Agglomeration and fair wages

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Abstract

This paper implements a fair wage constraint in the fashion of Akerlof and Yellen (1990) into an analytically tractable core-periphery agglomeration model. This enables us to study the role of imperfect labour markets for the pattern of agglomeration. We illustrate that, in the short run, a marginal increase in fair wage preferences leads to an unambiguous compression of the national factor price differential which involves an increase in the unemployment rate. In the long run, this mechanism tends to render full dispersion an unstable equilibrium already at higher trade costs than in the absence of fair wage preferences, that is, than with perfect labour markets. With asymmetric fair wage constraints, industrial production tends to agglomerate in the more constrained region at intermediate trade costs. Overall, we conclude that there is a tendency for fair wage preferences to enforce agglomeration.

JEL: F12, F15, F16, F21, R12
Keywords: Economic geography, Fair wage constraint, Imperfect labour markets

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1 Introduction

The seminal papers by Krugman (1991), Krugman and Venables (1995), and Venables (1996) have started off a sizable literature on the “new economic geography”, determining the agglomeration forces in general equilibrium with trade and international factor mobility. One major interest in this line of research is the analysis of the role of transport costs for the agglomeration of economic activity in the presence of firm-level economies of scale. The conceptual framework for this is also referred to as the core-periphery model (CPM). Forslid (1999) has developed an analytically solvable CPM that has been applied later by Andersson and Forslid (1999), Baldwin and Krugman (2000), and others. Pflüger (2004) provides a further simplified variant of this model that is based on the assumption of quasi-linear consumer preferences with Dixit and Stiglitz (1977) type differentiated manufactures and a homogeneous agricultural good.

As with related models of international trade, factor markets are typically assumed to be perfect. However, empirical stylised facts seem to be at odds with this presumption, at least for several of the developed economies. For instance, continental European labour markets show a higher rigidity in many respects and basically higher unemployment rates than, say, that of the US. In part, this imperfection may be related to workers’ preferences for fair wages, suggesting that workers’ effort positively depends on the ratio between actual wages and ones that they perceive as fair. This mechanism has been introduced in the seminal work of Akerlof and Yellen (1990). More recent work based on surveys among managers and workers, on firm-level studies analysing the nexus between wages and employment spell lengths, and on experimental evidence seems to be strongly supportive of the fair wage mechanism (see Howitt, 2002, and Bewley, 2005, for reviews of the literature). Beyond that, continental European labour markets are characterised by a much more stable dispersion of earnings than the US or the UK over the last 25 years (see OECD, 2004).¹ The reason for this may be seen in a preference in continental Europe to limit the gap in the wages of high-skilled and low-skilled workers, which comes at the expense of higher unemployment there.

It is this paper’s task to study the role of imperfect labour markets for agglomeration. For

¹The level of the US and the UK earnings dispersion was already higher than in continental Europe 25 years ago, and it even increased since then.
this, we implement a fair wage constraint into a core-periphery agglomeration model. Related work on the impact of fair wage preferences in trade models with immobile factors encompasses Kreickemeier (2004) and Kreickemeier and Nelson (2006). Here, the manufacturing sector employs two factors, capital and labour, where the former yields a higher return than the latter. An increase in earnings dispersion would thus be represented by higher relative capital rewards in our framework. Individuals have a preference to receive fair wages, which means that workers attribute some weight to the capital owners’ remuneration. As a consequence, the fair wage settles above the market clearing level. Firms can choose to pay either the market clearing wage or the fair wage. However, if the remuneration is below the fair level, workers reduce their effort proportionately thus increasing effective labour costs for firms. As in Akerlof and Yellen (1990) and Kreickemeier (2004), firms choose to pay the fair wage if doing so does not diminish their profits. In this model, both factor returns and the unemployment rate are endogenously determined in equilibrium.

Whereas both factors are immobile across borders in the short run, capital is mobile in the long run, but labour is not. Accordingly, fair wage preferences affect relative capital rentals and unemployment rates in the short run and the agglomeration pattern in the long run. For instance, the response of the capital rental to a marginal increase in the fair wage constraint is unambiguously negative. This works via a reduction in income and demand that is triggered by unemployment. A higher domestic fair wage parameter always increases domestic unemployment while foreign unemployment may decline. With symmetric fair wage parameters and identical countries, a marginal increase in this parameter exhibits an unambiguously positive impact of both countries’ unemployment rate. In the long run with two symmetric countries, a full dispersion equilibrium becomes unstable already at a higher level of trade costs than with perfect labour markets. With asymmetric fair wage constraints, workers in one region possess a higher preference for equal remuneration than workers in the other. Then, capital tends to flee the more constrained labour market at intermediate trade costs.

In the next section, we present the model and the fair wage constraint. Section 3 describes the short-run equilibrium where both capital and labour are assumed to be internationally immobile. In Section 4, we treat capital as mobile and analyse the long-run equilibrium. Section 5 concludes

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2Capital may be interpreted either as human capital (skilled labour) or even as physical capital.
with a summary of the most important findings.

2 The model

2.1 An analytically tractable core-periphery model

The model builds on a variant of the core-periphery agglomeration model. Household utility is characterised by a quasi-linear upper-tier utility function as in Pflüger (2004). There are two sectors, a homogeneous agricultural and a differentiated industrial sector. Consumers are characterised by a love of variety as in Dixit and Stiglitz (1977). Furthermore, there are two countries $i$ and $j$ where two factors are supplied, labour $L$ and capital $K$, respectively. $L$ is used in the production of both homogeneous and differentiated goods and immobile across borders. $K$ is only used for firm set-up (see Flam and Helpman, 1987; Pflüger, 2004) and mobile across borders in the long run. $L$’s supply is bound by a fair wage constraint (see Akerlof and Yellen, 1990; Kreickemeier, 2004; Kreickemeier and Nelson, 2006).

There are $L_i + K_i$ households in country $i$. Labour (capital) owners supply one unit of labour (capital) each. Household preferences in country $i$ are determined by

$$O_i = \alpha \ln C_{X_i} + C_{Y_i}, \quad C_{X_i} = \left( \sum_{i=1}^{n_i} x_{ii}^{\sigma-1} + \sum_{j=1}^{n_j} x_{ji}^{\sigma-1} \right)^{\frac{\sigma}{\sigma-1}}, \quad \alpha > 0, \quad \sigma > 1, \quad (1)$$

where $C_Y$ is aggregate demand for the agricultural good and $C_X$ is aggregate demand for industrial goods. $x_{ii}$ is demand for a single variety that is locally supplied and $x_{ji}$ denotes demand for a single foreign manufacture. There are $n_i$ ($n_j$) firms operating in the industrial sector in country $i$ ($j$). Each firm supplies a single variety. The varieties are substitutable at a constant elasticity of substitution of $\sigma$. Iceberg trade costs impede cross-border transactions of differentiated goods. In particular, producers have to send $\tau$ units so that one unit arrives. In equilibrium, the mill price is identical for differentiated goods that are supplied locally and ones that are exported. Accordingly, the consumer price differs between locally sold varieties (the corresponding consumer price in $j$ is $p_j$) and exported varieties (the price of the same variety for consumers in country $i$ is $p_j \tau$). To simplify the notation, we account already for the fact that all
firms producing and supplying at the same market set identical mill prices in equilibrium due to their access to identical technologies and the assumption of symmetric utility (hence, the mill price $p_i$ is identical across all firms producing in $i$).

Income in country $i$ is determined by

$$E_i = P_i C_{X_i} + C_{Y_i}, \quad P_i = \left[ n_i p_i^{1-\sigma} + n_j \phi p_j^{1-\sigma} \right] ^{\frac{1}{1-\sigma}}, \quad 0 \leq \phi = \tau^{1-\sigma} \leq 1, \quad (2)$$

where $P_i$ reflects the CES price index for consumers in country $i$. Utility maximisation determines demand and indirect utility, $V_i$:

\begin{align*}
  C_{X_i} &= \frac{\alpha}{P_i}, \quad C_{Y_i} = E_i - \alpha, \\
  x_{ii} &= \alpha p_i^{-\sigma} P_i^{\sigma-1}, \quad x_{ij} = \alpha(\tau p_i)^{-\sigma} P_j^{\sigma-1} \\
  V_i &= -\alpha \ln P_i + E_i + \alpha (\ln \alpha - 1). \quad (3)
\end{align*}

We adopt the common assumptions that there is perfect competition in sector $Y$ and trade of $Y$ is not subject to any barriers. Production of one unit of the agricultural good requires a single unit of labour such that $Y_i = L_{Y_i}$. Choosing the price of the agricultural good as the numéraire then guarantees that wages are $w_i = w_j = 1$ as long as diversification prevails. Market clearing for domestic varieties of the manufacture implies that aggregate supply at the firm level $X_i$ equals aggregate demand of a single variety: $X_i$.

The production of one unit of the manufacturing good requires $c$ units of labour such that $X_i = cL_{X_i}$, with $c > 0$. Accordingly, parameter $c$ reflects the marginal costs of producing one unit of manufactures. The production of manufactures is characterised by economies of scale. By assumption, each firm operating in the $X$-sector of country $i$ has to incur fixed costs $r_i$ associated with the use of a single unit of capital $K$ in firm set-up. Profits of firms are then determined by

$$\pi_i = (p_i - c)X_i - r_i. \quad (5)$$

In equilibrium, mill prices are set so that their relation to variable costs corresponds to a fixed mark-up

$$p_i = c \frac{\sigma}{\sigma - 1}. \quad (6)$$
Equilibrium profits are zero due to free entry which yields equilibrium firm size as

\[ \pi_i = \frac{c}{\sigma - 1} X_i - r_i = 0 \Rightarrow X_i = r_i \frac{\sigma - 1}{c}. \] (7)

### 2.2 The fair wage constraint

In contrast to previous research, supply of labour is elastic in this framework. As indicated before, workers’ remuneration is bound by a fair wage constraint. By fair wage we mean that workers attribute some weight to capital’s remuneration. Since we focus on the case where the capital rental is higher than the wage rate, the perceived fair wage always exceeds its market clearing counterpart. If employees in country \( i \) are paid less than their fair wage, they reduce effort according to the following effort function

\[ e_i = \min \left( \frac{w_i}{w_i^*}, 1 \right), \]

where \( w_i \) is the market wage in country \( i \) and \( w_i^* \) denotes the fair wage. Normal effort is set to unity. The functional form implies that workers reduce their effort proportionately if the market wage falls short of the fair wage. Hence, effective labour costs cannot be influenced by firms. As in Akerlof and Yellen (1990) and Kreickemeier (2004), firms choose to pay the fair wage if doing so does not diminish their profits. \( w_i^* \) is determined by

\[ w_i^* = \beta r_i + (1 - \beta) (1 - U_i) w_i. \] (8)

The preference for an equal income distribution rises with \( \beta \), a preference parameter workers attribute to the return to capital. The second component, \((1 - U_i) w_i\), describes the expected wage of workers outside their firm (where \( U_i \) denotes the unemployment rate) and is weighted by \( 1 - \beta \). It is straightforward to show that in equilibrium

\[ w_i^* > w_i \text{ and } U_i > 0, \]

By applying the zero-profit condition, we neglect the integer problem and treat firm numbers as continuous (see Baldwin, 1988).

Recall that capital may be associated with skilled labour.

For now, we assume that domestic and foreign workers are identical with respect to their fair wage preferences. This condition will be relaxed in an extension of the model, below.
unless $\beta = 0$. A similar relationship as eq. (8) exists for capital owners. However, since their market income always exceeds market clearing wages by assumption the fair capital return will always be lower than its market clearing counterpart and thus be non-binding. Consequently, capital is always fully employed.\textsuperscript{6} Using the equilibrium outcome and the assumption that workers are always paid their fair wage, we can derive the fair wage constraint as

$$\frac{w_i}{r_i} = \frac{\beta}{\beta + (1 - \beta)U_i}.$$  \hspace{1cm} (9)

As long as $\beta > 0$, unemployment arises in equilibrium and exerts a dampening effect on the relative wage-capital return ratio $w_i/r_i$. Under diversification, $w_i = 1$ so that the unemployment rate in country $i$ is determined by

$$U_i = \frac{\beta}{1 - \beta}(r_i - 1)$$

which implies $r_i \in [1, \beta^{-1}]$.

According to our assumptions, the factor constraints in country $i$ are

$$K_i = n_i$$
$$L_i = U_iL_i + (1 - U_i)L_i$$
$$= U_iL_i + a_{X_i}n_i x_i + \frac{Y_i}{L_{X_i}}$$

where $a_{X_i}$ describes the labour input coefficient for the manufacturing good, that is $cL/x_i$. For the employed workers, we then obtain\textsuperscript{7}

$$(1 - U_i) L_i = \frac{1 - \beta r_i}{1 - \beta} L_i.$$ \hspace{1cm} (10)

In the sequel, we assume countries to be symmetric not only with respect to $\beta$ but also with respect to $L_i = L_j \equiv L$, but we focus on differences in the allocation of capital ($K_i, K_j$) across economies.

\textsuperscript{6}Again, associating capital with skilled labour, this is consistent with the stylised fact that the risk of becoming unemployed is predominantly important for unskilled labour.

\textsuperscript{7}Since there is no unemployment benefit in the model, aggregate demand for a single variety is determined by $X_i = ((1 - U_i)L_i + K_i)x_{ii} + ((1 - U_j)L_j + K_j)x_{ij}$. 

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3 Short-run equilibrium

In the short run, both capital and labour are immobile across borders, but goods can be traded internationally. By using eqs. (2), (3), (5), (6) and the condition for employed workers (10), zero profits imply that the capital return in country \( i \) reads

\[
r_i = \frac{\kappa_i}{\sigma \kappa_i + \alpha \beta L} \left( \frac{\alpha \left( \frac{L - P}{K_i} + K_i \right)}{\kappa_i} + \phi \alpha \left( \frac{L - P}{K_j} (1 - \beta r_j + K_j) \right) \right),
\]

where \( \tilde{\beta} = \beta/(1 - \beta) \), \( \kappa_i = K_i + \phi K_j \) and \( \kappa_j = K_j + \phi K_i \). If the fair wage parameter \( \beta \) equals zero, the zero profit condition for the short run boils down to the one in Pflüger (2004) who assumes labour markets to be perfectly flexible.

In the short run, we are interested in the impact of the fair wage constraint on the equilibrium capital rental for a given allocation of capital among countries. We summarise the corresponding findings in the following proposition.

**Proposition 1.** An increase in \( \beta \) exhibits an unambiguously negative impact on the capital return \( r_i \). There is both a negative direct effect on \( r_i \) from an increase in the domestic fair wage parameter and a negative indirect effect from a simultaneous increase abroad.

*Proof.* See Appendix A.

The intuition is the following. A higher \( \beta \) attributes a higher weight of the capital return to the determination of the fair wage. As the firms always pay the fair wage and we set this wage to unity, a higher fair wage necessarily implies a lower capital return. The reduction in \( r_i \) can be inferred from the zero profit condition in eq. (5). With \( w = 1 \), prices are constant and, hence, the reduction in \( r_i \) causes a decline in demand, partly triggered by unemployment – that is, partly due to fewer workers that are able to purchase varieties of the differentiated good. As firms are faced with both domestic and foreign demand for their product, the decline in demand of one country’s households reduces both the domestic and the foreign capital return. The following proposition states the impact of the fair wage parameter on the domestic unemployment rate and thus sheds further light on this mechanism.
Proposition 2. An increase in $\beta$ raises world-wide unemployment. While the direct effect of an increase in the domestic fair wage constraint unambiguously increases $U_i$, the indirect effect lowers domestic unemployment.

Proof. See Appendix B.

The latter effect stems from the fact that a reduction of foreign demand for national varieties can be compensated by more national demand through higher employment. However, the direct impact of an increase in $\beta$ on domestic unemployment can be eventually overturned by the indirect one from rising fair wage preferences abroad. The direct effect is more likely overturned the more the foreign capital rental exceeds the domestic one. If $K_i = K_j$, the direct effect always dominates, that is, an increase in $\beta$ necessarily leads to higher unemployment in two symmetric countries (see Appendix B for further details). However, what matters here is that the effect on world-wide unemployment is positive so that the demand for varieties declines in $\beta$ as stated before.

In the previous analysis, capital was immobile across borders. In the following long-run perspective, this will be relaxed and our focus is then on the impact of a marginal increase in the fair wage parameter on the pattern of agglomeration.

4 Long-run equilibrium

In the long run, we consider a given allocation of capital as being unstable if capital owners would be better off after relocating their firm to the other country. We assume that they will prefer to do so as long as their indirect utility would be higher abroad. The indirect utility of capital owners in country $i$ is determined by $V_{Ki} = \alpha \ln(P_j/P_i) + (r_i - r_j)$. Notice that a necessary condition for a long-run equilibrium is that the difference between the indirect utility for capital owners residing in $i$ and the ones in $j$ is zero:

$$V_{Ki} - V_{Kj} = \alpha \ln(P_j/P_i) + (r_i - r_j) = 0.$$
Furthermore, we may rewrite the price indices for countries $i$ and $j$

\[ P_i = \left( \frac{\sigma}{\sigma - 1} \right) (K_i + \phi K_j)^{1-\sigma} ; \quad \frac{P_j}{P_i} = \left( \frac{\phi K_i + K_j}{K_i + \phi K_j} \right)^{1-\sigma}. \]

Using the definition $\lambda \equiv K_i / (K_i + K_j)$, we determine

\[ \alpha \ln \left( \frac{P_j}{P_i} \right) = \frac{\alpha}{1-\sigma} \ln \left( \frac{\phi \lambda + 1 - \lambda}{\lambda + \phi (1 - \lambda)} \right) \]

and define $\rho \equiv L / (K_i + K_j)$ to obtain

\[ V_{Ki} - V_{Kj} = \frac{\alpha}{1-\sigma} \ln \left( \frac{\phi \lambda + 1 - \lambda}{\lambda + \phi (1 - \lambda)} \right) + \alpha \left( 1 - \phi^2 \right) \frac{\alpha \beta \rho (2\lambda - 1)}{\sigma [\phi \lambda + (1 - \lambda)] + \alpha \beta \rho}. \tag{11} \]

The CPM contains two agglomeration forces and one dispersion force which appear in this setup as well. The first term in eq. (11) captures the **price index effect** (or **cost-of-living effect**), which always works in favour of agglomeration. There, an increase in $\lambda$ is associated with a higher share of firms being active in market $i$. This lowers the price index, since fewer varieties have to be imported. This by itself increases the real capital return, generating a capital inflow and, hence, an influx of firms. This price index effect represents the first of the two agglomeration forces in the model.

The second term in eq. (11), namely $r_i - r_j$, captures the impact of a change in the allocation of firms $\lambda$ on the nominal capital return differential. This term is the sum of two effects: the **market access effect** as the second agglomerative force in the model, and the **market crowding effect** as the dispersion force. The market access effect suggests that firms earn higher profits in the larger market. Therefore, a deviation from the symmetric equilibrium with $\lambda = 0.5$ stimulates a further capital outflow to the larger region. The market crowding effect indicates that the agglomeration of firms in one region implies tougher competition and thus lower capital rentals there. Accordingly, the net effect of the two is ambiguously related to $\lambda$, depending on the level of $\phi$.

We need to determine under which circumstances a symmetric equilibrium at $\lambda = 0.5$ (also referred to as **full dispersion**) is stable. Stability of such an equilibrium is achieved if the $V_{Ki} - V_{Kj}$-locus intersects with the abscissa and it is negatively sloped at $\lambda = 0.5$. Then, capital
owners in country $i$ do not have an incentive to move one further firm abroad, because they would encounter lower real capital rentals there. However, foreign capital owners would not want to locate one further unit of capital in market $i$ either, since they would then face lower real capital rentals abroad. The stability of the dispersion equilibrium depends on the level of trade costs. This is illustrated in Figure 1, where we use similar parameter values as Pflüger (2004), namely $\alpha = 0.3$ and $\sigma = 6$. However, we have to employ a sufficiently high share of labour relative to capital ($\rho = 11$) to ensure that $r_i, r_j \geq 1$. Otherwise, the fair wage constraint would not be binding. In Figure 1, countries are identical in all respects except $\lambda$.

Figure 1: Indirect utility differentials for different trade costs and $\beta_i = \beta_j = 0.1$

According to Figure 1, full dispersion is a stable equilibrium at a sufficiently high level of $\tau$, for example, at $\tau = 1.0402$. If trade costs are sufficiently low, the agglomeration forces outweigh the dispersion force and equilibria at $\lambda \neq 0.5$ become stable in the long run. A deviation from the symmetric allocation renders capital owners in the target country better off, thus generating incentives for other firms to follow the initial mover. At very low trade costs full agglomeration becomes stable (at $\lambda = 0$ and $\lambda = 1$). At intermediate trade costs, there are two interior stable
equilibria (referred to as partial agglomeration).

By using the condition \( \frac{\partial}{\partial \lambda} (V_{K_i} - V_{K_j}) = 0 \), it is possible to determine analytically the highest possible level of trade costs (i.e., the lowest possible \( \phi \)) that is consistent with a long-run equilibrium \( \lambda = 0.5 \):

\[
\phi^* = \frac{2(\sigma - 1) \left( \frac{\sigma - \alpha \beta}{1 - \beta} \right) - \sigma^2 - 4\alpha \beta \rho \left( \sigma + \alpha \beta \rho \right)}{2(\sigma - 1) \left( \sigma + \frac{\sigma + \alpha \beta}{1 - \beta} \right) + \sigma^2 - 4\alpha^2 \beta^2 \rho^2}.
\] (12)

If we increase \( \phi \) (lower trade costs) any further when starting at \( \phi^* \), the full dispersion equilibrium is not tenable any more in the long run. Although the determination of \( \phi^* \) is crucial to know whether and where full dispersion is possible in the long run, we do not know yet whether the fair-wage-constrained CPM is still characterised by a supercritical pitchfork bifurcation (hence, whether \( \lambda = 0.5 \) is a unique long-run equilibrium at \( \phi^* \)). The following proposition provides an answer on this question.

**Proposition 3.** A supercritical pitchfork bifurcation requires that \( \frac{\partial^2}{\partial \lambda^2} (V_{K_i} - V_{K_j}) \big|_{\phi=\phi^*,\lambda=0.5} = 0 \) and \( \frac{\partial^3}{\partial \lambda^3} (V_{K_i} - V_{K_j}) \big|_{\phi=\phi^*,\lambda=0.5} < 0 \). Whereas \( \frac{\partial^2}{\partial \lambda^2} (V_{K_i} - V_{K_j}) \big|_{\phi=\phi^*,\lambda=0.5} = 0 \) is generally fulfilled in the fair-wage-constrained CPM, \( \frac{\partial^3}{\partial \lambda^3} (V_{K_i} - V_{K_j}) \big|_{\phi=\phi^*,\lambda=0.5} < 0 \) is not. Only the unconstrained model (where \( \beta = 0 \)) is necessarily of the supercritical pitchfork bifurcation type. Such a bifurcation arises in the constrained model only if

\[
\sigma^2 (\sigma - 1) (1 - \phi^2) (1 + \phi)^2 \left( \sigma + \frac{2\alpha \beta \rho}{1 - \beta} \right) < \sigma^2 (\sigma - 1) (1 - \phi)^4 \left( \sigma + \frac{2\alpha \beta \rho}{1 - \beta} \right) + 4 (1 - \phi)^2 \left[ \frac{\sigma}{2} (1 + \phi) + \alpha \beta \rho \right]^2.
\]

**Proof.** See Appendix C. \( \square \)

Most importantly, we are interested in how the fair wage preference parameter \( \beta \) influences \( \phi^* \). Hence, we need to determine whether a marginal increase in \( \beta \) fosters agglomeration versus dispersion. We have seen that the corresponding effect on the capital rental in the short run is unambiguously negative. The following proposition sheds light on the relationship between \( \beta \) and \( \phi^* \).

**Proposition 4.** The break-point for the full dispersion equilibrium (\( \phi^* \)) is ambiguously related to \( \beta \) in general. However, a marginal increase in the fair wage parameter in an unconstrained
initial equilibrium with $\beta = 0$ leads to an unambiguous decline in $\phi^*$. Then, a marginal increase in $\beta$ renders full dispersion at the original level of $\phi^*$ untenable. In turn, a full dispersion equilibrium can then only be maintained at higher trade costs in the long run.

Proof. See Appendix D.

Figure 2: Bifurcation diagram for $\beta = 0$ (dashed) and $\beta = 0.1$ (solid)

By extending the exercise of Figure 1 to cover the entire range of trade costs, one arrives at the bifurcation diagram which depicts all stable long-run equilibria in relation to $\tau$. Figure 2 shows simulation results for the identical set of parameters as above whereas the constrained scenario with $\beta = 0.1$ – described by the solid (blue) line – is compared to the unconstrained case – described by the dashed (orange) line. The diagram clearly illustrates the pitchfork bifurcation for these parameters and shows that the break point shifts towards a higher level of $\tau$ as $\beta$ increases.

The intuition behind this result is the following. Recall that the price index effect (i.e.,
the cost-of-living effect) works in favour of agglomeration as $\lambda$ rises. However, an increase in $\beta$ at $\beta = 0$ will increase $r_i - r_j$ and lead to a reduction of the sensitivity of the capital rental differential with respect to $\lambda$. Therefore, the agglomeration forces will dominate the dispersion forces already at higher trade costs. Figure 3 sheds further light on their interaction.

Figure 3: Agglomeration and dispersion forces in relation to freeness of trade

Both agglomeration and dispersion forces decline in $\phi$. While the dispersion force generally dominates at a low freeness of trade, there is one well-defined $\phi$ at which agglomeration forces outweigh the market crowding effect. According to this model, Figure 3 illustrates the price index effect (as one agglomeration force) and the sum of market crowding and market access effects (henceforth referred to as the net dispersion force) as two separate loci. With perfect labour markets, $\phi^*_0$ reflects the break point at which full dispersion is no longer a stable equilibrium. There, the net dispersion force exactly offsets the price index effect. For all $\phi < \phi^*_0$, $\lambda = 0.5$ is stable. The net dispersion force becomes zero at $\phi^*_0$. There, market crowding and market access effects offset each other and the corresponding locus in Figure 3 crosses the abscissa. At
\( \phi = 1 \) the magnitude of all forces becomes zero since trade costs do not impede trade any longer. Then, the location of production has no influence on real capital rentals.

Let us now introduce a symmetric fair wage constraint – captured by \( \beta > 0 \) – starting from an initial equilibrium with \( \beta = 0 \). This rotates the net dispersion force locus counter-clockwise with its origin remaining at \( \phi = 1 \). While the price effect does not depend on \( \beta \), the market crowding effect loses strength relative to the market access effect. This is caused by the compression of capital rentals. Such a marginal increase in \( \beta \) leads to \( \phi^*_\beta < \phi^*_0 \) where the agglomeration forces exactly offset the dispersion force, and it also leads to \( \bar{\phi}_\beta < \bar{\phi}_0 \) (see Appendix E for a proof of the latter result).

Figure 4: Bifurcation diagram for \( \beta_i = \beta_j = 0 \) (dashed) and \( \beta_i = 0.1 \) and \( \beta_j = 0 \) (solid)

As a further exercise it is interesting to look at the asymmetric case where one region possesses a stricter fair wage constraint than the other. Although the non-linearities in the model prevent an analytical discussion in this case, we can provide tentative insights by means of numerical simulations. For the above parameters and \( \beta_i = 0.1 \) and \( \beta_j = 0 \) Figure 4 presents the results. Comparing the asymmetric fair wage scenario - again described by the solid (blue) parts - to
the unconstrained one with $\beta_i = \beta_j = 0$, we find that the fair wage constraint tends to drive capital to the unconstrained country at intermediate trade costs.

The intuition behind this outcome is the following. Note that $r_i < r_j$ at $\beta_i > \beta_j = 0$. This by itself generates an incentive for capital owners to reside in the unconstrained country $j$. Lower trade costs and, hence, more integrated commodity markets lead to an increasing share of firms locating in country $j$. Then, only partial or even full agglomeration in $j$ is stable in the long run. The direction of capital mobility is then unambiguous at moderate trade costs and the bifurcation breaks down. Full agglomeration prevails at an even higher level of trade costs than in the symmetric case. Only at extremely low trade costs may full agglomeration prevail even in the fair wage constrained country. There, it is not very important where to locate production since trade impediments are negligible. Both agglomerative and dispersive forces settle on a lower level and, thus, the disadvantage of having a fair-wage-constrained labour market loses its severeness.

5 Conclusions

This paper analyses the impact of imperfect labour markets on the pattern of agglomeration. We introduce a fair wage constraint as in Akerlof and Yellen (1990) into an analytically tractable core-periphery agglomeration model. In this model, both factor rewards and unemployment are endogenously determined in general equilibrium. Unemployment distorts market-clearing factor prices which generates agglomeration patterns that differ from the ones in the unconstrained core-periphery model. The most important insights can be summarised as follows.

If both countries’ labour markets are equally constrained by fair wages, the capital rentals in both economies are reduced in the short run. This also leads to a decline in demand, being triggered by higher world-wide unemployment. In the long run, the break point is shifted towards a higher level of trade costs. Hence, full dispersion of economic activity across economies is sustainable only at higher trade costs than with unconstrained wages. If individuals in one country have a preference for fair wages but those in the other country have not, then capital will be driven to the less constrained region at intermediate trade costs. In this case, trade
costs need to be much lower than in the unconstrained equilibrium to render full agglomeration sustainable in the long run.

Appendix

A Proof of Proposition 1

Appendix A lays out the proof of Proposition 1.

\[
\frac{\partial r_i}{\partial \beta} = \left( \alpha L \sigma \kappa_j + \alpha^2 \beta L^2 \right) \left( 1 - \phi^2 \right) \frac{1 - r_i}{(1 - \beta)^2} + \phi \alpha L \sigma \kappa_i \frac{1 - r_j}{(1 - \beta)^2} \left( \sigma \kappa_i + \alpha \beta \right) \left( \sigma \kappa_j + \alpha \beta \right) - \phi^2 \alpha^2 \beta^2 L^2 < 0
\]

\[
\frac{\partial r_i}{\partial \beta} \text{ is unambiguously negative as both } r_i \text{ and } r_j \text{ must be larger than one by assumption.}
\]

This result stems from the fact that both the direct domestic and the indirect foreign effect point in the same direction:

\[
\frac{\partial r_i}{\partial \beta} = \frac{1 - r_i}{(1 - \beta_i)^2} \left( \alpha L \sigma \kappa_j + \alpha^2 \beta_j L^2 \left( 1 - \phi^2 \right) \right) \left( \sigma \kappa_i + \alpha \beta_i \right) \left( \sigma \kappa_j + \alpha \beta_j \right) - \phi^2 \alpha^2 \beta_i \beta_j L^2 < 0
\]

\[
\frac{\partial r_i}{\partial \beta} = \frac{1 - r_j}{(1 - \beta_j)^2} \left( \alpha L \sigma \kappa_i \right) \left( \sigma \kappa_j + \alpha \beta_j \right) \left( \sigma \kappa_i + \alpha \beta_i \right) - \phi^2 \alpha^2 \beta_i \beta_j L^2 \leq 0.
\]

The indirect effect from an increase in the foreign fair wage parameter only exhibits a negative impact on } r_i \text{ if trade costs are not prohibitively high, that is } \phi > 0.\]

B Proof of Proposition 2

In Appendix B, we determine the impact of fair wage preferences on the unemployment rate } U_i \text{.}

\[
\frac{\partial U_i}{\partial \beta} = \left( \frac{\sigma^2 \kappa_i \kappa_j + \alpha \beta L \sigma \kappa_i}{\left( \sigma \kappa_i + \alpha \beta \right) \left( \sigma \kappa_j + \alpha \beta \right)} - \phi \alpha \beta L \sigma \kappa_i \frac{1 - r_j}{1 - \beta_j} \right) < 0
\]

The impact of a simultaneous rise in both } \beta_i \text{ and } \beta_j \text{ on home’s unemployment rate is positive iff}

\[
\left( \sigma^2 \kappa_i \kappa_j + \alpha \beta L \sigma \kappa_i \right) \left( r_i - 1 \right) - \phi \alpha \beta L \sigma \kappa_i \left( r_j - 1 \right) > 0,
\]
that is if the positive direct effect
\[
\frac{\partial U_i}{\partial \beta_i} = \frac{r_i - 1}{(1 - \beta_i)^2} + \tilde{\beta}_i \frac{\partial r_i}{\partial \beta_i}
\]
\[
= \frac{r_i - 1}{(1 - \beta_i)^2} \left( \frac{\sigma^2 \kappa_i \kappa_j + \alpha \tilde{\beta}_i L \kappa_i}{\sigma \kappa_i + \alpha \tilde{\beta}_i L} \right) - r_i \frac{\phi^2 \alpha^2 \beta_i \beta_j L^2}{\sigma \kappa_i + \alpha \tilde{\beta}_i L} > 0
\]
outweighs the indirect effect
\[
\frac{\partial U_i}{\partial \beta_j} = \tilde{\beta}_i \frac{\partial r_i}{\partial \beta_j}
\]
\[
= - \frac{r_j - 1}{(1 - \beta_j)^2} \left( \frac{\sigma \kappa_j + \alpha \tilde{\beta}_j L - \phi^2 \alpha^2 \beta_i \beta_j L^2}{\sigma \kappa_j + \alpha \tilde{\beta}_j L} \right) \leq 0.
\]

\[\boxed{}\]

\section{C Proof of Proposition 3}

In Appendix C, we derive the second and third derivatives of the country \(i\)-based capital owners’ indirect utility differential with respect to \(\lambda\). For the subsequent derivations, it will be useful to define
\[
T_{1i} \equiv \alpha \ln(P_j/P_i) = \frac{\alpha}{1 - \sigma} \ln \left( \frac{\phi \lambda + 1 - \lambda}{\lambda + \phi (1 - \lambda)} \right)
\]
\[
T_{2i} \equiv \frac{\sigma [\lambda + \phi (1 - \lambda)] + \alpha \tilde{\beta} \rho}{\alpha} \left( \frac{\phi \lambda + 1 - \lambda}{\lambda + \phi (1 - \lambda)} \right) - \alpha^2 \phi^2 \beta_j L^2
\]
\[
T_{3i} \equiv \left( \frac{\rho}{1 - \beta} + \lambda \right) \left( \sigma [\phi \lambda + 1 - \lambda] + \alpha \tilde{\beta} \rho \right) + \left( \frac{\rho}{1 - \beta} + 1 - \lambda \right) \phi \sigma [\phi \lambda + 1 - \lambda]
\]
\[
T_{3j} \equiv \left( \frac{\rho}{1 - \beta} + 1 - \lambda \right) \left( \sigma [\lambda + \phi (1 - \lambda)] + (1 - \phi^2) \alpha \tilde{\beta} \rho \right) + \left( \frac{\rho}{1 - \beta} + \lambda \right) \phi \sigma [\phi \lambda + 1 - \lambda]
\]
Note that
\[
\frac{\partial^2 (V_K - V_{Kj})}{\partial \lambda^2} = \frac{\partial^2 T_{1i}}{\partial \lambda^2} + \frac{\partial^2 T_{2i}}{\partial \lambda^2} (T_{3i} - T_{3j}) + 2 \frac{\partial T_{2i}}{\partial \lambda} \frac{\partial (T_{3i} - T_{3j})}{\partial \lambda} + T_{2i} \frac{\partial^2 (T_{3i} - T_{3j})}{\partial \lambda^2}
\]
Furthermore, we have

\[
\frac{\partial^2 T_{3i}}{\partial \lambda^2} = -\frac{\alpha}{1-\sigma} \frac{(1-\phi)^4 (1-2\lambda)}{\left([\lambda + \phi (1-\lambda)] [\phi\lambda + 1-\lambda]\right)^2} \implies \\
\frac{\partial^2 T_{3i}}{\partial \lambda^2} \bigg|_{\lambda=0.5} > 0 \\
T_{3i} \bigg|_{\lambda=0.5} = T_{3j} \implies \\
\frac{\partial^2 T_{3i}}{\partial \lambda^2} (T_{3i} - T_{3j}) \bigg|_{\lambda=0.5} = 0 \\
\frac{\partial T_{2i}}{\partial \lambda} \frac{\partial (T_{3i} - T_{3j})}{\partial \lambda} = -\frac{\alpha\sigma^2 (1-\phi)^2 [1-2\lambda] \left[(1-\phi)^2 \left(\sigma + 2\alpha\beta\rho\right) - (1-\phi)^2 \left(\frac{2\rho\sigma}{1-\beta} + \sigma\right)\right]}{\left\{\sigma[\lambda + \phi (1-\lambda)] + \alpha\beta\rho\right\} \left\{\sigma[\phi\lambda + 1-\lambda] + \alpha\beta\rho\right\} - \alpha^2 \phi^2 \beta^2 \rho^2} \\
2 \frac{\partial T_{2i}}{\partial \lambda} \frac{\partial (T_{3i} - T_{3j})}{\partial \lambda} \bigg|_{\lambda=0.5} = 0 \\
T_{2i} \frac{\partial^2 (T_{3i} - T_{3j})}{\partial \lambda^2} = 0
\]

Hence, \(\frac{\partial^2 (V_{Ki} - V_{Kj})}{\partial \lambda^2} \bigg|_{\lambda=0.5} = 0\).

Analogously, we can determine

\[
\frac{\partial^3 (V_{Ki} - V_{Kj})}{\partial \lambda^3} \bigg|_{\lambda=0.5} = \frac{\alpha}{1-\sigma} \frac{8 (1-\phi)^4}{(1+\phi)^2} \left\{\frac{\sigma^2 (\sigma - 1) (1-\phi^2) \left(\sigma + \frac{2\alpha\beta\rho}{1-\beta}\right)}{2\alpha^2 \phi^2 \beta^2 \rho^2} - \frac{\sigma^2 (\sigma - 1) (1-\phi^2) \left(\sigma + \frac{2\alpha\beta\rho}{1-\beta}\right) + 4 (1-\phi)^2}{(1+\phi)^2} \right\} \\
\sigma^2 (\sigma - 1) (1-\phi^2) (1+\phi)^2 \left(\sigma + \frac{2\alpha\beta\rho}{1-\beta}\right) < \sigma^2 (\sigma - 1) (1-\phi^2) \left(\sigma + \frac{2\rho\sigma}{1-\beta}\right) \\
+ 4 (1-\phi)^2 \left\{\frac{\sigma^2 (\sigma - 1) (1-\phi^2) \left(\sigma + \frac{2\rho\sigma}{1-\beta}\right)}{2\alpha^2 \phi^2 \beta^2 \rho^2} - \alpha^2 \phi^2 \beta^2 \rho^2\right\}^2
\]

For \(\beta = 0\), we obtain

\[
\frac{\partial^3 (V_{Ki} - V_{Kj})}{\partial \lambda^3} \bigg|_{\lambda=0.5,\beta=0} = \frac{1}{1-\sigma} \frac{8 (1-\phi)^4}{(1+\phi)^2} + \frac{8\sigma^2 (1-\phi)^2 \left(1-\phi^2\right) \sigma - (1-\phi)^2 \sigma (2\rho + 1)}{\sigma^2 (1+\phi)^2}
\]
There, $\partial^3 (V_{K_i} - V_{K_j}) / \partial \lambda^3 < 0$ requires

$$\sigma (\sigma - 1) (1 + \phi) < (1 - \phi) \sigma (\sigma - 1) (2\rho + 1) + (1 - \phi) (1 + \phi)^2$$

$$\implies -2\sigma (\sigma - 1) [\phi (1 + \rho) - \rho] < (1 - \phi) (1 + \phi)^2$$

Inserting $\phi^*$ at $\beta = 0$, we derive

$$-\frac{\sigma (2\rho + 1)}{2(\sigma - 1)(1 + \rho) + \sigma} < (1 - \phi) (1 + \phi)^2$$

which is generally fulfilled. Hence, the unconstrained CPM is characterised by a supercritical pitchfork bifurcation, but the fair-wage-constrained one is not in general. Hence, the fair wage constrained CPM exhibits a supercritical pitchfork bifurcation pattern only in the neighbourhood of $\beta = 0$. At high levels of $\beta$ both full dispersion and full agglomeration may be an equilibrium at a given level of $\phi^*$ (hence, there may be three stable equilibria at this and higher levels of trade costs).

\section*{D Proof of Proposition 4}

In Appendix D, we derive the sign of $\partial \phi^* / \partial \beta$:

$$\frac{\partial \phi^*}{\partial \beta} = \frac{2(\sigma - 1)p(\sigma - \alpha) - 4\alpha \sigma - 8\sigma^2 \beta \rho^2}{(1 - \beta)^2} \left[ \frac{2(\sigma - 1)\left( \sigma + \frac{\rho(\sigma + \alpha \beta)}{1 - \beta} \right) + \sigma^2 - 4\alpha^2 \beta^2 \rho^2}{2(\sigma - 1)\left( \sigma + \frac{\rho(\sigma - \alpha \beta)}{1 - \beta} \right) - \sigma^2 - 4\alpha^2 \beta^2 \rho^2} \right]$$

In general, $\partial \phi^* / \partial \beta$ is ambiguous. However, it is useful to evaluate $\partial \phi^* / \partial \beta$ at $\beta = 0$ to derive

$$\left. \frac{\partial \phi^*}{\partial \beta} \right|_{\beta=0} = \frac{[2(\sigma - 1)\sigma (\rho + 1) + \sigma^2][2(\sigma - 1)\rho (\sigma - \alpha) - 4\alpha \sigma \rho]}{[2(\sigma - 1)\sigma (\rho + 1) + \sigma^2]^2} - \frac{[2(\sigma - 1)\sigma \rho - \sigma^2][2(\sigma - 1)\rho (\sigma + \alpha)]}{[2(\sigma - 1)\sigma (\rho + 1) + \sigma^2]^2}$$

Still, the sign of $\partial \phi^* / \partial \beta |_{\beta=0}$ is not obvious. However, it can be determined after taking into account the following three conditions.
First, diversification of production in equilibrium requires
\[
\frac{\rho}{(\sigma - 1)} > r = \frac{\alpha (2\rho + 1)}{\sigma} \quad \implies \\
\alpha < \frac{\rho \sigma}{(\sigma - 1) (2\rho + 1)}.
\]
Second, \( r \) must be at least as high as unity for the fair wage approach to be meaningful:
\[
r = \frac{\alpha (2\rho + 1)}{\sigma} > 1 \quad \implies \\
\alpha > \frac{\sigma}{2\rho + 1}.
\]
Hence, for \( \alpha \) we know that
\[
\frac{\sigma}{2\rho + 1} < \alpha < \frac{\rho \sigma}{(\sigma - 1) (2\rho + 1)}.
\]
Whereas \( \phi^* \leq 1 \) is an unbinding constraint, a third condition emerges from \( 0 \leq \phi^* \), which yields
\[
\rho \geq \frac{\sigma}{2(\sigma - 1)}.
\]
Note that \( \frac{\partial \phi^*}{\partial \beta}|_{\beta = 0} < 0 \) requires
\[
[2 \sigma - 1 (\rho + 1 + \sigma)] [(\sigma - 1) (\sigma - \alpha) - 2\alpha \sigma] < [2 \sigma - 1 \rho - \sigma] [\sigma - \alpha] (\sigma + \alpha)].
\]
Subtracting \( 2(\sigma - 1) \rho (\sigma - 1) \sigma \) from both sides yields
\[
2(\sigma - 1)^2 \sigma - 2(\sigma - 1)^2 (\rho + 1) \alpha + \sigma (\sigma - 1) (\sigma - \alpha) - 4\alpha \sigma (\sigma - 1) (\rho + 1) - 2\alpha \sigma^2 \\
< 2(\sigma - 1)^2 \alpha \rho - \sigma (\sigma - 1) (\sigma + \alpha).
\]
By rearranging and collecting all \( \alpha \)-terms, we derive
\[
2(\sigma - 1)^2 \sigma + 2(\sigma - 1) \alpha \sigma^2 < \alpha \left[ 2(\sigma - 1)^2 (\rho + 1) + 4\sigma (\sigma - 1)(\rho + 1) + 2\sigma^2 + 2(\sigma - 1)^2 \rho \right].
\]
Hence,
\[
\alpha > \frac{(\sigma - 1)^2 \sigma + (\sigma - 1) \sigma^2}{(\sigma - 1)^2 (\rho + 1) + 2\sigma (\sigma - 1)(\rho + 1) + \sigma^2 + (\sigma - 1)^2 \rho}.
\]
The second condition \( (r > 1) \) must hold, so that it is sufficient to show that
\[
\frac{(\sigma - 1)^2 \sigma + (\sigma - 1) \sigma^2}{(\sigma - 1)^2 (\rho + 1) + 2\sigma (\sigma - 1)(\rho + 1) + \sigma^2 + (\sigma - 1)^2 \rho} < \frac{\sigma}{2\rho + 1}
\]
which holds if
\[
[(\sigma - 1)^2 + (\sigma - 1) \sigma] (2\rho + 1) < (\sigma - 1)^2 (2\rho + 1) + 2\sigma (\sigma - 1)(\rho + 1) + \sigma^2
\]
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$$\Leftrightarrow (\sigma - 1) \sigma (2 \rho + 1) - 2 \sigma (\sigma - 1) (\rho + 1) - \sigma^2 < 0$$
$$\Leftrightarrow -\sigma (\sigma - 1) - \sigma^2 < 0.$$ Hence, a marginal increase of $\beta$ in an initial equilibrium with $\beta = 0$ generally shifts the breakpoint towards higher levels of trade costs, that is $\partial \phi^*/\partial \beta \big|_{\beta=0} < 0$.

E Proof of $\partial \phi/\partial \beta < 0$

In Appendix E, we derive $\bar{\phi}$ (the value of $\phi$ where the market crowding effect exactly offsets the market access in $r_i - r_j$) and the sign of $\partial \bar{\phi}/\partial \beta$. Note that

$$\frac{\partial (r_i - r_j)}{\partial \lambda} = \frac{(1 - \phi^2) \left( \sigma + 2 \alpha \beta \rho \right) - \left( \frac{2 \rho}{1 - \beta} + 1 \right) \sigma (1 - \phi)^2}{\left[ \frac{\sigma}{\beta} (1 + \phi) + \alpha \beta \rho \right]^2 - \alpha^2 \phi^2 \beta^2 \rho^2} = 0$$

at

$$\bar{\phi} = \rho \frac{\sigma - \alpha \beta}{\sigma (1 - \beta + \rho) + \alpha \beta \rho}$$

Then,

$$\frac{\partial \bar{\phi}}{\partial \beta} = -\rho \frac{(2 \rho + 1) \alpha - \sigma}{\left[ \sigma (1 - \beta + \rho) + \alpha \beta \rho \right]^2}.$$ Note that $\partial \bar{\phi}/\partial \beta < 0$ requires that $\alpha > \sigma / (2 \rho + 1)$ which holds generally whenever $r_i, r_j > 1$ at $\beta = 0$.

References


