

Occupational Choice, Specific vs. General Human Capital Investment and Uncertainty from International Trade*

Toshihiro Ichida

Waseda University

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Abstract

I provide a dynamic extension of the model of occupational choice. Given that the individual agents face uncertainty in terms of trade, what are the optimal strategy for individual agents in human capital investments? When individuals are heterogeneous in their innate capabilities in multi-dimensions, do they invest in what they were good at when they were born? Do they invest in general human capital? Or do they invest in what they were not good at originally? I find the conditions about when agents will invest in general skills and when they specialize their skills. Depending on parameter values, it is quite possible for all individuals to invest in their innately strong skills even if there is no insurance market.

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1 Introduction

In the previous paper by the author, skills of the individual agents are exogenously given. Talents of individuals are assumed to be innate. The assumption was made because of analytical convenience and of the nature of the problems at hand, namely, the analysis on gain or loss from trade liberalization and the exploration of the reasons for difficulty of compensation. Of course this assumption is somewhat restrictive because, in the real world, people's skills change over time. In fact, it is widely known that human capital investment activities such as education and on-the-job training are prevalent. (Becker 1993) The purpose of this paper is to endogenize the skills of individual agents and to analyze the dynamic model of occupational choice. I will investigate the individuals' incentives to invest in their human capital skills. In particular, we are interested in whether the heterogeneous individuals want to invest in their specific skills or in general skills in the presence of uncertainty caused by trade liberalization.

The importance of specialization has been widely recognized from long time ago. (See, for example, Adam Smith 1776) The division of labor and specialization are the key sources of higher productivity of the modern world. The division of labor may be the result of individual endowed differences in comparative advantages. (Rosen 1978) Specialization, however, may also be a result of specialized human capital investments. (Rosen 1983) When there is no uncertainty about which sector individuals are employed, individual agents have incentives to specialize. (Murphy 1986, p.37)

When the world is uncertain, ex-ante specific human capital investments may be risky because specialization makes individual agents inflexible as factors of production ex post. To borrow the expressions from Grossman and Shapiro (1982), the investment toward specialization is like reducing "the degree of intersectoral mobility" in later period. A general (as opposed to specific) human capital investment is a form of self-insurance if the insurance markets are absent. Murphy (1986, section 5.2) also looked at the case where there is uncertainty about which sector will be good. He also concluded that less specialization exposes workers to less risk. Both Grossman and Shapiro (1982) and Murphy (1986) examined the individuals' incentives to general versus specific human capital investment where every agent is *identical* before he conducts an investment in human capital. So, what happens to these results if we start from the situation where agents are different in their innate comparative advantage before they conduct human capital investments? In this paper, I would like to analyze incentives of individual agents to invest in human capital skills when the role of endowed differences is important.

From this dynamic extension of the model of occupational choice, I could address the following research questions:

- What is the role of endowed differences of individuals in human capital investments?
- Do people specialize or generalize their skills when the world is uncertain?
- Do people invest in their innately strong skills? Do people specialize in what they were good at when they were born? If so, under what conditions?
- Do agents invest their time (and money) to enhance their skills in the socially inefficient way? Do they go to schools in which they will learn something they are innately poor at?
- Will there be the same amount of job-switching individuals if we allow dynamic development of human capitals?

Although there is a large amount of literature on human capital investment to date, to my knowledge there does not exist previous works that start from multi-dimensional heterogeneity. I will introduce such a model in the next section.

2 The Model

We consider a simple two-period and two-sector model of a small open economy that faces exogenously given international output prices. In period 0, agents are endowed with multi-dimensional heterogeneous skills and invest in their human capital to enhance their innate skills. Uncertainty about terms of trade will be realized in period 1 and agents choose their occupations and engage in production. Output markets for sectors X and Y are assumed to be competitive, both internationally and domestically. In making the investment decision in period 0, each agent is assumed to have rational expectations concerning the prices that will prevail in period 1.

The economy consists of a continuum of self-employed agents $j \in J$, each of whom is endowed with an individual-specific occupational skill vector (θ^j, τ^j) jointly distributed over a unit square $[0, 1] \times [0, 1] \subset \mathbb{R}^2$ in period 0. Let Θ_t and $F_t(\theta, \tau)$ denote the space and the joint distribution function of human capital skills for each period $t = 0, 1$. Thus, we know that $\Theta_0 = [0, 1]^2$. Let $f_t(\theta, \tau) > 0$ denote the joint density function for $F_t(\theta, \tau)$, and assume that f_t is integrable over any partition of the human capital skill space Θ_t , for $t = 0, 1$. Agents are price takers in the output markets. Each element of the skill vector (θ^j, τ^j) represents a sector-specific human capital skill; their magnitudes measure the innate capabilities of the agent j in the production of X and Y in efficiency units.

Once the terms of trade become known in period 1, each agent decides either to produce X using θ , or Y using τ . Each individual undertakes only one occupation at a time because human capital skills are embodied.

(Murphy 1986) An element of the skill vector (θ^j, τ^j) is indivisible and non-transferable. The sizes of an skill vector in each period $(\theta^j, \tau^j) \in \Theta_t$ is private information for j , but its aggregate distribution F_t is publicly known.

Both outputs are assumed to be produced with symmetrical production functions¹ that exhibit constant returns to scale in occupational skill:

$$\begin{cases} x^j(\theta^j) &= \theta^j \\ y^j(\tau^j) &= \tau^j \end{cases} \quad (1)$$

where x^j and y^j are individual level outputs, where θ^j and τ^j represent the efficiency units of individual's occupational skills.

Let P_X and P_Y denote the output prices for X and Y .² Then, the individual j 's occupational-choice decision-problem will be written as

$$E(\theta^j, \tau^j; P_X, P_Y) = \max_{X,Y} (P_X \cdot x^j(\theta^j), P_Y \cdot y^j(\tau^j)) \quad (2)$$

where $E(\theta^j, \tau^j; P_X, P_Y)$ represents earnings for best job given the prices and skills. Suppose for now that the individuals are risk-neutral and maximizes their earnings. The individual j will compare their total revenue in different occupations to determine in which sector he will produce:

$$P_X \cdot x^j(\theta^j) \gtrless P_Y \cdot y^j(\tau^j) \Leftrightarrow P_X \cdot \theta^j \gtrless P_Y \cdot \tau^j$$

Thus, the job dividing line will be represented by the equation.

$$\tau = \frac{P_X}{P_Y} \cdot \theta \quad (3)$$

Agents above line (3) will produce in sector Y and below line (3) will produce in sector X . Suppose that the demand condition and expected investments by individuals in the autarky equilibrium is represented by

$$P_X^A = P_Y^A. \quad (4)$$

Let P denote the vector (P_X, P_Y) . Let us choose appropriate units so that we can normalize the autarky price vector to be $P^A = (1, 1)$. The autarky equilibrium is such that the investment decisions of individual agents are also consistent with the price vector $P^A = (1, 1)$.

Timing of the model is as follows:

¹This symmetry of the production functions is not essential to my results. It is just that by delegating all the heterogeneity to the endowment side, we are able to simplify the algebraic calculations.

²Note that, because of production functions, output prices correspond one to one to wages per efficiency unit of skills for each sector.

1. In period 0, the type of each individual $(\theta^j, \tau^j) \in \Theta_0$ is given.
2. An individual choose to invest in either sector X skill or Y skill or both.
3. In period 1, a new relative price P_1 is realized.
4. An individual chooses occupation and produce.

Now let us think about the possible human capital investments by individual agents in period 0. There are two types of human capital investment: (1) General Human Capital Investment and (2) Sector Specific Human Capital Investment. Let investment decisions in period 0 be denoted by $Inv_0 \in \{G, S_X, S_Y\}$.

Assumption 1 *General investment $Inv_0 = G$ for an individual with (θ, τ) will be represented by*

$$(\alpha\theta, \alpha\tau) \quad \cdots \quad G \tag{5}$$

while specific investments, $Inv_0 = S_i$, for $i = X, Y$, for an individual with (θ, τ) will be represented by

$$\begin{cases} (\beta\theta, \tau) \text{ if invested in sector } X & \cdots & S_X \\ (\theta, \beta\tau) \text{ if invested in sector } Y & \cdots & S_Y \end{cases} \tag{6}$$

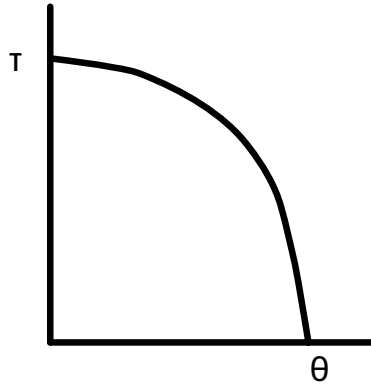
where α and β represent parameters such that

$$\beta > \alpha > 1. \tag{7}$$

From (7), both parameters are larger than 1, so human capital investments, regardless of specificity or generality, are effective. The size of investment efficiency for specific investment β is assumed to be larger than the size of α . Otherwise, every one will invest in general skills alone because specific skill investment will be dominated by general investment in either direction.

Another peculiarity about investment technology is that the effect of investments is proportional to the individual's original strength (innate capabilities) in each sector. If your innate skill θ in sector X production is very large, then your post investment skill in the same sector will be proportionally large. This assumption is appropriate if all the agents in the economy are young. For example, young Michael Jordan can be trained to be a superstar because of his exceptional innate talent as a basketball player. The effect of training Jordan is much larger than the effect of training a mediocre player. The assumption of proportional effectiveness of human capital skill investments will create a theoretical problem if we start thinking the economy with matured (and old) agents who already have invested in their skills. However, let us focus on this particular case of proportionality in this paper.

Figure 1: Human Capital Investments (Training) Possibilities Frontier.



Compared to more general investment frontier depicted in Murphy (1986) and Grossman and Shapiro (1982), (See, for example, Fig. 1 that is reproduced from the previous works.) this specification of human capital investment given by (5) and (6) is somewhat restrictive by not allowing varieties of intermediate cases in general investments. This restrictive assumptions are made in favor of simplicity. The fact that β is larger than α means an implicit assumption about some form of increasing returns to specialization in human capital investments. Because β is larger than α , we can conclude that $\Theta_1 \subset [0, \beta]^2$.

Assume that the uncertainty about the terms of trade, which will be resolved in period 1, takes the following form of two equally-likely states of nature.

Assumption 2 *Uncertainty about the terms of trade in period 1 takes the following form:*

$$P_1 = \begin{cases} (p, 1) & \text{with probability } \frac{1}{2} \\ (1, p) & \text{with probability } \frac{1}{2} \end{cases} \quad (8)$$

where p is a positive parameter larger than 1.

We concentrate on the case with same probability for the two states of nature. When the states of nature is such that $P_1 = (p, 1)$, producers in sector X will be benefitted in period 1 because P_X will be more expensive relative to P_Y . When $P_1 = (1, p)$ occurs, producers in sector Y will be benefitted because P_Y will be more expensive relative to P_X . As we have seen in the previous paper, the realization of a particular terms of trade may induce some workers to take different jobs once the uncertainty is resolved. Thus, some agents will choose to work in a favorable sector while others may stay in the sector at which they were innately good.

Let us further assume that the demand condition is symmetric in sectors X and Y , so that the consumer price index (CPI) in period 1 is the same for the different states of the world, namely $CPI_1(p, 1) = CPI_1(1, p)$. This assumption will allow us to compare welfare by directly looking at the earnings in period 1 without

worrying about substitution effects in consumption. (See Appendix for an explanation.) Assume also that no income-insurance market exists. (See Grossman and Shapiro [1982] for the comparison between self-insurance versus insurance markets.)

From the structure of uncertainty given by (8), there are two states of nature in period 1, namely, given by $P_1 \in \{(p, 1), (1, p)\}$. When the realized state of nature is $P_1 = (p, 1)$, the individual decision about occupational choice, for those who invested $Inv_0 \in \{G, S_X, S_Y\}$ in period 0, will be represented by the earnings equations $E[Inv_0; P_1 = (p, 1)]$: Three cases are given here as (9), (10), and (11) below.

1. Earning opportunities for those who conducted specific investment in sector X in period 0:

$$E[Inv_0 = S_X; P_1 = (p, 1)] = \max\{p\beta\theta, \tau\} \quad (9)$$

2. Earning opportunities for those who conducted general investment in both sectors in period 0:

$$E[G; (p, 1)] = \max\{p\alpha\theta, \alpha\tau\} \quad (10)$$

3. Earning opportunities for those who conducted specific investment in sector Y in period 0:

$$E[S_Y; (p, 1)] = \max\{p\theta, \beta\tau\} \quad (11)$$

When the state of nature is $P_1 = (1, p)$, the individual decision about occupational choice will be represented by the earnings equations (12), (13), and (14) below.

1. Earning opportunities for those who conducted specific investment in sector X in period 0:

$$E[S_X; (1, p)] = \max\{\beta\theta, p\tau\} \quad (12)$$

2. Earning opportunities for those who conducted general investment in both sectors in period 0:

$$E[G; (1, p)] = \max\{\alpha\theta, p\alpha\tau\} \quad (13)$$

3. Earning opportunities for those who conducted specific investment in sector Y in period 0:

$$E[S_Y; (1, p)] = \max\{\theta, p\beta\tau\} \quad (14)$$

Above cases represented by (9)-(14) summarize the earning possibilities of individual agents with innate skill vector (θ, τ) who can invest in either S_X , G , or S_Y . Now we can state the main theorem of the paper. There will be two cases we need to distinguish: (I) when the terms-of-trade risk is larger than the impact of investment efficiency from sector specific human capital skills investment: $p > \beta$ and (II) when the risk is smaller than the investment efficiency: $p < \beta$.

Theorem 1 (Case I) *When the terms-of-trade risk is larger than the impact of investment efficiency from sector specific human capital skills investment: $p > \beta$, the following three situations will occur depending on the size of the parameter value of α .*

- (i) *All the agents in the economy will conduct specific investments in the direction of their innate comparative advantage; agents $(\theta, \tau) \in \Theta_0$ with $\tau < \theta$ will conduct S_X (Specific investment in sector X) and agents $(\theta, \tau) \in \Theta_0$ with $\tau > \theta$ will conduct S_Y (Specific investment in sector Y), if the parameter of general investment is smaller than a particular threshold value, or*

$$\alpha < \frac{\beta + 1}{2}.$$

- (ii) *There will be some agents who will conduct general investments G . In particular, agents $(\theta, \tau) \in \Theta_0$ with*

$$\frac{\beta - \alpha}{\alpha - 1} \cdot \theta < \tau < \frac{\alpha - 1}{\beta - \alpha} \cdot \theta$$

will conduct G (General investment in both sectors), agents $(\theta, \tau) \in \Theta_0$ with $\tau < \frac{\beta - \alpha}{\alpha - 1} \cdot \theta$ will conduct S_X , and agents $(\theta, \tau) \in \Theta_0$ with $\tau > \frac{\alpha - 1}{\beta - \alpha} \cdot \theta$ will conduct S_Y , if the parameter of general investment is within a particular threshold values, or

$$\frac{\beta + 1}{2} < \alpha < \frac{(p + 1)\beta}{p + \beta}.$$

- (iii) *There will be some agents who will conduct general investments G . In particular, agents $(\theta, \tau) \in \Theta_0$ with*

$$\frac{\beta}{p} \cdot \theta < \tau < \frac{p}{\beta} \cdot \theta$$

will conduct G (General investment in both sectors), agents $(\theta, \tau) \in \Theta_0$ with $\tau < \frac{\beta}{p} \cdot \theta$ will conduct S_X , and agents $(\theta, \tau) \in \Theta_0$ with $\tau > \frac{p}{\beta} \cdot \theta$ will conduct S_Y , if the parameter of general investment is larger than a particular threshold value, or

$$\frac{(p + 1)\beta}{p + \beta} < \alpha.$$

Now we would also state the theorem for the case II where the risk is smaller than the investment efficiency: $p < \beta$.

Theorem 2 (Case II) *When the terms-of-trade risk is smaller than the impact of investment efficiency from sector specific human capital skills investment: $p < \beta$, the following three situations will occur depending on the size of the parameter value of α .*

- (i) *All the agents in the economy will conduct specific investments in the direction of their innate comparative advantage; agents $(\theta, \tau) \in \Theta_0$ with $\tau < \theta$ will conduct S_X (Specific investment in sector X) and agents $(\theta, \tau) \in \Theta_0$ with $\tau > \theta$ will conduct S_Y (Specific investment in sector Y), if the parameter of general investment is smaller than a particular threshold value, or*

$$\alpha < \frac{p + 1}{2p} \cdot \beta.$$

(ii) There will be some agents who will conduct general investments G . In particular, agents $(\theta, \tau) \in \Theta_0$ with

$$\frac{p\beta + \beta - p\alpha}{p\alpha} \cdot \theta < \tau < \frac{p\alpha}{p\beta + \beta - p\alpha} \cdot \theta$$

will conduct G (General investment in both sectors), agents $(\theta, \tau) \in \Theta_0$ with $\tau < \frac{p\beta + \beta - p\alpha}{p\alpha} \cdot \theta$ will conduct S_X , and agents $(\theta, \tau) \in \Theta_0$ with $\tau > \frac{p\alpha}{p\beta + \beta - p\alpha} \cdot \theta$ will conduct S_Y , if the parameter of general investment is within a particular threshold values, or

$$\frac{p+1}{2p} \cdot \beta < \alpha < \frac{p+1}{p(p+\beta)} \cdot \beta^2.$$

(iii) There will be some agents who will conduct general investments G . In particular, agents $(\theta, \tau) \in \Theta_0$ with

$$\frac{1}{p} \cdot \theta < \frac{p\beta + \beta - p\alpha}{p\alpha} \cdot \theta < \tau < \frac{p\alpha}{p\beta + \beta - p\alpha} \cdot \theta < p \cdot \theta.$$

will conduct G (General investment in both sectors), agents $(\theta, \tau) \in \Theta_0$ with $\tau < \frac{\beta}{p} \cdot \theta$ will conduct S_X , and agents $(\theta, \tau) \in \Theta_0$ with $\tau > \frac{\beta}{p} \cdot \theta$ will conduct S_Y , if the parameter of general investment is larger than a particular threshold value, or

$$\frac{p+1}{p(p+\beta)} \cdot \beta^2 < \alpha.$$

We will prove these theorems in the following sections. First, given these individual decisions faced in period 1 in (9)-(14), we will introduce finer partitions of the skill space.

2.1 The case (I) with $p > \beta$

When $p > \beta$ holds, then we can create 8 partitions from the skill space Θ_0 in the following manner:³

$$\left\{ \begin{array}{l} K_1 : (\theta, \tau) \in \Theta_0 \cap \{ 0 < \tau < \frac{1}{p\beta}\theta \} \\ K_2 : (\theta, \tau) \in \Theta_0 \cap \{ \frac{1}{p\beta}\theta < \tau < \frac{1}{p}\theta \} \\ K_3 : (\theta, \tau) \in \Theta_0 \cap \{ \frac{1}{p}\theta < \tau < \frac{\beta}{p}\theta \} \\ K_4 : (\theta, \tau) \in \Theta_0 \cap \{ \frac{\beta}{p}\theta < \tau < \theta \} \\ K_5 : (\theta, \tau) \in \Theta_0 \cap \{ \theta < \tau < \frac{p}{\beta}\theta \} \\ K_6 : (\theta, \tau) \in \Theta_0 \cap \{ \frac{p}{\beta}\theta < \tau < p\theta \} \\ K_7 : (\theta, \tau) \in \Theta_0 \cap \{ p\theta < \tau < p\beta\theta \} \\ K_8 : (\theta, \tau) \in \Theta_0 \cap \{ p\beta\theta < \tau < \infty \} \end{array} \right. \quad (15)$$

When the state of nature is $P_1 = (p, 1)$, agents who had invested S_X in period 0 and whose skill vector (in period 1) is within the following partition.

$$\bigcup_{i=1}^7 K_i = K_1 \cup K_2 \cup K_3 \cup K_4 \cup K_5 \cup K_6 \cup K_7 \quad (16)$$

³Because measure on the line is zero, we used $<$ in stead of \leq throughout.

will produce in sector X and agents in K_8 will produce in sector Y . Under the same state of nature, if the agents had invested in G , then those who are now in $\bigcup_{i=1}^6 K_i$ will produce in sector X and agents in $K_7 \cup K_8$ will produce in sector Y . If invested in S_Y , then agents who are in $\bigcup_{i=1}^5 K_i$ will produce in sector X and agents in $\bigcup_{i=6}^8 K_i$ will produce in sector Y .

Under the state of nature $P_1 = (1, p)$, agents who invested S_X and who are in $\bigcup_{i=1}^3 K_i$ will produce in sector X and agents in $\bigcup_{i=4}^8 K_i$ will produce in sector Y . Under the same state of nature, if the agents had invested in G , then those who are in $K_1 \cup K_2$ will produce in sector X and agents in $\bigcup_{i=3}^8 K_i$ will produce in sector Y . If invested in S_Y , then agents who are in K_1 will produce in sector X and agents in $\bigcup_{i=2}^8 K_i$ will produce in sector Y .

Now ex-ante expected earnings $E(Inv_0)$ from the investment schedules $Inv_0 \in \{G, S_X, S_Y\}$ when the uncertainty is represented by (8) are written by the following equations.

$$\begin{cases} E(S_X) &= \frac{1}{2}E[S_X; (p, 1)] + \frac{1}{2}E[S_X; (1, p)] \\ E(G) &= \frac{1}{2}E[G; (p, 1)] + \frac{1}{2}E[G; (1, p)] \\ E(S_Y) &= \frac{1}{2}E[S_Y; (p, 1)] + \frac{1}{2}E[S_Y; (1, p)] \end{cases} \quad (17)$$

For each partition K_i , the choice about which sector the agents produce is clear, and (17) will have real values once we decide the partition. Let such expected value denoted by $E(K_i; Inv_0)$, then we write the expected earnings for the agents in partition K_1 in the following manner.

$$\begin{cases} E(K_1; S_X) &= \frac{1}{2}E[S_X; X, (p, 1)] + \frac{1}{2}E[S_X; X, (1, p)] = \frac{1}{2}p\beta\theta + \frac{1}{2}\beta\theta \\ E(K_1; G) &= \frac{1}{2}E[G; X, (p, 1)] + \frac{1}{2}E[G; X, (1, p)] = \frac{1}{2}p\alpha\theta + \frac{1}{2}\alpha\theta \\ E(K_1; S_Y) &= \frac{1}{2}E[S_Y; X, (p, 1)] + \frac{1}{2}E[S_Y; X, (1, p)] = \frac{1}{2}p\theta + \frac{1}{2}\theta \end{cases} \quad (18)$$

where $E[Inv_0; i_1, P_1]$ denotes ex-post earnings in period 1 when the investment made in period 0 is Inv_0 , their working sector is $i_1 \in \{X, Y\}$ and the state of nature is given by $P_1 \in \{(p, 1), (1, p)\}$. From (18), it is clear that $E(K_1; S_X) > E(K_1; G) > E(K_1; S_Y)$. Thus, agents in K_1 will choose S_X in period 0 unambiguously.

For partition K_2 , we can show that agents in K_2 will choose S_X in period 0.

$$\begin{cases} E(K_2; S_X) &= \frac{1}{2}E[S_X; X, (p, 1)] + \frac{1}{2}E[S_X; X, (1, p)] = \frac{1}{2}p\beta\theta + \frac{1}{2}\beta\theta \\ E(K_2; G) &= \frac{1}{2}E[G; X, (p, 1)] + \frac{1}{2}E[G; X, (1, p)] = \frac{1}{2}p\alpha\theta + \frac{1}{2}\alpha\theta \\ E(K_2; S_Y) &= \frac{1}{2}E[S_Y; X, (p, 1)] + \frac{1}{2}E[S_Y; Y, (1, p)] = \frac{1}{2}p\theta + \frac{1}{2}p\beta\tau \end{cases} \quad (19)$$

It is clear that $E(K_2; S_X) > E(K_2; G)$ always holds true. $E(K_2; S_X) > E(K_2; S_Y)$ also holds true for K_2 , since

$$E(K_2; S_X) - E(K_2; S_Y) = \frac{1}{2}p\beta \left\{ \theta \cdot \left(\frac{1}{p} + 1 - \frac{1}{\beta} \right) - \tau \right\} > 0 \quad (20)$$

for K_2 : $\frac{1}{p\beta}\theta < \tau < \frac{1}{p}\theta$ and $1 - \frac{1}{\beta} = \frac{\beta-1}{\beta} > 0$.

For partition K_3 , again we can show that agents in K_3 will choose S_X in period 0.

$$\begin{cases} E(K_3; S_X) &= \frac{1}{2}E[S_X; X, (p, 1)] + \frac{1}{2}E[S_X; X, (1, p)] &= \frac{1}{2}p\beta\theta + \frac{1}{2}\beta\theta \\ E(K_3; G) &= \frac{1}{2}E[G; X, (p, 1)] + \frac{1}{2}E[G; Y, (1, p)] &= \frac{1}{2}p\alpha\theta + \frac{1}{2}p\alpha\tau \\ E(K_3; S_Y) &= \frac{1}{2}E[S_Y; X, (p, 1)] + \frac{1}{2}E[S_Y; Y, (1, p)] &= \frac{1}{2}p\theta + \frac{1}{2}p\beta\tau \end{cases} \quad (21)$$

It can be shown that $E(K_3; S_X) > E(K_3; S_Y)$ holds true for K_3 since RHS of (20) is the same as $E(K_3; S_X) - E(K_3; S_Y)$. In order to make sure that all agents in K_3 satisfy this condition, let us check the following:

$$\frac{\beta}{p} < \left(1 + \frac{1}{p} - \frac{1}{\beta}\right) \iff 1 < \beta < p. \quad (22)$$

Inequality (22) holds true because of assumptions made in this subsection. Now I also claim that $E(K_3; S_X) > E(K_3; G)$. In order to check if this is true, we would like to check the sign of the following expression,

$$E(K_3; S_X) - E(K_3; G) = \frac{p\alpha}{2} \cdot \left\{ \left(\frac{\beta - \alpha}{\alpha} + \frac{\beta}{\alpha} \cdot \frac{1}{p} \right) \theta - \tau \right\}. \quad (23)$$

To show that (23) is positive, we claim the following:

$$\left(\frac{\beta - \alpha}{\alpha} + \frac{\beta}{\alpha} \cdot \frac{1}{p} \right) > \frac{\beta}{p} \quad (24)$$

and this holds since

$$\left(\frac{\beta - \alpha}{\alpha} + \frac{\beta}{\alpha} \cdot \frac{1}{p} \right) - \frac{\beta}{p} = \frac{1}{p\alpha} \{ (\beta - \alpha) + (p - \alpha)\beta \} > 0$$

is obvious. If we summarize these results, we can state the following proposition.

Proposition 1 *When the terms of trade parameter satisfies $p > \beta$, all agents in partition $K_1 \cup K_2 \cup K_3$ or people who are in the set*

$$\left((\theta, \tau) \in \Theta_0 \cap \left\{ 0 < \tau < \frac{\beta}{p}\theta \right\} \right) \quad (25)$$

will choose to invest their human capital skills specifically for sector X. And because these agents invest in specific human capital of sector X in period 0, they will choose to work in sector X after the uncertainty is resolved in period 1 regardless of the realized state of nature.

Before we analyze the cases for partitions K_4 and K_5 , let us first see the partitions K_6 , K_7 , and K_8 . In a sense, these partitions are symmetric case to the partitions K_1 , K_2 , and K_3 . We will start from K_8 .

Expected value for partition K_8 : $p\beta\theta < \tau < \infty$ will be represented by the following equations.

$$\begin{cases} E(K_8; S_X) &= \frac{1}{2}E[S_X; Y, (p, 1)] + \frac{1}{2}E[S_X; Y, (1, p)] &= \frac{1}{2}\tau + \frac{1}{2}p\tau \\ E(K_8; G) &= \frac{1}{2}E[G; Y, (p, 1)] + \frac{1}{2}E[G; Y, (1, p)] &= \frac{1}{2}\alpha\tau + \frac{1}{2}p\alpha\tau \\ E(K_8; S_Y) &= \frac{1}{2}E[S_Y; Y, (p, 1)] + \frac{1}{2}E[S_Y; Y, (1, p)] &= \frac{1}{2}\beta\tau + \frac{1}{2}p\beta\tau \end{cases} \quad (26)$$

Agents in partition K_8 are the die-hard producers for sector Y . They all will produce in sector Y regardless of their investment decisions in period 0. By looking at the equations in (26), it is easy to show that $E(K_8; S_Y) > E(K_8; G) > E(K_8; S_X)$ and agents in K_8 will choose S_Y in period 0.

For partition K_7 : $p\theta < \tau < p\beta\theta$ expected earning equations will be represented by the following equation.

$$\begin{cases} E(K_7; S_X) &= \frac{1}{2}E[S_X; X, (p, 1)] + \frac{1}{2}E[S_X; Y, (1, p)] &= \frac{1}{2}p\beta\theta + \frac{1}{2}p\tau \\ E(K_7; G) &= \frac{1}{2}E[G; Y, (p, 1)] + \frac{1}{2}E[G; Y, (1, p)] &= \frac{1}{2}\alpha\tau + \frac{1}{2}p\alpha\tau \\ E(K_7; S_Y) &= \frac{1}{2}E[S_Y; Y, (p, 1)] + \frac{1}{2}E[S_Y; Y, (1, p)] &= \frac{1}{2}\beta\tau + \frac{1}{2}p\beta\tau \end{cases} \quad (27)$$

It is easy to show that $E(K_7; S_Y) > E(K_7; G)$ because it is the same as $E(K_8; S_Y) > E(K_8; G)$ above. Now we will show that $E(K_7; S_Y) > E(K_7; S_X)$.

$$E(K_7; S_Y) - E(K_7; S_X) = \frac{p\beta}{2} \left(\tau \left(1 + \frac{1}{p} - \frac{1}{\beta} \right) - \theta \right) \quad (28)$$

If we can show that

$$p > \frac{1}{\left(1 + \frac{1}{p} - \frac{1}{\beta} \right)} \iff p + 1 - \frac{p}{\beta} > 1 \iff p - \frac{p}{\beta} = p \left(\frac{\beta - 1}{\beta} \right) > 0 \quad (29)$$

Then for all agents in K_7 , $E(K_7; S_Y) > E(K_7; S_X)$ will hold.

For partition K_6 : $\frac{p}{\beta}\theta < \tau < p\theta$, we can show that agents in K_6 will choose S_Y in period 0. If we compare the expected earning equations, we have the following results.

$$\begin{cases} E(K_6; S_X) &= \frac{1}{2}E[S_X; X, (p, 1)] + \frac{1}{2}E[S_X; Y, (1, p)] &= \frac{1}{2}p\beta\theta + \frac{1}{2}p\tau \\ E(K_6; G) &= \frac{1}{2}E[G; X, (p, 1)] + \frac{1}{2}E[G; Y, (1, p)] &= \frac{1}{2}p\alpha\theta + \frac{1}{2}p\alpha\tau \\ E(K_6; S_Y) &= \frac{1}{2}E[S_Y; Y, (p, 1)] + \frac{1}{2}E[S_Y; Y, (1, p)] &= \frac{1}{2}\beta\tau + \frac{1}{2}p\beta\tau \end{cases} \quad (30)$$

It is clear that $E(K_6; S_Y) > E(K_6; S_X)$ always holds true since the value of $E(K_6; S_Y) - E(K_6; S_X)$ is the same as the RHS of the equation (28), we just have to check the similar condition as in (29) holds for all agents in this partition K_6 :

$$\frac{p}{\beta} > \frac{p\beta}{(p\beta + \beta - p)} \iff 1 < \beta < p. \quad (31)$$

Thus, it is shown because RHS of the inequality (31) holds true by assumption. Now, $E(K_6; S_Y) > E(K_6; G)$ also holds true for K_6 , since

$$E(K_6; S_Y) - E(K_6; G) = \frac{1}{2}p\alpha \left\{ \tau \cdot \left(\frac{\beta}{p\alpha} + \frac{\beta}{\alpha} - 1 \right) - \theta \right\} > 0 \quad (32)$$

for K_6 : $\frac{p}{\beta}\theta < \tau < p\theta$. To show the above, we must show that

$$\frac{p}{\beta} > \frac{p\alpha}{(\beta + p\beta - p\alpha)} \iff \frac{\beta}{p} < \frac{(\beta + p\beta - p\alpha)}{p\alpha} = \frac{\beta}{p\alpha} + \frac{\beta - \alpha}{\alpha} \quad (33)$$

The latter holds true as

$$(\beta + p\beta - p\alpha) - \alpha\beta = p(\beta - \alpha) + (1 - \alpha)\beta > 0. \quad (34)$$

Therefore, for all agents in K_6 , specific investment in sector Y , that is, S_Y is chosen.

If we summarize these results, we can state the following proposition.

Proposition 2 *When the terms of trade parameter satisfies $p > \beta$, for all agents in partition $K_6 \cup K_7 \cup K_8$ or people who are in the set*

$$\left((\theta, \tau) \in \Theta_0 \cap \left\{ \frac{p}{\beta}\theta < \tau < \infty \right\} \right) \quad (35)$$

will choose to invest their human capital skills specifically for sector Y . And because these agents invest in specific human capital of sector Y in period 0, they will choose to work in sector Y after the uncertainty is resolved in period 1 regardless of the realized state of nature.

Now that we finished describing the extreme cases, let's go back to the intermediate cases. Under the parameter values $p > \beta$, those who are in partition K_4 and K_5 are the agents who will produce in sector X under the states of nature $P_1 = (p, 1)$, and who will produce in sector Y under the states of nature $P_1 = (1, p)$ regardless of their prior investment decisions. Thus, their expected earnings equations are given by the following equations for $i = 4, 5$.

$$\begin{cases} E(K_i; S_X) &= \frac{1}{2}E[S_X; X, (p, 1)] + \frac{1}{2}E[S_X; Y, (1, p)] &= \frac{1}{2}p\beta\theta + \frac{1}{2}p\tau \\ E(K_i; G) &= \frac{1}{2}E[G; X, (p, 1)] + \frac{1}{2}E[G; Y, (1, p)] &= \frac{1}{2}p\alpha\theta + \frac{1}{2}p\alpha\tau \\ E(K_i; S_Y) &= \frac{1}{2}E[S_Y; X, (p, 1)] + \frac{1}{2}E[S_Y; Y, (1, p)] &= \frac{1}{2}p\theta + \frac{1}{2}p\beta\tau \end{cases} \quad (36)$$

We can conclude that $E(K_i; S_X) > E(K_i; S_Y)$ holds true for $i = 4$ and the opposite $E(K_i; S_X) < E(K_i; S_Y)$ holds true for $i = 5$. To prove this claim, let us calculate $E(K_i; S_X) - E(K_i; S_Y)$

$$E(K_i; S_X) - E(K_i; S_Y) = \frac{p}{2} \cdot (\beta - 1) (\theta - \tau) \quad (37)$$

Now, from (7) we can see that $\beta - 1 > 0$ and we know $p > 1$ by assumption. Clearly, RHS of the equation (37) is positive when $\theta > \tau$ (the case for K_4) and it is negative when $\theta < \tau$ (the case for K_5). Thus, we can conclude that

$$\text{for } i = 4, 5, E(K_i; S_X) \begin{matrix} \geq \\ \leq \end{matrix} E(K_i; S_Y) \iff \theta \begin{matrix} \geq \\ \leq \end{matrix} \tau. \quad (38)$$

As we know that $K_4 : \frac{\beta}{p}\theta < \tau < \theta$ and $K_5 : \theta < \tau < \frac{p}{\beta}\theta$, this proves the claim. The specific investment in what they were poor at in the autarky is strictly dominated by the investment in what they were good at in the autarky. This result is summarized as a proposition.

Proposition 3 *When the terms of trade parameter satisfies $p > \beta$, if specific investment occurs at all for agents in K_4 and K_5 , then it is in the direction of strengthening the agent's innate comparative advantage.*

Now we shall compare the general investment $E(K_i; G)$ with the specific investment in sector X represented by $E(K_i; S_X)$. Let us first calculate the following:

$$E(K_i; G) - E(K_i; S_X) = \frac{p}{2} \cdot \{(\alpha - 1)\tau - (\beta - \alpha)\theta\}. \quad (39)$$

From the condition in (7), we know $(\beta - \alpha) > 0$ and $(\alpha - 1) > 0$. Thus, I can conclude that

$$E(K_i; G) > E(K_i; S_X) \quad \text{if and only if} \quad \tau > \frac{\beta - \alpha}{\alpha - 1} \cdot \theta. \quad (40)$$

In the similar manner, we can also conclude that

$$E(K_i; G) > E(K_i; S_Y) \quad \text{if and only if} \quad \tau < \frac{\alpha - 1}{\beta - \alpha} \cdot \theta. \quad (41)$$

Given the condition (40) and (41), I can state the following proposition.

Proposition 4 *When the terms of trade parameter satisfies $p > \beta$, general investment occurs for agents in K_4 and K_5 only when the following condition holds.*

$$\alpha > \frac{\beta + 1}{2} \quad (42)$$

Epecially, if α is larger than $\frac{(p+1)\beta}{p+\beta}$, then all agents in K_4 and K_5 will engage in general investment. When the parameter of general human capital investment α must be within a certain range given by

$$\frac{\beta + 1}{2} < \alpha < \frac{(p + 1)\beta}{p + \beta},$$

agents who will invest in general skills and who will invest in specific skills coexist.

To prove above proposition, let us first consider the region of K_4 . The region K_5 is symmetric to this case because $\frac{\alpha-1}{\beta-\alpha}$ is a reciprocal of $\frac{\beta-\alpha}{\alpha-1}$. Now, let the general investment line be denoted by the equation:

$$\tau = \frac{\beta - \alpha}{\alpha - 1} \cdot \theta \quad (43)$$

where agents above this line (43) will find general investment better than specific investment and agents below the line (43) will find specific investment in sector X to be better than general investment. Since partition K_4 is the area between the line whose equation is

$$\tau = \frac{\beta}{p} \cdot \theta \quad (44)$$

and the line whose equation is

$$\tau = \theta. \quad (45)$$

By comparing the location of these lines (43), (44), and (45), we can examine the following three cases:

1. When the general investment line (43) is below the division line between K_3 and K_4 (44), this case will be represented by inequality:

$$\frac{\beta - \alpha}{\alpha - 1} < \frac{\beta}{p} \iff \frac{p}{\beta} < \frac{\alpha - 1}{\beta - \alpha} \iff \frac{(p + 1)\beta}{p + \beta} < \alpha \quad (46)$$

When the condition (46) holds true, the intersection of set K_4 and the set of agents who prefer general investment is equal to set K_4 . Therefore, when the inequality (46) holds, only general investment occurs in partition K_4 . This case corresponds to the reciprocal case in the partition K_5 where the general investment line for partition K_5 , $\tau = \frac{\alpha - 1}{\beta - \alpha} \cdot \theta$ (in this case, agents below this line prefers general investment) is above the division line between K_5 and K_6 . (The middle term in (46) represents this inequality. Similar analogy carries over to the next two cases where I will omit the explanation.)

2. When the general investment line (43) is within the area K_4 , which is between the line (44) and the line (45), this case will be represented by the following inequality:

$$\frac{\beta}{p} < \frac{\beta - \alpha}{\alpha - 1} < 1 \iff 1 < \frac{\alpha - 1}{\beta - \alpha} < \frac{p}{\beta} \iff \frac{(p + 1)\beta}{p + \beta} > \alpha > \frac{\beta + 1}{2}. \quad (47)$$

And in this case, partition $K_4 : \frac{\beta}{p}\theta < \tau < \theta$ can be further partitioned into two.

$$\left\{ \begin{array}{l} K_4(S_X) \quad : \quad \frac{\beta}{p}\theta < \tau < \frac{\beta - \alpha}{\alpha - 1}\theta \\ K_4(G) \quad \quad : \quad \frac{\beta - \alpha}{\alpha - 1}\theta < \tau < \theta \end{array} \right. \quad (48)$$

where agents in $K_4(S_X)$ will conduct specific investment in sector X and agents in $K_4(G)$ will conduct general investment.

3. When the general investment line (43) is above the division line between K_4 and K_5 , represented by (45), this case will be represented by the following inequality:

$$1 < \frac{\beta - \alpha}{\alpha - 1} \iff \frac{\alpha - 1}{\beta - \alpha} < 1 \iff \alpha < \frac{\beta + 1}{2} \quad (49)$$

When the condition (49) holds true, the intersection of set K_4 and the set of agents who prefer general investment is empty. Therefore, when (49) is satisfied, only specific investment for sector X occurs in partition K_4 .

We can state the following corollary to the above proposition:

Corollary 1 *When the terms of trade parameter satisfies $p > \beta$, all agents will specialize in the sector which they have innate comparative advantage if*

$$\alpha < \frac{\beta + 1}{2} \quad (50)$$

is satisfied. That is, everyone who are in $\bigcup_{i=1}^4 K_i$ will invest S_X , everyone who are in $\bigcup_{i=5}^8 K_i$ will invest S_Y and no one will invest G . The condition (50) means that the parameter for general investment efficiency α is very small compared to the parameter for the specific investment efficiency β .

Even if there is no income insurance market, risk neutral agents prefer to specialize in what they are born to be good at given that the specific investment is very strong compared to general investment. But this is when terms of trade risk is large: $p > \beta$. Let us now examine the case where the terms of trade risk is smaller: $p < \beta$.

2.2 The case (II) with $p < \beta$

When $p < \beta$ holds, then we can create 8 partitions in the following manner:

$$\left\{ \begin{array}{l} H_1 : (\theta, \tau) \in \Theta_0 \cap \left\{ 0 < \tau < \frac{1}{p\beta}\theta \right\} \\ H_2 : (\theta, \tau) \in \Theta_0 \cap \left\{ \frac{1}{p\beta}\theta < \tau < \frac{1}{p}\theta \right\} \\ H_3 : (\theta, \tau) \in \Theta_0 \cap \left\{ \frac{1}{p}\theta < \tau < \frac{p}{\beta}\theta \right\} \\ H_4 : (\theta, \tau) \in \Theta_0 \cap \left\{ \frac{p}{\beta}\theta < \tau < \theta \right\} \\ H_5 : (\theta, \tau) \in \Theta_0 \cap \left\{ \theta < \tau < \frac{\beta}{p}\theta \right\} \\ H_6 : (\theta, \tau) \in \Theta_0 \cap \left\{ \frac{\beta}{p}\theta < \tau < p\theta \right\} \\ H_7 : (\theta, \tau) \in \Theta_0 \cap \left\{ p\theta < \tau < p\beta\theta \right\} \\ H_8 : (\theta, \tau) \in \Theta_0 \cap \left\{ p\beta\theta < \tau < \infty \right\} \end{array} \right. \quad (51)$$

Note that the division lines between H_3 and H_4 and lines between H_5 and H_6 are different from the case in (15).

When the state of nature is $P_1 = (p, 1)$, agents who had invested S_X in period 0 and whose skill vector (in period 1) is within the following partition $\bigcup_{i=1}^7 H_i$ will produce in sector X and agents in H_8 will produce in sector Y . Under the same state of nature, if the agents had invested in G , then those who are now in $\bigcup_{i=1}^6 H_i$ will produce in sector X and agents in $H_7 \cup H_8$ will produce in sector Y . If invested in S_Y , then agents who are in $\bigcup_{i=1}^3 H_i$ will produce in sector X and agents in $\bigcup_{i=4}^8 H_i$ will produce in sector Y . Note that, under $P_1 = (p, 1)$, only the case for S_Y differs from the pattern from $p > \beta$.

Under the state of nature $P_1 = (1, p)$, agents who invested S_X and who are in $\bigcup_{i=1}^5 H_i$ will produce in sector X and agents in $\bigcup_{i=6}^8 H_i$ will produce in sector Y . Under the same state of nature, if the agents had invested in G , then those who are in $H_1 \cup H_2$ will produce in sector X and agents in $\bigcup_{i=3}^8 H_i$ will produce in sector Y . If invested in S_Y , then agents who are in H_1 will produce in sector X and agents in $\bigcup_{i=2}^8 H_i$ will produce in sector Y . Note that, under $P_1 = (1, p)$, only the case for S_X differs from the pattern from $p > \beta$.

The analysis for H_1, H_2, H_7 , and H_8 are exactly the same as the cases for K_1, K_2, K_7 , and K_8 , therefore, they are omitted. The analysis for H_3, H_4 and H_5, H_6 are symmetrical, therefore, let us focus on the analysis for H_3 and H_4 in this section.

For partition H_3 , expected earnings from different investment decisions in period 0 will be represented by the following equations:

$$\begin{cases} E(H_3; S_X) &= \frac{1}{2}E[S_X; X, (p, 1)] + \frac{1}{2}E[S_X; X, (1, p)] &= \frac{1}{2}p\beta\theta + \frac{1}{2}\beta\theta \\ E(H_3; G) &= \frac{1}{2}E[G; X, (p, 1)] + \frac{1}{2}E[G; Y, (1, p)] &= \frac{1}{2}p\alpha\theta + \frac{1}{2}p\alpha\tau \\ E(H_3; S_Y) &= \frac{1}{2}E[S_Y; X, (p, 1)] + \frac{1}{2}E[S_Y; Y, (1, p)] &= \frac{1}{2}p\theta + \frac{1}{2}p\beta\tau \end{cases} \quad (52)$$

It can be shown that $E(H_3; S_X) > E(H_3; S_Y)$ holds true for $H_3 : \frac{1}{p}\theta < \tau < \frac{p}{\beta}\theta$ because we can show the following:

$$E(H_3; S_X) - E(H_3; S_Y) = \frac{1}{2} \{p\beta(\theta - \tau) + (\beta - p)\theta\} > 0 \quad (53)$$

because $\theta - \tau > 0$ and $\beta - p > 0$. Now we want to compare $E(H_3; S_X)$ and $E(H_3; G)$. To determine, we will calculate the difference between the two and try to figure out the sign. However,

$$E(H_3; S_X) - E(H_3; G) = \frac{1}{2} \{p\beta\theta + \beta\theta - p\alpha\theta - p\alpha\tau\} \geq 0 \quad (54)$$

is not clear. Let us postpone the analysis until we look at the case for H_4 and we will look both cases together.

For partition H_4 , expected earnings from different investment decisions in period 0 will be represented by the following equations.

$$\begin{cases} E(H_4; S_X) &= \frac{1}{2}E[S_X; X, (p, 1)] + \frac{1}{2}E[S_X; X, (1, p)] &= \frac{1}{2}p\beta\theta + \frac{1}{2}\beta\theta \\ E(H_4; G) &= \frac{1}{2}E[G; X, (p, 1)] + \frac{1}{2}E[G; Y, (1, p)] &= \frac{1}{2}p\alpha\theta + \frac{1}{2}p\alpha\tau \\ E(H_4; S_Y) &= \frac{1}{2}E[S_Y; Y, (p, 1)] + \frac{1}{2}E[S_Y; Y, (1, p)] &= \frac{1}{2}\beta\tau + \frac{1}{2}p\beta\tau \end{cases} \quad (55)$$

It can be shown that $E(H_4; S_X) > E(H_4; S_Y)$ holds true for $H_4 : \frac{p}{\beta}\theta < \tau < \theta$ because we can show the following:

$$E(H_4; S_X) - E(H_4; S_Y) = \frac{1}{2} \{\beta(p+1)(\theta - \tau)\} > 0 \quad (56)$$

because $\theta - \tau > 0$. Now we want to compare $E(H_4; S_X)$ and $E(H_4; G)$, however, the result will be analyzed together with the case for H_3 since both cases have the same expected returns.

Let us rewrite the difference between $E(H_i; S_X)$ and $E(H_i; G)$, for $i = 3, 4$.

$$E(H_i; S_X) - E(H_i; G) = \frac{1}{2} \{p\beta\theta + \beta\theta - p\alpha\theta - p\alpha\tau\} \text{ for } i = 3, 4 \quad (57)$$

From the equation (57), we can conclude that the general investment line will be represented by the following equation.

$$\tau = \frac{p\beta + \beta - p\alpha}{p\alpha} \cdot \theta \quad (58)$$

Above the line (58), general investment is preferred since $E(H_i; S_X) < E(H_i; G)$, and below the line (58), specific investment in sector X is preferred since $E(H_i; S_X) > E(H_i; G)$. We now want to compare the location of the line (58) with the border lines for the partitions H_3 and H_4 , namely, the following three equations:

$$\begin{cases} \tau = \frac{1}{p}\theta \\ \tau = \frac{p}{\beta}\theta \\ \tau = \theta \end{cases} \quad (59)$$

The area surrounded by the first two equations in (59) is the partition H_3 , and the second and third equations will make the partition H_4 .

Claim 1 $\frac{p\beta + \beta - p\alpha}{p\alpha} > \frac{1}{p}$ must always hold. Therefore, it is not possible to have the case where all the agents in H_3 and H_4 will prefer G to S_X .

The above claim can be proved by calculating the following.

$$\frac{p\beta + \beta - p\alpha}{\alpha} - 1 = \frac{p\beta}{\alpha} + \frac{\beta}{\alpha} - p - 1 = (p+1) \left(\frac{\beta}{\alpha} - 1 \right) > 0 \quad (60)$$

holds true since the condition (7) says $\beta > \alpha$ and this implies $\left(\frac{\beta}{\alpha} - 1 \right) > 0$.

When the inequality

$$\frac{1}{p} < \frac{p\beta + \beta - p\alpha}{p\alpha} < \frac{p}{\beta} \quad (61)$$

is satisfied, then the general investment line will cut through the partition H_3 . Because of the above claim, LHS of the inequality (61) is satisfied automatically. Let us check the condition for the RHS of the inequality (61).

$$\frac{p\beta + \beta - p\alpha}{p\alpha} < \frac{p}{\beta} \iff \alpha > \frac{(p+1)\beta^2}{p(p+\beta)} \quad (62)$$

Therefore, if we summarize these results, we can state the following proposition.

Proposition 5 When the terms of trade parameter satisfies $p < \beta$, and when the parameter α is larger than $\frac{(p+1)\beta^2}{p(p+\beta)}$, then the condition $\frac{1}{p} < \frac{p\beta + \beta - p\alpha}{p\alpha} < \frac{p}{\beta}$ holds true and all agents in H_4 prefer general investment G , a part of agents in H_3 who satisfy

$$\frac{1}{p}\theta < \tau < \frac{p\beta + \beta - p\alpha}{p\alpha}\theta \quad (63)$$

will prefer S_X to G and the other part of agents in H_3 who satisfy

$$\frac{p\beta + \beta - p\alpha}{p\alpha}\theta < \tau < \frac{p}{\beta}\theta \quad (64)$$

will prefer G to S_X .

We can also conclude the case where some of agents in H_4 will prefer S_X to G .

Proposition 6 *When the terms of trade parameter satisfies $p < \beta$, and when the parameter α satisfies the following condition:*

$$\frac{p+1}{2p} \cdot \beta < \alpha < \frac{p+1}{p(p+\beta)} \cdot \beta^2, \quad (65)$$

The condition $\frac{p}{\beta} < \frac{p\beta+\beta-p\alpha}{p\alpha} < 1$ holds true and all agents in H_3 will prefer S_X to G and a part of agents in H_4 who satisfy

$$\frac{p}{\beta} \theta < \tau < \frac{p\beta + \beta - p\alpha}{p\alpha} \theta \quad (66)$$

will prefer S_X to G and the other part of agents in H_4 who satisfy

$$\frac{p\beta + \beta - p\alpha}{p\alpha} \theta < \tau < \theta \quad (67)$$

will prefer G to S_X .

We can state the following corollary, which concludes the case where all agents in H_3 and H_4 will prefer S_X to G , to the above proposition:

Corollary 2 *When the terms of trade parameter satisfies $p < \beta$, all agents will specialize in the sector which they have innate comparative advantage if*

$$\alpha < \frac{p+1}{2p} \cdot \beta, \quad (68)$$

is satisfied. Then $1 < \frac{p\beta+\beta-p\alpha}{p\alpha}$ holds true and all agents in H_3 and H_4 will prefer S_X to G . That is, everyone who are in $\bigcup_{i=1}^4 H_i$ will invest S_X , everyone who are in $\bigcup_{i=5}^8 H_i$ will invest S_Y and no one will invest G . The condition (68) means that the parameter for general investment efficiency α is very small compared to the parameter for the specific investment efficiency β and with relation to the terms of trade parameter p .

Even if there is no income insurance market, risk neutral agents prefer to specialize in what they are born to be good at given that the specific investment is very strong compared to general investment.

This concludes the proof of Theorem 2.

3 Conclusion

In this paper, I extended the model of occupational choice to a dynamic case. I now allow human capital skills investment by individual agents and will examine the incentives by individuals when they start out from different innate capabilities. In general, specialization enhances productive capabilities of the society through division of labor. Investing into specific skills, however, may be a risky strategy if there is no insurance market under uncertainty. Human capital investment in general skills are usually considered to be a form of self-insurance. Previous works in this field deal with the case where every individual is identical before his human capital investment. The model introduced in this paper analyzed the investment decision problem

when individual agents are heterogeneous in the sense of both absolute and comparative advantages in different sectors. The paper analyzed the case in which we encounter terms of trade uncertainty.

I find the conditions about when agents will invest in general skills and when they specialize their skills in a particular sector. Depending on parameter values: α , β , and p , it is quite possible for all individuals to invest in their innately strong skills even if there is no insurance market. For the agents with smaller degree of comparative advantage (those who are close to the 45 degree line in the unit square), general investment may occur when the parameter α is stronger. I did not find anyone who invested in human capital skills in the direction of their innate comparative disadvantage.

Our results are favored toward specific investment in the skills which agents have innate comparative advantages probably because of the following reasons. First, the agents are assumed to be risk neutral rather than risk averse. Risk neutral agents simply try to maximize expected income rather than expected utility. Therefore, given the set up of the model in this paper, agents find it worthwhile to enhance their strong skills rather than compensating their weaker skills. Second, investment technology is assumed to be the one of constant returns to scale. The investment coefficients α and β are multiplied proportionately by θ and τ , agents' skill levels prior to investment. Thus, agents who had already high θ (before investment) will have a better effect of training either from general or specific investment than the agents with lower value of θ . Agents with stronger comparative advantage in one sector will find themselves better off when they invest in the skills they can get most out of the investments. Third, the structure of uncertainty in this paper is limited to a very specific case of equal probabilities over two states of nature. The choice of this uncertainty structure is done in favor of simplicity, but it is true that individual incentives will change if we change the probability distribution. Future research can address these extensions.

References

- Becker, Gary S.**, *Human Capital*, third edition ed., Chicago and London: The University of Chicago Press, 1993.
- Grossman, Gene M. and Carl Shapiro**, "A Theory of Factor Mobility," *Journal of Political Economy*, October 1982, *90* (5), 1054–1069.
- Murphy, Kevin M.**, "Specialization and Human Capital." PhD dissertation, University of Chicago, Department of Economics August 1986.
- Rosen, Sherwin**, "Substitution and Division of Labour," *Economica*, August 1978, *45* (179), 235–250.
- , "Specialization and Human Capital," *Journal of Labor Economics*, January 1983, *1* (1), 43–49.