mu \theta^\mu d^{\frac{\nu}{\alpha}} \right)^{\frac{1}{1-\mu}} \exp \left[ \frac{\mu}{1-\mu} \int_I e(k) \ln \left( \frac{b(k)}{w(i)} \right) dk \right] \]  

is the optimal input quantity contributed by the agent \( A(i) \) in the second stage. If all input suppliers contribute optimal inputs \( m^*(i) \), total revenue is given by

\[ R^* = d^{\frac{\nu}{\alpha}} \theta^\mu \exp \left[ \mu \int_I e(k) \ln \left( \frac{m^*(k)}{e(k)} \right) dk \right] \]

The first order condition of an agent \( A(i) \) in the 2nd stage is

\[ \frac{\partial \pi(i)}{\partial m(i)} = b(i) \frac{\partial R}{\partial m(i)} - w(i) = b(i) R \frac{e(i)}{m^*(i)} - w(i) = 0. \]  

I now have to show that

\[ b(i) R^* \frac{e(i)}{m^*(i)} = w(i). \]  

is fulfilled. I plug in \( R^* \) (18) and \( m^*(i) \) (17) into the LHS of (20) to get

\[ b(i) \mu e(i) \frac{d^{\frac{\nu}{\alpha}} \theta^\mu \mu^{\frac{\nu}{\alpha}} \exp \left[ \frac{\mu}{1-\mu} \int_I e(k) \ln \left( \frac{b(k)}{w(i)} \right) dk \right]}{w(i) \left( \mu \theta^\mu d^{\frac{\nu}{\alpha}} \right)^{\frac{1}{1-\mu}} \exp \left[ \frac{\mu}{1-\mu} \int_I e(k) \ln \left( \frac{b(k)}{w(k)} \right) dk \right]} = w(i) \]

Therefore \( m^*(i) \) solves the profits maximization problem of a representative agent \( A(i) \).
Proposition 2

The total value of the relationship is given by total profits, i.e.

\[
\pi = R - \int_I w(k) m(k) \, dk - \int_I f(k) \, dk
\]  

(23)

Plug in I plug in \( R^* \) (18) and \( m^* (i) \) (17) to get total value of the relationships is all agents contribute \( m^* (i) \)

\[
\begin{align*}
\pi^* &= \left[ d^{\frac{\mu-\alpha}{\alpha}} \theta^{\frac{\mu}{1-\mu}} \mu^{\frac{\mu}{1-\mu}} \exp \left[ \frac{\mu}{1 - \mu} \int_1^R I e(k) \ln \left( \frac{b(k)}{w(k)} \right) \, dk \right] ight. \\
&\quad - \left. d^{\frac{\mu-\alpha}{\alpha}} \theta^{\frac{\mu}{1-\mu}} \mu^{\frac{1}{1-\mu}} \exp \left[ \frac{\mu}{1 - \mu} \int_1^R I e(k) \ln \left( \frac{b(k)}{w(k)} \right) \, dk \right] \int_1^R I e(k) b(k) \, dk \right] \\
&\quad - \int_I f(k) \, dk \\
&= d^{\frac{\mu-\alpha}{\alpha}} (\theta \mu)^{\frac{\mu}{1-\mu}} \exp \left[ \frac{\mu}{1 - \mu} \int_I e(k) \ln \left( \frac{b(k)}{w(k)} \right) \, dk \right] \left[ 1 - \mu \int_I e(k) b(k) \, dk \right] \\
&\quad - \int_I f(k) \, dk
\end{align*}
\]

(24)

With \( \psi = \exp \left[ \frac{\mu}{1 - \mu} \int_I e(k) \ln \left( \frac{b(k)}{w(k)} \right) \, dk \right] \left[ 1 - \mu \int_I e(k) b(k) \, dk \right] \)  

(25)

Since I am interested in the profit maximizing distribution of revenue \( b^* \) the following notation of \( \pi^* \) is convenient:

\[
\pi^* = d^{\frac{\mu-\alpha}{\alpha}} \theta^{\frac{\mu}{1-\mu}} \psi - \int_I f(k) \, dk
\]  

(24)

with

\[
\psi = \exp \left[ \frac{\mu}{1 - \mu} \int_I e(k) \ln \left( \frac{b(k)}{w(k)} \right) \, dk \right] \left[ 1 - \mu \int_I e(k) b(k) \, dk \right]
\]  

(25)
Optimal Distribution of Revenue

In the following it is convenient to use the assumptions $\int_I b(k) \, dk = 1$ and $\int_e b(k) \, dk = 1$ to reformulate $\psi$:

$$
\tilde{\psi} = \exp \left[ \frac{\mu}{1 - \mu} \left( \int_I e(k) \ln \frac{b(k)}{w(k)} \, dk \right) \right] \exp \left[ \frac{\mu}{1 - \mu} \left( \ln \int_I b(k) \, dk \right) \right] \left[ 1 - \frac{\mu \int_I e(k) b(k) \, dk}{\int_I b(k) \, dk} \right] \left[ 1 - \frac{\mu \int_I e(k) b(k) \, dk}{\int_I b(k) \, dk} \right]
$$

(26)

is maximized. I now show that the distribution of revenue $b^*$ given in (12) maximizes total profits in the first stage. To do so, I have to proof that

$$
\left. \frac{d\tilde{\psi}}{db (i)} \right|_{b^*} = 0
$$

(27)

holds. Using extensively the product rule I get

$$
\frac{d\tilde{\psi}}{db (i)} = \left[ 1 - \frac{\mu \int_I e(k) b(k) \, dk}{\int_I b(k) \, dk} \right] \frac{d}{db (i)} \exp \left[ \frac{\mu}{1 - \mu} \left( \int_I e(k) \ln \frac{b(k)}{w(k)} \, dk \right) \right] \\
+ \exp \left[ \frac{\mu}{1 - \mu} \left( \ln \int_I b(k) \, dk \right) \right] \frac{d}{db (i)} \left[ 1 - \frac{\mu \int_I e(k) b(k) \, dk}{\int_I b(k) \, dk} \right] \\
= \left[ 1 - \frac{\mu \int_I e(k) b(k) \, dk}{\int_I b(k) \, dk} \right] \exp \left[ \frac{\mu}{1 - \mu} \left( \int_I e(k) \ln \frac{b(k)}{w(k)} \, dk \right) \right] \\
\times \frac{d}{db (i)} \frac{\mu}{1 - \mu} \left( \int_I e(k) \ln \frac{b(k)}{w(k)} \, dk - \ln \int_I b(k) \, dk \right) \\
- \left[ 1 - \frac{\mu \int_I e(k) b(k) \, dk}{\int_I b(k) \, dk} \right] \exp \left[ \frac{\mu}{1 - \mu} \left( \ln \int_I b(k) \, dk \right) \right] \\
\times \mu \frac{d}{db (i)} \frac{\mu \int_I e(k) b(k) \, dk}{\int_I b(k) \, dk}
$$
The first order condition \( \frac{d\tilde{\psi}}{db(i)} = 0 \) simplifies to

\[
0 = \left[ 1 - \frac{\mu \int_I e(k) b(k) dk}{\int_I b(k) dk} \right] \frac{d}{db(k)} \frac{\mu}{1 - \mu} \left( \int_I e(k) \ln \frac{b(k)}{w(k)} dk - \ln \int_I b(k) dk \right)
\]

\[
= \left[ \int_I b(k) dk - \mu \int_I e(k) b(k) dk \right] \frac{1}{1 - \mu} \left( \frac{e(i)}{b(i)} \int_I b(k) dk - 1 \right)
\]

\[
- e(i) \int_I b(k) dk + \int_I e(k) b(k) dk
\]

\[
= \left[ \int_I b(k) dk - \mu \int_I e(k) b(k) dk \right]
\]

\[
- (1 - \mu) e(i) b(i) \int_I b(k) dk + (1 - \mu) b(i) \int_I e(k) b(k) dk
\]

\[
e(i) \left( \int_I b(k) \right)^2 - \mu e(i) \int_I b(k) dk \int_I e(k) b(k) dk
\]

\[
- b(i) \int_I b(k) dk + \mu b(i) \int_I e(k) db(k) dk
\]

\[
- (1 - \mu) e(i) b(i) \int_I b(k) dk + (1 - \mu) b(i) \int_I e(k) b(k) dk
\]

\[
= e(i) - b(i) - (1 - \mu) e(i) b(i) + [b(i) - \mu e(i)] \int_I e(k) b(k) dk
\]

(28)

Last, I provide the derivation of \( b^* \) for the Antràs and Helpman (2004) case. In this two input case (27) is given by

\[
0 = e(1) - b(1) - (1 - \mu) e(1) b(1) + [b(1) - \mu e(1)]
\]

\[
[e(1) b(1) + (1 - e(1))(1 - b(1))] \]

(29)

Solve for \( b(1) \) leads to:

\[
b^*(1) = \frac{e(1) (\mu e(1) + 1 - \mu) - \sqrt{e(1)(1 - e(1))(1 - \mu e(1)) (\mu e(1) + 1 - \mu)}}{2e(1) - 1}
\]

and is equivalent to (10) in Antràs and Helpman (2004) for \( \eta = e(1), \beta^* = b^*(1) \) and \( \alpha = \mu. \)
References


Figure 7: Revenue and Profits