

Multinational Firms, Monopolistic Competition and Foreign Investment Uncertainty.¹

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Abstract :

This paper develops a general equilibrium model of multinational firms operating under monopolistic competition and foreign investment uncertainty. Starting from a pure trading equilibrium and solving for the optimal foreign investment rule gives a scale-up factor which implies existence of a wedge between markup revenues and foreign investment costs. Greater volatility and risk aversion increase this scale-up factor over foreign investment costs, implying a delay in the exercise of FDI option. Growing market size facilitates early exercise. The model is extended to include a poisson jump process, which has policy implications for FDI reforms. This explains the ‘wait and watch’ behaviour of multinational firms better than a pure comparative advantage-trade cost framework, and suggests investment wooing behaviour of national governments is not an exercise in futility.

keywords: Multinational firm, monopolistic competition, foreign investment uncertainty, FDI option.

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1. Introduction :

More than two-third of world trade today is determined by activities of multinational enterprises, a phenomenon not well explained by the traditional trade theory. For many years the economics of relationship between trade and investment was studied as complements or substitutes (Mundell, 1957, Lipsey and Weiss,1981, Blomstrom et al,1988). More recent studies indicate such a generalization is not possible and that both trade and investment flows are determined simultaneously by location decisions of multinational firms (UNCTAD,1996).

‘New trade theory’ has developed general equilibrium models for multinational firms in the presence of imperfect competition (Venables et al,1996, Markusen and Venables,1998, Markusen,1995, Helpman,1984). While vertical multinationals are explained by the factor proportion analysis, horizontal multinational activity is explained by proximity-concentration trade-off (Brainard,1993, Brainard,1997). More recently, Export vs FDI cut off has been derived in the presence of firm heterogeneity (Helpman,Melitz and Yeaple,2003).

Real life behaviour is somewhat more complex than what can be explained by conventional cost-benefit analysis. While most of the work above is based on a standard marshallian kind of economic analysis, evidence indicates that firm’s investment decisions, and even more so, foreign investment decisions are taken in the face of uncertainty. Studies indicate that if we fail to take this into account, then even for reasonable parameter values, we may be up to two times off the mark compared to a routine cost-benefit analysis (Dixit and Pindyck,1994).

Even when comparative advantage indicates foreign investment should flow in or when economic liberalization removes barriers to foreign investment, they do not automatically flow in. There is a considerable wait-and-watch, a kind of time lag or inertia followed by a herd behaviour when investments actually start flowing in. This implies existence of some value-for-waiting or in other words, some opportunity costs beyond what are accounted for in a pure comparative advantage-trade costs analysis.

Similarly, it is common place to see national governments and industrial lobbies all over the world spend considerable time and energy trying to attract foreign investment into their respective countries. How do explain such behaviour? In this paper I present a framework, which will suggest it is optimal and in fact, rational to do so.

Almost simultaneously as the new trade theory was taking shape,

a strand of literature was growing out of the core finance theory to model investment decisions of individual firms under uncertainty. Starting with the work of McDonald and Siegel (1986) on the value of waiting to invest and Dixit's (1989) work on firm's investment decisions under uncertainty, a rich literature has developed on investment under uncertainty a la Dixit and Pindyck (1994). Baldwin and Krugman (1989) modelled hysteresis in trade under assumptions of large exchange rate shocks. Dixit (1989) applied it to exchange rate pass-through under perfect competition. Smets (1991) modelled export vs FDI in a leader-follower duopoly framework, while Rafael and Vettas (2003) modelled export vs FDI in a setting of monopoly with growing demand.

I take a step forward from the existing literature and develop a general equilibrium model of multinational firms operating under foreign investment uncertainty within the framework of a Dixit-Stiglitz type monopolistic competition. I also explicitly solve for policy driven FDI liberalization as a mixed poisson-jump brownian motion stochastic process. Another interesting feature of this model is that while the investment under uncertainty literature is based on the theory of call options, I solve 'FDI option' as a put option thereby enriching the theory of real options.

Section 2 presents the underlying static model. Section 3 introduces foreign investment uncertainty and develops its inter-temporal counterpart. Section 4 elaborates some comparative experiments. Section 5 extends the model by explicitly solving for a mixed poisson jump-brownian motion stochastic process. Section 6 concludes.

2. The Static Model :

There are two countries- i and j . There are two goods - Y , a homogeneous good; and X , a differentiated good with imperfectly substitutable varieties in a Dixit-Stiglitz fashion. Skilled labour (S) is the only factor of production and all costs are expressed in units of this factor.

Good Y , the homogeneous good, is produced by a perfectly competitive industry using a constant returns to scale technology:

$$Y_i = \frac{1}{a_y} S_{iy} \quad (1)$$

Its transport is costless, and it will be treated as numeraire: $P_y = 1$.

Good X , the differentiated good, is produced by a monopolistically competitive industry. Varieties of his good can be produced by national (indexed

by n) or multi-national (indexed by m) firms. Let $N_i^n, N_i^m, N_j^n, N_j^m$ stand for the number of firms of each type headquartered in country i or j that are active in equilibrium.

Sector X has a linear cost function with fixed costs. This implies economies of scale.

Total production costs for a national firm located in country j are :

$$S_{jx}^n = H^n + G + cX_{jj}^n + cX_{ji}^n \quad (2)$$

where H^n are the headquarter level fixed costs and G are the plant level fixed costs. Both of these are at least partly irreversible or sunk.

'c' are the constant marginal costs which are same for both national and multinational firms.

Total production costs for a multinational firm located in country j are :

$$S_{jx}^m + S_{ix}^m = H^m + G + cX_{jj}^m + G^m + cX_{ji}^m \quad (3)$$

where first part on the right hand side is cost of operation within the home country and second part on the right hand side is cost of operation within the foreign country.

Headquarter costs of a multi-national firm (H^m) are typically different from that of a national firm (H^n) because of a greater need for headquarter services. The costs of foreign investment (G^m) are different compared to a similar initiative by domestic firms (G) as almost all countries have individual FDI regimes, meaning thereby that routes and mechanisms prescribed for foreign investment are different from those for domestic firms. Costs of foreign investment are also different between the two countries ($G_i^m \neq G_j^m$) and depend on the local foreign investment environment.

As compared to a more straightforward trading (exporting) decision, FDI typically takes place in the face of foreign investment uncertainty. This in real life cannot also be easily hedged. I will explicitly model this later.

Assuming diversified production⁴ wage, w, is pinned down by the numeraire sector :

$$w = \frac{1}{a_y} \quad (4)$$

Total income in both countries depends on their factor endowments:

$$M_i = wS_i \quad (5)$$

⁴I assume labour endowment and value of demand parameters is such that both countries produce both the goods. Symmetric countries always have diversified production.

$$M_j = wS_j \quad (6)$$

On the demand side, there is a representative consumer in each country with a Cobb Douglas utility function, which for countries i and j are:

$$U_i = X_{ic}^\beta Y_{ic}^{1-\beta} \quad (7)$$

$$U_j = X_{jc}^\beta Y_{jc}^{1-\beta} \quad (8)$$

Here, X_{ic} is the CES aggregate of x-varieties in the familiar Dixit-Stiglitz fashion, and

$$X_{ic} = [N_i^n (X_{ii}^n)^\alpha + N_j^n (X_{ji}^n)^\alpha + N_i^m (X_{ii}^m)^\alpha + N_j^m (X_{ji}^m)^\alpha]^\frac{1}{\alpha}, \text{ such that } 0 < \alpha < 1. \quad (9)$$

$\epsilon = \frac{1}{1-\alpha}$ is then the elasticity of substitution between any two x-varieties.

I assume symmetry within each of the categories. This utility function permits two stage budgeting;

In first stage budgeting, the consumer allocates his total income to good Y and X-aggregate through the following demand functions:

$$Y_{ic} = (1 - \beta)M_i \quad (10)$$

$$X_{ic} = \beta \frac{M_i}{e_i} = \frac{M_{ix}}{e_i} \quad (11)$$

Here, M_{ix} is the share of national income of country i spent on X-aggregate, e_i is the unit expenditure function for X_{ic} (also called the price index) and good Y has been used as numeraire.

In second stage budgeting the consumer solves the sub-utility maximization problem for individual varieties.

Demand for an individual variety is then a solution to the sub-utility maximization problem given by:

$$x_k = p_k^{-\epsilon} e_i^{\epsilon-1} M_{ix} \quad (12)$$

where subscript k stands for an individual variety.

In this large group monopolistic competition each individual firm takes the price index e_i and country income M_i as given. The proportional mark-up of price over variable costs is given by the relationship:

$$p = \left(\frac{\epsilon}{\epsilon - 1}\right)wc \quad (13)$$

This pricing equation comes from the first order condition which is also called the ‘marginal revenue equal to marginal cost condition’. As we can see, the proportional markup of price over marginal costs is a constant independent of market shares. For constant marginal costs and equal wages this implies each x-variety is produced for the same price p in equilibrium. Let us call it $P_x = p$. However, x-varieties produced by national firms are sold abroad for a price $p\tau$, where τ are the iceberg trade costs.

As in the trade cost literature, ‘ τ ’ are the inclusive trade costs. They include not only transport costs but all intermediate costs like tariff barriers, non-tariff barriers, border costs, information costs, time costs, currency costs etc. According to this literature, trade costs are fairly large, an average estimate being 170% of the value of output (Anderson and Wincoop, 2003). Currency costs are only a small proportion of it—about 8-14% out of a total 170 %, and most of it is purely transaction cost. Trading horizon is typically short-term (few days to few weeks) and currency risks over such short periods are easily hedged in financial markets today, say through a simple, short-term forward, which is either costless or has costs which are very small. I do not assume large exchange rate shocks as in Baldwin and Krugman (1989), but medium to long term exchange rate risk which, given the illiquidity of futures markets beyond the short-term (Layard et al, 2002), is not easy to hedge against in the forward foreign exchange market and therefore matters for foreign direct investment among other sources of aggregate uncertainty detailed below.

Production regime for X-sector is determined by a set of conditions called the zero-profit conditions as detailed below:

$$\begin{aligned}
 pX_{ii}^n + pX_{ij}^n &\leq wcX_{ii}^n + wcX_{ij}^n + w(H^n + G) && (N_i^n) \quad (14) \\
 pX_{ii}^m + pX_{ij}^m &\leq wcX_{ii}^m + wcX_{ij}^m + w(H^m + G) + wG_j^m && (N_i^m) \\
 pX_{jj}^n + pX_{ji}^n &\leq wcX_{jj}^n + wcX_{ji}^n + w(H^n + G) && (N_j^n) \\
 pX_{jj}^m + pX_{ji}^m &\leq wcX_{jj}^m + wcX_{ji}^m + w(H^m + G) + wG_i^m && (N_j^m)
 \end{aligned}$$

These are written as inequalities in the complementary slackness form, meaning thereby that an equation will hold with equality if the output of corresponding firm is positive, otherwise the output of corresponding firm is zero. These conditions relate markup revenues to investment costs and number of firms is the endogenous variable. Depending on whether markup revenues cover investment costs or not, firms decide whether to operate as a national firm exporting to the foreign market or undertake a foreign direct

investment abroad and become multinational.

Let us assume for a moment that each type of firm is active in equilibrium. Demand functions for varieties produced by each of these firms as derived from the respective sub-utility maximization problem are given below:

$$\begin{aligned}
X_{ii}^n &= X_{ii}^m = X_{ji}^m = p^{-\epsilon} e_i^{\epsilon-1} M_{ix} = \beta \cdot p^{-\epsilon} e_i^{\epsilon-1} M_i = \beta \cdot p^{-\epsilon} e_i^{\epsilon-1} w S_i & (15) \\
X_{jj}^n &= X_{jj}^m = X_{ij}^m = p^{-\epsilon} e_j^{\epsilon-1} M_{jx} = \beta \cdot p^{-\epsilon} e_j^{\epsilon-1} M_j = \beta \cdot p^{-\epsilon} e_j^{\epsilon-1} w S_j \\
X_{ji}^n &= p^{-\epsilon} \tau^{1-\epsilon} e_i^{\epsilon-1} M_{ix} = \beta \cdot p^{-\epsilon} \tau^{1-\epsilon} e_i^{\epsilon-1} M_i = \beta \cdot p^{-\epsilon} \tau^{1-\epsilon} e_i^{\epsilon-1} w S_i \\
X_{ij}^n &= p^{-\epsilon} \tau^{1-\epsilon} e_j^{\epsilon-1} M_{jx} = \beta \cdot p^{-\epsilon} \tau^{1-\epsilon} e_j^{\epsilon-1} M_j = \beta \cdot p^{-\epsilon} \tau^{1-\epsilon} e_j^{\epsilon-1} w S_j
\end{aligned}$$

Iceberg trade costs imply, if a quantity X_{ji}^n is shipped by a national (exporting) firm, only $\frac{X_{ji}^n}{\tau}$ arrives in the foreign country at price $p\tau$. M_{ix} or M_{jx} is the share of income spent in aggregate on good X which can be further substituted out in terms of the demand parameters and factor endowments.

It is possible to simplify zero-profit conditions by using the pricing equation and marshallian demand functions for x-varieties above. Some algebra (see appendix 1) yields a simplified set of conditions, which determine the production regime for the X-sector, and these equations written compactly for country j firms are:

$$\begin{aligned}
\beta \cdot p^{1-\epsilon} \tau^{1-\epsilon} e_i^{\epsilon-1} S_i + \beta \cdot p^{1-\epsilon} e_j^{\epsilon-1} S_j &\leq \epsilon(H^n + G) & (N_j^n) & (16) \\
\beta \cdot p^{1-\epsilon} e_i^{\epsilon-1} S_i + \beta \cdot p^{1-\epsilon} e_j^{\epsilon-1} S_j &\leq \epsilon(H^m + G) + \epsilon G_i^m & (N_j^m) &
\end{aligned}$$

and similarly for country i firms.

As before, all equations will not hold with equality at any one time. If one of them holds with equality, the corresponding number of firms is positive, otherwise the corresponding number of firms is zero.

3. The Inter-temporal Model:

For inter-temporal analysis it is necessary to specify the starting and the end points.

Let us start at time $t = 0$ with a national production regime where only exporting firms are operational in both the countries and there is diversified production. This implies a pure trading equilibrium (no FDI) with both intra-industry and inter-industry trade. Let us assume that the representative consumer lives and that firms potentially operate forever. National firms in the pure trading equilibrium have an option to undertake a foreign

direct investment and start multinational production abroad. At sometime t^* between $t = 0$ and ∞ this option could be exercised and the production regime would switch from national to multinational provided it is optimal to do so. I will explicitly solve for this optimal rule.

As compared to a more straight-forward trading (export) decision, foreign investment is undertaken in the face of greater uncertainty. The foreign investment uncertainty is an aggregate uncertainty typically arising from foreign environment and could be because of statutory FDI policies, corporate governance regimes, legal-contractual issues, medium-to-long-term exchange rate costs, economic liberalization, FDI reforms, political instability etc. This uncertainty cannot be easily hedged and exists even when a multinational firm undertakes a foreign investment into a seemingly similar economy. It is this uncertainty which is of interest here and is represented by a stochastic shift variable R_i , multiplicative with the costs of foreign investment. Say for country i , $G_i^m = GR_i$. In the first instance, I will assume it to follow geometric brownian motion, which is an important theoretical benchmark. Later in the paper, I will extend the model by introducing poisson jumps and formulating a mixed process, which provides a more direct way of modelling FDI reforms and is realistic from a policy point of view.

As of now, let the stochastic process underlying foreign investment uncertainty be represented by:

$$dR_i = \mu_i R_i dt + \sigma_i R_i dW_t \quad (17)$$

where μ_i is the drift, σ_i is the volatility and dW_t is a Gauss-Wiener process representing brownian motion and at any instant satisfying $E(dW) = 0$ and $E(dW^2) = dt$.

Notice the two components of foreign investment—a certain portion (G) and an uncertain portion (R_i), which is driven by foreign investment uncertainty. This being a real model (there is no money here), costs of uncertainty associated with ‘trade-in-invisibles’ (which are an integral part of any foreign direct investment) are included in the process R_i . Such ‘trade-in-invisibles’ include head-quarter services, royalty payments, repatriation of profits, acquisition of shares etc. which are subject to regulatory regimes, capital controls and medium to long term exchange rate risks.

Any uncertainty which is equally faced by both national and multinational firms does not generate an option value between trading and FDI, but foreign investment uncertainty which is faced only by multi-national firms

implies existence of a real option, say for country j's exporting firms to either undertake a foreign direct investment in country i or to keep exporting as national firms as they were doing at time $t = 0$. I will henceforth call this the 'FDI option'.

To simplify exposition of the model, I will focus on country i and assume that country j follows a restrictive FDI regime in that it does not permit any foreign direct investment in X sector. Relaxation of this assumption is trivial and the same formulation would apply to the other country.

For an *individual* firm in country j, production decision at any time t^* between $t = 0$ and ∞ is given by the present value formulation:

$$\begin{aligned}
& \int_{t^*}^{\infty} e^{-\rho t} (\beta \cdot p^{1-\epsilon} \tau^{1-\epsilon} e_{i_0}^{\epsilon-1} S_i + \beta \cdot p^{1-\epsilon} e_j^{\epsilon-1} S_j) \cdot dt & (18) \\
= & \int_{t^*}^{\infty} e^{-\rho t} [\epsilon(H^n + G)] \cdot dt \\
& OR \\
& E \int_{t^*}^{\infty} e^{-\rho t} (\beta \cdot p^{1-\epsilon} e_{i_1}^{\epsilon-1} S_i + \beta \cdot p^{1-\epsilon} e_j^{\epsilon-1} S_j) \cdot dt \\
= & E \left[\int_{t^*}^{\infty} e^{-\rho t} [\epsilon(H^m + G)] \cdot dt + \epsilon GR_0 + \int_{t^*}^{\infty} e^{-\rho t} \epsilon (GR_i \cdot e^{(\mu_i - \frac{1}{2}\sigma_i^2)t} \cdot e^{\sigma W_i}) \cdot dt \right]
\end{aligned}$$

where ρ is the discount factor, β is the demand parameter, τ are the trade costs, $e_{i \text{ or } j}$ is the aggregate price index, e_{i_0} to e_{i_1} is the expected change in price index by undertaking FDI in country i, $S_{i \text{ or } j}$ are the factor endowments, ϵ is the elasticity of substitution between any two x-varieties, H^n or m are the headquarter costs, and further that in writing the present value formulation I have used the stochastic differential equation from above. Foreign investment costs are split into two parts - GR_0 , the setup costs which are revealed at time t^* (hence $E[GR_{t^*}] = GR_0$, no discounting needed) and GR_i , the subsequent costs over which expectations are formed and need to be explicitly solved for.

An individual firm takes prices and incomes as given. It is already operating as a national firm in country j (the first equality above), but has to form expectations over what would happen if it decided to switch to a multinational mode of production (the second equality above). 'OR' between the

two equalities indicates existence of a ‘real option’ between the two choices. Demand parameters and total factor endowments are also given and assumed not to change with time.

The firms know their operating characteristics and market structure well. All firms are assumed to be identical (homogeneous) and rational. Thus, when it becomes optimal for one firm in country j to switch from national to multinational mode of production, it is also optimal for other firms in country j to do so. Under assumption of rational expectations this forward looking behaviour implies an individual firm can fully anticipate the change in aggregate price index that would be caused by this switch in production regime from national to multinational as the producer price of an individual x -variety (p) and trade costs (τ) remain unchanged. This implies

$$E\{e_{i1}\} = e_{i1} = [N_i^n p^{1-\epsilon} + N_j^m p^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \quad (19)$$

Therefore, the expectation on left hand side of the second equation is easily taken care of as rational expectation implies expected present discounted value of mark-up revenues is same as its present discounted value:

$$\begin{aligned} & E \int_{t^*}^{\infty} e^{-\rho t} . (\beta . p^{1-\epsilon} e_{i1}^{\epsilon-1} S_i + \beta . p^{1-\epsilon} e_j^{\epsilon-1} S_j) . dt \\ &= \int_{t^*}^{\infty} e^{-\rho t} . (\beta . p^{1-\epsilon} e_{i1}^{\epsilon-1} S_i + \beta . p^{1-\epsilon} e_j^{\epsilon-1} S_j) . dt \\ &= \frac{\beta}{\rho} . p^{1-\epsilon} e_{i1}^{\epsilon-1} S_i + \frac{\beta}{\rho} p^{1-\epsilon} e_j^{\epsilon-1} S_j \end{aligned} \quad (20)$$

The expectation on right hand side of second equation is more tricky as it implies solving the stochastic integral:

$$E \int_{t^*}^{\infty} e^{-\rho t} . \epsilon \{GR_i . e^{(\mu_i - \frac{1}{2}\sigma_i^2)t}\} \{e^{\sigma W_t}\} . dt \quad (21)$$

A crucial part in this integral is the expectation of the Wiener process $E_t\{e^{\sigma W_t}\}$,

I undertake a change of variable by defining $Z_t = e^{\sigma W_t}$

By Ito's lemma

$$dZ_t = \sigma e^{\sigma W_t} dW_t + \frac{1}{2} \sigma^2 e^{\sigma W_t} dt \quad (22)$$

Writing it in the integral form :

$$Z_t = Z_0 + \sigma \int_0^t e^{\sigma W_s} dW_s + \int_0^t \frac{1}{2} \sigma^2 e^{\sigma W_s} ds \quad (23)$$

Taking expectations of both sides and using the fact that $E[Z_0] = 1$ (because by definition $W_0 = 0$); and that $E[\int_0^t e^{\sigma W_s} dW_s] = 0$ (increments of Wiener process are independent of the observed past), this expectation simplifies to :

$$E[Z_t] = 1 + \int_0^t \frac{1}{2} \sigma^2 E[e^{\sigma W_s}] ds \quad (24)$$

I now define another change of variable $E[Z_t] = x_t$

The expression is now equivalent to an ordinary differential equation $\frac{dx_t}{x_t} = \frac{1}{2} \sigma^2 x_t ds$ with initial condition $x_0 = 1$.

Its solution is $x_t = E[Z_t] = e^{\frac{1}{2} \sigma^2 t}$

Substituting the changed variables back into the stochastic integral, the $\frac{1}{2} \sigma^2 t$ terms cancel out and the stochastic expectation simplifies to :

$$E\{GR_i \cdot e^{(\mu_i - \frac{1}{2} \sigma_i^2)t}\} \{e^{\sigma W_t}\} = GR_i \cdot e^{\mu t} \quad (25)$$

The expected present discounted value of fixed costs for the multinational firm therefore becomes :

$$\begin{aligned} E \int_{t^*}^{\infty} e^{-\rho t} \cdot [\epsilon(H^m + G)] \cdot dt + \epsilon GR_0 + E \int_{t^*}^{\infty} e^{-\rho t} \epsilon (GR_i \cdot e^{(\mu_i - \frac{1}{2} \sigma_i^2)t} \cdot e^{\sigma W_t}) \cdot dt \\ = \frac{\epsilon}{\rho} (H^m + G) + \epsilon GR_0 + \frac{\epsilon GR_i}{\rho - \mu} \end{aligned} \quad (26)$$

Present value of markup revenues and fixed costs for a national firm in country j was simply:

$$\frac{\beta}{\rho} \cdot p^{1-\epsilon} \tau^{1-\epsilon} e_{i_0}^{\epsilon-1} S_i + \frac{\beta}{\rho} \cdot p^{1-\epsilon} e_j^{\epsilon-1} S_j$$

and

$$\frac{\epsilon}{\rho} (H^n + G)$$

respectively.

We still need to take into account the opportunity cost of real option between exporting and FDI as indicated by the term ‘OR’ in the decision making problem of an individual firm above. While a firm knows the trade costs it saves fairly well from its account books, the foreign investment costs are at best only an estimate.

Let us call the first state as $V(\text{ex})$ and the second as $V(\text{fdi})$. By moving from state $V(\text{ex})$ to $V(\text{fdi})$ the firm gains the present value of trade costs saved but also expects to lose the expected present value of foreign investment costs that need to be undertaken. The exercise of this real option can be interpreted as a trade-off between the expected gain and loss in the value of the firm in moving from one state (exporting as national firm) to the other (undertaking FDI as a multinational firm). Since output of these firms (i.e. good X) is freely traded in the open market, we can assume stochastic changes in V are sufficiently spanned by assets(goods) traded in the market. This assumption of completeness is not essential to the results here, but makes analysis simple.

To simplify notation, let T be the present value of trade costs saved when a country j 's exporting firm undertakes foreign direct investment in country i and \widehat{R} be the present value of foreign investment costs that will be incurred when this happens. Formally:

$$T = \frac{\beta}{\rho} \cdot p^{1-\epsilon} (\tau^{1-\epsilon} e_{i_0}^{\epsilon-1} - e_{i_1}^{\epsilon-1}) S_i \quad (27)$$

$$\widehat{R} = \epsilon G R_0 + \frac{\epsilon G R_i}{\rho - \mu} \quad (28)$$

Let $F(\widehat{R}, t; T)$ be the value of this option to switch production regime from exporting to FDI. Payoff from exercising this option at any time t is

given by the indicator function:

$$g(\widehat{R}, t; T) = \max[T - \widehat{R}, 0] \quad (29)$$

Taking analogy from finance theory, this is like an American Put Option, a class of options that are harder to solve and typically do not have closed form solutions. Such functions are also called free boundary problems. These are essentially variational problems in stochastic mathematics. Fortunately, this real option is not exactly like its financial counterparts. I will use an original idea from Merton (1973), which states that if time to maturity is infinite, the option pricing function becomes time independent and a closed form solution exists. Such options are called ‘perpetual puts’ and its option pricing function is written as $F(\widehat{R}, t = \infty; T)$ or simply, $F(\widehat{R}; T)$.

There are two equivalent ways of solving this - either through contingent claim analysis using the arbitrage theory or through stochastic dynamic programming. Because of its expositional neatness I will hereby use the arbitrage theory.

Using an application of second order Taylor series and Ito’s lemma gives us the following partial differential equation for the option pricing function:

$$(\rho - \delta)RF'(R)dt + \frac{1}{2}\sigma^2R^2F''(R)dt = \rho Rdt \quad (30)$$

where $\delta = \rho - \mu$ (a la Dixit, Pindyck,1984). δ is also called the rate of convenience yield.

This combines with the following boundary conditions :

1. $F(\infty; T) = 0$, a ‘terminal’ condition which means this option is of no value if foreign investment costs are extremely high.
2. $F(R^*; T) = T - R^*$, a ‘value matching’ condition which describes payoffs when it becomes optimal to undertake a foreign direct investment.
3. $\frac{\partial F(R^*; T)}{\partial R} = -1$, a ‘smooth-pasting’ or ‘high contact’ boundary condition which implies that slope of the payoff and option pricing function match at the exercise boundary and if not, it is not optimal to exercise. There is thus a continuity or smooth pasting at the optimal exercise boundary.

The general solution for this differential equation is :

$$F(R, \infty; T) = a_1R + a_2R^{-\gamma} \quad (31)$$

where

$$\gamma = \frac{\frac{1}{2}\sigma^2 - (\rho - \delta) + \sqrt{\left((\rho - \delta) - \frac{1}{2}\sigma^2\right)^2 + 2\rho\sigma^2}}{\sigma^2} \quad (32)$$

is the positive root of the fundamental quadratic :

$$Q = \frac{1}{2}\sigma^2(\gamma)(\gamma - 1) + (\rho - \delta)(\gamma) - \rho = 0 \quad (33)$$

The first boundary condition implies $a_1 = 0$.

The second or the value matching condition implies $a_2 = (T - R^*) \cdot R^{*\gamma}$

Now, from the smooth pasting condition, general solution evaluated at R^* , the optimal value, is :

$$\frac{\partial F(R^*, \infty; T)}{\partial R} = -\gamma a_2 R^{*\gamma-1} = -1 \quad (34)$$

Substituting for a_2 :

$$-\gamma (T - R^*) \cdot R^{*\gamma} R^{*\gamma-1} = -1 \quad (35)$$

With a bit of algebra this simplifies to

$$T = \frac{1 + \gamma}{\gamma} \widehat{R}^* \quad (36)$$

This gives us the ‘optimal foreign investment rule’ which is a deterministic, time-independent solution. R^* is the present value of foreign investment costs when the FDI option is exercised.

Intuitively, it says, ‘uncertainty combined with irreversibility drives a wedge between present value of trade costs saved and the critical value of foreign investment costs that will need to be incurred’. This wedge implies ‘hysteresis’. If the foreign investment costs are falling, the firms would like to wait longer before undertaking foreign investment, than they would in the absence of uncertainty.

If $\sigma \rightarrow 0$ (no uncertainty), the positive root $\gamma \rightarrow \infty$ and the optimal scale-up factor $\frac{1+\gamma}{\gamma} \rightarrow 1$, implying thereby that there is no ‘hysteresis’. If on the other hand $\sigma \rightarrow \infty$, the positive root $\gamma \rightarrow 0$ and the optimal scale-up

factor $\frac{1+\gamma}{\gamma} \rightarrow \infty$, implying thereby that the FDI option would never be exercised or that foreign investment would never take place.

Thus, parametrized in time, the total effect of foreign investment uncertainty is determined through its effect on present discounted value and through the opportunity cost of real option between exporting and FDI. ‘Perpetual Put’ makes our life simple because we can solve for the general equilibrium recursively at any point in time and the optimal investment rule remains unchanged (i.e. a closed form solution exists).

Equilibrium conditions for country j firms can now be written in the complementary slackness form as follows:

$$\begin{aligned} \frac{\beta}{\rho} p^{1-\epsilon} \tau^{1-\epsilon} e_{i_0}^{\epsilon-1} S_i + \frac{\beta}{\rho} p^{1-\epsilon} e_j^{\epsilon-1} S_j &\leq \frac{\epsilon}{\rho} (H^n + G) & (N_j^n) \\ \frac{\beta}{\rho} p^{1-\epsilon} e_{i_1}^{\epsilon-1} S_i + \frac{\beta}{\rho} p^{1-\epsilon} e_j^{\epsilon-1} S_j &\leq \frac{\epsilon}{\rho} (H^m + G) + \left(\frac{1+\gamma}{\gamma}\right) \left[\epsilon G R_0 + \frac{\epsilon G R_i}{\rho - \mu}\right] & (N_j^m) \end{aligned} \quad (37)$$

Aggregate price index which is endogenous may be further substituted out using the following expressions:

$$\begin{aligned} e_{i_0} &= [N_i^n p^{1-\epsilon} + N_j^n (p\tau)^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \\ e_{i_1} &= [N_i^n p^{1-\epsilon} + N_j^m p^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \\ e_j &= [N_i^n (p\tau)^{1-\epsilon} + N_j^n p^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \end{aligned} \quad (38)$$

We have seen that while foreign investment uncertainty is driven by an exogenous stochastic shift variable, the real option between exporting and FDI is an indicator function, meaning thereby that the optimal foreign investment rule is deterministic and so are the other endogenous variables which after substituting out the price index essentially mean the number of firms of each type in equilibrium. This defines the equilibrium production regime. If there is no uncertainty, there would be no option value, and hence, no scale-up of foreign investment costs and we would be back to a standard marshallian kind of revenue-cost analysis.

It is also pertinent to mention that what is modelled here is only the option decision relating to switching of production regime from trading to FDI after having started at time $t = 0$ with exporting (national) firms in both the countries. This has been called the ‘FDI option’ and it ceases

to have an option value once it is exercised. The reverse is usually not a symmetric phenomenon, but rather a pure exit decision for which a separate option problem exists. Such exit options have been adequately modelled in the literature (Dixit and Pindyck,1994).

To summarize, we start at time $t = 0$ with a pure trading equilibrium and diversified production. Let us say that foreign investment environment in country i is not conducive to start with. Exporting firms from country j , which are identical and rational, will solve for the foreign investment rule and say, optimally decide to continue as national firms. These firms wait and watch and thereby retain an option to undertake a foreign investment abroad which is valued in terms of its opportunity cost. Say later, because of some FDI reforms, foreign investment environment in country i becomes favourable and at some point in time, in accordance with the foreign investment rule, it becomes optimal for country j 's exporting firms to exercise the FDI option. They will now rush to undertake foreign investment in country i . There will be both partial and general equilibrium effects. While partial equilibrium effects are reflected in the costs and savings for individual firms, general equilibrium effects are reflected in the changes in the aggregate price index and the type of firms operating in equilibrium which are determined endogenously in this model. The price of an individual x -variety remains unchanged, as also the equilibrium wage which is pinned down by the numeraire sector. The factor market undergoes a simultaneous adjustment as part of skilled labour in country j freed up by its national firms starting multinational production abroad is used up in headquarter services and the remaining shifts to numeraire good (Y) sector whose production expands. Exactly the opposite happens in country i , where numeraire good sector contracts and some skilled labour moves into multinational production within the X sector.

4. Comparative Experiments:

As we have seen above, foreign investment uncertainty drives a wedge between trade costs saved and the foreign investment costs incurred, thereby delaying foreign direct investment into country i beyond what is predicted by pure comparative advantage. This is 'hysteresis'.

I will now conduct some thought experiments to answer the questions of "when"- that is to analyze the effect of uncertainty on timing of foreign investment given some comparative advantage and trade costs; and of "where", that is, in the presence of foreign investment uncertainty, where would it be

optimal to undertake a foreign investment amongst alternative locations, given some comparative advantage and trade costs?

Effect of Volatility : Let us say country i has higher a volatility compared to country j , $\sigma_i > \sigma_j$.

Volatility affects the optimal decision through the scale-up factor

$$\frac{1 + \gamma}{\gamma}. \quad (39)$$

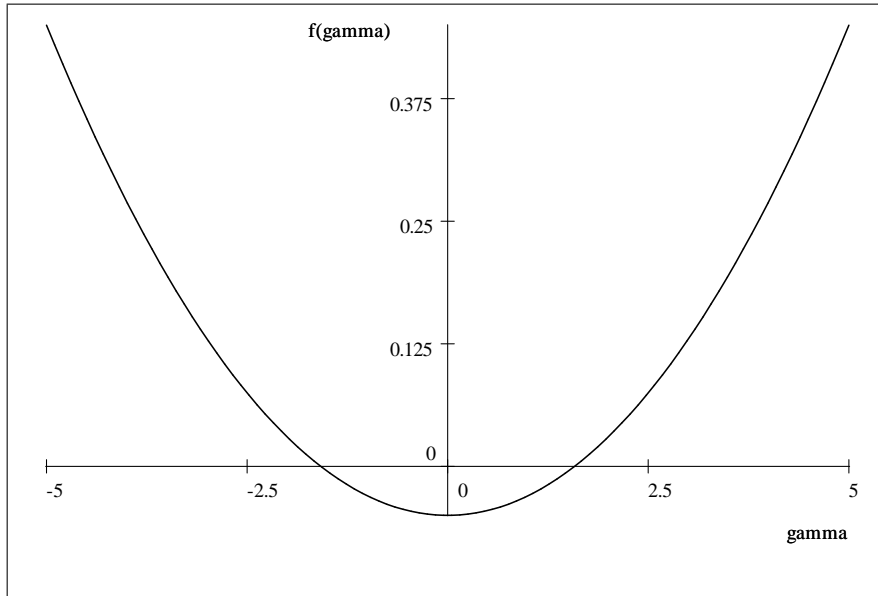
By totally differentiating the fundamental quadratic above with respect to volatility σ , holding drift constant (a mean preserving spread):

$$\frac{\partial Q}{\partial \gamma} \frac{\partial \gamma}{\partial \sigma} + \frac{\partial Q}{\partial \sigma} = 0 \quad (40)$$

Now, $Q(1) = -\delta$, that is the fundamental quadratic valued at $\gamma = 1$ is negative. Therefore, positive root γ is greater than one.

$Q(0) = -\rho$, that is fundamental quadratic valued at $\gamma = 0$ is negative.

This helps us plot the fundamental quadratic, which is a function of γ . Say for $\sigma = 0.2, \rho = 0.05$ and $\delta = 0.03$ this plot is:



Further that $\frac{\partial Q}{\partial \sigma} = (\gamma) (\gamma - 1) .\sigma$ is positive given that $\sigma > 0$ and γ , which is the positive root of fundamental quadratic, is greater than 1.

$\frac{\partial Q}{\partial \gamma} > 0$ as fundamental quadratic is increasing at its positive root.

This implies that for the total differentiation equality to hold, $\frac{\partial \gamma}{\partial \sigma} < 0$, i.e. derivative with respect to the volatility parameter is negative.

Therefore, if σ increases, γ decreases and therefore the optimal scale up factor $(\frac{1+\gamma}{\gamma})$ increases.

Hence, comparing two countries i and j , if $\sigma_i > \sigma_j$, then we either need a higher saving of trade costs from country j to i , or given trade costs, firms in country j need to wait longer for foreign investment costs in country i to fall low enough to trigger a FDI.

We have proved the following :

Proposition 1 : A mean preserving increase (higher volatility) in the aggregate uncertainty associated with foreign investment drives a greater wedge between trade costs that need to be saved and the foreign investment costs that need to be incurred at the optimal trigger point between exporting and FDI.

Effect of drift : The effect of drift is two fold - effect on the expected present value and effect on the opportunity cost of real option between exporting and FDI.

Again, by total differentiation,

$$\frac{\partial Q}{\partial \gamma} \frac{\partial \gamma}{\partial \delta} + \frac{\partial Q}{\partial \delta} = 0 \quad (41)$$

$\delta = \rho - \mu$ is the difference between the discount factor and the drift of the stochastic process.

$\frac{\partial Q}{\partial \delta} = -\gamma$ is < 0 (negative) as γ is the positive root of the fundamental quadratic.

$\frac{\partial Q}{\partial \gamma}$ is > 0 , positive from the properties of the fundamental quadratic as proved above.

Therefore, for the total differentiation equality to hold, $\frac{\partial \gamma}{\partial \delta}$ is > 0 (i.e. positive).

Thus, a lower drift (lower μ) implies a higher δ , which from the derivative above means a higher γ . This in turn implies a lower wedge or a lower scale-up factor, $(\frac{1+\gamma}{\gamma})$, between trade costs that would be saved and foreign investment costs that would be incurred. This is the opportunity cost or implicit insurance premium of holding the 'FDI option'.

At the same time, a lower μ implies a lower $\frac{e^{GR_i(t_0)}}{\rho - \mu}$ and hence a lower expected present value of foreign investment costs given that both ρ and μ

are positive fractions less than one (they are percentages) and that $\rho > \mu$ given the assumption of a convergent solution.

We have thus established the second proposition:

Proposition 2: A lower drift of foreign investment uncertainty has a two fold effect on the FDI decision of exporting firms. It has a direct effect through a decrease in the expected present value of foreign investment costs and an indirect effect through decrease in the size of the wedge or optimal scale-up factor between trade costs saved and the foreign investment costs incurred. Put together, this increases the chances of an early exercise of FDI option in the presence of falling foreign investment costs.

Effect of Risk aversion: Till now the FDI-option was analyzed assuming firms are risk-neutral, a procedure called risk-neutral valuation. Suppose now that firms (investors and/or managers) are risk averse. While earlier volatility affected the optimal decision of risk-neutral firms directly, it now has an additional effect through the ‘drift’ term. For a ‘put’ option (unlike a call) it is the positive deviations of uncertainty which matter, because they decrease the payoff from exercising this option. The simplest way to allow risk-aversion is to replace ρ with an appropriate risk-adjusted discount rate from the capital asset pricing model. If the firms undertaking FDI are risk-averse, the risk-adjusted rate of return for this option becomes:

$$\bar{\rho} = \rho - [E(\rho_m) - \rho] \frac{Cov\left(\frac{dF}{F}, \rho_m\right)}{Var(\rho_m)} \quad (42)$$

where ρ_m is the rate of return on some market portfolio, ρ is the risk-neutral rate and F is the option value function. The sign of covariance could be positive or negative.

The meaning of δ term introduced above will now become clearer. Till now σ and μ were treated as independent parameters. Risk-aversion essentially means we allow the drift to adjust as σ changes. Each unit increase in σ now requires an increase in δ by a coefficient term containing the correlation coefficient and the market price of risk :

$$\delta = \bar{\rho} - \mu = \rho - \phi \eta_{xm} \sigma - \mu \quad (43)$$

where ϕ is the market price of risk, η_{xm} is the correlation coefficient between foreign investment and some market portfolio, while other terms are as defined earlier.

Volatility of project value (here foreign direct investment) matters even for a risk-neutral firm (see proposition 1 above), but now in the presence of risk-aversion, it matters even more. Higher volatility now also means a lower δ , which in accordance with proposition 2 above further increases the size of wedge or optimal scale-up $\frac{1+\gamma}{\gamma}$. between trade costs and the foreign investment costs. This establishes the next proposition:

Proposition 3 : If firms are risk-averse they need either lower foreign investment costs or greater saving of trade costs before they can undertake a foreign direct investment as compared to when they are risk-neutral. This implies a greater wait and watch before FDI is undertaken in the presence of falling foreign investment costs.

Country Size and Income :

Since income is endogenous in general equilibrium, the variable of interest for modelling its effect is the total factor endowment S_i . Let us say skilled factor endowment of country i is growing with time at a defined time rate:

$$S_i(t) = S_{i0} \cdot e^{\eta t} \quad (44)$$

Equilibrium conditions for country j's firms would now be:

$$\frac{\beta}{\rho - \eta} \cdot p^{1-\epsilon} \tau^{1-\epsilon} e_{i_0}^{\epsilon-1} S_{i_0} + \frac{\beta}{\rho} \cdot p^{1-\epsilon} e_j^{\epsilon-1} S_j \leq \frac{\epsilon}{\rho} (H^n + G) \quad (N_j^n)$$

$$\frac{\beta}{\rho - \eta} \cdot p^{1-\epsilon} e_{i_1}^{\epsilon-1} S_{i_0} + \frac{\beta_j}{\rho} \cdot p^{1-\epsilon} e_j^{\epsilon-1} S_j \leq \frac{\epsilon}{\rho} (H^m + G) + \left(\frac{1+\gamma}{\gamma}\right) \left[\epsilon GR_0 + \frac{\epsilon GR_i}{\rho - \mu}\right] \quad (N_j^m)$$

(45)

This raises the strike price (and therefore payoffs) of the FDI option :

$$T = \frac{\beta}{\rho - \eta} \cdot p^{1-\epsilon} (\tau^{1-\epsilon} e_{i_0}^{\epsilon-1} - e_{i_1}^{\epsilon-1}) S_{i_0} \quad (46)$$

Country i is therefore more likely to attract foreign direct investment.

Proposition 4 : Country with a larger income and/or greater endowment of skilled factors is more likely to attract foreign direct investment given a level of trade costs and foreign investment costs. In the presence of growing market size or falling foreign investment costs this would imply an early exercise of the FDI option.

I have focussed on aggregate investment uncertainty here for the simple reason that it is relevant to facts-in-issue and is closely related to FDI policy

in the real world. Idiosyncratic productivity shocks, under the assumption that national and multinational firms use the same manufacturing process, is not likely to create an option value between trading and FDI. For simplicity I also assume there are no demand or aggregate productivity shocks.

I will now proceed to extend the model further by introducing poisson jumps in foreign investment costs. This provides a better way of modelling policy related uncertainty and the impact of ‘FDI reforms’, a term popularly used in emerging economies today.

5. Poisson Jump Process: Geometric brownian motion is an important theoretical benchmark, but for a more comprehensive analysis I will extend this model by examining a mixed brownian motion-poisson jump process. Foreign investment uncertainty arises from a variety of sources, including but not limited to, private sector expectations of public policy, sudden policy shifts like economic liberalization or FDI reforms, changes in taxation or corporate governance regimes, legal-contractual issues, exchange rate costs related to repatriation of profits, royalty, headquarter services or investment flows, industry wide or economy wide macro-shocks, political instability, lobbying by domestic firms etc. A poisson jump process can better account for many such alternative sources of uncertainty.

Let us allow for the possibility of a downward jump ϕ that can suddenly bring down the foreign investment costs in country i . Let λ be the probability that such a downward jump can arrive in any time-period. If it arrives the foreign investment costs would fall to $(1 - \phi)$ times the original value. The stochastic shift process driving the foreign investment costs will now be represented by a mixed brownian motion- poisson jump process given below:

$$dR_i = \mu_i R_i dt + \sigma_i R_i dW_t - R_i dq, \quad (47)$$

where $dq = \phi$ with probability λ and

$$dq = 0 \text{ with probability } 1 - \lambda \text{ in any time-period } dt.$$

Other terms are as before.

The expected percentage change in foreign investment costs in country i is now given by :

$$E[dGR_i] = (\mu_i - \lambda\phi).GR_i.dt \quad (48)$$

and expected variance of this change is :

$$Var[dGR_i] = \sigma^2.(GR_i)^2.dt + \lambda\phi^2.(GR_i)^2.dt \quad (49)$$

As before we need to solve for the optimal foreign investment rule for an individual firm in two steps. First, find the expected present value of foreign investment costs and second, find the scale-up factor that implies implicit insurance premium of holding the FDI option.

Firstly, the expected present value of fixed costs for a firm considering the exercise of this FDI option are given by (see appendix 2 for details):

$$\frac{\epsilon}{\rho}(H^m + G) + \epsilon GR_0 + \frac{\epsilon GR_i(t_0)}{\rho - \mu + \lambda\phi} \quad (50)$$

Intuitively, it says, higher probability of FDI reforms (i.e. higher λ) and higher impact of FDI reforms (i.e. higher ϕ implying a greater percentage fall in foreign investment costs) decrease the present value of foreign investment costs making foreign direct investment more likely.

Secondly, we need to solve for the opportunity cost of holding this FDI option.

The partial differential equation will now be written as:

$$(\rho - \delta)RF'(R)dt + \frac{1}{2}\sigma^2 R^2 F''(R)dt - \lambda[F(R) - F(R(1 - \phi))]dt = \rho Rdt \quad (51)$$

The boundary conditions remain the same:

1. $F(\infty; T) = 0$
2. $F(R^*; T) = T - R^*$
3. $\frac{\partial F(R^*; T)}{\partial R} = -1$

The general solution is again of the form

$$F(R, \infty; T) = a_1 R + a_2 R^{-\gamma}$$

As before, the first boundary condition implies

$$a_1 = 0$$

The second or the value matching condition implies

$$a_2 = (T - R^*) . R^{*\gamma}$$

γ is the positive solution (negative solution is ruled out by boundary conditions) to the following characteristic non-linear equation:

$$\frac{1}{2}\sigma^2(\gamma)(\gamma-1) + (\rho-\delta)(\gamma) + \lambda(1-\phi)^\gamma - (\rho+\lambda) = 0 \quad (52)$$

This equation does not have an analytic solution and so it needs to be solved numerically.

For $\sigma = 0.2, \rho = 0.05, \delta = 0.03, \lambda = 0.05$ and $\phi = 0.2$, the graph drawn for this function is shown in appendix 3. $\gamma = 1.8267$ is its positive solution.

The rest of the work required in obtaining the optimal foreign investment rule can be done analytically.

Smooth pasting implies

$$\frac{\partial F(R^*, \infty; T)}{\partial R} = -\gamma a_2 R^{*\gamma-1} = -1 \quad (53)$$

Substituting for a_2 gives us :

$$-\gamma(T - R^*) \cdot R^{*\gamma} R^{*\gamma-1} = -1 \quad (54)$$

Solving for the optimal scale-up factor again gives us:

$$T = \frac{1 + \gamma}{\gamma} \widehat{R}^* \quad (55)$$

As we can see the optimal foreign investment rule or the scale-up factor for foreign investment costs remains the same. And as before, it is deterministic and time independent. However, ‘ γ ’ now has a different value as compared to the last section and this is obtained as a numerical solution to the characteristic non-linear equation mentioned above.

As before, equilibrium conditions for country j’s firms can now be written in the complementary slackness form as:

$$\begin{aligned} \frac{\beta}{\rho} p^{1-\epsilon} \tau^{1-\epsilon} e_{i_0}^{\epsilon-1} S_i + \frac{\beta}{\rho} p^{1-\epsilon} e_{j_0}^{\epsilon-1} S_j &\leq \frac{\epsilon}{\rho} (H^n + G) & (N_j^n) \\ \frac{\beta}{\rho} p^{1-\epsilon} e_{i_1}^{\epsilon-1} S_i + \frac{\beta}{\rho} p^{1-\epsilon} e_{j_1}^{\epsilon-1} S_j &\leq \frac{\epsilon}{\rho} (H^m + G) + \left(\frac{1 + \gamma}{\gamma}\right) \left[\epsilon G R_0 + \frac{\epsilon G R_i(t_0)}{\rho - \mu + \lambda \phi}\right] & (N_j^m) \end{aligned} \quad (56)$$

I will again perform some comparative experiments. The difference is that now I will solve numerically for γ each time an exogenous parameter changes.

Effect of Volatility: As volatility of foreign investment uncertainty increases, the value of γ decreases. This solution for various values of σ is shown in the table below:

σ	ρ	δ	λ	ϕ	γ
0.05	0.05	0.03	0.05	0.2	3.3263
0.10	0.05	0.03	0.05	0.2	2.5772
0.15	0.05	0.03	0.05	0.2	2.1146
0.20	0.05	0.03	0.05	0.2	1.8267
0.25	0.05	0.03	0.05	0.2	1.6367
0.30	0.05	0.03	0.05	0.2	1.5046
0.35	0.05	0.03	0.05	0.2	1.4091
0.40	0.05	0.03	0.05	0.2	1.3378

This is the same result as Proposition 1 above. Higher volatility implies a lower γ and therefore a higher wedge or higher scale-up of foreign investment costs. Thus, to undertake foreign direct investment the firms now need either a higher saving of transport costs or in the presence of falling foreign costs they would wait and watch longer before the FDI option is exercised.

Effect of Drift: An increase in δ , which implies a lower drift, leads to an increase in the value of γ as shown in the table below:

σ	ρ	δ	λ	ϕ	γ
0.20	0.05	0.01	0.05	0.2	1.3432
0.20	0.05	0.02	0.05	0.2	1.5666
0.20	0.05	0.03	0.05	0.2	1.8267
0.20	0.05	0.04	0.05	0.2	2.1223

This decreases the optimal scale-up factor $\left(\frac{1+\gamma}{\gamma}\right)$ for foreign investment costs. Besides, it also implies a lower expected present value of foreign investment costs. Both these effects together imply Proposition 2 above, meaning thereby that falling foreign investment cost increases chances of foreign investment and leads to an early exercise of the FDI option.

Probability of FDI reforms: A higher probability of FDI reforms, as measured by the factor λ , increases the value of γ as shown in the table below:

σ	ρ	δ	λ	ϕ	γ
0.20	0.05	0.03	0.00	0.2	1.5811
0.20	0.05	0.03	0.05	0.2	1.8267
0.20	0.05	0.03	0.10	0.2	2.0887
0.20	0.05	0.03	0.15	0.2	2.3602
0.20	0.05	0.03	0.20	0.2	2.6356

This decreases the optimal scale-up factor $\left(\frac{1+\gamma}{\gamma}\right)$ over foreign investment costs. It also decreases the expected present value of foreign investment costs as already proved above. The combined effect is summarized in Proposition 5 below:

Proposition 5: An increase in the probability of a sudden drop in foreign investment costs decreases the optimal scale-up over foreign investment costs and also decreases the expected present value of foreign investment costs. This dual effect facilitates foreign direct investment by increasing chances of an early exercise of the FDI option.

Impact of FDI reforms: A larger impact of FDI reforms as measured by the percentage parameter ϕ also increases the value of γ as can be seen from the table below:

σ	ρ	δ	λ	ϕ	γ
0.20	0.05	0.03	0.05	0.00	1.5811
0.20	0.05	0.03	0.05	0.10	1.7063
0.20	0.05	0.03	0.05	0.20	1.8267
0.20	0.05	0.03	0.05	0.30	1.9356
0.20	0.05	0.03	0.05	0.40	2.0280
0.20	0.05	0.03	0.05	0.50	2.1018

This also decreases the optimal scale-up factor $\frac{1+\gamma}{\gamma}$ and the expected present value of foreign investment costs. The effect on foreign direct investments is summarized in Proposition 6 below.

Proposition 6: An increase in size of the percentage downward jump in foreign investment costs decreases the optimal scale-up over foreign investment costs and also decreases the expected present value of foreign invest-

ment costs. This dual effect facilitates foreign direct investment by increasing chances of an early exercise of the FDI option.

6. Conclusion: Real life behaviour of multinational firms is somewhat more complex than what can be explained by conventional comparative advantage-trade cost analysis. Almost all countries, including the developed ones, have specific FDI regimes which are driven by policy changes. Even when FDI reforms bring down foreign investment costs, multinational firms do not immediately rush in. There is a considerable ‘wait and watch’, a kind of time lag or inertia before investments actually start flowing in. This is even more common with developing economies, where foreign investment uncertainty is expected to be high. This model shows that, by indulging in such cautious behaviour, multinational firms are actually seeking additional compensation for some real costs over and above what are accounted for in a conventional cost-benefit analysis.

I solve for the optimal foreign investment rule. Fortunately, it is deterministic and time-independent. Foreign investment costs are scaled up by a factor which depends on the parameters of foreign investment uncertainty. This implies ‘hysteresis’, because in the presence of falling foreign investment costs, multinational firms will wait longer than they would in the absence of uncertainty. Similarly, given comparative advantage and trade costs between alternative locations, firms prefer the ones with less uncertain investment environments.

Greater volatility and risk aversion delays foreign investment, while growing market size and income facilitates it. A particularly interesting aspect of this paper is the mixed poisson jump-brownian motion process, which explicitly models policy driven FDI reforms. It shows how a sudden drop in foreign investment costs brought about by a policy shift, as also a greater probability of it, can facilitate early exercise of the FDI option. National governments all over the world, which spend considerable time and energy trying to woo foreign investments, are actually exerting a positive effect through the expectations of foreign investment uncertainty.

To summarize, this paper enriches the existing general equilibrium models of multinational firms by providing a better explanation for their observed behaviour in uncertain foreign environments. It embeds the theory of real options into a framework of Dixit-Stiglitz type monopolistic competition. It explicitly solves for policy driven FDI liberalization as a mixed poisson-jump brownian motion stochastic process. Further that, while the investment

under uncertainty literature is based on the theory of call options, I solve 'FDI option' as a put option thereby also enriching the theory of real options.

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Appendix 1:

1) Zero profit condition for national firms in country j :

$$pX_{jj}^n + pX_{ji}^n \leq wcX_{jj}^n + wcX_{ji}^n + w(H^n + G) \quad (N_j^n)$$

Bring the quantities produced of each variety to the left hand side:

$$(p - wc)X_{jj}^n + (p - wc)X_{ji}^n \leq w(H^n + G) \quad (N_j^n)$$

Substitute using the mark-up or pricing equations :

$$\frac{p}{\epsilon}X_{jj}^n + \frac{p}{\epsilon}X_{ji}^n \leq w(H^n + G) \quad (N_j^n)$$

Substitute using the marshallian demand functions:

$$\beta.p^{1-\epsilon}e_j^{\epsilon-1}wS_j + \beta.p^{1-\epsilon}\tau^{1-\epsilon}e_i^{\epsilon-1}wS_i \leq w\epsilon(H^n + G) \quad (N_j^n)$$

Cancelling out wages from both sides gives the required equation :

$$\beta.p^{1-\epsilon}e_j^{\epsilon-1}S_j + \beta.p^{1-\epsilon}\tau^{1-\epsilon}e_i^{\epsilon-1}S_i \leq \epsilon(H^n + G) \quad (N_j^n)$$

2) Zero profit condition for multinational firms in country j :

$$pX_{jj}^m + pX_{ji}^m \leq wcX_{jj}^m + wcX_{ji}^m + w(H^m + G) + wG_i^m \quad (N_j^m)$$

Bring the quantities produced of each variety to the left hand side:

$$(p - wc)X_{jj}^m + (p - wc)X_{ji}^m \leq w(H^m + G) + wG_i^m \quad (N_j^m)$$

Substitute using the mark-up or pricing equations :

$$\frac{p}{\epsilon}X_{jj}^m + \frac{p}{\epsilon}X_{ji}^m \leq w(H^m + G) + wG_i^m \quad (N_j^m)$$

Substitute using the marshallian demand functions:

$$\beta.p^{1-\epsilon}e_j^{\epsilon-1}wS_j + \beta.p^{1-\epsilon}e_i^{\epsilon-1}wS_i \leq w\epsilon(H^m + G) + w\epsilon G_i^m \quad (N_j^m)$$

Cancelling out wages from both sides gives the required equation :

$$\beta.p^{1-\epsilon}e_j^{\epsilon-1}S_j + \beta.p^{1-\epsilon}e_i^{\epsilon-1}S_i \leq \epsilon(H^m + G) + \epsilon G_i^m \quad (N_j^m)$$

The same can be repeated for national and multinational firms in country i.

Appendix 2: Present Value of Fixed Costs for the Poisson Jump Process
The stochastic shift process with poisson jumps is given by

$$dR_i = \mu_i R_i dt + \sigma_i R_i dW_t - R_i dq$$

This implies dR_i will take the following different values, which along with their respective probabilities, are :

$$\begin{aligned} dR_i &= \mu_i + \sigma R_i \sqrt{dt} \text{ with probability } \frac{1}{2}(1 - \lambda) .dt \\ &\mu_i - \sigma R_i \sqrt{dt} \text{ with probability } \frac{1}{2}(1 - \lambda) .dt \\ &\mu_i - \phi R_i \text{ with probability } \lambda .dt \end{aligned}$$

The poisson jump is assumed to be much bigger as compared to a single increment in the Wiener process.

We know from the properties of brownian motion that $E[dW_t] = 0$, i.e. expected change in Wiener process for any time period dt is zero.

Expected change in foreign fixed costs over any time period dt is therefore given by the equation:

$$E[GR_i] = (\mu_i - \lambda\phi) .GR_i .dt$$

The (strong) solution to the given stochastic differential equation for mixed poisson jump-brownian motion process is :

$$GR_i = GR_{t=0} . \int_0^t \exp\left[\left(\mu_i - \lambda\phi - \frac{1}{2}\sigma^2\right)t + \sigma W_t\right] .dt$$

To solve for the expected foreign fixed costs, we need to solve the expectation of the stochastic exponential on right hand side:

$$E_t[GR_i] = E_t[GR_{t=0} \int_0^t \exp\left\{\left(\mu_i - \lambda\phi - \frac{1}{2}\sigma^2\right)t\right\} . \exp\{\sigma W_t\} .dt]$$

which implies

$$E_t [GR_i] = GR_{t=0} \cdot \int_0^t \exp\left\{\left(\mu_i - \lambda\phi - \frac{1}{2}\sigma^2\right) t\right\} \cdot E_t[\exp\{\sigma W_t\}] \cdot dt$$

Solving the stochastic exponential as in section 3 of the paper above:

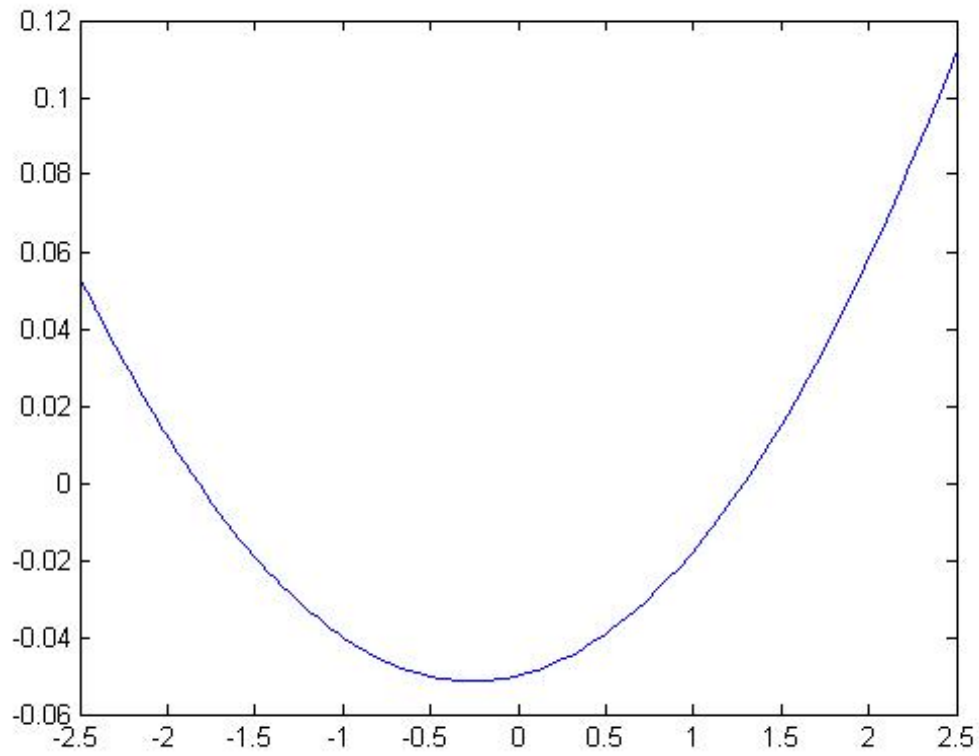
$$E_t[\exp\{\sigma W_t\}] = \exp\left(\frac{1}{2}\sigma^2 t\right)$$

We now get rid of the expectation term on the right hand side as everything else is deterministic:

$$E_t [GR_i] = GR_{t=0} \int_0^t \exp\{(\mu_i - \lambda\phi) t\} \cdot dt$$

Thus, starting at a time t_0 between 0 and ∞ and potentially operating forever, expected present value of fixed costs for a country j multinational firm undertaking foreign direct investment in country i are:

$$\begin{aligned} & E_{t^*} \left\{ \int_{t^*}^{\infty} e^{-\rho t} \cdot [\epsilon(H^m + G)] \cdot dt + \epsilon GR_0 + \int_{t^*}^{\infty} e^{-\rho t} \epsilon (GR_i \cdot e^{(\mu_i - \lambda\phi - \frac{1}{2}\sigma_i^2)t} e^{\sigma W_t}) dt \right\} \\ = & \int_{t^*}^{\infty} e^{-\rho t} \cdot [\epsilon(H^m + G)] \cdot dt + \epsilon GR_0 + E_{t^*} \left\{ \int_{t^*}^{\infty} e^{-\rho t} \epsilon (GR_i \cdot e^{(\mu_i - \lambda\phi - \frac{1}{2}\sigma_i^2)t} e^{\sigma W_t}) dt \right\} \\ = & \frac{\epsilon}{\rho} (H^m + G) + \epsilon GR_0 + \frac{\epsilon GR_i}{\rho - \mu + \lambda\phi} \end{aligned}$$



Appendix 3:

Graph for the characteristic equation for mixed poisson jump-brownian motion stochastic process with endogenous solution for parameter γ on the horizontal axis.