Is an Export Subsidy a Robust Trade Policy Recommendation towards a Unionized Duopoly?*

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Abstract

Bandyopadhyay et al. (2000) and Brander and Spencer (1988) imply that the robust trade policy recommendation towards a unionized duopoly is an export subsidy. In this paper, we show that we cannot get such a result even in the linear case if the opportunity cost of public funds is sufficiently high. However, if we introduce political ingredients to the model, i.e., considering the case where the domestic firm and the trade union lobby the government for setting their favorable trade policies by giving the government political contributions (modeled in a common agency setting), then the result of robustness will be restored if the government cares about political contributions sufficiently relative to national welfare.

Key Words: Unionized duopoly, Strategic trade policy, Opportunity cost of public funds, Special interest politics

JEL Classification: F13, D72

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1 Introduction

This paper studies whether an export subsidy is a robust trade policy recommendation towards a unionized duopoly.

It is well known that the fundamental problem faced by strategic trade policy models is that the \textit{ex ante} policy recommendation is very sensitive to the \textit{ex post} market conduct. For instance, the optimal trade policy is an export subsidy if firms compete as Cournot competitors in product market (see Brander and Spencer (1985)); whilst it is an export tax if firms engage in Bertrand competition (see Eaton and Grossman (1986)).

Recently Bandyopadhyay \textit{et al.} (2000) point out: demand linearity ensures that an export subsidy is the optimal trade policy towards a unionized Bertrand duopoly.\footnote{See Bandyopadhyay \textit{et al.} (2000), Proposition 1.} This paper and Brander and Spencer (1988), which show that the optimal trade policy towards a unionized Cournot duopoly is an export subsidy, together imply that an export subsidy is a robust trade policy recommendation towards a linear unionized duopoly.

Is this result a robust one? The aim of this paper is to answer this question.

We begin with a linear model in which following Brander and Spencer (1988), we introduce a trade union to a Brander-Spencer third-market model for one of the two exporting countries, say, the ‘domestic country’, and consider the case of unilateral intervention. In this model, we reproduce the result of robustness implied by the above two papers in a clear-cut way: the optimal trade policy is an export subsidy irrespective of the form of market conduct. This serves as a benchmark case. Then, we introduce an opportunity cost of public funds to the above setting. Now even in the linear case, an export subsidy is not a robust trade policy recommendation if this cost is sufficiently high. Then, we consider another variation: to allow the domestic firm and the trade union to lobby for their favorable policies by giving the domestic government political contributions prior to the government setting trade policy. This is modeled as a common agency framework due to Bernheim and Whinston (1986), and Grossman and Helpman (1994). We show that an export subsidy is a robust policy recommendation irrespective
of the form of market conduct if the government cares about political contributions sufficiently relative to national welfare.  

So, what is the main lesson that we have learnt from this simple exercise? First of all, an export subsidy can hardly be a robust trade policy recommendation towards a unionized duopoly, if we consider this problem from a purely economic perspective: it is very sensitive to the opportunity cost of public funds even in the simplest setting. However, an export subsidy as a robust policy recommendation can be supported by political reasons, for instance, special interest lobbying.

This research is related to several strands of literatures.

Since Brander and Spencer (1988), many papers have explored the trade policy implications of a unionized duopoly, notably Bandyopadhyay et al. (2000), and so on.

Matsuyama (1990) introduces a social cost of public funds to the strategic trade policy literature, and is followed notably by Neary (1994). As far as we know, our paper is the first paper introducing an opportunity cost of public funds to the research of strategic trade policy towards a unionized duopoly.

To the best of our knowledge, Fung and Lin (2000) is the only other paper that uses a common agency approach to studying strategic trade policy from a political economy perspective. But they do not introduce an opportunity cost of public funds to their research, whilst we do.

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\(^2\)See Helpman (1997) for an excellent introduction to political economy of trade policy.

\(^3\)Neary (1994) introduces an opportunity cost of public funds to Cournot setting in a Brander-Spencer third-market model, whilst introducing an opportunity cost of public funds to Bertrand setting in the Carmichael-Gruenspecht model. See Carmichael (1987) and Gruenspecht (1988). They consider the case when firms move first, then governments design trade policies, and emphasize the importance of timing in decisions.

We introduce an opportunity cost of public funds to both Cournot and Bertrand settings in a Brander-Spencer third-market model, in which governments can commit to trade policies.

\(^4\)A second-order difference between our paper and their paper is as follows. In a partial equilibrium version of Grossman and Helpman (1994), they show that “even with political pressure, the … politically determined export subsidy is identical to the Brander-Spencer rent-shifting export subsidy”, and this “highlights the possibility that lobbying can restore the level of trade intervention to a more efficient one in the absence of the benevolent
The paper is organized as follows. Section 2 sets out the basic model. In section 3, we do equilibrium analysis. Next, we introduce an opportunity cost of public funds and special interest politics to the basic model in sequence. In section 6, we discuss the relationship between results obtained in this paper and those in existing literatures. The final section concludes.

2 The Basic Model

There are three countries: domestic, foreign and a third country. A domestic firm and a foreign firm produce differentiated goods and sell the goods in the third market. There is no consumption of the goods either in the domestic or in the foreign country.

Technology:

In each country, labor is the only input for production. The domestic firm and the foreign firm share the same Ricardian technology: to produce one unit of output needs one unit of labor. However, there are two differences between the two countries. First, the opportunity wage rate for workers in the domestic country is \( w_0 \); whilst the opportunity wage rate for workers in the foreign country is \( w_0^* \); they are not necessarily equal. Second, domestic workers are organized and form a trade union, whilst this is not the case in the foreign country.\(^5\) (This implies that the wage rate for the domestic workers is greater than the opportunity wage rate, whilst the wage rate for the foreign workers is the same as the opportunity wage rate.)

\textsuperscript{5} As indicated above, they do not consider the question of whether an export subsidy is a robust trade policy.

The question of robustness is the main focus of our paper, and in our paper the politically determined export subsidy is always greater than the rent-shifting export subsidy set by a benevolent government. Why do we observe this difference? The answer is that these two papers use different modeling techniques. As indicated above, their model is a slight variation of Grossman and Helpman (1994), whilst we model explicitly the domestic firm and the trade union as special interest groups. In their model, the special interest group that gains from an export subsidy has the same lobbying power as the special interest group that loses from an export subsidy, whilst in our model, both the trade union and the domestic firm gain from an export subsidy. Of course, which modeling technique is appropriate is an empirical question.
Preferences:

In the third country, the representative consumer’s preference is given by

\[ U(x, x^*; m) = u(x, x^*) + m, \]

where

\[ u(x, x^*) = a(x + x^*) - \frac{1}{2} \left( bx^2 + 2xx^* + bx^*^2 \right), \]

and \( b > 1 \), which is a parameter that measures the degree of product differentiation. The consumption of domestic products is given by \( x \), and the consumption of foreign products, \( x^* \); \( m \) represents the consumption of a numeraire good.

The indirect demand system is given by

\[ p = a - bx - x^*, \quad p^* = a - x - bx^*, \]

where \( p \) is the price for domestic products, and \( p^* \), the price for foreign products. The direct demand system is given by

\[ x = \alpha - \beta p + \gamma p^*, \quad x^* = \alpha + \gamma p - \beta p^*, \]

where

\[ \alpha = \frac{a}{b + 1}, \quad \beta = \frac{b}{b^2 - 1}, \quad \gamma = \frac{1}{b^2 - 1}. \]

Timing:

This is a three-stage game.

In the first stage, the domestic government sets trade policy, \( s \). If \( s > 0 \), this is an export subsidy; if \( s < 0 \), this is an export tax; if \( s = 0 \), this is the non-intervention policy.

In the second stage, the domestic wage rate and the domestic employment levels are determined. The trade union moves first and sets the wage rate. After observing the wage rate, the domestic firm decides how much labor to employ. (We use a Leontief model to characterize the strategic interactions in this stage.)

In the third stage, the domestic firm and the foreign firm engage in product market competition in the third country either as Cournot competitors or as Bertrand competitors.
Then the game is over.

**Payoffs:**

The domestic firm and the foreign firm receive their profits respectively. The trade union receives its economic rents. The economic rents are defined as the product of the difference between the actual wage rate and the opportunity wage rate and the employment level. The government receives national welfare, which is given by the sum of the domestic firm’s profits and the trade union’s economic rents subtracting the costs of subsidy.

The solution concept is a Subgame Perfect Nash Equilibrium (SPNE).\(^6\)

Next, we use generalized backward induction to solve this model with the help of the following Assumption.

**Assumption 1**

\[
(2b^2 - 1) (a - w_0) - b (a - w_0) > 0.
\]

Assumption 1 says that without the government’s intervention, the domestic firm can survive in product market competition: either Cournot or Bertrand.\(^7\)

### 3 Equilibrium Analysis

#### 3.1 Cournot competition

First of all, let us analyze the case where the domestic firm and the foreign firm compete as Cournot competitors in product market.

In the third stage of the game, the domestic firm maximizes its profits:

\[
\pi = (a - bx - x^* - w + s) x,
\]

\(^{6}\)It should be noted that Bandyopadhyay *et al.* (2000) consider only Bertrand competition, and Brander and Spencer (1988) consider only Cournot competition. We consider both Bertrand and Cournot cases, though in a simplest setting. See discussion in Conclusion.

\(^{7}\)See the first subsection of Appendix.
whilst the foreign firm maximizes its profits:

\[ \pi^* = (a - x - bx^* - w_0^*) x^*. \]

The domestic firm’s first-order condition for profit maximization and the foreign firm’s first-order condition for profit maximization determine simultaneously the Nash equilibrium: \(^8\)

\[
\begin{align*}
x &= \frac{2b(a - w + s) - (a - w_0^*)}{4b^2 - 1}, \\
x^* &= \frac{2b(a - w_0^*) - (a - w + s)}{4b^2 - 1}.
\end{align*}
\]

Notice that these are also equilibrium employment levels of the domestic firm and the foreign firm.

In the second stage of the game, the trade union chooses wage rate, \(w\), to maximize its economic rents:

\[
\omega = (w - w_0) x(w) = (w - w_0) \left[ \frac{2b(a - w + s) - (a - w_0^*)}{4b^2 - 1} \right].
\]

From the first-order condition for maximization, we can solve for the equilibrium wage rate given the domestic trade policy: \(^9\)

\[ w = \frac{2b(a + s + w_0) - (a - w_0^*)}{4b}. \]

Notice that \(\frac{dw}{ds} = \frac{1}{2}\). This means that the trade union skims off one half of the trade policy.

Using expression (3), we can show

\[
\begin{align*}
x &= \left[ \frac{2b(a - w_0 + s) - (a - w_0^*)}{2(4b^2 - 1)} \right], \\
\pi &= b \left[ \frac{2b(a - w_0 + s) - (a - w_0^*)}{2(4b^2 - 1)} \right]^2.
\end{align*}
\]

\(^8\) Notice that the first-order conditions are also sufficient in this standard Cournot game.

\(^9\) It is straightforward to show

\[
\frac{\partial^2 \omega}{\partial w^2} = -\frac{4b}{4b^2 - 1} < 0.
\]

Therefore, there is a unique interior solution.
\[
\omega = \frac{[2b(a - w_0 + s) - (a - w_0^*)]^2}{8b(4b^2 - 1)}.
\] (6)

In the first stage of the game, the government chooses trade policy, \(s\), to maximize national welfare:

\[
G = \pi + \omega - sx,
\]
where \(\pi\) is given by expression (5), \(\omega\) is given by expression (6), and \(x\) is given by expression (4). From the first-order condition for national welfare maximization, we can solve for the optimal trade policy.\(^{10}\) It is an export subsidy:

\[
s = \frac{b [2b(a - w_0) - (a - w_0^*)]}{2b^2 - 1} > 0.
\] (7)

How do we explain this result?

Consider the case when domestic workers are not organized. So, the unit production cost is \(w_0\). And we go back to the classic profit-shifting setting of Brander and Spencer (1985). National welfare is defined as

\[
G = \pi - sx = (a - bx - x^* - w_0 + s)x - sx.
\] (8)

It is easy to show that the optimal trade policy is an export subsidy being equal to

\[
s = \frac{[2b(a - w_0) - (a - w_0^*)]}{4b(2b^2 - 1)}.
\]

Now the unit production cost decreases to \(w_0 - \frac{[2b(a - w_0) - (a - w_0^*)]}{4b(2b^2 - 1)}\). This gives the domestic firm a Stackelberg quantity leadership position in product market competition.

When domestic workers are organized, the unit production cost is \(w\). As shown above, the optimal trade policy is an export subsidy given by expression (7). It can be shown that the unit production cost decreases to \(w - s = w_0 - \frac{[2b(a - w_0) - (a - w_0^*)]}{4b(2b^2 - 1)}\), which is equal to the unit production cost decreases to \(w_0 - \frac{[2b(a - w_0) - (a - w_0^*)]}{4b(2b^2 - 1)}\). This gives the domestic firm a Stackelberg quantity leadership position in product market competition.

\[^{10}\text{It is straightforward to show}
\]

\[
\frac{d^2(\pi + \omega - sx)}{ds^2} = \frac{b(2b^2 - 1)}{(4b^2 - 1)^2} < 0.
\]

Therefore, there is a unique interior solution.
cost after the domestic trade policy intervention in the case of no trade union. National welfare is defined as

\[ G = \pi + \omega - sx \]

\[ = (a - bx - x^* - w + s) x + (w - w_0) x - sx \]

\[ = (a - bx - x^* - w_0 + s) x - sx, \]

which is the same as expression (8). So, again, the government wants to use trade policy to give the domestic firm a Stackelberg quantity leadership position in product market competition while taking the trade union’s best response to the trade policy intervention into account. Since given the trade policy, the trade union skims off a part of it (in the linear case, one half), the government chooses a higher subsidy than in the case of no trade union in order to make the domestic firm commit to a Stackelberg leader’s output in product market competition.

See Figure 1.

Figure 1: Optimal trade policy towards a linear unionized Cournot duopoly

\( R \) denotes the domestic firm’s reaction function when domestic workers are not organized.
\( R' \) denotes its reaction function when domestic workers are organized. \( R'' \) denotes its export-subsidy-augmented reaction function when domestic workers are organized. \( R^* \) denotes the foreign firm’s reaction function. The intersection of \( R \) and \( R^* \) is denoted by \( C \), representing a Cournot equilibrium when domestic workers are not organized. The intersection of \( R'' \) and \( R^* \) is denoted by \( S \), representing a Stackelberg equilibrium, in which the domestic firm has a leadership position. When domestic workers are organized, without the trade policy intervention, the domestic firm’s unit production cost increases and its reaction function moves inward, (see the left arrow in Figure 1). When designing an export subsidy, the government knows that the trade union skims off a part of it. So, the government chooses an export subsidy, which is bigger than the export subsidy in the case when domestic workers are not organized, in order to offset this effect and give the domestic firm a Stackelberg quantity leadership position (see the right arrow in Figure 1).

### 3.2 Bertrand competition

Next let us analyze the case where the domestic firm and the foreign firm compete as Bertrand competitors in product market.

In the third stage of the game, the domestic firm maximizes its profits:

\[
\pi = (p - w + s) (\alpha - \beta p + \gamma p^*),
\]

whilst the foreign firm maximizes its profits:

\[
\pi^* = (p^* - w_0^*) (\alpha + \gamma p - \beta p^*).
\]

The domestic firm’s first-order condition for profit maximization and the foreign firm’s first-order condition for profit maximization determine simultaneously the Nash equilibrium:\(^{11}\)

\[
p = \frac{(2\beta + \gamma) \alpha + 2\beta^2 w - 2\beta^2 s + \beta \gamma w_0^*}{4\beta^2 - \gamma^2}, \quad (9a)
\]

\[
p^* = \frac{(2\beta + \gamma) \alpha + \beta \gamma w - \beta \gamma s + 2\beta^2 w_0^*}{4\beta^2 - \gamma^2}. \quad (9b)
\]

\(^{11}\)Notice that the first-order conditions are also sufficient in this standard Bertrand game.
Substituting the equilibrium prices into the direct demand for domestic products, we get the domestic firm’s equilibrium outputs, and hence equilibrium employment levels:

\[ x(w) = \beta \frac{(2\beta + \gamma)\alpha - (2\beta^2 - \gamma^2)w + (2\beta^2 - \gamma^2)s + \beta\gamma w_0^*}{4\beta^2 - \gamma^2}. \]  

(10)

In the second stage of the game, the trade union chooses wage rate, \( w \), to maximize its economic rents:

\[ \omega = (w - w_0) x(w) \]

\[ = (w - w_0) \left\{ \beta \frac{(2\beta + \gamma)\alpha - (2\beta^2 - \gamma^2)w + (2\beta^2 - \gamma^2)s + \beta\gamma w_0^*}{4\beta^2 - \gamma^2} \right\}. \]  

(11)

From the first-order condition for maximization, we can solve for the equilibrium wage rate given the domestic trade policy: \( s \).

\[ w = \frac{(2\beta + \gamma)\alpha + (2\beta^2 - \gamma^2)s + (2\beta^2 - \gamma^2)w_0 + \beta\gamma w_0^*}{2(2\beta^2 - \gamma^2)}. \]  

(12)

Notice that \( \frac{dw}{ds} = \frac{1}{2} \). This means that the trade union skims off one half of the trade policy.

Using expression (12), we can show

\[ x = \beta \frac{(2\beta + \gamma)\alpha + (2\beta^2 - \gamma^2)s - (2\beta^2 - \gamma^2)w_0 + \beta\gamma w_0^*}{2(2\beta^2 - \gamma^2)}, \]  

(13)

\[ \pi = \beta \left[ \frac{(2\beta + \gamma)\alpha + (2\beta^2 - \gamma^2)s - (2\beta^2 - \gamma^2)w_0 + \beta\gamma w_0^*}{2(2\beta^2 - \gamma^2)} \right]^2, \]  

(14)

\[ \omega = \beta \left[ \frac{(2\beta + \gamma)\alpha + (2\beta^2 - \gamma^2)s - (2\beta^2 - \gamma^2)w_0 + \beta\gamma w_0^*}{4(2\beta^2 - \gamma^2)(4\beta^2 - \gamma^2)} \right]^2. \]  

(15)

In the first stage of the game, the government chooses trade policy, \( s \), to maximize national welfare:

\[ G = \pi + \omega - sx, \]

\[ \frac{\partial^2 \omega}{\partial w^2} = -\frac{2\beta(2\beta^2 - \gamma^2)}{4\beta^2 - \gamma^2} < 0. \]

Therefore, there is a unique interior solution.

\[ \text{It is straightforward to show} \]

\[ \frac{\partial^2 \omega}{\partial w^2} = -\frac{2\beta(2\beta^2 - \gamma^2)}{4\beta^2 - \gamma^2} < 0. \]
where $\pi$ is given by expression (14), $\omega$ is given by expression (15), and $x$ is given by expression (13). From the first-order condition for national welfare maximization, we can solve for the optimal trade policy.\footnote{It is straightforward to show}

$$s = \frac{(2\beta + \gamma) \alpha - (2\beta^2 - \gamma^2) w_0 + \beta \gamma w_0^*}{2\beta^2}$$

(16)

$$= \frac{(2\beta^2 - 1) (a - w_0) - b (a - w_0^*)}{2b^2} > 0,$$

since $\alpha = \frac{a}{b+1}$, $\beta = \frac{b}{b^2 - 1}$, $\gamma = \frac{1}{b^2 - 1}$.

How do we explain this result?

Consider the case when domestic workers are not organized. So, the unit production cost is $w_0$. And we go back to the classic profit-shifting setting of Eaton and Grossman (1986). National welfare is defined as

$$G = \pi - sx = (p - w_0 + s) (\alpha - \beta p + \gamma p^*) - s (\alpha - \beta p + \gamma p^*).$$

(17)

It is easy to show that the optimal trade policy is an export tax being equal to

$$s = -\frac{\gamma^2}{4b^2} \left[ (2\beta + \gamma) \alpha - (2\beta^2 - \gamma^2) w_0 + \beta \gamma w_0^* \right].$$

Now the unit production cost increases to $w_0 + \frac{\gamma^2(2\beta + \gamma) \alpha - (2\beta^2 - \gamma^2) w_0 + \beta \gamma w_0^*}{4b^2(2\beta^2 - \gamma^2)}$. This gives the domestic firm a Stackelberg price leadership position in product market competition.

When domestic workers are organized, the unit production cost is $w$. As shown above, the optimal trade policy is an export subsidy given by expression (16). It can be shown that the unit production cost decreases to $w - s = w_0 + \frac{\gamma^2(2\beta + \gamma) \alpha - (2\beta^2 - \gamma^2) w_0 + \beta \gamma w_0^*}{4b^2(2\beta^2 - \gamma^2)}$, which is equal to the unit production cost after the domestic trade policy intervention in the case of no trade.
union. National welfare is defined as

\[ G = \pi + \omega - sx \]

\[ = (p - w + s) (\alpha - \beta p + \gamma p^*) + (w - w_0) (\alpha - \beta p + \gamma p^*) - s (\alpha - \beta p + \gamma p^*) \]

\[ = (p - w_0 + s) (\alpha - \beta p + \gamma p^*) - s (\alpha - \beta p + \gamma p^*), \]

which is the same as expression (17). So, again, the government wants to use trade policy to give the domestic firm a Stackelberg price leadership position in product market competition while taking the trade union’s best response to the trade policy intervention into account. Since given the trade policy, the trade union skims off a part of it (in the linear case, one half), the government chooses an export subsidy in order to make the domestic firm commit to a Stackelberg leader’s price in product market competition.

See Figure 2.

![Figure 2: Optimal trade policy towards a linear unionized Bertrand duopoly](image)

\( R \) denotes the domestic firm’s reaction function when domestic workers are not organized. \( R' \) denotes its reaction function when domestic workers are organized. \( R'' \) denotes its export-
subsidy-augmented reaction function when domestic workers are organized. \( R^* \) denotes the foreign firm’s reaction function. The intersection of \( R \) and \( R^* \) is denoted by \( B \), representing a Bertrand equilibrium when domestic workers are not organized. The intersection of \( R' \) and \( R^* \) is denoted by \( S \), representing a Stackelberg equilibrium, in which the domestic firm has a leadership position. When domestic workers are organized, without the trade policy intervention, the domestic firm’s unit production cost increases and its reaction function moves outward, (see the right arrow in Figure 2). When designing an export subsidy, the government knows that the trade union skims off a part of it. So, the government chooses an export subsidy rather than an export tax in the case when domestic workers are not organized, to offset this effect and give the domestic firm a Stackelberg price leadership position (see the left arrow in Figure 2).

Based on the arguments in the above two subsections, we have the following Proposition.

**Proposition 1** An export subsidy is the optimal trade policy towards a linear unionized duopoly irrespective of both the form of market conduct and the degree of product differentiation.

So far, we have reproduced the result implied by Brander and Spencer (1988), and Bandyopadhyay et al. (2000) in a clear-cut way. Is this result a robust one?

### 4 Opportunity Cost of Public Funds

In this section, we introduce an opportunity cost of public funds, \( \delta > 1 \), to the basic model. Nothing is changed except national welfare function. We have the following Proposition.

**Proposition 2** (Opportunity Cost of Public Funds) When the opportunity cost of public funds is strictly greater than 1, the necessary and sufficient condition for the government to set an export subsidy irrespective of the form of market conduct is given by

\[
\delta < \left( \frac{2b^2 - 1}{4b^2 - 1} + 1 \right).
\]
Proof. See Appendix.

Compared to Proposition 1, now we see that an export subsidy cannot always be a robust policy recommendation towards a linear unionized duopoly when the opportunity cost of public funds is strictly greater than 1.

See Figure 3.

Figure 3: Opportunity cost of public funds and optimal trade policy

Along the downward sloping curve, \( \left( \frac{2\delta^2}{\delta^2 - 1} + 1 \right) - \delta = 0 \). Along the upward sloping curve, \( \left( \frac{2\delta^2 - 1}{\delta^2 - 1} + 1 \right) - \delta = 0 \). If parameter configurations are strictly below the upward sloping curve, then the optimal trade policy is an export subsidy irrespective of the form of market conduct. Notice that the horizontal axis, \( \delta = 1 \), represents the case that we discussed in the last section. If parameter configurations are between the two curves, then the optimal trade policy is an export subsidy if firms compete as Cournot competitors, whilst it is an export tax if firms compete as Bertrand competitors. If parameter configurations are strictly above the downward sloping curve, then the optimal trade policy is an export tax irrespective of the form of market conduct.
As a result, if the opportunity cost of public funds is sufficiently high, then we would not have an export subsidy as a robust trade policy recommendation. This is mainly because that the gains from the ‘strategic use of trade policy’ cannot offset the costs of using public funds.

5 Special Interest Politics

However, what governments set trade policy to please a particular industry is a frequently observed phenomenon. For instance, see case studies done by Goldstein and McGuire (2004), and Pritchard and MacPherson (2004). This suggests that trade policy making can be influenced by politics, such as special interest lobbying. In particular, the domestic firm and the trade union may lobby the government to shift resources from other sectors to the export sector prior to the stage during which the government sets trade policy.

In this section, we use a common agency approach developed by Bernheim and Whinston (1986), and Grossman and Helpman (1994) to studying the impact of special interest lobbying on trade policy towards a unionized duopoly. We want to know whether an export subsidy is a robust policy recommendation when special interest lobbying is present.

We introduce an initial stage to the basic model during which the domestic firm and the trade union simultaneously make political contributions, which are contingent on trade policies, to the government. The domestic firm’s political contributions are denoted by $C^F(s)$. The trade union’s political contributions are denoted by $C^T(s)$. Then follows the three-stage game described in the basic model. Notice that now the domestic firm receives its profits minus its political contributions, $\pi - C^F(s)$. The trade union receives its economic rents minus its political contributions, $\omega - C^T(s)$. The government receives

$$G = \sum_{i \in \{F,T\}} C^i(s) + \lambda(\pi + \omega - \delta sx), \lambda \geq 0.$$ 

$\lambda$ is a parameter that represents the marginal rate of substitution between national welfare and political contributions. The larger is $\lambda$, the more weight is placed on national welfare relative to
political contributions.\textsuperscript{14} Hence, the larger is $\lambda$, the less the government will be influenced by the domestic firm and the trade union. When $\lambda \to \infty$, the government receives national welfare, and cannot be influenced by the domestic firm and the trade union.

Notice that the domestic firm and the trade union have an incentive to engage in lobbying. When the domestic firm and the foreign firm engage in Cournot competition in the third market, its profits are given by expression (5), and
\[
\left. \frac{d\pi}{ds} \right|_s = \frac{b^2}{4b^2 - 1} [2b(a - w_0) - (a - w_0^*)] \left[ \frac{\delta}{2\delta - \left(1 + \frac{\delta}{4(\beta^2 - \gamma^2)}\right)} \right] > 0;
\]
the trade union’s economic rents are given by expression (6), and
\[
\left. \frac{d\omega}{ds} \right|_s = \frac{1}{2(4b^2 - 1)} [2b(a - w_0) - (a - w_0^*)] \left[ \frac{\delta}{2\delta - \left(1 + \frac{\delta}{4(\beta^2 - \gamma^2)}\right)} \right] > 0,
\]
where $s$ is given by expression (24).\textsuperscript{15} So, in the Cournot case, both the domestic firm and the trade union have an incentive to engage in lobbying.

When the domestic firm and the foreign firm engage in Bertrand competition in the third market, its profits are given by expression (14), and
\[
\left. \frac{d\pi}{ds} \right|_s = \beta \frac{(2\beta^2 - \gamma^2)}{2(4\beta^2 - \gamma^2)} \left[ (2\beta + \gamma) \alpha - (2\beta^2 - \gamma^2) w_0 + \beta \gamma w_0^* \right] \left[ \frac{\delta}{2\delta - \left(1 + \frac{\delta}{4(\beta^2 - \gamma^2)}\right)} \right] > 0;
\]
the trade union’s economic rents are given by expression (15), and
\[
\left. \frac{d\omega}{ds} \right|_s = \frac{\beta}{2(4\beta^2 - \gamma^2)} \left[ (2\beta + \gamma) \alpha - (2\beta^2 - \gamma^2) w_0 + \beta \gamma w_0^* \right] \left[ \frac{\delta}{2\delta - \left(1 + \frac{\delta}{4(\beta^2 - \gamma^2)}\right)} \right] > 0,
\]
where $s$ is given by expression (26).\textsuperscript{16} So, in the Bertrand case, both the domestic firm and the trade union have an incentive to engage in lobbying.

Now, let us turn to how the government design its trade policy. Bernheim and Whinston (1986) uses a Truthful Equilibrium as the solution concept to a common agency game. According

\textsuperscript{14}See Grossman and Helpman (1994).
\textsuperscript{15}See Proof of Proposition 2 in Appendix.
\textsuperscript{16}See Proof of Proposition 2 in Appendix.
to their Theorem 2, in a truthful equilibrium the government chooses trade policy, \( s \), to maximize the sum of the domestic firm’s payoff, the trade union’s payoff and its own payoff.\(^{17}\)

\[
(\pi - C^F(s)) + (\omega - C^T(s)) + G
\]

\[
\Leftrightarrow
\]

\[
\pi + \omega + \lambda(\pi + \omega - \delta sx).
\]

Notice that when the domestic firm and the foreign firm engage in Cournot competition in product market, there exists a unique interior solution to the government’s maximization problem if and only if\(^{18}\)

\[
\left(\frac{2b^2}{4b^2 - 1} + 1\right)\left(\frac{\lambda + 1}{\lambda}\right) < 2\delta.
\]

When the domestic firm and the foreign firm engage in Bertrand competition in product market,

\(^{17}\)The government chooses trade policy, \( s \), to maximize \( G \). The first-order condition for maximization is given by

\[
\frac{dC^F}{ds} + \frac{dC^T}{ds} + \lambda \frac{d(\pi+\omega-\delta sx)}{ds} = 0.
\]

In a truthful equilibrium, the domestic firm’s contribution schedule satisfies

\[
\frac{dC^F}{ds} = \frac{d\pi}{ds}.
\]

The trade union’s contribution schedule satisfies

\[
\frac{dC^T}{ds} = \frac{d\omega}{ds}.
\]

Given these, the first-order condition for maximizing \( G \) becomes

\[
\frac{d\pi}{ds} + \frac{d\omega}{ds} + \lambda \frac{d(\pi+\omega-\delta sx)}{ds} = 0.
\]

This is the first-order condition for maximizing \( \pi + \omega + \lambda(\pi + \omega - \delta sx) \).

\(^{18}\)Notice that in Cournot case, we have

\[
\frac{d^2}{ds^2} \frac{[\pi + \omega + \lambda(\pi + \omega - \delta sx)]}{ds^2} = \frac{b\lambda}{4b^2 - 1} \left[ \left(\frac{2b^2}{4b^2 - 1} + 1\right) \left(\frac{\lambda + 1}{\lambda}\right) - 2\delta \right].
\]

So, \( \left(\frac{2b^2}{4b^2 - 1} + 1\right) \left(\frac{\lambda + 1}{\lambda}\right) < 2\delta \) is both necessary and sufficient for the second-order condition to be satisfied.
there exists a unique interior solution if and only if\(^{19}\)
\[
\left(\frac{2b^2 - \gamma^2}{4b^2 - \gamma^2} + 1\right) \left(\frac{\lambda + 1}{\lambda}\right) = \left(\frac{2b^2 - 1}{4b^2 - 1} + 1\right) \left(\frac{\lambda + 1}{\lambda}\right) < 2\delta.
\] (20)

Combining these two facts, there exists a unique interior solution irrespective of the form of market conduct, if and only if\(^{20}\)
\[
\left(\frac{2b^2}{4b^2 - 1} + 1\right) \left(\frac{\lambda + 1}{\lambda}\right) < 2\delta.
\] (21)

We have the following Proposition.

**Proposition 3** (Special Interest Politics) In an interior solution, the necessary and sufficient condition for the domestic government to set an export subsidy is as follows:

1. \(\left(\frac{2b^2}{4b^2 - 1} + 1\right) \left(\frac{\lambda + 1}{\lambda}\right) < 2\delta;\)
2. \(\delta < \left(\frac{2b^2 - 1}{4b^2 - 1} + 1\right) \left(\frac{\lambda + 1}{\lambda}\right).\)

**Proof.** See Appendix. ■

Look at the above Proposition. The first condition just repeats condition (21), it guarantees that an interior solution exists. Comparing the second condition to condition (18), it is easy to see that the presence of special interest politics weakens the effect of an opportunity cost of public funds on trade policy making.

**Remark 1** (i) When \(\delta > \left(\frac{2b^2}{4b^2 - 1} + 1\right) \left(\frac{\lambda + 1}{\lambda}\right),\) the optimal trade policy is an export tax irrespective of the form of market conduct. (ii) When \(\left(\frac{2b^2 - 1}{4b^2 - 1} + 1\right) \left(\frac{\lambda + 1}{\lambda}\right) < \delta < \left(\frac{2b^2}{4b^2 - 1} + 1\right) \left(\frac{\lambda + 1}{\lambda}\right) < 2\delta,\)

\(^{19}\)In Bertrand case, we have
\[
\frac{\partial^2}{\partial s^2}\left[\pi + \omega + \lambda (\pi + \omega - \delta s t)\right] = \frac{\beta (2b^2 - \gamma^2) \lambda}{2 (4b^2 - \gamma^2)} \left[\frac{2b^2 - \gamma^2}{4b^2 - \gamma^2} + 1\right] \left(\frac{\lambda + 1}{\lambda}\right) - 2\delta.
\]
So, \(\left(\frac{2b^2}{4b^2 - 1} + 1\right) \left(\frac{\lambda + 1}{\lambda}\right) = \left(\frac{2b^2 - 1}{4b^2 - 1} + 1\right) \left(\frac{\lambda + 1}{\lambda}\right) < 2\delta\) is both necessary and sufficient for the second-order condition to be satisfied.

\(^{20}\)When \(\lambda\) goes to infinity, the second-order condition is satisfied automatically irrespective of the form of market conduct. This is the case that we discussed in the above section.
the optimal trade policy is an export subsidy for the Cournot case; whilst it is an export tax for the Bertrand case. 21

So far, we have finished discussion of the case when there exists a unique interior solution irrespective of the form of market conduct. However, notice that when special interest lobbying is present, there may not exist an interior solution.

Remark 2 (i) When \( \frac{2b^2 - 1}{b^2 - 1} + 1 \left( \frac{\lambda + 1}{\lambda} \right) < 2\delta < \left( \frac{2b^2}{b^2 - 1} + 1 \right) \left( \frac{\lambda + 1}{\lambda} \right) \), there exists a unique interior solution in the Bertrand case there does not exist an interior solution in the Cournot case. The optimal trade policy is an infinite export subsidy for the Cournot case, whilst it is a finite export subsidy for the Bertrand case. (ii) When \( \left( \frac{2b^2 - 1}{b^2 - 1} + 1 \right) \left( \frac{\lambda + 1}{\lambda} \right) > 2\delta \), there does not exist an interior solution both in Bertrand case and in Cournot case. So, the optimal trade policy is an infinite export subsidy irrespective of the form of market conduct. 22

The results obtained in the above discussion are summarized in the following Corollary.

Corollary 1 The necessary and sufficient condition for the domestic government to choose an export subsidy irrespective of the form of market conduct is given by the second condition in Proposition 3.

Proof. Proposition 3 and Remark 2 together implies the Corollary immediately. ■

21 See Proof of Proposition 3 in Appendix.
22 This Remark presents unpleasant results derived from the current model, in which the government’s budget constraint is not included. When this constraint is explicitly modeled, an infinite export subsidy will not be an optimal solution: the government chooses an export subsidy such that the total costs of subsidy hit its budget constraint. However, notice that the results obtained in the current model cannot be changed qualitatively.

(Also notice that in the current model, the domestic firm and the trade union do not pay for an export subsidy. Now, what will happen when they need to pay for it? It can be shown that again we can find parameter configurations for which the second-order condition for maximization is not satisfied.)
Corollary 1 says that whether there is an interior solution, for a pair of $b$ and $\delta$,$^{23}$ if the extent to which the government is influenced by the domestic firm and the trade union is sufficiently great, then the government will use an export subsidy irrespective of the form of market conduct. This is a sharp contrast to the result obtained in the last section. To see an extreme example, consider the case when $\lambda = 0$.

6 Discussion

So far, we have finished three exercises and derived a number of results. What is the relationship between these results and those obtained in existing literatures? See the following Figure.

\[ \text{Figure 4: Summary of results} \]

Define $z = \frac{\lambda}{\lambda + 1}$, since $\lambda$ is nonnegative, $0 \leq z \leq 1$. The horizontal axis represents $z$, whilst the vertical axis represents $\delta$, $\delta \geq 1$. Fixing a $b$, curve F1 represents the case when condition

\[ ^{23}\text{We treat $\delta$ as a finite number. For example, according to Ballard et al. (1985), $\delta \in [1.17, 1.56]$.} \]
(29) holds with equality: \( (2b^2 - 1 + 1) \left( \frac{\lambda + 1}{\lambda} \right) - \delta = 0 \Leftrightarrow \left( \frac{2b^2 - 1}{4b^2 - 1} + 1 \right) \frac{1}{2} - \delta = 0 \). Curve F2 represents the case when condition (31) holds with equality: \( (2b^2 - 1 + 1) \left( \frac{\lambda + 1}{\lambda} \right) - \delta = 0 \Leftrightarrow \left( \frac{2b^2 - 1}{4b^2 - 1} + 1 \right) \frac{1}{2} - \delta = 0 \). Curve F3 represents the case when in condition (19), an equality holds: \( \left( \frac{2b^2 - 1}{4b^2 - 1} + 1 \right) (\frac{\lambda + 1}{\lambda}) - 2\delta = 0 \Leftrightarrow \left( \frac{2b^2 - 1}{4b^2 - 1} + 1 \right) \frac{1}{2} - 2\delta = 0 \). Curve F4 represents the case when in condition (20), an equality holds: \( \left( \frac{2b^2 - 1}{4b^2 - 1} + 1 \right) (\frac{\lambda + 1}{\lambda}) - 2\delta = 0 \Leftrightarrow \left( \frac{2b^2 - 1}{4b^2 - 1} + 1 \right) \frac{1}{2} - 2\delta = 0 \). These four curves divide the \((z, \delta)\) plane into five regions.

In region I, the optimal trade policy is a finite export tax irrespective of the form of market conduct. See part (i) of Remark 1.

In region II, the optimal trade policy is a finite export subsidy for the Cournot case, whilst it is a finite export tax for the Bertrand case. See part (ii) of Remark 1.

In region III, the optimal trade policy is a finite export subsidy irrespective of the form of market conduct. See Proposition 3.

In region IV, the optimal trade policy is an infinite export subsidy for the Cournot case, whilst it is a finite export subsidy for the Bertrand case. See part (i) of Remark 2.

In region V, the optimal trade policy is an infinite export subsidy irrespective of the form of market conduct. See part (ii) of Remark 2.

In summary, In region III, IV and V, an export subsidy is a robust policy recommendation towards a linear unionized duopoly.

So far, we have explored all of the possibilities in the \((z, \delta)\) plane.

The results of Brander and Spencer (1988), and Bandyopadhyay et al. (2000) can be represented by point A with coordinate \((1, 1)\) in the above Figure: they study optimal trade policy towards a unionized duopoly without an opportunity cost of public funds and political economy. We reproduce their results in a clear-cut way in Proposition 1 as a benchmark case.

Neary (1994) can be represented by point C: he studies optimal strategic trade policy for the Cournot case in a Brander-Spencer third-market model with an opportunity cost of public funds but without political economy. We reproduce his result for the Cournot case and go further to

\(^{24}\text{See Proof of Proposition 3 in Appendix.}\)

\(^{25}\text{See Proof of Proposition 3 in Appendix.}\)
consider the Bertrand case, which is represented by point B. See Proposition 2.

Fung and Lin (2000) can be represented by the horizontal axis: they study optimal strategic trade policy from a political economy perspective but without introducing an opportunity cost of public funds to their model. See Proposition 3. Setting $\delta = 1$ in the two conditions, we get their results.\footnote{Comparing expression (28) to expression (24), expression (30) to expression (26), (see Appendix), it is easy to see that in our paper, in an interior solution, the politically determined export subsidy is always greater than the rent-shifting export subsidy set by a benevolent government. This result is different from Fung and Lin (2000). We also discuss this point in Introduction.}

In summary, all of the results of previous literatures can be viewed as a special case of our research. And our paper explores fully optimal trade policy towards a linear unionized duopoly with an opportunity cost of public funds and special interest lobbying.

7 Conclusion

We have studied whether an export subsidy is a robust trade policy recommendation towards a linear unionized duopoly, and two main messages have been derived. First of all, an export subsidy can hardly be a robust trade policy recommendation towards a unionized duopoly, if we consider this problem from a purely economic perspective: it is very sensitive to the opportunity cost of public funds even in the simplest setting. Second, an export subsidy as a robust policy recommendation can be supported by political reasons, for instance, special interest lobbying.

These are fairly robust results. As to the first result, if the result of robustness cannot be obtained in the simplest case, it can hardly be obtained in more sophisticated cases. As to the second result, on the one hand, in reality we observe that governments set trade policies to please a particular industry. (See Goldstein and McGuire (2004).) This means that governments can be very sensitive to special interest lobbying. On the other hand, special interest groups, such as domestic firms and trade unions have an incentive to engage in lobbying. To understand this, notice that when the export subsidy is set sufficiently big, then the foreign firm would be
driven out of the third-country market, and domestic firms and trade unions would gain from a monopoly market structure. So, our result derived from the simplest model could be reproduced in a more sophisticated model.

References


### A Appendix (Not Intended for Publication)

#### A.1 Discussion of Assumption 1

In this subsection, we discuss the case of no intervention and show the reason why we made Assumption 1 in the text.

##### A.1.1 Cournot competition

It is straightforward to show that the SPNE is given by

\[
x = \frac{2b(a - w) - (a - w_0^*)}{4b^2 - 1}, \quad x^* = \frac{2b(a - w_0^*) - (a - w)}{4b^2 - 1};
\]

\[
w = \frac{2b(a + w_0) - (a - w_0^*)}{4b}.
\]

In an equilibrium,

\[
x = \frac{2b(a - w_0) - (a - w_0^*)}{2(4b^2 - 1)},
\]

\[
\pi = b \left[ \frac{2b(a - w_0) - (a - w_0^*)}{2(4b^2 - 1)} \right]^2,
\]

\[
\omega = \frac{2b(a - w_0) - (a - w_0^*)}{8b(4b^2 - 1)}.
\]
We assume that in an equilibrium, \( x > 0 \). It is straightforward to show

\[
x > 0
\]

\[\Leftrightarrow\]

\[
2b \left( a - w_0 \right) - (a - w_0^*) > 0.
\]  

(22)

I.e., without the government’s intervention, the domestic firm can survive in product market competition.

A.1.2 Bertrand competition

It is straightforward to show that the SPNE is given by

\[
p = \frac{(2\beta + \gamma) \alpha + 2\beta^2 w + \beta \gamma w_0^*}{4\beta^2 - \gamma^2}, \quad p^* = \frac{(2\beta + \gamma) \alpha + \beta \gamma w + 2\beta^2 w_0^*}{4\beta^2 - \gamma^2};
\]

\[
w = \frac{(2\beta + \gamma) \alpha + (2\beta^2 - \gamma^2) w_0 + \beta \gamma w_0^*}{2(2\beta^2 - \gamma^2)}.
\]

In an equilibrium,

\[
x = \beta \frac{[(2\beta + \gamma) \alpha - (2\beta^2 - \gamma^2) w_0 + \beta \gamma w_0^*]}{2(4\beta^2 - \gamma^2)},
\]

\[
\pi = \beta \left[ \frac{(2\beta + \gamma) \alpha - (2\beta^2 - \gamma^2) w_0 + \beta \gamma w_0^*}{2(4\beta^2 - \gamma^2)} \right]^2,
\]

\[
\omega = \beta \frac{[(2\beta + \gamma) \alpha - (2\beta^2 - \gamma^2) w_0 + \beta \gamma w_0^*]^2}{4(2\beta^2 - \gamma^2)(4\beta^2 - \gamma^2)}.
\]

We assume that in an equilibrium, \( x > 0 \). It is straightforward to show

\[
x > 0
\]

\[\Leftrightarrow\]

\[
(2\beta + \gamma) \alpha - (2\beta^2 - \gamma^2) w_0 + \beta \gamma w_0^* > 0
\]

\[\Leftrightarrow\]

\[
(2b^2 - 1) \left( a - w_0 \right) - b \left( a - w_0^* \right) > 0.
\]  

(23)

I.e., without the government’s intervention, the domestic firm can survive in product market competition.
Finally, notice that \((2b^2 - 1) (a - w_0) - b (a - w_0^*) > 0\) is sufficient for \(2b (a - w_0) - (a - w_0^*) > 0\).

### A.2 Proof of Proposition 2

#### A.2.1 Cournot competition

Now, national welfare is given by

\[
G = \pi + \omega - \delta sx.
\]

And it is straightforward to show that the optimal trade policy is given by\(^{27}\)

\[
s = \frac{\left[2b (a - w_0) - (a - w_0^*)\right] \left(\frac{2b^2}{4b^2 - 1} + 1\right) - \delta}{2b \left[2\delta - \left(\frac{2b^2}{4b^2 - 1} + 1\right)\right]}.
\]

(24)

By Assumption 1, \(2b (a - w_0) - (a - w_0^*) > 0\). Since \(\delta > 1\), \(2\delta - \left(\frac{2b^2}{4b^2 - 1} + 1\right) > 0\). Therefore,

\[
\text{sign } s = \text{sign} \left[\left(\frac{2b^2}{4b^2 - 1} + 1\right) - \delta\right].
\]

(25)

#### A.2.2 Bertrand competition

Now, national welfare is given by

\[
G = \pi + \omega - \delta sx.
\]

\(^{27}\)Notice that first of all, since \(\delta > 1\), we have

\[
\frac{d^2 (\pi + \omega - \delta sx)}{ds^2} = -\frac{b \left[2\delta - \left(\frac{2b^2}{4b^2 - 1} + 1\right)\right]}{4b^2 - 1} < 0,
\]

so, there is a unique interior solution. Next, when \(\delta = 1\), \(s\) is given by expression (7).
And it is straightforward to show that the optimal trade policy is given by

\[ s = \left( \frac{(2\beta + \gamma) \alpha - (2\beta^2 - \gamma^2) w_0 + \beta \gamma w_0^*}{(2\beta^2 - \gamma^2) \left[ 2\delta - \left( \frac{2\beta^2 - \gamma^2}{4\beta^2 - \gamma^2} + 1 \right) \right]} \right). \]  

(26)

By Assumption 1, \((2\beta + \gamma) \alpha - (2\beta^2 - \gamma^2) w_0 + \beta \gamma w_0^* = (2\delta^2 - 1) (a - w_0) - b (a - w_0^*) > 0.\)

Since \(\delta > 1, 2\delta - \left( \frac{2\beta^2 - \gamma^2}{4\beta^2 - \gamma^2} + 1 \right) > 0.\) Therefore,

\[ \text{sign } s = \text{sign} \left[ \left( \frac{2\beta^2 - \gamma^2}{4\beta^2 - \gamma^2} + 1 \right) - \delta \right] \]

(27)

Notice that Condition (25) and (27) imply the Proposition immediately. ■

A.3 Proof of Proposition 3

A.3.1 Cournot competition

It is straightforward to show that the optimal trade policy is given by

\[ s = \left( \frac{2b (a - w_0) - (a - w_0^*)}{2b \left[ 2\delta - \left( \frac{2b^2}{4b^2 - 1} \right) \left( \frac{\lambda + 1}{\lambda} \right) \right]} \right). \]  

(28)

By Assumption 1, \(2b (a - w_0) - (a - w_0^*) > 0,\) and since condition (21) holds,

\[ \text{sign } s = \text{sign} \left[ \left( \frac{2b^2}{4b^2 - 1} + 1 \right) \left( \frac{\lambda + 1}{\lambda} \right) - \delta \right]. \]  

(29)

A.3.2 Bertrand competition

It is straightforward to show that the optimal trade policy is given by

\[ s = \left( \frac{(2\beta + \gamma) \alpha - (2\beta^2 - \gamma^2) w_0 + \beta \gamma w_0^*}{(2\beta^2 - \gamma^2) \left[ 2\delta - \left( \frac{2\beta^2 - \gamma^2}{4\beta^2 - \gamma^2} + 1 \right) \right]} \right). \]  

(30)

Notice that first of all, since \(\delta > 1,\) we have

\[ \frac{d^2 (\pi + \omega - \delta sx)}{ds^2} = \frac{\beta (2\beta^2 - \gamma^2)}{2 \left( 4\beta^2 - \gamma^2 \right)} \left[ 2\delta - \left( \frac{2\beta^2 - \gamma^2}{4\beta^2 - \gamma^2} + 1 \right) \right] < 0, \]

so, there is a unique interior solution. Next, when \(\delta = 1, s\) is given by expression (16).
By Assumption 1, \((2\beta + \gamma) \alpha - (2\beta^2 - \gamma^2) w_0 + \beta \gamma w_0^* = (2b^2 - 1)(a - w_0) - b(a - w_0^*) > 0\), and since condition (21) holds,

\[
\begin{align*}
\text{sign } s &= \text{sign} \left[ \left( \frac{2\beta^2 - \gamma^2}{4\beta^2 - \gamma^2} + 1 \right) \left( \frac{\lambda + 1}{\lambda} \right) - \delta \right] \\
&= \text{sign} \left[ \left( \frac{2b^2 - 1}{4b^2 - 1} + 1 \right) \left( \frac{\lambda + 1}{\lambda} \right) - \delta \right].
\end{align*}
\] (31)

Combining the arguments in the above two subsections, we establish the Proposition. ■