Second Thoughts on Exporter Productivity

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Abstract

Empirical literature has established a positive link between firm productivity and export status, yet notable exceptions exist. The present paper shows that the underlying theory (Melitz, 2003) is in fact able to accommodate the rule as well as the exception. The fulcrum of the argument is the tension between empirical work, measuring productivity based on average cost information and theoretical work, representing productivity via marginal cost. In a heterogeneous firms trade model, we compute productivity based on average cost and find that around the export-indifferent firm exporters will be less productive than non-exporters. Furthermore, simulation results illustrate that this effect may feed through to the aggregate level.

JEL: F12, F13, F15

Key Words: Intra-industry trade, firm productivity, monopolistic competition, heterogeneous firms.

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1 Introduction

Intra-industry trade models with heterogeneous firms have in recent years narrowed the gap between the theory and stylized facts of international trade. One of the central empirical findings is that export-active firms are more productive than their non-exporting counterparts, see the seminal contribution by Bernard and Jensen (1995); see Greenaway and Kneller (2007) and Wagner (2007) for comprehensive surveys of the literature that followed. The empirical regularity of productivity differences and heterogeneous firms is reconciled with the theory of international trade in the breakthrough paper of Melitz (2003).\(^1\)

Despite its status as a stylized fact of modern trade, the finding of an exporter productivity premium is subject to some notable exceptions. For example, Bernard and Wagner (1997) and Wagner (2002) for Germany, Liu et al. (1999) for Taiwan, Aw et al. (2000) for Korea, Head and Ries (2003) for Japan, Damijan et al. (2004) for Slovenia, Hansson and Lundin (2004) for Sweden, Girma et al. (2004) for Ireland, Girma et al. (2005) for the UK, Castellani and Zanfei (2007) for Italy, – inter alia – include findings where the hypothesis of exporters having higher productivity than non-exporters can not be supported for all the covered sectors, firm groupings, years, included productivity measures or estimation specifications. In parallel, a central debate in the empirical literature deals with pre- and post-entry differences, related to the causality between export status and firm productivity. Within this literature, pre-entry differences are almost always established, yet for post-entry differences (learning effects) the evidence is rather mixed, see the surveys of Greenaway and Kneller (2007) and Wagner (2007). For example for Sweden Greenaway et al. (2005) can not support a clear learning effect from exporting.

The present paper argues that the existing theory following Melitz (2003) is in fact fully capable of capturing both the rule as well as the exception. We investigate the concept of productivity in heterogeneous firms models. The underlying issue is that while empirical work is forced to measure productivity based on average cost information, theoretical work represents firm productivity more precisely by marginal costs. The switch between theoretical marginal cost and average cost empirical data will most likely be unproblematic for the majority of applications, yet it contains room for a settle ambiguity. We show that once the theoretical framework is used to compute ‘observed’ productivity based on average costs, the relative produc-

\(^1\)Simultaneously Bernard et al. (2003) provide a Ricardian model with similar features and addressing the same issue. Earlier theoretical contributions on the issue are made by Schmitt and Yu (2001), Montagna (2001) and Jean (2002).
tivity of exporters to non-exporters may go either way and will ultimately depend on the specific distribution of productivity draws in the economy. In particular, in the proximity to the export-indifferent firm exporters will have higher average costs and therefore lower observed productivities than non-exporters. Thus, if productivity is computed from average cost information, the theory does not in general predict that exporters are more productive than non-exporters.

We establish our result in a Meltiz (2003)-type framework. Since the critical driver of our finding is the decision of the export-indifferent firm and thus contingent on trade costs, we include additional trade costs apart from the customary iceberg costs. The iceberg cost specification in a marginal cost heterogeneity setting has the undesirable side effect that the lower marginal cost firm is not only more productive in producing goods, but is also more productive in transporting goods. This biases the export market self-selection of firms. Accordingly, we include – in addition to iceberg costs – homogeneous unit trade costs and ad valorem trade costs. The ranking of non-exporters displaying the higher productivity than exporters around the export-indifferent firm is established for all these cases.

Obviously, the effect highlighted here only matters in the proximity to the export-indifferent firm, and will thus for large enough productivity differentials not feed through to the aggregate level. The actual distribution of marginal costs matters. For sufficiently narrow distributions of marginal cost draws in the economy, the exporter productivity premium will become negative. In this case, the class of exporters will have higher average costs, and thus lower productivity, than their non-exporting counterparts, even though they have lower marginal costs.

The importance of the underlying distribution for the aggregate results may explain why previous work has not pointed at this issue. Furthermore, a central question in the theoretical literature has been on effects of trade liberalisation on economy wide productivity, not the relative productivity of exporters and non-exporters. For example Baldwin (2005) computes observed productivity in a similar fashion to the present paper, going beyond the weighted marginal cost-ranking contained in Melitz (2003). But since the measure is provided for a specific distribution function, namely a Pareto distribution, the effect highlighted in the present paper does not show up. If one computes the exporter productivity premium in Baldwin (2005), the large density of very unproductive non-exporters in the Pareto case dominates the

\[^2\text{In fact the finding is independent of the source of heterogeneity in the model. The inverse ranking occurs both for the usual marginal cost heterogeneity but also for the cases of fixed export costs heterogeneity among firms (e.g. Schmitt and Yu, 2001, Jørgensen and Schröder, 2008). See the Appendix for an illustration.}\]
cluster of relatively less-productive exporters close to the export-indifferent firm.

The remainder of the paper is structured as follows. The next section establishes the ranking of exporters and non-exporters in terms of observed productivity and in the proximity to the export-indifferent firm. Section 3 provides simulation results for a Melitz (2003) specification and shows that the effect may feed through to the aggregated level. Section 4 discusses results and derives implications for empirical research. Section 5 concludes.

2 Firm-level Productivity

In this section, we consider the Melitz (2003) framework, augmented to include ad valorem and homogeneous unit trade costs apart from the customary iceberg trade costs. Despite the extension the analysis remains highly tractable, since the central results of the Section rely solely on simple qualitative properties of firm-specific productivity measures for the export-indifferent firm.

All results presented are in fact independent of the specific source of heterogeneity in the model, as long as partitioning into exporters and non-exporters occurs. Thus, the source of heterogeneity could, for example, be differences in variable trade costs or fixed export costs, as in Schmitt and Yu (2001). However, in order to stay within the familiar Melitz (2003) setting, we only present results with reference to marginal cost heterogeneity.

Profit Expressions

As in Melitz (2003), demand for each variety is $q = Q \left( \frac{p}{P} \right)^{-\sigma}$ where $p$ is the price of the good, $Q$ aggregate demand and $P$ the price index. Monopolistic firms have production technologies with increasing returns due to fixed costs ($f$) and constant marginal productivity ($\phi$). To enter the industry firms invest ($f_E$) in developing a blue-print.3

Firms entering the export market face fixed export market access costs ($f_x$), iceberg costs ($\tau \geq 1$), as well as ad valorem ($t$) and unit ($T$) trade costs. Variation in blue-prints determines firm heterogeneity, i.e. variation in blue-prints is represented by differences in marginal productivity ($\phi$).

Given constant elasticity of substitution, firms set prices as a constant markup $\left( \frac{\sigma}{\sigma-1} \right)$ on marginal costs. Profits on the domestic and foreign markets

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3Firms enter the industry until the expected value of profits equals the sunk investment cost. Following the literature, we ignore time discounting, instead firms face a constant probability of death ($\delta$).
are

\[ \Pi^{\text{Dom}} = \left( p - \frac{w}{\varphi} \right) Q \left( \frac{p}{P} \right)^{-\sigma} - w f \]
\[ = \frac{1}{\sigma - 1} \left( \frac{w}{\varphi} \right)^{1-\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} Q P^\sigma - w f \] (1)

\[ \Pi^{\text{Exp}} = \left( p^* (1 - t) - T - \frac{w}{\varphi} \right) Q^* \left( \frac{p^*}{P^*} \right)^{-\sigma} - w f_x \]
\[ = \frac{1}{\sigma - 1} \left( T + \frac{w}{\varphi} \right)^{-\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} Q^* P^{*\sigma} - w f_x . \] (2)

where * denotes foreign market variables and \( w \) is the wage rate.\(^4\)

The only sunk costs are those to develop a blue-print, and thus export-indifferent firms are defined by \( \Pi^{\text{Exp}} = 0 \) and firms indifferent to leave the industry are defined by \( \Pi^{\text{Dom}} = 0.\)\(^5\)

**Observed Productivity**

Productivity rankings of firms in the theoretical literature are usually provided in terms of cost heterogeneity, typically marginal costs which are unique to each firm. Here we depart from this representation and introduce – in line with the empirical approaches – a productivity measure that is based on average costs, more precisely value added over factor use (see also Baldwin, 2005). In the specific model at hand, this reads value added per worker, since labour is the only factor of production. Moreover, under the above assumptions value added equals revenue less trade costs. Thus observed productivity, depending on the firms market presence, reads

\[ \rho = \begin{cases} 
\frac{p Q(\frac{p}{P})^{-\sigma} - \epsilon}{\frac{1}{\sigma} Q(\frac{p}{P})^{-\sigma} + f} & \text{if not exporting} \\
\frac{p Q(\frac{p}{P})^{-\sigma} + (p^*(1-t)-T)Q^* \frac{p^*}{P^*}^{-\sigma}}{\frac{1}{\sigma} Q(\frac{p}{P})^{-\sigma} + \epsilon \frac{1}{\sigma} Q^*(\frac{p^*}{P^*})^{-\sigma} + f + f_x} & \text{if exporting} 
\end{cases} \]

**Lemma 1.** Observed productivity \((\rho)\), contingent on export status, is continuous and increasing in marginal productivity \((\varphi)\)

\(^4\)Firms take the wage rate as given.

\(^5\)Following the literature we impose parameter restrictions such that there is partitioning and no firm chooses to operate on the export market alone.
Proof. Insert prices (e.g. \( p = \frac{w}{\varphi} \sigma \sigma -1 \) on the home market) and differentiate wrt. \( \varphi \).

The empirical relevant productivity measure (value added per worker) is positively related to marginal productivity, which is the common theoretical productivity measure in the literature. However, this is only the case for a given export status (pure domestic or export-active). Indeed for the export-indifferent firm productivity drops, when switching status.

**Proposition 1.** The export-indifferent firm’s observed productivity \( (\rho) \) over all sold units is lower when being export-active compared to being a pure domestic firm.

**Proof.** Denote variables of the export-indifferent firm by \( \tilde{\cdot} \). Using the productivity expressions and that \( \tilde{\Pi}^{\text{Exp}} = 0 \iff (p^* (1 - t) - T) Q^* (\frac{p^*}{p}) - \sigma = wf_x + \tau \frac{w}{\varphi} Q^* (\frac{p^*}{p}) - \sigma \) from (2), it follows for the observed productivity of a pure domestic, \( d \), and export-active, \( x \), export-indifferent firm that

\[
\tilde{\rho}^d > \tilde{\rho}^x
\]

\[
\iff \frac{p Q (\frac{p}{p}) - \sigma}{w Q (\frac{p}{p}) - \sigma + wf} > \frac{w Q (\frac{p}{p}) - \sigma + \tau \frac{w}{\varphi} Q^* (\frac{p^*}{p}) - \sigma}{w Q (\frac{p}{p}) - \sigma + wT + wf_x}
\]

\[
\iff \frac{1}{\sigma - 1} \left( \frac{w}{\varphi} \right)^{1-\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} Q P^\sigma - wf > 0
\]

\[
\iff \tilde{\Pi}^{\text{Dom}} (\varphi) > 0
\]

**Corollary 1.** Ranked by marginal productivity, \( \varphi \), there exists a cluster of firms around the export-indifferent firm such that all exporters in the cluster have lower observed productivity than the included non-exporters.

**Proof.** Follows from Lemma 1 and Proposition 1.

The crucial element for the above results is that non-variable costs of exports \( (f_x) \) enter the productivity measure, i.e. they are fixed but not sunk. These fixed costs, in the usual interpretation of the literature, are overhead market access costs that reoccur in each period, in contrast to the initial entry costs \( (f_E) \), which are sunk. The above results imply, that the productivity measure ‘value added per worker’ increases in marginal productivity \( (\varphi) \) contingent on export status but has a discrete fall when firms start to export. Thus the theory does not general predict that exporters have higher observed productivity than non-exporters.
Figure 1: Average costs of firms, ranked by marginal costs.

Figure 1 illustrates how the presence of fixed costs pushes a wedge between the ranking of firms according to marginal costs and average costs.

The intuition for our finding is straightforward. First, the firm that is just indifferent towards starting to export makes positive profits on the home market, else it would have exited to start with. The same is true for several of its neighboring non-exporting firms that have higher marginal costs but still make positive profits on the home market. They all have an operating surplus that more than covers their fixed costs of home production. Second, consider a situation where the indifferent firm exactly switches from non-exporting to exporting. Then we have added a zero-profit activity (namely exporting) to an otherwise profitable firm. In particular, the operating surplus that the export-indifferent firm can make on the foreign market exactly suffices to cover the fixed costs of exporting. But, building the average across all sold units (i.e. the profitable home sales and the zero-profit foreign sales), the indifferent firm’s average costs must have gone up when switching status. Thus, average profitability across all produced units must have gone down and hence its observed productivity must now be lower than that of the neighboring non-exporting firms.
3 Aggregate Exporter Productivity

In the previous section, we have considered rankings of firms’ observed productivity in the proximity to the export-indifferent firm; and we have shown that they go against the common empirical finding. This case occurs if productivity is – in line with the empirical measures – expressed via average cost information. We will now examine if also the average observed productivity among all exporters may fall below the average observed productivity of all non-exporters in equilibrium. From above we know that observed productivity increases in marginal productivity and features a one-off drop when firms switch status from pure domestic to export-active. A negative aggregate export productivity premium is thus more likely to occur when the distribution of marginal productivities of active firms is concentrated around the export cut-off value.

We verify this intuition numerically in a two-country Melitz (2003) model, i.e. taking account of general equilibrium effects, including endogenously determined cut-off values and firms entry decisions. One distribution of marginal productivity, that results in a negative export productivity premium at the aggregate level is given by the following: firms in the innovation process obtain viable blue-prints with probability $\frac{1}{2}$, and else draw blanks; the specific marginal productivities associated with the viable blue-prints are normally distributed with mean 1. In Figure 2, we plot the export productivity premium in the economy (i.e. the difference between average observed productivity of exporters and non-exporters, weighted by their respective size in the economy) for different levels of dispersion in marginal productivities, i.e. the standard deviation of the normal distribution.

The numerical analysis supports our intuition that the economies’ export productivity premium decreases with lower dispersion. Eventually the premium may turn negative for a sufficiently narrow distribution, i.e. for such cases the class of non-exporting firms is more productive than their exporting counterparts.

In contrast to the analytical approach in Section 2, where non-variable sunk costs are excluded from the productivity measure in order to obtain tractable results, the above numerical examples for a negative exporter productivity premium are obtained also when including sunk fixed cost.\textsuperscript{8}

\textsuperscript{6}The simulation codes are provided in a separate Appendix.
\textsuperscript{7}Other parameters of the model are given by $L = 1$, $f = f_x = \delta = 0.1$, $f_e = 0.02$, $\sigma = 1.2$ and $\tau = 1.25$.
\textsuperscript{8}In the estimations underlying Figure 2, we include both the market entry costs $f$ and $f_x$ as well as the initial entry costs $f_e$ in the observed productivity measure, thus biasing results against our finding. Since the sunk innovation costs are spread over more units...
Cost heterogeneity may manifest itself along several dimensions. We have in the main body of the paper focused on marginal cost heterogeneity, following Melitz (2003). However, the presented results are more general, and the identified mechanisms are also at work for the case of fixed cost heterogeneity (e.g. Schmitt and Yu, 2001, Jørgensen and Schröder, 2008) or variable trade cost heterogeneity, thus the possibility of a negative export premium is not confined to a world of marginal cost heterogeneity. We provide a simple analytically solved example of a negative export premium in a model with heterogeneous fixed export costs and homogeneous marginal costs in the Appendix.

4 Discussion

By examining an average cost-based productivity measure in a Melitz (2003) type setting, we have shown that the existing theory permits the productivity rankings between exporters and non-exporters to be inverted. The key point in exporting than non-exporting firms, their inclusion pushes the exporter productivity premium up. Nevertheless, we obtain numerical examples of negative export premia, see Figure 2.
is that empirically measured productivity is computed from average cost information, while the theory ranks firms according to marginal costs. Thus, predictions derived from theory have to take account of fixed costs, before they are compatible with the empirical approach. Fixed costs, for example the fixed costs of exporting, are a central element in the new theory. Such fixed costs vary in size, differ across firms and are well established in the empirical literature, e.g. Roberts and Tybout (1997); Das et al. (2001).

The findings of the present paper add some novel perspectives to the empirical discussion. In particular, we have shown that the heterogeneous firms trade theory following Melitz (2003) contains further and rich results for predictions on exporter productivity. In fact the theory suggests that for certain countries, industries, or periods it might well be the case that groups of non-exporters display higher measured productivity than groups of exporters. The actual distribution of productivity draws in the economy will matter crucially for the theoretical prediction.

In this light, the exceptions to the empirical regularity may deserve further examination. For example, Hanson and Lundin (2004) find for Sweden that exporters may be less productive than non-exporters and Greenaway et al. (2005) find that post-entry productivity gains do not materialize. These results for the Swedish economy have previously been thought to go against the theoretical prediction and have been explained with the greater openness of the Swedish economy. The current paper suggests, that these types of ‘exceptions’ could be partly driven by a more narrow (more homogeneous) productivity distribution among Swedish manufacturers.

Obviously, the present paper has not the mission or the space to verify such an alternative explanation empirically. Yet it is noteworthy that for example in Girma et al. (2004) an ambiguous finding in ranking pure domestic and export-active firms for Ireland contrasts with a clear ranking vis a vis multinationals, and that multinationals feature a broader spread in the productivity distribution.

Finally, our results may also provide a new perspective on the empirical pre- and post-entry productivity differences discussion (see e.g. the survey by Greenaway and Kneller, 2007). When expressing productivity via average costs, the theory predicts pre-entry productivity advantages of future exporters compared to future non-exporters, but post-entry productivity – on the individual firm level – should drop. Such an effect would distort measures of learning effects. The central reason is, that exporting firms may face reoccurring fixed export costs, which by definition will depress their observed productivity compared to their previous non-exporting periods.
5 Conclusion

In recent years, the empirical international economics literature has established a positive link between firm productivity and export status. Yet some notable exceptions exist. The present paper shows that the workhorse model of heterogeneous firms trade, i.e. Melitz (2003), does in fact accommodate both the rule as well as the exceptions.

The key point is that empirically observable productivity is computed from average cost information, while the theory ranks firms according to marginal costs. Thus, predictions derived from theory must take account of fixed costs, before they are compatible with the empirical approach.

The findings of the paper have two central implications for future empirical research. Firstly, in trade models with heterogeneous firms it will not generally be true that exporters are more productive than non-exporters, given that productivity measures include fixed costs of production and market access. The actual predictions of the theory for the aggregate level – the sign of the export productivity premium – will depend on the distribution of marginal productivity in the economy. Secondly, possible firm-level learning effects from engaging with foreign markets will to some extent be blurred by a drop in measured productivity that stems from the fixed export costs incurred by exporters.
Appendix: Negative export premium example under heterogeneous fixed export costs

Step 1: Consider a case where fixed export costs heterogeneity determines export status. Assume that countries are symmetric and that fixed costs of export are either prohibitive (infinite) or $f_e > 0$ with equal probability. Firms obtain a blueprint from a costly innovation process with probability $x \in (0, 1)$. Assume for the moment that firms with finite fixed costs of export actually export and that firms with prohibitive high fixed costs of exports serve the domestic market, then

$$
\rho = \begin{cases} 
\frac{\sigma}{\sigma - 1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} QP^\sigma - f 
& \text{if not exporting} \\
\frac{1}{\sigma - 1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} \tau^{1-\sigma} QP^\sigma - f_e 
& \text{if exporting}
\end{cases}
$$

and $\rho^d > \rho^e \iff f_x \tau^{\sigma-1} > \delta f_e + f$. Thus, if the weighted geometric average of fixed and variable trade costs are sufficiently high, exporting firms are less productive.

Step 2: Show that this situation can arise in equilibrium. We normalize wages and marginal costs equal to unity then

$$
\Pi^{\text{Dom}} = \frac{1}{\sigma - 1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} QP^\sigma - f \\
\Pi^{\text{Exp}} = \frac{1}{\sigma - 1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} \tau^{1-\sigma} QP^\sigma - f_e
$$

Expected profit equals $E\pi = x \left[ \frac{1}{2} (\Pi^{\text{Dom}} + \Pi^{\text{Exp}}) + \frac{1}{2} \Pi^{\text{Dom}} \right]$ and the zero-profit condition $E\pi = \delta f_e$ can be written as $\frac{1}{\sigma - 1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} QP^\sigma = \frac{\frac{3}{2} \delta f_e + f + 2f}{2 + \tau^{\sigma-1}}$. Now we verify that firms with prohibitive fixed export costs stay in the market and that firms with finite fixed export costs actually export. Inserting the zero-profit condition into the profit expression, we have $\Pi^{\text{Exp}} \geq 0 \iff f_x \tau^{\sigma-1} \leq \frac{1}{2} \delta f_e + f$ and $\Pi^{\text{Dom}} \geq 0 \iff f_x \tau^{\sigma-1} \geq f - \frac{3}{2} \delta f_e \tau^{\sigma-1}$. Hence for $f_x \tau^{\sigma-1} \in \left( \delta f_e + f, \frac{1}{2} \delta f_e + f \right]$ the export premium in the economy is negative.

Intuitively, this result says, there exist exporting costs sufficiently low to ensure positive export profits but sufficiently high to ensure exporter productivity is lower than non-exporter productivity.
Appendix: Ox code used in Section 3

//This programme calculates the average productivity among exporting and non-exporting firms in a Melitz(2003) type model//
#include<oxstd.h> #include<oxdraw.h> #include<oxprob.h>
#include<maximize.h> #import<maximize> #include<oxfloat.h>

//Model parameters//
const decl sigma=1.2; //Elasticity of substitution//
const decl L=1000; //Country size - makes no difference//
const decl fe=0.02; //Entry costs//
const decl fx=0.10; //Fixed costs of export//
const decl f=0.1; //Fixed costs of production//
const decl delta=0.1; //Probability of firm death - "discounting"//
const decl draws=10000; //Data points to "estimate" distribution of marginal productivity const
const decl n=1; //Number of foreign countries - if n>1 should fixed costs of exports then be fx or n*fx - in this programme fx is choosen sofar//
const decl obs=90; //Number of data points//

decl t,phi,raa,vp;

//The distribution function of marginal productivity and "average" marginal productivity//
of domestic firms given productivity is above a certain threshold//

fPhitilde(threshold) {
  decl i,sum,j,phitilde,out,G;
  j=0; sum=0;
  for (i=0;i<draws;++i)
  {
    if(phi[i]>threshold)
    {
      sum=phi[i]^(sigma-1)+sum;
      j=j+1;
    }
  }
  phitilde=(sum/j)^(1/(sigma-1)); G=1-j/draws;
  out=zeros(2,1); out[0]=phitilde; out[1]=G;
  return out;
}

//****************************************************************
//This bloc finds the thresholds//

//Expected profits//
fExpectedProfit(threshold) {
decl thresholdx, pi, h1, h2, px, pin, out;
thresholdx=threshold*t*(fx/f)^(1/(sigma-1));
h1=fPhitilde(threshold); h2=fPhitilde(thresholdx);
px=(1-h2[1])/(1-h1[1]); pin=1-h1[1];
pi=f*((h1[0]/threshold)^(sigma-1)-1)+px*n*fx*
    ((h2[0]/thresholdx)^(sigma-1)-1);
out=zeros(3,1); out[0]=pi; out[1]=pin; out[2]=px;
return out;
}

//Free entry or zero profit condition/
fFreeentry(const vP, const adFunc, const avScore, const amHessian) {
    decl h, eq, pi, pin;
    h=fExpectedProfit(vP[0]);
    pi=h[0]; pin=h[1];
    eq=pi-delta*fe/pin;
    adFunc[0]=-eq^2;
    return 1;
}

//Solving for the entry threshold/
fThreshold() {
    decl ir, dfunc;
    MaxControlEps(0.0000000001, 0.000001);
    ir=MaxBFGS(fFreeentry, &vp, &dfunc, 0, TRUE);
    return vp;
}

//This block derives the mass of firms and the price index

//"Average" marginal productivity of firms operating in a
given market/
fPhitildet(threshold, M) {
    decl phitildet, out, Mt, Mx, px, pin, h1, h2, thresholdx;
    thresholdx=threshold*t*(fx/f)^(1/(sigma-1));
    h1=fPhitilde(threshold);
    h2=fPhitilde(thresholdx);
    px=(1-h2[1])/(1-h1[1]); pin=1-h1[1];
    Mx=px*M; Mt=M+n*Mx;
    phitildet=((M*h1[0]^(sigma-1)+n*Mx*(h2[0]/t)^(sigma-1))/Mt)
        ^((1/(sigma-1));
    out=zeros(3,1);
    return out;
}

//Mass of firms//
\[ f(M, \text{threshold}) \] 
\[
\begin{align*}
&\text{decl } M, h, \pi, px; \\
&h = f\text{ExpectedProfit}(\text{threshold}); \\
&\pi = h[0]; \ px = h[2]; \\
&M = (L/\sigma)/(\pi+f+n*fx*px); \\
&\text{return } M;
\end{align*}
\]

//Price index//
\[
\begin{align*}
f(\text{Priceindex}, \text{threshold}, M) \{ \\
&\text{decl } P, h, \text{welfare}; \\
&h = f\text{Phitildet}(\text{threshold}, M); \\
&P = (h[1]^{(1/(1-\sigma))})/(\text{raa}\cdot h[0]); \\
&\text{return } P;
\end{align*}
\]

//*******************************************************************

//Average productivities (revenue/costs)//
\[
f(\text{AvgProd}(\text{threshold}) \{ \\
&\text{decl } \text{thresholdx}, \text{Avg}, \text{Avgx}, i, M, \text{out}, P, \text{rev}, \text{revx}, \text{labour}, \text{labourx}, \\
&m, k, \text{avg}, \text{avgx}, \text{avg1}, \text{avgx1}, R; \\
\text{thresholdx} = \text{threshold}\cdot t\cdot (fx/f)^{(1/(\sigma-1))}; \\
M = f(M(\text{threshold})); \\
P = f\text{Priceindex}(\text{threshold}, M); \\
R = L\cdot P^{(\sigma-1)}; \\
m = 0; \ k = 0; \ \text{avg} = 0; \ \text{avgx} = 0; \ \text{rev} = 0; \ \text{revx} = 0; \ \text{labour} = 0; \ \text{labourx} = 0; \\
\text{avgx1} = 0; \ \text{avg1} = 0; \\
\text{for } (i=0; i<\text{draws};++i) \\
\{ \\
\text{if } (\text{phi}[i] > \text{threshold} && \text{phi}[i] < \text{thresholdx}) \\
\{ \\
\text{rev} = R\cdot (\text{raa}\cdot \text{phi}[i])^{(\sigma-1)} + \text{rev}; \\
\text{labour} = (1/\text{phi}[i])\cdot R\cdot (\text{raa}\cdot \text{phi}[i])^{\sigma}\cdot f + \delta\cdot f + \text{labour}; \\
\text{avg} = \text{avg} + (R\cdot (\text{raa}\cdot \text{phi}[i])^{(\sigma-1)})/(1/\text{phi}[i])\cdot R\cdot \\
\quad (\text{raa}\cdot \text{phi}[i])^{\sigma}\cdot f + \delta\cdot f; \\
\text{avg1} = \text{avg1} + (R\cdot (\text{raa}\cdot \text{phi}[i])^{(\sigma-1)})/(1/\text{phi}[i])\cdot R\cdot \\
\quad (\text{raa}\cdot \text{phi}[i])^{\sigma}\cdot f; \\
\} \\
\text{if } (\text{phi}[i] > \text{thresholdx}) \\
\{ \\
\text{revx} = (R\cdot (\text{raa}\cdot \text{phi}[i])^{(\sigma-1)})\cdot (1+n\cdot t^{(1-\sigma)}) + \text{revx}; \\
\text{labourx} = (1+n\cdot t^{(1-\sigma)})/(1/\text{phi}[i])\cdot R\cdot (\text{raa}\cdot \text{phi}[i])^{\sigma}\cdot f + \delta\cdot f + \text{labour}; \\
\text{k} = \text{k} + 1; \\
\text{avgx} = \text{avgx} + (R\cdot (\text{raa}\cdot \text{phi}[i])^{(\sigma-1)})((1+n\cdot t^{(1-\sigma)})/(1/\text{phi}[i])\cdot R\cdot (\text{raa}\cdot \text{phi}[i])^{\sigma}\cdot f + \delta\cdot f + \text{labour}); \\
\text{avgx1} = \text{avgx1} + (R\cdot (\text{raa}\cdot \text{phi}[i])^{(\sigma-1)})((1+n\cdot t^{(1-\sigma)})/(1/\text{phi}[i])\cdot R\cdot (\text{raa}\cdot \text{phi}[i])^{\sigma}\cdot f + \delta\cdot f + \text{labour}); \\
\} \\
\} 
\]
avgx1=avgx1+((R*(raa*phi[i])^(sigma-1))*(1+n*t^(1-sigma)))/(1+n*t^(1-sigma))*(1/phi[i])*R*(raa*phi[i])^sigma+f+fx;

avg=avg/m; avgx=avgx/k;
avg1=avg1/m; avgx1=avgx1/k;
Avg=rev/labour; Avgx=revx/labourx;
out=zeros(4,1);
return out;

main() { decl i,f0,data,f1; raa=(sigma-1)/sigma; t=1.25; //Iceberg trade friction// vp=1.001; data=zeros(obs,5);
//Constructing marginal productivities f1=round(ranu(draws,1)); f0=rann(draws,1);

//Calculating variables of interest//
for (i=0;i<obs;++i)
{
    decl h,c,b,a;
    println(i);
    b=0.1-0.001*i; c=1;
    phi=f1.*(b*f0+c);
    h=fThreshold();
    data[i][0:3]=fAvgProd(h)';
    data[i][4]=b;
}

//Generating output//
DrawTitle(0, "Average productivity (black: non exporters, red: exporters"); DrawTitle(1, "Export productivity premium"); DrawX(0, data[] [0],data[] [4]'); DrawAdjust(ADJ_COLOR,1,TP_SOLID); DrawX(0, data[] [1],data[] [4]'); DrawX(1, data[] [2],data[] [4]');
ShowDrawWindow();
}
References


