Expansion of trade at the extensive margin: welfare and trade-volume consequences

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Abstract

A workhorse tool of international trade theory, the world Edgeworth box, suffers from the limitation that it considers two-country cases only (every point is a division of the world endowment between two countries), and is awkward for handling trade costs. Here we build further on a tool introduced in our earlier paper (JIE 2007), in which a matrix of countries all trade simultaneously. One axis indexes a country’s factor endowment and the other indexes a country’s trade cost. We also continue our earlier analysis of fragmentation, and inquire about the effects of adding trade in intermediates to trade in final goods. Our setup in the present paper is better suited to seeking general gains-from-fragmentation results and is somewhat closer to recent literature such as Grossman-Rossi-Hansberg (2007). Simulation results give a general intuition about who gains and who loses from fragmentation, and an analytical approach gives more rigorous (but limited) sufficient conditions for gains and necessary conditions for losses.
1. Introduction

There has been a lot of recent interest in the expansion of trade at the extensive margin, in which innovations in communications, transportation, and institutions have permitted a wider range of goods and services to be traded. This added trade at the extensive margin goes by a variety of names including “fragmentation”, “vertical specialization”, and “trade in tasks”. These added goods and services have generally been modeled as intermediates and, while there is no compelling reason to make such an assumption, we will continue in that tradition here.

The analysis and analytical tools needed to address fragmentation are rather different from those of traditional trade theory, in which a liberalization results in less costly and therefore generally more trade in an existing set of goods and services. Here we are thinking of a discrete change in which one good or service, used by many countries, switches from a non-traded status to one in which trade costs are more or less the same to/from all locations.

We think that it is fair to say that general results and indeed even a general intuition about fragmentation are lacking. There has been a great deal of theoretical analysis and a good amount of empirical work, but the theory has been confounded by an inability to solve for world general equilibrium price changes that are fundamental to important questions such as trade-volume and welfare effects. Much of the analysis continues in a two-country tradition. Similarly, there are no gains-from-trade theorems for countries going from some trade to more trade, whether this is at the intensive or extensive margins. The possibility of adverse price changes (terms-of-trade) effects rules this out.

The purpose of this paper is to extend our earlier work (Markusen and Venables 2007) in which we propose a new geometric tool to analyze problems such as fragmentation in the
presence of country-specific trade costs. Our “box” is a matrix of countries, with a country’s factor endowments on one axis and a country’s trade costs on the other. Every cell in the matrix is a distinct country, and all countries trade together simultaneously. Thus not only can a capital abundant country be compared to a labor-abundant country, a high-trade-cost capital-abundant country can be compared to a low-trade-cost capital-abundant country.

Our extensions here are useful for a more systematic consideration of gains from trade questions: in particular, what are sufficient conditions for a country to gain from fragmentation and/or what are necessary conditions for it to lose. Our framework is somewhat closer to the “trade in tasks” formulation of Grossman and Rossi-Hansberg (2007) but still retains much continuity from early analyses of fragmentation.

We begin with a very symmetric model that is a classic two-good, two-factor Heckscher-Ohlin model, except with a three-good intermediate-goods structure. Each final good uses two of the intermediates, with one of the intermediates used symmetrically in both final goods. This structure allows us to consider both fragmentations (introduction of trade in one of the intermediates) that are symmetric or neutral in the sense of affecting both finals goods symmetrically and ones that are asymmetric, or biased in favor of one final good. “Symmetric” and “neutral” are defined more formally as fragmentations allowing new trade in intermediates that leave world relative prices of final consumption goods unchanged.

Some of our results are as follow. First, since we begin with a very symmetric two-good model, central countries with the world average factor endowment do not trade when only final goods can be traded. The fragmentations we consider always lead to welfare gains to both fringe (endowments remote from the world average) countries and central countries, and the latter may
trade a large amount in the post-fragmentation equilibrium. But high-trade-cost counties with endowments moderately different from the world average also gain as they are brought into trade from autarky and/or are able to import just the fragment of the good they are bad at rather than the whole package.

In the simulations, symmetric (neutral) fragmentations always imply that no country is worse off. Many countries, however, may experience a sharp drop in trade volume. The intuition is that given in the last sentence of the previous paragraph. A country may be very bad at making auto engines but really good at every other component. If the country can only trade complete cars, then it imports cars. But with fragmentation, it can import engines and perhaps even export cars. This substantially increases its welfare and may significantly reduce its trade volume.

Asymmetric fragmentations that target one sector may reduce the welfare of some countries. This occurs for countries that are ideally suited to the production of an integrated good that is affected by fragmentation. The situation is more or less the converse of that discussed in the previous paragraph. Suppose a country is ideally suited to integrated auto production. Now there is fragmentation in the auto sector only. Our country now finds that there are countries better suited to engine production and countries better suited to all other components. The price of cars falls in equilibrium relative to other final goods and our country is worse off. Note that this cannot happen unless the relative price of cars falls, meaning that the fragmentation must be asymmetric.

A more formal analysis of gains from trade follows the simulations. We show that a sufficient condition for every country to be no worse off following fragmentation is that world
relative prices of final goods do not change. This is not trivial, since the domestic prices of some goods often change even if world prices are constant; for example, an export good may become non-trade or imported. A necessary condition for a country to lose is that the domestic price of at least one final good exported (imported) prior to fragmentation falls (rises). We conclude with a more general sufficient condition for gains expressed in terms of correlations or “on average” relationships.

2. Modeling framework

Our model assumes constant return and perfect competition, and the absence of any other distortions such as tariffs or quotas (although these could be easily added). We will concentrate on a simple case to illustrate our techniques and also to provide intuition about more general phenomena. The structure of production in this simple example is illustrated in Figure 1 and summarized in the following points.

(1) There are two final goods, $X_1$ and $X_2$.

(2) There are three intermediate goods: A, B, and C. A and B are inputs into $X_1$ production and B and C are inputs into $X_2$ production. B is thus a shared intermediate or “task” (Grossman Rossi-Hansberg). A and C are sector-specific tasks.

(3) Intermediate goods are produced from fixed endowments of capital and labor and these factors are assumed internationally immobile throughout the paper.

(4) A great deal of symmetry will be assumed in order to set up the intuition for the later gains-from-trade analysis. All production functions are Cobb-Douglas. A and B each have 50% shares in $X_1$ and B and C each have 50% shares in $X_2$. B has primary factor shares of (50/50). A
is capital intensive with capital/labor shares $\alpha/\beta > 1$ and C is labor intensive with capital/labor shares $\alpha/\beta < 1$.

(5) All countries have identical and homothetic preferences, with shares 50/50.
Countries’ endowments are evenly and symmetrically distribution along a line, with the most capital abundant country having endowments $K = 0.90$ $L = 0.10$ and the most labor abundant country having endowments $K = 0.10$ $L = 0.9$. There are an odd number of countries with the central country having an endowments $K = 0.5$ $L = 0.5$.

(6) Trade costs are country specific and apply to imports and exports from/to all countries. We could think of trade costs as being port costs only. The marginal cost of added distance is zero. Bilateral trade flows will thus not be determined, a limitation of the model.

(7) In addition to an endowment index $j$, a country has a trade-cost index $i$, which is common to all imports and exports. There are exactly $i$ countries with endowment index $j$. Our countries form an $i \times j$ matrix, with exactly one country in each cell of the matrix.

We assume that the final goods $X_1$ and $X_2$ are always tradable at a country’s country-specific trade cost (although autarky is computed as a benchmark). None, some, or all of the intermediates may be tradable, at each country’s country-specific trade cost depending on the experiment. Primary factors are not tradable as noted above. “Fragmentation” is short-hand for the introduction of trade in some previously-non-traded intermediate.

Important to some results is the notion of a “symmetric” or “neutral” fragmentation, in the sense that it is not biased toward one final good or the other. In the present case, allowing trade in B, trade in A and C, or trade in A, B, and C are neutral or symmetric fragmentations. Allowing trade in A and B but not C is an asymmetric fragmentation. As your intuition will
likely suggest, the latter will increase the efficiency of $X_1$ production and will lower the relative price of $X_1$ relative to $X_2$ in equilibrium.

3. **The numerical model**

The reason that many readers will not have seen a model like this (especially referring to the multi-country, multi-trade-cost features) is because it largely analytically intractable. Often curves separating two “regime” (e.g., which goods a country produces or trades) can be solved for, but the expressions are implicit functions of endowments and trade costs and are not that helpful.

We solve the model numerically, using the powerful complementarity and set features of GAMS and Rutherford’s higher-level language MPS/GE in particular. The model decomposes into three large blocks of inequalities, where each inequality has an associated non-negative variable. Activities refer to the production of final goods, intermediate goods, export and import activities and the “production” of utility. Commodities refer to final goods, intermediate goods, factors of production, and utility. The blocks are as follows.

<table>
<thead>
<tr>
<th>Inequalities</th>
<th>Complementary Variables</th>
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<td>Zero-profit inequalities</td>
<td>Activity levels</td>
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<td>Market-clearing inequalities</td>
<td>Commodity prices</td>
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If an inequality is strict in equilibrium, the complementary variable is zero. So, for example, if the marginal cost of an activity is greater than price in equilibrium, then that activity is slack (not operated) in equilibrium. In our simulations, a great many activities are slack. Most
countries do not produce all five (final and intermediate) goods, and they never both import and export the same good (each alternative being represented by an activity). The simulations shown below involve a model with 36,864 weak inequalities in 36,864 complementary variables - minus one, due to Walras’ Law.

Five scenarios or regimes are computed. “Trade” always refers to the ability to trade at each country’s country-specific trade costs. These are as follows:

- **AT** - autarky
- **F** - trade in final goods only
- **B** - allows B to be traded as well
- **AC** - allows A and C but not B to be traded.
- **ABC** - allows A, B and C to be traded
- **AB** - allows A and B but not C to be traded (asymmetric fragmentation)

We will now turn to results on welfare and on the volume of trade in these alternative regimes.

4. **Simulation Results**

Figures 2 and 3 illustrate the use of box, henceforth the MV box. The horizontal (front) axis gives a country’s endowment, and the other axis of the base gives a country’s trade costs, high-trade cost countries in the front. These first two diagrams consider the movement from autarky to trade (at each country’s trade cost) in $X_1$ and $X_2$ only. Figure 2 gives each country’s gains from trade as a share of its autarky welfare level. As is probably intuitive, a country’s gains are increasing in the difference between its endowment and the world average (henceforth
“endowment remoteness”) and decreasing in its trade cost.

Figure 3 gives the volume of trade in final goods, imports plus exports, measured as a proportion of a country’s real income. The valley where there is no trade are countries in autarky despite (costly) trading opportunities. The sloping walls are countries producing both $X_1$ and $X_2$. The plateaus or mesas, are counties that are completely specialized and hence exporting half of their gdp and importing all of the good not produced, which is half of the value of consumption and hence half of gdp.

Figures 4 and 5 give the added effects of introducing trade in intermediate good B, again at each country’s country-specific trade cost. Specifically, Figure 4 gives welfare with trade in B minus welfare with goods trade only, divided by the latter. Countries with endowments remote from the world average gain a lot: they can now specialize in A or C and import B, whereas previously they had to produce both A and B (or B and C) to make $X_1$ or $X_2$ which they could then export.

The more interesting thing about Figure 4 is the gains for central low-trade-cost countries. This is a general equilibrium effect that raises the price of B relative to $X_1$, $X_2$, A and C with intuition as follows. With final-goods trade only, the central countries are relatively indifferent to trading (the central countries is completely indifferent), with prices differences from autarky being small. When trade in B is permitted, the fringe countries on both sides want to import B, but the central countries are indifferent to exporting B at the old prices. The relative price of B must rise to restore equilibrium (trade balance in B) and so the central countries benefit.

The added volume of trade is shown in Figure 5: specifically, the volume of trade as a
share of income with goods-trade only minus the volume of trade as a share of income with trade in B allowed. The fringe countries’ total trade is three times gdp (or 200% more than trade in final-goods only). Take the countries on the left specializing in A, so A production is 100% of gdp. An equal value of B imports is required to match A production, so imports of B are 100% of gdp. But all of X₂ consumption is also imported, which is 50% of gdp. X₁ production is 200% of gdp of which 50% is consumed domestically and 150% is exported. The central low-trade-cost countries trade twice their gdp. All domestic production, composed of B only, is exported and all final consumption, also equal to gdp, is imported.

Figures 6, 7a, 7b and 7c show effects of allowing trade in A and C but not B. The welfare effects relative to goods-trade only for low-trade cost countries are very similar to those in Figure 4. This is once again a relative-price effect, except now it is better thought of as an increase in the excess supply of A and C by the fringe countries rather than an increase in the excess demand for B from central countries. Thus for low-trade-cost countries, central countries benefit from the price change while fringe countries benefit from being able to specialize more in A or C.

But unlike Figure 4, we now have gains for high-trade cost countries that have endowments that are moderately remote from the world average. Take countries on the left, ideally suited to A production. Gains are due to a combination of the following effects, depending on the exact country in question: (a) the ability to import just C instead of importing X₂ or being in autarky, and/or (b) produce less or eliminate production of C, and/or (c) specializing more in and export A instead of exporting integrated X₁ (requiring more B production).
Figure 7a shows the volume of trade under trade in A and C relative to goods-trade only. This is one diagram which is better viewed from the low-trade-cost end, so this is shown in Figure 7b. What is interesting about this case is that there are large groups of countries that trade less when fragmentation is permitted and for many of these trade falls from 100% of gdp to only 50%, a fall of half.

The intuition for this effect is captured in Figure 7c, which divides the countries by the effect of fragmentation on production specialization; specifically, changes in the number of goods produced, five being the maximum. The view is from the low-trade-cost end, corresponding to Figure 7b. Regions 1 and 4 in Figure 7c are countries which do not change their specialization. Regions 2, 5, and 6 are regions in which specialization increases. Region 3 is the interesting one, and corresponds to the areas of decreased trade volume in Figures 7a, and 7b. Prior to fragmentation, countries in this region specialized in integrated $X_1$ production, exporting $X_1$ in exchange for all of their $X_2$ consumption ($X_2$ and C not produced). Fragmentation allows these countries to just import C, combining it with domestic B production to produce $X_2$, rather than importing the complete good $X_2$. In other words, instead of importing a bundle of B and C (good $X_2$), the country just imports C and so trade volume falls.

Suppose that shirt production requires fabric production and cut-and-sew (corresponding to textile and garment industries). You might be quite good at one of these, but if you are very bad at the other, then your only option is to import complete shirts and specialize in the other industry. But if fragmentation is permitted, then you can just import the thing that you are bad at and reduce your specialization and exports in the other sector to pay for that intermediate.

Allowing trade in A, B and C produces figures qualitatively very similar to those with
trade in A and C allowed, and so we will skip these in the exposition.

Readers may have noticed that in the cases considered so far, no country loses from fragmentation. We will show in the next section that this is no coincidence, but in fact it is a general result arising from symmetry of the fragmentation B or A and C across final-goods sectors. In order to make the point more clearly, Figures 8 and 9 consider allowing trade in A and B, but not C. This is an asymmetric fragmentation with effects concentrated on $X_1$ production.

Figures 8 shows that there is a set of counties, moderately capital-abundant and with moderate trade costs, that are made worse off by this fragmentation (losses are quantitatively quite small, but they are losses). The intuition to this is as follows. These are countries that are initially well suited to integrated $X_1$ production. After fragmentation, these countries find that some countries to the left are better at A production and some countries to the right are better at B production. The consequence is that the price of $X_1$ relative to $X_2$ falls in the new equilibrium. This makes countries ideally suited to integrated $X_1$ production worse off.

Figure 9 shows the change in the volume of trade in this asymmetric fragmentation. Interestingly, the countries that are worse off do not change their specialization and trade volume, they just get a worse terms of trade. Countries that reduce their trade volume get gains, due to effect mentioned above: the ability to import just that part of a final good (intermediate good A) that they are bad at.
5. **Gains from Trade: Analytical Results**

General gains-from-trade results in trade theory are hard to obtain. The basic textbook result only proves that free trade is better for a country than autarky or, somewhat more generally, any level of restricted (but not subsidized!) trade is better than autarky. However, there is no general result that says that more trade is better than less (but positive) trade. This is due to the possibility of adverse price changes; e.g., further liberalization by a large country can make it worse off. But this is exactly the type of result we are seeking here: under what conditions will the introduction of an additional set of trade possibilities to an already-existing set of traded goods improve welfare? Our approach follows the classic “revealed preference” methodology.

X denotes a vector of final goods quantities, with the vector p denoting their prices. Similarly, Z denotes a vector of intermediate goods with prices q. In our explicit model above, X (now denoting X₁, X₂) and Z (now denoting A, B, and C) were disjoint sets. The proof to follow does not preclude a final good also having intermediates uses: X and Z can share elements. The distinction is that (a) only X goods enter into welfare-producing consumption and (b) we will *define* the absence of fragmentation to mean that goods cannot be traded for intermediate use.

Superscript w will denote world prices. Superscript f denotes prices with fragmentation, trade in intermediates, while subscript n denotes prices in the no-fragmentation equilibrium. Thus vectors of world final goods prices with and without fragmentation are $p^{wf}$ and $p^{wn}$ respectively. We will also use this notation for intermediate goods prices, although they are not defined for the no-fragmentation case: $q^{wf}$ and $q^{wn}$. 

We have to allow for the fact that a domestic prices differs from the world price due to the country-specific trade costs, and that difference depends on whether the good is exported, non-traded, or imported. With $t \geq 1$ being the gross trade costs, the domestic price of a particular good $i$ must fall in the interval

$$p_i^{\text{df}} \geq p_i^{f} \geq p_i^{\text{df}/t}, \quad p_i^{\text{mn}} \geq p_i^{n} \geq p_i^{\text{mn}/t}$$  \hspace{1cm} (1)$$

The domestic price will lie at the left-hand end of the interval if $i$ is an import, in between if it is a non-traded good, and at the right-hand end if it is an export good. We have similar expressions for the intermediates.

$$q_i^{\text{df}} \geq q_i^{f} \geq q_i^{\text{df}/t}, \quad q_i^{\text{mn}} \geq q_i^{n} \geq q_i^{\text{mn}/t}$$  \hspace{1cm} (2)$$

Domestic prices for a particular country will henceforth be denoted by the vectors $p$ and $q$ for final and intermediate goods respectively. Not country designator is used, since we are trying to show gains from fragmentation for all countries, regardless of their endowments and trade costs.

A subscript $o$ on a quantity denotes production of the good and a subscript $c$ denotes consumption. For an intermediate good, “consumption” means its use as a domestic input in a final good. Thus for both final and intermediate goods, a positive value for production minus consumption of the good indicates it is an export good, and so forth.

We assume a competitive undistorted economy, and so the first theorem of welfare economics applies and the value-added of the economy in regime $j$ ($j = f, n$) must be maximized at that regime’s prices. Specifically, the value added in regime $f$ must be greater than or equal to regime-$n$ value added when the latter is evaluated at regime-$f$ prices. Total value added for the
economy is given by the value of final goods production minus intermediate usage
(“consumption” of intermediates) plus the value of intermediate goods production. The
production efficiency condition is then given by
\[
(p^f X_o^f - q^f Z_c^f) + q^f Z_o^f \geq (p^f X_o^n - q^f Z_c^n) + q^f Z_o^n
\]
Which rearranges to
\[
p^f X_o^f + (q^f Z_o^f - q^f Z_c^f) \geq p^f X_o^n + (q^f Z_o^n - q^f Z_c^n)
\]
The last term on the right-hand side is zero since the intermediates are not traded in the n-regime,
so production and consumption are the same for each good.
\[
p^f X_o^f + (q^f Z_o^f - q^f Z_c^f) \geq p^f X_o^n
\]
Now we introduce the balance of trade constraints for each regime, which require that the
value added in production equals the value of final consumption.
\[
p^f X_o^f + (q^f Z_o^f - q^f Z_c^f) = p^f X_c^n
\]
\[
p^n X_o^n = p^n X_c^n
\]
Substitute the right-hand side of (6) for the left-hand side of (5)
\[
p^f X_c^n \geq p^f X_o^n
\]
Now add the right-hand side of (7) and subtract the left-hand side, and also add and subtract the
term \(p^f X_c^n\). (8) then becomes
The classic revealed-preference criterion for gains from fragmentation requires that the left-hand side of (11) is greater than or equal to the first term on the right-hand side: the value of regime-f consumption is revealed preferred to regime-n consumption valued at regime-f prices. So a sufficient condition for this to be true is that the second additive term on the right-hand side of (11) is non-negative. So we will now focus on showing conditions under which this term is non-negative.

Suppose that fragmentation leaves relative world prices of final goods unchanged. Then if we adopt a normalization (numeraire rule) that the sum of world final goods prices is constant, then it also follows that the absolute level of each final good price is unchanged following fragmentation. Thus \( p^f = p^n = p^* \). We will refer to this as a “neutral” or “symmetric” fragmentation, emphasizing that these are definitions (e.g., a symmetric fragmentation is one that leaves relative final-goods prices unchanged).

The quantities term at the right-hand end of (11) is net exports of X in regime n. Without loss of generality, consider any good i and suppose that it is initially an export good in regime n.

\[
\begin{align*}
    p^f X^f_i &\geq p^f X^n_i + p^n X^n_i - p^n X^n_i + p^f X^n_i - p^f X^n_i \\
    p^f X^f_i &\geq p^f X^n_i + p^f (X^n_i - X^n_i) - p^n (X^n_i - X^n_i) \\
    p^f X^f_i &\geq p^f X^n_i + (p^f - p^n) (X^n_i - X^n_i)
\end{align*}
\]

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The quantities term at the right-hand end of (11) is net exports of X in regime n. Without loss of generality, consider any good i and suppose that it is initially an export good in regime n.

\[
X^n_i > X^n_i \quad \text{or} \quad (X^n_i - X^n_i) > 0
\]

Three cases need to be considered. First, good i remains an export good in regime f. Then the domestic price of good i is unchanged and the term for good i in the right-hand summation of
(11) is zero. Second, suppose that good i becomes non-traded in regime f. Then from (1), its
domestic price is greater than or equal to its regime-n price: \( p_i^f \geq \frac{p_i^{nf}}{t} = p_i^n \), so the term for
good i in the right-hand summation in (11) is non-negative. Finally, suppose that good i
switches to being an import good in regime f. Then its domestic price *rises* from \( p_i^n = \frac{p_i^{w}}{t} \) to
\[ p_i^f = p_i^{w} t > p_i^n \] and now the term for good i in the right-hand summation of (11) is positive.

Summarizing, we have

\[
(p_i^f - p_i^n)(X_{a}^n - X_{c}^n) = 0 \quad \text{good i continues to be exported} \tag{14}
\]

\[
(p_i^f - p_i^n)(X_{a}^n - X_{c}^n) \geq 0 \quad \text{good i becomes non-traded}
\]

\[
(p_i^f - p_i^n)(X_{a}^n - X_{c}^n) > 0 \quad \text{good i switches to an import}
\]

Thus for any final good i that is initially exported, that term in the right-hand summation
in (11) must be non-negative. An identical argument can be made that for any good i that is
initially imported, the same must be true (if it switches to being exported, its domestic price must
fall). And, of course, for any good i that is initially non-traded, that term in the summation is
zero. This establishes that

\[
(p^f - p^n)(X_{c}^n - X_{c}^n) \geq 0 \tag{13}
\]

From (11), this is in turn a sufficient condition for

\[
p^f X_{c}^f \geq p^n X_{c}^n \tag{14}
\]

which is a sufficient condition for gains from fragmentation.
Proposition 1

A sufficient condition for every country to gain from adding trade in intermediates is that the world prices of final consumption goods do not change.

Proposition 2

A necessary condition for a country to lose from adding trade in intermediates is that the world price of a good it exported (imported) under no intermediates trade falls (rises).

The condition that world prices do not change is very restrictive and “overly” sufficient. Consider again the last term in (11), also given as (13): if this is non-negative, then this is sufficient for gains from fragmentation. Let $X^n = (X^n - X^n)$ denoted the vector of net exports in regime n (no trade in intermediates), also referred to as the “initial” net export vector. (13) can be written as any of the following

$$ (p^f - p^n)X^n \geq 0 \quad \text{or} \quad p^fX^n \geq 0 \quad \text{since} \quad p^nX^n = 0 $$

The first expression is just a simple correlation between the changes in domestic final-goods prices and the initial net export vector. We will have gains from fragmentation if “on average” the prices of initially-exported goods rise and the prices of initially-imported goods fall. Noting that the value of the initial trade vector at initial prices is zero (trade balance: equation (7)), we have the second expression. If, after fragmentation, the country were to retain its initial net export vector, it would run a trade surplus if the inequality is strict. It could then improve its welfare by cutting some exports and/or increasing some imports to restore trade balance.
Proposition 3

The following conditions are equivalent *sufficient* conditions for a country to *gain* from adding world trade in intermediates: (a) the correlation between domestic price changes and the initial net export vector is non-negative; (b) the value of the initial net export vector at post-fragmentation prices is non-negative (the original net export vector would now yield a trade surplus).

While these are about the best we can do from a theory point of view, they are not all that helpful to a policy maker. The immediate question is: under what sort of conditions is the sufficient condition likely to be met? I think that our previous analysis with a specific model gives us at least some intuition behind the answer to this question. When a fragmentation is relatively neutral or symmetric, in the sense of affecting most final goods in a similar way, then the fragmentation is more likely to be welfare improving. In the case where we only allowed trade in B, this is an input into both $X_1$ and $X_2$ with equal shares in each, and so is neutral and symmetric. When we allow trade in A and C (and by extension A, B, and C), each is used in only one final good but they have, respectively, equal shares in those two final goods. So again, the fragmentation is neutral and no country loses.

Conversely, permitting trade in A and B but not C is clearly not symmetric and neutral. It pushes down the world price of good $X_1$ in equilibrium. As noted above, this results in a welfare loss for low-trade-cost countries that were ideally suited to producing and exporting integrated $X_1$ before fragmentation.
Summary and Conclusions

We conclude with a quick recap of some of our results.

(1) Symmetric or neutral fragmentations which leave the world (not domestic) prices of final goods unchanged benefit all countries, or at least leave no one worse off.

(2) Asymmetric fragmentations in the sense of affecting one sector more can reduce the welfare of some countries. If A and B can be traded, for example, then this lowers the relative price of X₁ and reduces the welfare of countries which were ideally suited to producing integrated X₁ initially.

(3) These first two findings help give some general intuition for the analytical gains-from-trade analysis that follows. A necessary condition for a country to lose from fragmentation is that it suffer a negative terms-of-trade shock. On average, the domestic prices of goods initially exported (imported) must fall (rise). An equivalent way of stating the same result is to say that the value of the initial net export vector at post-fragmentation prices must be negative: the initial export vector would now yield a trade deficit at the new prices.

(4) Any fragmentation may reduce the volume of trade for many countries. The intuition is that a country may be good at B but really bad at C, for example, and so has to specialize in X₁ and import X₂ in equilibrium. But if C can be traded, it can just import C and not integrated X₂ and export a smaller amount of X₁ to pay for it. Imagine that, whenever your car breaks, you have to buy a whole new car instead of just replacing the part that is broken.
REFERENCES


$TITLE: MODEL5
* model dimensions: 36,864 weak inequalities in 36,864 unknowns
* minus one for numeraire (price index for central lowest-trade-cost country)

SETS  I       countries trade costs    /1*31/,
       J       countries factor endowments /1*41/,
       F       factors of production    /L, S/;

PARAMETERS
   TC(I), TCA(I), TCB(I), TCC(I) trade costs
   ENDOW(I,J,F) factor endowments
   AX, BX, BY, CY intermediate goods factor shares
   FA(F), FB(F), FC(F) primary factor intensities;

* set primary-factor and intermediate-goods' intensities (shares)

   AX = 50;
   BX = 50;
   BY = 50;
   CY = 50;
   FA(“L”) = 10;
   FA(“S”) = 40;
   FB(“L”) = 25;
   FB(“S”) = 25;
   FC(“L”) = 40;
   FC(“S”) = 10;

$ONTEXT
$MODEL: MULTI

$SECTORS:
   X(I,J)   ! production activity for X
   Y(I,J)   ! production activity for Y
   A(I,J)   ! production activity for A
   B(I,J)   ! production activity for B
   C(I,J)   ! production activity for C
   EA(I,J)  ! export activity for A
   IA(I,J)  ! imports activity for A
   EB(I,J)  ! export activity for B
   IB(I,J)  ! import activity for B
   EC(I,J)  ! export activity of C
   IC(I,J)  ! import activity for C
   EX(I,J)  ! export activity for X
   IX(I,J)  ! import activity for X
   EY(I,J)  ! export activity for Y
   IY(I,J)  ! import activity for Y
   XX(I,J)  ! supply of X to own market
   YY(I,J)  ! supply of Y to own market
   W(I,J)   ! welfare index

$COMMODITIES:
   PW(I,J)   ! utility price index
   PX(I,J)   ! domestic producer price of X (marginal cost)
   PY(I,J)   ! domestic producer price of Y (marginal cost)
   PA(I,J)   ! domestic producer price of A (marginal cost)
   PB(I,J)   ! domestic producer price of B (marginal cost)
   PC(I,J)   ! domestic producer price of C (marginal cost)
   PCX(I,J)  ! domestic consumer price of X
   PCY(I,J)  ! domestic consumer price of Y
   PF(I,J,F) ! factor prices
   PFA PFB PFC ! world prices of intermediate goods
   PFX PFY ! world prices of final goods
$\text{CONSUMERS:}$

\begin{align*}
\text{CONS}(I,J) &\quad \text{! income of the representative consumer} \\
\text{PROD}:X(I,J) &\quad s:1 \\
O:PX(I,J) &\quad Q:100 \\
I:PA(I,J) &\quad Q:AX \\
I:PB(I,J) &\quad Q:BX \\
\text{PROD}:Y(I,J) &\quad s:1 \\
O:PY(I,J) &\quad Q:100 \\
I:PB(I,J) &\quad Q:BY \\
I:PC(I,J) &\quad Q:CY \\
\text{PROD}:A(I,J) &\quad s:1 \\
O:PA(I,J) &\quad Q:50 \\
I:PF(I,J,F) &\quad Q:FA(F) \\
\text{PROD}:B(I,J) &\quad s:1 \\
O:PB(I,J) &\quad Q:50 \\
I:PF(I,J,F) &\quad Q:FB(F) \\
\text{PROD}:C(I,J) &\quad s:1 \\
O:PC(I,J) &\quad Q:50 \\
I:PF(I,J,F) &\quad Q:FC(F) \\
\text{PROD}:EX(I,J) &\quad \\
O:PFX &\quad Q:100 \\
I:PX(I,J) &\quad Q:(100*TC(I)) \\
\text{PROD}:EY(I,J) &\quad \\
O:PFY &\quad Q:100 \\
I:PY(I,J) &\quad Q:(100*TC(I)) \\
\text{PROD}:IX(I,J) &\quad \\
O:PCX(I,J) &\quad Q:100 \\
I:PFX &\quad Q:(100*TC(I)) \\
\text{PROD}:IY(I,J) &\quad \\
O:PCY(I,J) &\quad Q:100 \\
I:PFY &\quad Q:(100*TC(I)) \\
\text{PROD}:XX(I,J) &\quad \\
O:PCX(I,J) &\quad Q:100 \\
I:PX(I,J) &\quad Q:100 \\
\text{PROD}:YY(I,J) &\quad \\
O:PCY(I,J) &\quad Q:100 \\
I:PY(I,J) &\quad Q:100 \\
\text{PROD}:EA(I,J) &\quad \\
O:PFA &\quad Q:100 \\
I:PA(I,J) &\quad Q:(100*TCA(I)) \\
\text{PROD}:IA(I,J) &\quad \\
O:PA(I,J) &\quad Q:100 \\
I:PFA &\quad Q:(100*TCA(I)) \\
\text{PROD}:EB(I,J) &\quad \\
O:PFB &\quad Q:100 \\
I:PB(I,J) &\quad Q:(100*TCB(I)) \\
\text{PROD}:IB(I,J) &\quad \\
O:PB(I,J) &\quad Q:100 \\
I:PFB &\quad Q:(100*TCB(I))
\end{align*}
$PROD: EC(I,J) 
  O: PFC  Q: 100 
  I: PC(I,J)  Q: (100*TCC(I))

$PROD: IC(I,J) 
  O: PC(I,J)  Q: 100 
  I: PFC  Q: (100*TCC(I))

$PROD: W(I,J) s:1 
  O: PW(I,J)  Q: 200 
  I: PCX(I,J)  Q: 100 
  I: PCY(I,J)  Q: 100

$DEMAND: CONS(I,J) 
  D: PW(I,J)  Q: (SUM(F, ENDOW(I,J,F))) 
  E: PF(I,J,F)  Q: ENDOW(I,J,F)

$OFFTEXT
$SYSINCLUDE mpsgeset MULTI

OPTION MCP=PATH;
OPTION SOLPRINT = OFF;

* chose numeraire: utility price index for lowest cost central country
PW.FX("31","21") = 1;

* set endowments for the 31x41 = 1271 countries
LOOP(I,
  LOOP(J,
    ENDOW(I,J,"S") = (180 + 4 - 4*ORD(J));
    ENDOW(I,J,"L") = (20- 4 + 4*ORD(J));
  );
);

* first compute autarky solution
* by setting all trade costs to prohibitive levels
TC(I) = 100;
TCA(I) = 100;
TCA(I) = 100;
TCB(I) = 100;
TCB(I) = 100;
TCC(I) = 100;
TCC(I) = 100;
);

* help the solver by setting initial values of all
* trade activities to zero
EA.L(I,J) = 0;
IA.L(I,J) = 0;
EB.L(I,J) = 0;
IB.L(I,J) = 0;
EC.L(I,J) = 0;
IC.L(I,J) = 0;
EX.L(I,J) = 0;
IX.L(I,J) = 0;
EY.L(I,J) = 0;
IY.L(I,J) = 0;
* solve for autarky equilibrium

MULTI.workspace = 25;
$INCLUDE MULTI.GEN
SOLVE MULTI USING MCP;

Remainder of program solves for different scenarios (regimes) and processes output for the figures.
Figure 1: Structure of Production

Utility

FINAL GOODS
(50/50 shares in U)

INTERMEDIATE GOODS
(50/50 shares in X,Y)

PRIMARY FACTORS
(20) (80) (50) (50) (80) (20)

FINAL GOODS ALWAYS TRADABLE AT COUNTRY-SPECIFIC TRADE COST
INTERMEDIATE GOODS NONE / SOME / ALL TRADABLE AT COUNTRY-SPECIFIC TRADE COST
PRIMARY FACTORS NOT TRADABLE
Figure 2: Gains from trade: autarky to trade in final goods only
Figure 3: Volume of trade: trade in final goods only

Country i's endowment of labor (capital = 1 - labor endowment)
Figure 4: Additional gains from allowing trade in B
Figure 5: Additional volume of trade from allowing trade in B

Country i’s endowment of labor (capital = 1 - labor endowment)
Figure 6: Additional gains from allowing trade in A and C (no trade in B)
Figure 7a: Additional volume of trade from allowing trade in A and C (no trade in B)
Figure 7b: Additional volume of trade from allowing trade in A and C (no trade in B)
Figure 7c: Introduce trade in A and C to existing trade in X and Y

1. No change in specialization, produce all 5 goods
2. Increase in specialization, stop producing C
3. Decrease in specialization, start producing Y using imported C
4. No change in specialization, produce A, X and B
5. Increase in specialization, stop producing X and B, specialize in A
6. Increase in specialization, stop producing A and C
Figure 8: Additional gains from allowing trade in A and B (no trade in C)
Figure 9: Additional volume of trade from allowing trade in A and B (no trade in C)