Information Costs, Networks and Intermediation in International Trade*

Dimitra Petropoulou†

February 2006‡
CEP Working Paper No. 1442

Abstract

This paper considers the role of information intermediaries in facilitating international trade and examines the part information costs play in explaining that role. A pairwise matching model with two-sided information asymmetry is developed to analyse the impact of information costs on the incentives for network building and matching by information intermediaries. The model is used to show how information costs affect the way in which trade is organised, either directly between traders or indirectly through trade intermediaries. When information costs are high, direct and indirect trade are shown to coexist in equilibrium. Moreover, intermediaries are shown to raise trade volume and welfare. The framework sheds light on the effect of information and communication technology (ICT) improvements on both the level and means of organisation of trade. In the repeated game, where cooperative symmetric equilibria with multiple intermediaries are sustainable, ICT improvements strengthen the incentives for network building by intermediaries but lower the number of intermediaries in the market due to downward pressure on commission rates. The implications of a small tariff and a small ethnic network on the incentives for intermediation, trade and welfare are also examined.

Keywords: International Trade, Pairwise Matching, Information Cost, Intermediation, Networks

JEL Classification Numbers: F10, C78, D43, D82, D83, L10

*I would like to thank the ESRC for their financial support and Tony Venables, Alejandro Cunat, Steve Redding, Henry Overman, Gilles Duranton and Kwok Tong Soo for their invaluable comments. Moreover, I would also like to thank all seminar and conference participants at the London School of Economics, and elsewhere, for their feedback. Any errors are mine.

†London School of Economics, Department of Economics, Houghton Street, London WC2A 2AE, United Kingdom. E-mail address: d.petropoulou@lse.ac.uk

‡This paper was presented at the CEPR European Research Workshop on International Trade (ER-WIT), June 2005, the 4th Conference on Research in Economic Theory and Econometrics (CRETE), July 2005, the Association of Southern European Economic Theorists (ASSET), October 2005, as well as at the London School of Economics International Trade Seminar, University of Cyprus, the Aristotle University of Thessaloniki, Greece and the University of Ioannina, Greece as well as at the Centre for Economic Performance (CEP) annual Stoke Rochford Conference.
1 Introduction

Over 90% of world trade is carried out by sea, for which almost all trade transactions include an intermediary at some stage. Trade intermediaries can serve many functions, and there is a broad literature on the role of middlemen. They have been shown to reduce search costs (Rubinstein and Wolinksy, 1987; Yavas, 1992, 1994), to offer expertise in markets with adverse selection (Biglaiser, 1993), to operate as guarantors of quality under producer moral hazard (Biglaiser and Friedman, 1993), as well as to operate as investors in quality-testing technology (Li, 1998). More recently, Shevchenko (2004) endogenises the number of intermediaries who buy and sell goods and examines the optimality of the size and composition of their inventories.

This literature explores the role of middlemen as buyers and sellers of goods. In contrast, this paper explores the role of intermediaries as brokers of information rather than goods. We consider the role of information intermediaries in facilitating international trade and examine the part information costs play in explaining that role.

Information is required to identify profitable trading opportunities and locate suitable trading partners, particularly where goods are differentiated and information about product characteristics is important. Information asymmetries, coupled with costs of acquiring information, can hinder the matching of agents with opportunities and prevent prices from allocating scarce resources across countries. Portes and Rey (1999) point to a lack of information about international trading opportunities and the need to tap into ‘deep knowledge’. In such a setting, international trade can be facilitated through intermediaries who invest in information networks or contacts and match agents with suitable opportunities for a fee.

Rauch and Watson (2002) present some summary statistics from a Pilot survey of international trade intermediaries based in the US. Despite the small number of observations, their evidence suggests that 50% of intermediation in trade of differentiated products does not involve taking title of goods and reselling, while the figure for intermediation in homogeneous-goods is only 1%. Moreover, 36% of the revenue of differentiated-product intermediaries is reported to come from success fees based on the value of transactions, while the figure for homogeneous-good intermediation is only 1%. This is consistent with the search based or network view of trade, pioneered by Rauch (1999), Rauch and Trindade (1999) and others, that posits that the information requirements for differentiated goods are much greater due to the need to match specific characteristics. The evidence to date supports this, pointing to a more pronounced role for information intermediaries and ethnic networks in the trade of differentiated goods.

The facilitation of trade through information networks has only recently begun to be formally developed. Recent literature on networks in international trade (Casella and Rauch, 2001) focuses on gaining insight on how information-sharing networks among internationally dispersed ethnic minorities or business groups can overcome informal trade barriers such as inadequate information about trading opportunities and weak enforcement of international contracts (Anderson and Marcouiller, 2002).

---

1 Source: International Maritime Organisation (IMO)
Casella and Rauch (2001) develop a model where output is produced through a joint venture where agents with complete information domestically cannot judge the quality of their match abroad. Introducing a subset of agents with social ties that have complete information in international matches with other group members increases aggregate trade and income, but hurts the anonymous market by depriving it disproportionately of the groups more productive members. The authors assume costless matching in the international market, coupled with a lack of information about the quality of foreign matches, to address the effects of pre-existing social ties or group membership on trade. As such, contact-building, the costs of locating a foreign match or scope for trade intermediation are not addressed.

Rauch and Watson (2002) model the supply of ‘network intermediation’ where agents endogenously choose whether to be producers or intermediaries, depending on their endowment of contacts. The authors show that incentives to form network intermediaries may be sub-optimal. Again, contact-building is not modelled, rather, agents have an endowment of contacts from a known distribution that allows the choice to become an intermediary to be analysed.

Caillaud and Julien (2003) examine imperfect competition between information intermediation service providers on the internet, determine the market structures likely to emerge and characterise intermediaries’ pricing strategies. The emphasis of this paper is to provide an explanation for specific features of internet information intermediation, such as price discrimination and the use of exclusive or non-exclusive contracts.

This paper explicitly examines the role of information costs on the incentives for information intermediaries to emerge as trade facilitators and addresses a broad range of questions in a tractable, unified framework. First, how do information costs affect trade patterns and the way trade is organised, either directly or indirectly through intermediaries? Second, how might network structures help overcome information barriers? Third, how might intermediation affect trade and welfare and how might direct links between traders, such as through ethnic networks affect trade and welfare? Finally, how might improvements in information and communication technology (ICT) that lower information costs affect the way in which trade is organised?

The pairwise matching model developed in this paper contributes to the literature in several ways. First, the framework shows how information costs affect the realisation and organisation of trade transactions, for a given set of trade opportunities. Second, information intermediation and network building are endogenous to the model. Third, the framework provides an explanation for the emergence of information intermediaries when information barriers are high and examines the impact of declining information costs on the pattern of intermediation. Finally, co-ethnic ties between traders are introduced and the effects on intermediation and the anonymous market examined.

The model is particularly applicable to international trade in differentiated goods for which information about product characteristics is important. The model can also be applied more broadly to intermediated markets where contact-building and matching are key. Examples may include headhunters in the job market, real estate agents in the housing or rental market, charterers in the transportation market, matchmakers in the marriage market (in some cultures) and others.

The rest of the paper is organised as follows. Section 2 presents the model with
a single intermediary. Section 3 extends the model to include multiple intermediaries. Section 4 discusses the main findings of the model and concludes.

2 The Model with a Single Intermediary

We begin by building the framework with a single intermediary. We then add to the model by allowing free entry of intermediaries and a time dimension in Section 3. This step by step approach is beneficial to the reader as it offers a clearer explanation of the structure and intuition behind the model through which the complexities arising from interaction between intermediaries can best be understood. Moreover, comparative statics and the effect of trade policy on the incentives for network building and intermediation are illustrated more clearly with one firm.

2.1 The Basic Framework

Consider a pairwise matching model with a continuum of exporters on the interval \([0, 1]\) in the Home country and a continuum of importers on the interval \([0, 1]\) in the Foreign country. Importers and exporters are distributed uniformly along the line with unit density. For a given trader, there is a unique matching partner on the other side of the market. Further, importers and exporters match in pairs to trade 1 unit of output, where each match generates a joint surplus \(S > 0\). If agents fail to locate their match they do not trade and therefore receive a payoff of 0. Assume all market participants are risk-neutral.
Suppose the importer and exporter distributions are placed facing each other, as in Figure 1, and that any given exporter can only match and trade with his importing counterpart on the opposite side. The framework can best be seen to reflect trade in differentiated goods where specific characteristics have to be matched. Alternatively, the goods traded may be homogeneous with the differentiation over the interval [0,1] reflecting differences in the desired future date of delivery, where traders sign forward contracts in advance. With no trade frictions, importers and exporters identify each other costlessly and all trade opportunities are exploited generating a total surplus of S.

Let there be two-sided information asymmetry. That is, traders on each side of the market do not know where their partner lies on the opposing interval [0,1]. A given trader has zero probability of locating his matching partner by just picking randomly from the opposite population of traders. We assume the trader can, however, match by chance with probability \( q(i) \), where parameter \( i \in [0,1] \) reflects the level of information costs or barriers in the economy. This is the probability of a double-coincidence match, given the level of information technology, \( i \). Let \( q'(i) < 0 \), so that improvements in information and communication technology, reflected by a fall in \( i \), give rise to an increase in the probability of matching pairs coming together. Further, let \( q(1) = 0 \) and \( q(0) = 1 \), so information cost level \( i = 1 \) prohibits any matching, while \( i = 0 \) corresponds to the perfect information case where all trade opportunities are exploited. Imagine that where information barriers are not prohibitive, useful information about a portion of traders filters through allowing some direct trade to take place. Thus, \( q(i) \) is the expected direct trade volume and \( q(i)S \) the expected joint surplus generated from direct trade.

Further suppose, for concreteness\(^2\), that \( q(i) = 1 - i \), so the expected joint surplus of direct trade is \( (1 - i)S \). Note that the current set-up with \( (1 - i) \) expected matches each generating \( S \), is isomorphic to a framework where all matches take place but the search process required to achieve a match erodes the gains from trade by a proportion \( i \). Throughout the paper we maintain the former interpretation but results can easily be re-interpreted using the latter.

Now consider a single intermediary with access to a particular technology for building contacts with importers and exporters. An information network can be defined as a group of importers and exporters and the information links between them. Suppose the intermediary can invest in an information network, which takes the form of a list of exporters and a list of importers. The intermediary randomly contacts importers and exporters and collects the pertinent information for trade (location, product characteristics, delivery date), while making himself known to the traders. The traders do not, however, find out information about each other.

Setting up the network requires a fixed cost \( F \). Let the marginal cost of seeking out an additional trader to join the network be \( c(i) \), a convex function with \( c'(i) > 0 \) and \( c''(i) > 0 \). It is therefore increasingly costly for the intermediary to form a new contact as the level of information costs rises. Further, let \( c(0) = 0 \) making it costless to

\(^2\)Note that any example in which \( q(i) \) is declining in \( i \), and in which \( q(1) = 0 \) and \( q(0) = 1 \), yields qualitatively similar results. The rate of decline of \( q(i) \) with \( i \) does affect the level of thresholds and network size, but not the pattern of results.

Hence, the choice of \( q(i) = 1 - i \) does not restrict the model in any way and is made purely to simplify the derivation of results.

5
make contacts under perfect information. Crucially, the barriers to information flow, as captured by parameter \( i \), affect both traders seeking each other and the intermediary’s marginal cost for building the network. That is, more efficient information flow, as captured by a lower value of \( i \), improves the search technology of traders, but also facilitates more efficient network-building by the intermediary.

Let \( P_X \) and \( P_M \) be the proportion of importers and exporters on each list respectively, where \( P_X \in [0, 1] \) and \( P_M \in [0, 1] \). Hence the intermediary’s total cost of building a network is given by (1).

\[
C = F + c(i)(P_X + P_M)
\]  

(1)

The intermediary withholds the details of the list and so the importers and exporters that match via the network can only do so with the assistance of the intermediary, a service for which the intermediary commands a share of the surplus. As will be shown later in the section, the commission charged by the intermediary is pinned down exclusively by the level of information costs in the economy, which determines the probability of a direct match in the absence of intermediation. The intermediary maximises profit subject to the participation constraint of traders contacted and so designs a contract that depends exclusively on \( i \).

The timing of the game is as follows. In stage 1 the intermediary contacts a proportion of importers and exporters forming an information network and offers them a take-it-or-leave-it contract specifying \( \alpha_f(i) \), the intermediary’s commission rate for matching them with their trading partner. In stage 2, traders choose whether to accept or reject the contract. Indirect trade matches amongst those who accept take place through the network in stage 3 and the intermediary retains his success fee. In stage 4, any unmatched traders (those not contacted in stage 1, those that reject the intermediary’s offer in stage 2 and those who accept but do not find their match through the network in stage 3) have the opportunity to trade directly with probability \( q(i) = 1 - i \). We proceed to solve for the subgame perfect Nash equilibrium of the game by backwards induction.

In the final stage of the game, any unmatched traders face a probability \( q(i) \) of matching directly and generating expected joint gains from trade at \((1 - i)S\) that are split between the importer and exporter. This probability of a direct match represents the likelihood that the pertinent information required for trade filters through, despite trade barriers reflected by \( i \). The greater are trade barriers, the less likely it is for the information to filter through.

Since \( i \) is exogenous, the probability of any pair trading directly is also exogenous to the model and is assumed to be unaffected by the intermediary’s choice of network size, \( P \). That is, the likelihood of any particular pair matching directly in the final stage of the game is unaffected by the number of matches made through the intermediary earlier in the game. For the sake of parsimony, there is no explicitly modelled search process carried out by traders in the model. The focus of this paper is to examine the incentives for intermediation in the presence of information barriers, for which the simplifying assumption of an exogenous direct trade option is sufficient.

Alternatively, we may imagine there is some search process, the probability of success of which is summarised by \( q(i) \). Since unmatched traders in the final stage have no
information about which particular traders have matched through the intermediary in stage 3, even if they have an expectation about the proportion that has matched, their search must still span the entire range of possible partners. Hence, we believe the assumption that $q(i)$ is unaffected by $P$ is a reasonable one given the inherent lack of information in the model.

Let $\alpha_X$ and $\alpha_M$ be the exporter’s and importer’s surplus share, respectively, where $\alpha_X + \alpha_M = 1$. For simplicity, assume that both parties have equal bargaining power and so split the surplus equally between them. That is, $\alpha_X = \alpha_M = \frac{1}{2}$. Exporters’ and importers’ expected payoffs from direct trade ($E^{DT}(\Pi_X)$ and $E^{DT}(\Pi_M)$, respectively) are expressed in equation (2).

$$E^{DT}(\Pi_X) = E^{DT}(\Pi_M) = \frac{1}{2} q(i) S = \frac{1}{2} (1 - i) S$$

(2)

In stage 3, the intermediary matches the importers and exporters in his information network who have accepted the contact in stage 2, generating $S$ per match. Since all traders are identical, they will either all accept or all reject the take-it-or-leave-it offer. As such, the intermediary maximises his expected profit in stage 1 subject to the participation constraints of importers and exporters. In equilibrium, therefore, all traders accept the offer in stage 2.

The probability of any particular trade match occurring indirectly depends on the intermediary’s choice of information network size ($P_X$ and $P_M$) and is less than 1 if not all traders are part of the intermediary’s network. Let $\alpha_j(i)$ denote the share of $j$, given the level of information costs $i$, where $j = \{X, M, I\}$. For simplicity and comparability with the direct trade case, allow exporters and importers equal power i.e. $\alpha_X(i) = \alpha_M(i) \equiv \alpha_I(i)$. Equation (3) follows, where $\alpha_I(i)$, the surplus share of the intermediary, is determined endogenously.

$$2\alpha_I(i) + \alpha_I(i) = 1$$

(3)

The maximum number of matches that can be achieved through the network is $\min\{P_X, P_M\}$, the minimum number is $\max\{P_X + P_M - 1, 0\}$ while the expected number of intermediated matches is $P_X P_M$. In equilibrium, $P_X = P_M \equiv P$ since equalising the size of the lists maximises the expected number of intermediated matches for any given level of investment. This implies the maximum number of matches is $P$, while the minimum number of matches is $2P - 1$ for $\frac{1}{2} \geq P \geq 1$ and 0 for $P < \frac{1}{2}$. For an exporter (importer) deciding whether to trade via the network, the probability of her partner also being in the network is $P$, the proportion of importers (exporters) on the intermediary’s list. The expected number of intermediated matches given a network of size $P$ is $P^2$.

Consider any pair $j$ of trade partners ($X_j, M_j$). The expected payoff of exporter $X_j$ (or $M_j$) if she signs up to the intermediary, conditional on being contacted by the intermediary in stage 1, is expressed in equation (4).

---

3 This is a simplifying assumption designed to make exporters’ and importers’ payoffs symmetric for the purpose of computational ease. The intermediary’s choice of network size is unaffected by the way in which residual surplus is split between the traders, provided these shares are the same under direct and indirect trade.

4 The probability of any pair matching integrated over the range of possible pairs.
\[ E(\Pi_{X_j} \mid X_j \in P) = E(\Pi_{M_j} \mid M_j \in P) = \frac{1}{2} S \{ P[1 - \alpha_I(i)] + (1 - P)q(i) \} \]  

(4)

To ensure trader participation in stage 2, the intermediary must set \( \alpha_I(i) \) sufficiently low such that the expected payoff from signing up to the network is at least as large as the expected payoff from direct trade. This requires that the expected payoff of equation (4) is at least as large as that of (2). Thus, the network participation constraint is given by the following inequality:

\[ \alpha_I(i) \leq 1 - q(i) = i \]  

(5)

Anticipating the behaviour of traders in stage 2, the intermediary extracts as much surplus as possible subject to trader participation. It follows that in equilibrium \( \alpha_I(i) = 1 - q(i) = i \) is chosen in stage 1 for all values of \( P \) and in stage 2 all traders accept\(^5\). That is, the intermediary offers the traders a contract with commission rate \( s_i \), which leaves the risk-neutral traders indifferent between signing up and trying their luck directly, irrespective of \( P \).

We need not make any assumptions about the traders’ beliefs regarding network size, \( P \), or about the truthfulness of any announcements of network size made by the intermediary, since ultimately \( P \) drops out of the participation constraint and it is the traders’ outside option of direct trade, pinned down by \( i \), which determines the maximum commission rate the intermediary can charge.

At the outset of the game, there are four possible positions for any pair \((X_j, M_j)\). First, both partners lie outside the intermediary’s network and so match with probability \( q(i) \). This event occurs with probability \((1 - P)^2\) and generates an expected payoff of \( \frac{1}{2}q(i) S \) for each trader. Second, both partners are inside the network, an event that occurs with probability \( P^2 \). The payoff to each trader when matched by the intermediary is \( \frac{1}{2}S [1 - \alpha_I(i)] \). Third, \( M_j \) lies inside the network and \( X_j \) outside. This occurs with probability \( P(1 - P) \) and, since a match cannot be made through the intermediary, generates an expected payoff to each of \( \frac{1}{2}q(i) S \). Finally, \( X_j \) lies inside the network and \( M_j \) outside, also with probability \( P(1 - P) \) and with an expected payoff to each of \( \frac{1}{2}q(i) S \).

It follows that the \textit{ex ante} expected payoff to any trader \( j \) at the outset of the game, given a network size \( P \), can be expressed by equation (6):

\[ E(\Pi_{X_j} \mid P) = E(\Pi_{M_j} \mid P) = \frac{1}{2} S \{ q(i)(1 - P^2) + [1 - \alpha_I(i)] P^2 \} \]  

(6)

Given that \( q(i) = 1 - i \) and anticipating that \( \alpha_I(i) = i \), it follows that \( E(\Pi_X \mid P) = E(\Pi_M \mid P) = \frac{1}{2}(1 - i)S = E^{DT}(\Pi_X) = E^{DT}(\Pi_M) \). That is, traders are indifferent between having an intermediary in the market operating a network of size \( P \), or not. To evaluate the welfare implications of introducing an intermediary as well as the impact

\[^5\text{Assume that when indifferent between the two modes of trade, traders sign up with the intermediary. Alternatively, assume the intermediary offers an infinitesimally small additional amount, } \varepsilon, \text{ to ensure traders sign up to the network.}\]
on total trade, the equilibrium choice of $P$ needs to be examined. The intermediary chooses $P$ to maximise expected profits subject to $\alpha_i^*(i) = 1 - q(i) = i$. The expected profits of the intermediary at the outset of the game are described by equation (7):

$$E(\Pi_I) = iSP^2 - 2Pc(i) - F$$  \hspace{1cm} (7)

For a given level of information costs $i$, and appropriate parameter values for $S$ and $F$, the intermediary has an incentive to maximise network size. The optimal network size, conditional on covering fixed costs, is therefore 1. It follows that for the range of information costs under which fixed costs are covered, the network includes all traders, so $P^* = 1$. With $F > 0$ there is a range of values of $i$ under which the intermediary cannot make non-negative profits. In this case the information network is not viable and so $P^* = 0$.

The intermediary’s expected profit with a complete network is given by (8).

$$E(\Pi_I|P = 1) = iS - 2c(i) - F$$  \hspace{1cm} (8)

Let $\hat{i}$ be the threshold level of information costs above which $E(\Pi_I|P = 1) \geq 0$. It therefore follows that for sufficiently high levels of information costs $\hat{i} \leq i \leq 1$, where $0 \leq F \leq iS - 2c(i)$, the network includes all traders and the intermediary extracts a portion $i$ of the gains from trade such that $P^* = 1$ and $\alpha_i^*(i) = i$. All trade matches occur indirectly through the intermediary. Alternatively, under sufficiently low levels of information cost $0 \leq i \leq \hat{i}$, where $F > iS - 2c(i)$, the fixed cost is prohibitively high relative to information costs and so the intermediary is inactive. That is, $P^* = 0$ and $q(i)$ trade matches occur directly.

Hence:

**Proposition 1** Under the assumptions of the model, the following strategies for a single intermediary and traders constitute the unique, subgame perfect equilibrium of the game: Traders choose to accept the intermediary’s offer if $\alpha_I(i) \leq i$, and reject otherwise. The intermediary chooses $[P^*, \alpha_I(i)] = [1, i]$ if $0 \leq F \leq iS - 2c(i)$, and $[P^*, \alpha_I(i)] = [0, i]$ otherwise.

Figure 2 illustrates the equilibrium network size ($P$) against information cost ($i$) for parameter values $F = 5$ and $S = 16$ and marginal cost function $c(i) = i^2$. The functions and parameter values have been chosen to illustrate the case in which the intermediary has an incentive to operate an information network within the parameter range $i \in [0, 1]$. The values chosen for $F$ and $S$ are important since the intermediary is inactive, for the entire range of $i$, when $F$ and $c(i)$ are particularly high relative to $S$. The outcome is less sensitive to the particular functional form of $c(i)$.

The figure depicts a series of isoprofit contours for the intermediary. The lowest contour is the zero isoprofit contour, with higher contours reflecting positive profit levels. For any given level of $i$ the intermediary chooses network size $P$ such that the highest possible isoprofit contour is attained.

The threshold value $\hat{i}$ where profit is zero, above which $P^* = 1$ is $\hat{i} = 4 - \frac{3}{2} \sqrt{6}$ in this example. For $4 - \frac{3}{2} \sqrt{6} \leq i \leq 1$, network-building is profitable and the network includes
Figure 2: $F = 5$, $S = 16$, $c(i) = i^2$

all traders. All trade matches therefore occur indirectly. As information costs decline with ICT improvements, so does the share of the gains from trade appropriated by the intermediary. The lower is $i$, the higher is the expected profit from direct trade, which implies a lower commission for the intermediary. Below $\hat{i} = 4 - \frac{3}{2}\sqrt{6}$, the intermediary can no longer sustain the network. Furthermore, the smaller is $F$, or the higher is $S$, the lower is the threshold level $\hat{i}$ below which the network stops being viable. Figure 3 illustrates the expected profit of the intermediary, $E(\Pi_I)$ as a function of information costs $i$, and network size $P$. The isoprofit contours of Figure 2 are cross-sections of the $E(\Pi_I)$ function. Both figures illustrate that the intermediary’s expected profit is increasing in $i$, for any given level of $P$. Moreover, $E(\Pi_I)$, increases with $P$, for any given $i$. There is no interior maximum path under these assumptions and so the intermediary chooses to operate an exhaustive network, conditional on making non-negative profits. The expected trade volume under direct search is depicted in Figure 4. The expected number of units traded, $q(i) = 1 - i$, falls from 1 to 0, and the gains from trade, $q(i)S$, fall from $S$ to 0 as information costs rise. With an intermediary, expected intermediated trade is $P^2$ and expected direct trade is $(1 - P^2)q(i) = (1 - P^2)(1 - i)$, yielding a total of $[(1 - P^2)(1 - i) + P^2]$. It follows that the expected number of matches not made is $(1 - P^2)i$. Total expected trade volume is therefore 1 if $P^* = 1$ and $q(i)$ if $P^* = 0$, as illustrated in Figure 5 below. There is a dip in trade volume when $i$ falls below the threshold. In a sense, the traders’ outside option of direct trade is ‘too good’ and the intermediary is unable to compensate traders’ accordingly and cover costs. Hence,

**Corollary 1:** Under the assumptions of the model, equilibrium expected direct trade volume is $(1 - i)$ if $F \geq iS - 2c(i)$ and 0 otherwise. Indirect trade is 1 if $0 \leq F \leq$
Figure 3: $E(\Pi_I)$ under $F = 5, S = 16, c(i) = i^2$

Figure 4: Expected direct trade
Recall that we can re-interpret the direct trade search technology such that all trade matches occur, but a proportion \( i \) of the gains from trade are eroded due to the costs of search. Under this interpretation, all matches take place over the entire range of possible values of \( i \). The expected gains from trade are invariant to the interpretation we choose, however, the ex post distribution of the gains differ. With \( q(i) \) defined as a probability, only a proportion of trade matches occur generating the entire gains from trade. With the alternative interpretation of \( i \), all traders trade but generate a smaller gain.

Now consider the welfare implications of introducing the intermediary. When \( i > \hat{i} \), the intermediary leaves the traders indifferent relative to the direct trade case, and gains an expected profit \( E(\Pi_I) = iS - F - 2c(i) > 0 \). Hence there are positive welfare gains from introducing an intermediary. This is due to the fact that information costs are a real resource cost. Given network viability, the intermediary’s technology allows for more efficient matching than under direct trade. Despite the dip in expected trade volume depicted in Figure 4, there is no corresponding discontinuity in expected welfare as information costs decline. As \( i \) falls towards \( \hat{i} \), expected intermediary profit is squeezed as the commission declines. At \( \hat{i} \), \( E(\Pi_I) = 0 \) and the joint trade surplus generated through expected direct trade is \( (1 - \hat{i})S \). Further declines in \( i \) below \( \hat{i} \) smoothly increase expected welfare towards \( S \).

### 2.2 Loading of the Intermediary

The results in the simple case of subsection 2.1 are extreme in the sense that the information network either includes all traders or none at all. For parameters such that network
building is profitable, it pays for the intermediary to expand the network to include all traders. In practice, however, networks are not exhaustive and both direct and indirect trade is observed. This section endogenises the intermediary’s costs by examining the case where the intermediary’s choice of network size affects this fixed cost of network building\(^6\). The introduction of an additional set of assumptions regarding the nature of fixed costs gives rise to an interior solution for the choice of network size under which both direct and indirect trade co-exist in equilibrium.

It is economically reasonable to assume that the costs of operating an information network, or managing a set of contacts, depend on the size of the information network. Furthermore, one may suppose that information costs affect the magnitude of the additional burden a larger information network has on the costs of operating it. That is, one may reasonable expect that a larger network gives rise to loading of the intermediary and, moreover, that the degree of loading depends on the level of information costs.

In the context of the framework introduced in subsection 2.1, such loading of the intermediary can be modelled by introducing an additional loading cost that is a function of information costs, \(i\), and network size, \(P\). Let the loading cost of operating a network of size \(P\), given \(i\), be \(L(i, P)\), described by (9) - (11). Fixed cost remains \(F\).

\[
\begin{align*}
L_i &= \frac{\partial L(i, P)}{\partial i} > 0 \text{ and } L_{ii} = \frac{\partial^2 L(i, P)}{\partial i^2} > 0 \\
L_P &= \frac{\partial L(i, P)}{\partial P} > 0 \text{ and } L_{PP} = \frac{\partial^2 L(i, P)}{\partial P^2} > 0 \\
L_{iP} &= L_{Pi} = \frac{\partial^2 L(i, P)}{\partial P \partial i} > 0
\end{align*}
\]

(9) - (11)

Loading cost is increasing in both information costs and network size, with positive second derivatives and a positive cross derivative\(^7\). The latter implies that loading of the intermediary through \(P\) is increasing in the level of information costs, which is the driving assumption behind the result in Proposition 2 that for high levels of intermediation costs the intermediary chooses an interior solution for network size\(^8\).

With the added assumptions described by the (9) - (11) the intermediary chooses \(P\) to maximise expected profits (12) subject to constraint (5):

\[
E(\Pi_i | P) = ISP^2 - 2PC(i) - L(i, P) - F
\]

(12)

The first order condition of the intermediary’s maximisation problem is given by equation (13)

\(^6\)One may argue that when fixed costs depend on \(P\) they are no longer fixed costs but variable costs! For the purpose of consistency I will continue to call the function \(F(i, P)\) ‘fixed cost’ and the component of this function that is independent of \(P\), the ‘pure fixed cost component’.

\(^7\)The same qualitative results that follow from these additional assumptions can also be found by assuming \(P\) increases the marginal cost of network building such that \(c = c(i, P)\), where \(c_i > 0, c_P > 0, c_{ii} > 0, c_{PP} > 0\) and \(c_{iP} = c_{Pi} > 0\). It is, however, intuitively more appealing to suppose that overhead costs are increasing in \(P\) and \(i\), rather than marginal costs.

\(^8\)The case where \(F_{iP} = 0\) is discussed in Section 2.2.1. The interaction between \(i\) and \(P\) in fixed cost is severed resulting in a pattern of intermediation qualitatively similar to that in Section 1.
\[ \frac{\partial E(\Pi_I|P)}{\partial P} = 2iSP - 2c(i) - LP = 0 \]  

(13)

The second order condition is given by (14):

\[ \frac{\partial^2 E(\Pi_I|P)}{\partial P^2} = 2iS - LP_P \]  

(14)

The sign of the second order condition depends on the level of information costs in the economy and the functional form of \( L \). Typically the first order condition gives rise to a profit maximising and profit minimising path of \( P \) with respect to \( i \). Let \( \tilde{P}(i) \) correspond to the profit maximising path. Higher information costs give rise to a greater degree of loading of the intermediary through \( L_iP = L_{Pi} > 0 \) and so the profit maximising network size \( \tilde{P}(i) \) is decreasing in \( i \). Substituting \( \tilde{P}(i) \) into the intermediary’s expected profit equation gives rise to the expected profit level under this path, which depends on \( i \) and is given by equation (15).

\[ E(\tilde{\Pi}_I) = iS \left[ \tilde{P}(i) \right]^2 - 2c(i) \left[ \tilde{P}(i) \right] - L \left[ i, \tilde{P}(i) \right] - F \]  

(15)

If \( E(\tilde{\Pi}_I) \geq 0 \), given that \( P \in (0, 1) \), then the intermediary will choose \( P^* = \min(\tilde{P}(i), 1) \) to maximise expected profits. If \( E(\tilde{\Pi}_I) < 0 \), however, the intermediary will be inactive.

Hence:

**Proposition 2** Under the assumptions of the model and \( L(i, P) \) described by (9)-(11), the following strategies for a single intermediary and traders constitute the unique, subgame perfect equilibrium of the game: Traders choose to accept the intermediary’s offer if \( \alpha_I(i) \leq i \), and reject otherwise. The intermediary chooses \([P^*, \alpha_I(i)] = \left[ \min(\tilde{P}(i), 1), i \right]\) if \( E(\tilde{\Pi}_I) \geq 0 \) and \([P^*, \alpha_I(i)] = [0, i] \) otherwise.

Let \( \hat{i} \) be the threshold level of information costs at which \( E(\tilde{\Pi}_I) = 0 \). This corresponds to the level of information costs above which the intermediary becomes active. Further, let \( \hat{i} \) be the threshold level above which the intermediary no longer chooses an exhaustive network, but instead follows path \( P^* = \tilde{P}(i) \). If follows that for \( 0 \leq i < \hat{i} \), all trade takes place directly and the expected trade volume is \( q(i) \). For \( \hat{i} \leq i \leq i \), the information network is exhaustive and so all trade takes place indirectly through the intermediary and trade volume is 1. Finally, for \( \hat{i} < i \leq 1 \), the intermediary’s network is inexhaustive and so direct and indirect trade take place simultaneously in the subgame perfect equilibrium. Expected trade volume through the intermediary is \( \left[ \tilde{P}(i) \right]^2 \) and expected indirect trade is \( q(i) \left[ 1 - \left( \tilde{P}(i) \right)^2 \right] = (1 - i) \left[ 1 - \left( \tilde{P}(i) \right)^2 \right] \). It follows that total expected trade is \( 1 - i \left[ 1 - \tilde{P}(i) \right] \).
Figure 6 illustrates the intermediary’s equilibrium choice of $P$ under $S = 16$ and cost functions $c(i) = i^2$, $L(i, P) = 10i^2P^7$ and $F = 2$. The functions and parameter values have been chosen to illustrate the interesting case in which the intermediary has an incentive to operate an inexhaustive information network within the parameter range $i \in [0, 1]$.

The figure depicts a series of isoprofit contours for the intermediary, the lowest of which is the zero isoprofit contour. Higher contours reflect positive profit levels. For any given level of $i$ the intermediary chooses network size $P$ such that the highest possible isoprofit contour is attained. The optimal path, $\hat{P}(i)$, derived from the first order condition of the intermediary’s profit maximisation problem, is negatively sloped for high values of information costs. Figure 7 depicts the expected profit of the intermediary, $E(\Pi_I)$, in which the maximising path $\hat{P}(i)$ is clearly visible.

The additional assumptions of this section have yielded two further results. First, for high values for $i$ the intermediary’s network size is not exhaustive and so both direct and indirect trade take place in equilibrium. Second, the network size of the intermediary expands as information costs decline. That is, ICT improvements result give rise to a larger equilibrium information network and thus more intermediated trade.

Eventually, the intermediary’s network becomes exhaustive with further reductions in $i$ and all trade is intermediated. Additional reductions of $i$ lower the profit level of the intermediary until it is no longer possible to cover fixed costs and the intermediary becomes inactive.

The model gives rise to the potentially testable prediction that more intermediation takes place under middle of the range information costs than when information costs are very high. The intuition behind this result is as follows. A lower level of $i$ improves the direct trade option of traders by raising the probability of matching directly. This lowers the commission rate the intermediary can command per match, thereby lowering the intermediary’s expected revenue for any given network size, $P$. The lower level of $i$ also reduces the fixed costs of operating an information network of size $P$. By easing the loading of the intermediary, a lower $i$ gives the intermediary some slack to expand his network size. The resulting increase in the equilibrium choice of network size increases the number of expected matches and improves expected revenue. Overall, the expected profit of the intermediary is lower for lower values of $i$.

The volume of trade corresponding to the pattern of intermediary of Figure 6 is illustrated in Figure 8. As $i$ falls from 1, the network expands yielding a larger number of intermediated trade matches, while at the same time, $q(i)$ is positive, albeit very small, generating a small number of direct matches. Intermediated matches rise to 1 as $i$ falls. For high values of $i$ both direct and indirect trade takes place in equilibrium, although direct trade is small relative to indirect trade both due to the relatively large size of the network and the low probability of a match through direct search when $i$ is very high. More generally, the figure points to a non-monotonic relationship between information costs and expected trade volume which remains to be explored empirically.

It follows from the analysis that,

**Corollary 2:** Under the assumptions of the model and $L(i, P)$ described by (9)-(11), equilibrium expected direct trade volume is $(1 - i)$ for $0 \leq i < \hat{i}$ where $E(\Pi_I) < 0$,
Figure 6: $S = 16$, $c(i) = i^2$, $L(i, P) = 10i^2P^7$, $F = 2$

Figure 7: $S = 16$, $c(i) = i^2$, $L(i, P) = 10i^2P^7$, $F = 2$
Figure 8: Expected trade volume with intermediation

and $(1 - i) \left[ 1 - \min \left( \widetilde{P}^2(i), 1 \right) \right]$ for $\hat{i} \leq i < 1$ where $E(\Pi_I) \geq 0$. Indirect trade is $\min \left[ \widetilde{P}^2(i), 1 \right]$ for $\hat{i} \leq i < 1$, and 0 otherwise.

Finally, we turn to the welfare effects of intermediation under the additional assumptions introduced in this section. As in subsection 2.1, the intermediary adds to welfare since network building and information brokering is more resource-efficient than direct trade. Since the intermediary keeps the traders he matches indifferent between direct and indirect trade, the intermediary’s expected profit is a pure resource gain. Notice that for high $i$ welfare is maximised by restricting network size and thus expected trade volume. It is, therefore, efficient for some trade matches not to take place in equilibrium due to the resource burden arising from loading of the intermediary.

### 2.2.1 Comparative Statics

Comparative statics are carried out in this subsection to illustrate how the equilibrium intermediation pattern changes with lower trade surplus, $S$, as well as for different specifications of fixed and loading costs.

Suppose $F = 0$. In this case, all costs associated with setting up and operating the network stem from the information requirements of running an information network and so when $i = 0$ the fixed cost of operating a network of any given size drops to 0. For example, consider $S = 16$ and cost functions $c(i) = i^2$, $L(i, P) = 10i^2 P^7$, as depicted in Figure 9. With no pure fixed cost element in $F$, the intermediary operates an exhaustive network even for very small values of $i$, under which the expected commission is very low. Formally, the intermediary’s equilibrium strategy is to choose $[P^*, \alpha_I(i)] =$
Figure 9: $S = 16, c(i) = i^2, L(i, P) = 10i^2P^7, F = 0$

$$\left[\min\left(\bar{P}(i), 1\right), i\right]$$ for all $i$. This implies that $\hat{i} = 0$ and so the intermediary is always active.

Now consider the effects of an increase in fixed cost$^9$ $F$ from 2, as illustrated in Figure 6, to 5.5, as illustrated in Figure 10. This shifts down the expected profit function by 3.5 and restricts the range of $i$ for which the intermediary is viable. In this example, the intermediary’s information network is never exhaustive since the increase in $F$ is large enough to raise threshold $\hat{i}$ to the right of $\hat{i}$.

Consider the alternative loading cost specification $L(i, P) = 3(i^2 + P^2)$, for which assumptions (9) and (10) continue to hold but $L_{iP} = 0$. The zero cross derivative severs the link between the level of information costs and the degree of loading arising from a network of size $P$. This causes the negative relationship between $P$ and $i$ for high levels of $i$ to disappear, as shown in Figure 11, and so the pattern of networking is qualitatively similar to that found in subsection 2.1.

Finally, we consider the implications of a lower trade surplus, $S$, on the pattern of networking and trade. This case is of particular interest since trade policy can give rise to a decline in $S$, such as through the implementation of a tariff. Consider a tariff, $t$, which lowers the available trade surplus per match from $S$ to $S(1 - t)$, as well as the intermediary’s expected profits, for all values of $i$. With lower expected profits, the intermediary is viable for a smaller range of $i$ and invests in a smaller network in the upper range of $i$.

Figure 12 illustrates the effects of a tariff $t = 25\%$ under $S = 16, c(i) = i^2, L(i, P) =$

---

$^9$Note that a decrease in the density of importers and exports over $(0, 1)$ is equivalent to an increase in fixed cost, $F$. 

18
Figure 10: $S = 16$, $c(i) = i^2$, $L(i, P) = 10i^2P^7$, $F = 5.5$

Figure 11: $S = 16$, $c(i) = i^2$, $L(i, P) = 3(i^2 + P^2)$, $F = 2$
Figure 12: $S = 16, c(i) = i^2, L(i, P) = 10i^2P^7, F = 2$ and $t = 25\%$

The tariff reduces the available surplus from 16 to 12, increasing threshold $\tilde{i}$ to $\tilde{\tau}$ and lowering $\hat{i}$ to $\hat{\tau}$. This implies less intermediation and lower expected total trade for high $i$, while the network remains exhaustive for a smaller range of intermediate $i$.

Interestingly, the welfare effects of the tariff depend on the level of $i$. For the range of $i$ for which expected total trade is unchanged, the tariff results in a pure transfer from the traders and intermediary to the government, without reducing overall welfare. For the range of $i$ for which expected total trade declines, however, the tariff is welfare reducing since it induces the intermediary to operate a smaller network size.

Hence, Proposition 3 Under the assumptions of the model with $L(i, P)$ described by (9)-(11), a small tariff, $t$, raises threshold $\hat{i}$ while lowering threshold $\tilde{i}$ and $\tilde{\tau}(i)$. The tariff reduces expected total trade and welfare where $i \in \left[ \hat{i}, \tilde{\tau} \right]$ and $i \in \left[ \frac{\tau}{\hat{i}}, 1 \right]$, and leaves expected total trade and welfare unchanged where $i \in \left[ 0, \hat{i} \right]$ and $i \in \left[ \tilde{\tau}, \frac{\tau}{\hat{i}} \right]$.

2.2.2 Co-Ethnic Ties

Now suppose a subset $k$ of the pairwise matches are between importers and exporters who know each other and can thus match costlessly\(^{10} \), where $k \in (0, 1)$. A possible
explanation for this could be that these traders have the same ethnicity and so form an informal co-ethnic network. Alternatively, importers and exporters may have a longstanding trading relationship, or other historical ties, that have brought the traders in contact with each other.

Assume the $k$ pairs match frictionlessly in an extra stage 0, directly preceding stage 1, generating a joint surplus of $kS$ and exiting the market. The co-ethnic network, therefore, effectively reduces the populations of importers and exporters available to the intermediary for contact-building from 1 to $(1-k)$. Furthermore, suppose the intermediary observes this decline before choosing his optimal network size and commissional rate. The game then proceeds through stages 1 to 4, as before.

When a continuum of exporters and importers of length 1, $P$ has the dual interpretation as being both the absolute number of importers and exporters and the proportion of importers and exporters contacted by the intermediary. With length $(1-k)$, $P$ is the absolute number of traders contacted on each side of the market, while $P$ is the proportion of the importers and exporters $P$ represents. For a given $P$, the probability of any pair matching through the intermediary is now $P^2/(1-k)$ while the expected number of intermediated matches is $P^2/(1-k) > P^2$. It follows that with a network of size $k$, the expected profit of the intermediary is given by equation (16):

$$E(\Pi_I|k) = iS \frac{P^2}{(1-k)} - 2Pc(i) - L(i, P) - F \text{ subject to } P \in (0, 1-k)$$

There are two conflicting effects to consider. First, the population of importers and exporters available to the intermediary is smaller and so the maximum network size the intermediary can operate is $1-k$ rather than 1. Thus for relatively low values of $i$ where the intermediary chooses to operate a full network of 1, the network is restricted to $1-k$. For very large $k$ the network lacks the necessary scale to cover fixed costs and the ethnic network eclipses the information network.

Second, a given investment to build contacts with $P$ traders now corresponds to a larger proportion of the set of traders and generates a larger number of expected matches. Crucially, the absolute number of traders contributes to the cost of network building, while the relative number of traders determines expected matches and thus expected revenue. This has the effect of increasing expected revenue for any given $P$ and hence improving the effectiveness of network building.

Figure 13 depicts the case where $S = 16$, $c(i) = i^2$, $L(i, P) = 10i^2P^2$ and $F = 2$. The figure illustrates the equilibrium network path in the absence of an ethnic network, as well as the lower schedule depicting the pattern of intermediation where $k = 0.1$. The intermediary’s optimal schedule shifts up as the profitability of network building improves, $\tilde{P}(i)|_{k=0.1} > \tilde{P}(i)|_{k=0}$, thereby raising $\tilde{i}$ to $\hat{i}$. Moreover, the intermediary is active above a lower threshold level of information costs $\hat{i}|_{k=0.1} < \hat{i}|_{k=0}$.

For the middle range of $i$, however, the intermediary is constrained by $P \leq 1-k$ and operates a smaller network size in absolute terms. The network continues to be exhaustive in relative terms. For high $i$, however, the number of intermediated matches rises case the expected number of ethnic matches is $k^2$, rather than $k$. This alternative approach is more complex computationally and offers similar results so we restrict our attention to the former.
with small \( k \), as does the proportion of expected total trade that is intermediated. Moreover, intermediation is profitable from a lower threshold of \( i \), generating intermediation in the range \( [\hat{i}|_{k=0.1}, \hat{i}|_{k=0}] \).

When information cost are high, the intermediary’s network is not exhaustive so direct, intermediated and co-ethnic trade co-exist in equilibrium. In particular, ethnic trade is \( k \), expected intermediated trade volume is \( \frac{1}{P} (1 - k) \) and the small amount of expected direct trade is \( \left[ 1 - k - \frac{P}{(1 - k)} \right] q(i) \). Expected total trade with a co-ethnic network can be shown to be \( 1 - i(1 - k) + \frac{iP^2}{(1 - k)} \). If \( k = 0 \), expected total trade reduces to \( 1 - i + \bar{P}(i) \), as before. Furthermore, for intermediate values of \( i \), only ethnic and intermediated trade take place, while for low \( i \) only ethnic and direct trade take place.

In terms of welfare, the frictionless trade of coethnic ties is beneficial in several ways. First, it raises the expected surplus accruing to the subset \( k \) of pairs. Moreover, it improves the profitability of the intermediary’s network-building technology in the anonymous market, thereby raising the expected profit of the active intermediary. Finally, the equilibrium expect payoff of the remaining \( 1 - k \) importers and \( 1 - k \) exporters is unchanged. Hence, the effects of introducing a small subset of pairs that match frictionlessly are welfare-improving.

### 3 The Model with Multiple Intermediaries

The analysis of Section 2 shows that in the simplest case with only one intermediary, the intermediary makes positive profits in equilibrium. This may trigger entry of further in-
termediaries each building their own network of contacts and competing in the brokering of information. This section extends the framework to analyse multiple intermediaries, \( n \), first in the one-shot game, \( G \), and then in the infinitely-repeated game, \( G_\infty \). The main example illustrated in subsection 2.2 is extended to permit free entry and the candidate equilibria are characterised. The resulting implications for contact-building and expected trade are investigated.

### 3.1 Multiple Intermediaries in the One-Shot Game

It is beneficial to the reader to begin with the case of two intermediaries before analysing free entry of intermediaries. The arguments are similar with any number of intermediaries, \( n \), but the intuition behind the competitive interaction of the intermediaries is clearer in the case of two firms. This subsection extends the model of subsection 2.2 to include a second intermediary in the one-shot game.

#### 3.1.1 Two Intermediaries: \( n = 2 \)

Suppose there are two intermediaries, \( A \) and \( B \) with access to the same network-building technology. In stage 1, the intermediaries randomly contact a proportion of importers and exporters and offer those traders they can match together a take-it-or-leave-it contract specifying their commission rates, \( \alpha_A \) and \( \alpha_B \), respectively. The traders choose whether to accept or reject the contract(s) in stage 2. All possible trade matches between those who accept take place through the networks in stage 3 and the intermediaries retain their respective success fees. In stage 4, any unmatched traders trade directly with probability \( q(i) = 1 - i \).

In stage 1, intermediaries always contact the same proportion of each side of the market, in order to maximise the number of matches for any given level of network investment. Hence, \( P_{XA} = P_{MA} = P_A \) and \( P_{XB} = P_{MB} = P_B \). With more than one information network, there is an expected degree of overlap between the networks. Some matches can be made by either \( A \) or \( B \). Since the contacts of each intermediary are randomly selected and private to each intermediary, \( A \) and \( B \) can calculate the expected size of the overlap but do not know which matches they face competition for. Moreover, since the intermediaries cannot select particular traders to be in their information network, it is not possible for firms to cooperate in order to prevent network overlap.

Consider any pair \( j \) of trade partners \((X_j, M_j)\). The pair may match through \( A \) only, through \( B \) only, through both networks, or through neither. The probability of each event is listed in equations (17) to (20):

\[
\begin{align*}
\text{Prob} (j \text{ can match via } A \text{ or } B) &= P_A^2 P_B^2 \\
\text{Prob} (j \text{ can match via } A \text{ only}) &= P_A^2 (1 - P_B^2) \\
\text{Prob} (j \text{ can match via } B \text{ only}) &= P_B^2 (1 - P_A^2) \\
\text{Prob} (j \text{ cannot match by } A \text{ or } B) &= (1 - P_A^2) (1 - P_B^2)
\end{align*}
\]
The direct trade option available to traders in stage 4 implies that $\alpha_A$ and $\alpha_B$ cannot exceed $i$. As in Section 2, traders served exclusively by one intermediary accept the contract in stage 2 provided the success fee does not exceed $i$.

The traders in the area of overlap between the networks of $A$ and $B$ are known to both intermediaries. In order for a particular match to be possible through both $A$ and $B$, the importer and exporter must lie in both information networks. The timing of the game is such that upon building their information networks, the intermediaries observe which matches they can make and approach the respective parties to secure a contract. That is, a trader that is approached by both intermediaries can infer that her trading partner is also known to both intermediaries.

Suppose also that the contracts offered by the intermediaries are non-exclusive\(^{11}\), so traders are free to sign either or both contracts offered to them. The non-exclusive contracts specify the commission rate that is applied if the particular intermediary brings about the match. If the traders sign both contracts, then the match takes place through either intermediary with probability $\frac{1}{2}$.

In the event that a trader receives two take-it-or-leave-it contracts, and $\alpha_A \neq \alpha_B$, then she will accept the contract with the lowest commission rate. All traders will choose in this manner and so all common matches are made through the intermediary with the lowest commission rate.

If, however $\alpha_A = \alpha_B \leq i$, the traders are indifferent between the two contracts. In order to ensure they match with their trading partner, they accept both contracts and so each intermediary expects to gain half of the common matches.

Consider the expected matches made by intermediary $A$ depending on the chosen success fee. The three possible cases, where $A$’s success fee is lower than, higher than, or equal to that of $B$, are described by equations (21) to (23) below:

\begin{align}
E(\text{matches by } A)_{\alpha_A < \alpha_B, \alpha_A \leq i} &= P_A^2 \\
E(\text{matches by } A)_{\alpha_B < \alpha_A \leq i} &= P_A^2 - P_A^2 P_B^2 \\
E(\text{matches by } A)_{\alpha_A = \alpha_B \leq i} &= P_A^2 - \frac{1}{2} P_A^2 P_B^2
\end{align}

Similarly, equations (24) to (26) describe those for $B$:

\begin{align}
E(\text{matches by } B)_{\alpha_B < \alpha_A, \alpha_B \leq i} &= P_B^2 \\
E(\text{matches by } B)_{\alpha_A < \alpha_B \leq i} &= P_B^2 - P_A^2 P_B^2 \\
E(\text{matches by } B)_{\alpha_A = \alpha_B \leq i} &= P_B^2 - \frac{1}{2} P_A^2 P_B^2
\end{align}

\(^{11}\)Alternatively, the intermediaries may offer exclusive contracts, whereby the traders must choose whether to sign up exclusively with $A$ or $B$. With exclusivity, the possibility emerges that both traders required for a match are known to both intermediaries, yet the trade match does not take place since the importer and exporter have signed exclusive contracts with different intermediaries. This scenario is computationally more cumbersome, yet gives rise to the same overall pattern of intermediation. The added feature is that a proportion of common matches fail to take place, while with non-exclusive contracts, this is avoided.
Hence, the expected profit levels for $A$ and $B$ are given by equations (27) to (29) and (30) to (32), respectively:

$$E(\Pi_A)|_{\alpha_A<\alpha_B, \alpha_A \leq i} = \alpha_A SP_A^2 - 2P_A c(i) - L(i, P_A) - F$$

$$E(\Pi_A)|_{\alpha_B<\alpha_A, \alpha_B \leq i} = \alpha_A S \left[ P_A^2 - P_A^2 P_B^2 \right] - 2P_A c(i) - L(i, P_A) - F$$

$$E(\Pi_A)|_{\alpha_A=\alpha_B \leq i} = \alpha_A S \left[ P_A^2 - \frac{1}{2} P_A^2 P_B^2 \right] - 2P_A c(i) - L(i, P_A) - F$$

$$E(\Pi_B)|_{\alpha_B<\alpha_A, \alpha_B \leq i} = \alpha_B SP_B^2 - 2P_B c(i) - L(i, P_B) - F$$

$$E(\Pi_B)|_{\alpha_A<\alpha_B, \alpha_A \leq i} = \alpha_B S \left[ P_B^2 - P_A^2 P_B^2 \right] - 2P_B c(i) - L(i, P_B) - F$$

$$E(\Pi_B)|_{\alpha_A=\alpha_B \leq i} = \alpha_B S \left[ P_B^2 - \frac{1}{2} P_A^2 P_B^2 \right] - 2P_B c(i) - L(i, P_B) - F$$

Although the configuration $\alpha_A = \alpha_B = i$ maximises the total profit of the intermediation sector, it is not an equilibrium of the one-shot game if intermediaries have an incentive to undercut each other. Intermediaries cannot observe which trade matches are common and which exclusive since the information network of each intermediary is private information. Price discrimination is therefore, impossible, and each intermediary sets a universal success fee.

$\alpha_A = \alpha_B = i$ is not an equilibrium if each intermediary gains by undercutting marginally and gaining all the matches that can be served by both intermediaries. Undercutting gives rise to a decline in the marginal revenue per expected match, but an increase in the number of expected matches. This drives down the equilibrium success fee\textsuperscript{12} to $\alpha_A^{\ast} = \alpha_B^{\ast} = \alpha^{\ast}$, where $\alpha^{\ast}$ is defined as the surplus share at which $E(\Pi_A)|_{\alpha_A^{\ast}=\alpha_B^{\ast}=\alpha^{\ast}} \geq E(\Pi_A)|_{\alpha_A^{\ast} < \alpha_B^{\ast} = \alpha^{\ast}}$ and $E(\Pi_A)|_{\alpha_A^{\ast} > \alpha_B^{\ast} = \alpha^{\ast}} \leq E(\Pi_A)|_{\alpha_A^{\ast} = \alpha_B^{\ast} = \alpha^{\ast}}$, as well as $E(\Pi_B)|_{\alpha_A^{\ast}=\alpha_B^{\ast}=\alpha^{\ast}} \geq E(\Pi_B)|_{\alpha_A^{\ast} < \alpha_B^{\ast} = \alpha^{\ast}}$ and $E(\Pi_B)|_{\alpha_A^{\ast} > \alpha_B^{\ast} = \alpha^{\ast}} \leq E(\Pi_B)|_{\alpha_A^{\ast} = \alpha_B^{\ast} = \alpha^{\ast}}$, where $\alpha_A^{\ast} = \alpha_B^{\ast} = \alpha^{\ast} - \Delta \alpha$ and $\alpha_A^{1} = \alpha_B^{1} = \alpha^{\ast} + \Delta \alpha$.

If intermediaries make losses at $\alpha^{\ast}$, the equilibrium outcome is that both $A$ and $B$ cannot survive in the market as a result of the competitive pressure driving down the success fees. If the expected profits of both $A$ and $B$ are positive, then both are active and charge $\alpha_A^{\ast} = \alpha_B^{\ast} = \alpha^{\ast}$. Intermediaries still share the common matches, but there is no incentive to deviate as the loss from further reducing the success fee exceeds the gain from more expected matches.

In equilibrium:

\textsuperscript{12} A configuration of intermediary surplus shares where $\alpha_A^{\ast} \neq \alpha_B^{\ast}$ cannot be an equilibrium. The intermediary with the higher share always has an incentive to undercut or match the other intermediary.
Similarly, (33) and (34) imply that \( \alpha^* \) and \( \alpha^+ \) imply that traders accept and from which neither cannot extract any surplus in equilibrium due to the competitive forces driving their pricing behaviour. Anticipating this, they do not invest in network building in stage 1.

Also:

\[
E(\Pi_A)|_{\alpha^+_A=\alpha^+_B=\alpha^*} \geq E(\Pi_A)|_{\alpha^+_A<\alpha^+_B=\alpha^*} \\
\implies \alpha^* S \left[ P^2_A - \frac{1}{2} P^2_A P^2_B \right] - 2P_A c(i) - L (i, P_A) - F \\
\geq \alpha^+_A S P^2_A - 2P_A c(i) - L (i, P_A) - F \\
\implies \alpha^* \leq \frac{2\Delta \alpha}{P^2_B} \tag{33}
\]

(33) and (34) imply that \( \alpha^* \) must lie within the band of width \( 2\Delta \alpha \) defined by (35) in order for intermediary \( A \) to have no incentive to deviate from \( \alpha^* \):

\[
\frac{2\Delta \alpha \left[ 1 - P^2_B \right]}{P^2_B} \leq \alpha^* \leq \frac{2\Delta \alpha}{P^2_B} \tag{35}
\]

Similarly, \( E(\Pi_B)|_{\alpha_A^+=\alpha_B^+=\alpha^*} \geq E(\Pi_B)|_{\alpha_A^+<\alpha_B^+=\alpha^*} \) and \( E(\Pi_B)|_{\alpha_A^+>\alpha_B^+=\alpha^*} \leq E(\Pi_B)|_{\alpha_A^+=\alpha_B^+=\alpha^*} \), define the band of width \( 2\Delta \alpha \) given by (36) in which \( \alpha^* \) must lie in order for \( B \) to have no incentive to deviate:

\[
\frac{2\Delta \alpha \left[ 1 - P^2_A \right]}{P^2_A} \leq \alpha^* \leq \frac{2\Delta \alpha}{P^2_A} \tag{36}
\]

Moreover, (37) defines the range in which \( \alpha^* \) must lie in order for the traders to accept the contract offered in stage 2:

\[
0 \leq \alpha^* \leq i \tag{37}
\]

(35), (36) and (37) must hold simultaneously in order for an equilibrium \( \alpha^* \) to exist such that traders accept and from which neither \( A \) nor \( B \) has any incentive to deviate\(^{13}\).

It follows that the existence and level of \( \alpha^* \) depends on \( P^2_A, P^2_B, i \) and \( \Delta \alpha \), the marginal change in \( \alpha \) with which intermediaries can change the success fee, i.e. the unit of measurement of \( \alpha_A \) and \( \alpha_B \). If intermediaries can change \( \alpha_A \) and \( \alpha_B \) by infinitesimal amounts, then \( \Delta \alpha \to 0 \). It follows that as \( \Delta \alpha \to 0, \alpha^* \to 0 \). Since shares are continuous variables ranging from 0 to 1, \( \Delta \alpha \to 0 \) and so \( \alpha^* \to 0 \) in equilibrium. Intermediaries cannot extract any surplus in equilibrium due to the competitive forces driving their pricing behaviour. Anticipating this, they do not invest in network building in stage 1.

Hence:

\(^{13}\)The equilibrium is not defined when either or both networks have 0 measure.
Proposition 4 In the one shot game, $G$, with two intermediaries, the following strategies for the traders and intermediaries constitute the unique, subgame perfect equilibrium of the game: Intermediaries choose $[P^*_I, \alpha_I(i)] = (0, 0)$, where $I = A, B$. If traders receive only one offer, accept if $\alpha_I(i) \leq i$, where $I \in \{A, B\}$; reject otherwise. If traders receive two offers, where $\alpha_A(i) \neq \alpha_B(i)$ accept the $\min[\alpha_A(i), \alpha_B(i)]$, if $\min[\alpha_A(i), \alpha_B(i)] \leq i$; reject otherwise. If traders receive two offers, where $\alpha_A(i) = \alpha_B(i) \leq i$, accept both (non-exclusive) contracts; reject otherwise.

3.1.2 Three Intermediaries and Beyond: $n \geq 3$

Now consider three intermediaries $A$, $B$ and $C$ with access to the same network-building technology. In stage 1, each intermediary randomly contacts a proportion of importers and exporter and builds a network. The intermediaries offer the traders they can bring together a non-exclusive, take-it-or-leave-it contract specifying their commission rate, $\alpha_A$, $\alpha_B$ and $\alpha_C$, respectively. The traders choose whether to accept or reject the contract(s) in stage 2. All possible trade matches between those who accept take place through the networks in stage 3 and the intermediaries retain their respective success fees. In stage 4, any unmatched traders trade directly with probability $q(i) = 1 - i$.

The intermediaries always contact the same proportion of each side of the market, in order to maximise the number of matches for any given level of network investement. Hence, $P_{XA} = P_{MA} = P_A$, $P_{XB} = P_{MB} = P_B$ and $P_{XC} = P_{MC} = P_C$.

The arguments with $n \geq 3$ firms are similar to those with two. With three intermediaries, there is an expected degree of overlap between networks $A$ and $B$, $A$ and $C$ and $B$ and $C$, as well as an expected overlap between all three networks, $A$, $B$ and $C$. The traders in the areas of network overlap are offered a contract by multiple firms. Traders choose to sign up with the intermediary with the lowest commission rate, provided it does not exceed $i$. If there is no single intermediary with a lower commission rate, but two or more firms offering identical commission rates not exceeding $i$, then the traders accept these contracts and the matches in the overlap are split equally between the intermediaries. For example, if $\alpha_A = \alpha_B = \alpha_C = i$, then the matches in the overlap common to all three firms are split between the three intermediaries, the matches common to $A$ and $B$ are shared between them, the matches common to $A$ and $C$ are shared between them etc.

It is never a Nash equilibrium of the one-shot game for the three intermediaries to set non-zero commission rates since each intermediary has an incentive to marginally undercut the commission rate of the other intermediaries, in order to gain all the common matches where the information networks overlap. By undercutting, the intermediary suffers a marginal reduction of the expected revenue from exclusive matches but gains the full commission on additional common sales. This incentive to undercut yields $\alpha^*_A = \alpha^*_B = \alpha^*_C = 0$ as the unique equilibrium configuration of intermediary commission rates.

Hence:

Proposition 5 In the one shot game, $G$, with three intermediaries, the following strategies for the traders and intermediaries constitute the unique, subgame perfect equilibrium of the game: Intermediaries choose $[P^*_I, \alpha_I(i)] = (0, 0)$, where $I = A, B, C$. If
traders receive only one offer, accept if \( \alpha_I(i) \leq i \), where \( I \in (A, B, C) \); reject otherwise. If traders receive two offers from \( j, k \in (A, B, C) \), where \( \alpha_j(i) \neq \alpha_k(i) \) accept the min \( \{\alpha_j(i), \alpha_k(i)\} \), if \( \min\{\alpha_j(i), \alpha_k(i)\} \leq i \); reject otherwise. If traders receive two offers, where \( \alpha_j(i) = \alpha_k(i) \leq i \), accept both \( j \) and \( k \); reject otherwise. If traders receive three offers from \( j, k, l \in (A, B, C) \), where \( \alpha_j(i) = \alpha_k(i) = \alpha_l(i) \leq i \), then accept all offers; reject otherwise. If \( \alpha_j(i) = \alpha_k(i) < \alpha_l(i) \), then accept \( j \) and \( k \) if \( \alpha_j(i) = \alpha_k(i) \leq i \); reject otherwise. If \( \alpha_j(i) < \alpha_k(i) = \alpha_l(i) \), then accept \( j \) if \( \alpha_j(i) \leq i \); reject otherwise.

See Appendix A for the proof of Proposition 5.

Similar arguments may be employed in the case of four or more intermediaries in the one-shot game. The expected overlap between information networks pushes the non-cooperative equilibrium commission rate down to zero and drives the intermediaries out of the market. It follows that in the one-shot game with \( n \geq 2 \) intermediaries, the Nash equilibrium is always characterised by zero commission rates. Hence, even with free entry of intermediaries only a single intermediary is viable in the market.

### 3.2 Multiple Intermediaries in the Repeated Game

The analysis of Section 3.1 shows that in the one-shot game, where intermediaries have no future interactions, the cooperative outcome where firms charge a non-zero commission rate cannot be supported as a Nash equilibrium of the game. This section turns to the repeated game where the expectation of future interactions with rival intermediaries allows for a subgame perfect equilibrium in which multiple intermediaries cooperate. For the sake of clarity, the arguments are presented first in the case of two intermediaries and subsequently for \( n > 2 \).

#### 3.2.1 Two Intermediaries: \( n = 2 \)

Despite the scope for profitable network building, intermediaries compete each other out of the market in the one-shot game. If intermediaries could agree to cooperate and charge \( \alpha_A(i) = \alpha_B(i) = i \), the maximum surplus per trade match would be appropriated. In the one-shot game the cooperative outcome is unsustainable since each intermediary has the incentive to defect from the agreement. Though not a subgame perfect equilibrium of the constituent game \( G \), the cooperative agreement is a subgame perfect equilibrium of the infinitely repeated game for a sufficiently high discount factor.

Define \( G_\infty \) to be the infinitely repeated game with constituent game \( G \) with two intermediaries. In each period, new populations of exporters and importers are distributed uniformly along the unit lines, with unit density. Each intermediary chooses whether or not to invest in a new information network in each period, and the stages in each period are as in \( G \).

If \( A \) and \( B \) agree to charge \( \alpha_A(i) = \alpha_B(i) = i \) in each period, the cooperative expected profits per period are described by (38) and (39):
If one of the intermediaries, say \( A \), defects, the expected profits in the defection period are, where \( \alpha_A^D = i - \Delta \alpha, \Delta \alpha \to 0 \):

\[
E(\Pi^C_a)|_{\alpha_A^D} = iS \left[ \frac{P_A^2 - \frac{1}{2} P_A^2 P_B^2}{2} \right] - 2P_A c(i) - L(i, P_A) - F 
\] (38)

\[
E(\Pi^C_b)|_{\alpha_A^D} = iS \left[ \frac{P_B^2 - \frac{1}{2} P_A^2 P_B^2}{2} \right] - 2P_B c(i) - L(i, P_B) - F 
\] (39)

Thereafter, the cooperative agreement breaks down and the intermediaries revert to the non-cooperative outcome where \( \alpha_A^{NC} = \alpha_B^{NC} = 0 \) and \( E(\Pi^C_a)|_{\alpha_A^{NC}} = E(\Pi^C_b)|_{\alpha_B^{NC}} = 0 \).

The discounted expect profit for \( A \) under the cooperative agreement is

\[
\frac{1}{1 - \delta} \left[ iS \left( \frac{P_B^2 - \frac{1}{2} P_A^2 P_B^2}{2} \right) - 2P_B c(i) - F(i, P_B) \right], \text{ where } \delta \text{ is the discount factor.}
\]

The discounted expect profit for \( A \) under defection is \( \alpha_A^D P_A^2 - 2P_A c(i) - F(i, P_A) \). The cooperative agreement is an equilibrium of the infinitely repeated game, where \( \Delta \alpha \to 0 \):

\[
\frac{1}{1 - \delta} \left[ iS \left( \frac{P_A^2 - \frac{1}{2} P_A^2 P_B^2}{2} \right) - 2P_A c(i) - L(i, P_A) - F \right] \geq \alpha_A^D P_A^2 - 2P_A c(i) - L(i, P_A) - F
\] (42)

That is, if intermediaries’ discount factor satisfies the inequality:

\[
\delta \geq \frac{2 i S P_A^2 P_B^2}{2 [i S P_A^2 - 2P_A c(i) - L(i, P_A) - F]}
\] (43)

Hence:

**Proposition 6** In the infinitely repeated game \( G_\infty \) with two intermediaries \( A \) and \( B \), the cooperative equilibrium \( \alpha_A(i) = \alpha_B(i) = i \) is a subgame perfect equilibrium of \( G_\infty \), if \( \delta \geq \frac{i S P_A^2 P_B^2}{2 [i S P_A^2 - 2P_A c(i) - L(i, P_A) - F]} \). Otherwise, the non-cooperative equilibrium of \( G \) is the subgame perfect equilibrium in every subgame of \( G_\infty \).

Now assume that \( \delta \geq \frac{i S P_A^2 P_B^2}{2 [i S P_A^2 - 2P_A c(i) - L(i, P_A) - F]} \) so the cooperative equilibrium is a subgame perfect equilibrium of \( G_\infty \). Intermediaries \( A \) and \( B \) set \( \alpha_A(i) = \alpha_B(i) = i \) and their expected profits are described by equations (38) and (39). The first order conditions of the intermediaries’ expected profit maximisation problems are described by (44) and (45), respectively:

\[
\frac{\partial E(\Pi^C_A)|_{P_A}}{\partial P_A} = iSP_A (2 - P_B^2) - 2c(i) - L_{PA} = 0
\] (44)

\[
\frac{\partial E(\Pi^C_B)|_{P_B}}{\partial P_B} = iSP_B (2 - P_A^2) - 2c(i) - L_{PA} = 0
\] (45)
The first order conditions can be solved to find the subgame perfect equilibrium network size of the two intermediaries. (44) and (45) yield the best response functions $P_A^* (P_B)$ and $P_B^* (P_A)$, respectively, which can be drawn for a given level of information costs, $i$.

To illustrate, consider the example where $S = 16$, $c(i) = i^2$, $L(i, P) = 10i^2P_T^2$ and $F = 2$. The first order conditions are now:

\[
16iP_A (2 - P_B^2) - 2i^2 - 10i^2P_A - 2 = 0 \quad (46)
\]
\[
16iP_B (2 - P_A^2) - 2i^2 - 10i^2P_B - 2 = 0 \quad (47)
\]

The best response functions $P_A^* (P_B)$ and $P_B^* (P_A)$ are drawn for $i = 0.7$ in Figure 14. They cross four times at points $v$, $x$, $y$, and $z$. Point $v$, ‘low symmetric’, is the configuration of $P_A$ and $P_B$, $P_A^* = P_B^* = 0.043792$, at which profit is minimised. At $v$, $E(\Pi_A^C)_{i=0.7, P_A^* = P_B^* = 0.043792} = E(\Pi_B^C)_{i=0.7, P_A^* = P_B^* = 0.043792} = -2.0215$, so this point will never be chosen.

Point $x$, ‘high symmetric’, is the configuration of $P_A$ and $P_B$, $P_A^* = P_B^* = 0.82993$, where profit is maximised. The expected profit levels, $E(\Pi_A^C)_{i=0.7, P_A^* = P_B^* = 0.82993} = E(\Pi_B^C)_{i=0.7, P_A^* = P_B^* = 0.82993} = 0.9154$.

Points $y$ and $z$ are the asymmetric configurations of $P_A$ and $P_B$ where one intermediary operates a large network, and the other a small network. Consider $y$ where $P_A = 0.9$ and $P_B = 0.07353$. This yields an expected profit of $E(\Pi_A^C)_{i=0.7, P_A^* = 0.9, P_B^* = 0.07353} = 3.8218$ for large $A$ and a loss for $B$, $E(\Pi_B^C)_{i=0.7, P_A^* = 0.9, P_B^* = 0.07353} = -2.036$. The converse is true at point $z$. These are also not subgame perfect equilibria, as the small firm makes a loss\(^14\). For these parameter values, the unique subgame perfect cooperative Nash equilibrium is that where $P_A^* = P_B^* = 0.82993$.

Plotting similar reaction functions for different values of $i$ charts the equilibrium schedule of $P_A^*(i) = P_B^*(i) = P^*(i)$ depicted in Figure 15.

The upper schedule is the equilibrium path with a monopolist intermediary, and the lower schedule that of each of the two firms. With two firms sharing the market, there is a degree of overlap, which results in common matches split equally between the two intermediaries in equilibrium. This implies fewer matches for a given level of network investment when compared to a monopolist intermediary. The schedules for $A$ and $B$ lie below that of the monopolist, while the two firms can only survive in the market for $i \geq 0.4$. For high levels of information costs, both intermediaries are active in the market. Improvements in ICT cause them to expand their network size as a result of the interaction of $i$ and network size in fixed cost. Lower information costs also imply a lower cooperative commission rate charged by intermediaries and so expected profits are lower. Below $i = 0.4$, only one firm survives in the market. Without competition through expected network overlap, the monopolist expands the information network.

What about the impact of competition on trade volume? Recall that the expected total trade volume with a single intermediary is $[1 - i(1 - P_T^2)]$. The expected total

\(^14\)Only if $S$ is very large can the smaller firm ever make positive profits in the asymmetric points. That is, an equilibrium where one intermediary operates a very large network and the other a small one can only exit in highly lucrative markets.
Figure 14: $S = 16, c(i) = i^2, L(i, P) = 10i^2P^7, F = 2$, drawn for $i = 0.7$

Figure 15: $S = 16, c(i) = i^2, L(i, P) = 10i^2P^7, F = 2$
trade volume with two intermediaries is now \([1 - i(1 - 2P^2 + P^4)]\). Comparing the two indicates suggests conflicting effects. There are two intermediaries in operation, but they are smaller in network size than the monopolist intermediary as a result of network overlap. Despite the overlap, total trade volume typically rises with a second intermediary as the scale effect outweighs the overlap effect.

3.2.2 Three Intermediaries and Beyond: \(n \geq 3\)

Now consider three intermediaries \(A, B\) and \(C\) with access to the same network-building technology. For a sufficiently high discount factor, \(\delta\), a cooperative subgame perfect equilibrium exists\(^{15}\), where \(\alpha_A(i) = \alpha_B(i) = \alpha_C(i) = i\), for the infinitely repeated game \(G_\infty\) with constituent game \(G\). In particular, it can be shown that:

**Proposition 7** In the infinitely repeated game \(G_\infty\) with three intermediaries \(A, B\), and \(C\), the cooperative equilibrium \(\alpha_A(i) = \alpha_B(i) = \alpha_C(i) = i\) is a subgame perfect equilibrium of \(G_\infty\), if \(\delta \geq \frac{\mathrm{iSP}^2(3P_A^2 + 3P_B^2 - 2P_B^2 - 3P_A^2 - 2P_B^2 P_C^2)}{6[iSP_A^2 - 2P_A C(i) - L(i, P_A) - F]}\). Otherwise, the non-cooperative equilibrium of \(G\) is the subgame perfect equilibrium in every subgame of \(G_\infty\).

See Appendix B for the proof of Proposition 7.

Let us focus on the case where \(\delta \geq \frac{\mathrm{iSP}^2(3P_B^2 + 3P_C^2 - 2P_B^2 P_C^2)}{6[iSP_A^2 - 2P_A C(i) - L(i, P_A) - F]}\) so the cooperative equilibrium, \(\alpha_A^*(i) = \alpha_B^*(i) = \alpha_C^*(i) = i\), is a subgame perfect equilibrium of \(G_\infty\). The equilibrium network path of each intermediary, given \(i\) and the network size of its rival, is derived from the first order conditions of intermediaries’ profit maximisation. Focusing only on the symmetric equilibrium of the three-firm case, where \(P_A^* = P_B^* = P_C^* = P^*\), the common network size, conditional on \(i\), is depicted in Figure 16 for the case where \(S = 16\), \(c(i) = i^2\), \(L(i, P) = 10i^2P^T\) and \(F = 2\).

Now consider four intermediaries, \(A, B, C\) and \(D\). With \(n = 4\) the intermediaries’ information networks can overlap with up to three other firms, hence the expected matches per intermediary with cooperative commission rates is lower with \(n = 4\) than \(n = 3\), for any given \(i\).

The probability of any particular pair \(j\) of trade partners \((X_j, M_j)\) trading through a particular intermediary, or set of intermediaries, under \(\alpha_A(i) = \alpha_B(i) = \alpha_C(i) = \alpha_D(i)\), yields the expected number of matches by \(A, B, C\) and \(D\) given by equations (48) to (51):

\[
E(\text{matches by } A) |_{\alpha_A = \alpha_B = \alpha_C = \alpha_D} = P_A^2 - \frac{1}{2} \left( P_A^2 P_B^2 + P_A^2 P_C^2 + P_A^2 P_D^2 \right) + \frac{1}{3} \left( P_A^2 P_B^2 P_C^2 + P_A^2 P_B^2 P_D^2 + P_A^2 P_C^2 P_D^2 \right) - \frac{1}{4} P_A^2 P_B^2 P_C^2 P_D^2
\]

\(^{15}\)For a sufficiently high discount factor, \(\delta\), any configuration of commission rates such that \(\alpha_A(i) = \alpha_B(i) = \alpha_C(i) \leq i\) can be supported as a subgame perfect Nash equilibrium in the repeated game. We restrict our attention to the case where firms cooperate to set \(\alpha_A(i) = \alpha_B(i) = \alpha_C(i) = i\), the maximum possible commission rate.
For a sufficiently high discount rate, \( \delta \), the cooperative agreement where \( \alpha^*_A(i) = \alpha^*_B(i) = \alpha^*_C(i) = \alpha^*_D(i) = i \) is sustainable. Restricting our attention to symmetric candidate equilibria, consider \( P^*_A = P^*_B = P^*_C = P^*_D = P^* \). For sufficiently large \( S \), there exists a range of information costs for which the four intermediaries can cover fixed costs and are active. This is not possible in our example, where \( S = 16, c(i) = i^2, L(i, P) = 10i^2P^7 \) and \( F = 2 \). Four intermediaries cannot achieve non-negative profits even for very high levels of \( i \), and so with free entry the maximum number of firms that invest in information networks is \( n^* = 3 \).
Only three intermediaries are viable when the information costs, and hence the maximum cooperative commission rate, is high. The equilibrium network size of each of the three firms is lower than that which would prevail if there were only two intermediaries in the market as a result of overlap between two or more of the intermediaries, and the level of $i$ above which each can cover fixed costs is much higher than with two firms.

The equilibrium network paths with $n = 1$, $n = 2$, and $n = 3$ firms in Figure 16 chart the evolution of the symmetric, equilibrium network size of active intermediaries as $i$ declines under free entry. For levels of information frictions above $i_3$ three intermediaries are active. As information costs decline intermediaries extract a lower surplus from each match. At the same time, lower $i$ implies lower costs of running the network through $L(i, P)$. The latter effect dominates and so each firm expands its network size. When $i$ falls below $i_3$ only two firms survive in the market, each on a higher path. Further ICT improvements cause each of the two intermediaries to expand network size until $i$ falls below $i_2$ and so only a monopolist intermediary is viable. For these parameter values, the intermediary’s network captures the entire market\textsuperscript{16}. Once information costs fall below $\hat{i}$, no intermediaries operate in the market and all trade is direct.

Hence,

**Proposition 8** With free entry of intermediaries, a maximum (finite) number of symmetric firms, $n^*$, can survive in the market while behaving cooperatively in the repeated game $G_\infty$. The number of active firms is increasing with $i$. $n^*$ is defined where $E(\Pi_{n^*})|_{i=1} \geq 0$ and $E(\Pi_{n^*+1})|_{i=1} < 0$. Active firms operate larger networks with lower information costs. When $i$ falls below the threshold level for the profitability of $n^*$ firms, $i_{n^*}$, the industry consolidates leaving $n^* - 1$ firms operating larger networks. The $n^* - 1$ firms expand their networks symmetrically with further reductions in $i$, until the industry consolidates at $i_{n^*-1}$ leaving $n^* - 2$ firms, each with a larger network etc.

### 3.2.3 Expected Trade and Welfare

The pattern of network-building and firm entry in the cooperative symmetric equilibrium of the repeated game $G_\infty$ is illustrated in Figure 16 for $S = 16$, $c(i) = i^2$, $L(i, P) = 10i^2 P^7$ and $F = 2$. The figure illustrates that higher information frictions, which give rise to higher cooperative commission rates, permit more firms to make non-negative profits and so induces entry of intermediaries. Although there are more firms in the market for high $i$, each firm chooses to operate a smaller information network in the cooperative equilibrium. This is due to the expected overlap between information networks that curtails expected matches, and thus expected intermediary profits, for a given choice of network size.

The implications for expected trade and welfare are discussed in this section in the context of the example depicted in Figure 16. Recall that $i_1$ is the threshold level of information costs above which one intermediary can make non-negative profits. $i_2$ and $i_3$ are the threshold levels above which two and three intermediaries, respectively, can

\textsuperscript{16}For different parameter values and choice of cost functions, it is possible to show that network size never reaches 1 in equilibrium.
survive in the market. Moreover, let $P_1$, $P_2$ and $P_3$ be the proportion of the population of importers and exporters intermediaries form contacts with in the cooperative, symmetric equilibrium\footnote{These depend on $i$ and are derived from the first order condition of the intermediaries’ expected profit maximisation problem.} with one, two and three active intermediaries, respectively, where $P_1 > P_2 > P_3$.

It can easily be calculated that expected total trade volume in the cooperative, symmetric equilibrium is described by the following expressions.

$$E(\text{Trade Volume}) = \begin{cases} 1 - i, & \text{if } i < i_1 \\ 1 - i (1 - P_1^2), & \text{if } i_1 \leq i < i_2 \\ 1 - i (1 - 2P_2^2 + P_2^4), & \text{if } i_2 \leq i < i_3 \\ 1 - i (1 - 3P_3^2 + 3P_3^4 - P_3^6), & \text{if } i_3 \leq i \leq 1 \end{cases} \quad (52)$$

The expected trade volume corresponding to the example of Figure 16 is illustrated in Figure 17. When $i < i_1$, there are no active intermediaries so all expected trade is direct trade. For $i_1 \leq i < i_2$, one intermediary is active and operates a full network. With $P_1 = 1$, $E(\text{Trade Volume}) = 1$ so with a monopolist intermediary all trade matches may take place.

For $i_2 \leq i < i_3$, two intermediaries are active and operate a less than full network each. Although each network is smaller, there are now two networks and so the expected trade schedule with two firms lies above that with a single firm.

For $i_3 \leq i \leq 1$, all three intermediaries are active and operate smaller networks each. Again, the expected trade path with three firms lies above that with two firms for the range of $i$ for which all three are active.

The outer envelope of these expected trade schedules is that which prevails in the cooperative, symmetric equilibrium under free entry. Hence, despite the decline in expected matches per intermediary as $n^*$ rises, total expected intermediated trade is larger as the overall catchment area of intermediaries widens. There is also a small amount of direct trade between those who cannot match through the intermediaries.

The relationship between information costs and expected trade volume is clearly non-monotonic and subject to discontinuities as the number of intermediaries in the market changes.

In the cooperative equilibrium, all intermediaries extract the larger possible share of trade surplus, $i$. Traders are therefore indifferent between direct and indirect trade and so enjoy the same surplus level as in the absence of intermediaries. It follows that examining the expected profits of intermediaries is sufficient to make statements about welfare.

With three active firms, the expected per period profit of each firm in the symmetric equilibrium, $iS(P_3^2 - P_3^3 - \frac{1}{3}P_3^6) - 2P_3c(i) - L(i, P_3) - F$, is lower than the $iS(P_2^2 - \frac{1}{2}P_2^4 - 2P_2c(i) - L(i, P_2) - F$ earned with two firms, which in turn is lower than $iSP_1^2 - 2P_1c(i) - L(i, P_1) - F$ earned by the single intermediary. Although expected profit per intermediary falls with competition, expected total profits in the intermediation industry rise as the industry intermediates a larger number of trade matches. Hence, the free entry of competing intermediaries is strictly welfare improving.
Therefore,

**Proposition 9** The free entry of intermediaries increases expected total trade volume in the market, for a given $i$, and is strictly welfare improving.

## 4 Concluding Remarks

This paper develops a pairwise matching model with a continuum of importers and exporters with two-sided information asymmetry to explore the role of information intermediaries in facilitating international trade and examine the part information costs play in explaining that role.

Under the simplest set of assumptions with a single intermediary in the market, an exhaustive information network emerges when information costs are high, and all trade takes place indirectly.

When the model is extended to allow for loading of the intermediary, the incentives for network building are curtailed when information costs are very high and the intermediary operates an inexhaustive network. Direct and indirect trade are shown to coexist in equilibrium when information costs are above a certain threshold, while the presence of information intermediaries raises both expected trade volume and welfare compared to direct trade. Interestingly, ICT improvements that lower information costs provide incentives for network expansion and more intermediated trade. Moreover, the network intermediary is largest for intermediate values of information costs. The effects of a small tariff on expected trade and welfare depend on the level of information costs. For
high information costs where the intermediary operates an inexhaustive network, the
tariff reduces the intermediary’s expected revenue lowering the intermediary’s optimal
network size and lowering trade volume and welfare.

Furthermore, a small ethnic network gives rise to more intermediation for high and
low levels of information costs, respectively, by improving the effectiveness of contact-
building and thus profitability. For intermediate level of information costs, the ethnic
network crowds out intermediated trade.

There is a growing literature on the importance of face-to-face contact for economic
relationships. Storper and Venables (2004) describe face-to-face contact as a communi-
cation technology that assists the transmission of uncodifiable information. Moreover,
face-to-face contact may generate trust that can facilitate the exploitation of trade oppor-
tunities in the face of contract enforcement problems. Since it is impossible to have
face-to-face contact with all possible trading partners, agents sometimes rely on infor-
mal networks for information transmission such as ethnic and business networks (Rauch
2001; Rauch and Trindade 1999). Rauch and Trindade (1999) find that ethnic Chinese
networks increased bilateral trade in differentiated products within Southeast Asia, be-
tween 1980 and 1990, by at least 150%. This suggests that the informal trade barriers
these networks help overcome are important.

The model is also extended to allow for free entry of intermediaries, first in the
one-shot game and then in the infinitely repeated game. In the former, intermediaries
compete through the choice of the commission fee and drive each other out of the market.
A range of cooperative outcomes are sustainable as subgame perfect equilibria of the
infinitely repeated game. Focusing on the case where firms cooperate to set the maximum
sustainable commission rate, the pattern of intermediation with symmetric multiple
firms is characterised.

The framework sheds light on how information and communication technology (ICT)
improvements affect both the level and means of organisation of trade. In the symmetric
cooperative equilibrium, ICT improvements that lower the cost of acquiring information
strengthen the incentives for network building by information intermediaries but lower
the number of intermediaries in the market through downward pressure on commissions.
There is a non-monotonic relationship between the information costs and expected total
trade. Generally speaking, however, we may conclude that ICT improvements tend to
increase expected total trade.

A number of empirical works assess the impact of information and communication
costs on trade flows. For example, Freund and Weinhold (2000) find an increasing
and significant impact of the internet on total trade flows from 1997-1999, with a 10%
increase in the relative number of web hosts in one country resulting in 1% greater
communication costs into a model of bilateral trade and find that international variations
in communication costs have a significant influence on trade patterns. Moreover, the
impact of communication costs on trade is larger for trade in differentiated goods, by as
much as one third. Harris (1995) also finds that information and communication needs
are much greater for differentiated goods and that trade in these products is likely to be
more sensitive to variations in the costs of communication\textsuperscript{18}. For high values of information costs where the intermediaries’ networks are not exhaustive, some trades in the market are expected not to take place. Some transactions, albeit relatively few, are lost as a result of the information frictions in the model. Indeed, there is less trade than standard models would predict; Trefler (1995) finds that trade is ‘missing’ relative to the Heckscher-Ohlin-Vanek prediction by up to 50\%. Trade costs have been posited as a possible solution to Trefler’s ‘missing trade’ puzzle, of which information costs are a component. Anderson and van Wincoop (2003) calculate the tariff equivalent estimate for total trade barriers to be 170\% for industrialised countries, and estimate information barriers at 6\% and language barriers at 7\% of this total.

The model applies to trade in differentiated goods for which information about product characteristics is important. More broadly, the model may be applied to intermediated markets where contact-building and matching are key. Examples may include headhunters in the job market, real estate agents in the housing market, charterers in the transportation market, matchmakers in the marriage market (in some cultures) and others. Each of these examples is particularly appropriate as the function of the intermediary is a matching one only. Real estate agents do not buy the properties they advertise, nor do charterers buy the ships they arrange transports for.

The commission fee of the intermediary is shared by both the importer and exporter in the model, while in practice it may be paid by only one side of the market, for example by the letter in the real estate rental market. The model may be adjusted relatively easily to take this into account by assuming the costs relating to finding a match fall on one side of the market. Rather than a probability of matching directly, let \( q(i) = 1 - i \) be the factor by which trade surplus diminishes as a result of information costs in direct trade. With this interpretation, \( iS \) is the cost of matching directly. If this cost, as well as the fee of matching through an intermediary, is borne by one side of the market, then the results of the model regarding network building and trade are identical to those described in this paper. All traders remain indifferent between matching directly or indirectly in equilibrium, but those on the side of the market that bears the cost have a lower expected payoff.

A further possible extension is to see whether intermediaries can deter entry of further intermediaries through the strategic choice of the cooperative commission level. Moreover, if it is possible, to examine whether it is optimal for intermediaries to do so.

A key feature of the model is that importers and exporters can only trade in pairs. Traders generate surplus \( S \) from matching with their perfect partner, and 0 otherwise. In practice, we may expect there to be several trade matches, of varying quality, available to

\textsuperscript{18} Whether the communication costs are seen to affect fixed or variable costs of trade depends on the role that communication costs are assumed to play in the transaction. If communication is primarily relevant in facilitating search for trading partners, then its costs could be seen as affecting the fixed or sunk costs of trading. This is the view take by Harris (1995) and that of Freund and Weinhold (2000) about the impact of the Internet. Fink, Mattoo and Neagu (2002) assume, however, that communication costs affect trade primarily by influencing variable costs between two nations, resulting in a gravity equation.
importers and exporters. Introducing a range of possible matches or several destinations for goods may provide have important implications for both the level and means of organisation of trade.

Finally, this paper focuses on international trade facilitation through information intermediaries where information frictions exist in identifying trading partners. There is considerable scope for further research on how information frictions may affect international trade as information frictions in other stages of the trade process are also likely be important.

5 References


6 Appendix A

Proposition 5 In the one shot game, G, with three intermediaries, the following strategies for the traders and intermediaries constitute the unique, subgame perfect equilibrium of the game: Intermediaries choose \([P^*_I, \alpha_I(i)] = (0, 0)\), where \(I = A, B, C\). If traders receive only one offer, accept if \(\alpha_I(i) \leq i\), where \(I \in (A, B, C)\); reject otherwise. If traders receive two offers from \(j, k \in (A, B, C)\), where \(\alpha_I(i) \neq \alpha_k(i)\) accept the \(\min[\alpha_I(i), \alpha_k(i)]\), if \(\min[\alpha_I(i), \alpha_k(i)] \leq i\); reject otherwise. If traders receive two offers, where \(\alpha_I(i) = \alpha_k(i) \leq i\), accept both \(j\) and \(k\); reject otherwise. If traders receive three offers from \(j, k, l \in (A, B, C)\), where \(\alpha_I(i) = \alpha_k(i) = \alpha_l(i) \leq i\), then accept all offers; reject otherwise. If \(\alpha_I(i) = \alpha_k(i) < \alpha_l(i)\), then accept \(j\) and \(k\) if \(\alpha_j(i) = \alpha_k(i) \leq i\); reject otherwise. If \(\alpha_I(i) < \alpha_k(i) = \alpha_l(i)\), then accept \(j\) if \(\alpha_j(i) \leq i\); reject otherwise.

Proof. Consider any pair \(j\) of trade partners \((X_j, M_j)\). The pair may match through \(A\) only, \(B\) only, \(C\) only, through \(A\) or \(B\), \(A\) or \(C\), \(B\) or \(C\), all three networks, or through neither. The probability of each event is listed in equations (53) to (59):

\[
\text{Prob}(j \text{ can match via } A, B \text{ or } C) = P_A^2 P_B^2 P_C^2
\]

(53)

\[
\text{Prob}(j \text{ can match via } A \text{ or } B) = P_A^2 P_B^2 (1 - P_C^2)
\]

(54)

\[
\text{Prob}(j \text{ can match via } A \text{ or } C) = P_A^2 P_C^2 (1 - P_B^2)
\]

(55)

\[
\text{Prob}(j \text{ can match via } B \text{ or } C) = P_B^2 P_C^2 (1 - P_A^2)
\]

(56)

\[
\text{Prob}(j \text{ can match via } A \text{ only}) = P_A^2 (1 - P_B^2) (1 - P_C^2)
\]

(57)

\[
\text{Prob}(j \text{ can match via } B \text{ only}) = P_B^2 (1 - P_A^2) (1 - P_C^2)
\]

(58)

\[
\text{Prob}(j \text{ can match via } C \text{ only}) = P_C^2 (1 - P_A^2) (1 - P_B^2)
\]

(59)

From these we can compute the expected matches made by intermediary \(A\) conditional on \(A\)'s choice of commission rate relative to that of intermediaries \(B\) and \(C\). Where the commission rates are identical, the common matches are split across the intermediaries. The seven possible cases are described by (60) to (66):

\[
E(\text{matches by } A)_{\alpha_A < \alpha_B, \alpha_C; \alpha_A \leq i} = P_A^2
\]

(60)

\[
E(\text{matches by } A)_{\alpha_A = \alpha_B < \alpha_C, \alpha_A \leq i} = P_A^2 - \frac{1}{2} P_A^2 P_B^2
\]

(61)

\[
E(\text{matches by } A)_{\alpha_A = \alpha_C < \alpha_B, \alpha_A \leq i} = P_A^2 - \frac{1}{2} P_A^2 P_B^2
\]

(62)

\[
E(\text{matches by } A)_{\alpha_A > \alpha_B, \alpha_C, \alpha_A \leq i} = P_A^2 - P_A^2 P_B^2 - P_A^2 P_C^2 + P_A^2 P_B^2 P_C^2
\]

(63)

\[
E(\text{matches by } A)_{\alpha_A = \alpha_B > \alpha_C, \alpha_A \leq i} = P_A^2 - \frac{1}{2} (P_A^2 P_B^2 - P_A^2 P_B^2 P_C^2) - P_A^2 P_C^2
\]

(64)

\[
E(\text{matches by } A)_{\alpha_A = \alpha_C > \alpha_B, \alpha_A \leq i} = P_A^2 - \frac{1}{2} (P_A^2 P_C^2 - P_A^2 P_B^2 P_C^2) - P_A^2 P_B^2
\]

(65)

\[
E(\text{matches by } A)_{\alpha_A = \alpha_B = \alpha_C \leq i} = P_A^2 - \frac{1}{2} (P_A^2 P_B^2 + P_A^2 P_C^2) + \frac{1}{3} P_A^2 P_B^2 P_C^2
\]

(66)

As with two firms, the three intermediaries have an incentive to undercut each other. It follows that a configuration of intermediary commission rates where the rates are
not equal cannot constitute a pure strategy subgame perfect equilibrium of the game. The intermediary with the higher share always has an incentive to undercut or match the other intermediary. Let the symmetric equilibrium success fees be \( \alpha_A^* = \alpha_B^* = \alpha_C^* = \alpha^* \), where \( \alpha^* \) is defined as the surplus share at which \( E(\Pi_A) | \alpha_A^* = \alpha_B^* = \alpha_C^* = \alpha^* \geq E(\Pi_A) | \alpha_A^* < \alpha_B^* = \alpha_C^* = \alpha^* \) and \( E(\Pi_A) | \alpha_A^* > \alpha_B^* = \alpha_C^* = \alpha^* \leq E(\Pi_A) | \alpha_A^* = \alpha_B^* = \alpha_C^* = \alpha^* \), and similarly for \( B \) and \( C \), where \( \alpha_A^* = \alpha_B^* = \alpha^* - \Delta \alpha \) and \( \alpha_C^* = \alpha_B^* = \alpha^* + \Delta \alpha \).

The conditions under which \( \alpha^* \) does not deviate from \( \alpha^* \) yield the following band:

\[
\frac{6\Delta\alpha (1 - P_B^2 - P_C^2 + P_B^2 P_C^2)}{3P_B^2 + 3P_C^2 - 4P_B^2 P_C^2} \leq \alpha^* \leq \frac{6\Delta\alpha}{3P_B^2 + 3P_C^2 - 2P_B^2 P_C^2} \tag{67}
\]

Similar bands can be derived for intermediaries \( B \) and \( C \). Moreover, for traders to accept the contract in stage 2, \( \alpha^* \leq i \). If intermediaries can change \( \alpha_A \) and \( \alpha_B \) by infinitesimal amounts, however, then \( \Delta \alpha \to 0 \), from which it follows that \( \alpha^* \to 0 \). Thus the intermediaries cannot extract any surplus in equilibrium due to the competitive forces driving their pricing behaviour. Anticipating this, they do not invest in network building in stage 1. ■

7 Appendix B

**Proposition 7** In the infinitely repeated game \( G_\infty \) with three intermediaries \( A \), \( B \), and \( C \), the cooperative equilibrium \( \alpha_A(i) = \alpha_B(i) = \alpha_C(i) = i \) is a subgame perfect equilibrium of \( G_\infty \), if \( \delta \geq \frac{\text{iSP}_A^3(3P_B^2 + 3P_C^2 - 2P_B^2 P_C^2)}{6(\text{iSP}_A^3 - 2\text{PA}\alpha(i) - L(i, PA) - F)} \). Otherwise, the non-cooperative equilibrium of \( G \) is the subgame perfect equilibrium in every subgame of \( G_\infty \).

**Proof.** In order for \( \alpha_A(i) = \alpha_B(i) = \alpha_C(i) = i \) to be a subgame perfect equilibrium of \( G_\infty \), there must be no incentive for any intermediary to defect from the cooperative agreement. Due to the symmetric nature of the intermediaries, we need only examine the incentives for one of the intermediaries, say \( A \). If \( A \) continues to cooperate, the expected profit under the cooperative agreement, \( \alpha_A(i) = \alpha_B(i) = \alpha_C(i) = i \), is generated in each period.

Assuming cooperative behaviour from intermediaries \( B \) and \( C \), consider the implications of a defection by \( A \). In the period in which defection occurs, \( A \) undercuts \( B \) and \( C \) gaining all the common trade matches and generating a larger expected profit. \( B \) and \( C \) observe \( A \)'s defection and cooperation breaks down for all subsequent periods. The commission rates and intermediaries' expected profits for all subsequent periods, therefore, fall to 0.

Hence, the discounted expect profit for \( A \) under the cooperative agreement is given by:

\[
\frac{1}{1 - \delta} \left[ \text{iS} \left( P_A^2 - \frac{1}{2} P_A^2 P_B^2 - \frac{1}{2} P_A^2 P_B^2 + \frac{1}{3} P_A^2 P_B^2 P_C^2 \right) - 2P_A c(i) - L(i, PA) - F \right] \tag{68}
\]

The discounted expect profit under defection is given by:
\[ \alpha_A^D SP_A^2 - 2P_Ac(i) - L(i, P_A) - F, \text{ where } \alpha_A^D = i - \Delta \alpha \] (69)

It follows that for the cooperative agreement to be sustainable, where \( \Delta \alpha \rightarrow 0 \):

\[
\frac{1}{1 - \delta} \left[ iS \left( P_A^2 - \frac{1}{2} P_A^2 P_B^2 - \frac{1}{2} P_A^2 P_C^2 + \frac{1}{3} P_A^2 P_B^2 P_C^2 \right) - 2P_Ac(i) - L(i, P_A) - F \right] \\
\geq \alpha_A^D SP_A^2 - 2P_Ac(i) - L(i, P_A) - F
\] (70)

That is:

\[
\delta \geq \frac{iSP_A^2 (3P_B^2 + 3P_C^2 - 2P_B^2 P_C^2)}{6 [iSP_A^2 - 2P_Ac(i) - L(i, P_A) - F]} \] (71)