How Strong is the Love of Variety?*

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Abstract

Because of its tractability in general equilibrium framework, constant elasticity of substitution (CES) preference structure is widely employed in trade models of product differentiation. It assumes that the representative consumer *loves* variety in the sense that each additional variety is as valuable as the last. Consequently, the monopolistic competition model with CES preferences, while highly tractable, predicts that variety grows faster than observed in the data. The prediction implies large variety gains from trade liberalization for markets large or small. This paper develops a model that can generate the slower rate of variety growth seen in the data. It incorporates a more general, still tractable, CES preference structure that nests two extreme versions of trade models: Krugman (1980) and Armington (1969) style models. With limited love of variety the consumer faces a trade-off between buying more varieties or higher quantities per variety and; in equilibrium it yields a slower variety growth rate. The empirics confirm that consumer's *"love of variety"* is lower by 42% relative to the one assumed by commonly used CES and reject both Krugman's and Armington's model. In a simple symmetric welfare calculation, the *"love of variety"* estimates reduce variety gains by approximately 40% relative to the standard case.

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I. Introduction

First introduced in international trade theory by Krugman (1979, 1980), Dixit-Stiglitz (1977) monopolistic competition model is widely used in general equilibrium modeling of trade flows with product differentiation. In its standard form, the model employs constant elasticity of substitution (CES) preferences to gain tractability in general equilibrium framework. Consequently, it exhibits stark predictions on the number of varieties, prices and output per variety.

Krugman's monopolistic competition model assumes each country specializes in a range of varieties and predicts that variety expansion is proportional to country size. The prediction implies that larger economies export more only on the extensive margin (a greater range of varieties). But, the data suggests the extensive margin accounts for only two-thirds of greater exports of larger economies (Hummels and Klenow – 2002, 2005). Thus, the variety growth rate seems to be lower than the one predicted by the theory.

Alternatively, Armignton(1969)'s model, which dominates Computable General Equilibrium (CGE) analyses of trade policy, assumes varieties to be differentiated by country of origin (national product differentiation). In contrast with Krugman's model, in equilibrium, a larger country expands only through the intensive margin in the sense that it produces higher quantities of its variety sold at lower prices on the world market.

These predictions have important welfare implications. In Krugman's model, greater variety represents the only source of gains from trade liberalization. In contrast, in Armington's model, trade liberalization yields unfavorable terms of trade effects since the number of varieties cannot adjust (no variety gains). CGE models suggest that both terms of trade and variety gains are important consequences of trade liberalization. Thus, Armington's model may understate the gains from trade because it lacks the variety adjustment margin and Krugman's model may overstate them because it features no terms of trade effects.

This paper develops a model that can generate the slower rate of variety growth seen in the data. It incorporates a more general CES preference structure that nests the two extreme versions of trade models: Krugman's and Armington's model. In Krugman's model, varieties are differentiated by origin but also within each country. Any two varieties originating from a country are equally substitutable as any two varieties from different countries. In Armington's model each country produces one variety or the consumer perceives varieties originating from the same country as perfect substitutes. The general CES generalizes the elasticity of substitution across same country's varieties and its lower and upper bounds are the elasticity of substitution in Krugman's and Armington's model.

Intuitively, the consumer regards same country's varieties as more substitutable than varieties originating from different countries. Why are varieties more similar within a country? It could be country specific comparative advantage that makes a country's varieties more alike. For instance, French wine varieties are more similar to each other than to Chilean wine varieties because of country specific vineyard areas, vineyard techniques, technology for enhancing quality during fermentation, or ageing methods. Thus, the consumer has decreasing marginal valuation for same country's varieties. Put it another way, the general CES preference introduces flexibility in consumer's love of variety which attains its highest and lowest level in Krugman's and Armington's model.

A simple trade model shows that consumer's limited love of variety can slow down the variety growth rate. On the demand side, the consumer faces a trade-off between buying more varieties or higher quantities per variety. The elasticity of imports with respect to the number of varieties equals consumer's love of variety. In equilibrium, without factor price equalization, countries with higher GDP expand production of new varieties at a rate equal to consumer's love of variety. Intuitively, any level of consumer's love of variety lower than in Krugman's model limits the extent to which larger economies allocate their additional resources towards producing new varieties and thus they also produce and export higher quantities per variety at lower prices. But, for any level of consumer's love of variety higher than in Armington's model, the terms of trade effects are less adverse.

In the empirical work, I employ the "U.S. Imports of Merchandise" bilateral trade data for 1991-2004, which provides more commodity detail; and UN's COMTRADE data for 1999 with more geographic coverage to identify and estimate consumer's love of variety. The consumer's love of variety represents the elasticity of relative imports with respect to the extensive margin. The extensive margin represents the cross-section equivalent of the variety growth measure derived by Feenstra (1994). Building on Feenstra (1994)'s methodology, I derive the variety adjusted price index corresponding to the general CES and separate it into the extensive margin and traditional price index. The general CES variety adjusted price index nests the CES price index when the love of variety is the highest. The love of variety estimates reject both Krugman's and Armington's models and provide evidence for a hybrid trade model that blends together features of both models to reconcile the theory with empirical evidence.

This work relates and adds to two lines of research. First, my work relates to the literature that develops richer models of product differentiation that predict a slower variety growth rate. This literature assumes different preference structures characterized by variable price elasticity of demand: quadratic utility function (Ottaviano and Thisse - 1999, Ottaviano et. all - 2000) and the ideal variety approach (Lancaster- 1979, Hummels and Lugovskyy -

2005). A monopolistic competition model with variable price elasticity of demand predicts that the variety price decreases and the variety output increases in market size. Thus, the economy expansion takes place not only through the extensive margin, but also through the intensive margin yielding a less than proportional relationship between the number of varieties and country size. Variable price elasticity of demand makes these models harder to work with in a general equilibrium framework, and as a result there are only a few trade applications of these models.

Despite its stark features, CGE and economic geography models widely use CES preference structure to gain tractability in general equilibrium framework. The general CES preferences, while being tractable in general equilibrium, is more flexible than CES. This paper's approach maintains the tractability of CES preferences and generates the same predictions on the number of varieties, prices and output per variety as the models with variable price elasticity of demand do.

Second, my work builds on and adds to the literature that calibrates or estimates the welfare impact of import varieties in the CES framework. Feenstra (1994) shows that the consumer perceives the introduction of new varieties as a decrease in prices and thus the variety adjusted import price indexes are lower than the traditional price indexes. Broda and Weinstein (2004) estimates also the impact of new imported varieties on U.S. welfare and finds that greater product variety increased U.S. consumer's welfare by 3% of U.S. GDP from 1972 to 2001.

These results hinge heavily on modeling consumers' preferences using CES utility. CGE models find the use of CES preferences inappropriate because of the implausibly large variety gains as a result of trade liberalization and because of the "… potential instability into the markets, since the expansion of an industry via entry adds varieties and thus makes the product of the industry as a whole more desirable" (Brown, Deardorff and Stern - 1995).

This paper derives the variety adjusted price index corresponding to a general CES and estimates consumer's love of variety. The love of variety estimates can be introduced in computable general equilibrium (CGE) models to evaluate the variety effects of trade liberalization. A simple calibration in Appendix 2 shows that love of variety estimates could have a major impact on welfare calculations. Moreover, if product varieties are industrial inputs, a lower love of variety has strong implications for economic growth as well as on the strength of agglomerations.

The rest of the paper is organized as follows. Section II describes a simple trade model to illustrate how consumer's love of variety can explain the slower variety growth rate observed in the data. Section III uses the model structure to derive the equation taken to the data to estimate consumer's love of variety in section IV and V. Section VI concludes.

II. Diminishing returns to national varieties

This section describes a simple open economy model to illustrate how consumer's love of variety can explain the slower variety growth rate observed in the data. The model follows closely Hummels and Klenow (2002)'s setup and it represents a hybrid trade model of product differentiation nesting Krugman's and Armington's models as two extreme versions of trade models. Without price equalization, the hybrid model predicts that larger economies expand the production of new varieties at a rate equal to the consumer's love of variety. Intuitively, the consumer's limited love of variety puts an upper bound on a country's resource allocation towards producing more varieties and thus larger economies produce more varieties as well as higher quantities per variety at lower prices. The elasticity of the number of varieties with respect to GDP is lower than in Krugman's model and higher than in Armington's model.

2.1. Preference structure

The representative consumer's preferences are identical across countries and are represented by a nested general CES[1] utility function. First, the representative consumer allocates the income Y_i across all exporters and then she chooses the quantity per variety to import from each exporter. For a given product, the consumer perceives varieties as differentiated by firm as well as by national origin.

$$(2.1) \quad U_i = \left[\sum_j n_j^{\frac{\beta-1}{\sigma}} \left(\sum_{l=1}^{n_j} x_{jl}^{\frac{\sigma-1}{\sigma}} \right) \right]^{\frac{\sigma}{\sigma-1}}$$

Subject to $\sum_j \sum_{l=1}^{n_j} p_{jl} x_{jl} = w_i L_i = Y_i$; where w_i is workers' wage and L_i is the size of the labor force in country *i*.

The parameter $\sigma > 1$ represents the elasticity of substitution across varieties *l* exported by country *j*; x_{jl}, p_{jl} and n_j denote the quantity, prices per variety and number of varieties bought from country *j* (including from country i). The parameter $\beta \in [0,1]$ represents the consumer's *love of variety* – the marginal valuation of a variety.

[1] In the working paper of their seminal work, Dixit and Stiglitz(1975) proposed a general CES utility function that allows for different degrees of *"love of variety"* by introducing diversity multiplicatively as an externality into the CES utility function. In their specification the *love of variety* parameter takes negative or positive values. Positive/negative values of the preference for diversity parameter could be interpreted as diversity being a positive/negative externality or public good/bad. Other theoretical work used different forms of the general CES: Benassy(1996), Montagna(1999) and Either(1982). This specification of the general CES function comes from Brown, Deardorff and Stern(1995).

At the extremes, if $\beta = 1$ the consumer values equally varieties originating from an exporter and varieties from different exporters as in Krugman's model and if $\beta = 0$ the consumer values only varieties originating from different exporters as in Armington's model, while she perceives same country's varieties as perfect substitutes[2]:

Krugman: $U_i = \left[\sum_j n_j x_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$; Armington: $U_i = \left[\sum_j \left(n_j x_j \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$

Thus, the general CES utility generalizes the degree of substitutability across an exporter's varieties. The elasticity of substitution - σ - represents the substitutability across varieties originating from different countries. If $\beta = 1$, it also represents the substitutability across an exporter's varieties, but as β decreases towards 0, an exporter's varieties become perfect substitutes. To formalize it, the elasticity of substitution across same country's varieties can be written as a function of the love of variety parameter:

$\mu(\beta) : \mu : \ [0,1] \rightarrow [\sigma, \infty), \mu = \mu(\beta); \ \mu'(\beta) < 0$.

Put it another way, for any $\beta < 1$, the consumer regards same country's varieties as more substitutable than varieties originating from different countries. Why are varieties more similar within a country? It could be country specific comparative advantage that makes a country's varieties more alike. For instance, French wine varieties are more similar to each other than to Chilean wine varieties because of country specific vineyard areas, vineyard techniques, technology for enhancing quality during fermentation, or ageing methods.

[2] To illustrate better how the general CES nests Krugman's and Armington's preference structure, I assume that varieties originating from the same country are symmetric in quantities: $x_{jl} = x_j$

2.2. Demand

The demand for exporter *j*'s variety is[3]:

$$x_{jl} = \frac{p_{jl}^{-\sigma} n_j^{\beta-1}}{\sum_j n_j^{\beta-1}\left(\sum_{l=1}^{n_j} p_{jl}^{1-\sigma}\right)} Y_i \tag{2.2}$$

For $\beta = 1$ the demand becomes the CES demand. For any values of $\beta < 1$, the consumer faces a trade-off between the quantity per variety and the number of varieties imported. In other words, as varieties become less valuable at the margin than in the CES framework, the consumer would rather buy a higher quantity per variety than more varieties. For $\beta = 0$ an increase in the number of varieties is offset by a decrease in the quantity per variety. That is, the consumer becomes indifferent between buying more varieties or more per variety from an exporter as long as the total quantity stays the same.

Taking sum across all varieties exported by country *j* in (2.2) and rearranging, I obtain the relative imports from exporter *j* as a function of the general CES price indexes (i.e. the general CES minimum cost of obtaining one unit of utility from varieties *l* exported by country *j* or sold domestically by country *i*):

$$\frac{M_j}{M_i} = \left(\frac{n_j^{\frac{\beta-1}{1-\sigma}} \left(\sum_l p_{jl}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}{n_i^{\frac{\beta-1}{1-\sigma}} \left(\sum_l p_{il}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}} \right)^{1-\sigma} = \left(\frac{P_j}{P_i}\right)^{1-\sigma} \tag{2.3}$$

Assuming, for simplicity, all varieties originating from a country are symmetric in prices, the relative total demand becomes:

[3] In the rest of section 1, I drop the importer subscript *i* and whenever it appears it denotes the importing country as a domestic producer.

(2.4) $$\frac{M_j}{M_i} = \left(\frac{p_j}{p_i}\right)^{1-\sigma} \left(\frac{n_j}{n_i}\right)^{\beta}$$

The elasticity of relative imports with respect to the relative number of varieties equals the consumer's love of variety. An increase in the number of varieties exported by *j*, ceteris paribus, yields a less than proportional increase in relative imports for any $\beta < 1$.

2.3. Market equilibrium

Each firm incurs a marginal cost of production and a fixed cost of production ($\alpha > 0$):

(2.5) $l_j = \alpha + \frac{q_j}{A_j}$, where A_j - labor productivity

Each firm has monopoly power in its own market and the firm's profit maximization problem yields the solution for the price of each variety as a constant markup over marginal cost:

(2.6) $$p_j = \frac{\sigma}{\sigma - 1} \frac{w_j}{A_j}.$$

For simplicity, I assume no transport costs or fixed costs of exporting and symmetry in prices of an exporter's varieties; and thus the *zero-profit condition* for each exporter yields the supplied quantity per variety:

(2.7) $$q_j = \frac{\alpha(\sigma - 1)}{w_j / A_j}$$

From (2.6) and (2.2) it follows:

(2.8) $\frac{p_j}{p_i} = \frac{w_j / A_j}{w_i / A_i}$; (2.9) $\frac{x_j}{x_i} = \left(\frac{p_j}{p_i}\right)^{-\sigma} \left(\frac{n_j}{n_i}\right)^{\beta - 1}$

Equation (2.9) represents the inverse relative general CES demand for each country's variety. For $\beta = 1$, the relative quantities demanded depend only on variety prices, i.e. inverse relative CES demand. For any value of $\beta < 1$, the relative quantities demanded depend on variety prices but also on the number of varieties in the market. Everything else equal, the relationship reflects the trade-off the consumer faces between buying higher quantities per variety or more varieties. The trade-off represents the novelty introduced in the model by the general CES.

Using (2.8) and (2.9), the *market clearing condition for each variety* $\left(x_j/x_i = q_i/q_j\right)$ gives:

(2.10) $$\left(\frac{n_j}{n_i}\right)^{1-\beta} = \left(\frac{w_j}{w_i}\right)^{1-\sigma}\left(\frac{A_j}{A_i}\right)^{\sigma-1}$$

Intuitively, as the number of varieties increases the quantity demanded per variety decreases at a rate depending on consumer's love of variety but the quantity supplied per variety has to verify the zero profit condition. Thus, new varieties enter until the quantity demanded equals quantity supplied. For higher values of β, the quantity demanded per variety decreases at a lower pace and thus more varieties enter until it equals the quantity supplied.

The *full employment condition* yields the quantity supplied per variety as a function of labor force size (i.e. inverse labor supply for each variety):

(2.11) $$L_j = n_j l_j = n_j\left(\alpha + \frac{q_j}{A_j}\right) \Rightarrow q_j = A_j\left(\frac{L_j}{n_j} - \alpha\right)$$

The *labor market clearing condition for each variety* $\left(x_j/x_i = q_i/q_j\right)$ using inverse labor supply (2.11) yields:

$$(2.12)\quad \frac{w_j}{w_i} = \left(\frac{L_j}{L_i}\right)^{-\frac{1}{\sigma}} \left(\frac{A_j}{A_i}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{n_j}{n_i}\right)^{\frac{\beta}{\sigma}}$$

Plugging (2.10) into (2.12) yields the labor demand equation:

$$(2.13)\quad \frac{w_j}{w_i} = \left(\frac{L_j}{L_i}\right)^{\frac{\beta-1}{\sigma-\beta}} \left(\frac{A_j}{A_i}\right)^{\frac{\sigma-1}{\sigma-\beta}}$$

Equation (2.13) suggests that the slope of the relative labor demand is increasing in β.

In a comparison between a large and a small country, for $\beta = 1$, the relative wage reflects only their productivity differences and not their labor force sizes (i.e. $w_j/w_i = A_j/A_i$). For $\beta = 0$, it depends both on productivity differences and labor force sizes ($w_j/w_i = \left(L_j/L_i\right)^{-\frac{1}{\sigma}} \left(A_j/A_i\right)^{\frac{\sigma-1}{\sigma}}$).

Figure 1 illustrates the relative wage determination as a function of love of variety. The lower is consumer's love of variety the lower the wage. Intuitively, lower β slows down the variety growth rate and increases the quantity per variety produced. Higher quantities can be sold at lower prices and thus the value of marginal product of labor decreases, yielding lower wages.

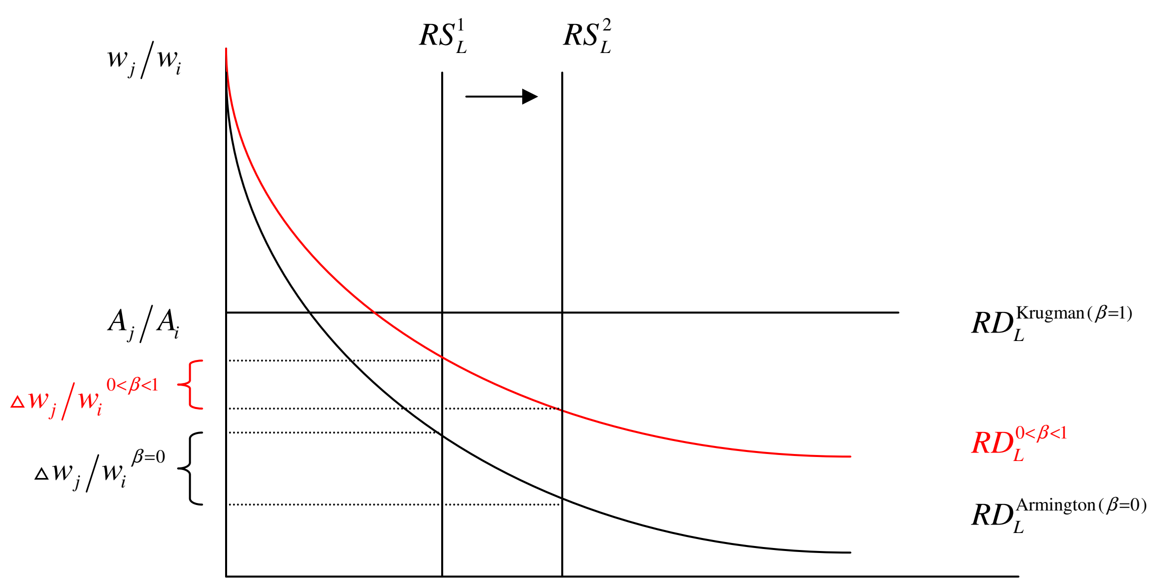

Figure 1: Relative wage determination

L_j/L_i

The terms of trade is a crucial mechanism in this model. Acemoglu and Ventura(2002) use the same mechanism in a model with $\beta = 1$, endogenous capital accumulation and fixed labor endowment. In their model, the production of each variety uses a fixed labor requirement and it features constant returns to capital. Since the fixed cost of production is in terms of the scarce factor, as countries accumulate more capital, the number of varieties is bounded above by the labor endowment. Thus, larger countries produce also higher quantities per variety and they face adverse terms of trade effects. In the limited love of variety model the number of varieties is bounded above by consumer's marginal valuation for a variety.

The relative GDPs are:

$$(2.14)\quad \frac{Y_j}{Y_i} = \frac{w_j L_j}{w_i L_i} = \left(\frac{A_j}{A_i}\right)^{\frac{\sigma-1}{\sigma-\beta}} \left(\frac{L_j}{L_i}\right)^{\frac{\sigma-1}{\sigma-\beta}}$$

Using (2.13) and (2.14) into (2.8), the relative variety prices and quantities are:

$$(2.15)\quad \frac{p_j}{p_i} = \left(\frac{Y_j}{Y_i}\right)^{\frac{\beta-1}{\sigma-1}};\quad \frac{x_j}{x_i} = \left(\frac{p_j}{p_i}\right)^{-\sigma} = \left(\frac{Y_j}{Y_i}\right)^{\frac{(\beta-1)(-\sigma)}{\sigma-1}}.$$

That is, a larger country produces and exports higher quantities at lower prices with an elasticity decreasing in β. Furthermore, the *trade balance condition*: $n_j p_j x_j / n_i p_i x_i = Y_j / Y_i$ pins down the relative number of traded varieties:

$$(2.16)\quad \frac{n_j}{n_i} = \left(\frac{Y_j}{Y_i}\right)^{\beta}.$$

Consumer's love of variety determines the rate at which a larger country produces and exports more varieties. This relationship nests Krugman for $\beta = 1$ and thus the variety growth rate is proportional to country size. Also, for $\beta = 0$ (Armington model), a larger country produces no more varieties than a smaller country (see Figure 2).

Table 1 provides a summary of hybrid model's predictions and a comparison to Krugman and Armington predictions on the number of imported varieties, quantity and price per variety. In Krugman's model, countries with higher GDP and labor force size export more varieties with no terms of trade effects. In contrast, Armington's model shuts down the variety expansion channel and predicts that countries with higher GDP and with more workers export higher quantities at lower prices and thus face adverse terms of trade effects.

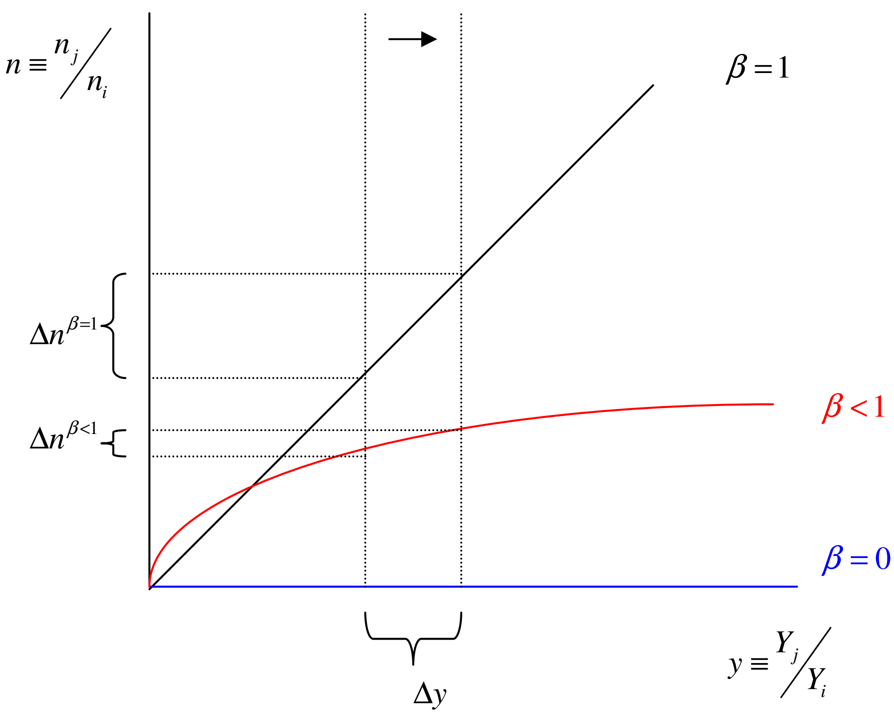

Figure 2: ***The relative GDP and number of exported varieties***

The model with limited love of variety predicts that larger countries produce and export more varieties as well as higher value per variety. The love of variety represents the elasticity of number of varieties with respect to GDP. In the limited love of variety model the strength of the terms effects is lower than in Armington's model for any $\beta > 0$. That is, if the consumer values varieties at the margin then larger economies export more varieties and they face less adverse terms of trade effects for each variety.

Table 1: The elasticity of *Z* with respect to country's GDP and labor force size (L)

Z	**Krugman** $(\beta=1)$		**Limited LoV**		**Armington** $(\beta=0)$	
	GDP	*L*	*GDP*	*L*	*GDP*	*L*
Number of varieties	1	1	β	$\beta\frac{\sigma-1}{\sigma-\beta}$	0	0
Export value per variety:	0	0	$(1-\beta)$	$(1-\beta)\frac{\sigma-1}{\sigma-\beta}$	1	$\frac{\sigma-1}{\sigma}$
- quantity per variety	0	0	$(1-\beta)\frac{\sigma}{\sigma-1}$	$\sigma\frac{1-\beta}{\sigma-\beta}$	$\frac{\sigma}{\sigma-1}$	1
- price per variety	0	0	$-\frac{1-\beta}{\sigma-1}$	$-\frac{1-\beta}{\sigma-\beta}$	$-\frac{1}{\sigma-1}$	$-\frac{1}{\sigma}$

The hybrid model predictions match several features of the data[4]. It predicts a less than proportional export extensive margin and export intensive margin with respect to labor force size as well as GDP. Larger economies export higher quantities per variety but with a lower elasticity with respect to labor force size and GDP than in Armington's model. This paper's model fails to explain the variety price facts observed in the data. The model can match these facts if larger countries improve their technologies for producing each variety (the model assumes that a country's technology level is exogenous). Moreover, the model lacks the import extensive margin, but introducing the fixed costs of exporting together with variable trade costs could easily generate it. Acemoglu and Ventura (2002)'s model features also only an export extensive margin. It has the same predictions on the intensive margin but it predicts that the number of varieties is proportional to the country's employment and constant with respect to

[4] Hummels and Klenow (2002, 2005)

its GDP. Thus, the limited love of variety model can match better the empirical facts on the export extensive margin.

III. Empirical model

Next, I structurally identify and estimate consumer's love of variety and test whether it is lower than the one implicitly assumed in Krugman's model. Following the model described in the previous section, an obvious identification would relate the relative number of varieties to the relative GDP. However, deriving this relationship required imposing some strong assumptions: symmetry in prices and quantities across an exporter's varieties, no ad-valorem trade costs or fixed costs of exporting and an inelastic labor supply. Under weaker assumptions, I structurally identify consumer's love of variety as the elasticity of imports with respect to the number of varieties by estimating a difference-in-difference relative import demand (i.e. a difference-in-difference equivalent of (2.3)):

$$(3.1)\qquad \frac{M_j}{M_k} = \left(\frac{n_j^{\frac{\beta-1}{1-\sigma}} \left(\sum_l p_{jl}{}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}}{n_k^{\frac{\beta-1}{1-\sigma}} \left(\sum_l p_{kl}{}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}} \right)^{1-\sigma} = \left(\frac{P_j}{P_k} \right)^{1-\sigma} \equiv \left(P_{jk} \right)^{1-\sigma}$$

The difference-in-difference import demand helps to control for domestic varieties by dividing both the relative demand for exporter *j*'s varieties and the relative demand for exporter *k*'s varieties to domestic varieties. The logarithm of relative import demand as given by (3.1) is non-linear in the number of varieties and thus requires burdensome estimation techniques. This section uses Feenstra (1994)'s methodology to log-linearize the relative import demand by decomposing the relative general CES price index into a price component and a number of varieties component.

3.1. Price index decomposition

The CES price index P_{jk} (i.e. variety-adjusted price index) can be decomposed in two components, the *traditional price index* $\widetilde{P}_{jk}$ and *extensive margin* (i.e. a weighted count of the number of varieties) following Feenstra (1994)'s methodology. The methodology separates the extensive margin and the traditional price index without assuming that an exporter's varieties have equal prices and quantities. Feenstra (1994) shows the consumer perceives the introduction of new varieties as a decrease in prices such that the CES price index decreases in the number of varieties. The more substitutable varieties are, the lower impact they have on the price index.

If the set of varieties is the same across exporters (*j* and *k*), the cross section equivalent of the CES price index equals the traditional price index and can be written as[5]:

$$(3.2) \qquad \widetilde{P}_{jk} = \prod_{l\in I}\left(\frac{p_{jl}}{p_{kl}}\right)^{\omega_{jl}(I)}$$

$$s_{rl}(I) \equiv \frac{p_{rl}x_{rl}}{\sum_{l\in I} p_{rl}x_{rl}} \; for \;\; r = j,k\,; \qquad \omega_{jl}(I) \equiv \frac{\left(\dfrac{s_{jl}(I)-s_{kl}(I)}{\ln s_{jl}(I)-\ln s_{kl}(I)}\right)}{\sum_{l\in I}\left(\dfrac{s_{jl}(I)-s_{kl}(I)}{\ln s_{jl}(I)-\ln s_{kl}(I)}\right)}$$

The weights used in constructing the price index are the logarithmic mean of the cost shares of each variety *l* in country *j*'s exports. But, the traditional price index is not appropriately defined if the set of varieties varies across exporters. For a pair of countries, there are some varieties in

[5] Sato(1976) and Vartia(1976)

the common set (I) and some varieties outside the common set. In this case, the traditional price index needs to be adjusted by the relative share of varieties outside the common set. The construction of the variety-adjusted price index requires two conditions. First, exporter j and k should export at least one common variety ($I \neq \varnothing$). Second, the varieties in the common set should be identical such that the relative variety prices in (3.2) are meaningful. That is, any demand shifter should affect proportionally the varieties originating from different countries in the common set.

Proposition 1 formalizes the extension of Feentra (1994)'s methodology for decomposing the general CES price index.

Proposition 1:[6] If $b_{jl} = b_{kl}$ for $l \in I \subseteq (I_j \cap I_k),\;\; I \neq \varnothing$, then the general CES price index can be written as $P_{jk} = \widetilde{P}_{jk} \left(\frac{\lambda_j}{\lambda_k} \right)^{\frac{\beta}{1-\sigma}}$

where b_{jl}, b_{kl} - unobservable demand shifters and

$$(3.3) \quad \lambda_r \equiv \frac{\sum_{l \in I} p_{rl} x_{rl}}{\sum_{l \in I_r} p_{rl} x_{rl}} \quad for \quad r = j,k \;\; \square$$

I define the extensive margin as:

$$(3.4) \qquad EM_{jk} \equiv \frac{\lambda_j}{\lambda_k} = \frac{\sum_{l \in I_j} p_{jl} x_{jl} \Big/ \sum_{l \in I} p_{jl} x_{jl}}{\sum_{l \in I_k} p_{kl} x_{kl} \Big/ \sum_{l \in I} p_{kl} x_{kl}}$$

If the set of varieties imported from j is a subset of the set of varieties imported from k ($I = I_j$), then the extensive margin simplifies to:

[6] The proof of the proposition can be found in appendix 1.

(3.5) $$EM_{jk} = \frac{\sum_{l \in I_j} p_{kl} x_{kl}}{\sum_{l \in I_k} p_{kl} x_{kl}}$$

And, the variety-adjusted price index can be written as follows:

(3.6) $$P_{jk} = \left(EM_{jk} \right)^{\frac{\beta}{1-\sigma}} \widetilde{P}_{jk}$$

The extensive margin of country *j* represents the weighted count of varieties relative to exporter *k*'s varieties. The varieties are weighted by their importance in *k*'s exports. If I assign equal weight to each variety, the extensive margin represents the simple count of varieties exported by *j* to an importer as a share of the number of varieties exported by *k*.

To sum up, Proposition 1 extends Feenstra (1994)'s methodology of constructing the variety-adjusted price index for any values of $\beta \in [0,1]$. In the extension, the new varieties lower the price index at a rate that depends on both σ and β. A lower love of variety, ceteris paribus, dampens the effect of new varieties on the price index. That is, if the consumer values new varieties less at the margin, they have a lesser effect on the price index.

3.2. Relative import demand with asymmetric varieties

Using decomposition (3.6) I can re-write equation (3.1) as:

(3.7) $$\frac{M_j}{M_k} = \left(EM_{jk} \right)^{\beta} \left(\widetilde{P}_{jk} \right)^{1-\sigma}$$

The observed relative bilateral imports are a function of relative bilateral variety-adjusted price indexes. Equation (3.7) is the asymmetric equivalent of (2.4). An increase in the number of imported varieties acts in the same way as a decrease in prices: it will draw resources towards the exporter's products and the higher is the love of variety the larger will be the shift.

The love of variety parameter represents the elasticity of relative imports with respect to extensive margin:

$$(3.8)\ \frac{\partial M_j / M_k}{\partial EM_{jk}} \frac{EM_{jk}}{M_j / M_k} = \beta$$

The price elasticity of demand remains $1-\sigma$ as in the standard CES framework. The empirical analysis structurally identifies and estimates the love of variety parameter by taking (3.7) to the data.

IV. Estimation and results

In this section I structurally estimate U.S.'s love of variety using (3.7) for each product.

4.1. Data and estimation

I use U.S. data from the "U.S. Imports of Merchandise" CD-ROM for the period 1991-2004, published by the U.S. Bureau of the Census. The dataset contains U.S. imports collected from electronically submitted Customs forms, covering an average of 223 exporters and commodity detail at 10 digit level Harmonized System (HS) classification. The data includes country of origin, value, quantity, freight and duties paid.

Identifying and estimating the love of variety by exploiting the time series variation in the U.S. data has some advantages. The U.S. data provides detailed information on trade costs. Also, it is more disaggregated at the commodity level which allows a finer measurement of "unique" products.

The empirical implementation defines a product as a 2 digit level HS category (denoted by *h*) and a variety as a 10 digit level HS category (denoted by *l*) within a 2 digit level HS category. For each U.S. – trade partner data point in a HS 2 category for a year, I construct the relative imports, extensive margin and prices according to the decomposition methodology outlined in section 3.1. I choose the ROW (rest-of-the-world) as the comparison country *k*. That is to say, the comparison country consists of all the exporters other than *j* taken together. The ROW is a convenient comparison country because I can exploit all the information available in the data. An additional advantage of using ROW is that, conditional on a time period, the common set of varieties between any exporter *j* and ROW is the set of HS 10 categories exported by *j*. This property allows a more intuitive construction of the extensive margin (i.e. a weighted count of varieties) as in (3.5) which weighs each variety with its ROW trade value.[7]

The price index $\widetilde{P}_{jkt}$ can be written as (where *t* indexes time periods):

(4.1) $\widetilde{P}_{jkt} = \underbrace{\prod_{l \in I_{jt}} \left(\frac{\tau_{jtl}}{\tau_{ktl}} \right)^{\omega_{jtl}(I_{jt})}}_{\tau_{jkt}} \underbrace{\prod_{l \in I_{jt}} \left(\frac{p_{jl}}{p_{kl}} \right)^{\omega_{jtl}(I_{jt})}}_{\widetilde{P}_{jk}^{FOB}}$. And, equation (3.7) in logarithmic scale becomes:

(4.2) $$\log \frac{M_{jt}}{M_{kt}} = \beta \ \log EM_{jkt} + (1-\sigma)\log \tau_{jkt} \ + (1-\sigma)\log \widetilde{P}_{jk}^{FOB}$$

I measure the relative trade costs (τ_{jkt}) using ad-valorem trade costs (i.e. 1+ the share of duties and freight paid in the import value) for each HS 10. For each product, the ROW trade costs represent a weighted average of trade costs, where the weights are the share of each exporter's variety into the ROW exports to U.S. for each time period. I include an exporter fixed effect (implemented by mean-differencing) to capture the relative fob variety prices. Thus, the estimating equation for each product *h* becomes:

---

[7] The choice of ROW as a comparison country was inspired by Hummels – Klenow (2005).

(4.3) $\log IMPSHR^{h}_{jkt} = \delta^{h}_{j} + \beta_{h} \log EM^{h}_{jkt} + (1-\sigma_{h}) \log \tau^{h}_{jkt} + \varepsilon^{h}_{jkt}$

The extensive margin varies across exporters because of exporter's size (as shown in the model presented in section II) or because of other reasons outside the model such as trade costs combined with fixed costs of exporting. The love of variety estimation exploits these sources of variation. By estimating a specification in relative terms, the time-shifters common to all exporters such as importer's market size are differenced out.

I estimate β_h with the null hypothesis that $\beta_h = 1$ and I seek to reject the null in order to show that $\beta_h < 1$. I expect the elasticity of substitution between varieties to be greater than one ($\sigma_h > 1$) and with the magnitudes in line with previous results obtained in the literature. The love of variety estimate measures the degree to which the U.S. values new varieties.

Before I proceed to results, a discussion on the variable construction is in order. The decomposition of the variety-adjusted price indexes into extensive margin and price component requires the existence of a common set of varieties between exporter *j* and *k*. Theoretically a variety in the common set features an equal unobservable demand shifter for both exporters which can be interpreted as the same number of hidden varieties, the same quality or taste parameter. Previous studies (Hummels-Klenow- 2005, Broda and Weinstein - 2004) have empirical defined variety at different level of data aggregation imposed by data availability. In the empirical implementation, I define the common set of varieties as the set of HS 10 categories within a HS 2 category in which both exporters have positive exports to a given importer.

This could represent a mis-measurement problem if there are multiple hidden varieties within each HS 10 category. But, in the paper's specification, this issue does not represent a problem if the relative number of hidden varieties is proportional to the relative number of observed varieties. I can use the US data to test the statement. Consider that HS 10 categories represent the hidden varieties within an observed HS 6 category. For each HS 2 category, the following is true: $n^j_{\text{HS10}} / n^k_{\text{HS10}} = n^j_{\text{HS10/HS6}} / n^k_{\text{HS10/HS6}} * N^j_{\text{HS6}} / N^k_{\text{HS6}}$, where n^j_{HS10}, $n^j_{\text{HS10/HS6}}$ and N^j_{HS6} represent the number of HS 10 categories within an HS 2, the number of HS 2 categories within an HS 6 category and the number of HS 6 category within an HS 2 exported by j. Testing whether varieties defined at HS 6 level in the common set feature the same number of hidden varieties (i.e. $n^j_{\text{HS10/HS6}} / n^k_{\text{HS10/HS6}} = 1$) is equivalent to testing whether the relative number of hidden varieties ($n^j_{\text{HS10}} / n^k_{\text{HS10}}$) is proportional to relative number of observed varieties ($N^j_{\text{HS6}} / N^k_{\text{HS6}}$). Figure 7 confirms that hidden varieties do not represent problem in the specification in relative terms and the deviations from the 45 degree line are captured by exporter fixed effects.

Also, I argue that the estimates could reveal the extent to which this problem may be of concern. If the common set of varieties is inappropriately defined (and as a result the extensive margin is inappropriately defined), I should observe a shift in the relative product demand instead of a movement along. In other words, the elasticity of substitution estimate becomes less than unity. The results suggest that this issue is not of concern.

4.2. Results

I estimate specification (4.3) for each product. Pooling across products restricts the elasticity of substitution to be equal across products which based on the estimates in the literature is clearly a strong assumption (Hummels -1999 and Broda and Weinstein- 2004). Thus, I consider product regressions results more reliable.

The results by product are summarized by figure 3 and 4. Table 2 and 3 provides a summary of the estimates. All U.S. $\widehat{\beta_h}$ are significantly lower than one. The estimates show that, on average, a 10% increase in the extensive margin leads to 4% increase in relative bilateral imports. 99% of $\widehat{\sigma_h}$ are significantly different than unity at 5% level with a weighted average of 5.33, in line with the estimates in the literature. The magnitudes of $\widehat{\sigma_h}$ suggest that the mis-measurement in variables due to hidden variety does not introduce a bias in the estimates.

V. Cross-importer love of variety

In this section, I use cross-importer variation that provides more geographic coverage than U.S. data but it is less disaggregated at the commodity level and it has no data on trade costs. Despite of these limitations, the estimates provide further support for this paper's conjecture that the consumer values less new varieties than the monopolistic competition with CES assumes.

5.1. Data and estimation

I use data from UN's COMTRADE data for 1999. The COMTRADE data was obtained through UNCTAD/ World Bank WITS data system, which yields bilateral import data collected by the national statistical agencies of 143 importing countries, covering 224 exporters and 5015 6 digit level Harmonized System (HS) classification categories. After merging it with great circle distance data, I obtain a dataset covering 132 importers and 185 exporters for a total of 4,328,408 data points.

I define a product as a 2 digit level HS category (denoted by *h*) and a variety as a 6 digit level HS category (denoted by *l*) within a 2 digit level HS category. For each bilateral pair in each HS 2 category, I construct the relative imports, extensive margin and prices according to the decomposition methodology outlined in section 3.1.

Since detailed data for trade costs is not readily available for many importers, I use distance as a crude proxy for trade costs. I model trade costs as:

$$(4.4) \qquad \tau_{ijl} = t_{il} * \left(d_{ij}\right)^{\gamma}$$

where t_{il} represents the ad-valorem tariff and d_{ij} represents the distance between the pair of countries *i* and *j*. Conditional on an importer, the ad-valorem tariff for a variety can be safely assumed to be equal across exporting countries (Hummels and Lugovskyy - 2005). The price index becomes:

$$(4.5) \qquad \widetilde{P}_{ijk} = \prod_{l \in I_{ij}} \left(\frac{d_{ij}}{d_{ik}} \right)^{\omega_{ijl}(I_{ij})\gamma} \underbrace{\prod_{l \in I} \left(\frac{p_{jl}}{p_{kl}} \right)^{\omega_{ijl}(I_{ij})}}_{\widetilde{P}_{jk}^{FOB}}$$

where d_{ik} represents the weighted average distance of ROW exports to country *i*, the weights being the share of each trade partner in world trade.

The estimating equation represents the cross-section equivalent of (4.3) in which I substitute the time subscript (*t*) for the importer subscript (*i*):

$$(4.6) \qquad \log IMPSHR_{ijk}^{h} = \delta_{j}^{h} + \beta_{h} \log EM_{ijk}^{h} + \gamma(1-\sigma_{h}) \log d_{ijk}^{h} + \varepsilon_{ijk}^{h}$$

The specification includes exporter fixed effects (δ_j^h) common to all importers that capture the exporters' fob variety prices. Note that importer specific factors common to all exporters such as market size and income are differenced out by estimating a specification in relative terms. Note also that I cannot separately identify the elasticity of substitution (σ) from the elasticity of transport costs with respect to distance (γ). Hummels (2001) provides estimates of the elasticity of transport costs with respect to distance ($\hat{\gamma} = 0.26$) which can be used to back out the elasticity of substitution estimates. Recall that the estimates of σ can indicate whether the mis-measurement in variables induced by hidden varieties represents a concern for the consistency of love of variety estimates.

As for U.S., I estimate β_h with null hypothesis that $\beta_h = 1$ and I seek to reject the null to show that $\beta_h < 1$. I expect the elasticity of substitution across varieties to be greater than one ($\sigma_h > 1$) and with the magnitudes in line with previous results obtained in the literature. The love of variety parameter measures the degree to which importers value an exporter's varieties.

5.2. Results

I estimate specification (4.6) for each product.

The results are summarized by figure 5 and figure 6. Table 2 and 3 provide summary statistics of the estimates. All $\widehat{\beta_h}$ are significantly lower than one. The estimates show that, on average, a 10% increase in the extensive margin leads to 5.6% increase in relative bilateral imports. All the price elasticity of demand estimates ($(1-\hat{\sigma}_h)\hat{\gamma}$) are negative and significant at 5% level. Moreover, the average of $\widehat{\sigma_h}$ [8]'s is 3.79 which suggest that the mis-measurement in variables caused by hidden variety does not introduce a bias in the estimates.

[8] calculated using $\hat{\gamma} = 0.26$ (Hummels - 2001)

VI. Conclusion

This paper describes a simple trade model which incorporates a more general CES preference structure that nests Krugman's and Armington's model. The model illustrates how consumer's limited love of variety can explain the slower variety growth rate observed in the data. The general CES introduces a trade-off that the consumer faces between buying more varieties or higher quantities per variety. In equilibrium, without factor price equalization, a larger country exports more varieties at a rate equal to consumer's love of variety and higher quantities per variety sold at lower prices on the world markets. For any values of the love of variety lower than in Krugman's model, the variety expansion is less than proportional to country size as observed in the data. Introducing a more general CES preference structure in a monopolistic competition model matches better the empirical facts while still remaining tractable in general equilibrium.

The empirics structurally identify and estimate consumer's love of variety as the elasticity of relative imports to extensive margin and find that it is lower than the one assumed by commonly used CES. Consumer's limited love of variety has important implications for welfare calculations. A simple calibration in Appendix 2 shows that a love of variety estimate of 0.6, ceteris paribus, reduces the variety gains from trade liberalization by 40%. Moreover, if product varieties are industrial inputs, a lower love of variety has strong implications for economic growth as well as on the strength of agglomerations.

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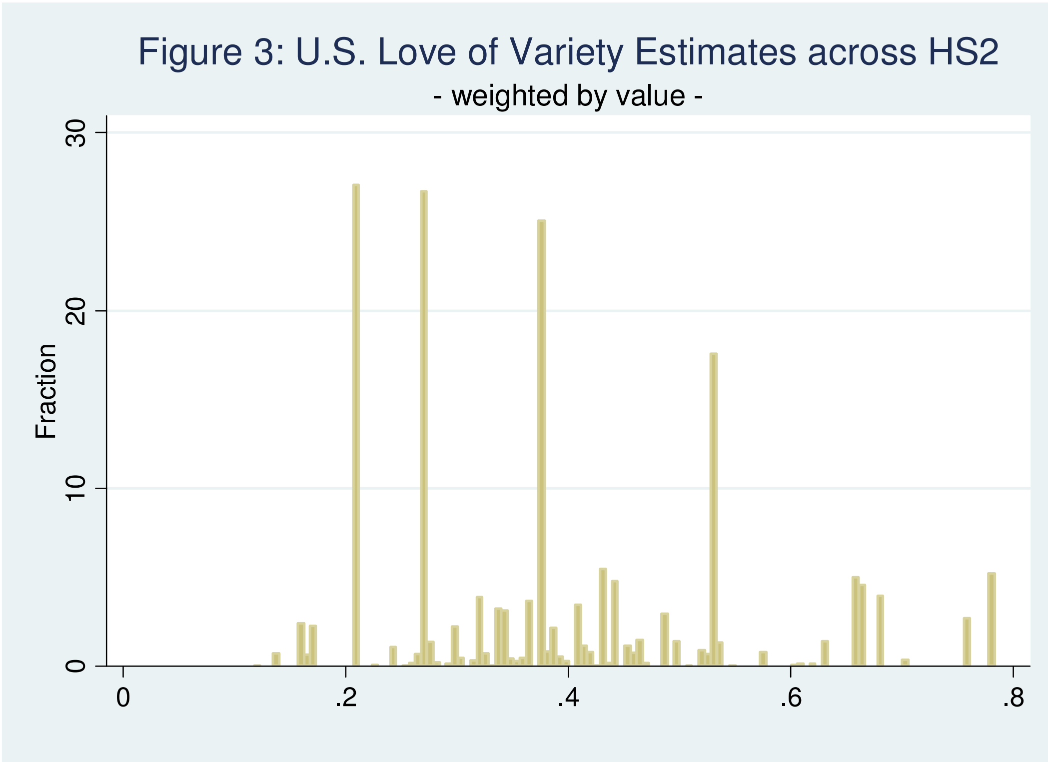
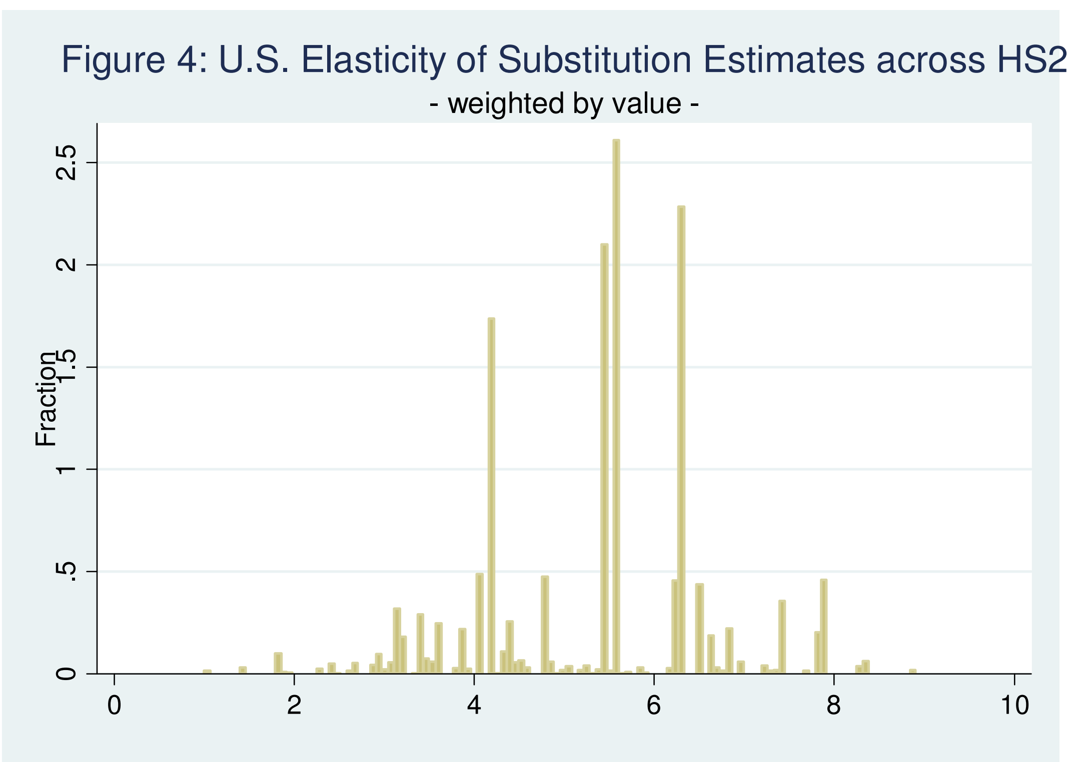
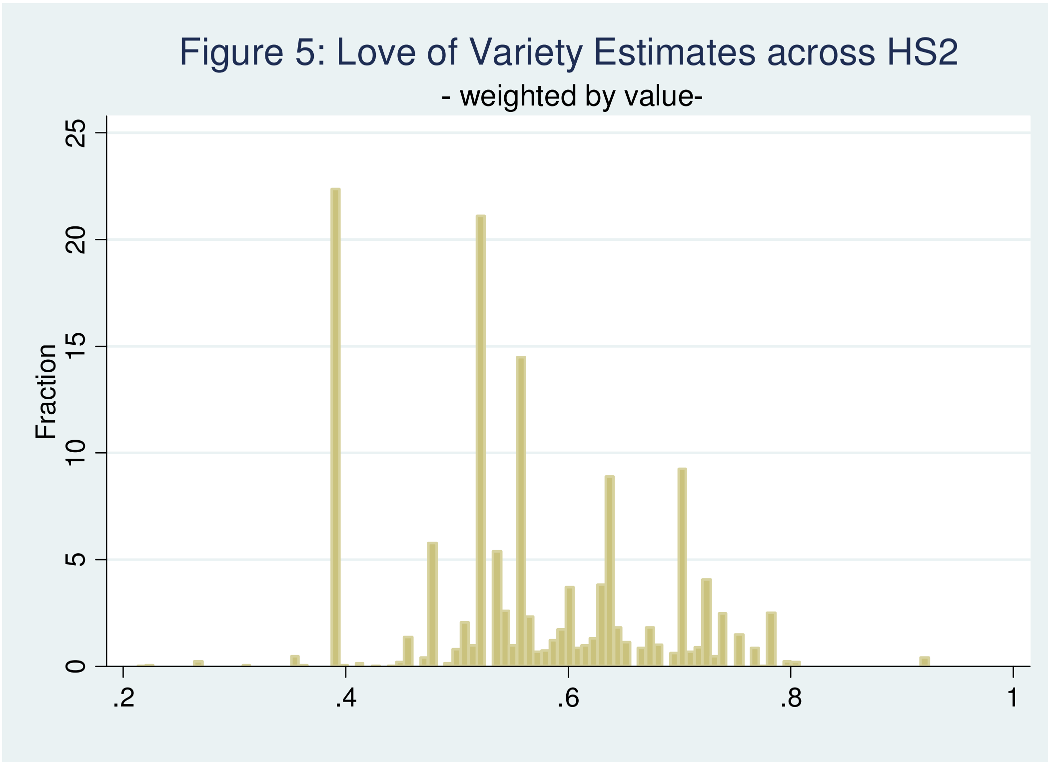
U.S. Love of Variety and Elasticity of Substitution Estimates across Products





Note: The weight represents the average HS 2 import value across 1991-2004.

Cross-importer Love of Variety and Elasticity of Substitution Estimates across Products



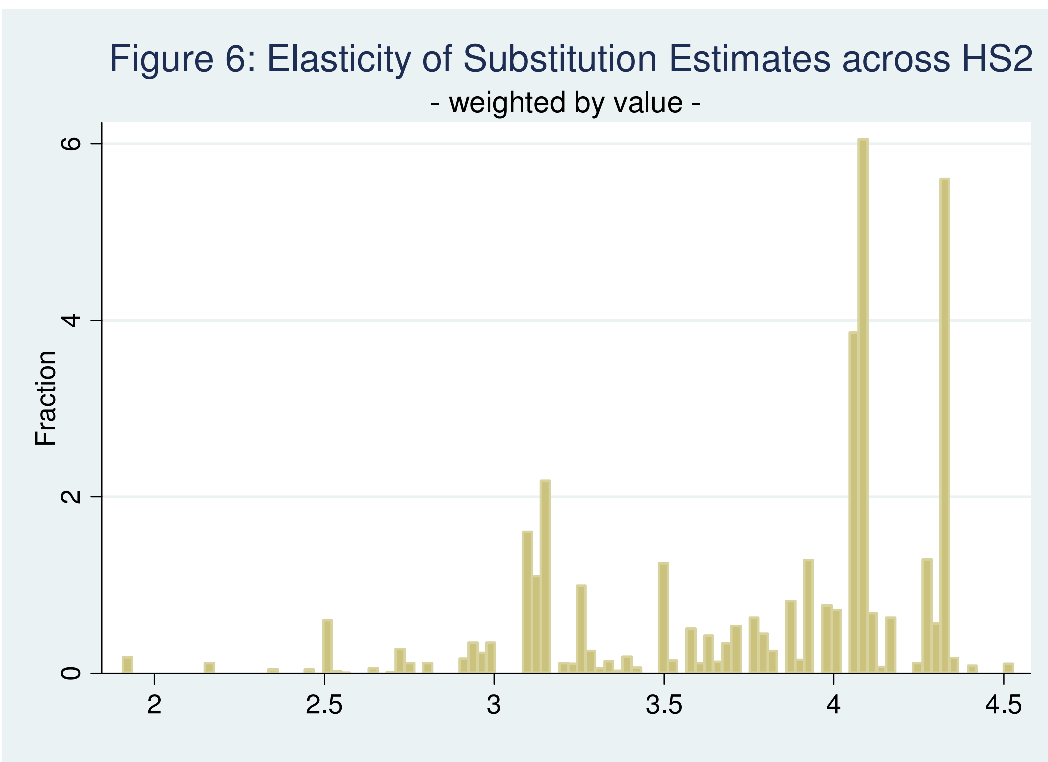


Table 2. Love of Variety Estimates by HS 2
Summary Statistics

Specification	Weighted Mean	Simple Mean	Std. Deviation	Min.	Max.
U.S.	0.4	0.41	0.14	0.12	0.78
Cross-importer	0.56	0.58	0.13	0.21	0.91

Notes: 1.The cross-importer estimates are weighted by the world trade value of each HS 2 category
2. The U.S.estimates are weighted by the average HS 2 trade value across 1991-2004
3. All estimates significantly different from one.

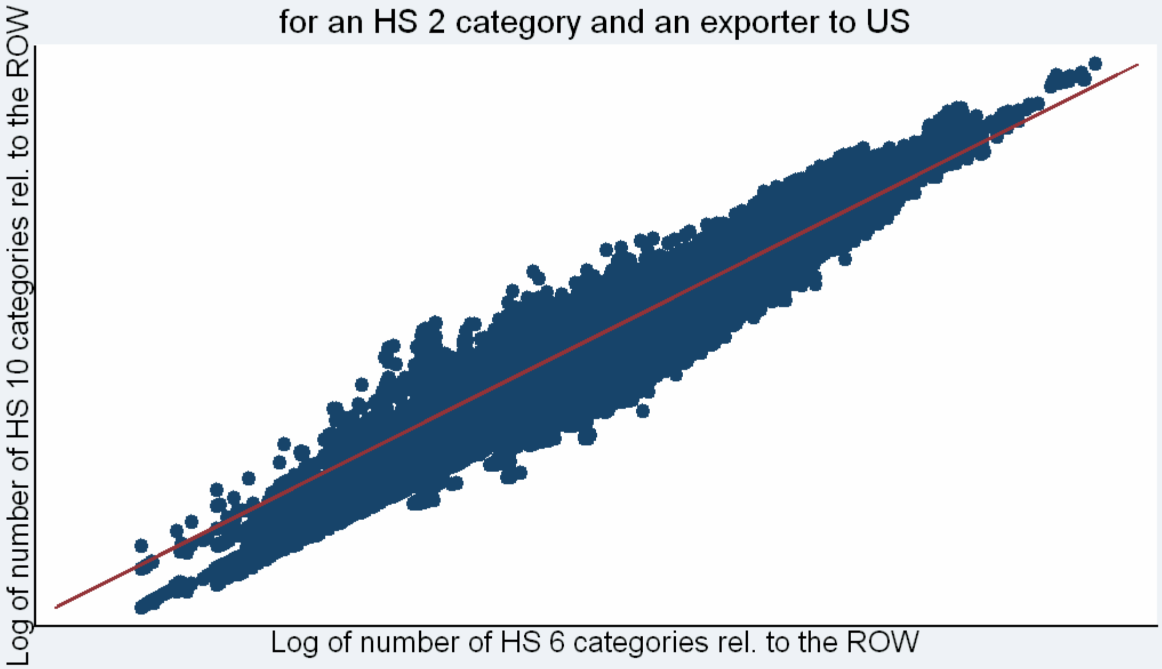
Table 3. Elasticity of Substitution Estimates by HS 2
Summary Statistics

Specification	Weighted Mean	Simple Mean	Std. Deviation	Min.	Max.
U.S. *(using trade costs)*	5.33	4.68	1.7	1.2	8.88
Cross-importer *(using distance)*	3.79	3.42	0.58	1.9	4.5

Notes: 1. The cross-importer estimates are weighted by the world trade value of each HS 2 category
2. The U.S. estimates are weighted by the average HS 2 trade value across 1991-2004
3. 99% of U.S. estimates are significantly different from one at 5% level
4. The cross-importer estimates are calculated using the estimate of elasticity of trade costs with respect to distance of 0.26 (Hummels - 2001)

Figure 7: Log of number of HS 10 categories rel. to the ROW and log of number of HS 6 categories rel. to the ROW

for an HS 2 category and an exporter to US



The line is the 45 degree line

Appendix 1. Price index decomposition[9]

The general CES utility function:

$$(5.1)\qquad U = n_j^{\frac{\beta-1}{\sigma-1}} \left(\sum_{l \in I_j} b_{jl}^{\frac{\beta}{\sigma}} x_{jl}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

The minimum cost of obtaining one unit of utility from varieties *l* of a product corresponding to the above utility function:

$$(5.2)\qquad P_j = n_j^{\frac{1-\beta}{1-\sigma}} \left(\sum_{l \in I_j} b_{jl}{}^{\beta} p_{jl}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

where σ is the elasticity of substitution between varieties and $I_j = \{1,..., N_j\}$ is the set of imported varieties from country *j* with the quantity per variety $x_{jl} > 0 \quad \forall\, l \in I_j$, prices $p_{jl} > 0 \;\; \forall l \in I_j$ and the unobservable demand shifter $b_{jl} > 0$.

This setup is equivalent to Feenstra(1994)'s when $\beta = 1$ corresponding to the upper bound of the "love of variety" parameter. I preserve Feenstra(1994)'s notation for the minimum cost of obtaining one unit of utility from varieties *l* of a product when $\beta = 1$ with lower case *c*. In the following, I extend the price index decomposition derived by Feenstra(1994) to allow for different degrees of preference for variety.

First, I define the variety-adjusted price index based on the assumption that the number of varieties is identical between country *j* and *k* ($I_j = I_k = I$) and the unobservable demand

[9] The notation is adapted to this paper even though I follow closely Feenstra(1994).

shifter is the same for the common set of varieties ($b_{jl} = b_{kl} = b \quad \forall l \in I$). The price index as defined by Diewert(1976)[10] is:

$$(5.3) \quad \widetilde{P}_{jk} = \frac{P_j\left(p_j, I, b\right)}{P_k(p_{k,} I, b)} = \frac{c_j\left(p_j, I, b\right)}{c_k(p_{k,} I, b)}$$

The second equality comes from plugging (5.2) into (5.3) and using the assumption that the number of varieties is the same in both countries.

Sato(1976)[11] shows that the price index corresponding to the CES unit cost function can be written as:

$$(5.4) \quad \widetilde{P}_{jk} = \prod_{l \in I} \left(\frac{p_{jl}}{p_{kl}} \right)^{\omega_{jl}(I)}$$

which is a geometric mean of variety prices with weights $\omega_{jl}(I)$. The weights are defined as follows:

$$(5.5) \quad \omega_{jl}(I) \equiv \frac{\left(\frac{s_{jl}(I) - s_{kl}(I)}{\ln s_{jl}(I) - \ln s_{kl}(I)} \right)}{\sum_{l \in I} \left(\frac{s_{jl}(I) - s_{kl}(I)}{\ln s_{jl}(I) - \ln s_{kl}(I)} \right)}$$, where the cost shares $s_{jl}(I)$ are:

$$(5.6) \quad s_{rl}(I) \equiv \frac{p_{rl} x_{rl}}{\sum_{l \in I} p_{rl} x_{rl}} \quad for \quad r = j, k \, .$$

Proposition 1: If $b_{jl} = b_{kl}$ for $l \in I \subseteq (I_j \cap I_k), \; I \neq \varnothing$, then $\frac{P_j}{P_k} = \widetilde{P}_{jk} \left(\frac{\lambda_j}{\lambda_k} \right)^{\frac{\beta}{1-\sigma}}$

where $(5.7) \quad \lambda_r \equiv \frac{\sum_{l \in I} p_{rl} x_{rl}}{\sum_{l \in I_r} p_{rl} x_{rl}} \quad for \quad r = j, k$

[10] I adapt the time series result of this paper to cross section f.
[11] I adapt the time series result to cross section.

Proof:

The expenditure shares of each variety *l of* country *r=j,k* can be derived as the elasticity of unit cost function with respect to the price of variety *l*:

$$(5.8)\quad s_{lr}(I_r) = \frac{\partial P_r\left(p_r, n_r, I_r\right)}{\partial p_{rl}} \frac{p_{rl}}{P_r\left(p_r, n_r, I_r\right)} = c_r\left(p_r, n_r, I_r\right)^{\sigma-1} b_{rl}{}^{\beta} p_{rl}{}^{1-\sigma} \quad for\ r = j,k$$

Rearranging, I can obtain:

$$(5.9)\qquad c_r(p_r, n_r, I_r) = s_{rl}\left(I_r\right)^{\frac{1}{\sigma-1}} b_{rl}{}^{\frac{\beta}{1-\sigma}} p_{rl} \quad for \quad r = j,k$$

The price index associated with the general CES unit cost function can be written using (5.9) as:

$$(5.10)\quad \begin{aligned} \frac{P_j\left(p_j, b_j, I_j\right)}{P_k\left(p_k, b_k, I_k\right)} &= \frac{n_j^{\frac{1-\beta}{1-\sigma}} c_j(p_j, b_j, I_j)}{n_k^{\frac{1-\beta}{1-\sigma}} c_k\left(p_k, b_k, I_k\right)} = \frac{n_j^{\frac{1-\beta}{1-\sigma}} s_{jl}\left(I_j\right)^{\frac{1}{\sigma-1}} b_{jl}^{\frac{\beta}{1-\sigma}} p_{jl}}{n_k^{\frac{1-\beta}{1-\sigma}} s_{kl}\left(I_k\right)^{\frac{1}{\sigma-1}} b_{kl}^{\frac{\beta}{1-\sigma}} p_{kl}} \overset{for\ \ l\in I}{=} \\ &\overset{for\ \ l\in I}{=} \frac{n_j^{\frac{1-\beta}{1-\sigma}} s_{jl}\left(I_j\right)^{\frac{1}{\sigma-1}} p_{jl}}{n_k^{\frac{1-\beta}{1-\sigma}} s_{kl}\left(I_k\right)^{\frac{1}{\sigma-1}} p_{kl}} \end{aligned}$$

The expenditure shares of each variety can be written:

$$(5.11)\quad s_{rl}\left(I_r\right) = \underbrace{\frac{p_{rl} x_{rl}}{\sum_{l\in I} p_{rl} x_{rl}}}_{s_{rl}(I)} \underbrace{\frac{\sum_{l\in I} p_{rl} x_{rl}}{\sum_{l\in I_r} p_{rl} x_{rl}}}_{\lambda_r} \quad for\ r = j,k$$

I can define the number of varieties as:

$$(5.12)\quad \frac{n_j}{n_k} \equiv \frac{\sum_{l\in I_j} p_{jl} x_{jl} \Big/ \sum_{l\in I} p_{jl} x_{jl}}{\sum_{l\in I_k} p_{kl} x_{kl} \Big/ \sum_{l\in I} p_{kl} x_{kl}} = \frac{\lambda_k}{\lambda_j}$$

Rewriting the variety expenditure shares as in (5.11) and using (5.12), (5.10) becomes:

$$(5.13)\quad \frac{P_j}{P_k} = \frac{\lambda_j^{\frac{\beta}{1-\sigma}} s_{jl}(I)^{\frac{1}{\sigma-1}}\, p_{jl}}{\lambda_k^{\frac{\beta}{1-\sigma}} s_{kl}(I)^{\frac{1}{\sigma-1}}\, p_{kl}}$$

Taking the geometric mean across varieties in (5.13) and using the weights $\omega_{jl}(I)$, I get:

$$(5.14)\quad \frac{P_j}{P_k} = \left(\frac{\lambda_j}{\lambda_k}\right)^{\frac{\beta}{1-\sigma}} \widetilde{P_{jk}} \prod_{l\in I}\left(\frac{s_{jl}(I)}{s_{kl}(I)}\right)^{\frac{\omega_{jl}(I)}{\sigma-1}}$$

It is easy to prove that the product in (5.14) equals 1. q.e.d

So, the CES price index can be written as:

$$(5.15)\quad \frac{P_j}{P_k} = \left(\frac{\lambda_j}{\lambda_k}\right)^{\frac{\beta}{1-\sigma}} \widetilde{P_{jk}}$$

The price index defined by (5.15) is equivalent to the CES price index derived by Feenstra(1994) when $\beta = 1$.

Appendix 2. Variety gains – a simple calculation

The general CES utility function:

$$U(x_l) = n^{\frac{\beta-1}{\sigma-1}} \left(\sum_{l=1}^{n} x_l^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad \overset{x_l = x,\ \forall l = \overline{1,n}}{\Rightarrow} \quad U(x) = n^{\frac{\beta}{\sigma-1}}(nx)$$

where x_l and n represent the quantity per variety and number of variety consumed.

In a symmetric world, I can perform a simple calculation of the impact of the *"love of variety"* strength on the calculated gains from greater variety independent of the total quantity consumed:

$$\frac{U_1(n_1,x)\big/ n_1 x - U_0(n_0,x)\big/ n_0 x}{U_0(n_0,x)\big/ n_0 x} = \frac{n_1^{\frac{\beta}{\sigma-1}} - n_0^{\frac{\beta}{\sigma-1}}}{n_0^{\frac{\beta}{\sigma-1}}} = \left(\frac{n_1}{n_0}\right)^{\frac{\beta}{\sigma-1}} - 1$$

"Love of variety"	**%U change** *for a 10% increase in n*	**Decrease in variety gains** *(LoV=1 as base)*
1.0	4.88%	
0.9	4.38%	10.22%
0.8	3.89%	20.38%
0.7	3.39%	30.50%
0.6	2.90%	40.57%
0.5	2.41%	50.60%
0.4	1.92%	60.57%
0.3	1.44%	70.50%
0.2	0.96%	80.38%
0.1	0.48%	90.21%
0.0	0.00%	100.00%

Note: The calculations assume the elasticity of substitution to be equal to 3. Even though magnitudes change as the elasticity of substitution changes, the message of the calculations remains robust.

Column 3 of the above table shows the impact of the “love of variety” on variety gains. The lower the preference for diversity, the smaller are the variety gains relative to the case when “love of variety” equals one.