Public Pollution Abatement, Regional Capital Mobility and Tax Competition

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Abstract

The environmental economics literature has ignored the effects of the interaction between capital mobility, private and public pollution abatement and cross-border pollution on environmental policies. We analyze these effects in the context of a general equilibrium model. We find, among other things that i) when countries are symmetric the presence of public pollution abatement makes Nash equilibrium pollution taxes efficient, something that is not true in its absence, ii) when countries are asymmetric the Nash equilibrium pollution taxes depend mainly on the direction of capital mobility, factor intensity and the degree of cross-border pollution and iii) the impact of capital mobility on the effectiveness of pollution taxes in reducing pollution depends on the marginal utilities of income and on whether the countries cooperate or not.

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JEL Classification: F18, F22, and H21
1 Introduction

In response to growing environmental concerns, governments and international organizations have designed policies of pollution abatement and control (PAC). In a 2003 report the OECD defines PAC activities as

"... the purposeful activities aimed directly at the preservation, reduction, and elimination of pollution nuisances arising as a residual of production processes or the consumption of goods and services ...... In total, PAC expenditure comprises the flow of investment, internal current expenditure, subsidies and fees that is directly aimed at pollution abatement and control, and which is incurred by the public sector, the business sector, private households and specialized producers of PAC services..." (Linster and Zegel (2003))

In the same report PAC expenditures in OECD countries vary from 0.7 (Portugal 1994) to 2.6% (Austria 1998) of GDP per annum in the period 1990-2000. A revealing stylized fact of this report is that a significant part of these expenditures are undertaken by the public sector. For most countries public expenditures account for about 40-60% of total PAC. Tables 1 and 2 present these statistics in detail. These statistics reveal two important stylized facts. First, PAC expenditures as a percentage of GDP are sizeable and second a significant part of these is incurred by the public sector.¹

Thus, it is important that both private and public sectors abatement are taken into consideration in analyzing environmental policies, especially in light of the fact that emission tax revenue is often earmarked for pollution abatement activities by governments. For example, Brett and Keen (2000) note that, in the US, it is quite customary for environmental taxes to be earmarked for specific environment related public expenditure. In particular, such tax proceeds are commonly paid into trust funds that finance various clean-up activities, or are spend on road and public transport networks. Yet, the literature on pollution abatement has, by and large, assumed that pollution abatement is entirely abated by the private sector in response to emission taxes on private producers

¹In particular, table 3b of the same report provides evidence that in many countries a significant part of the financing of cross border pollution abatement is undertaken by the government. For example, in 1994 the US government spent 0.3 percent of GDP on abating air pollution which accounted for 33 percent of total expenditure on air pollution abatement.
Hatzipanayotou, Lahiri, and Michael (2002), Hatzipanayotou, Lahiri, and Michael (2005) and Chao and Yu (1999) are some of the very few studies that explicitly consider the simultaneous provision of pollution abatement by the private and public sectors. Chao and Yu (1999) examine the welfare implications of international transfers when public pollution abatement is financed by foreign aid and emission tax revenue. Hatzipanayotou, Lahiri, and Michael (2002) examine optimal policies for the donor and recipient countries in a similar framework but, also, incorporating cross-border pollution. Hatzipanayotou, Lahiri, and Michael (2005) examine the optimal policy implications of a number of multilateral reforms in a two country model with cross-border pollution where public sector abatement is financed through a fraction of environmental tax revenue. These studies, however, ignore an important feature of open economies, that of international capital mobility. On the other hand, there is a large literature examining various aspects, including optimal environmental policies, of the interaction between international capital mobility and the environment, but without accounting for the simultaneous abatement of pollution by the private and public sectors. (e.g. Copeland (1994), Copeland and Taylor (1997) and Rauscher (1997))

The present paper bridges the gap in the literature, by incorporating both capital mobility and public pollution abatement. To this end, we construct a general equilibrium model of a regional block (RB) with two non-identical countries and free commodity and capital flows. We assume that pollution, a by-product of production, generated in each country is transmitted across borders, and it is abated partly by the private producers, in response to an emissions tax, and partly by the local governments. Governments finance their public pollution abatement activities using lump-sum and pollution tax revenue. We derive the cooperative and Nash optimal pollution taxes and relate them to the marginal cost of public pollution abatement.

The paper offers two innovations. The first is the generalization of the existing models which incorporate simultaneous provision of public and private pollution abatement. In this more general model we incorporate all the features that different papers have stressed.

\(^2\)The OECD in a 2003 workshop report provides evidence that many countries impose emission taxes. For example, most EU countries, including the UK, Germany, France and Italy, impose energy and CO\(_2\) taxes (see Bygrave and Ellis (2003)).
as important features in studying environmental policies such as cross-border pollution and asymmetries between countries. The second innovation is the analysis of these policies in the presence of capital mobility. Changes in environmental policies in the presence of public pollution abatement create externalities between neighboring countries because of cross-border pollution. At the same time, such changes affect tax revenue and therefore public pollution abatement. On the other hand, environmental policies affect capital mobility which in turn affects emissions.

This generalized model allows us to identify interactions between these features that have been ignored by the literature so far. Capital mobility affects the optimal choice of pollution taxes in the two countries since higher taxes lower the return to capital and drive capital out of the country. That restricts the ability of governments to finance public pollution abatement. We show that the coexistence of all these features affect optimal pollution taxes by comparing the general results to the special cases where: 1) there is no cross-border pollution, 2) countries are identical, 3) there is no capital mobility and 4) there is no public pollution abatement. Finally, we examine how capital mobility affects the ability of pollution taxes to reduce pollution.

2 The Analytical Framework

2.1 The Model

We develop a general equilibrium model of a regional block (RB) comprising two small open economies, Home and Foreign, which trade freely with each other and the rest of the world.\(^3\) As a result, commodity prices in the two countries are constant and equal to the world commodity prices. In both countries pollution of the eyesore type is generated as a by-product of production, and it is transmitted across national borders. Identical residents, inhabiting each country, are adversely affected and suffer disutility from locally generated pollution and from pollution emitted by foreign producers and transmitted across borders. With respect to the flows of factors of production, it is assumed that capital is freely mobile within the RB, but immobile between the region and the rest of the world. Other factors of production, such as labor, are intra-regionally

\(^3\)Following the standard convention we denote all the variables of the foreign country with an asterisk.
and internationally immobile.\textsuperscript{4}

Without loss of generality we call the capital-importing country Home; the model of Foreign, the capital-exporting country, follows analogously. Home’s maximum value of production of private goods is denoted by the revenue function, $R(p, v, t, K)$, defined as:

$$R(p, v, t, K) = \max_{x, z, K} \{ p' x - t z : (x, z, K) \in \Phi(v, K) \text{ and } K \leq K + k_f \},$$

where $p$ is the vector of exogenously given world commodity prices, $\Phi(v, K)$ is the country’s aggregate technology set denoting private production and abatement technologies, $v$ is the endowment vector of the immobile factors, $K$ is Home’s capital endowment, $k_f$ is the amount of foreign capital operating in Home and thus $K$ is the domestic supply of capital, $x$ is the vector of net outputs, and $z$ is the amount of pollution emission by the private sector, net of the amount abated by the private sector.\textsuperscript{5} In the present analysis, since $(v)$ and $(p)$ are invariant, for notational simplification the revenue function is written as $R(t, K)$. We assume that the $R(t, K)$ function is strictly concave in $K$ ($R_{KK} < 0$) and strictly convex in $t$ ($R_{tt} > 0$). The latter assumption implies that a higher emission tax level lowers the amount of pollution emissions by the private sector. By the envelop theorem, the partial derivative of the revenue function with respect to $K$, (i.e., $R_K$) is the marginal revenue product of capital, and by the same theorem, the level of pollution, $z$, generated by the private sector is given by\textsuperscript{6}

$$z = -R_t(t, K).$$

We also assume that the polluting activity is capital intensive, that is, $R_{tK} < 0$ and $R_{tK}^{**} < 0$.

Accounting for both private and public sector pollution abatement, the overall net

\textsuperscript{4}We conjecture that the model may resemble the case of a region—either with all its members developed (e.g., EU) or some developed and some developing (e.g., NAFTA) or two regions in a federal state—vis-à-vis the rest of the world. In such a context, there is free commodity trade within the region, and nearly free commodity trade between the region and the rest of the world.

\textsuperscript{5}For simplicity we assume only one type of pollution emission is generated in one sector. A prime (‘$'$’) denotes a transposed vector or matrix, and $p x - t z$ is the value of factor income. Finally, $\Phi(v, K)$ includes production technologies and abatement technologies in various private sectors, as they carry out some pollution abatement in response to the emission tax ($t$).

\textsuperscript{6}Copeland (1994) and Turunen-Red and Woodland (2004), among others, define pollution in the same way.
pollution \( r \), affecting the home country residents is:

\[
r = z - g + \Theta(z^* - g^*),
\]

where the parameter \( \Theta \in [0, 1] \) is the rate of cross-border pollution or the spillover parameter, \( g \) is the level of public pollution abatement in the home country, and \( z^* \) and \( g^* \) denote the levels of pollution net of private abatement and the level of public pollution abatement, respectively, in the foreign country.\(^7\)

As for the country’s public sector, we assume that it imports from the rest of the world, at a constant price \( P_g \), a commodity used to provide public pollution abatement at the level \( g \). The cost of the imported good (\( i.e., P_g g \)), used for public pollution abatement, is financed through the emission tax revenue (\( i.e., -tR_t(t, K) \)), and lump-sum taxes (\( T \)). Thus, the government’s budget constraint is written as:

\[
P_g g = -tR_t(t, K) + T. \tag{4}
\]

This formulation reflects the requirement in many countries that pollution tax revenues are used for environmental clean-up. We also allow governments to use lump-sum taxes to finance public pollution abatement. We abstract from all other activities of the government in an effort to isolate the effects that relate to optimal environmental policies.

Turning to the demand side of the economy, we assume that each country comprises identical individuals. Utility is adversely affected by both local and foreign pollution transmitted across borders. Let \( E(u, r) \) denote the minimum expenditure required to achieve a level of utility, \( u \), at constant prices \( p \), omitted from the expenditure function for reasons noted earlier, and at the given level of net pollution \( r \). The partial derivative of the expenditure function with respect to \( u \), \( E_u \), denotes the reciprocal of the marginal utility of income. Since pollution adversely affects household utility, the partial derivative of the expenditure function with respect to \( r \), \( E_r \), is positive denoting the households’ marginal willingness to pay for a reduction in pollution (\( e.g. \) see Chao and Yu (1999)).\(^8\)

\(^7\)This formulation of additive level of net pollution, \( r \), implies that the two countries emit the same pollutant. Generalizing the present specification to one where the two countries emit different types of pollutants only results to unwarranted algebraic complications without providing substantive analytical insight.

\(^8\)In Copeland (1994)’s terminology, \( E_r \) is a measure of the marginal damage to consumers from
That is, a higher level of net pollution requires a higher level of spending on private goods to mitigate its detrimental effects so that a constant level of utility is maintained.

Home’s, the capital-importing country’s, budget constraint requires that private spending $E(u, r)$ must equal factor income from the production of goods $R(t, K)$ minus repatriated earnings of foreign capital domestically employed $k^J R_K(t, K)$ and lump-sum taxes $T$. Thus, the income-expenditure identity for the home country is

$$E(u, r) = R(t, K) - k^J R_K(t, K) - T.$$

(5)

The model of Foreign, the capital-exporting country, is similarly developed. The corresponding equations for Foreign are

$$z^* = -R^*_t(t^*, K^*)$$

(6)

$$r^* = z^* - g^* + \Theta^*(z - g)$$

(7)

$$P^*_g g^* = -t^* R^*_t(t^*, K^*) + T^*$$

(8)

$$E^*(u^*, r^*) = R^*(t^*, K^*) + k^J R_K(t, K) - T^*,$$

(9)

where $r^*$ is the level of total net pollution for Foreign, $\Theta^*$ is the rate of cross-border pollution in that country and $K^*$ is the supply of capital. By the assumptions of the model $dK = dk^J = -dK^*$.

Finally, international capital mobility though non-existent between the RB and the rest of the world, is perfect within the RB, i.e., between Home and Foreign. Since it is assumed that capital earnings are untaxed by both countries, perfect regional capital mobility equalizes the factor’s reward in the two countries. That is, equilibrium in the RB’s capital market requires that

$$R_K(t, K) = R^*_K(t^*, K^*).$$

(10)

Appendix A of the paper lays out the complete comparative statics of the system.
2.2 Pollution Taxes, Public Abatement and Net Pollution

In this section we derive and briefly discuss the effects of raising pollution taxes \((t\) and \(t^*)\) on net pollution \((r\) and \(r^*)\). These preliminary results are of use in the analysis to follow and highlight the effects that other studies ignore by omitting either public pollution abatement or capital mobility from the analysis. In Home, the effect of a higher pollution tax \((t)\) on domestic net pollution \((r)\) can be derived as follows. Using equations 2, (3), and Appendix A we get that

\[
\frac{dr}{dt} = -\left(\frac{dg}{dt} + \Theta \frac{dg^*}{dt}\right) - (R_{tt} + R_{tK} \frac{dK}{dt}) + \Theta R^*_t \frac{dK}{dt}
\]

(11)

where \(H = R_{KK} + R^*_t K^*\) and \(\Delta = E_{uu} E_{u^*} HP_g P^g\) and are negative. Intuitively, equation (11) shows that a higher tax \((t)\) affects domestic net pollution \((r)\), first through its impact on public abatement in Home and Foreign. This effect is not captured by the literature that does not account for public pollution abatement.\(^9\) Second, it affects \((r)\) through changes in the two countries levels of gross pollution. In particular, changes in Home’s gross pollution \((z)\) are due to changes in the domestic pollution tax (direct effect) and changes in the domestic capital stock (indirect effect). Both effects lead to a reduction of \((z)\). On the other hand, the higher \((t)\) affects foreign gross pollution \((z^*)\) indirectly through changes in Foreign’s capital stock \((K^*)\). This effect increases \((z^*)\) and through cross-border pollution it increases \((r)\).\(^{10}\) The reduced form of equation (11) is given by equation (29) in Section 5. The combination of these effects also highlights the importance of including both capital mobility and public pollution abatement in the model. The presence of capital mobility affects the choice of pollution taxes since higher pollution taxes lead to an outflow of capital \((\frac{dK}{dt} < 0)\). That, in turn, affects tax revenues which determine how much pollution the government is able to abate.

Equivalently, the effect of the higher pollution tax \((t)\) on Foreign’s net pollution \((r^*)\) is shown to be

\(^9\)The effects of environmental policies on pollution, without public sector abatement, in the presence of capital mobility are examined in other studies such as Rauscher (1991) and Copeland and Taylor (1997).

\(^{10}\)Hatzipanayotou, Lahiri, and Michael (2002) ignore these last two effects on pollution because they examine this issue in a model with \(\Theta = 0\) and no capital mobility.
\[
\frac{dr^*}{dt} = -(\frac{dg^*}{dt} + \Theta^* \frac{dg}{dt}) - \Theta^*(R_{tt} + R_{tK} \frac{dK}{dt}) + R^*_{t_2} \frac{dK}{dt}
\] (12)

In the absence of public sector pollution abatement and of cross-border pollution in both countries, i.e., \(\Theta = \Theta^* = 0\), we unambiguously obtain that \(dr/dt < 0\) and \(dr^*/dt > 0\). In the absence of public sector abatement but in the presence of cross-border pollution, i.e. \(\Theta > 0\) and \(\Theta^* > 0\), \(dr/dt < 0\) and \(dr^*/dt < 0\) if \(R_{tK} = R^*_{t_2} K^*\). The reduced form of equation (12) is given in Appendix A. Analogous results are inferred for an increase in Foreign’s pollution tax \((t^*)\) on the home and foreign countries’ levels of net pollution.

3 Taxes and Welfare

In this section we examine the effect of a higher domestic pollution tax \((t)\) on levels of national welfare \((u)\) and \((u^*)\). Analogous results are stated for the effects of a higher tax \((t^*)\) on the aforementioned variables. We also examine the effects of higher lump-sum taxes, \(T\) and \(T^*\) for each country’s level of national welfare.

We first analyze the welfare effects of small changes in policy variables and we show how the coexistence of public pollution abatement and capital mobility alter the existing results. Differentiating equation (5) gives

\[
E_u du = E_r dg + \Theta E_r dg^* - [-E_r (R_{tK} - \Theta R^*_{t_2} K^*)]dK
+ [E_r R_{tt} + R_t k^f R_{K1}]dt + \Theta E_r R^*_{t_2} dt^* - dT.
\] (13)

3.1 Lump-sum Taxes and Welfare

Using Appendix A, the effect of an increase in the domestic (foreign) lump-sum taxes on domestic (foreign) welfare is given by

\[
\frac{du}{dT} = \frac{S_g}{E_u p_g},
\]
\[
\frac{du^*}{dT^*} = \frac{S^*}{E^*_u p^*_g}.
\] (14)
where \( S_g \equiv (E_r - P_g) \) and \( S^*_g \equiv (E^*_r - P^*_g) \). We say that the public pollution abatement is locally under(over)-provided in Home if \( S_g > 0 (< 0) \), and in Foreign if \( S^*_g > 0 (< 0) \). Therefore, raising lump-sum taxes is unambiguously welfare improving (deteriorating) if the public pollution abatement is locally under (over)-provided. Public pollution abatement is locally optimally provided in Home (Foreign) if \( S_g = 0 \) (\( S^*_g = 0 \)), that is, if \( E_r = P_g \) (\( E^*_r = P^*_g \)). This is the Samuelson rule for optimal public good provision within each country. In this context public pollution abatement is a public good.

### 3.2 Pollution Taxes and Welfare

Using equations (13) and (11), the welfare effect of an increase in Home’s pollution tax \((t)\) on its own welfare is given by

\[
\frac{du}{dt} = \frac{1}{E_u} \{ R_t - E_r \frac{dr}{dt} - k^f (R_{Kt} + R_{KK} \frac{dK}{dt}) \}.
\]  

(16)

The reduced form of equation (16) is given in Appendix A. Equation (16) shows that the increase in \((t)\) affects Home’s level of welfare in three ways. The higher \((t)\) induces, first, a transfer of additional resources from production of goods to pollution abatement by private producers. As a result real income, and, therefore, welfare is reduced (i.e., \( E_u^{-1} R_t < 0 \)). Second, it affects \((u)\) through changes in domestic net pollution (i.e., \( -E_u^{-1} E_r \frac{dr}{dt} \)). Namely, since \( E_r \) is the households’ marginal willingness to pay for pollution abatement, then \( -E_u^{-1} E_r \frac{dr}{dt} \) is a measure of the marginal benefit/damage of changes in \((r)\) due to the increase in \((t)\) on households’ utility. Through this term, the increase in \((t)\) increases \((u)\) if \( \frac{dr}{dt} < 0 \). Third, the term \( -E_u^{-1} k^f (R_{Kt} + R_{KK} \frac{dK}{dt}) \) captures the effect of \((t)\) on \((u)\) through changes in payments to foreign capital operating at home. This change in payments to \( k^f \) is due to changes in the domestic marginal revenue product of capital, \( R_K \), induced by the higher \((t)\). Namely, by assumption, a higher \((t)\) reduces \( R_K \) and thus payments to \( k^f \). In addition, as previously discussed, \( \frac{dK}{dt} < 0 \) causing an increase in the marginal revenue product of capital and thus an increase in payments to \( k^f \). It can be shown, however, that the positive direct effect \( -E_u^{-1} k^f R_{Kt} \) always dominates the negative indirect effect \( -E_u^{-1} k^f R_{KK} \frac{dK}{dt} \). Thus, the overall impact of \((t)\) on \((u)\) through
changes in payments to $k^f$ is positive\textsuperscript{11}.

The effect of an increase in $(t)$ on Foreign’s level of welfare is given by

$$\frac{du^*}{dt} = \frac{1}{E_{u^*}} \left\{ -E_{r^*} \frac{dr^*}{dt} + k^f (R_{Kt} + R_{KK} \frac{dK}{dt}) \right\}$$ \hspace{1cm} (17)

The reduced form of equation (17) is given in Appendix A. Equation (17) shows that an increase in $(t)$ affects $(u^*)$, through, first, its effect on net pollution, $(r^*)$, and second through its effect on repatriated payments of its capital operating in the home country.

The discussion of the first effect follows the discussion of equation (12), and the discussion of the second one follows that of equation (16). Analogously, using Appendix A we get the reduced form expressions of an increase in $(t^*)$ on welfare in Foreign $(u^*)$ and in Home $(u)$.

## 4 Optimal Lump-sum and Pollution Taxes

In this section we turn to the derivation of the optimal pollution taxes, $(t)$ and $(t^*)$, and lump-sum taxes, $T$ and $T^*$, in the presence of all these interactions generated by the coexistence of public pollution abatement, capital mobility and cross-border pollution.

We first derive the optimal tax rates under the assumption of policy cooperation between the two countries, and then, under the assumption of lack of such a cooperation between them.

### 4.1 Cooperative Taxes

A standard result in the literature of environmental economics is that in the presence of cross-border pollution externalities optimal policy requires either the adoption of cooperative policies among regions or the mandate of policies by a central (e.g., federal) authority. Here, we begin our analysis of tax policy choices by presenting the first-best policy choices of the RB. This regime entails the simultaneous cooperative choice of lump-sum and pollution taxes that maximize the two countries’ joint welfare. This

\textsuperscript{11}In Appendix A, this is shown by the last term (i.e., $-\Delta^{-1} k^f R_{K} R_{K}^* \cdot P^*_g P_g^*$), of the reduced form of equation (16).
regime constitutes a benchmark solution to which the Nash equilibrium results to follow are compared.

4.1.1 Cooperative Lump-sum Taxes

The maximization of the countries’ joint welfare requires setting \( \frac{du}{dT} + \frac{du^*}{dT} = 0 \) and \( \frac{du}{dT^*} + \frac{du^*}{dT^*} = 0 \), where \( \frac{du}{dT} \) and \( \frac{du^*}{dT^*} \) are given by equations (14) and (15), respectively. Moreover, using the appendix A we get

\[
\frac{du}{dT^*} = \frac{\Theta E_r}{E_u P_g^*} \quad \text{and} \quad \frac{du^*}{dT} = \frac{\Theta^* E_r^*}{E_u^* P_g^*}. \tag{18}
\]

\[
\frac{du^*}{dT^*} = \frac{\Theta^* E_r^*}{E_u^* P_g^*} \quad \text{and} \quad \frac{du}{dT} = \frac{\Theta E_r}{E_u P_g^*}. \tag{19}
\]

From equations (14), (15), (18) and (19) we get that the cooperative first-best policy choice for provision of public abatement requires that:

\[
\frac{E_r^*}{E_r^* + \lambda E_r} = P_g^* \quad \text{and} \quad \frac{E_r}{E_r + \frac{1}{\lambda} E_r^*} = P_g^*, \tag{20}
\]

where, \( \lambda = \left( \frac{E_u}{E_u^*} \right) \) denotes the ratio of Foreign to Home’s marginal utility of income. Intuitively, a unit of pollution generated by Home causes \( E_r \) damage domestically and \( \Theta^* E_r^* \) damage in Foreign. Thus \( \tilde{E}_r \) is the global damage caused by a unit of locally generated pollution weighted by the relative marginal utilities of income, \( \left( \lambda \right) \). Similarly \( \tilde{E}_r^* \) is the weighted global damage caused by a unit of pollution generated in Foreign.

Therefore, \( \tilde{E}_r \) (\( \tilde{E}_r^* \)) is the weighted global marginal willingness to pay for pollution abatement of the Home (Foreign) generated pollution. When \( \tilde{E}_r - P_g > 0 (< 0) \) we say that the public pollution abatement in Home is globally under-provided (over-provided), and when \( \tilde{E}_r = P_g \), the public pollution abatement by Home is globally optimally provided. Similar definitions apply to Foreign.

Equations (20) and (21) indicate that maximizing joint welfare requires that lump-sum taxes in each country are set at a level where the weighted global marginal willingness

\[\text{\footnote{Throughout the analysis we assume interior solutions for the policy instruments and for interior allocations of capital and pollution levels.}}\]
to pay for pollution abatement for pollution generated in each country equals the unit cost of providing it (i.e., $E_r = P_g$ and $E_r^* = P_g^*$). Note that these two equations represent the relevant Samuelson rule for optimal provision of regional public (pollution abatement) goods. Moreover, because of the existence of cross-border pollution, the relevant Samuelson rule accounts not only for the marginal willingness to pay for pollution abatement within the emitting country, but also for the marginal willingness to pay for it in the other country, weighted by the relative marginal utilities of income.

Note that in the absence of cross-border pollution, i.e., $\Theta = \Theta^* = 0$, changes in one country’s lump-sum taxes have no effect on the other country’s welfare level, and that the first-best cooperative choice for public good provision reduces to $E_r \equiv E_r = P_g$ and $E_r^* \equiv E_r^* = P_g^*$. This result is achieved without cooperation between the two countries.

### 4.1.2 Cooperative Pollution Taxes

Deriving the cooperative first-best choice of pollution taxes requires setting $\frac{d\bar{u}}{dt} + \frac{d\bar{u}^*}{dt} = 0$ and $\frac{d\bar{u}}{dt} + \frac{d\bar{u}^*}{dt} = 0$, where the expressions for $\frac{d\bar{u}}{dt}$, $\frac{d\bar{u}^*}{dt}$, $\frac{d\bar{u}}{dt}$, and $\frac{d\bar{u}^*}{dt}$ are given in Appendix A. In general, the cooperative pollution taxes, $t^c$ for Home and $t^{cs}$ for Foreign, assuming that the two countries also cooperate in lump-sum taxes are given by

$$
t^c = P_g - (1 - \lambda) \frac{k^f R_{Kt} P_g (R_{K}^2 R_{KK} + HR_{tt}, R_{K}^* R_{K}^* - R_{K}^2 R_{K}^* R_{K}^*)}{HE_r (HR_{tt} R_{tt} - R_{K}^2 R_{K}^* R_{K}^*)} \quad \text{and (22)}
$$

$$
t^{cs} = P_g^* + (1 - \lambda) \frac{k^f P_g^* R_{K}^* (R_{K}^2 R_{KK} + HR_{tt} R_{KK} - R_{K}^2 R_{KK})}{HE_r^* (HR_{tt} R_{tt} - R_{K}^2 R_{K}^* R_{K}^*)}. \quad \text{(23)}
$$

From equations (22) and (23) note that if $\lambda = 1$ we get that $t^c = P_g$ and $t^{cs} = P_g$. In this case, the cooperative optimal policies require that $t^c = P_g = \bar{E}_r$ and are independent of capital mobility. In general, however, even when countries cooperate, capital mobility affects the cooperative pollution taxes. This is because payments to Foreign’s capital operating in Home constitute a direct income transfer from the latter to the former. Since, in general the marginal utility of income differs in the two countries (i.e., $\lambda \neq 1$), the transfer of repatriated capital income affects global welfare. In other words, despite the fact that the income lost by Home is exactly the same as that gained by Foreign, the utilities that correspond to that income are different, and therefore they affect the maximization of their joint welfare. The sign of the coefficient of $(1 - \lambda)$ in the right-
hand-side of equations (22) and (23) is assumed to be positive.\(^{13}\) Thus, from equation (22) for \(\lambda > 1\) we get that \(t^c > P_g\) and from equation (23) that \(t^c < P_{g^*}\). For \(\lambda > 1\) Home’s marginal utility of income is lower than Foreign’s which means that Home is relatively richer, which calls for a higher pollution tax in Home and a lower pollution tax in Foreign. The intuition is as follows. Pollution taxes allow countries to redistribute income between them through their effect on repatriated income of Foreign’s capital in Home. As \(t^c\) increases and \(t^c\) decreases the return to capital in Home decreases while that in Foreign increases causing capital outflow from Home to Foreign. This increases the return to capital and thus increases the repatriated capital income going from Home to Foreign. If \(\lambda > 1\) the marginal utility of income is higher in Foreign. This means that joint welfare can increase if income is transferred from Home to Foreign. The following proposition summarizes the result.

**Proposition 1** If the marginal utilities of income across countries are the same (\(\lambda = 1\)) and both countries cooperate in the choice of both policy instruments then the first-best policy choice, is achieved when \(t^c = P_g = \bar{E}_r\) and \(t^{sc} = P_{g^*} = \bar{E}_{r^*}\). If, however, \(\lambda > 1\) the first-best is achieved when \(t^c > P_g = \bar{E}_r\) and \(t^{sc} < P_{g^*} = \bar{E}_{r^*}\). If \(\lambda < 1\), then the first best is achieved when \(t^c < P_g = \bar{E}_r\) and \(t^{sc} > P_{g^*} = \bar{E}_{r^*}\).

To highlight the role of public pollution abatement, consider the case where all pollution abatement is undertaken by the private producers, in response to the emissions tax, and where pollution tax revenue is lump-sum distributed to local households in each country. In this case, the cooperative first-best choice of pollution taxes is \(t^c = \bar{E}_r\) and \(t^{sc} = \bar{E}_{r^*}\). Note that this cooperative policy choice is different from \(t^c = P_g = \bar{E}_r\) and \(t^{sc} = P_{g^*} = \bar{E}_{r^*}\), since in the latter \(P_g\) and \(P_{g^*}\), the cost of public sector abatement in each country, is exogenous, and it is to its specific value that the pollution tax and the global damage of a unit of pollution that are equated. Also, since \(dr/dT < 0\), when \(g > 0\) we have less \(r\) for the same \(t\), thus \(t^c(g = 0) > t^c(g > 0)\). This highlights the way in which public pollution abatement affects optimal taxes.

\(^{13}\)Combining the first two terms in brackets in the numerator of equation (22) we get that \(R_{K^*}^2 R_{K^*} - HR_{K^*}^2 + R_{K^*}^2 - R_{K^*}^2 = R_{K^*}^2 R_{K^*} - R_{K^*}^2 R_{K^*} - R_{K^*} R_{K^*} \) which is positive if \(R_{K^*}^2 - R_{K^*} R_{K^*} < 0\) (sufficient but not necessary condition). That is, the total effect of an increase in capital on its return, direct and indirect through changes in pollution, is negative.
4.2 Nash Equilibrium Lump-sum and Pollution Taxes

We now derive the optimal Nash equilibrium lump-sum and pollution taxes for Home and Foreign and compare them to the benchmark cooperative case. The two countries choose these taxes simultaneously.

4.2.1 Nash Equilibrium Lump-sum Taxes

Setting equations (14) and (15) equal to zero, we derive the Nash equilibrium lump-sum taxes. The emerging Nash equilibrium conditions require that lump-sum taxes are chosen such that \( E_r = P_g \) for Home and \( E_r^* = P_g^* \) for Foreign.

4.2.2 Nash Pollution Taxes

Setting \([du/dt] = 0 \) and \([du*/dt^*] = 0 \) we derive the following reaction functions:

\[
t = P_g + \frac{-S_g HR_t P_g^* + \Theta R_{tK} R_{t^*K^*} P_g E_r (P_g^* - t^*) - k^f R_{tK} R_{t^*K^*} P_g P_g^*}{E_r P_g^* (HR_{tt} - R_{tK}^2)}
\]

\[
t^* = P_g^* + \frac{-S_g^* HR_{t^*} P_g^* + \Theta^* R_{tK} R_{t^*K^*} P_g^* E_r^* (P_g^* - t^*) + k^f R_{t^*K^*} R_{tK} P_g P_g^*}{E_r^* P_g^* (HR_{t^*t^*} - R_{t^*K^*}^2)}.
\]

Given that the structure of the game is such that lump-sum taxes are locally optimally chosen (i.e., \( S_g = S_g^* = 0 \)), solving simultaneously equations (24) and (25) gives the following expressions for each country’s Nash equilibrium pollution taxes.\(^{14}\)

\[
t^N = P_g - k^f R_{tK} P_g^2 P_g^*[E_r^* R_{t^*K^*} (HR_{t^*t^*} - R_{t^*K^*}^2) + \Theta^* R_{tK} R_{t^*K^*} E_r] / \Delta^N
\]

\[
t^*^N = P_g^* + k^f R_{t^*K^*} P_g^2 P_g^*[E_r R_{tK} (HR_{tt} - R_{tK}^2) + \Theta R_{tK} R_{t^*K^*} E_r^*] / \Delta^N,
\]

where \( \Delta^N = E_r E_r^* P_g P_g^*[HR_{tt} R_{tK} (HR_{t^*t^*} - R_{t^*K^*}^2) - \Theta \Theta^* R_{tK} R_{t^*K^*} R_{t^*K^*}^2] \) and is positive. From equations (26) and (27) we note that when lump-sum taxes are locally optimally chosen, the effect of pollution taxes on payments to Foreign’s capital operating in Home constitute the only difference between the Nash and cooperative tax rates derived when \( \lambda = 1 \). Note that the Nash pollution taxes are independent of the marginal utilities

\(^{14}\)The general expressions for the Nash pollution taxes when lump-sum taxes are not chosen optimally are given in appendix A.
of income since in the Nash equilibrium each country is only concerned about its own welfare.

Observing the above expressions we note that in general the Nash pollution taxes can be greater or smaller than the unit cost of the public pollution abatement, as opposed to the benchmark case of cooperative choice of both instruments and $\lambda = 1$. We resolve some of this ambiguity by considering some special case as stated in the following Proposition:

**Proposition 2** Under the conditions of the model

1. If $\Theta = 0$ and $\lambda = 1$ then $t^N > t^c = P_g$.

2. If $\Theta^* = 0$ and $\lambda = 1$ then $t^{*N} < t^{*c} = P_g^*$.  

3. If the two countries are symmetric in the sense that $\lambda = 1$, $E_r = E_r^*$, and $R_{KK} = R_{K^K*}$, then $t^N > t^c$ and $t^{*N} < t^{*c}$. 

4. If $\lambda = 1$ and the two countries have identical factor endowments and production technologies, leading to $k^f = 0$, then the Nash and cooperative pollution taxes are the same and equal to the marginal cost of public pollution abatement, i.e., $t^N = t^c = P_g$, and $t^{*N} = t^{*c} = P_g^*$.  

5. If $\lambda < 1$ and $\Theta \approx 0$ then $t^N > P_g > t^c$. Similarly if $\lambda < 1$ and $\Theta^* \approx 0$ then $t^{*N} < P_g^* < t^{*c}$.  

6. If $\lambda > 1$ and $\Theta \approx 0$ then $t^N > P_g$ and $t^c > P_g$. Similarly if $\lambda > 1$ and $\Theta^* \approx 0$ then $t^{*N} < P_g^*$ and $t^{*c} < P_g^*$.  

The proof of Proposition (2) follows from equations (26) and (27). Note that the first 4 cases assume that the marginal utilities of income between the two countries are the same. Intuitively, the first two cases of Proposition (2) are directly derived from the assumption that Home is the capital-importing and Foreign is the capital-exporting country, and from the assumption that pollution is capital intensive in both countries. That is, the inflow of capital in Home raises pollution, thus leading to a higher domestic Nash pollution tax level. The reverse holds for Foreign, the capital-exporting country.
The intuition of the third case is as follows. Payments to Foreign’s capital operating in Home reduce real income, and thus lower the domestic households’ marginal willingness to pay for pollution abatement (i.e., $E_{ru} > 0$). As a result, at a constant pollution tax ($t$), net pollution generated in Home (i.e., $z - g$) rises. At the same time, the opposite holds in Foreign, i.e., $z^* - g^*$ falls. If the two countries are symmetric, and since $\Theta \leq 1$ and $\Theta^* \leq 1$, then net pollution ($r = z - g + \Theta(z^* - g^*)$) in Home rises, and net pollution ($r^* = z^* - g^* + \Theta^*(z - g)$) in Foreign falls. Therefore, we get that in Home $t^N > t^c$, and in Foreign $t^{*N} < t^{*c}$.

Case 4 of Proposition 2 highlights a counter-intuitive result. When countries are identical in the above sense, Nash pollution taxes are efficient ($t^N = t^c$, and $t^{*N} = t^{*c}$) while Nash lump-sum taxes are not. But, if countries cooperate in lump-sum taxes only, then Nash pollution taxes are reduced, i.e., $t^N < t^c = P_g$, and $t^{*N} < t^{*c} = P_g^*$, and hence become inefficient.\(^{15}\) Cases 5 and 6 show that the relationship between the Nash and cooperative pollution taxes depends on the relative marginal utilities of income in the two countries.\(^{16}\)

A key feature of our model is that, contrary to most of the literature, we allow countries to be non-identical. We next examine how differences between the two countries affect optimal taxes. To do so consider first the case where the two countries are identical. If countries are identical, the return on capital is identical and $k_f = 0$. In that case, if both taxes are chosen optimally $t^N = t^c = P_g$ and $t^{*N} = t^{*c} = P_g^*$. However from equations (20) and (21) $T^N \neq T^c$ and $T^{*N} \neq T^{*c}$. Therefore, if countries are identical and both taxes are chosen optimally there is no need for cooperation in pollution taxes since Nash taxes are efficient. On the other hand, there is scope for cooperation in lump-sum taxes. If, however, the two countries are non-identical and choose both taxes optimally, Nash pollution taxes are not efficient. From equations (20) and (21) we still get that $T^N \neq T^c$ and $T^{*N} \neq T^{*c}$. Proposition 2 summarizes the sufficient conditions for $t^N > t^c$ and $t^{*N} < t^{*c}$.

Once again, in order to highlight the analytical importance of public sector abatement, consider the case where pollution clean-up is undertaken only by the private sector, in

\(^{15}\) Cooperation in lump-sum taxes implies $\hat{S}_g = \hat{S}_g^* = 0$, which in turn implies that $S_g = -\Theta^*E_r < 0$ and $S_g^* = -\Theta E_r < 0$. Then from equations (24) and (25) when $k_f = 0$ then $t^N < t^c = P_g$, and $t^{*N} < t^{*c} = P_g^*$.

\(^{16}\) Case 5 states sufficient conditions for $t^N > P_g > t^c$ while in case 6 if $\lambda$ is large enough then $t^c > t^N > P_g$. 

16
response to the emission tax, while pollution tax revenue is lump-sum distributed to local households in each country. Using Appendix B we get that the Nash equilibrium pollution tax for home is

\[ t^N = E_r + \frac{(\Theta E_r R^*_{t,K *} - k^f R^*_{K*K*}) R_{tK}}{(HR_{tt} - R^2_{tK})} \]  \hspace{1cm} (28)

Comparing equations (26) and (28) observe that public pollution abatement affects Nash equilibrium pollution taxes in three ways. First, it makes the Nash equilibrium tax of Home depend directly on the corresponding tax of Foreign. This is due to the fact that tax receipts in Foreign are used for pollution abatement which affects the welfare of Home directly due to cross border pollution. Second, it complicates the impact of capital mobility on Nash equilibrium taxes. This is obvious when we compare the terms multiplied by \( k^f \) in the two expressions. If Foreign increases it’s pollution tax to increase public pollution abatement it drives capital towards Home. This additional interaction between pollution taxes and capital mobility has to be accounted for. Finally, the exogenous cost of public pollution abatement, the additional tool provided by lump-sum taxes and the existence of public pollution abatement wipe out the effect represented by \( (\Theta E_r R^*_{t,K *} R_{tK}/(HR_{tt} - R^2_{tK})) \) in (28). In particular, let the two countries have identical factor endowments and technologies, leading to \( k^f = 0 \), and let \( \Theta > 0 \ \Theta^* > 0 \) (two-way cross border pollution). The emerging equilibrium conditions indicate inefficient Nash pollution taxes, i.e. \( t^N < E_r \) and \( t^*N < E^*_r \) (see equation (28)). On the other hand, and in direct contrast to this result, case 4 of Proposition 2 indicates that Nash pollution taxes and Nash lump-sum taxes are efficient, in the presence of public pollution abatement. We state this result in the following proposition:

**Proposition 3** If the two countries have identical factor endowments and production technologies, leading to \( k^f = 0 \), then in the presence of cross-border pollution and:

i) in the presence of public pollution abatement Nash equilibrium pollution taxes are efficient, i.e. \( t^N = E_r \) and \( t^*N = E^*_r \).

ii) in the absence of public pollution abatement Nash equilibrium pollution taxes are inefficient, i.e. \( t^N < E_r \) and \( t^*N < E^*_r \).
5 Capital Mobility and the Effect of Pollution Taxes on Net Pollution

In this section we examine how capital mobility affects the effectiveness of pollution taxes in reducing net pollution evaluated at their Nash and cooperative levels. The term $dr/dt$ represents the effectiveness of pollution taxes in reducing net pollution. Therefore, for each case (Nash and cooperative equilibria) we evaluate $dr/dt$ with and without capital mobility. This is another analytical novelty of the paper. That is, while some of the previously reviewed studies examine the effects of capital mobility on pollution levels, to the best of our knowledge, none of them examines the impact of capital mobility on the effectiveness of the optimally chosen environmental policy instruments in reducing net pollution.\footnote{Damania (2000) defines the effectiveness of environmental policy in the same way but looks at the impact of environmental policy on the financial structure of firms.}

From the system in Appendix A we get that

$$\frac{dr}{dt} = -\Delta^{-1}P_g^* (HR_t - R_{tK}^2)(P_g - t) - \Delta^{-1}\Theta R_g R_{tK} R_{tK}^* (P_g^* - t^*) + \Delta^{-1} HP_g^* R_t \tag{29}$$

Consider first the benchmark case of no capital mobility. From equations (26), (27), (22) and (23) we get that $t^N = t^c = P_g$ and $t^{*N} = t^{*c} = P_g^*$. From this and equation (29) we get that

$$\frac{dr}{dt} = \Delta^{-1} HP_g^* R_t = \frac{R_t}{P_g E_u^* E_u^*}. \tag{30}$$

Note that $dr/dt$ is negative and therefore an increase in $t$ reduces net pollution. The effectiveness of $t$ in reducing net pollution increases as this value becomes more negative, or as $dr/dt$ is reduced. In that case the same reduction in pollution taxes reduces net pollution more.

Now consider the case of identical countries with capital mobility. If the two countries are identical then $k^f = 0$. From equations (26), (27), (22) and (23) we, once again, get that $t^N = t^c = P_g$ and $t^{*N} = t^{*c} = P_g^*$. This means that $dr/dt$ is the same as in equation (30) and therefore capital mobility has no impact on the effectiveness of pollution taxes in reducing net pollution. That, of course, applies to small changes from the Nash
equilibrium and small changes from the cooperative equilibrium.

If the two countries are different then capital mobility generally affects \( dr/dt \). The only exception is that of the cooperative equilibrium with \( \lambda = 1 \). In that case, from equations (22) and (23) we get that \( t^c = P_g \) and \( t^{cc} = P_g^* \) and therefore capital mobility does not affect the effectiveness of pollution taxes (see equation (29)). If \( \lambda > 1 \), from the same equations we get that \( t^c > P_g \) and \( t^{cc} < P_g^* \). From equation (29) we get that in this case \( dr/dt \) is higher than that in equation (30).\(^{18}\) This means that in this case the effectiveness of pollution taxes in reducing net pollution is lower as a result of capital mobility. If \( \lambda < 1 \) then \( t^c < P_g \) and \( t^{cc} > P_g^* \), which makes \( dr/dt \) lower than that in equation (30). Therefore, in this case capital mobility increases the effectiveness of pollution taxes.

**Proposition 4** At the cooperative equilibrium, the effectiveness of Home’s pollution taxes in reducing Home net pollution increases (decreases) if \( \lambda > 1 \) (\( \lambda < 1 \)). The effectiveness of Foreign’s pollution taxes in reducing Foreign net pollution decreases (increases) if \( \lambda > 1 \) (\( \lambda < 1 \)). If \( \lambda = 1 \), then the effectiveness of pollution taxes in reducing net pollution remains unaffected in both countries.

Next, we examine how capital mobility affects the impact of an increase in Home’s pollution tax on its net pollution evaluated at Nash equilibrium. The following Proposition summarizes the results:

**Proposition 5** The presence of capital mobility (i) in the capital-importing country decreases the effectiveness of a higher pollution tax in reducing net pollution evaluated at Nash\(^{19}\) if \( \Theta = 0 \) and \( \Theta^* \geq 0 \), and (ii) in the capital-exporting country increases the effectiveness of a higher pollution tax in reducing net pollution evaluated at Nash if \( \Theta^* = 0 \) and \( \Theta \geq 0 \).

From equations (24) and (25) when \( \Theta = 0 \) and \( \Theta^* \geq 0 \) we get that \( t^N > P_g \) and \( t^{*N} < P_g^* \). Therefore from equation (29) we get that \( dr/dt \) is higher than that given

\(^{18}\)The first two terms of equation (29) are positive as opposed to zero.

\(^{19}\)Nash equilibrium refers to the case where the two countries apply their Nash lump-sum taxes and their Nash pollution taxes simultaneously.
by equation (30). Therefore, capital mobility decreases the effectiveness of pollution taxes in reducing net pollution. Intuitively, when \( \Theta = 0 \) the residents of Home are only affected by changes in, \( z - g \). Capital mobility affects both \( z \) and \( g \). The increase in \( t \) leads to an outflow of capital from Home to Foreign, which in turn lowers \( z \). On the other hand the effect on \( g \) is negative since the capital outflow reduces pollution and thus pollution tax revenue. At Nash \( t^N > P_g \) and thus the effect on \( g \) is smaller than that on \( z \), and thus the effect on \( r = z - g \) is smaller. Similarly, if \( \Theta^* = 0 \) and \( \Theta \geq 0 \) from equations (24) and (25) we get that \( t^N < P_g \) and \( t^{*N} > P_g^* \), and from the equation for \( dr^*/dt^* \) in the Appendix we get that in this case the effectiveness of pollution taxes is increased for the capital exporting country.

6 Conclusion

There is ample real world evidence that the public sector is a key player in pollution abatement activities. Despite this stylized fact the literature has paid little attention to it. On the other hand, the few attempts at modelling public pollution abatement ignore another important feature of open economies, that of international capital mobility. This paper extends the existing literature in two ways, i) it offers a generalized model of simultaneous private and public pollution abatement which includes asymmetric countries and cross-border pollution and ii) it considers public pollution abatement in the presence of international capital mobility. Within this framework, we examine the cooperative and Nash pollution taxes of the countries involved and the impact of capital mobility on the effectiveness of pollution taxes to reduce pollution.

To address these issues, the paper presents a model of a RB with two non-identical countries with cross border pollution, free trade in goods and perfect capital mobility within the region. Pollution, a by-product of production adversely affects welfare and is abated by the private and public sectors in both countries. The government uses revenue collected from pollution and lump-sum taxes to finance public pollution abatement.

The major results of the paper are summarized in the various propositions and to avoid repetition we do not cite them here. A few general remarks, however, are worth emphasizing since they highlight effects that the literature has ignored so far. First, the cooperative lump-sum taxes need to account for the existence of cross-border pollution
and the relative marginal utilities of income between the two countries. Second, the Nash equilibrium pollution taxes depend on whether the country is a net capital importer or capital exporter, on the factor intensity of the polluting activity and on the degree of cross-border pollution. Contrary to the cooperative taxes Nash pollution taxes are independent of the relative marginal utilities of income. Third, when countries are symmetric and in the presence of public pollution abatement the Nash equilibrium taxes are efficient while in the absence of public pollution abatement they are not due to the effects of cross-border pollution. Finally, the impact of capital mobility on the effectiveness of pollution taxes in reducing net pollution depends on the relative marginal utilities of income and whether the two countries cooperate or not.
Appendix A: Comparative Statics

\[
\begin{bmatrix}
E_u & 0 & -E_r & -\Theta E_r & -E_r(R_tK - \Theta R_{t,K}^*) + k^f R_{KK}
\end{bmatrix} =
\begin{bmatrix}
du
\end{bmatrix}
\]

\[
\begin{bmatrix}
E_r^* & 0 & -\Theta E_r^* & -E_r^*(R_{t,K}^* - \Theta R_{t,K}) - k^f R_{KK}
\end{bmatrix} =
\begin{bmatrix}
du^*
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0 & P_g & 0 & tR_{tK}
0 & 0 & 0 & P_g^* & -t^* R_{t-K}^*
0 & 0 & 0 & 0 & H
\end{bmatrix} =
\begin{bmatrix}
dg
\end{bmatrix}
\]

\[
\begin{bmatrix}
E_r R_{tt} + R_t - k^f R_{tK}
\Theta E_r R_{tt} + k^f R_{tK}
-R_t - tR_{tt}
0
-R_{tK}
\end{bmatrix} =
\begin{bmatrix}
\Theta E_r R_{tt+1}
E_r^* R_{tt+1} + R_t^*
0
-R_t^* - t^* R_{t+1-K}^*
0
\end{bmatrix} dt +
\begin{bmatrix}
0
0
-1
1
0
\end{bmatrix} dT +
\begin{bmatrix}
0
0
-1
0
1
\end{bmatrix} dT^*
\]

\[\Delta \frac{dr^*}{dT^*} = -P_g^* (H R_{t+1}^* - R_{t+1-K}^*) (P_g^* - t^*) - \Theta^* R_{tK} R_{t+1-K}^* P_g^* (P_g - t) + H P_g R_{t-K}^*\]

\[\Delta \frac{dK}{dT} = \frac{-R_{tK}}{H} \]

\[\Delta \frac{dg^*}{dT^*} = -P_g^* t R_{t-K}^* - \Theta^* R_{tK} R_{t+1-K}^* P_g^* (P_g - t) + H P_g R_{t-K}^*\]

\[\Delta \frac{dK}{dT} = \frac{-R_{tK}}{H} \]

\[\Delta \frac{dg}{dT^*} = 0 \]

\[\Delta \frac{dr}{dT^*} = 0 \]

\[\Delta \frac{dg}{dT^*} = 0 \]

\[\Delta \frac{dK}{dT} = \frac{-R_{tK}}{H} \]

\[\Delta \frac{dr}{dT^*} = 0 \]

\[\Delta \frac{dg}{dT^*} = 0 \]

\[\Delta \frac{dK}{dT} = \frac{-R_{tK}}{H} \]

\[\Delta \frac{dr}{dT^*} = 0 \]

\[\Delta \frac{dg}{dT^*} = 0 \]

\[\Delta \frac{dK}{dT} = \frac{-R_{tK}}{H} \]

\[\Delta \frac{dr}{dT^*} = 0 \]

\[\Delta \frac{dg}{dT^*} = 0 \]

\[\Delta \frac{dK}{dT} = \frac{-R_{tK}}{H} \]

\[\Delta \frac{dr}{dT^*} = 0 \]

\[\Delta \frac{dg}{dT^*} = 0 \]

\[\Delta \frac{dK}{dT} = \frac{-R_{tK}}{H} \]

\[\Delta \frac{dr}{dT^*} = 0 \]

\[\Delta \frac{dg}{dT^*} = 0 \]
\[
\frac{du}{dt} = \Delta^{-1}E^*_u(E^*_r - t^*)R^r_{tK} + \Delta^{-1}E^*_uE^*_r(t^* - t)\]
\[
\frac{du}{dt} = \Delta^{-1}E^*_uE^*_r(P^*_g - t) - \Delta^{-1}E^*_uP^*_gR^r_{tK} + \Delta^{-1}E^*_uE^*_rP^*_gE^*_r(P^*_g - t^*)
\]
\[
\Delta \frac{du}{dt} = E^*_uE^*_rP^*_g(R^r_{tK} + R^r_{tK}) + \Delta^{-1}E^*_uE^*_rP^*_gE^*_rP^*_gE^*_r(P^*_g - t)
\]
\[
\Delta \frac{du}{dt} = \Theta E^*_uE^*_rP^*_g(R^r_{tK} + R^r_{tK}) + \Delta^{-1}E^*_uE^*_rP^*_gE^*_rP^*_gE^*_r(P^*_g - t)
\]
\[
\Delta \frac{du}{dt} = \Theta E^*_uE^*_rP^*_g(R^r_{tK} + R^r_{tK}) + \Delta^{-1}E^*_uE^*_rP^*_gE^*_rP^*_gE^*_r(P^*_g - t)
\]

Appendix B: The Model without Public Pollution Abatement

We present, as a special case of our general model, the case where pollution abatement is entirely undertaken by private producers in response to the government imposed emission taxes. The level of production generated pollution by the private sector in the two countries is as given by equations (2) and (6). Overall net pollution affecting Home and Foreign, respectively is given by:

\[
r = z + \theta z^* \quad \text{and} \quad r^* = z^* + \theta z \quad (31)
\]

The income-expenditure identities for the two countries are, respectively, given by:

\[
E(u, r) = R(t, K) - tR_t(t, K) - k^f R_K(t, K), \quad \text{and}
\]
\[
E^*(u^*, r^*) = R^*(t^*, K^*) - t^*R^r_{t^*}(t^*, K^*) + k^f R_K(t, K) \quad (32)
\]

Equilibrium in the Regional Block’s capital market is given by equation (10) in the text. Differentiating equations (32) using equations (2), (10) and (31) we obtain the following matrix system

\[
\begin{bmatrix}
E_u & 0 & -(E^*_r - t)R^r_{tK} + \theta E^*_rR^r_{tK} + k^f R^r_{tK} \\
0 & E^*_u & (E^*_r - t^*)R^r_{tK} - \theta E^*_rR^r_{tK} - k^f R^r_{tK} \\
0 & 0 & H
\end{bmatrix}
\begin{bmatrix}
\frac{du}{dt} \\
\frac{du^*}{dt} \\
\frac{dK}{dt}
\end{bmatrix}
\]
\[
\begin{bmatrix}
(E_t - t)R_{tt} - kR_{Kt} \\
\theta^* E_r^* R_{tt} + kR_{Kt} \\
-R_{Kt}
\end{bmatrix}
dt + \begin{bmatrix}
\theta E_r^* R_{t^*t^*} \\
(E_r^* - t^*)R_{t^*t^*} \\
R_{t^*K^*}
\end{bmatrix}
dt^*
\]

where \( H = R_{KK} + R_{K^*K^*} \cdot (<0) \), and \( \Delta = E_u E_u^* H < 0 \) is the determinant of the matrix of the unknowns.

**References**


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