Innovation, Licensing, and Imitation: The Effects of Intellectual Property Rights Protection and Industrial Policy

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Abstract

This paper examines long-run effects of intellectual property rights (IPRs) protection and industrial policies on innovation and technology transfer by using a North-South quality ladder model where licensing is a main mode of technology transfer to developing countries. We show that governments of the developing countries can promote innovation and technology transfer by strengthening IPRs protection, which is enforced by restricting imitation of products. Moreover, results of this paper imply that subsidies on the cost of license negotiation can promote innovation and technology transfer, whereas subsidies on the cost of R&D have no effect on them.

Keywords: Licensing; Innovation; Imitation; Intellectual property rights;
JEL classification: F43; O33; O34;

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1 Introduction

For developing countries which have only limited knowledge about advanced technologies, absorption of it from abroad is usually essential to promote their economic growth. Historically, for instance, Japanese economy after World War II was able to experience high economic growth by importing knowledge about advanced technologies from the United States and Western Europe. Today, as technologies are continually improved by numerous research and development efforts, the absorption of advanced knowledge from abroad must be more and more important for developing countries in order to catch up with the technological progress. For that reason, how developing countries can absorb the knowledge smoothly and attain high economic performance has been one of concerns in the field of economic growth theories for the last two decades.

Ordinarily, the knowledge is considered to be transferred from developed countries to developing ones through various channels. For example, importing machines and equipments enables firms in developing countries to access general knowledge and information embodied in them (See Coe, Helpman, and Hoffmaister, 1997). On the other hand, multinational firms in developed countries often contribute to convey knowledge about refined manufacturing techniques through their subsidiaries.¹

Local firms in developing countries also play an important role in diffusing the knowledge about advanced technologies through two major ways. First, some firms often sell products by copying the ideas and the technologies without permission of their patent holders in developed countries. In practice, the imitation activities are widely observed owing to weak protection of intellectual property rights (IPRs) in developing countries.² Second, other local firms aim to make licensing contracts with patent holders to use their ideas and

¹Lai (1998) investigated the long-term effects of intellectual property rights protection in developing countries on rate of innovation, rate of international production transfer, and world income distribution, when the multinational firms transferred their technologies through foreign direct investment.
²Helpman (1993) explored how strengthening IPRs in developing countries affect welfare, by using a dynamic general equilibrium model such that technological innovation takes place in developed countries while the technology are imitated by developing countries at an exogenous speed.
designs legally or to learn their know-how. In particular, firms in Japan and Korea have made many licensing contracts in development process of those countries after World War II to absorb advanced technologies.

In this paper, to analyze the role of the licensing activities in technology transfer, we construct a product-life-cycle type of general equilibrium model such that local firms in developing countries pay efforts to win license contracts under circumstances of prevailing imitation. The analyses will be very meaningful for many developing countries because development stories of Japan and Korea are suggestive to those countries’ growth. A key feature of our model is to explicitly take account of (illegal) copies of products’ ideas in addition to licensing activities as a means of technology diffusion in developing countries, and to regard a frequency of the copies as a measure of the strength of IPR protection.

By using the model, we investigate influences of strengthening IPRs in developing countries, of introducing subsidy in licensing and R&D processes, and of changing rent distribution between licensors and licensees. As a consequence of the analyses, we show the following three main results: (i) strengthening IPRs through restricting imitation unambiguously raises technology transfer and innovation; (ii) introducing subsidy to licensing activities promotes technology transfer and innovation, while subsidy to R&D activities has no effect on them; and (iii) an increase in the rate of license fees paid to the licensors is obstacle not only to technology transfer but also to R&D activities.

In spite of importance of licensing activities in technology transfer, only a few previous studies have incorporated them into a general equilibrium model. Yang and Maskus (2001) initially addressed this issue by using a “quality ladder” type of product-cycle model devel-

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3A famous example is the ‘pilgrimage to Montecatini’. Many Japanese firms visited Montecattini in Italy — an Italian company that succeeded in converting propylene into a fiber-forming propylene — in order to obtain a licensing agreement.

4See Peck (1976), Ozawa (1980), and Enos and Park (1988). One of the reason why the firms in those countries made many licensing contracts lies in that their governments desired to develop their domestic firms rather than to depend on foreign firms. In addition, the Japanese government authorities have also feared outflows of rents to abroad. In fact, the Japanese government have restricted foreign direct investment through a law established in 1950 (Foreign Capital Law).
oped by Grossman and Helpman (1991, ch.12) and concluded that stronger IPRs promote innovation and technology transfer. They considered that strengthening IPR protection decreases the negotiation cost of licensing contracts (the size effect) and raises rent distribution of licensor firms that have patents of advanced technologies (the distribution effect), both of which positively work on technology transfer and innovation. However, they assume that firms in developed countries use their resources to make licensing contracts, so that their model does not suit Japanese and Korean experiences well. In addition, the steady state of their model is dynamically unstable, so that the model has a problem that the economy cannot approach the steady state.\(^5\) Later, Tanaka, Iwaisako, and Futagami (2006) modified this point and constructed a technology licensing model. In their model, firms in developing countries are assumed to use their own resources to win licensing contracts. However, they did not examine effects of IPR protection policies.

Although our model is based on Yang and Maskus (2001) and Tanaka, Iwaisako, and Futagami (2006), our study extends the models in two respects. First, our model takes into consideration a possibility that the design of a product is copied by another firm, which Yang and Maskus (2001) and Tanaka, Iwaisako, and Futagami (2006) did not consider. As stated previously, the imitation of products is fairly prevailing in developing countries and one of main channels of technology diffusion, so that our model reflects this fact.

Second, and more importantly, our setting enables us to obtain a richer implication in a policy of IPR protection. Yang and Maskus (2001) did not specify a way how government authorities control the strength of IPR protection. Instead, they presumed the size effect and the distribution effect to be the consequence of the policy. As a result, it is likely to be difficult to derive an implication about a practical policy from the results. That is, the model leaves mechanisms that tighter IPR protection influences the cost of license negotiation and the rule of rent division unspecified, so that the study could not conclude for certain whether such a concrete policy as restriction on imitation is effective in encouragement of innovation.

\(^5\)A proof is available from the authors on request.
and technology transfer. In contrast, our model can draw a conclusion about the restrictive policy on imitation since we explicitly introduce imitation into the model as the index of the degree of IPR protection.

The rest of this paper is structured as follows. Section 2 introduces the model. Section 3 shows that there exists a unique steady state. Section 4 conducts comparative statics of the steady state. Section 5 gives concluding remarks.

2 The Model

This paper constructs a quality ladder type of dynamic North-South model in which licensing is the main mode of technology transfer to a developing country in order to examine the effects of governmental policies on innovation and technology transfer. In this model, we consider that imitation of products also prevails in the developing country because of incomplete IPR protection. We regard this imitation speed as an index reflecting the level of IPR protection in the developing country. We mainly base our model on Yang and Maskus (2001) and Tanaka, Iwaisako, and Futagami (2005).

Consider an economy consisting of two regions, North and South, which are labeled by $N$ and $S$, respectively. Continuum of goods indexed by $\omega \in [0, 1]$ exists in the economy and they are produced either in the North or in the South. Each product $\omega$ is classified by a countable infinite number of qualities $j = 0, 1, \cdots$. The product with one-grade higher quality than the current top-of-the-line quality of the product becomes available if innovation occurs in the industry. Therefore, product $\omega$ with quality $j$ can be produced after $j$th innovation in the industry $\omega$. We assume that the quality is provided by $q_j(\omega) = \lambda^j$, where the increment of quality, $\lambda > 1$, is identical to all products. As described below, research and development conducted by firms brings this quality improvement. We choose units appropriately so that the quality at time $t = 0$ is equal to unity in all industries.
2.1 Consumer’s Optimization

Consumers living in both regions have identical preferences

\[ U = \int_0^\infty e^{-\rho t} \log u(t) dt, \]  

(1)

where \( \rho \) is a common subjective discount rate and \( \log u(t) \) represents instantaneous utility at time \( t \). We specify the instantaneous utility function as Cobb-Douglas form:

\[ \log u(t) = \int_0^1 \log \left[ \sum_j q_j(\omega)d_{j,t}(\omega) \right] d\omega, \]

where \( d_{j,t}(\omega) \) denotes consumption of good \( \omega \) with quality \( j \) at time \( t \). The representative consumer maximizes his or her utility (1) under an intertemporal budget constraint

\[ \int_0^\infty e^{-\int_0^t r(s)ds} E_t dt = A_0, \]

where \( r_t \) is the interest rate which consumers in both countries face at time \( t \) and \( A_0 \) is the sum of initial asset holdings and discounted total labor income. The term \( E_t \) represents the flow of spending at time \( t \), namely,

\[ E_t = \int_0^1 \left[ \sum_j p_{j,t}(\omega)d_{j,t}(\omega) \right] d\omega, \]

where \( p_{j,t}(\omega) \) is the price of product \( \omega \) with quality \( j \) at time \( t \).

We can solve this representative consumer’s problem in two steps. In the first step, we think of the intratemporal maximization problem by computing the allocation of spending \( E_t \) to maximize \( \log u(t) \) given prices at time \( t \). As a result of the static maximization, the consumer allots identical expenditure shares to all products and chooses a single quality \( j = J_t(\omega) \) of each product that carries the lowest quality-adjusted price \( p_{j,t}(\omega)/q_{j,t}(\omega) \). This
implies the static demand function

\[ d_{j,t}(\omega) = \begin{cases} 
E_{t}/p_{j,t}(\omega) & \text{for } j = J_t(\omega), \\
0 & \text{otherwise}. 
\end{cases} \]

In the second step, we compute the time pattern of spending to maximize the consumer’s utility (1) subject to the dynamic budget constraint. This intertemporal utility maximization requires \( \dot{E}_t/E_t = r_t - \rho \) By taking the aggregate spending as the numeraire, we normalize \( E_t = 1 \) for all \( t \) so that the interest rate \( r_t \) is always equal to the subjective discount rate \( \rho \).

### 2.2 Production

Next, we consider the production side. We assume that each economy has a single primary production factor, labor. The amount of total labor supply is constant and depends on the country. We also assume that one unit of output requires one unit of labor input. In addition, research activities and license negotiations to win a license from a patent holder need also labor input while imitation is assumed to progress exogenously without any labor, as we shall describe further below.

There exist two types of firm, ‘leaders’ and ‘followers’, in the economy. Leaders are the firms that have the ability to produce the current highest quality of each good, while followers are the other firms. As a general feature in this kind of model, industrial leaders intend not to further research their products until the products are copied. Therefore, quality gap between leaders and closest rivals never exceed one step.

A firm is also distinguished in terms of its location, that is, whether it is in the North or in the South. We assume that only Northern firms have the ability to conduct R&D and bring state-of-the-art products in the market. Succeeding in innovation, a Northern firm can acquire the patent on the design of the product in the North. In addition, the firm can also export the good to the South without any transportation cost and tariffs.
On the other hand, to acquire the exclusive right to produce and sell a product of a Northern firm that has not been imitated yet, a Southern firm must propose a license contract to the Northern patent holder. Granted the license, the Southern firm can receive the blueprint about the product and acquires the enough knowledge to manufacture it. Moreover, the firm can make a monopoly of the product in the entire world legally. However, Southern licensees must keep paying a part of the rents from the sale to their licensors as a license fee, until the products are imitated or replaced by the next higher quality products. We assume that Southern licensees pay the exogenously determined share proportion of profits $\delta \in (0, 1)$, which reflects the bargaining power between a licensee and a licensor. All Southern licensees are forbidden by their licensors to imitate the blueprint and cancel the license contracts to avoid paying the license fee.

If a license of a product has been granted to a Southern firm, the design of the product becomes a target of imitation by other Southern firms. Then, for tractability, we assume that imitation occurs exogenously and that every industry produced in the developing country under license is equally exposed to the threat of imitation. In more detail, we assume that success of imitation in each industry follows exogenous Poisson process with rate $i\gamma$, where $i$ is an exogenous parameter determined by imitation technology while $0 \leq \gamma \leq 1$ is a policy parameter which Southern authority can control. Therefore, smaller $\gamma$ means stronger IPR protection in the South: when $\gamma$ is equal to zero, patent enforcement in the South is perfect so that no imitation occurs; when $\gamma$ is equal to one, IPRs are not protected in the South at all. In this paper, we assume that $i\gamma > \rho/(1 - \delta)$ to ensure the local stability of the steady state. If once imitation occurs in an industry, perfect competition prevails in the industry and the product is sold at unit cost $w^S$, where $w^S$ is the wage rate in the South. Therefore,

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6It may be more natural to consider that the rent sharing between licensors and licensees is determined endogenously according to their bargaining power. An alternative setting to endogenize the rent sharing is to assume that licensors and licensees decide the rent share $\delta$ by Nash bargaining. However, to simplify analysis, we regard share proportion $\delta$ as exogenously given, following Yang and Maskus (2001) and Tanaka, Iwaisako, and Futagami (2005).

7The assumption of exogenous imitation following the technology transfer to developing countries has been adopted also in Lai (1998).
the Southern licensee loses its monopoly rent and its stock turns to no value in the market. In addition, the Northern licensor cannot receive license fee any longer. We posit state-of-the-art products manufactured in the North own confidentiality, so that Southern firms are technologically or economically impractical to copy it.

To maximize profits, each leader firm whose product has not been imitated yet sets its product at the upper limit prices which can exclude its rival firms from the market. Assuming that patents of all products whose qualities are inferior to the each state-of-the-art product are in the public domain and that the Southern wage is less than the Northern wage, the strongest rivals of each leader firm whose product has not been imitated yet are always Southern followers who have an ability to produce the second highest quality of each product. As a leader firm can exclude its rival firms by setting the lowest quality-adjusted price on its product, optimal price setting of each Northern producer and each Southern licensee is the marginal cost of the strongest rival firms multiplied by the increment of quality:

\[ p = \lambda w^S. \]

This price setting of each leader yields demand per product of \( 1/\lambda w^S \). Hence, each Northern patent holder earns a flow of profits

\[ \pi_N = (\lambda w^S - w^N) \frac{1}{\lambda w^S} = 1 - \frac{w^N}{\lambda w^S}, \quad \text{(2)} \]

where \( w^N \) is the wage rate in the North, which must be restricted to be below \( \lambda w^S \) so that the Northern leaders can earn strictly positive profit. On the other hand, the profits of Southern licensees are

\[ \pi_L = (\lambda w^S - w^S) \frac{1}{\lambda w^S} = 1 - \frac{1}{\lambda}, \quad \text{(3)} \]
2.3 Research Activities and License Negotiations

We describe random success of innovation as a Poisson process. That is, \( a_N \tilde{I}_i \) units of labor input into research activities during an infinitesimal time interval \( dt \) lead an entrepreneur \( i \) to success of innovation with probability \( \tilde{I}_i dt \), where \( a_N \) is a parameter. We assume the productivity of research \( a_N \) to be the same in all firms, so that follower firms can compete with the incumbent leaders in regard to research and development of next higher quality products. Let \( V_N \) be the market value of a Northern leader that is the reward for innovation.

In any moment, each entrepreneur decides labor input to maximize his or her instantaneous profit \( (V_N - w^N a_N) \tilde{I}_i \). Hence, the following zero-profit condition in research activities is satisfied:

\[
V_{N,t} \leq w^N_t a_N \quad \text{with equality if } \tilde{I}_i > 0.
\]

(4)

Similarly, we assume that a formation of a license contract follows a Poisson process. We assume that Southern firm \( i \) that wishes to be licensed must input \( a_L \tilde{\iota}_i \) units of the labor per unit of time into negotiation activities for attaining success of negotiation with instantaneous probability \( \tilde{\iota}_i \), where \( a_L \) is a parameter that satisfies \( a_L < a_N \). Let \( V_L \) denote the expected present value of profits earned by an incumbent licensee firm. The expected gain obtained by winning a license for Southern firms is equal to \((1 - \delta)V_L\). Therefore, such firms optimally choose the intensity of license negotiation in order to maximize their instantaneous profit \([ (1 - \delta)V_L - w^S a_L ] \tilde{\iota}_i \). In the equilibrium, since the level of license negotiation must be positive but finite, the following zero-profit condition in the license negotiation is satisfied:

\[
V_{L,t} \leq w^S_t \frac{a_L}{(1 - \delta)} \quad \text{with equality if } \tilde{\iota}_i > 0.
\]

(5)

Although, in the above, we posit that patent holders are always willing to comply with offers of license contract, we need to consider the possibility that patent holders might refuse the offers. Northern patent holders will allow Southern firms to manufacture their products.
only when market value after licensing exceeds the current value, $V_{N,t}$; otherwise they will not admit a license. Reaching an agreement of a license, the Northern patent holder acquires a claim on a fraction of Southern licensee’s profit as the license fee. Since a Northern licensor can receive $100 \times \delta$ percent of the licensee’s profit at any moment, the expected value of that claim is expressed as $\delta V_{L,t}$. Assuming that a licensor is obliged to compete with its licensee in the Bertrand fashion if it also produces the product, no licensor has an incentive to continue to sell the product after the license agreement in the equilibrium. Therefore, the market value of a licensor firm is also $\delta V_{L,t}$. As a result, the equilibrium with positive licensing requires the condition\(^8\)

$$\delta V_{L,t} \geq V_{N,t}.$$ 

### 2.4 Equilibrium Conditions

In the equilibrium, there exist three possible categories of industry in the market: (i) the Northern patent holder produces the state-of-the-art variety, (ii) the Southern licensee produces the highest-quality product under a license, (iii) the Southern imitators produce the highest-quality product under perfect competition. We represent the measure of industries belonging to each category by $n_{N,t}$, $n_{L,t}$, and $n_{M,t}$, respectively. Because the measure of all industries is unity, we have $n_{N,t} + n_{L,t} + n_{M,t} = 1$. In the following part of this paper, we focus only on the symmetric equilibrium such that all industries in the same category are symmetric.

In the symmetric equilibrium, intensities of innovation and licensing are common to the all industries. Let $I$ denote this aggregate innovation intensity in each industry, namely, $I = \sum_i \tilde{I}_i$. Then, investment to innovation targeted at any industries yields same expected

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\(^8\)In our setting, Southern firms only incur license negotiation cost and Northern firms can enjoy the benefit of licensing agreement without any effort. Perhaps, it may be more realistic to assume that not only the Southern firms but also the Northern firms incur the negotiation cost. However, to make the analysis tractable and contrastive to the existing studies, we assume that only Southern firms throw their resources into license negotiation and bear all of the negotiation cost.
payoff because the patents of a second-highest-quality product is always in the public domain. In consequence, research efforts of entrepreneurs range over all \( \omega \) equally and innovation occurs in every industry with equal probability. Meanwhile Southern firms negotiate license contracts with Northern state-of-the-art patent holders whose products are neither licensed nor imitated. In the symmetric equilibrium, Southern firms choose equal efforts of negotiation among the industries. As a result, each Northern leader firm whose product is neither licensed nor imitated receives such offers with equal intensity in the equilibrium. Let \( \iota \) denote the aggregate intensity of license negotiation targeting at each industry, namely, \( \iota = \sum_i \iota_i \).

How does the measure of products in each category change over time? In an infinitesimal time interval of length \( dt \), Northern entrepreneurs succeed in upgrading \( I_t(n_{L,t} + n_{M,t})dt \) products which Southern licensees or imitators manufacture, whereas \( \iota_t n_{N,t} dt \) products are newly licensed and come to be manufactured in the South (see figure 1). Therefore, the measure of products manufactured in the North follows the equation of motion,

\[
\dot{n}_{N,t} = I_t(n_{L,t} + n_{M,t}) - \iota_t n_{N,t}.
\]

On the other hand, measure \( i \gamma n_{L,t} dt \) licensed products are newly copied in the time interval, while \( I_t n_{M,t} dt \) imitated products revert to Northern leaders because of success in innovation. Hence, we obtain the following equation of motion for the measure of imitated products:

\[
\dot{n}_{M,t} = i \gamma n_{L,t} - I_t n_{M,t}.
\]

Because \( n_{N,t} + n_{L,t} + n_{M,t} = 1 \), the measure of licensed products changes over time according to the following equation:

\[
\dot{n}_{L,t} = -\dot{n}_{N,t} - \dot{n}_{M,t}.
\]
Now we consider how the market value of each firm varies over time. First, we think about a stock of a Northern patent holder that has not granted a license of its product. During an infinitesimal time interval of length $dt$, each Northern incumbent leader is exposed to hazard of replacement by a higher quality product with probability $I_t dt$. If innovation occurs, the incumbent Northern patent holder suffers capital loss $V_{N,t}$. In the same time interval, the patent holder can reach a license agreement with a Southern firm with probability $\iota_t dt$. Then, the Northern leader acquires $\delta V_{L,t}$ instead of the current market value $V_{N,t}$. If neither innovation nor licensing occurs in the industry during the time interval, the patent holder can earn the profits $\pi_{N,t} dt$ and capital gain $\dot{V}_{N,t} dt$. The total sum of those is expected earnings of shareholders of a Northern leader firm. Provided that idiosyncratic risks arising from a holding of a stock are diversified by all investors, a stock should yield the exactly same expected rate of return as the risk-free interest rate, $r_t$. Then the no-arbitrage condition between the stock and a risk-free asset is

$$r_t V_{N,t} = \pi_{N,t} + \dot{V}_{N,t} - I_t V_{N,t} + \iota_t (\delta V_{L,t} - V_{N,t}).$$

(8)

In the same manner, we can consider about the total market value of a Southern firm operating under a license. A Southern licensee suffers capital loss $V_{L,t}$ if innovation occurs in the industry. In addition, a licensee firm is also faced with the risk of imitation by other Southern firms. Since a product is imitated with instantaneous probability $i\gamma$, a Southern licensee loses the monopolistic position with probability $(I_t + i\gamma) dt$ in the infinitesimal time interval of length $dt$. If neither innovation nor imitation occurs in the industry during the time interval, the licensee firm can earn the profits $\pi_{L,t} dt$ and capital gain $\dot{V}_{L,t} dt$. Since the sum of these risky rates of returns must be identical with risk-free interest rate, we obtain the following no-arbitrage condition:

$$r_t V_{L,t} = \pi_{L,t} + \dot{V}_{L,t} - (I_t + i\gamma) V_{L,t}. $$

(9)
We consider finally the labor market equilibrium conditions. Let $L^N$ and $L^S$ denote the fixed labor supply in the North and in the South, respectively. For analytical tractability, we assume that $L^S$ is equal to or greater than $(a_L/a_N)L^N$. Entrepreneurs which make an effort of R&D and Northern leader firms require labor in the North, while firms under negotiation with Northern patent holders and Southern manufacturers (licensees and competitive firms) demand labor in the South. Entrepreneurs in the North conduct R&D at the same aggregate intensity $I_t$ over all industries, so they employ $a_N I_t (n_{N,t} + n_{L,t} + n_{M,t})$ units of labor at each moment. In addition, each manufacturing firm in the North sells $1/\lambda w^S_t$ units of the product at time $t$, so that Northern manufactures employ $n_{N,t}/\lambda w^S_t$ units of labor at the time. Thus, the labor market clearing condition in the North is

$$a_N I_t (n_{N,t} + n_{L,t} + n_{M,t}) + \frac{1}{\lambda w^S_t} n_{N,t} = L^N. \quad (10)$$

In the South, follower firms employ $a_L I_t n_{N,t}$ units of labor for license negotiation at time $t$. Moreover, licensee firms require $n_{L,t}/\lambda w^S_t$ units of labor, while competitive firms that manufacture imitated products need $n_{M,t}/w^S_t$ units of labor. In consequence, the labor market clearing condition in the South is

$$a_L I_t n_{N,t} + \frac{1}{\lambda w^S_t} n_{L,t} + \frac{1}{w^S_t} n_{M,t} = L^S. \quad (11)$$

### 3 Steady-state Equilibrium

In the following part of this paper, we pay attention to only the steady state equilibrium in which innovation and licensing keep taking place at a constant speed over time.\footnote{We can verify that the equilibrium path converging to the steady state exists and is locally unique if aggregate innovation intensity in the steady state, $\bar{I}$, satisfies the condition (19), as stated below. The proof is given in the appendix.} In the steady state, the fraction of each type of industry, the market values of firms, and wage rates of both regions are all constant over time. Let variables with upper bar, e.g., $\bar{I}$, denote the
steady state values of the corresponding variables.

In the steady state, the market value of a firm is equal to the expected present value of profits. From equations (3) and (9), the value of a licensee firm is given by

$$\bar{V}_L = \frac{(\lambda - 1)/\lambda}{1 + i\gamma + \rho}.$$  \hspace{1cm} (12)

Moreover, zero-profit condition in licensing activities (5) implies that

$$\bar{w}^S = \frac{(1 - \delta)\bar{V}_L}{a_L}.$$  \hspace{1cm} (13)

Similarly, using the relation that

$$\bar{w}^N = \frac{\bar{V}_N}{a_N}$$  \hspace{1cm} (14)

which is derived from the zero-profit condition in innovation (4), together with equations (2), (8), and (13), we obtain

$$\bar{V}_N = \frac{1 + \delta i\bar{V}_L}{1 + i + \rho + \frac{a_L}{a_N(1 - \delta)\lambda V_L}}.$$  \hspace{1cm} (15)

For the sake of seeking the steady state values, we rewrite the condition on the Northern labor market. Combining equations (10), (12), and (13), we can derive pairs of $\bar{I}$ and $\bar{n}_N$ which are consistent with Northern labor market clearing as follows:

$$\bar{n}_N = \frac{(1 - \delta)(\lambda - 1)}{a_L}\Phi(\bar{I}; \gamma),$$  \hspace{1cm} (16)

where $\Phi(\bar{I}; \gamma) \equiv (L^N - a_N\bar{I})/(\bar{I} + i\gamma + \rho)$. This relationship is drawn as downward sloping curve on the $(\bar{I}, \bar{n}_N)$ plane (see figure 2). We name this curve NL. The NL curve means that the ratio of products manufactured in the North to all products must decrease with an increase in $\bar{I}$; otherwise the Northern labor market is never cleared owing to excess demand for labor of incumbent leaders and of firms researching higher quality products.
To determine $\bar{I}$ and $\bar{n}_N$, we need another equation with respect to those variables. We can derive this relationship from the Southern labor market clearing condition. First, from equations (6) and (7), the following relations are satisfied in the steady state:

$$\bar{\iota} \bar{n}_N = \bar{I}(1 - \bar{n}_N), \quad \bar{n}_L = \frac{\bar{I} + i\gamma}{I + i\gamma}(1 - \bar{n}_N), \quad \text{and} \quad \bar{n}_M = \frac{i\gamma}{I + i\gamma}(1 - \bar{n}_N). \quad (17)$$

Substituting (12), (13), and (17) into (11), we have

$$\bar{n}_N = 1 - \frac{(1 - \delta)(\lambda - 1)L^S}{a_L} \Psi(\bar{I}; \gamma), \quad (18)$$

where $\Psi(\bar{I}; \gamma) \equiv (\bar{I} + i\gamma)(\bar{I} + i\gamma + \rho)(\bar{I} + i\gamma \lambda) + (1 - \delta)(\lambda - 1)(\bar{I} + i\gamma)].$ Note that the right hand side of equation (18) is increasing with $\bar{I}$ under the assumption $i\gamma > \rho/(1 - \delta)$. We describe the combinations of $\bar{I}$ and $\bar{n}_N$ that satisfy equation (18) by SL curve of figure 2. The interpretation of the SL curve is similar to that of the NL curve. Namely, the proportion of products manufactured in the South to all products, $1 - \bar{n}_N$, must fall with a rise of $\bar{I}$ since the additional labor demand of leader firms manufacturing the products and of follower firms engaged in license negotiation prevents the Southern labor market clearing without such reduction.

Since SL curve is upward sloping under the assumption, there is at most one crossing point of the two curves in the region where both $\bar{I}$ and $\bar{n}_N$ are positive. To ensure that the unique and attainable steady state and the locally unique equilibrium path converging to this steady state exists, we focus our attention on the case in which $\bar{I}$ satisfies the following inequality:

$$\max \left\{ 0, \lambda \left( \frac{L^S}{a_L} - i\gamma \right) \right\} < \bar{I} < \frac{L^N}{a_N}. \quad (19)$$

If both curves cross once in the first quadrant, $\bar{n}_N$ takes the value that is between zero and one. Then $\bar{n}_L$, $\bar{n}_M$, and $\bar{\iota}$ are given by equations (17). Furthermore, $\bar{V}_L$, $\bar{V}_N$, $\bar{w}^S$, and $\bar{w}^N$ are determined by equations (12)–(15).
However, we must impose two additional conditions on parameters to assure the existence of the steady state. First is a condition about the incentive of patent holders to licensing. As stated above, a patent holder has no incentive to grant a license of its product if the expected value of the license fee is below the current market value. Therefore, the steady state with strictly positive licensing requires $\delta V_L \geq \bar{V}_N$. The second condition is concerned with the wage of both regions. Our above analysis presumes that the Northern wage is higher than the Southern one. Moreover, the Northern wage $w^N$ is restricted below $\lambda w^S$ in the equilibrium; otherwise Northern leaders would cease to operate because of negative profits. From equations (13), (14), and (15), we can restate these conditions as

$$\frac{a_N(1-\delta)}{a_L} < \frac{1 + \delta i \bar{V}_L}{(1+i+\rho)V_L + \frac{a_L}{a_N(1-\delta)\lambda}} < \min \left\{ \delta, \frac{a_N(1-\delta)\lambda}{a_L} \right\}.$$  

4 Comparative Statics

In this section, we consider how the intensities of innovation and licensing are affected by the stronger protection of intellectual property rights, the subsidy policy for license negotiation and innovation, and the change of profit division rule.

4.1 Effects of strengthening IPR protection

In section 2, we have assumed imitation speed to be directly controllable by Southern authorities. That is, we have thought of $\gamma$ as a policy parameter about IPR protection of the Southern government. Thus, we first conduct comparative statics with respect to $\gamma$ in this subsection.

First, totally differentiating equation (16), we obtain

$$d\bar{n}_N = -\frac{(1-\delta)(\lambda - 1)}{a_L(1+i+\rho)^2} \left[ (L^N + a_Ni\gamma + a_N\rho)dI + i(L^N - a_N\bar{I})d\gamma \right].$$  \hspace{1cm} (20)
Because $L^N - a_N \bar{I} > 0$ from equation (10), equation (20) means that $\bar{n}_N$ is required to increase with a fall of $\gamma$, if $\bar{I}$ were unchanged, in order to maintain equilibrium in the Northern labor market. In other words, NL curve in figure 2 rotates clockwise in response to a fall of $\gamma$ induced by tightening IPR protection. This is because, for given $\bar{I}$ and $\bar{n}_N$, a lower $\gamma$ raises the Southern wage through a rise of $\bar{V}_L$ (see equations (12) and (13)). The higher Southern wage enables each Northern patent holder to set higher price and leads to smaller product demand, which means smaller labor demand of each incumbent Northern leader. Therefore, Northern labor supply could afford to retain more producers in the North, that is, $\bar{n}_N$ must be higher than before to clear excess labor supply, if the labor demand of research sector, $a_N \bar{I}$, were unchanged by the Southern policy change.

Next, total differential of equation (18) is expressed as follows:

$$d\bar{n}_N = \frac{(1 - \delta)(\lambda - 1)L^S \left[ \Psi(\bar{I}; \gamma) \right]^2}{a_L} \left( A\bar{I} + B\bar{I} \right), \quad (21)$$

where $A = \delta + \lambda(1 - \delta) - \left[ i\gamma \rho(\lambda - 1)/(\bar{I} + i\gamma)^2 \right] > 0$ and $B = i\left[ \lambda + \left[ \bar{I}\rho(\lambda - 1)/(\bar{I} + i\gamma)^2 \right] \right] > 0$. Equation (21) shows that $\bar{n}_N$ is required to decrease with a fall of $\gamma$, if $\bar{I}$ were unchanged, in order to maintain equilibrium in the Southern labor market. Thus, SL curve in figure 2 moves downward by a decrease of $\gamma$.

Intuitively, if both $\bar{I}$ and $\bar{n}_N$ were unchanged, restrictions on imitation by the Southern government would decrease the Southern labor engaged in manufacturing by two reasons. First, by the same argument as Northern labor demand, the increase of the Southern wage through a rise of $\bar{V}_L$ which is due to reduction of threats of imitation decreases each incumbent leader’s demand for Southern labor with the fall of $\gamma$. Second, from equation (17), reduction of the proportion of imitated industries to licensed ones in the South, $\bar{n}_M/\bar{n}_L$, induced by the policy modification has a negative effect on labor demand in the South because a licensee firm behaving as a monopolist employs less Southern labor than competitive firms. The redundant labor originated from these two effects must be absorbed by follower firms.
under license negotiation to restore equilibrium in the Southern labor market. In consequence, tightening IPR protection positively affects measure of industries where a Southern follower firm reaches an agreement of a license contract with a patent holder, $\bar{n}_N$, at each moment. This means that, from equation (17), $\bar{n}_N$ must decrease with strength of IPR protection for given $\bar{I}$ in order to keep the Southern labor market clearing.

As tightening IPR protection shifts both NL and SL curve upward simultaneously, such modification of IPR protection policy moves intersection from $E$ to $E'$ in figure 2. Since the new intersection after the policy change locates more rightward than the original one, we can confirm that tightening IPR protection in the South increases $\bar{I}$ unambiguously.

However, the figure provides no information about whether $\bar{n}_N$ increases or not. Therefore, we next compute $\partial \bar{n}_N / \partial \gamma$ to know it. Using equations (20) and (21), we can eliminate the term $d\bar{I}$ and obtain

$$d\bar{n}_N = \frac{1}{(1 - \delta)(\lambda - 1) A a_L(\bar{I} + i\gamma + \rho)^2} \left[ \frac{L^N + a_N i\gamma + a_N \rho}{L^S A(\bar{I} + i\gamma + \rho)^2} \left[ \Psi(\bar{I}; \gamma) \right]^2 \right] d\gamma. \quad (22)$$

Because $(L^N + a_N i\gamma + a_N \rho) B - i(L^N - a_N \bar{I}) A$ is positive, we can confirm that $\partial \bar{n}_N / \partial \gamma > 0$ from equation (22). This result and equation (17) imply that both $\partial \bar{I}/\partial \gamma$ and $\partial (\bar{m}_N)/\partial \gamma$ are negative. In other words, license activities are stimulated by a strengthening IPR protection.

Hence, our above analysis is summarized into the next proposition.

**Proposition 1:** Strengthening intellectual property rights protection through restrictions on imitation promotes innovation and technology licensing to developing countries.

This proposition enhances the Yang and Maskus’ (2001) result of strengthening IPR protection. Instead of assuming a policy parameter of the Southern government authorities, Yang and Maskus (2001) have indirectly regarded both a rise of the probability of success in license negotiation under the same labor input (the cost-reducing effect) and a rise of royalty
payment to licensor (the distribution effect) as the effects of strengthening IPR protection. Based on such formulation, they concluded that stronger protection of IPRs promotes innovation and licensing through the two effects.

In contrast, our model explicitly takes into account the possibility of imitation by other firms, and assumes that government authority is able to control the imitation through observing whether imitated products are made or sold in the market. The present paper interprets low speed of the exogenous imitation as stronger IPR protection by the Southern government. As a result, we obtain proposition 1 which implies that the developing country’s authority that aims to encourage license of state-of-the-art products should regulate illegal copies targetting at licensed products.\(^\text{10}\)

We can also show that restrictions on imitation raise the Southern wage in the steady state. From equation (13), \(\frac{\partial \bar{w}^S}{\partial \gamma}\) satisfies the following equation:

\[
\frac{\partial \bar{w}^S}{\partial \gamma} = -\frac{(1 - \delta)(\lambda - 1)}{a_L \lambda(I + i\gamma + \rho)^2} \left( i + \frac{\partial I}{\partial \gamma} \right) < 0. \tag{23}
\]

To interpret this equation, recall that the Southern wage relates closely to profitability in negotiation activities of license because of free entry. Then, this equation shows that a rise of \(\gamma\) changes the Southern wage through two channels. First, tighter enforcement of IPR protection (a fall of \(\gamma\)) mitigates the threat of imitation by other firms, so that a licensee can enjoy longer expected duration of the monopoly. This effect causes the stock value of a licensee firm that has the one-to-one correspondence to the Southern wage to directly increase. However, since strengthening IPR protection activates innovation in the North as stated in proposition 1, the higher frequency of innovation shortens the expected duration of the monopoly. In consequence, this second indirect effect induces the Southern wage to decrease and depresses the first effect. But we can verify that the first positive effect

\(^{10}\)Another feature of our model is to confirm the local stability of the steady state. We prove in the appendix that the steady state of our model has a unique converging path. Meanwhile the steady state of Yang and Maskus (2001) is totally unstable as mentioned in the introduction.
dominates the second negative effect under restriction (19). Substituting equation (21) into (22) and rearranging the terms, we have

\[
\frac{\partial \bar{I}}{\partial \gamma} = -i \times \frac{(L^N - a_N \bar{I}) + L^S (B/i)(\bar{I} + i\gamma + \rho)^2 \left[ \Psi(\bar{I}; \gamma) \right]^2}{(L^N + a_N \bar{I} + a_N \rho) + L^S A(\bar{I} + i\gamma + \rho)^2 \left[ \Psi(\bar{I}; \gamma) \right]^2}.
\]

As we can show that the numerator of the fraction on the right hand side is smaller than the denominator from condition (19), \( \frac{\partial \bar{I}}{\partial \gamma} \) is larger than \(-i\). Thus, from equation (23), we can conclude that the Southern wage \( \bar{w}^S \) is decreasing with \( \gamma \) (that is, increasing with the degree of IPR protection).

4.2 Effects of subsidy policies

In this subsection, we explore effects of industrial policies through subsidies on R&D and technology transfer. In the former subsection, we have confirmed that the Southern government can encourage innovation and technology transfer by restricting imitation activities. However, government authorities may probably be concerned to subsidize license negotiation and R&D for the sake of more directly promoting innovation and technology transfer. Hence, we examine whether or not governments can promote such activities by partly bearing the cost of license negotiation and R&D.

4.2.1 Subsidies on license negotiation

We first consider the effects of subsidies to license negotiation by the Southern government. Let \( s_L \in [0, 1) \) be a subsidy rate on the cost of license negotiation. The subsidy improves profitability of Southern follower firms under license negotiation. That is, by engaging license negotiation, a Southern follower firm \( i \) can earn the instantaneous expected profit \([(1 - \delta)V_L - (1 - s_L)w^S a_L] \bar{I}_i \). Therefore, the introduction of subsidies modifies the zero
profit condition and yields the following relation:

$$\bar{w}^S = \frac{(1 - \delta)\bar{V}_L}{(1 - s_L)a_L}. \quad (24)$$

If $s_L$ is equal to zero, this equation reduces to equation (13). Moreover, this equation shows that a rise of subsidy rate, other things being equal, pushes the Southern wage up because Southern follower firms wish to harder negotiate a license contract with a Northern patent holder owing to reduction of the negotiation cost.

To determine a new pair of $\bar{I}$ and $\bar{n}_N$, we first compute new NL and SL curves. From equations (10), (12), and (24), we have the following new NL curve:

$$\bar{n}_N = \frac{(1 - \delta)(\lambda - 1)}{a_L}\hat{\Phi}(\bar{I}; s_L), \quad (25)$$

where $\hat{\Phi}(\bar{I}; s_L) \equiv (L^N - a_N\bar{I})/[(1 - s_L)(\bar{I} + i\gamma + \rho)]$. Similarly, substituting (12), (17), and (24) into (11), we obtain the new SL curve as follows:

$$\bar{n}_N = 1 - \frac{(1 - \delta)(\lambda - 1)L^S}{a_L}\hat{\Psi}(\bar{I}; s_L), \quad (26)$$

where $\hat{\Psi}(\bar{I}; s_L) \equiv (\bar{I} + i\gamma)/[(1 - s_L)(\bar{I} + i\gamma + \rho)(\bar{I} + i\gamma\lambda) + (1 - \delta)(\lambda - 1)\bar{I}(\bar{I} + i\gamma)]$. Clearing of the labor market of both countries requires $\bar{I}$ and $\bar{n}_N$ to satisfy both equations (25) and (26).

Next, we totally differentiate equations (25) and (26) and examine the effects of subsidies on license negotiation. Totally differentiating equation (25) implies that

$$d\bar{n}_N = \frac{(1 - \delta)(\lambda - 1)}{a_L(1 - s_L)} \left[ -\frac{L^N + a_Ni\gamma + a_N\rho}{(\bar{I} + i\gamma + \rho)^2} d\bar{I} + \hat{\Phi}(\bar{I}; s_L)ds_L \right]. \quad (27)$$

This equation shows that $\bar{n}_N$ must increase with a rise of subsidy rate $s_L$ for a given $\bar{I}$ to maintain equilibrium in the Northern labor market. Namely, NL curve rotates clockwise by
a rise of subsidy rate as in figure 2. This is because a rise of $s_L$ enables each Northern leader to set higher price through the rise of the Southern wage and leads to smaller labor demand of incumbent leaders for given $\bar{I}$ and $\bar{n}_N$. As a result, $\bar{n}_N$ must be higher than before to clear Northern labor market, if the labor demand of R&D sector, $a_N\bar{I}$, were unchanged.

Meanwhile, from equation (26), we have

$$d\bar{n}_N = \frac{(1 - \delta)(\lambda - 1)L^S}{a_L} \left[ \hat{\Psi}(\bar{I}; s_L) \right]^2 \times \left\{ [(1 - s_L)A + s_L(1 - \delta)(\lambda - 1)]d\bar{I} - \frac{(\bar{I} + i\gamma + \rho)(\bar{I} + i\gamma\lambda)}{\bar{I} + i\gamma}ds_L \right\}. \tag{28}$$

This equation shows that $\bar{n}_N$ must decrease with a rise of subsidy rate $s_L$ for a given $\bar{I}$ to maintain equilibrium in the Southern labor market. Thus, SL curve moves downward as in figure 2. This is due to the same reason as the Northern labor market: namely, a rise of $s_L$ decreases the labor demand of incumbent producers through the rise of the Southern wage for given $\bar{I}$ and $\bar{n}_N$, so that $\bar{m}_N$, which is equal to $\bar{I}(1 - \bar{n}_N)$ from equation (17), must be higher than before. Therefore, $1 - \bar{n}_N$ must increase with the rise of subsidy rate in order to clear the Southern labor market if $\bar{I}$ were unchanged by the policy modification.

A rise of subsidy rate moves the intersection in figure 2 from $E$ to $E'$. In consequence, we can immediately confirm that the innovation intensity increases in the new steady state. However, because whether $\bar{n}_N$ increases or not is again unclear from the figure, we next compute $\partial \bar{n}_N / \partial s_L$. For tractability, we suppose that initial subsidy rate is equal to zero and examine the effect of marginal rise of the rate. Substituting equation (28) into (27) and applying $s_L = 0$, we obtain

$$\left[ 1 + \frac{L^N + a_N i\gamma + a_N \rho}{AL^S(\bar{I} + i\gamma + \rho)^2 \left[ \hat{\Psi}(\bar{I}; 0) \right]^2} \right] d\bar{n}_N$$

$$= - \frac{(1 - \delta)(\lambda - 1)}{a_L} \left[ \frac{(L^N + a_N i\gamma + a_N \rho)(\bar{I} + i\gamma\lambda)}{A(\bar{I} + i\gamma)(\bar{I} + i\gamma + \rho)} - \hat{\Phi}(\bar{I}; 0) \right] ds_L.$$
This equation implies that $\partial \bar{m}_N / \partial s_L|_{s_L=0} < 0$ because $(L^N + a_N i \gamma + a_N \rho)(\bar{I} + i \gamma) > \Phi(\bar{I}; \gamma) = \Phi(\bar{I}; 0)$ from condition (19) and from the assumption that $L^S \geq (a_L/a_N)L^N$. Thus, from equation (17), we can conclude that $\partial \bar{v} / \partial s_L|_{s_L=0}$ and $\partial (\bar{m}_N) / \partial s_L|_{s_L=0}$ are positive: that is, marginal increase of subsidies on license negotiation from zero can also promote licensing. These results are summarized as follows.

**Proposition 2:** Suppose that the initial rate of a subsidy on license negotiation is zero. Then, a marginal subsidy on license negotiation can promote innovation and licensing.

How does the subsidy on license negotiation affect the Southern wage $\bar{w}^S$? The subsidy has an effect on the Southern wage $\bar{w}^S$ through two channels:

$$
\frac{\partial \bar{w}^S}{\partial s_L} = \frac{1 - \delta}{(1 - s_L)^2 a_L} \bar{V}_L + \frac{1 - \delta}{(1 - s_L) a_L} \frac{\partial \bar{V}_L}{\partial s_L} = \frac{1 - \delta}{(1 - s_L) a_L \lambda (I + i \gamma + \rho)} \left[ \frac{\bar{I} + i \gamma + \rho}{1 - s_L} - \frac{\partial \bar{I}}{\partial s_L} \right].
$$

(29)

The first term represents a direct effect, whereas the second term does an indirect effect through $\bar{V}_L$. Since a rise of $s_L$, other thing being equal, reduces the negotiation cost, the Southern wage must be higher than before to attain zero profit in the negotiation activities. On the other hand, the higher $s_L$ also raise the innovation intensity $\bar{I}$, so that it lowers the stock value of licensee firms $\bar{V}_L$ through increasing the danger of being replaced by a higher quality product. Hence, this second effect has a negative effect on the Southern wage.

Although the two effects have different signs each other, the direct positive effect always dominates the indirect negative effect if initial subsidy rate is zero and condition (19) is satisfied. Substituting (27) into (28) and applying $s_L = 0$, we have

$$
\frac{\partial \bar{I}}{\partial s_L}|_{s_L=0} = \frac{(\bar{I} + i \gamma + \rho) \left\{ (\bar{I} + i \gamma)(L^N - a_N \bar{I}) + L^S(\bar{I} + i \gamma \lambda)(\bar{I} + i \gamma + \rho)^2 \left[ \hat{\Psi}(\bar{I}; 0) \right]^2 \right\}}{(\bar{I} + i \gamma)(L^N + a_N i \gamma + a_N \rho) + L^S A(\bar{I} + i \gamma)(\bar{I} + i \gamma + \rho)^2 \left[ \hat{\Psi}(\bar{I}; 0) \right]^2}.
$$

Using this equation and condition (19), we can show that $\partial \bar{I} / \partial s_L|_{s_L=0} < \bar{I} + i \gamma + \rho$. Thus,
from equation (29), \( \frac{\partial \bar{w}^S}{\partial s_L}{|_{s_L=0}} > 0 \), namely, the marginal rise of the subsidy rate from zero raises the Southern wage \( \bar{w}^S \).

### 4.2.2 Subsidies on R&D

Now, we turn our analysis to the effects of subsidies to R&D by the Northern government. Let \( s_R \in [0, 1) \) denote a subsidy rate on R&D chosen by the Northern government. As in the subsidies on license negotiation, subsidies on R&D improve profitability of Northern firms engaging in R&D. Namely, a Northern follower firm \( i \) which undertakes R&D can earn the instantaneous expected profit \( [\bar{V}_N - (1 - s_R)w_N a_N] \bar{I}_i \). Hence, introduction of the subsidies alters the zero profit condition and replaces equation (14) into the following:

\[
\bar{w}^N = \frac{\bar{V}_N}{(1 - s_R)a_N}.
\]

However, this modification does not any change on both NL and SL curves because labor market clearing conditions in both countries are independent of the Northern wage (see equations (10) and (11)). Thus, neither \( \bar{I} \) nor \( \bar{\iota} \) is influenced by subsidies on R&D at all. We can summarize this result as the following proposition.

**Proposition 3:** Subsidies on R&D have no influence on innovation and licensing.

In addition, because innovation intensity is not influenced by the policy modification, the stock value of Southern licensee firms, \( \bar{V}_L \), is also unchanged. As a result, the subsidies on the cost of R&D has no influence on the Southern wage. Hence, the subsidy policy on R&D, which is intended for promoting innovation, can only affect the wage gap between the North and the South in this model.
4.3 Effects of a change in profit division

The determination of profit division rule between a licensor and a licensee has been assumed to be dependent on the bargaining power between a licensor and a licensee. The bargaining power may be affected by the change of contracting environment, for example, the revision of commercial law and the change of the enforcement of patent law. Hence, in this subsection, we examine how an alteration of the rate of license fee influences innovation and licensing.

Consider that the licensors’ share of profit $\delta$ rises marginally from the initial value. Because low expected return induced by the higher license fee becomes insufficient to pay the negotiation cost, the higher licensors’ share deteriorates the profitability of Southern firms engaging in negotiation. Therefore, to restore zero profit in negotiation activities, the Southern wage $\bar{w}^S$ must fall in new equilibrium if $\bar{I}$ were unchanged by the change. Because of the limit pricing strategy, the decrease of the Southern wage leads to higher labor demand of each incumbent leader. Hence, the improvement of licensors’ share of profit makes the fraction of products manufactured in the North, $\bar{n}_N$, impossible to retain the same level as before for a given $\bar{I}$, so that NL curve is pressured to rotate counterclockwise. In fact, by totally differentiating NL curve (16), we obtain

$$d\bar{n}_N = -\frac{\lambda - 1}{a_L} \left[ \frac{(1 - \delta)(L^N + a_N i\gamma + a_N \rho)}{(I + i\gamma + \rho)^2} d\bar{I} + \Phi(\bar{I}; \gamma) d\delta \right].$$

(30)

From this relationship, we can confirm that $\bar{n}_N$ must fall with a rise of $\delta$ for a given $\bar{I}$ to be consistent with equilibrium in the Northern labor market.

The similar argument to the NL curve is also applied to the SL curve. Totally differentiating SL curve (18) gives the following relation:

$$d\bar{n}_N = \frac{L^S(\lambda - 1)}{a_L} \left[ \Psi(\bar{I}; \gamma) \right]^2 \left[ (1 - \delta)Ad\bar{I} + \frac{(\bar{I} + i\gamma + \rho)(\bar{I} + i\gamma \lambda)}{I + i\gamma} d\delta \right].$$

(31)
Because of the pricing rule, the fall of the Southern wage generates additional labor demand of each Southern firm to manufacture the product. Therefore, the rise of $\delta$ makes the fraction of products manufactured in the South, $\bar{n}_L + \bar{n}_M$, inevitable to decrease for a given $\bar{I}$. In consequence, SL curve is required to shift upward.

Figure 3 describes how a rise of $\delta$ affects two curves and the intersection. As the figure shows, a rise of the rate of license fee $\delta$ induces the intersection to move leftward and the innovation intensity $\bar{I}$ to decrease unambiguously. However, figure 3 does not explain again whether the change raises $\bar{n}_N$ or not. Therefore, we must compute $\partial \bar{n}_N / \partial \delta$ to determine the signs of $\partial \bar{I} / \partial \delta$ and $\partial (\bar{I} \bar{n}_N) / \partial \delta$.

Substituting equation (31) into (30) to eliminate the term of $d\bar{I}$, we have

$$
1 + \frac{(L^N + a_N i\gamma + a_N \rho)}{A L^S (\bar{I} + i\gamma + \rho)^2 [\Psi(\bar{I}; \gamma)]^2} d\bar{n}_N = \left( \frac{\lambda - 1}{a_L} \right) \left[ \frac{(L^N + a_N i\gamma + a_N \rho)(\bar{I} + i\gamma \lambda)}{A(\bar{I} + i\gamma)(\bar{I} + i\gamma + \rho)} - \Phi(\bar{I}; \gamma) \right] d\delta.
$$

Recall that $(L^N + a_N i\gamma + a_N \rho)(\bar{I} + i\gamma \lambda)/[A(\bar{I} + i\gamma)(\bar{I} + i\gamma + \rho)] > \Phi(\bar{I}; \gamma)$ from condition (19) and from the assumption that $L^S \geq (a_L / a_N) L^N$. Hence, this equation implies that $\partial \bar{n}_N / \partial \delta > 0$. Moreover, from the result and equation (17), we obtain $\partial \bar{I} / \partial \delta < 0$ and $\partial (\bar{I} \bar{n}_N) / \partial \delta < 0$. Namely, a higher license fee rate $\delta$ reduces licensing activities.

We can summarize these results into the following proposition:

**Proposition 4**: A higher rate of license fee becomes an obstacle to both innovation and conclusion of license agreement.

This proposition contains a seemingly counterintuitive assertion: that is, higher licensor’s share of profit deters not only licensing but also innovation. However, in fact, the result is fairly natural in this model. A higher licensor’s share of profit, other things being equal, raises the stock value of a Northern leader owing to an improvement of expected payoff the leader can obtain by reaching a license agreement. This effect on the stock value must be
exactly offset by a change of the Northern wage since innovators always attain zero profit. However, these changes do not have any influence on the degree of innovation because innovation intensity depends neither on the stock value of a Northern leader nor on the Northern wage (see equation (10) and (11)). In addition, more Northern labor is devoted to manufacturing by incumbent leaders than before for two reasons. First, incumbent leaders become less likely to reach a license agreement because of the reduction of negotiation effort by Southern follower firms. In consequence, more leaders operate in the North than before. Second, a rise of $\delta$ forces the Southern wage down, which is verified later. As we pointed out previously, a decrease in the Southern wage leads to higher labor demand by each incumbent leader. Thus, less labor can engage in R&D activities than before, so that the economy incurs the situation of less innovation despite a favorable change to licensors.

We now examine how a rise of licensor’s profit share $\delta$ affects the Southern wage. As mentioned previously, this change lowers the Southern wage. We compute the partial derivative of $\bar{w}^S$ with respect to $\delta$ to verify it:

$$\frac{\partial \bar{w}^S}{\partial \delta} = \frac{-\bar{V}_L}{a_L} + \frac{1 - \delta}{a_L} \frac{\partial \bar{V}_L}{\partial \delta} = -\frac{(1 - \delta)(\lambda - 1)}{a_L \lambda (I + i\gamma + \rho)^2} \left[ \frac{\bar{I} + i\gamma + \rho}{1 - \delta} + \frac{\partial \bar{I}}{\partial \delta} \right].$$

(32)

This equation shows that a rise of $\delta$ brings two effects on the profitability of license negotiation. First, other things being equal, higher $\delta$ directly impinges on licensees’ profitability, which is expressed by the first term of equation (32). On the other hand, indirect effect through the value of a licensee firm improves the profitability, as represented by the second term. This is due to a rise of licensee’s stock value induced by the decrease in the danger exposed to replacement by a new invention. As a result, a rise of $\delta$ causes two effects conflicting with each other. However, by computing $\partial I/\partial \delta$, we can confirm that the second positive effect is insufficient to compensate the first negative effect. Substituting (30) into
(31) and eliminating the term $d\bar{n}_N$, we have

$$\frac{\partial \bar{I}}{\partial \delta} = -\frac{\bar{I} + i\gamma + \rho}{1 - \delta} \frac{(\bar{I} + i\gamma)(L^N - a_N \bar{I}) + L^S(\bar{I} + i\gamma)^2 \Psi(\bar{I}; \gamma)^2}{(\bar{I} + i\gamma)(L^N + a_N i\gamma + a_N \rho) + L^S A(\bar{I} + i\gamma)(\bar{I} + i\gamma + \rho)^2 \Psi(\bar{I}; \gamma)^2}.$$ 

Noting that $\tilde{\Psi}(\bar{I}; 0) = \Psi(\bar{I}; \gamma)$, we can show that $\partial \bar{I}/\partial \delta > -(\bar{I} + i\gamma + \rho)/(1 - \delta)$ in the same way as the proof of $\partial \bar{I}/\partial s_{L|sL=0} < \bar{I} + i\gamma + \rho$. Thus, we can conclude that the Southern wage is decreasing with the profit share of licensors from equation (32), that is, $\partial \bar{w}^S/\partial \delta < 0$.

## 5 Concluding Remarks

Licensing of patents has played an important role as a means of introduction of high technologies in the development process of Japan and Korea after World War II. In this paper, we have constructed a dynamic North-South model to investigate the effects of governments’ policies on innovation and technology transfer in the economy where the main channel of the transfer is such licensing activities. From the comparative statics analysis of the model, we have obtained the following three results. First, Southern government can promote innovation and technology transfer by restricting illegal imitation activities. Second, an increase in subsidies on the cost of license negotiation also promotes innovation and technology transfer, while subsidies on the cost of R&D have no effect on them. Third, policy change that induces higher rate of license fee reduces innovation and technology transfer.

A key feature of our model is that production of illegal copies is assumed to prevail in the South because of imperfect enforcement of IPR protection that is often observed in actual developing countries. As a result, the model well describes firms in developing countries which make an effort at negotiating with patent holders in the developed countries to acquire the licenses under the environment of imperfect IPR protection. In addition, the assumption enables us to draw a richer implication about IPR protection policy enforced by restricting
imitation activities than existing studies which dealt with licensing in general equilibrium framework because the studies simplified the setting about IPR protection policy.

However, our model is constructed on the following two assumptions in order to maintain simplicity. First, imitation of products is assumed to be exogenous process. But, it may be more realistic to consider copying as a profit-maximizing activity of follower firms.\footnote{Glass and Saggi (2002b) pointed out that strengthening IPR protection in the South decreases FDI flows to the South by using a model which endogenizes imitation activities as well as FDI.} Second, no alternative channel of technology transfer except licensing is incorporated in the model. In practice, not only licensing but also FDI is one of main channel of technology transfer to developing countries.\footnote{For example, Glass and Saggi (2002a) constructed a symmetric two country model which includes the firm’s choice of market mode: establishing a local subsidiary company or licensing its technology to another local firm. Antràs (2005) developed a North-South model in which Northern researcher can choose the mode of production between licensing and multinationalizing.} Therefore, one of directions for future research may be to develop a model which improves these two respects.

A Appendix: A Proof of Local Stability of the Steady State

In this appendix, we show that the steady state of the economy is a saddle point whose stable manifold is hyperplane of dimension two. Because our model has two state variables (and two jump variables), it ensures the steady state to be saddle-point stable and the equilibrium path to be (locally) determinate.

To show it, we first derive autonomous system of differential equations which describes the dynamics of the model. Because the system of this economy is completely described by four variables, $n_{N,t}$, $n_{L,t}$, $V_{L,t}$, $V_{N,t}$, we compute dynamic equations with respect to those variables. Substituting equations (5), (10), and (11) into (6) yields the first differential equation with respect to $n_{N,t}$ as

$$\dot{n}_{N,t} = -\frac{L^S}{a_L} + \frac{(\lambda - 1)n_{M,t}}{(1 - \delta)\lambda V_{L,t}} + \left[\frac{L^N}{a_N} + \frac{1}{(1 - \delta)\lambda V_{L,t}} \left(1 - \frac{a_L}{a_N}n_{N,t}\right)\right](1 - n_{N,t}). \tag{33}$$
Similarly, eliminating $n_{L,t}$ from equation (7) by using the relation $n_{N,t} + n_{L,t} + n_{M,t} = 1$ and substituting (5) and (10) into (7), we have the second differential equation with respect to $n_{M,t}$ as follows:

$$
\dot{n}_{M,t} = i\gamma (1 - n_{N,t}) - n_{M,t} \left[ \frac{L^{N}}{a_{N}} - \frac{a_{L}n_{N,t}}{a_{N}(1 - \delta)\lambda V_{L,t}} + i\gamma \right].
$$

(34)

These two variables are state ones since the measure of products belonging to each category, which is determined as a result of past innovation, licensing, and imitation, cannot adjust immediately.

The other two equations are derived from no-arbitrage conditions (8) and (9). From equations (3), (5), (9), and (10), and $r_{t} = \rho$, we obtain the third differential equation with respect to $V_{L,t}$:

$$
\dot{V}_{L,t} = \left( \rho + i\gamma + \frac{L^{N}}{a_{N}} \right) V_{L,t} - \left[ \frac{a_{L}n_{N,t}}{a_{N}(1 - \delta)\lambda} + \left( 1 - \frac{1}{\lambda} \right) \right].
$$

(35)

Moreover, substituting equations (2), (4), (5), and $r_{t} = \rho$ into (8), we have the last differential equation with respect to $V_{N,t}$:

$$
\dot{V}_{N,t} = \left( \frac{a_{L}}{a_{N}(1 - \delta)\lambda V_{L,t}} + I_{t} + \nu_{t} + \rho \right) V_{N,t} - 1 - \delta\nu_{t}V_{L,t},
$$

(36)

where $I_{t}$ and $\nu_{t}$ are given by equations (10) and (11) according to the values of $n_{N,t}$, $n_{L,t}$, and $V_{L,t}$. Note that the stock values of firms, $V_{L,t}$ and $V_{N,t}$, are jumpable.

The four equations give an autonomous system of differential equations and completely characterize dynamics of the model. The values of the other endogenous variables are determined from the four variables.

Next, we derive a characteristic equation associated with a Jacobian matrix of a system of equations linearized around the steady state. From equations (33) – (36), the system of
The linearized equations is given by

\[
\begin{pmatrix}
\dot{n}_{N,t} \\
\dot{n}_{M,t} \\
\dot{V}_{L,t} \\
\dot{V}_{N,t}
\end{pmatrix} =
\begin{pmatrix}
-J_{11} & J_{12} & -J_{13} & 0 \\
J_{21} & -J_{22} & -J_{23} & 0 \\
-J_{31} & 0 & J_{33} & 0 \\
J_{41} & J_{42} & J_{43} & J_{44}
\end{pmatrix}
\begin{pmatrix}
n_{N,t} - \bar{n}_{N} \\
n_{M,t} - \bar{n}_{M} \\
V_{L,t} - \bar{V}_{L} \\
V_{N,t} - \bar{V}_{N}
\end{pmatrix},
\]  

(37)

where

\[
J_{11} = -\frac{\partial \dot{n}_{N,t}}{\partial n_{N,t}} = \bar{I} + \frac{1}{(1 - \delta)\lambda V_L} + \frac{a_L(1 - \bar{n}_N)}{a_N(1 - \delta)\lambda V_L} > 0,
\]

\[
J_{12} = \frac{\partial \dot{n}_{N,t}}{\partial n_{M,t}} = \frac{\lambda - 1}{(1 - \delta)\lambda V_L} > 0,
\]

\[
J_{13} = -\frac{\partial \dot{n}_{N,t}}{\partial V_{L,t}} = \frac{(\lambda - 1)\bar{n}_M}{(1 - \delta)\lambda(V_L)^2} + \frac{(1 - \bar{n}_N)}{(1 - \delta)\lambda(V_L)^2} \left(1 - \frac{a_L}{a_N} \bar{n}_N\right) \\
= \frac{1}{V_L} \left[\frac{L^S}{a_L} - \frac{L^N}{a_N}(1 - \bar{n}_N)\right] > 0,
\]

\[
J_{21} = \frac{\partial \dot{n}_{M,t}}{\partial n_{N,t}} = \frac{a_L\bar{n}_M}{a_N(1 - \delta)\lambda V_L} - i\gamma,
\]

\[
J_{22} = -\frac{\partial \dot{n}_{M,t}}{\partial n_{M,t}} = \bar{I} + i\gamma > 0,
\]

\[
J_{23} = -\frac{\partial \dot{n}_{M,t}}{\partial V_{L,t}} = \frac{\bar{n}_M}{V_L} \left(\frac{L^N}{a_N} - \bar{I}\right) > 0,
\]
\[ J_{31} = -\frac{\partial \dot{V}_L, t}{\partial n_{N, t}} = \frac{a_L}{a_N(1 - \delta)\lambda} > 0, \]

\[ J_{33} = \frac{\partial \dot{n}_{N, t}}{\partial n_{N, t}} = \frac{L_N}{a_N} + i\gamma + \rho > 0, \]

\[ J_{44} = \frac{\partial \dot{n}_{N, t}}{\partial n_{N, t}} = \frac{a_L}{a_N(1 - \delta)\lambda V_L} + \bar{I} + \bar{\ell} + \rho > 0, \]

and \( J_{41}, J_{42}, \) and \( J_{43} \) are not relevant to the following analysis. Note that it is not clear whether \( J_{21} \) is positive or negative. Defining matrix \( J \) as

\[
J \equiv \begin{pmatrix}
-J_{11} & J_{12} & -J_{13} \\
J_{21} & -J_{22} & -J_{23} \\
-J_{31} & 0 & J_{33}
\end{pmatrix},
\]

we obtain the characteristic equation of the coefficient matrix on the right-hand side of equation (37) as follows:

\[
(J_{44} - x)(-x^3 + \text{Tr } J \ x^2 - B_J \ x + \text{Det } J) = 0,
\]

where

\[
\text{Tr } J = -J_{11} - J_{22} + J_{33},
\]

\[
B_J = J_{11}J_{22} - J_{11}J_{33} - J_{12}J_{21} - J_{13}J_{31} - J_{22}J_{33},
\]

\[
\text{Det } J = J_{11}J_{22}J_{33} + J_{12}J_{23}J_{31} + J_{13}J_{22}J_{31} - J_{12}J_{21}J_{33}.
\]
Since $J_{44}$ is positive, we must show that the polynomial

$$-x^3 + \text{Tr} \ J \ x^2 - B \ J \ x + \text{Det} \ J = 0$$  \hspace{1cm} (42)

has one root with a positive real part and two roots with negative real parts in order to prove that the steady state has a stable manifold of dimension two.

To show it, we exploit the following application of the “Ruth’s theorem” to a third-order polynomial, which is used in Benhabib and Perli (1994):

**Theorem (Benhabib and Perli, 1994):** *The number of roots of the polynomial in (42) with positive real parts is equal to the number of variations of sign in the scheme*

$$-1 \quad \text{Tr} \ J \quad - B \ J + \frac{\text{Det} \ J}{\text{Tr} \ J} \quad \text{Det} \ J.$$  \hspace{1cm} (43)

A proof of more general version of Ruth’s theorem is, for example, given by Gantmacher (1960, vol.2, ch.XV).

To make use of the theorem, and show that the scheme (43) has one variation of sign, we next verify that $\text{Det} \ J > 0$. From the components of Jacobian matrix, each term on the right-hand side of (41) is

$$J_{11}J_{22}J_{33} = \left[ \bar{I} + \frac{1}{(1-\delta)\lambda V_L} + \frac{a_L(1-\bar{n}_N)}{a_N(1-\delta)\lambda V_L} \right] \left( \bar{I} + i\gamma \right) \left( \frac{L^N}{a_N} + i\gamma + \rho \right)$$  \hspace{1cm} (42)

$$> \frac{a_L(1-\bar{n}_N)}{a_N(1-\delta)\lambda V_L} \left( \bar{I} + i\gamma \right) \left( \bar{I} + i\gamma + \rho \right),$$

$$J_{12}J_{23}J_{31} = \left[ \frac{\lambda - 1}{(1-\delta)\lambda V_L} \right] \times \frac{\bar{n}_M}{V_L} \left( \frac{L^N}{a_N} - \bar{I} \right) \times \frac{a_L}{a_N(1-\delta)\lambda}$$

$$= \frac{a_Li\gamma(1-\bar{n}_N)}{a_N(1-\delta)^2\lambda V_L(\bar{I} + i\gamma)} \left( \bar{I} + i\gamma + \rho \right) \left( \frac{L^N}{a_N} - \bar{I} \right),$$

33
\[
J_{13}J_{22}J_{31} = \left[ \frac{(\lambda - 1)\bar{n}_M}{(1 - \delta)\lambda(V_L)^2} + \frac{(1 - \bar{n}_N)}{(1 - \delta)\lambda(V_L)^2} \left( 1 - \frac{a_L}{a_N} \bar{n}_N \right) \right] \times \left( \bar{I} + i\gamma \right) \times \frac{a_L}{a_N(1 - \delta)\lambda} \\
> \frac{a_L i\gamma(1 - \bar{n}_N)}{a_N(1 - \delta)^2\lambda V_L} \left( \bar{I} + i\gamma + \rho \right),
\]

\[
-J_{12}J_{21}J_{33} = -\frac{\lambda - 1}{(1 - \delta)\lambda V_L} \left[ \frac{a_L\bar{n}_M}{a_N(1 - \delta)\lambda V_L} - i\gamma \right] \left( \frac{L}{a_N} + i\gamma + \rho \right) \\
> -\frac{a_L i\gamma(1 - \bar{n}_N)}{a_N(1 - \delta)^2\lambda V_L(\bar{I} + i\gamma)} \left( \bar{I} + i\gamma + \rho \right) \left( \frac{L}{a_N} + i\gamma + \rho \right). 
\]

Therefore, we have the following inequality:

\[
\text{Det } J > \frac{a_L(1 - \bar{n}_N)(\bar{I} + i\gamma + \rho)}{a_N(1 - \delta)^2\lambda V_L(\bar{I} + i\gamma)} \left[ (1 - \delta) \left( \bar{I} + i\gamma \right)^2 - i\gamma\rho \right].
\] (44)

Equation (44) together with the assumption that \( i\gamma > \rho/(1 - \delta) \) implies \( \text{Det } J > 0 \). Thus, we can conclude that the scheme (43) varies its signs an uneven number (one or three) of times. To show that the number of the variations is always one, we consider three cases according to the sign of \( \text{Tr } J \): \( \text{Tr } J < 0 \), \( \text{Tr } J = 0 \), and \( \text{Tr } J > 0 \).

(i) The case of \( \text{Tr } J < 0 \)

If \( \text{Tr } J < 0 \), then the scheme (43) must have only one variation of the sign regardless of signs of \( -B J + \text{Det } J/\text{Tr } J \). Hence, in this case, the characteristic equation (38) has always two positive solutions and two solutions with negative real parts, so that the steady state is saddle-point stable and the equilibrium path is locally determinate.

(ii) The case of \( \text{Tr } J = 0 \)

If \( \text{Tr } J = 0 \), then the sum of three solutions of equation (42) is equal to zero. Because the product of the three solutions, which is equal to \( \text{Det } J \), is strictly positive, it can occur only if equation (42) has one positive solution and two solutions with negative real parts.

Thus, in the case of \( \text{Tr } J = 0 \), the characteristic equation (38) has two positive solutions and two solutions with negative real parts.
(iii) **The case of** $\text{Tr} \ J > 0$

Now suppose that $\text{Tr} \ J > 0$. In this case, we must verify that $- B \ J + \text{Det} \ J/\text{Tr} \ J > 0$ to show that scheme (43) has one variation of signs. Since the sign of $- B \ J + \text{Det} \ J/\text{Tr} \ J$ coincides with that of $- B \ J \text{Tr} \ J + \text{Det} \ J$, we prove below that $- B \ J \text{Tr} \ J + \text{Det} \ J$ is positive.

Using equations (39) – (41), we can rewrite $- B \ J \text{Tr} \ J + \text{Det} \ J$ as follows:

$$- B \ J \text{Tr} \ J + \text{Det} \ J = \text{Tr} \ J \left[ J_{11} J_{33} + J_{13} J_{31} + J_{22} J_{33} \right] + J_{11} J_{22} (J_{11} + J_{22})$$

$$+ J_{31} (J_{12} J_{23} + J_{13} J_{22}) - J_{12} J_{21} (J_{11} + J_{22}). \quad (45)$$

Recall that the sign of $J_{21}$ is ambiguous, while the signs of the other components of Jacobian matrix are all positive. Equation (45) directly shows that $- B \ J \text{Tr} \ J + \text{Det} \ J$ is positive if $J_{21}$ is negative or zero, so that scheme (43) varies its sign only once in that case.

On the other hand, $- B \ J \text{Tr} \ J + \text{Det} \ J$ is still positive, even if $J_{21} > 0$. To verify it, we again rewrite $- B \ J \text{Tr} \ J + \text{Det} \ J$ into

$$- B \ J \text{Tr} \ J + \text{Det} \ J = \text{Tr} \ J \left[ J_{11} J_{33} + J_{12} J_{21} + J_{22} J_{33} \right] + J_{11} [J_{22} (J_{11} + J_{22}) - J_{13} J_{31}]$$

$$+ J_{33} (J_{13} J_{31} - J_{12} J_{21}) + J_{12} J_{23} J_{31}. \quad (46)$$

The second term on the right-hand side of equation (46) is positive because

$$J_{22} (J_{11} + J_{22}) - J_{13} J_{31} = \left( \bar{I} + i\gamma \right) \left[ \bar{I} + \frac{a_L (1 - \bar{n}_N)}{a_N (1 - \delta) \lambda V_L} + (\bar{I} + i\gamma) \right]$$

$$+ \frac{1}{(1 - \delta) \lambda V_L} \left[ (\bar{I} + i\gamma) - \frac{L^S}{a_N} + \frac{a_L L^N (1 - \bar{n}_N)}{(a_N)^2 (1 - \delta) \lambda V_L} \right],$$

where $(\bar{I} + i\gamma) - (L^S/a_N) > 0$ under condition (19). In addition, the third term on the right-hand side of equation (46) is also positive since

$$J_{13} J_{31} - J_{12} J_{21} = \frac{a_L (1 - \bar{n}_N)}{a_N (1 - \delta)^2 \lambda^2 (V_L)^2} \left( 1 - \frac{a_L}{a_N} \bar{n}_N \right) + \frac{i\gamma (\lambda - 1)}{(1 - \delta) \lambda V_L} > 0.$$
As a result, equation (46) implies that $-B \cdot \text{Tr } J + \text{Det } J$ is positive even if $J_{21} > 0$, so that scheme (43) turns its signs only once in this case also. Therefore, in the case of $\text{Tr } J > 0$, two solutions of the characteristic equation (38) are positive and the other two solutions are with negative real parts.

Thus, the proof to confirm saddle-point stability and local determinacy of the steady state has been completed.
References


imitated industries $n_{M,t}$  
licensed industries $n_{L,t}$  
industries in the North $n_{N,t}$

innovation: $I_t n_{M,t}$  
innovation: $I_t n_{L,t}$  
innovation: $I_t n_{N,t}$

imitation: $i \gamma n_{L,t}$  
license agreement: $t_t n_{N,t}$

Figure 1: The structure of industries
Figure 2: Determination of the steady state
Figure 3: The effects of the rise of $\delta$