A Model of Ideal Differentiation and Trade
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Abstract
The purpose of this paper is to explain the puzzling empirical observation by Mayer and Ottaviano (2007) that prices “defy gravity” by being higher in larger countries. To this aim I endogenize fixed costs in a model of monopolistic competition, where it is usually assumed that the firms’ technology is exogenous. My model assumes that firms differentiate their own product from that of others by spending more on fixed costs. The model solves for a symmetric equilibrium in fixed costs and the elasticity of substitution, with market size and trade cost effects that are congruent with the empirical evidence.

1 Introduction

The relationship between trade freeness, firm behavior, and the pattern of trade is one of the central questions facing international trade economists. This question has been approached in several veins of the literature. One important task is to understand the relationship between firm technology and trade liberalization. Recent literature has concentrated on the idea that "technology affects trade", mostly based on the Melitz (2003) mechanism whereby a firm’s productivity determines whether it exports or not. This paper aims to contribute to an understanding of the opposite idea, that "trade affects technology." This aspect of the technology/trade relationship has been less developed in the literature, but it surely is equally important to the understanding of firm behavior.

Another important task is to build a trade model that can explain some of
the new stylized facts about the pattern of trade. Mayer and Ottaviano (2007) found that larger economies export a greater variety of goods and export at higher prices. The authors also found that lower trade costs result in lower prices, but they did not have a theoretical explanation for this phenomenon. This paper offers a theoretical approach to explaining the pattern of prices in the data.

The purpose of this paper is thus to explain how trade liberalization affects firm technology, and how this in turn affects the pattern of trade. This paper’s innovation is to combine Sutton’s (1991) concept of endogenous sunk costs with the "new" models of trade. The model exhibits differences in the degree of product differentiation across countries, where larger countries produce manufacturing goods that are "more differentiated" than those of smaller countries. This is achieved in a horizontal product differentiation framework à la Krugman (1979, 1980). The innovation is to allow firms to first choose from a continuous set of technologies, with a more differentiated product requiring higher fixed cost outlays. The fixed cost outlays can be considered to be persuasive advertising or product development that differentiates one’s own product from that of other firms. Firms then compete via monopolistic competition. Since product differentiation is captured by the elasticity of substitution in this model, this provides endogenous markups characterized by "anti-competitive effects" (i.e. increasing markup). This contrasts the static markup of the Dixit-Stiglitz model of monopolistic competition and the "pro-competitive effects" (i.e. decreasing markup) of Lancaster (1979,1980).

While adding additional parameters of product differentiation to trade models is not a new concept, endogenizing product differentiation in this context appears to be novel. Modelling product differentiation based on country of origin has its origin in Armington (1969), which was adapted by Feenstra (1994) and
Broda and Weinstein (2006) to measure the gains from variety due to increased imports to the U.S. To the best of my knowledge there are no models in the literature incorporating the models of endogenous product differentiation with models of international trade in the manner that I propose.

The theoretical model in this paper provides results that agree with empirical studies of trade patterns. Namely, the model illustrates that larger countries export products at a higher price, and lower trade costs lead to lower prices. Moreover, larger countries export a greater variety of goods. All of these results are in accordance with the empirical findings of Mayer and Ottaviano (2007).

A particularly interesting result regarding trade liberalization in this model is that prices and fixed advertising outlays are highest at intermediate levels of trade costs. The model thus provides a theoretical explanation for why prices can increase due to trade liberalization, which can also help to make sense of the weak evidence in the literature that prices fall when trade liberalizes, as is found by Ravenga (1997), Treffer (2004), and Feenstra (2006).

The rest of the paper is organized as follows: The related literature is briefly overviewed in Section 2. The autarky model is presented in Section 3. The model is expanded to include two countries and iceberg trade costs in Section 4. Welfare effects are presented in Section 5. Testing the model is discussed in Section 6. Conclusions and suggestions for future research follow in Section 7.

2 Related Literature

The Dixit-Stiglitz (1977) model of monopolistic competition, which was applied to international trade by Krugman (1979, 1980), assumes exogenous technology. In response to this, there has been recent interest in creating models of international trade that endogenize firm technology. To this aim, dynamic models can explain the effect of trade on technological change. Dinopolous and
Segerstrom (1999), for example, show that trade liberalization increases firms’ R&D investment and the rate of technological change. Static models exploring the relationship between trade and technology can be sorted into two mechanisms: those where "trade affects technology" and those where "technology affects trade."

Concerning models of trade where "trade affects technology", the two main approaches in the literature are General Oligopolistic Equilibrium (Neary 2002, Neary 2003) and models based on Dixit-Stiglitz monopolistic competition. Endogenizing market structure in models of monopolistic competition in general equilibrium has been approached in several ways. These models can be categorized by the degree of firm heterogeneity. At one extreme there is the homogeneous firm model of Eckel (2006). Eckel modifies the standard Dixit-Stiglitz model by endogenizing fixed costs and assuming that variable costs are a decreasing function of fixed costs. Eckel also borrows the Lancasterian assumption that the elasticity of substitution is increasing in the number of products. Eckel finds that variety decreases as market size increases if variable costs are sufficiently sensitive to increases in fixed costs.

Moving towards models with more heterogeneity, there are the discrete-type heterogeneous firm models, including those by Markusen and Venables (1997) and Ekholm and Knarvik (2005). In those models it is assumed that there are two discrete types of firms, "small scale" firms (with high variable costs and low fixed costs) and "large scale" firms (with low variable costs and high fixed costs). Under certain restrictions on parameters, an equilibrium can arise where both domestic and multinational firms coexist in the market. Liberalization of the market enhances the profitability of multinationals compared to domestic firms, which leads to an increase in the number of multinational firms and a decrease in the number of domestic firms. Elberfeld and Götz (2002) come to a
similar conclusion using a model of monopolistic competition with two discrete technology types.

As for literature studying the mechanism of how "technology affects trade," Melitz (2003) is the most popular approach. In the Melitz model there is a continuum of heterogeneous firms that differ in their marginal productivity. Heterogeneity arises as firms randomly draw their productivity from a continuous distribution. Melitz and Ottaviano (2005) follow the original Melitz model closely, incorporating heterogeneous firms with endogenous markups. In their model, aggregate productivity and price markups respond to market size and the freeness of trade. However, while the markups in this model are endogenously determined, they are not a strategic variable of the firm. Instead they are determined by a firm's own random productivity draw. Akerman and Forslid (2007) modify the Melitz model by assuming that market entry costs are increasing in market size, which leads to predictions that match the stylized facts.

There is also recent work in the literature that attempts model the trade-technology relationship in both directions, incorporating heterogeneous firms or workers and endogenous technology choice. Yeaple (2005) incorporates a continuum of heterogeneous workers with endogenous technology choice. As in Ekholm and Knarvik (2005), discrete types of manufacturing-sector technologies are available and all technologies occur in equilibrium under certain parameter restrictions. Yeaple assumes a continuous distribution of heterogeneous workers that vary in ability, and firms are homogeneous. The model analyzes the effect of trade costs on entry, technology choice, whether or not to export, and the types of workers to employ. The results of the model are in accordance with many of the stylized facts of trade. Bustos (2005) combines the Melitz (2003) concept of heterogenous firm productivity with technology choice. Endogenous technology in these papers is a trade-off between fixed costs and variable costs.
and the technology choice is discrete.

Throughout all of the static general equilibrium models, there is a trade-off between higher fixed production costs and lower variable production costs. While this is a good way to model endogenous R&D, it cannot explain endogenous advertising, which is more likely to impact upon consumers’ preferences. This is the niche that the following model aims to fill. Furthermore, all but one of these models restricts technological choice to a discrete set. Arkolakis (2006), however, endogenizes market access costs such that the marginal cost of reaching additional consumers is increasing. Arkolakis uses the Butters (1977) theory of advertising which presumes that advertising’s function is purely informative. In contrast, my model uses the Sutton (1991) concept of persuasive advertising. While Sutton’s approach has been very influential in the IO literature, it has not been adapted for use in trade models.

There are also models of vertical product differentiation and trade, such as Flam and Helpman (1987) and Grossman and Helpman (1991), which predict that richer countries produce and export high-quality goods. Schott (2004) provides evidence that within a given product category, countries with higher GDP per-capita sell higher quality goods.

# 3 Autarky Model

## 3.1 Setting

I begin by describing the model in a closed economy. Assume there are two industries, with a homogeneous good $A$ exhibiting constant returns to scale (CRS), and a differentiated goods industry exhibiting increasing returns to scale (IRS). The IRS industry is composed of $N$ firms. I assume that $N$ is large so that there is no strategic interaction between differentiated firms when they set
their price or preference parameter. Labor mobile between industries. The price of
the CRS good is normalized to unity, which sets the wage to unity as well.
Any individual is endowed with one unit of labor.

3.2 Preferences (Consumers):
Consumers spend a portion \( \mu \) of their income on the differentiated good, and a
portion \( 1 - \mu \) on the homogeneous good. Utility for the differentiated goods is
additive and concave each good. Each good has its own "preference parameter,"
\( \sigma_i \), in the utility function.

A representative consumer’s utility maximization problem for differentiated
goods can be written as:

\[
\max U = \sum_{i=1}^{N} c_i^{\sigma_i-1} \quad \text{s.t.} \quad \sum_{i=1}^{N} p_i c_i = \mu
\]

where \( \sigma_i \in (1, \infty) \). The demand for a good by a representative consumer is
thus:

\[
c_i = \mu \frac{\left(\frac{\sigma_i}{\sigma_i - 1}\right)^{-\sigma_i} \lambda^{-\sigma_i} p_i^{-\sigma_i}}{\sum_{i=1}^{N} \left(\frac{\sigma_i}{\sigma_i - 1}\right)^{-\sigma_i} \lambda^{-\sigma_i} p_i^{-\sigma_i}} \quad i = 1, ..., N
\]

The price elasticity of demand is identical to that of a standard CES utility
function:

\[
\eta_i = \sigma_i
\]

One cannot obtain an analytical solution for the elasticity of substitution be-
cause the prices and quantity variables all have potentially different exponents.
However, in the symmetric equilibrium, where all the \( \sigma_i \)'s are the same, \( \sigma \) will
turn out to be the elasticity of substitution. One can also derive an expression
for the marginal utility of income:

\[ \lambda = \frac{\sigma_i - 1}{\sigma_i} c_i^{\sigma_i - 1} p_i^{-1} \]  

(2)

3.3 Technology (Firms) in IRS Industry:

3.3.1 Production Technology

Labor is the only input in this economy, and each IRS firm's total labor requirement \( l_i \) includes an endogenously determined fixed labor cost \( F_i \), and an exogenous variable amount of labor cost \( \beta \) in the production process.

\[ l_i = F_i + \beta x_i \]

The timing of differentiated good firms' decisions is as follows: In stage 1, firms decide whether or not to enter. In stage 2, firms optimally set their preference parameter \( \sigma_i \). In stage 3, firms set price via monopolistic competition. The equilibrium is found using backward induction.

3.3.2 Endogenous Technology

A technology equilibrium requires some sort of trade-off. Previous studies assuming a trade-off between fixed costs and variable costs, where higher fixed costs result in lower variable costs. The model in this paper departs from others in the literature by assuming that the fixed cost is a function of the "preference parameter," \( \sigma_i \),

\[ F_i = F_i(\sigma_i) \]  

(3)

where I assume that \( F'_i(\sigma_i) < 0, F''_i(\sigma_i) > 0 \). The concept that fixed costs affect a consumer preference parameter is inspired by the work of Sutton (1991). The idea is that the level of fixed costs are a function of the desired level of
product differentiation. Thus, differentiating one’s own product from others (i.e. lowering the preference parameter) would require higher fixed costs. These fixed costs could be persuasive advertising or product development that differentiates a firm’s own product from that of other firms. I do not assume a functional form at the moment, only that it is upward sloping and convex as $\sigma_i$ decreases, so that advertising expenditures exhibit decreasing returns. In addition, the domain of the function must lie within $(1, \infty)$, as this ensures that demand is elastic. An example of a particular advertising function is given in figure 1.

The notion that fixed costs can affect consumer preferences is not novel. Sutton’s (1991) partial equilibrium models endogenized fixed costs by assuming that they augmented demand. Arkolakis (2006) also uses an advertising function, but it exhibits decreasing returns to reaching consumers. In contrast, this paper’s advertising function exhibits decreasing returns to product differentiation. To the best of my knowledge, I cannot find any evidence in the literature of modelling an explicit relationship between the elasticity of substitution and fixed costs.

### 3.3.3 Stage 3: Setting price

In stage 3 the firm sets price in order to maximize profit, and takes endogenous fixed costs, $F_i$, as given from stage 2:

$$\max_{p_i} \Pi = p_i x_i - \beta x_i - F_i$$

The first order condition is:

$$p_i = \frac{\eta_i}{\eta_i - 1} \beta$$
Given the utility specification, one can substitute $\eta_i = \sigma_i$:

$$p_i = \frac{\sigma_i}{\sigma_i - 1}$$

(4)

One thus obtains markup pricing in stage 3, with a constant markup for a given $\sigma_i$. The markup is endogenous, however, since $\sigma_i$ is an endogenous variable chosen by the firm. Firms will take this into consideration when choosing $\sigma_i$ in stage 2.

3.3.4 Stage 2: Setting preference parameter:

In stage 2, each firm chooses their preference parameter, $\sigma_i$, to maximize profit:

$$\max_{\sigma_i} \pi_i = [p_i(\sigma_i) - \beta] x_i(\sigma_i) - F_i(\sigma_i)$$

Firm $i$’s demand is the sum of consumer demands, $x_i = Lc_i$. Using (1), the demand for a good in the manufacturing industry is:

$$x_i = \mu L \left( \sum_{i=1}^{N} \left( \frac{\sigma_i}{\sigma_i - 1} \right)^{-\sigma_i} \lambda^{-\sigma_i} p_i^{-\sigma_i} \right)$$

(5)

The large $N$ assumption means that firms ignore $\lambda$ and the denominator in (5) when setting their preference parameter. The first order condition for advertising in the symmetric equilibrium, after substituting in for $\lambda$ using (2) and expressing in elasticity form, is:

$$\frac{\sigma}{\sigma - 1} \frac{\ln x(\sigma)}{L} = \varepsilon_f(\sigma)$$

(6)

The left hand side of (6) is the elasticity of operating profits, $R = (p - \beta) x$,.
with respect to $\sigma$. The right hand side of (6) is the elasticity fixed costs with respect to $\sigma$, denoted as $\varepsilon_f (\sigma)$. Note that (6) equates the marginal revenue and marginal cost of increasing $\sigma$, which explains why both sides of (6) are negative. Given the assumptions about $F (\sigma)$, marginal cost increases and approaches infinity as $\sigma$ decreases to unity.

An illustration of the marginal revenue and marginal cost curves from (6) is provided in figure 1. Note that an isoelastic functional form of $F (\sigma)$ cannot exist since it is assumed that $F (\sigma)$ approaches infinity at $\sigma = 1$. The advertising function will thus have an elasticity that is negative and increasing in $\sigma$, i.e. $\frac{\partial \varepsilon_f (\sigma)}{\partial \sigma} > 0$.

It can be shown that (6) is a maximum under fairly general conditions (see Appendix A).

### 3.3.5 Stage 1: Entry decision

Firm entry occurs until profits equal zero. Combining the profit maximization condition and the zero profit condition, one obtains:

$$x = \frac{(\sigma - 1) F (\sigma)}{\beta}$$  \hspace{1cm} (7)

Inspection of (7) reveals that if the advertising function is $F (\sigma) = \frac{1}{\sigma - 1}$ then $x = \beta^{-1}$. The scale effect is thus absent in this particular form of the advertising function. However, $F (\sigma)$ can also be chosen such that $x$ is an increasing or decreasing function of $\sigma$. In the latter case, this gives a positive scale effect if greater market size leads to a lower $\sigma$. This is a useful property of this model, since one can obtain scale effects despite the CES utility specification.

The full employment of labor condition nails down the number of firms in the manufacturing industry:

$$N = \frac{\mu L}{F + \beta x}$$  \hspace{1cm} (8)
Overall, equations (2), (3), (4), (6), (7) and (8) make up the autarky model. This includes the same equations as a standard monopolistic competition model for profit maximization in price, zero profits, and full employment of labor, plus (3) and (6). The unknowns are \( p, x, N, \sigma, F, \) and \( \lambda. \)

This system of equations is not analytically solvable without further assumptions about the shape of the advertising function. The problematic equation is the first order condition for \( \sigma. \) Nonetheless, one can show without any further assumptions that \( \sigma \) is decreasing in market size. In addition, the system can be analytically solved for a special case of the advertising function, which is given further on in the paper.

A simulation of the autarky model for different market sizes is given in figure 2. The simulation uses an advertising function with a positive scale effect \( F(\sigma) = \frac{1}{(\sigma-1)^2}. \)

### 3.4 Market Size Effects

Substituting (2), (7) and (8) into (6) and using the implicit function theorem to solve \( \frac{\partial \sigma}{\partial L} \), provides the following condition:

\[
\frac{\partial \sigma}{\partial L} < 0 \iff \frac{x'(\sigma)}{x(\sigma)} < \frac{\partial \pi_f(\sigma)}{\partial \sigma} + (\sigma-1)^2 \frac{\sigma}{\partial L} > 0
\]

Thus it can be shown that \( \sigma \) decreases in market size under very general circumstances. This result means that goods become more differentiated as the market size increases.

The market size effect on \( \sigma \) affects all of the other endogenous variables in the model. As market size increases, fixed cost outlays will accordingly increase via the relationship specified in equation (3). Prices rise via the markup pricing rule (4). As for the the market size effect on variety, one can substitute (7) into
(8) to obtain a relationship between $N$ and $\sigma$:

$$N = \frac{\mu L}{\sigma F(\sigma)} \quad (9)$$

Equation (9) illustrates that the effect of market size on variety depends on how $F$ and $\sigma$ vary with market size. In contrast to Krugman (1980), the number of firms need not increase linearly with market size. Likewise, the effect of market size on firm scale also depends on the choice of advertising function, as can be seen in (7).

The predictions of this model agree with respect to market size effects agree with the stylized facts as discussed by Mayer and Ottaviano (2007). Moreover, it is generally accepted that firm size increases with market size (Schmalansee 1989 p.992). I now move on to discuss a particular specification of the advertising function that simplifies the analysis.

### 3.5 Solving the Autarky Model Fully

One problem with the model above is that an analytical solution cannot be obtained without further assumptions about the shape of the advertising function. This makes it difficult to interpret equation (6), the first order condition for $\sigma$. However, one can fully solve the autarky model using the following advertising function:

$$F(\sigma) = \frac{1}{\sigma - 1} \quad (10)$$

This form of the advertising is convex and downward sloping in $\sigma$, and is asymptotic to $\sigma = 1$ (see figure 1). This particular formulation has the special property of having no "scale effect", for reasons discussed below.

The first order and second order conditions using this particular advertising
function are:

\[ \frac{2\sigma}{\sigma - 1} = \ln \beta L \]

and

\[ \frac{-2}{(\sigma - 1)^2} < 0 \]

In this case the lagrange multiplier, \( \lambda \), can be removed from the system of equations and the analytical solution becomes:

\[
p = \frac{\ln \beta L}{2} \beta
\]

\[
x = \frac{1}{\beta}
\]

\[
N = \frac{2\mu L}{\ln \beta L}
\]

\[
\sigma = \frac{\ln \beta L}{\ln \beta L - 2}
\]

\[
F = \frac{\ln \beta L}{2} - 1
\]

This particular formulation of the advertising function is tractable because it eliminates the problem of having a logged \( \sigma \) term in the first order condition for \( \sigma \). It does this by forcing \( x \) to equal \( \frac{1}{\beta} \) instead of the usual \( \frac{(x-1)F(x)}{\beta} \). This means that there are no "scale effects" (market size effects on \( x \)) in this case. Fixed costs and price are increasing and concave in \( \beta \) and \( L \). Note also that \( \sigma \) is decreasing and convex in market size and marginal cost, and \( \lim_{L \to \infty} \sigma = 1 \). Variety increases concavely with market size, which is an intuitive result if one considers that \( N = \frac{\mu L}{px} \), where \( p \) increases concavely with market size. Any kind of positive scale effect would thus increase the concavity of variety growth.

Since the number of firms increases without bound one can posit that \( \lim_{N \to \infty} \sigma = 1 \). This is the opposite conclusion of Lancaster’s (1979, 1980) "Ideal Variety"
approach to modelling monopolistic competition, where \( \lim_{N \to \infty} \sigma = +\infty \). This result also contrasts with standard oligopoly theory, which generally asserts that markups decrease as the number of firms increase. The main reason for the contradictory result of this paper is that product differentiation is endogenous, whereas other models of trade take product differentiation as exogenous. The endogeneity of product differentiation expands the "product space" in the eyes of consumers, so that competition becomes less tough despite the fact that more firms are entering the market. This is the intuition behind the market size result that I obtain. One can obtain an analytical solution to the market size effect using the advertising function \( F(\sigma) = \frac{1}{\sigma^2} \):

\[
\frac{\partial \sigma}{\partial L} = \frac{-2L^{-1}}{(\ln \beta L - 2)^2} < 0
\]

### 3.6 A Closer Look at the Marginal Revenue Curve

The shape of the marginal revenue curve is difficult to interpret from (6), but it is illustrated in figure 1. Inspection of figure 1 reveals that the marginal revenue curve is decreasing, then increasing, as \( \sigma \) decreases. One can decompose the marginal revenue function into a "markup effect" and a "quantity effect" in order to understand this relationship. The operating profit of a firm can be given as:

\[
R = (p - \beta) x
\]

After log differentiating and simplifying one obtains:

\[
\frac{dR}{R} = \left( -\frac{\sigma}{\sigma - 1} \right) \frac{d\sigma}{\sigma} + \left[ \ln \frac{x}{L} + 2\frac{\sigma}{\sigma - 1} \right] \frac{d\sigma}{\sigma} \quad (11)
\]

The first term of (11) is the price effect, which is always positive for decreases in \( \sigma \) and simply equal to the markup pricing rule. The second term is the
quantity effect, which is either positive or negative for decreases in $\sigma$, depending on the sign of the term in the square brackets. One can see that $\ln \frac{x}{L}$ is strictly negative and $2 \frac{\sigma}{\sigma - 1}$ is strictly positive, and the relative size of these terms will determine the sign of the quantity effect. It is difficult to say anything about the this differential since $x$ and $\sigma$ are both endogenous. However, if one assumes that the advertising function takes the functional form $F(\sigma) = \frac{1}{\sigma - 1}$, then $x = \frac{1}{\beta}$, a constant, and one can see that decreasing marginal revenue occur can occur whenever $\frac{\sigma}{\sigma - 1} - \ln (\beta L) > 0$.

In sum, the price effect on the marginal revenue from decreases in $\sigma$ is always positive. The quantity effect on marginal revenue is also positive when $\sigma$ is large, but negative when $\sigma$ decreases. For low enough $\sigma$ the negative quantity effect outweighs the positive price effect, which results in a marginal revenue curve that is eventually downward sloping.

An alternative intuition is to recall that the quantity effect is driven by the term $\left(\frac{\sigma}{\sigma - 1}\right)^{-2\sigma_i}$ in the numerator of equation (5), while the price effect is driven by $\left(\frac{\sigma}{\sigma - 1}\right)^1$. The quantity effect dominates as $\sigma_i$ approaches unity.

4 Two Country Model

4.1 Setting, Preferences and Technology

Extending the autarky model to a two country model with iceberg trade costs yields interesting results regarding the firms’ technology choice and the pattern of trade. I begin by laying out the consumers’ and firms’ optimization problems and the relevant equations. I then interpret the effect of market size and trade liberalization on the endogenous variables.

The representative consumers at Home and Foreign solve the following utility maximization problems:
The $H$ and $F$ subscripts refer to Home and Foreign consumers respectively, while the asterisk superscripts refer to the foreign firm. Consumers are identical in both countries and spend $\mu$ of their income on differentiated goods and $1-\mu$ of their income on a homogeneous agricultural good. When differentiated goods are shipped between countries, $\tau$ units must be shipped in order for 1 unit to arrive. Within a country, all firms have identical sigmas, but they can differ between countries.

The relevant equations of the two country model are given in Appendix B. The two country model is not analytically solvable, but can be easily simulated.

### 4.2 Market Size Effects

Market size effects in the two country model work in the same way as they do in autarky, that is, $\sigma$ and $\sigma^*$ are both decreasing in $L$ and $L^*$. Firms in the country that face a greater market potential will have a lower sigma. If trade is costly, this means that firms in the larger country will produce goods that are more differentiated. This occurs since less of the world demand "melts away" from the perspective of firms in the larger country.

The model’s output under asymmetric country size is illustrated in figure 3. It follows from the asymmetric sigmas that firms in different countries will have asymmetric prices, fixed costs, and output per firm as well. Namely, the larger country will produce goods with higher markups, thus exporting higher priced manufactured goods and importing lower priced manufactured goods. Fixed costs will also be higher in the larger country, and output per firm in the larger
country will be greater if the advertising function induces scale effects, i.e. \( \frac{\partial x(\sigma)}{\partial \sigma} < 0 \). These predictions for prices and extensive margins are in accordance with the results of Mayer and Ottaviano (2007). The result that the elasticity of substitution is decreasing can also make some sense of Broda and Weinstein’s (2006) result that the elasticity of substitution has been decreasing over time despite the increase in "globalization" over the time of their sample.

If there are no trade costs, then firms in both countries face the same market potential and all the endogenous variables in both countries are identical.

### 4.3 Trade Cost Effects

The effect of trade costs on the endogenous variables is an interesting aspect of the model. It is illustrative to rearrange (24) under the special case where country sizes are identical (i.e. \( L = L^* \)):

\[
\frac{\sigma}{\sigma - 1} + \ln \left( \frac{x(\sigma)}{L(1 + \tau^{1-\sigma})} \right) = \frac{\tau^{1-\sigma} \ln \tau}{(1 + \tau^{1-\sigma})} = \frac{\partial f(\sigma)}{\partial L} \quad (12)
\]

This equation effectively divides the first order condition into two parts, a "market size effect" and a "trade friction effect".

The "market size effect" is almost identical to the left hand side of the first order condition in autarky, (6), except for the additional term \( 1 + \tau^{1-\sigma} \) multiplying \( L \) in the denominator. This term equals 1 under infinite trade costs and 2 under free trade, since free trade between two countries of equal size effectively doubles the market.

The "trade friction effect" is an additional term that is not present in the first order condition under autarky. Note that \( -\sigma \frac{\tau^{1-\sigma} \ln \tau}{(1 + \tau^{1-\sigma})} = \frac{\tau^{1-\sigma} \ln \tau}{(1 + \tau^{1-\sigma})} \frac{\partial L}{\partial \sigma} (1 + \tau^{1-\sigma}) \), which is the elasticity of market potential with respect to the elasticity of sub-
stitution. If trade costs are absent or infinite then the trade friction effect will disappear. This term is positive for intermediate trade costs, reaching a single maximum. The marginal revenue of decreasing $\sigma$ is thus maximized at some intermediate level of trade costs. This result illustrates that trade frictions affect more than just market potential; the friction itself enhances the marginal revenue of product differentiation. The intuition is that lowering $\sigma$ abates the loss of demand due to "melting", and the marginal benefit from this activity is greatest when "melting" is greatest (i.e. intermediate trade costs).

It can be helpful to analyze this result within a trade liberalization context. If two countries move from autarky to free trade, $\sigma$ first decreases, then increases as trade costs approach zero. Similarly, $F$, $p$, and possibly $x$ (if scale effects are present) first increase to a maximum at some intermediate level of trade costs, then decreases as trade costs approach zero. This contrasts with the monotonic market size effects that one observes in the autarky model.

When country sizes are symmetric the maximum trade friction effect occurs at the same level of trade costs in both countries. When the country sizes are asymmetric a similar result holds, but the extreme points do not occur at the same level of trade costs. Instead, the larger country reaches its minimum $\sigma$ and its maximum $F$, $p$, and $x$ at a higher trade cost level than the smaller country. Thus, smaller countries must liberalize trade more than larger countries before they experience lower prices. As trade costs decrease the endogenous variables "converge" to the same value across countries. Intuitively, this occurs because firms in both countries face an identical market potential under free trade and therefore respond by choosing the same level of product differentiation. These phenomena are illustrated in figure 3.
4.4 The Home Market Effect

In contrast to the standard model, the home market effect in this model involves more than just the number of goods produced in each country. Since prices and even quantities can differ across countries, a measurement of the home market effect must consider differences in the total value of exports of the manufactured good. Equations (26) and (27) can be converted into a variety share and market share formulation, where

\[ s_{pxN} = \frac{pxN}{pxN + pxN'} \quad \text{and} \quad s_L = \frac{L}{L + L'}; \]

\[ s_{pxN} = \frac{s_L}{1 - \phi (\lambda F)^{p^{+\sigma}} \Phi} + \frac{(1 - s_L) \phi}{\Phi (\lambda H)^{p^{+\sigma}} \Phi - \phi} \tag{13} \]

where \( \Phi = x^* \left( \frac{\sigma}{\sigma - 1} \right)^{\sigma - \sigma'} \left( \frac{p}{p^* - \sigma'} \right). \) If \( \sigma = \sigma^* \) then (13) reduces to

\[ s_{pxN, symmetric} = \frac{s_L (1 + \phi) - \phi}{1 - \phi}, \]

which is the same equation that is obtained in the standard model with exogenous technology. The condition where \( \sigma = \sigma^* \) only occurs in this model if the countries are of identical size.

Computing \( \frac{\partial s_{pxN}}{\partial s_L} \) is cumbersome, so I rely on simulation instead. Figure 4 illustrates that \( \frac{\partial s_{pxN}}{\partial s_L} \) is positive, which is the same result as the standard model with exogenous technology (i.e. Helpman and Krugman 1985). However, the home market magnification effect does not appear to be present in this model.

5 Welfare Effects

5.1 Autarky Case

This section analyzes the effect of market size on consumer welfare. I elect to use consumer’s indirect utility function for differentiated goods as a measure of
welfare. In autarky, the indirect utility function is clearly increasing in variety and decreasing in price. The effect of $\sigma$ on indirect utility, however, is unclear:

$$v = N^{\frac{1}{\sigma}} \left( \frac{\mu}{\bar{p}} \right)^{\frac{\sigma-1}{\sigma}}$$

The effect of market size on consumer welfare is thus not straightforward, since variety increases but the real wage decreases as the market expands. This contrasts with the standard model that assumes exogenous technology, where the real wage is constant and welfare effects occur exclusively via increased variety.

One can express the indirect utility function in terms of only $L$ and $\sigma$:

$$v(L, \sigma) = \mu L^{\frac{1}{\sigma}} \left[ \sigma F(\sigma) \right]^{-\frac{1}{\sigma}} \left( \frac{\sigma-1}{\sigma} \right)^{\frac{\sigma-1}{\sigma}}$$

The total differential of indirect utility with respect to market size is:

$$\frac{dv}{dL} = \frac{\partial v}{\partial L} + \frac{\partial v}{\partial \sigma} \frac{\partial \sigma}{\partial L}$$

The direct effect, $\frac{\partial v}{\partial L}$, is positive because of the increase in the number of varieties, but the indirect effect requires further calculation. It has already been shown that $\frac{\partial \sigma}{\partial L}$ is negative under fairly general conditions, so the effect of market size on utility depends on $\frac{\partial v}{\partial \sigma}$:

$$\frac{\partial v}{\partial \sigma} = N^{\frac{1}{\sigma}} \left( \frac{\sigma-1}{\sigma} \right)^{\frac{\sigma-1}{\sigma}} \left[ \ln \frac{\beta x}{L} - \frac{\sigma F'(\sigma)}{F(\sigma)} \right]$$

The condition for a negative sign on the derivative is:

$$\frac{\partial v}{\partial \sigma} < 0 \Leftrightarrow \ln \frac{\beta x}{L} - \frac{\sigma F'(\sigma)}{F(\sigma)} < 0$$  \hspace{1cm} (14)
It is not clear (14) holds, but if one assumes that the advertising function takes the form \( F(\sigma) = \frac{1}{\sigma^2} \) then the condition for a negative sign on the derivative is:

\[
\frac{\partial v}{\partial \sigma} < 0 \Leftrightarrow \ln \beta < \ln L
\]

This inequality will hold, which illustrates that market size has an unambiguously positive impact on utility in this case. Indeed, for any advertising function where (14) holds, market size positively affects consumer utility. Thus the positive "product differentiation" effect outweighs the negative real wage effect. The autarky simulation in figure 2 reveals that indirect utility is monotonically increasing in market size for an advertising function with positive scale effects.

5.2 Two Country Case - Welfare Effects

5.2.1 Welfare Effect of Market Size

The effect of changing market size or changing trade costs is not straightforward. However, it can be shown that larger markets experience greater utility, since the variety effect and the product differentiation effect dominate the real wage effect under fairly general conditions. Figure 4 illustrates that indirect utility is an increasing function of population share.

5.2.2 Welfare Effect of Trade Costs

The total differential of indirect utility with respect to market size is:

\[
\frac{dv}{d\tau} = \frac{\partial v}{\partial \tau} + \frac{\partial v}{\partial \sigma} \frac{\partial \sigma}{\partial \tau}
\]
Derivation of indirect utility with respect to $\tau$ reveals that the direct effect is clearly negative:

$$\frac{\partial}{\partial \tau} v(L, \tau, \sigma) = \mu L^{\frac{1}{\sigma}} [\sigma F(\sigma)]^{-\frac{1}{\sigma}} \left(\frac{\sigma - 1}{\sigma}\right)^{\frac{x+1}{\sigma}} \frac{1}{\sigma} \left(1 + \tau^{1-\sigma}\right)^{-\frac{1}{\sigma} - 1} (1 - \sigma)^{-\sigma} < 0$$

(15)

The indirect effect, $\frac{\partial \sigma}{\partial \tau}$, is more difficult to ascertain. It was shown in the previous section that $\frac{\partial \sigma}{\partial \tau}$ will be negative under fairly general circumstances. The problematic derivative is $\frac{\partial \tau}{\partial \tau}$, since (15) illustrates that $\frac{\partial \tau}{\partial \tau}$ is negative for lower trade costs and positive for higher trade costs, which makes prediction of the sign difficult. However, one can say that higher trade costs unequivocally result in lower utility when trade costs are high enough.

Figure 3 illustrates using a numerical simulation that indirect utility is monotonically increasing as trade costs decrease. Once again the positive variety differentiation effect outweighs the negative real wage effect.

5.3 Market Equilibrium versus Optimum

It can be useful to compare the market equilibrium with the constrained and unconstrained optimum, and also to compare to the results of the standard Dixit-Stiglitz model of monopolistic competition. The market equilibrium in autarky has already been described above, and differs from Krugman (1980) by pinning down the level of fixed costs and the elasticity of substitution.

The constrained optimum maximizes utility subject to firms breaking even. The constrained optimum in this paper differs from the standard result by also pinning down the level of fixed costs and the elasticity of substitution. The first order condition for the elasticity of subsitution in the constrained case is

$$\ln \frac{1}{\sigma} + \ln x = \varepsilon_f,$$
where $\varepsilon_f$ is the elasticity of fixed costs with respect to the elasticity of substitution.

This is very similar to the market equilibrium first order condition (6). The most notable difference is that market size does not matter in the constrained optimum. Assuming $F = \frac{1}{\sigma - 1}$, one can clearly see the condition for the market equilibrium to be more differentiated (i.e. lower $\sigma$) than constrained optimal:

$$\ln \frac{1}{\sigma} > \frac{\sigma}{\sigma - 1} - \ln L$$

Thus, the market equilibrium does not differentiate products enough in small markets, while it results in more than optimal product differentiation in large markets.

### 6 Testing the Model

This model predicts that fixed costs, markups, and output per firm are increasing functions of market size, and are maximized at some intermediate level of trade costs. Furthermore, the impact of trade liberalization on fixed costs, markups, and output per firm is more pronounced for smaller countries. These hypotheses may be empirically testable.

The most unique property of this model is the trade friction effect. A test of the model would be to derive a gravity model for prices. Mayer and Ottaviano (2007) estimated a gravity equation for prices with monotonic distance and market size effects. In this case, the gravity equation would have to allow for the non-monotonic trade friction effect to be evaluated. If such a gravity model revealed that prices are highest at intermediate trade costs, this would be compelling evidence for the existence of the trade friction effect.
7 Conclusion

The model presented in the paper takes a new look at product differentiation in the Dixit-Stiglitz model of monopolistic competition. Moreover, it has several attractive features that agree with the stylized facts on trade. First, the model allows for firms to endogenously choose from a continuous set of technologies by creating a trade-off between fixed costs and the elasticity of substitution. This is an appropriate assumption if fixed costs represent persuasive advertising or product development that differentiate one’s own product from others in the eyes of consumers. To the best of my knowledge, the idea of endogenizing the elasticity of substitution in order to endogenize technology is novel. Second, the model allows for scale effects, despite having CES utility properties in the symmetric equilibrium. Third, fixed costs, markups, and output per firm are increasing functions of market size, a characteristic that agrees with the literature. Fourth, the model produces "endogenous markups" that are a direct result of firms’ optimizing behavior.

The result that markups increase with market size may be considered somewhat controversial, since the "conventional wisdom" is that markups will decrease as market size increases, firms enter, and the competition becomes "tougher." This model offers an alternative explanation when product differentiation is endogenous and works via the elasticity of substitution parameter. As Tirole (1988, p.289) puts it, "Though it will be argued that advertising may foster competition by increasing the elasticity of demand (reducing "differentiation"), it is easy to find cases in which the reverse is true." It is hoped that this paper has given some theoretical foundation to this argument.
Appendix A

The second order condition is:

\[
\frac{\partial^2 \pi}{\partial \sigma^2} < 0 \Leftrightarrow \frac{x'(\sigma)}{x(\sigma)} < \frac{\partial \pi_f (\sigma)}{\partial \sigma} > 0 + (\sigma - 1)^{-2} > 0
\]

The second order condition is met as for any advertising function that exhibits non-negative scale effects (i.e. \( x'(\sigma) \leq 0 \)). This condition can also be met if \( x'(\sigma) > 0 \) as long as the inequality holds (i.e. as long as the scale effect is not too strong).

Appendix B

The marginal utilities of income for Home consumers can be expressed in two ways:

\[
\lambda_H = \frac{\sigma - 1}{\sigma} c_H^{\frac{1}{\sigma}} p^{-1}
\]

\[
\lambda_H = \frac{\sigma - 1}{\sigma} c_H^{\frac{1}{\sigma}} \tau^{-1} p^{-1}
\]

The demand for a Home firm’s good is:

\[
x = Le_H + \tau L^* c_F
\]

\[
= \mu L \frac{\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{\sigma}} \lambda_H^{\sigma} p^{-1}}{N \left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{\sigma}}} \lambda_H^{\sigma - \frac{\sigma}{\sigma-1} p^{1 - \sigma} + N^* \left(\frac{\sigma^*}{\sigma-1}\right)^{\frac{1}{\sigma^*}} \lambda_H^{\sigma^*} p^{1 - \sigma^*} \phi^*}
\]

\[
+ \mu L^* \frac{\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{\sigma}} \lambda_H^{\sigma - \frac{\sigma}{\sigma-1} \phi}}{N \left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{\sigma}}} \lambda_H^{\frac{\sigma}{\sigma-1} p - \sigma \phi} + N^* \left(\frac{\sigma^*}{\sigma-1}\right)^{\frac{1}{\sigma^*}} \lambda_H^{\sigma^*} p^{1 - \sigma^*}
\]
and the demand for a *Foreign* firm’s good is:

\[
x^* = \tau L c^*_H + L^* c^*_F \tag{17}
\]

\[
= \mu L \frac{\left(\frac{\sigma^*}{\sigma-1}\right)^{-\sigma^*} \lambda^{-\sigma^*}_H p^{1-\sigma^*} \phi^*}{N \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \lambda^{-\sigma}_H p^{1-\sigma} + N^* \left(\frac{\sigma^*}{\sigma-1}\right)^{-\sigma^*} \lambda^{-\sigma^*}_H p^{1-\sigma^*} \phi^*} + \mu L^* \frac{\left(\frac{\sigma^*}{\sigma^*-1}\right)^{-\sigma^*} \lambda^{-\sigma^*}_F p^{1-\sigma^*} \phi^*}{N \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \lambda^{-\sigma}_F p^{1-\sigma} + N^* \left(\frac{\sigma^*}{\sigma^*-1}\right)^{-\sigma^*} \lambda^{-\sigma^*}_F p^{1-\sigma^*} \phi^*}
\]

where \( \phi = \tau^{1-\sigma} \) and \( \phi^* = \tau^{1-\sigma^*} \).

As in the autarky model, firms set price in the last stage. The first order conditions for \( p \) and \( p^* \) are:

\[
p = \frac{\sigma}{\sigma - 1} \beta \tag{18}
\]

\[
p^* = \frac{\sigma^*}{\sigma^* - 1} \beta \tag{19}
\]

The zero profit conditions at *Home* and *Foreign* can be written as:

\[
x = \frac{(\sigma - 1) F(\sigma)}{\beta} \tag{20}
\]

and

\[
x^* = \frac{(\sigma^* - 1) F(\sigma^*)}{\beta} \tag{21}
\]

*Home* and *Foreign* firms choose the optimum level of \( \sigma, \sigma^*, F \) and \( F^* \) in stage 2. The advertising functions for *Home* and *Foreign* firms are:

\[
F = F(\sigma) \tag{22}
\]

\[
F^* = F(\sigma^*) \tag{23}
\]

Firms in both countries thus abide by the same advertising function, but the
fixed cost outlays will differ across countries if the equilibrium $\sigma$’s differ.

The first order conditions for $\sigma$ and $\sigma^*$ are:

\[
\mu L \left[ (\sigma - 1)^{-1} + 2 \ln \frac{\sigma - 1}{\sigma} - \ln \beta \lambda_H \right] + \mu L^* \left[ (\sigma^* - 1)^{-1} + 2 \ln \frac{\sigma^* - 1}{\sigma^*} - \ln \tau \lambda_F \right] = F'(\sigma)
\]

and

\[
\mu L \left[ (\sigma^* - 1)^{-1} + 2 \ln \frac{\sigma^* - 1}{\sigma^*} - \ln \tau \lambda_H \right] + \mu L^* \left[ (\sigma - 1)^{-1} + 2 \ln \frac{\sigma - 1}{\sigma} - \ln \beta \lambda_F \right] = F'(\sigma^*)
\]

where $\Psi = \frac{\beta \lambda_H}{\beta \lambda_F} - \frac{\sigma^* (\sigma^* - 1)}{(\sigma^* - 1)^{\sigma^*}}$ and $\Omega = \frac{(\beta \lambda_F)^{\sigma^* (\sigma^* - 1)} (\frac{\sigma^*}{\sigma^*})^{\sigma^*}}{\beta \lambda_H^{\sigma (\sigma - 1)} (\frac{\sigma}{\sigma})^{\sigma}}$.

The equilibrium expressions for $N$ and $N^*$ are derived from (16) and (17):

\[
N = \frac{\mu L}{p x (1 - \phi)} \left[ \frac{L^* \phi}{1 - \phi} - \frac{L \phi^*}{1 - \phi} \right]
\]

and

\[
N^* = \frac{\mu L^*}{p^* (1 - \phi^*)} \left[ \frac{L \phi^*}{1 - \phi^*} - \frac{L^* \phi}{1 - \phi} \right]
\]

where $\Phi = \frac{\sigma}{\sigma^*} \frac{(\sigma^* - 1)}{(\sigma - 1)^{\sigma^*}}$.

In order to illustrate the expressions in (26) and (27) to the standard model with exogenous technology, as can be found in Helpman and Krugman (1985), one can arbitrarily set $\sigma = \sigma^*$, which reduces (26) and (27) to

\[
N_{\text{symmetric}} = \frac{\mu L}{p x (1 - \phi)} (L - \phi L^*)
\]

and

\[
N^*_{\text{symmetric}} = \frac{\mu L^*}{p^* (1 - \phi)} (L^* - \phi L)
\]
Equations (28) and (29) are identical to those of the standard model with exogenous technology. The condition where $\sigma = \sigma^*$ only occurs in this model if the countries are of identical size. One crucial difference between (26) and (27) and the standard Helpman and Krugman expressions is that $px$ and $p^*x^*$ will increase as the market expands in (26) and (27), which negatively affects variety. On the other hand, $L$ and $L^*$ are divided by terms in the square brackets that will change as the market expands.

Overall, equations (18) through (27) make up the two country model.
References


Figure 1: Advertising, Marginal Revenue and Marginal Cost Curves, $F(\sigma) = \frac{1}{1-\sigma}$, $L = 100$, $\beta = 1$
Figure 2: Autarky simulation, $F(\sigma) = \frac{1}{(\sigma-1)^2}$, $\mu = 0.5, \beta = 1$
Figure 3: Two country simulation, varying trade costs, $L = 120$, $L^* = 100$, $F(\sigma) = \frac{1}{\sigma - 1}$, $\mu = 0.5$, $\beta = 1$
Figure 4: Two country simulation, varying population share, $\tau = 2$, $F(\sigma) = \frac{1}{\sigma - 1}$, $\mu = 0.5$, $\beta = 1$, $L + L^* = 220$