

Trade between symmetric countries, heterogeneous firms and the skill wage premium

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Abstract

This paper examines the effects of trade liberalisation between symmetric countries on the skill wage premium. I use a model of monopolistic competition with heterogeneous firms and two factors of production: skilled and unskilled labour. I introduce a correlation between productivity and skill intensity in the production process, which generates the empirically observed link between firm size, export status, wages and skill intensity. The entry and exit of firms following trade liberalisation has non-trivial effects on the demand for both types of labour, and therefore on their wages. I show that the impact of trade liberalisation on the skill wage premium depends on the type of trade costs considered, and on their initial size. A decrease in the fixed costs of trade has a potentially non-monotonic effect: it increases the skill premium in a world where the costs of international trade are high, but reduces it if trade costs are already low. On the other hand, a drop in the variable trade costs yields an unambiguous increase in the skill wage premium.

Keywords: Intra-industry trade, Trade and wages, Firm heterogeneity, Trade liberalisation, Skill Premium

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1 Introduction

The strong rise in wage inequalities has, in the last two decades, been one of the most prominent political issues in industrial countries. Many in the popular debate have emphasised the negative role of increased competition from developing countries and tended to blame globalisation as the main factor explaining this trend. Trade liberalisation has, however, never been *en vogue* among economists as a potential explanation. A major reason for this is that most trade studies have concentrated on models based on North-South heterogeneity¹, thereby leaving aside the bulk part of international trade, made of exchanges between industrialised countries. This may seem surprising to those familiar with the recent evolutions in the theory of international trade. The last twenty years have seen a dramatic surge of theories explaining North-North trade, based on monopolistic competition embedded in single industries, but only few conclusions have been drawn from this type of trade for wage inequalities.

It is only recently that trade economists have turned to intra-industry trade models as a potential determinant of the evolution of the skill wage premium. Epifani and Gancia (2007) assume a correlation between the scale and the skill intensity of a sector, and follow Dinopoulos et al. (2002) who assumed such a correlation at the firm level. With appropriate assumptions on preferences, the increase in scale inherent to trade liberalisation therefore exerts a bias towards skill demand and raises the skill wage premium. Yeaple (2005) uses a monopolistic competition model in which ex-ante homogeneous firms can choose between two production technologies with different complementarities to skill labour. A central feature of his model is that exporting firms use the high productivity technology, with which high skilled labour is more productive. Trade liberalisation expands the set of exporting firms and therefore pushes the demand for and the wage of highly skilled labour up. These three studies all show that trade liberalisation yields an increase in the skill wage premium.

Recent evidence however shows that the increase in the skill wage premium has been slowing down over the last decade², suggesting that its evolution may be less straightforward than previously suggested by most trade models. In order to explain this fact, I use a one-sector monopolistic competition framework with heterogeneous firms, largely building on the seminal work of Melitz (2003), and extend it to two factors of production:

¹see Heckscher-Ohlin type models, or more recently outsourcing models such as Feenstra and Hanson (2001, 1995)

²see Lemieux (2006) and Card and DiNardo (2002) for evidence on that matter.

skilled and unskilled labour. I assume that firms are heterogeneous in the relative productivity of skilled labour, in the sense that some technologies use skills more effectively than others. This establishes a correlation between productivity, skill intensity and exports: more productive firms are relatively skill intensive and export more than other firms. This correlation is analog to that obtained in Yavas (2006), and provides a similar condition to the link between skill intensity and scale in Epifani and Gancia (2007) and Dinopoulos et al. (2002). As in Melitz (2003), the present model is built on two types of trade costs: variable costs, which can be interpreted as transport costs or as tariffs, and fixed export costs, usually seen as administrative costs to export in a foreign country.

The novel result of this approach is that the evolution of the skill wage premium depends on two factors: (i) the kind of trade costs considered (ii) the size of the trade costs before liberalisation takes place. A reduction in the variable costs of trade unambiguously raises the skill wage premium. However, this tendency slows down as the economies become more open. This is a particularly interesting conclusion considering the recent evidence on the evolution of the skill wage premium. On the other hand, if there are no variable costs, a drop in the fixed costs of trade has a non-monotonic effect on the skill wage premium. If the initial costs of trade are high, liberalisation increases the skill wage premium, while liberalisation from initially low costs decreases the skill premium. I also show that a one percent reduction in the fixed costs of trade is always less detrimental for inequalities than a one percentage decrease in the variable costs. Furthermore, I provide sufficient conditions for existence and uniqueness of an equilibrium with costly trade.

The core mechanism yielding the results is the following: trade liberalisation makes it cheaper to export goods, so that relatively productive, skill intensive firms benefit from this and scale up their production. Relatively unproductive firms are, on the other hand, driven out of the market, releasing more unskilled labour. Both effects tend to raise the skill premium. The skill intensity of firms newly entering the export market is however undetermined. At low levels of initial trade openness, only productive firms, with a higher than average skill intensity enter the export market following liberalisation. This drives the skill premium further upwards. If the costs of trade are initially low, however, the marginal firms entering the export market are unskilled intensive, and their entry slows down the increase in the skill wage premium. This cannot overcompensate the first two effects, so that the skill wage premium unambiguously increases. On the other hand, if the fixed costs of exporting decrease, the first of the three effects above does not occur, and the skill wage premium can decrease if trade is initially open.

This paper is related to the nascent literature on the skill premium and intra-industry trade³, but extends it to a heterogeneous firms framework, thereby yielding richer conclusions. Yavas (2006) also builds on the Melitz (2003) model for her analysis, but restricts herself to the case of fixed export costs and therefore misses the variety of results I derive. I moreover provide sufficient conditions for existence and uniqueness of a costly trade equilibrium, which play a crucial role in the analysis.

The second strand of literature to which this paper relates is the rapidly expanding field of heterogeneous firm models of trade⁴. The present model matches some important empirical features about exporting firms emphasised in this literature. Not surprisingly since I largely build on Melitz (2003), the fact that exporting firms are bigger and more productive is preserved, and conform to the conclusions of Bernard and Jensen (1995), Bernard et al. (2003) and many others. Additionally, the model typically generates higher average wages for exporting firms⁵ due to their employing relatively more skilled labour, a feature which is conform to the results of Bernard and Jensen (1995). Bernard and Jensen (1997) provide a strong empirical case for my approach, arguing that "the between plant movement of workers and wages, which are especially important in the increases in the aggregate wage gap, are largely determined by demand shifts across plants, and in particular by export related demand movements"⁶.

The remainder of the paper is structured as follows. Section 2 develops the model. Section 3 derives the comparative statics of the relative wage and constitutes the core of this paper. Section 4 extends the results of the model to different or more general set-ups. Section 5 provides a numerical solution to the model for illustrative and quantitative purposes. Section 6 concludes.

³Dinopoulos et al. (2002), Yeaple (2005), Epifani and Gancia (2007) and Yavas (2006)

⁴Melitz (2003), Bernard et al. (2003), Helpman et al. (2004), Bernard et al. (2004) among others

⁵Under the sufficient and plausible condition that there is weakly less skilled than unskilled labour in the economy and that skilled labour is at least as productive as unskilled labour.

⁶Bernard and Jensen (1997) p.25-26

2 The model

2.1 The Firms

There is a continuum of firms, each producing a different variety. Production uses two factors, skilled (s) and unskilled (u) labour, both in fixed aggregate supply. They are combined in a constant elasticity of substitution (CES) production function:

$$y = \left[u^{\frac{\sigma-1}{\sigma}} + z^{\frac{1}{\sigma}} s^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (1)$$

$\sigma > 1$ is the elasticity of substitution between the two factors of production. This is an empirically founded assumption, as most studies estimate a parameter σ between 1 and 2.⁷ Firms are heterogeneous as to the productivity of skilled labour, indexed by z , which is the realisation of a random variable, drawn from an exogenously given continuous distribution. Intuitively, there exist different production technologies in a sector. To produce shoes, a firm can employ unskilled workers, using a rudimentary technology, for which skilled labour is no more productive than unskilled labour. On the other hand, a shoe producer can use a more advanced technology, requiring high-quality machines, and therefore engineers, who are more productive than unskilled labour. The randomness in this model is the technology, i.e. the productivity of skilled labour, drawn by each entrepreneur.

Acemoglu (2002) among others uses the same form of production function to study skill-biased technological change, where $z^{\frac{1}{\sigma}}$ increases exogenously over time. Note that z enters to the power $\frac{1}{\sigma}$ only to simplify the algebra at later stages. Any strictly increasing function of z would yield similar results.

Firms can sell their product on their domestic⁸ market (subscript d), as well as on the export market (subscript ex)⁹. Production in each market involves fixed costs, common to all firms of same origin. This generates increasing returns to scale, as in standard models of monopolistic competition. Fixed costs of domestic production (α) and of exporting (α_{ex}) are paid in terms of an internationally mobile asset in infinitely elastic supply, with a

⁷see Acemoglu (2002) p.20

⁸I use "domestic" when consumption takes place in the country of production. "domestic" is therefore different from "home", and symmetrically, "export" is different from "foreign".

⁹I assume throughout this article that there are only two countries. Extending this result to more countries is straightforward.

price normalised to one. I assume that $\alpha_{ex} \geq \alpha$, a rather innocuous assumption considering that doing business in another country requires additional costs of learning a legal system, customs, language, etc. There are moreover iceberg costs of exporting production. When a firm exports a unit of a good, a fraction $\tau < 1$ arrives at destination. These costs can be interpreted either as tariffs, or as transportation costs. They generate a set of firms which produce on their domestic market but do not export, because exporting is more expensive than selling on the domestic market.

The profits of selling on the domestic and on the export market are given by:

$$\pi_i(z) = p_i(z)c_i(z) - w_U u_i(z) - w_S s_i(z) - \alpha_i \quad \text{for } i = \{ex, d\} \quad (2)$$

$p_d(z)$ denotes the price at which a firm producing with characteristic z sells its product on the domestic market, and $p_{ex}(z)$, the price charged on the export market. w_U is the wage of unskilled labour and w_S the wage of skilled labour. Labour is perfectly mobile between firms in a country. This, and the assumption of symmetry between the two countries, guarantees that there is a single unskilled, and a single skilled wage for both countries. $c_{ex}(z)$ and $c_d(z)$ denote consumption respectively on the domestic and export market. Due to the iceberg costs of exporting, the goods market equilibrium implies:

$$y_d(z) = c_d(z) \quad (3)$$

$$y_{ex}(z) = \frac{c_{ex}(z)}{\tau} \quad (4)$$

Where y_d and y_{ex} are the production aimed at the domestic and export markets. Plugging this in (2), profit maximisation yields the following condition:

$$s_i(z) = \left(\frac{w_S}{w_U} \right)^{-\sigma} z u_i(z) \quad \text{for } i = \{d, ex\} \quad (5)$$

The higher the z drawn by a firm, the higher its skilled to unskilled labour ratio. More productive firms are therefore more skilled intensive. This is a plausible correlation, empirically found by a number of studies, such as Idson and Oi (1999), Haltinwanger et al. (1999) and Bernard and Jensen (1995). The relative demand for skilled and unskilled labour also depends on their relative wage.

$$p_d(z) = \frac{\epsilon}{\epsilon - 1} (w_U^{1-\sigma} + zw_S^{1-\sigma})^{\frac{1}{1-\sigma}} \quad (6)$$

$$p_{ex}(z) = \frac{p_d}{\tau} \quad (7)$$

Where ϵ is the consumers' elasticity of substitution between goods in a CES utility function. Price is therefore the unit cost of production times a markup.

Nothing guarantees that all firms make positive profits. On the contrary, those having drawn a low z may not produce if they find that they cannot cover the fixed costs of production. I turn here to the study of the cutoff firm, i.e. that having drawn a technology (z) for which it produces and breaks even. I define z^* as the cutoff level of z for which profits are zero in the domestic market, and z_{ex}^* , the level for which profits on the export market are zero, i.e.:

$$\begin{aligned} \pi_d(z^*) &\equiv 0 \\ \pi_{ex}(z_{ex}^*) &\equiv 0 \end{aligned}$$

Plugging (5) into (1), (2), and using (6), employment and production of the cutoff firm are given by:

$$u(z^*) = \alpha(\epsilon - 1)w_U^{-\sigma}(w_U^{1-\sigma} + z^*w_S^{1-\sigma})^{-1} \quad (8)$$

$$s(z^*) = \alpha(\epsilon - 1)z^*w_S^{-\sigma}(w_U^{1-\sigma} + z^*w_S^{1-\sigma})^{-1} \quad (9)$$

$$y(z^*) = \alpha(\epsilon - 1)(w_U^{1-\sigma} + z^*w_S^{1-\sigma})^{\frac{1}{\sigma-1}} \quad (10)$$

These three equations also hold for the export sector when replacing α with α_{ex} and z^* with z_{ex}^* . They allow to express the cutoffs as a function of the endogenous variables: z^*, z_{ex}^*, w_U and w_S .

The expression for the cutoff production level parallels the original results of Melitz (2003), which also contains these three components¹⁰. Employment and production of the cutoff firms increase with the fixed costs. Indeed,

¹⁰In Melitz (2003), though the author does not explicitly write it, the cutoff level of production can be expressed as: $\bar{q} = (\sigma - 1)f\varphi$, where σ corresponds to my ϵ , f to α , and φ to productivity

higher fixed costs require to sell more in order to break even. A higher elasticity of substitution (ϵ) in the utility function puts downward pressures on the markup, also forcing firms to sell more. A lower unit cost of production, which corresponds to an increase in the last term of (10) decreases the sale-price more than the costs due to the markup, and forces the cutoff firm to produce more.

2.2 The consumers and the goods market

The aim of this section is to express the production and employment of each firm as a function of that of the cutoff firms. This is a standard approach which simplifies the analysis and allows to describe the whole employment and production schedule in the economy.

Consumers have a CES utility function with an elasticity of substitution between goods equal to $\epsilon > 1$. Their maximisation problem gives the standard condition:

$$\frac{c_i(z)}{c_j(z')} = \left(\frac{p_i(z)}{p_j(z')} \right)^{-\epsilon} \quad \text{for } i, j = \{d, ex\} \quad (11)$$

For all goods for which firms have drawn z , z' and produce.

In particular, for $z' = z^*$, (11) becomes:

$$\frac{c_i(z)}{c_d(z^*)} = \left(\frac{p_i(z)}{p_d(z^*)} \right)^{-\epsilon} \quad \text{for } i = \{d, ex\} \quad (12)$$

This property allows me to rewrite the employment and production level of each firm on the market as a function of the cutoff firm. For this, I use the goods market equilibrium (3) for each produced z as well as (12) and (6). Production for the domestic market, y_d , of a firm z is:

$$\frac{y_d(z)}{y_d(z^*)} = \left(\frac{w_U^{1-\sigma} + zw_S^{1-\sigma}}{w_U^{1-\sigma} + z^*w_S^{1-\sigma}} \right)^{\frac{\epsilon}{\sigma-1}} \quad \text{for } z \geq z^* \quad (13)$$

Since the two countries are perfectly symmetric:

$$y_d^F(z^*) = y_d^H(z^*) \equiv y_d(z^*) \quad (14)$$

Superscripts H and F stand for Home and Foreign. The symmetry assumption ensures that two firms having drawn the same z will produce the same amount of output for domestic purposes, whatever their location.

I now turn to the production for export purposes $y_{ex}(z)$. From (7) and (11):

$$\frac{c_d(z)}{c_{ex}(z')} = \left(\frac{p_d(z)\tau}{p_d(z')} \right)^{-\epsilon} \quad (15)$$

Using (4), I can additionally write:

$$\frac{y_d(z)}{y_{ex}(z')} = \frac{c_d(z)\tau}{c_{ex}(z')} = \left(\frac{p(z)}{p(z')} \right)^{-\epsilon} \tau^{1-\epsilon} \quad (16)$$

The level of production for the export market is therefore:

$$\begin{aligned} y_{ex}(z) &= \tau^{\epsilon-1} y_d(z) \quad \forall z \geq z_{ex}^* \\ y_{ex}(z) &= 0 \quad \text{otherwise} \end{aligned} \quad (17)$$

Due to the assumption of iceberg transportation costs, prices of imports are relatively high, and therefore, for a given z , less consumed. The higher the consumers' elasticity of substitution ϵ , the higher the impact of τ on demand, and the lower the production aimed at the export market. On the other hand, the loss of goods during transportation forces exporting firms to send relatively more goods to the foreign country. These two effects are summarised in the $\tau^{\epsilon-1}$ term above. The net effect is however negative, as, by assumption, $\epsilon > 1$, so that the production of a firm for the export market is smaller than the production for domestic sales. Due to the assumption that $\alpha_{ex} > \alpha$, there exist technology levels at which firms produce for domestic purposes, but do not export. This is the meaning of the second equality in (17).

Using (13), as well as the cutoff level of production (10), I rewrite the domestic production of a z -firm as follows:

$$y_d(z) = \left(\frac{w_U^{1-\sigma} + zw_S^{1-\sigma}}{w_U^{1-\sigma} + z^*w_S^{1-\sigma}} \right)^{\frac{\epsilon}{\sigma-1}} \alpha(\epsilon-1)(w_U^{1-\sigma} + z^*w_S^{1-\sigma})^{\frac{1}{\sigma-1}} \quad (18)$$

The first term in bracket on the right-hand side indexes the demand for each good on the demand for the cutoff good through the relative prices. The remaining part is the cutoff level of domestic production. The higher the z a firm draws, the more it produces. The same procedure can be applied to the two inputs:

$$u_d(z) = (w_U^{1-\sigma} + zw_S^{1-\sigma})^{\frac{\sigma-\epsilon}{1-\sigma}} (w_U^{1-\sigma} + z^*w_S^{1-\sigma})^{\frac{1-\epsilon}{\sigma-1}} \alpha(\epsilon-1)w_U^{-\sigma} \quad (19)$$

$$s_d(z) = (w_U^{1-\sigma} + zw_S^{1-\sigma})^{\frac{\sigma-\epsilon}{1-\sigma}} (w_U^{1-\sigma} + z^*w_S^{1-\sigma})^{\frac{1-\epsilon}{\sigma-1}} \alpha(\epsilon-1)zw_S^{-\sigma} \quad (20)$$

From the above expressions for $y_d(z)$, $u_d(z)$, $s_d(z)$ and from (6) and (7), profits can be rewritten as:

$$\pi_d(z) = \alpha \left[\left(\frac{w_U^{1-\sigma} + zw_S^{1-\sigma}}{w_U^{1-\sigma} + z^*w_S^{1-\sigma}} \right)^{\frac{\epsilon-1}{\sigma-1}} - 1 \right] \quad (21)$$

A similar equation holds for π_{ex} , with α_{ex} and z_{ex}^* replacing α and z^* . This expression increases in z and is equal to zero for the cutoff firm. This confirms that all firms drawing a z higher than z^* want to produce for the domestic market, since this gives them positive profits. On the other hand, all firms drawing a z under the cutoff level decide not to produce, and similarly for the export market. This ensures that only relatively productive, skilled intensive firms export. Exporting firms employ proportionately more skilled labour than non-exporting firms and pay therefore higher wages on average¹¹ to their workers. This is conform to the empirical evidence presented by Bernard and Jensen (1995). Exporting firms also produce more, and have a higher z , which is consistent with the evidence given by Bernard and Jensen (1997) that they are more technology intensive.

The relationship between the two cutoff levels z^* and z_{ex}^* is a particularly useful one. I have up to now expressed employment and production for the domestic market and for the export market respectively as functions of z^* and z_{ex}^* . Using the following relationship allows to express everything as a function of the domestic cutoff firm, and completes the description of the production and employment schedules as a function of w , z and z^* .

Using (13) for z^* and z_{ex}^* , (10) and its equivalent for z_{ex}^* , as well as (17) yields:

$$\frac{\alpha}{\alpha_{ex}} = \left(\frac{w_U^{1-\sigma} + z^*w_S^{1-\sigma}}{w_U^{1-\sigma} + z_{ex}^*w_S^{1-\sigma}} \right)^{\frac{\epsilon-1}{\sigma-1}} \tau^{1-\epsilon} \quad (22)$$

The interval (z^*, z_{ex}^*) in which firms only produce for their domestic market increases in the difference between the fixed costs. Higher transportation costs (a lower τ) also have a positive impact on the interval. Both effects are very intuitive: higher barriers to trade results in a lower proportion of exporters.

¹¹I assume for this point that there is weakly more unskilled than skilled labour in each country, and that $F(z)=0$ for $z < 1$. These two conditions are sufficient to ensure that $w_S > w_U$.

Equivalently:

$$z_{ex}^* = \left[\tau^{1-\sigma} \left(\frac{\alpha_{ex}}{\alpha} \right)^{\frac{\sigma-1}{\epsilon-1}} (w_U^{1-\sigma} + z^* w_S^{1-\sigma}) - w_U^{1-\sigma} \right] w_S^{\sigma-1} \quad (23)$$

2.3 The Equilibrium conditions

As in Melitz (2003), the number of firms on the market is endogenous¹². There is an unbounded pool of ex-ante identical entrants on the market. Before drawing their z -type, entrepreneurs need to make an initial investment, modelled in terms of a fixed sunk cost β , in order to enter the market. This sunk cost is, similarly to the fixed cost of production, paid in terms of the international asset. After this investment, entrepreneurs learn their type z , drawn from a common distribution $F(z)$ with upper bound Z , and decide whether to produce and export. I assume that f has support $\Theta = [1, Z]$, which ensures that skilled labour is in each firm at least as productive as unskilled labour. This is not a necessary assumption but simplifies the intuition. There is entry on the market up to the point where the expected profits of entering are equal to the sunk entry cost. It must therefore hold, in equilibrium that:

$$E(\pi) = \int_{z^*}^Z \pi_d(z) dF(z) + \int_{z_{ex}^*}^Z \pi_{ex}(z) dF(z) = \beta \quad (24)$$

From now on, denote the ratio of unskilled to skilled wage (the inverse of the skill premium) by w . The labour markets for unskilled and skilled labour have to clear. I denote U and S as the total unskilled and skilled labour force in a country, and solve the model from the labour market equations. Using (17) and the fact that the production function exhibits constant returns to scale:

$$u_{ex}(z) = u_d(z) \tau^{\epsilon-1} \text{ for } z \geq z_{ex}^* \quad (25)$$

$$u_{ex}(z) = 0 \text{ otherwise}$$

$$s_{ex}(z) = s_d(z) \tau^{\epsilon-1} \text{ for } z \geq z_{ex}^* \quad (26)$$

$$s_{ex}(z) = 0 \text{ otherwise}$$

I therefore write the equilibrium on the labour market as:

$$U = M \left(\int_{z^*}^Z u_d(z) dF(z) + \tau^{\epsilon-1} \int_{z_{ex}^*}^Z u_d(z) dF(z) \right) \quad (27)$$

¹²I consider a one-period version of Melitz' original framework. This is a minor change.

The first and second term in brackets in (27) represent the unskilled labour used respectively for domestic production and exports. M is the endogenous mass of entrepreneurs having paid the fixed entry costs.

Equivalently for the skilled labour market:

$$S = M \left(\int_{z^*}^Z s_d(z) dF(z) + \tau^{\epsilon-1} \int_{z_{ex}^*}^Z s_d(z) dF(z) \right) \quad (28)$$

Definition 1 *An equilibrium is a vector (z^*, z_{ex}^*, w, M) such that consumers maximise their utility, firms maximise their profits, make zero expected profits, and all markets clear.*

I now turn to the conditions under which an equilibrium exists and is unique.

Assumption 1 *f , the density function of z is monotonically decreasing over its support Θ and has the following property:*

$$\frac{2\sigma}{\sigma-1} \geq \frac{x_1 f(x_1)}{1-F(x_1)} \geq \frac{x_2 f(x_2)}{1-F(x_2)} \quad \text{for all } x_1, x_2 \in \Theta \text{ with } x_2 \geq x_1 \quad (29)$$

Assumption 1 requires that $f(z)$ should be decreasing fast enough. The proof is instructive in understanding why this has to be the case. A change in w or z^* have both direct effects on the equilibrium conditions, but also indirect effects through the change in the cutoff export productivity level. Assumption 1 ensures that the indirect effect does not dominate the direct effect. For example, an increase in the relative wage of unskilled labour should intuitively depreciate the demand for unskilled labour in the economy by increasing its relative price. However, this will decrease the export cutoff level z_{ex}^* which may have a countervailing positive effect on the demand for unskilled labour. Under Assumption 1, this second effect will not dominate, because the density of firms entering the export market will be small enough, and an increase in w will depreciate the demand for unskilled labour.

Assumption 2 $\epsilon \geq \sigma$

I do not consider the assumption that $\epsilon \geq \sigma$ to be very restrictive. Empirical estimates for σ are usually comprised between 1 and 2¹³ whereas ϵ

¹³Acemoglu (2002) p.20

tends to be higher, between 3 and 6¹⁴. These estimates naturally depend on the sectors, but it is intuitive that in most cases, $\epsilon > \sigma$. It appears indeed easier for a consumer to substitute a smaller car for a bigger car, than for a car manufacturer to substitute a chain worker for an engineer.

Assumption 3 $\frac{U}{S}$ and β are sufficiently small.

Proposition 1 Under Assumptions 1-3, there exists a unique equilibrium.

Proof. See Appendix ■

Note that Assumptions 1 to 3 are sufficient, but not necessary.

Since the Pareto density plays an important role in empirical studies of that type of models, it is of particular interest to check that this density fulfills the sufficient conditions for existence and uniqueness of an equilibrium.

Corollary 1 Under Assumption 2 and 3, there exists a unique equilibrium if f is a Pareto density function with support $\Theta' = [1, \infty]$ defined as follows:

$$f(z) = az^{-a-1} \tag{30}$$

with $a \leq \frac{2\sigma}{\sigma-1}$

Proof. See Appendix ■

3 Results

The aim of this model is to study the impact of a real perturbation in the costs of exporting on the wage gap between unskilled and skilled labour. I use two definitions of trade liberalisation: a decrease in the iceberg costs of transportation, and a reduction in the fixed costs of exporting.

3.1 Trade liberalisation as a drop in the iceberg transport costs

I first consider the effect of a decrease in the iceberg costs, i.e. as an increase in τ , on the relative wage w . This can be interpreted as the result of a

¹⁴Bernard et al. (2003) and others use 3,8, while a reasonable markup of 20 per cent would require 6. Both σ and ϵ naturally depend on the sector.

policy choice of decreasing tariffs or as a consequence of technical progress, which lowers transportation costs. I use a total differential argument on the equilibrium conditions to study the comparative statics of the model. I refer to the ‘initial’ situation for the equilibrium prevailing before any perturbation in the costs of trade.

Lemma 1 *Under Assumptions 1-3, the sign of $\frac{dw}{d\tau}$ is given by the sign of:*

$$\gamma \left(\int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} (Bw^{-\sigma} - Az) dF(z) \right) + \eta (Az^* - Bw^{-\sigma}) - \xi (Az_{ex}^* - Bw^{-\sigma}) \quad (31)$$

where $\gamma, \eta, \xi > 0$ and:

$$Bw^{-\sigma} - Az' = w^{-\sigma} \left[\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} (z - z') dF(z) + \tau^{\epsilon-1} \int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} (z - z') dF(z) \right] \quad (32)$$

for all $z' \in \Theta$

Proof. *See Appendix* ■

For a constant w , trade liberalisation has three effects on firms, as shown by the three components of (31). These are essentially identical to those highlighted in Melitz (2003), and are reprinted in figure 2.

First, following a decrease in the marginal costs of exporting, the most productive firms, which are initially exporting, find it profitable to scale up their production aimed at the export market (arrow A in figure 2). Second, the most productive among the initially non-exporting firms can now make weakly positive profits on the export market, in which they enter (arrow B). This decreases the export cutoff level z_{ex}^* . As these first two channels tend to increase expected profits, the cutoff level of domestic production z^* needs to rise in order to leave expected profits constant as required by (24). The least productive producing firms in the economy therefore drop out of the market (arrow C), which is the third effect of trade liberalisation on firms.

These three effects have different consequences for the unskilled and skilled labour markets. Two of them have a clear positive effect on the relative demand for skilled labour. First, initially exporting firms are productive and relatively skilled intensive. An expansion of their production thus increases the demand for skilled labour more than that of unskilled labour, as shown by the negative sign of the first term in (31). This tends to depreciate w . Second, firms dropping out of the domestic market are relative unproductive and unskilled intensive. They therefore release much unskilled labour, and their effect on (31), represented by the second term, is also negative.

The impact of firms newly entering the export market is however undetermined. This depends on the relative unskilled intensity of the cutoff export firms, represented by the third term in (31). A positive (negative) sign for this term means that these firms are relatively unskilled (skilled) intensive. From (32), it is immediate that this sign depends on the initial z_{ex}^* , i.e. on the initial degree of trade openness. If trade is initially expensive, only very productive firms are able to export, and the cutoff export firm is relatively skilled intensive. In this case, firms newly entering the export market following a marginal trade liberalisation are also skilled intensive, and increase the demand for skilled labour proportionately more. On the other hand, if trade is initially cheap, relatively unproductive, unskilled intensive firms can benefit from the trade liberalisation by becoming able to export. Firms newly entering the export market after a small increase in τ demand relatively much unskilled labour, and $Az_{ex}^* - Bw^{-\sigma}$ is negative. The entry of firms with a lower than average skill intensity on the export market attenuates the rise in the skill wage premium.

Crucial for the analysis is to determine whether the third effect described above can overcompensate the other two and yield a decrease in the skill premium. It appears that, under Assumption 1, this cannot be the case.

Proposition 2 *Under Assumptions 1-3, a decrease in the variable costs of trade unambiguously increases the skill wage premium.*

Proof. *See Appendix* ■

This is due to the sufficient condition for existence of an equilibrium, which requires that the density function $f(z)$ be decreasing at a high enough speed. The impact of the marginal firms which start exporting is therefore small and cannot compensate for the low productive, unskilled intensive firms dropping out of the market. From Corollary 1, this is the case under the widely used Pareto distribution.

This result is even stronger if we consider that the variable costs were modeled as iceberg transportation costs. Indeed, exported goods are on average skill intensive, so that iceberg trade costs, which is a loss of a fraction of the production, are also skill intensive. A rise in τ therefore reduces the amount of skilled labour that is used for trade costs, and tends to decrease the skill wage premium. Alternatively, trade costs could be paid as an amount of numeraire asset for every exported unit of a good. This does not change the qualitative results of the model, and the above proposition also holds.

3.2 Trade liberalisation as a drop in the fixed costs of exporting

Trade liberalisation can also be defined as a drop in the fixed costs of exporting α_{ex} . Clearer legislations, better fluency in English or convergence in tastes around the world are significant elements pleading for the plausibility of a drop in these fixed costs.

Lemma 2 *If Assumption 1-3 hold, the sign of $\frac{dw}{d\alpha_{ex}}$ is given by the sign of:*

$$\eta'(Bw^{-\sigma} - Az^*) - \xi'(Bw^{-\sigma} - Az_{ex}^*) \quad \text{where: } \eta', \xi' > 0 \quad (33)$$

Proof. *See Appendix* ■

The mechanism at stake is very similar to that highlighted in the case of variable costs of trade. There are however two main differences. First, the first term in (31) disappears, because a decrease in the fixed costs of exporting, though it raises the profits of all exporting firms, does not change their level of production and employment, which only depends on the marginal costs of exporting. Initially exporting firms therefore do not scale up their production. The only two effects remaining are the marginal effects of firms dropping out of the domestic market and firms entering the export market. Second, the adjustment of z^* that is necessary to maintain constant expected profits after a drop in the fixed costs of export is smaller than in the case of a decrease in variable costs. The reason is again that initially exporting firms do not benefit from liberalisation as massively as if the variable costs were decreasing, so that z^* does not have to rise as much to maintain the expected profit condition. These two differences with the case of variable trade liberalisation both reduce the forces driving the skill premium up.

Proposition 3 *Under Assumptions 1-3,*

1. *If the fixed costs of trade are initially high, a marginal decrease in the fixed costs of trade (decrease in α_{ex}) unambiguously raises the skill wage premium.*
2. *If the fixed costs of trade (α_{ex}) are initially low, and if the variable costs of trade are sufficiently low (τ close enough to 1), a marginal decrease α_{ex} decreases the skill wage premium.*

Proof. *See Appendix* ■

A decline in the skill wage premium can here happen when the fixed costs of trade decrease from an already low level. This result is similar to Yavas (2006), with the additional qualification that the variable trade costs should be sufficiently low. Otherwise, the marginal firms entering the market may still be more skill intensive than the average.

It is possible to draw the comparison even further between the two types of liberalisation. For this I define:

$$\epsilon_{\tau} \equiv \frac{\tau}{w} \frac{dw}{d\tau} \quad (34)$$

$$\epsilon_{\alpha_{ex}} \equiv -\frac{\alpha_{ex}}{w} \frac{dw}{d\alpha_{ex}} \quad (35)$$

ϵ_{τ} ($\epsilon_{\alpha_{ex}}$) stands for the elasticity of the relative wage of unskilled labour with respect to the variable (fixed) trade costs.

Assumption 4 $\epsilon \geq 3$

Proposition 4 *Assumption 4 is sufficient to ensure that a one percent decrease in the fixed costs of trade is always less detrimental for inequalities than a one percent decrease in the variable trade costs.*

The intuition for this is much the same as that described for Proposition 3. It is again worth noting that the assumption of iceberg trade costs is a worst case for the above proposition, and that it would also hold if the variable trade costs were paid in terms of the numeraire asset, with the weaker assumption that $\epsilon \geq 2$. The reason is, as mentioned for the robustness of Proposition 2, that a decrease in iceberg transportation costs releases a high proportion of skilled labour, and tends to limit the rise in the skill wage premium.

4 Numerical solutions

In this section, I parameterise the model to solve it numerically. This serves the purpose of illustrating the results and testing their quantitative importance. Of particular interest in this section is to see whether a trend reversal in the evolution of relative wages can happen for plausible parameter values.

The literature on heterogeneous firms suggests that domestic sales of firms approximately follow a Pareto distribution. Axtell (2001) estimates the parameter of the Pareto distribution for sales to be around 1, which is conform

to Zipf's law, or slightly higher, up to 1.25. Other studies, such as Bernard et al. (2004) or Ghironi and Melitz (2005) take a parameter of 0.6, in order to be conform with the variance of the logarithm of sales estimated by Bernard et al. (2003). The original model of Melitz (2003) has the nice property that a Pareto distribution of productivity yields a Pareto distribution for sales. Unfortunately, this property is not preserved for the distribution of z in this model. I however assume that z is Pareto distributed, since this yields a sales distribution which is relatively close to Pareto benchmark of Axtell (2001), and I assume that Z (the upper bound of the support) goes to infinity. For all parameters of interest, I use values which are now well established in the literature. Following Bernard et al. (2003) and subsequent studies, I assume that the consumers' elasticity of substitution (ϵ) is equal to 3.8, which fulfills Assumption 4. I set the elasticity of substitution between factors in the production function (σ) to 2, in the range suggested by Acemoglu (2002). I set $U = S = 1$, assuming for simplicity that there are as many skilled as unskilled in the economy, and set the fixed entry costs (β) equal to 0.8, without loss of generality.

I parameterise the model so as to obtain a relative unskilled to skilled wage of 0.65, which is in the range estimated in the last decade, as well as a proportion of exporters among firms equal to 20 percent¹⁵. I also want to obtain a distribution of sales which is approximately conform to that of Axtell (2001).

4.1 Liberalisation as a decrease in variable costs

I equate α and α_{ex} to 1, in order to concentrate on the effect of a change in iceberg costs¹⁶. The ratio of α over β is therefore close to the one used by Bernard et al. (2004). In order to obtain the appropriate relative wage and share of exporters, I set the coefficient a of the Pareto distribution of z equal to 3.4 and τ equal to 0.79. This can be interpreted as the current actual situation. The generated τ is indeed very close to estimates of Obstfeld and Rogoff (1996) of 0.77. The resulting distribution of sales is compared to a Pareto distribution in figure 3, and is relatively close to empirical observations. Note that the coefficient of the Pareto distribution fulfills Assumption 1. Using this, I let the iceberg costs vary from 0.65 to 1 and compute the relative wage of unskilled labour along this path, as shown in figure 4. Trade liberalisation has, as expected, a negative effect on the relative wage of unskilled labour,

¹⁵see Ghironi and Melitz (2005)

¹⁶in this, I follow Bernard et al. (2004)

which decreases from 0.67 to 0.62 along the path. Of particular interest is the slower rate of growth of the skill premium as τ increases. This is all the more striking in percentage points. At high initial values of τ , a marginal decrease in the variable costs of trade represents a proportionately large liberalisation, and is accompanied by a low rise in the skill wage premium. This numerical exercise shows results that are in line with the recent empirical evolution of the skill wage premium in Western countries.

4.2 Liberalisation as a decrease in the fixed costs of exporting

Liberalisation as a decrease in fixed costs of exporting may be of particular relevance for the European Union, which has a common market, and relatively small transport costs due to small distances. Differences in languages, laws and culture remain however important. In order to concentrate on the effect of a drop in fixed costs, I set variable costs equal to zero ($\tau = 1$), and $\alpha = 1$ as before. I calibrate the model again to have a relative wage of 0.65 and a proportion of exporters of 0.2. I obtain $a = 3.55$, a value that is very close to the one obtained above, and $\alpha_{ex} = 1.8$. Figure 5 shows the change in relative wages due to a decrease in the fixed costs of exporting. Liberalisation has a negative effect on w at low degrees of trade openness, but this effect is very small. On the other hand, when fixed costs of exporting become smaller than twice the domestic fixed costs of production, further trade liberalisation leads to a substantial decrease in the wage skill premium.

5 Conclusion

I find that trade liberalisation between symmetric countries can have an important impact on the skill premium, and therefore complement the nascent literature on the link between intra-industry trade and the skill wage premium. I use a monopolistic competition framework with heterogeneous firms and two factors, skilled and unskilled labour. Firms are heterogeneous as to the relative productivity of skilled labour in the production process. In this model, productive firms are skill intensive and export. Trade liberalisation implies the reallocation of market shares from unskilled intensive unproductive firms to skill intensive productive firms, driving the wage premium upward. The entrance of new firms on the export market has however an ambiguous effect on the relative wage, which depends on whether the cutoff

export firm is more skill intensive than average or not. A marginal trade costs reduction from a low (high) level causes the entrance of lower (higher) than average skill intensive firms on the export market, and raises the relative demand for unskilled (skilled) labour. Crucial for the analysis is the definition of trade liberalisation used: a reduction in the variable costs of international trade unambiguously increases the skill premium, while a reduction in the fixed costs of trade may have non-monotonic effects on the skill premium.

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Appendix

Proof of Proposition 1

The endogenous variables M and z_{ex}^* can be uniquely expressed as a function of z^* and w . Looking for the equilibrium therefore amounts to looking for the tuple (z^*, w) that satisfies the equilibrium conditions. To prove existence and uniqueness, I proceed in three steps, summarised in the following three lemmas. Figure 1 should help the intuition behind the three steps.

For convenience, I define A and B as follows:

$$A = \int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} w^{-\sigma} dF(z) + \tau^{\epsilon-1} \int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} w^{-\sigma} dF(z) \quad (36)$$

$$B = \int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} z dF(z) + \tau^{\epsilon-1} \int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} z dF(z) \quad (37)$$

Lemma 3 *The equilibrium condition for expected profits (24) establishes a monotonically increasing relationship between w and z^* .*

Proof.

I use the Implicit function theorem on (24). The partial derivatives of $E(\pi)$ with respect to z^* and w are:

$$\frac{\partial E(\pi)}{\partial z^*} = \frac{\epsilon-1}{1-\sigma} \alpha (w^{1-\sigma} + z^*)^{\frac{\epsilon-1}{1-\sigma}-1} \left[\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-1}{\sigma-1}} dF(z) + \tau^{\epsilon-1} \int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-1}{\sigma-1}} dF(z) \right] < 0 \quad (38)$$

$$\frac{\partial E(\pi)}{\partial w} = \alpha (\epsilon-1) (w^{1-\sigma} + z^*)^{\frac{1-\epsilon}{\sigma-1}-1} (Bw^{-\sigma} - Az^*) > 0 \quad (39)$$

Therefore:

$$\frac{dz^*}{dw} > 0 \quad (40)$$

■

Lemma 4 *Under Assumptions 1 and 2, the labour market equilibrium conditions (27) and (28) establish a monotonic negative relationship between w and z^* .*

Proof. This is the longest, and heaviest part of the proof. I use the implicit function theorem on the ratio of (27) and (28):

$$\frac{U}{S} = \frac{\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} w^{-\sigma} dF(z) + \tau^{\epsilon-1} \int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} w^{-\sigma} dF(z)}{\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} z dF(z) + \tau^{\epsilon-1} \int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} z dF(z)} \equiv G(z^*, w) \quad (41)$$

- Step 1: $\frac{\partial G(z^*, w)}{\partial z^*} < 0$

$$\begin{aligned} \frac{\partial G(z^*, w)}{\partial z^*} &= (w^{1-\sigma} + z^*)^{\frac{\sigma-\epsilon}{1-\sigma}} f(z^*) \left[\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} (z^* - z) dF(z) + \tau^{\epsilon-1} \int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} (z^* - z) dF(z) \right] \\ &+ (w^{1-\sigma} + z^*)^{\frac{\sigma-\epsilon}{1-\sigma}} f(z_{ex}^*) \frac{\alpha_{ex}}{\alpha} \left[\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} (z_{ex}^* - z) dF(z) + \tau^{\epsilon-1} \int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} (z_{ex}^* - z) dF(z) \right] \end{aligned}$$

Using Assumption 2, the following inequalities hold:

$$\int_i^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} (j-z) dF(z) \leq \frac{\int_i^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} dF(z)}{1 - F(i)} \int_i^Z (j-z) dF(z) \text{ for } j, i = \{z^*, z_{ex}^*\} \quad (44)$$

Moreover:

$$\frac{\int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-\sigma}{\sigma-1}} dF(z)}{1 - F(z_{ex}^*)} \geq \left(\frac{\alpha_{ex}}{\alpha} \right)^{\frac{\epsilon-\sigma}{\sigma-1}} \tau^{\sigma-\epsilon} \frac{\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-\sigma}{\sigma-1}} dF(z)}{1 - F(z^*)} \quad (45)$$

Using (44) and (45):

$$\begin{aligned} \frac{\partial G(z^*, w)}{\partial z^*} \frac{1 - F(z^*)}{\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-\sigma}{\sigma-1}}} &\leq f(z^*) \left[\gamma \int_{z^*}^Z (z^* - z) dF(z) + \tau^{\sigma-1} \left(\frac{\alpha_{ex}}{\alpha} \right)^{\frac{\epsilon-\sigma}{\sigma-1}} \int_{z_{ex}^*}^Z (z^* - z) dF(z) \right] \\ &+ f(z_{ex}^*) \frac{\alpha_{ex}}{\alpha} \left[\gamma \int_{z^*}^Z (z_{ex}^* - z) dF(z) + \tau^{\sigma-1} \left(\frac{\alpha_{ex}}{\alpha} \right)^{\frac{\epsilon-\sigma}{\sigma-1}} \int_{z_{ex}^*}^Z (z_{ex}^* - z) dF(z) \right] \end{aligned}$$

The second term in each square bracket is unambiguously negative. Therefore:

$$\frac{\partial G(z^*, w)}{\partial z^*} \frac{1 - F(z^*)}{\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-\sigma}{\sigma-1}}} \leq (1 - F(z_{ex}^*)) f(z^*) \tau^{\sigma-1} \left(\frac{\alpha_{ex}}{\alpha} \right)^{\frac{\epsilon-\sigma}{\sigma-1}} (z^* - z_{ex}^*) + f(z_{ex}^*) \frac{\alpha_{ex}}{\alpha} (z_{ex}^* - z^*) (1 - F(z^*)) \quad (46)$$

The right hand side is negative if:

$$\frac{f(z_{ex}^*)}{1 - F(z_{ex}^*)} \tau^{1-\sigma} \left(\frac{\alpha_{ex}}{\alpha} \right)^{\frac{\sigma-1}{\epsilon-1}} \leq \frac{f(z^*)}{1 - F(z^*)} \quad (47)$$

From (22), and since w is positive, the above inequality holds under Assumption 1

- Step 2: $\frac{\partial G(z^*, w)}{\partial w} < 0$

$\frac{\partial U}{\partial w}$ consists of two parts, C and D.

1) Sign of C

$$CB^2 = w^{-\sigma}(\epsilon - \sigma) \left[\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}-1} (Az - Bw^{-\sigma}) dF(z) + \tau^{\epsilon-1} \int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}-1} (Az - Bw^{-\sigma}) dF(z) \right] \quad (48)$$

Rearranging, and assuming that $\epsilon \geq \sigma$ (Assumption 2), C has the same sign as:

$$Aw^\sigma \left[\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}-1} z dF(z) + \tau^{\epsilon-1} \int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}-1} z dF(z) \right] - B \left[\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}-1} dF(z) + \tau^{\epsilon-1} \int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}-1} dF(z) \right] \quad (49)$$

High values of z receive more weight in the second part than in the first part of the expression above. The whole expression is therefore weakly negative, and C as well.

2) *Sign of D*

$$DB^2 = -\sigma w^{-1} AB + (Bw^{-\sigma} - Az_{ex}^*) \tau^{\epsilon-1} (\sigma - 1) w^{-\sigma} \left(\tau^{1-\sigma} \left(\frac{\alpha_{ex}}{\alpha} \right)^{\frac{\sigma-1}{\epsilon-1}} - 1 \right) (w^{1-\sigma} + z_{ex}^*)^{\frac{\sigma-\epsilon}{1-\sigma}} f(z_{ex}^*) \quad (50)$$

Rearranging:

$$\frac{BDw}{A} = -\sigma + (\sigma - 1) \tau^{\epsilon-1} w^{1-\sigma} \left(\tau^{1-\sigma} \left(\frac{\alpha_{ex}}{\alpha} \right)^{\frac{\sigma-1}{\epsilon-1}} - 1 \right) (w^{1-\sigma} + z_{ex}^*)^{\frac{\sigma-\epsilon}{1-\sigma}} f(z_{ex}^*) \left(\frac{w^{-\sigma}}{A} - \frac{z_{ex}^*}{B} \right) \quad (51)$$

A sufficient condition for the right hand side to be negative is:

$$\frac{\sigma}{\sigma - 1} \geq \tau^{\epsilon-1} w^{1-\sigma} \left(\tau^{1-\sigma} \left(\frac{\alpha_{ex}}{\alpha} \right)^{\frac{\sigma-1}{\epsilon-1}} - 1 \right) (w^{1-\sigma} + z_{ex}^*)^{\frac{\sigma-\epsilon}{1-\sigma}} f(z_{ex}^*) \frac{w^{-\sigma}}{A} \quad (52)$$

For this to hold, it is again sufficient that:

$$\frac{\sigma}{\sigma - 1} \geq \frac{w^{1-\sigma} \left(\tau^{1-\sigma} \left(\frac{\alpha_{ex}}{\alpha} \right)^{\frac{\sigma-1}{\epsilon-1}} - 1 \right) f(z_{ex}^*)}{2(1 - F(z_{ex}^*))} \quad (53)$$

A sufficient condition for this is:

$$\frac{\sigma}{\sigma - 1} \geq \frac{z_{ex}^* f(z_{ex}^*)}{2(1 - F(z_{ex}^*))} \quad (54)$$

Assumption 1 and 2 therefore guarantee that C and D are weakly negative, and therefore that $\frac{\partial G(z^*, w)}{\partial w} \leq 0$. The implicit theorem using Step 1 and 2 ends the proof of the lemma.

■

Lemma 5 *Under Assumptions 1-3:*

1. For $z^* = 1$, the wage given by the labour market condition (41) is higher than the wage given by the expected profit condition (24).
2. As $w \rightarrow \infty$, the z^* defined by the expected profit condition (24) converges to a value strictly greater than 1.
3. As $w \rightarrow 0$, the z^* defined by the labour market condition (41) converges to a value strictly greater than 1.

Proof.

1. Proof of part 1

For convenience, define w_1 and w_2 as the wage needed respectively for (24) and (41) to hold when $z^* = 1$. Hereafter, I prove that under Assumptions 1-3, $w_1 \leq w_2$.

From the proof of Lemma 3, $\frac{\partial E(\pi)}{\partial w} > 0$. This means that the smaller the β , the smaller the w_1 . As $\beta \rightarrow 0$, $w_1 \rightarrow 0$ from (24).

From the proof of Lemma 4, if Assumptions 1 and 2 hold, $\frac{\partial G(w, z^*)}{\partial w} < 0$. Hence, the smaller the $\frac{U}{S}$, the bigger the w_2 . As $\frac{U}{S} \rightarrow 0$, $w_2 \rightarrow \infty$ from (41).

This proves that there exist a β and a $\frac{U}{S}$ small enough such that $w_1 \leq w_2$. Under Assumptions 1-3, part 1 of Lemma 5 therefore holds

2. Proof of part 2

If $w \rightarrow \infty$, (24) becomes:

$$\beta = \alpha \left[\int_{z^*}^Z \left(\frac{z}{z^*} \right)^{\frac{\epsilon-1}{\sigma-1}} - 1 dF(z) \right] + \alpha_{ex} \left[\int_{z_{ex}^*}^Z \left(\frac{z}{z_{ex}^*} \right)^{\frac{\epsilon-1}{\sigma-1}} - 1 dF(z) \right] \quad (55)$$

The right hand side is decreasing in z^* . As β decreases, the z^* defined by the above equation will approach Z. Under Assumption 3, the second part of Lemma 5 therefore holds.

3. Proof of part 3

As $w \rightarrow 0$, $G(z^*, w)$ goes to infinity. Under Assumptions 1 and 2, $G(z^*, w)$ is decreasing in z^* , so that z^* will go to Z in order for the labour market condition to hold.

■

Combining Lemma 3 to 5 proves Proposition 1.

Proof of Corollary 1

If z is Pareto distributed on $\Theta' = [1, \infty]$ such that for all $x \in \Theta'$: $f(x) = ax^{-a-1}$, then for all $x \in \Theta'$:

$$\frac{xf(x)}{1-F(x)} = a \quad (56)$$

As long as

$$a \leq 2 \frac{\sigma}{\sigma-1} \quad (57)$$

the Pareto distribution fulfills Assumption 1.

Proof of Lemma 1

I use a total differential argument on the equilibrium conditions (24) and (41):

$$\begin{bmatrix} \frac{\partial E(\pi)}{\partial w} & \frac{\partial E(\pi)}{\partial z^*} \\ \frac{\partial G(z^*, w)}{\partial w} & \frac{\partial G(z^*, w)}{\partial z^*} \end{bmatrix} \begin{bmatrix} dw \\ dz^* \end{bmatrix} = \begin{bmatrix} -\frac{\partial E(\pi)}{\partial \tau} \\ -\frac{\partial G(z^*, w)}{\partial \tau} \end{bmatrix} d\tau \quad (58)$$

Using Cramer's rule:

$$\frac{dw}{d\tau} = \frac{\frac{\partial E(\pi)}{\partial z^*} \frac{\partial G(z^*, w)}{\partial \tau} - \frac{\partial E(\pi)}{\partial \tau} \frac{\partial G(z^*, w)}{\partial z^*}}{\frac{\partial E(\pi)}{\partial w} \frac{\partial G(z^*, w)}{\partial z^*} - \frac{\partial E(\pi)}{\partial z^*} \frac{\partial G(z^*, w)}{\partial w}} \equiv \frac{det1}{det2} \quad (59)$$

The denominator of the above equation is negative under Assumptions 1-3. This is immediate from the steps of the proof of Lemmas 3 and 4.

Differentiating and appropriately rearranging, I can rewrite the numerator as:

$$det1 = \frac{1}{B^2} \left[-\gamma \left(\int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\sigma-\epsilon}{1-\sigma}} (Bw^{-\sigma} - Az) dF(z) \right) - \eta (Az^* - Bw^{-\sigma}) + \xi (Az_{ex}^* - Bw^{-\sigma}) \right] \quad (60)$$

where:

$$\gamma = \frac{(\epsilon - 1)^2}{\sigma - 1} \alpha (w^{1-\sigma} + z^*)^{\frac{1-\epsilon}{\sigma-1}-1} \left[\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-1}{\sigma-1}} dF(z) + \tau^{\epsilon-1} \int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-1}{\sigma-1}} dF(z) \right] \tau^{\epsilon-2} \quad (61)$$

$$\eta = \alpha (w^{1-\sigma} + z^*)^{-1} (\epsilon - 1) f(z^*) \tau^{\epsilon-2} \int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-1}{\sigma-1}} dF(z) \quad (62)$$

$$\xi = (\epsilon - 1) \alpha_{ex} (w^{1-\sigma} + z^*)^{-1} f(z_{ex}^*) \tau^{-1} \left[\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-1}{\sigma-1}} dF(z) \right] \quad (63)$$

Lemma 1 follows directly.

■

Proof of Proposition 2

The first two terms in (31) are positive, while the third can be negative. I here prove that the sum of the second and the third term is always positive, which is sufficient for (31) to be positive. If z_{ex} approaches z^* , which is its lower limit, the sum of the last two terms is $(Az^* - Bw^{-\sigma})(\xi - \eta)$. A sufficient condition for the whole sum to be positive is therefore that $\xi \leq \eta$.

From equations (62) and (63):

$$\xi = \frac{\alpha_{ex} f(z_{ex}^*)}{\alpha f(z^*)} \tau^{1-\epsilon} \frac{\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-1}{\sigma-1}} dF(z)}{\int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-1}{\sigma-1}} dF(z)} \eta \quad (64)$$

From Assumption 1:

$$\frac{\int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-1}{\sigma-1}} f(z) d(z)}{(1 - F(z_{ex}^*)) (w^{1-\sigma} + z_{ex}^*)^{\frac{\epsilon-\sigma}{\sigma-1}}} \geq \frac{\int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-1}{\sigma-1}} f(z) d(z)}{(1 - F(z^*)) (w^{1-\sigma} + z^*)^{\frac{\epsilon-\sigma}{\sigma-1}}} \quad (65)$$

The above inequality, combined with (22), yields:

$$\frac{\xi}{\eta} \leq \left(\frac{w^{1-\sigma} + z_{ex}^*}{w^{1-\sigma} + z} \right) \frac{f(z_{ex}^*)}{f(z^*)} \frac{1 - F(z^*)}{1 - F(z_{ex}^*)} \quad (66)$$

From Assumption 1: $\xi < \eta$ therefore holds.

Proof of Lemma 2

The Proof of Lemma 2 is very similar to that of Lemma 1. The exact same argument on total differentiation and Cramer's rule holds, and the sign of $\frac{dw}{d\alpha_{ex}}$ is therefore given by the sign of $-det3$, where $det3$ is defined as follows:

$$det3 \equiv \frac{\partial G(z^*, w)}{\partial \alpha_{ex}} \frac{\partial E(\pi)}{\partial z^*} - \frac{\partial G(z^*, w)}{\partial z^*} \frac{\partial E(\pi)}{\alpha_{ex}} \quad (67)$$

Appropriately differentiating and rearranging yields:

$$det3 = \frac{Az^* - Bw^{-\sigma}}{B^2} \eta' + \frac{Bw^{-\sigma} - Az_{ex}^*}{B^2} \xi' \quad (68)$$

where:

$$\eta' = (1 - F(z_{ex}^*)) f(z^*) (w^{1-\sigma} + z^*)^{\frac{\sigma-\epsilon}{1-\sigma}} \quad (69)$$

$$\xi' = \frac{f(z_{ex}^*)}{w^{1-\sigma} + z^*} \left[-\tau^{\epsilon-1} (1 - F(z_{ex}^*)) (w^{1-\sigma} + z_{ex}^*)^{\frac{\epsilon-1}{\sigma-1}} + \int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-1}{\sigma-1}} dF(z) + \tau^{\epsilon-1} \int_{z_{ex}^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-1}{\sigma-1}} dF(z) \right] \quad (70)$$

Proof of Proposition 3

I use a limit argument to show that for $\tau = 1$ (no variable costs), a reduction in the fixed costs of trade will be accompanied - for low enough α_{ex} - by a decrease in the skill

wage premium. As $z_{ex}^* \rightarrow z^*$, the right hand side of (67) becomes negative: $det3 \rightarrow (Az^* - Bw^{-\sigma})(\eta' - \xi')$, where:

$$\eta' - \xi' \rightarrow 2(1 - F(z^*))f(z^*)(w^{1-\sigma} + z^*)^{\frac{\sigma-\epsilon}{1-\sigma}} - 2\frac{f(z^*)}{w^{1-\sigma} + z^*} \int_{z^*}^Z (w^{1-\sigma} + z)^{\frac{\epsilon-1}{\sigma-1}} dF(z) < 0$$

For low enough α_{ex} and τ close enough to one such that z_{ex}^* converges to z^* , the skill wage premium decreases with trade liberalisation.

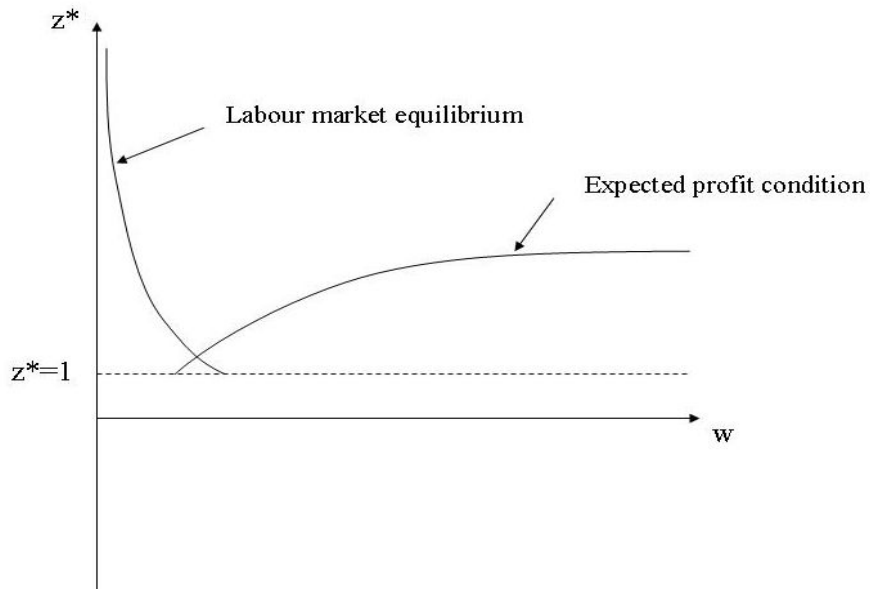


Figure 1: The equilibrium

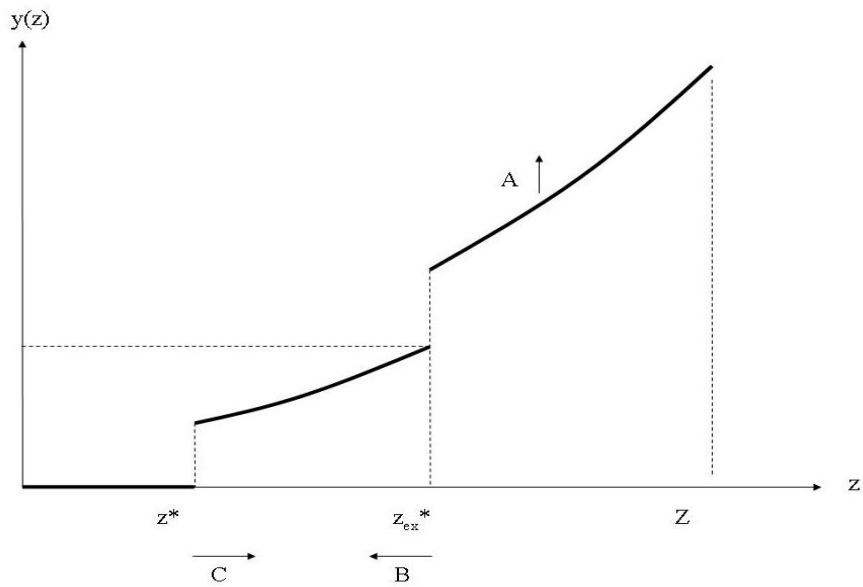


Figure 2: Effects of an increase in τ on the market structure for a fixed w

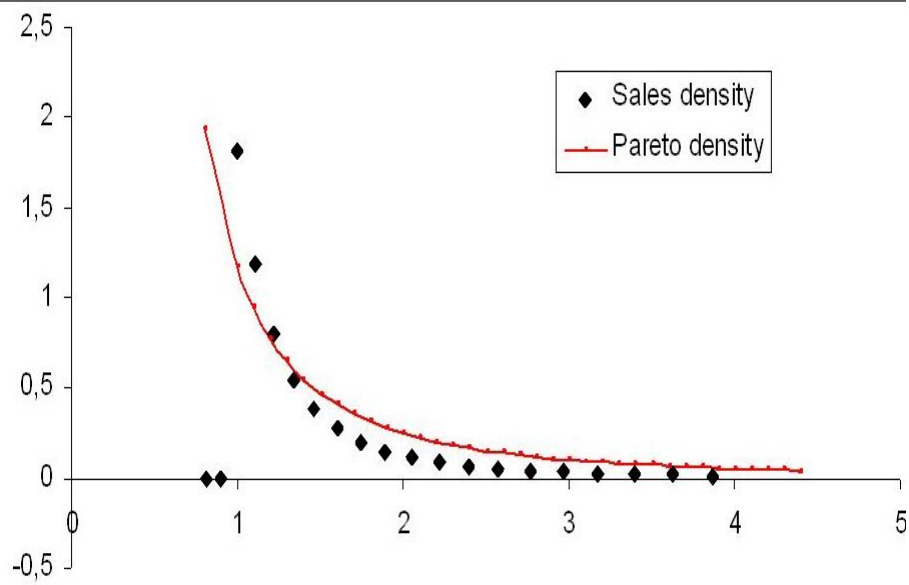


Figure 3: Model generated domestic sales distribution of producing firms compared to empirical Pareto distribution with parameter 1,2

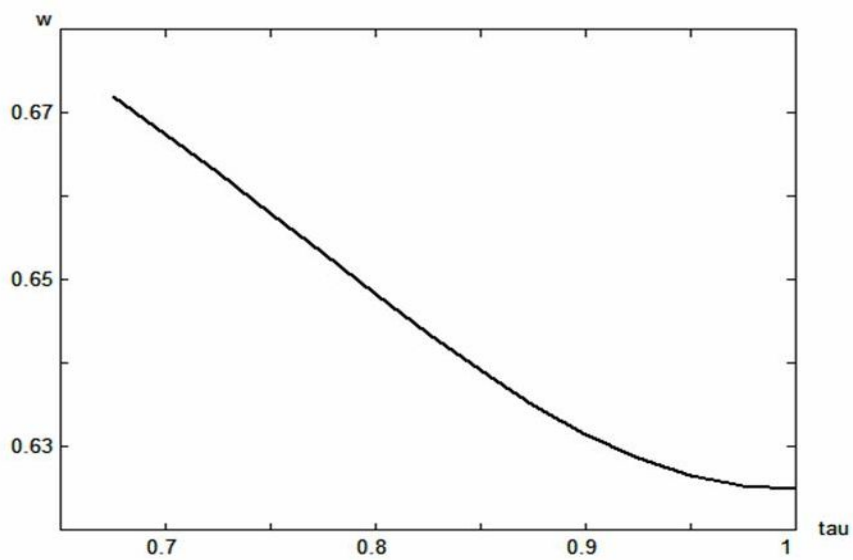


Figure 4: Impact of a change in τ on w for $\epsilon = 3.8, \sigma = 2, a = 3.4, \alpha = \alpha_{ex} = 1, \beta = 0.8$

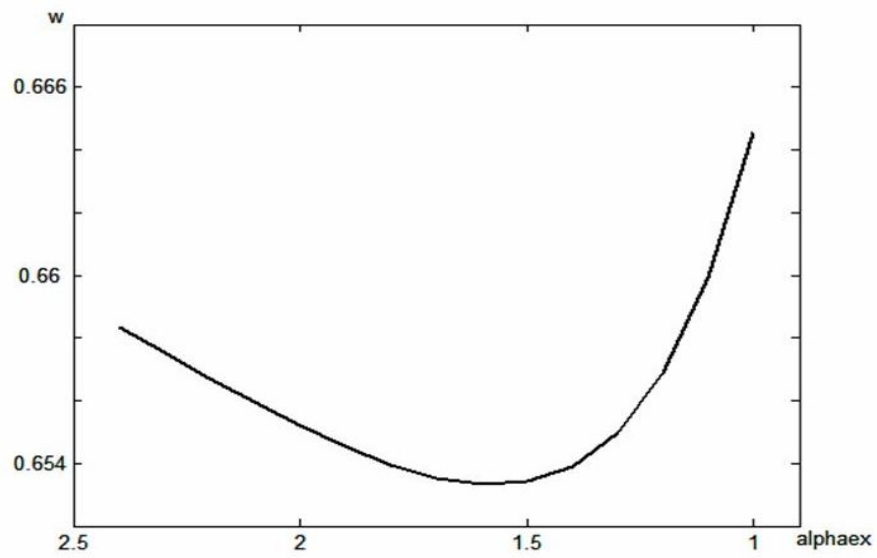


Figure 5: Impact of a change in α_{ex} on w for $\epsilon = 3.8, \sigma = 2, a = 3.55, \alpha = 1, \tau = 1, \beta = 0.8$